EMITTANCE EVOLUTION OF THE DRIVE ELECTRON BEAM IN A HELICAL UNDULATOR FOR THE ILC POSITRON SOURCE *

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Abstract

The effect of the ILC positron source's helical undulator on the drive electron beam is of great interest. People have been looking into the effects of wakefields, quad misalignment and radiation damage. In this paper we report an emittance damping effect of the ILC positron source undulator on the drive electron beam and our QUAD-BPM error simulation results. For a 100m RDR undulator, the emittance of the drive electron beam will be damped by about 1% instead of growing because the damping is stronger than the quantum excitation for this RDR undulator configuration with the RDR drive electron beam. Quad-BPM misalignment simulations show that a 20µm rms misalignment error in a 250m long undulator beamline can cause about 5% emittance growth in the drive electron beam. Taking into consideration the damping effect of the undulator, the net emittance growth will be smaller than this.

INTRODUCTION

There are many aspects of the ILC positron source undulator which will affect the emittance evolution of the drive electron beam passing through the undulator. There have been numerous reports on this issue. Duncan Scott and James Jones studied the effect of transverse resistive wall wakefields from the undulator beam tube and beam optics effects [1]. They found that the kick from wakefields is very small and that the emittance growth is due to the optics. Their study shows that a 10µm resolution in the BPM-Quads will result in ~8% emittance growth in the vertical plane. Kiyoshi Kubo from KEK has reported the effects of BPM-Quad misalignment and the synchrotron radiation from the undulator [2]. He found that the effect of undulator radiation and QUAD-BPM misalignment on the emittance growth can be tolerated.

We performed numerical simulations on the effect of radiation from a helical undulator using the ELEGANT code [3] and found that there is a damping of the normalized emittance of the drive electron beam caused by the radiation from the helical undulator. We also did QUAD-BPM error simulations using elegant and our results agree with the results of the previous study [ref?].

NUMERICAL SIMULATION

Simulations were done using ELEGANT[3], a particle tracking code, of undulators with the parameter sets listed in table 1. The drive beam energy is 150GeV as in the ILC baseline. The normalized emittance of the drive electron beam is 10 mm-mrad in x (horizontal) and 0.04 mm-mrad in y (vertical). With the assumption of 12.5m quad spacing and a 72.6 degree FODO array, the β_{max} for the undulator input will be 41 m. The initial beam spot size and divergence can be estimated as: $\sigma_x=3.7\text{x}10^{-5}$ m, $\sigma_y=2.4\text{x}10^{-6}$ m, $\sigma_x'=0.9\text{x}10^{-6}$ rad and $\sigma_y'=0.06\text{x}10^{-6}$ rad.

Table 1: Undulator parameters used in our simulations.

	K	**λ_u(cm)**
UK1	0.92	1.15
UK2	0.79	1.1
UK3	0.64	1.05
CO1	0.42	1.0
CO2	0.72	1.2
CO3	0.3	0.7

Results with on-axis injection

For a 100m undulator with no FODO optics and no energy spread in the drive beam, the ELEGANT simulation results are given in Table 2.

Table 2. ELEGANT simulation result for 100m undulator.

	$\Delta\varepsilon_{nx}/\varepsilon_{nx}$ (%)	**$\Delta\varepsilon_{ny}/\varepsilon_{ny}$ (%)**	**$\Delta E/E$ (%)**
UK1	-1.37464	-1.06	-1.3756
UK2	-1.10608	-0.912	-1.112
UK3	-0.79802	-0.679	-0.804
CO1	-0.38277	-0.395	-0.383
CO2	-0.77138	-0.652	-0.789
CO3	-0.39768	-0.382	-0.399

As shown in table 2, the normalized emittance of the drive electron beam is damped as a result of radiation in the undulators. The rate of damping is roughly proportional to the rate of energy loss.

Results with off-axis injection

Simulations with beam passing through a 100m RDR baseline undulator with no FODO lattice or other optics were done to study the effect of jitter of the beam center. No initial energy spread is present in the injected drive electron beam. The results are presented in Table 3.

Table 3. ELEGANT simulation result for 100m RDR baseline undulator with off-axis injection

Offset	$\Delta\varepsilon_{nx}/\varepsilon_{nx}$ (%)	$\Delta\varepsilon_{ny}/\varepsilon_{ny}$ (%)
0,10µm,50µm	-1.37	-1.06
1mm in x	-1.59	-1.13

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1mm in y	-1.59	-1.14

As shown in Table 3, for offsets smaller than 50 microns, there is no noticeable effect on the emittance evolution. When the offset is 1mm, the damping in both x and y direction increases because the beam now sees a stronger magnetic field and is thus losing more energy due to radiation.

