

top FCNC after the LHC

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Outline

1. Why top FCNC?
2. A framework for top FCNC physics - the effective operator approach
3. The story so far I - LEP, Hera and the Tevatron
4. The story so far II - limits from B physics
5. The future (at the LHC)
6. Is there top FCNC left to explore?
7. Conclusion

1. Why top FCNC

Process	SM	QS	2HDM	MSSM	\mathcal{R} SUSY	TC2
$t \rightarrow uZ$	$\mathcal{O}(10^{-17})$	$\mathcal{O}(10^{-4})$	–	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$
$t \rightarrow u\gamma$	$\mathcal{O}(10^{-16})$	$\mathcal{O}(10^{-9})$	–	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-5})$
$t \rightarrow ug$	$\mathcal{O}(10^{-14})$	$\mathcal{O}(10^{-7})$	–	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-5})$
$t \rightarrow cZ$	$\mathcal{O}(10^{-14})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-7})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$
$t \rightarrow c\gamma$	$\mathcal{O}(10^{-14})$	$\mathcal{O}(10^{-9})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-5})$
$t \rightarrow cg$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(10^{-7})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-5})$

Any hint of FCNC related to the top is a signal for new physics

2. DIY

- a minimal set of effective operators for top physics

1. Write all possible dimension 6 operators that have at least one top quark

Complete list of dimension 6 effective operators:

Buchmüller e Wyler, *Nucl. Phys. B*268 (1986) 621.

2. Use equations of motion to further reduce the number of operators

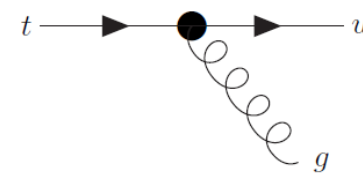
$$\mathcal{O}_{itG}^L = i \frac{\alpha_{it}^{S,L}}{\Lambda^2} \bar{q}_L^i \lambda^a \gamma_\mu D_\nu q_L^3 G^{\mu\nu}$$

$$\mathcal{O}_{itG}^R = i \frac{\alpha_{it}^{S,R}}{\Lambda^2} \bar{u}_R^i \lambda^a \gamma_\mu D_\nu t_R G^{\mu\nu}$$

$$\mathcal{O}_{itG\phi} = \frac{\beta_{it}^S}{\Lambda^2} (\bar{q}_L^i \lambda^a \sigma^{\mu\nu} t_R) \tilde{\phi} G_{\mu\nu}^a$$

$$\mathcal{O}_{itG}^L = F^L (\mathcal{O}_{itG\phi}, (\mathcal{O}_{itG\phi})^\dagger, \mathcal{O}_{NC}, \mathcal{O}_{4F})$$

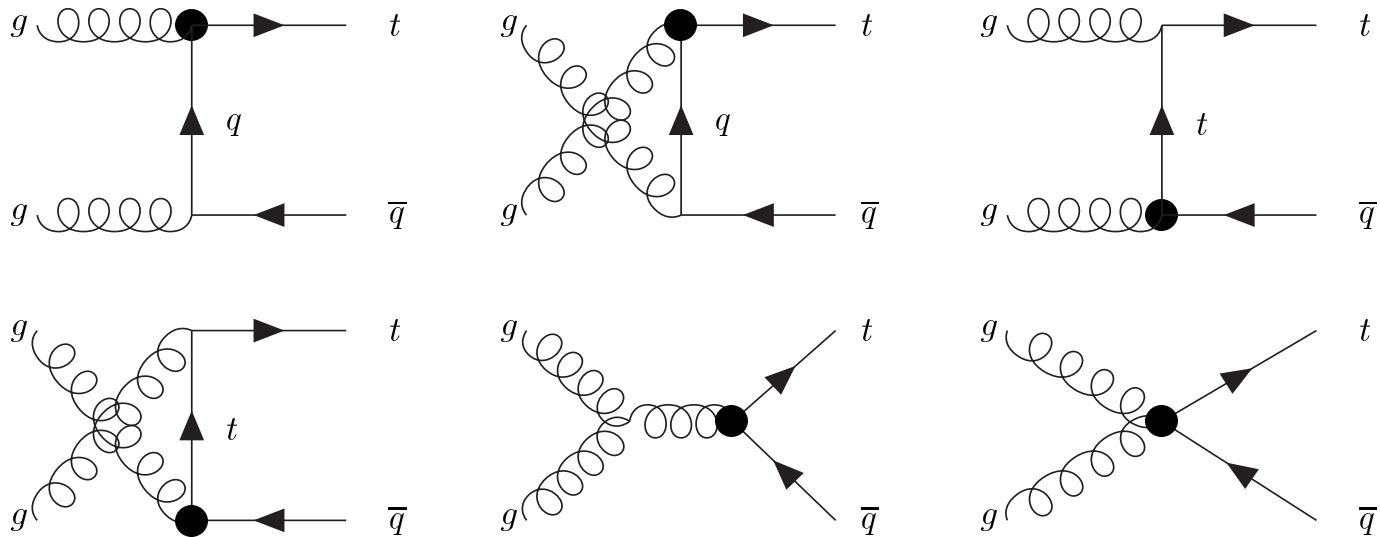
$$\mathcal{O}_{itG}^R = F^R (\mathcal{O}_{itG\phi}, (\mathcal{O}_{itG\phi})^\dagger, \mathcal{O}'_{4F})$$



If there are no 4-fermion operators involved in the physical process we can keep one type of operator only!

Now let us see this happening!