Results with increasing undulator length

Table 4. ELEGANT simulation results for RDR baseline undulator at different undulator lengths.

Length of undulator	$\Delta\varepsilon_{nx}/\varepsilon_{nx}$ (%)	$\Delta\varepsilon_{ny}/\varepsilon_{ny}$ (%)
~100m	-1.36	-1.11
~200m	-2.66	-0.65
~300m	-3.91	1.73

Shown in Table 4 is the emittance evolution of the drive e^- beam after passing through the RDR undulator without FODO lattice or any other optics. In this set of simulations, the input drive e^- beam has an rms energy spread of 750MeV. As shown in the table, the emittance in the horizontal plane continues damping down while the undulator length is increased from 100m to 300m. For the vertical plane however, the emittance damped about by 1.11% after 100m. After 200m it had damped only by about 0.65% and it had grown by 1.73% after 300m of undulator. We will explain this in the next section.

Results with increasing length of RDR undulator and FODO lattice

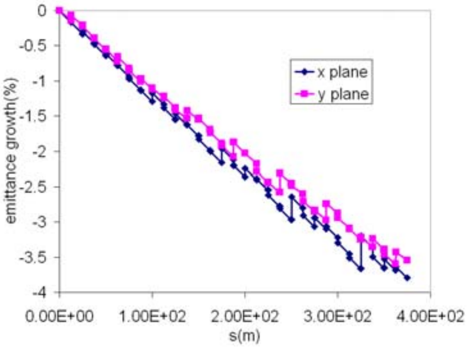


Figure 1. Emittance growth in the undulator with K=0.92, λ=1.15cm and FODO lattices.

Shown in Figure 1 is the emittance growth in an undulator with RDR parameters. The beamline has a quad every 12.4m. The thickness of the quad is assumed to be 0.1m. An undulator section about 12m long with K=0.92, λ=1.15cm is inserted in between the quads. The FODO lattice has a 90 degree phase advance in the y plane and 70 degrees in the x plane. Unlike the results in Table 4, the emittance continues damping with the increase of the total length of the undulator.

ANALYSIS

With some approximations, we can obtain the change of emittance after an electron beam passing through helical undulator as:

$$\Delta\varepsilon_n = -\varepsilon_n \frac{|\Delta E|}{E} + \left(\beta_0 + \frac{L^2}{3\beta_0}\right)\frac{K}{\gamma}\frac{1}{E^2}\frac{\hbar\omega_{\max}}{2}|\Delta E| \tag{1}$$

where $\omega_{\max}$ is the cutoff frequency of the first harmonic, L is the length of the undulator, ΔE is the energy lost in the undulator, and E is the energy of injected beam. Using equation (1), a simple calculation shows that for a 100m long RDR undulator, the damping/excitation ratios are 3 in vertical and 600 in horizontal. If the undulator length increases, this ratio will continue to decrease and eventually reach equilibrium. The emittance will then remain at its equilibrium value determined by the undulator parameters.

In order to determine the equilibrium of emittance, we rewrite equation (1) as

$$\Delta\varepsilon_n = \frac{|\Delta E|}{E}\left[-\varepsilon_n + \left(\beta_0 + \frac{L^2}{3\beta_0}\right)\frac{K}{\gamma}\frac{1}{E}\frac{\hbar 4\pi\gamma^2 c}{2\cdot(1+K^2)\lambda_u}\right] \tag{2}$$

When L<<β, this can be further simplified to

$$\Delta\varepsilon_n \sim= \frac{|\Delta E|}{E}\left[-\varepsilon_n + \beta_0\frac{K\hbar 4\pi c}{2m_0(1+K^2)\lambda_u}\right] \tag{3}$$

from which we observe that the excitation term is fixed and only determined by the undulator parameters and beta function of the beam. When the normalized emittance of beam is larger than this term, the emittance will be damped until it is equal to this term. If the emittance is smaller than this term, then the emittance will grow until it is equal to this term. The value of this term is the approximate equilibrium emittance of the corresponding helical undulator.

From equation (3), the approximate equilibrium emittance of the RDR undulator can be obtained as about 4×10^{-9}m-rad which is about 10% of the emittance of the injected e^- beam.

Now consider the results shown in Table 4. As discussed above, the emittance should continue damping or growing until it reaches equilibrium and the equilibrium of the RDR undulator is about 10% of the injected beam. In that case, why did the emittance grow in the vertical plane after the 300m long undulator in Table 4? Because in that simulation the undulator is inserted as a single unit. There are no FODO optics and the beam spot size will keep growing so the beam will see a bigger and bigger spread in the undulator field and thus cause the emittance growth. In a real undulator, the FODO lattice will maintain the beam spot size and thus the emittance will keep damping until it reaches equilibrium. The results in Figure 1 also confirm this.

QUAD-BPM ERROR SIMULATION

To study the effect of Quad-BPM misalignment in the undulator, we performed a set of simulations with the FODO lattice only. The conditions are as follows:

- Beamline:
  - Quad magnet every 12.4 meters, thickness of the quad is assumed to be 10cm.
  - 70 degrees phase advance in the horizontal plane and 90 degrees phase advance in vertical plane is assumed.
  - 10 FODO periods, totaling 250 meters of beamline.
- Quad-BPM misalignment dy in vertical plane
  - The σ of dy: 5μm -- 100 μm
  - 50 random seeds used when generating errors.
- Beam injected:
  - emittance: 4x10^{-8}m-rad in vertical, 10x10^{-6}m-rad in horizontal, well matched with the lattice.
  - Energy: 150GeV, rms energy spread 0.5%.

The beta function of the beamline is given in Figure 2. In the horizontal plane, β_{max} is about 45m and β_{min} is about 13m. In the vertical plane, β_{max} is about 40m and β_{min} is about 10m. Using the beta functions shown in Figure 2, we calculated the phase advance in both the vertical and horizontal planes. The phase advance obtained from the simulation is 90 degrees in the vertical plane and 70 degrees in the horizontal plane.

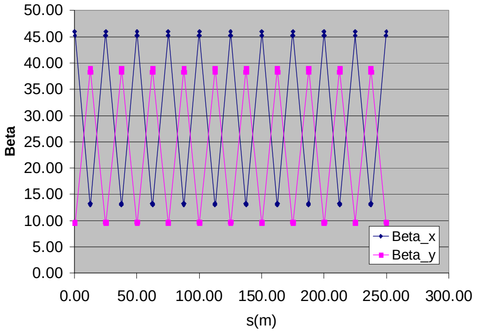
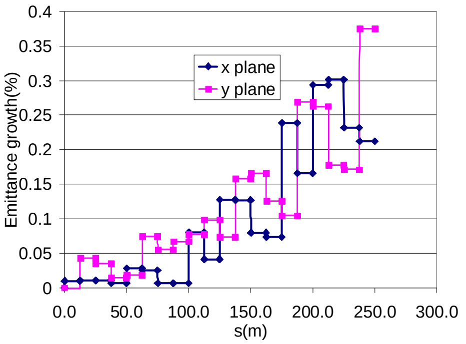


Figure 2. Beta function of beamline



As shown in Figure 3, the emittance growth due to energy spread is small.

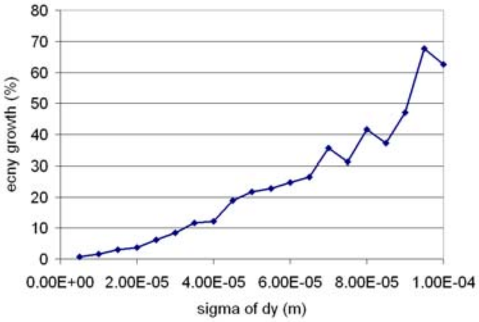


Figure 4. Emittance growth in the vertical plane due to Quad-BPM errors.

The emittance growth in the plane caused by Quad-BPM errors is very small. The emittance growth in the vertical plane is shown in Figure 4. For the results shown in Figure 4, we have used 50 random seeds for the Quad-BPM errors. A 20μm rms misalignment error in this 250m undulator beam line will cause about 5% emittance growth in the vertical plane.

SUMMARY

For the 100m RDR undulator, the emittance of the drive electron beam will be damped by about 1% instead of growing as the damping is stronger than the quantum excitation for this RDR undulator with the RDR drive electron beam.

Quad-BPM misalignment will cause the emittance to grow. A 20μm rms misalignment error in a 250m long undulator beamline can cause about 5% emittance growth in the drive electron beam. Taking into consideration the damping effect of the undulator, the net emittance growth will be smaller.

REFERENCES

[1] Duncan Scott and James Jones, Transverse Resistive Wall Wakefields of the ILC Positron Undulator Beam Tube and their Effects on an Orbit Dependant Emittance Growth, EUROTeV-Report-2007-007

[2] Kiyoshi Kubo, Simulation of electron beam in undulatorsof e^{+} source of ILC -Emittance and orbit angle with quad misalignment and corrections-No wakefiled, ILC-LET meeting, Daresbury, 2007.01

[3] M. Borland, "elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation," Advanced Photon Source LS-287, September 2000.

[4] ILC Reference Design Report

Figure 3. Emittance growth due to energy spread only