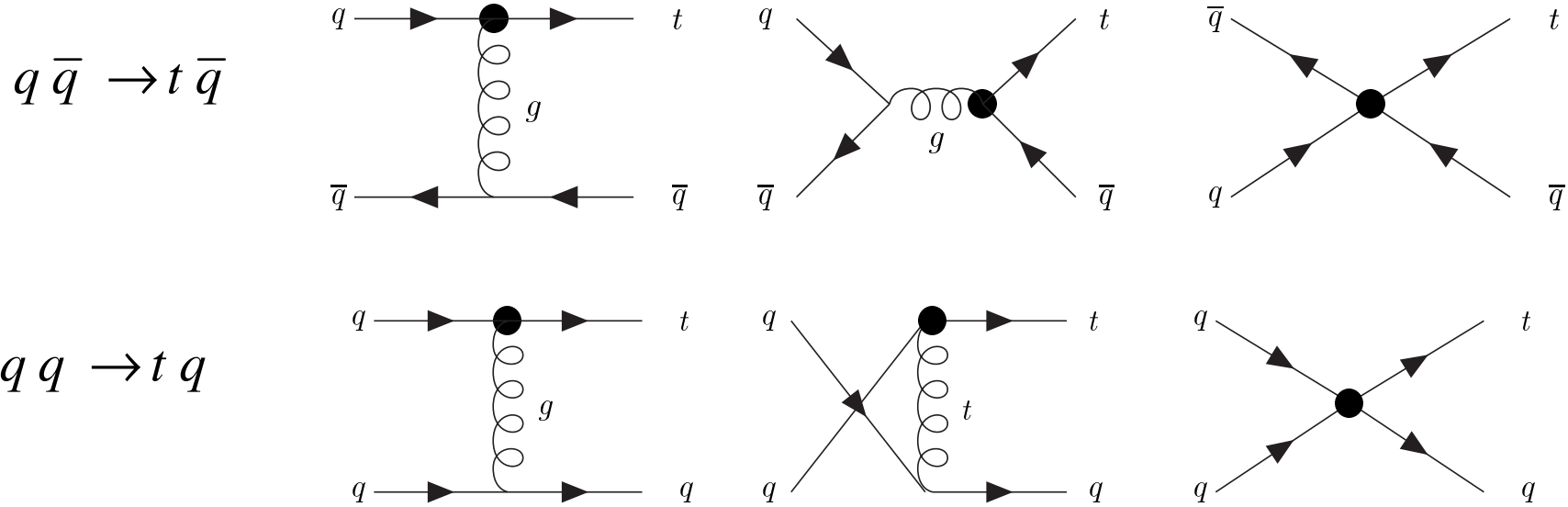
3a. without 4F operators (contribution to $pp \rightarrow tj$)



$$\frac{d\sigma(gg \rightarrow t\bar{u})}{dt} = -\frac{g_s^2}{4m_t^3} \frac{F_{gg}}{ut s^3 (s+t)^2 (s+u)^2} \Gamma(t \rightarrow ug)$$

$$\begin{aligned} F_{gg} = & 4s^2 t (s+t)^3 (s^2 + 2st + 2t^2) + s(s+t)^2 (4s^4 + 11s^3 t + 48s^2 t^2 + 52st^3 + 18t^4) u \\ & + 2(s+t) (10s^5 + 27s^4 t + 69s^3 t^2 + 90s^2 t^3 + 45st^4 + 9t^5) u^2 \\ & + (s+t) (44s^4 + 115s^3 t + 203s^2 t^2 + 162st^3 + 36t^4) u^3 \\ & + 2(26s^4 + 85s^3 t + 135s^2 t^2 + 99st^3 + 27t^4) u^4 + 4(2s+t) (4s^2 + 9st + 9t^2) u^5 \\ & + 2(4s^2 + 9st + 9t^2) u^6 \end{aligned}$$

3b. with 4F operators (contribution to $pp \rightarrow tj$)



$$\sigma_{4F}^{(u)} = [32|\alpha_{ut}|^2 + 32|\alpha_{tu}|^2 - 19 \operatorname{Re}(\alpha_{ut}\alpha_{tu}) + 90 \operatorname{Im}(\alpha_{ut}\beta_{tu}) - 90 \operatorname{Im}(\alpha_{tu}\beta_{tu}^*) + 178(|\beta_{tu}|^2 + |\beta_{ut}|^2) - 21 \operatorname{Re}(\alpha_{ut}\gamma_{u_1}) + 21 \operatorname{Re}(\alpha_{tu}\gamma_{u_1}^*) + \operatorname{Im}(\beta_{tu}\gamma_{u_1}^*) + 3 \operatorname{Re}(\alpha_{ut}\gamma_{u_2}) - 1 \operatorname{Re}(\alpha_{tu}\gamma_{u_2}^*) - \operatorname{Im}(\beta_{tu}\gamma_{u_2}^*) + 56|\gamma_{u_1}|^2 + 26|\gamma_{u_2}|^2 + 35|\gamma_{u_3}|^2] \frac{1}{\Lambda^4} \text{ pb}$$

4-fermion operators spoil the beautiful proportionality to the BR

$$\mathcal{O}_{u_1} = \frac{g_s \gamma_{u_1}}{\Lambda^2} (\bar{t} \lambda^a \gamma^\mu \gamma_R u) (\bar{q} \lambda^a \gamma_\mu \gamma_R q) + \text{h.c.}$$

$$\mathcal{O}_{u_2} = \frac{g_s \gamma_{u_2}}{\Lambda^2} [(\bar{t} \lambda^a \gamma_L u') (\bar{u}'' \lambda^a \gamma_R u) + (\bar{t} \lambda^a \gamma_L d') (\bar{d}'' \lambda^a \gamma_R u)] + \text{h.c.}$$

$$\mathcal{O}_{u_3} = \frac{g_s \gamma_{u_3}^*}{\Lambda^2} [(\bar{t} \lambda^a \gamma_L u) (\bar{d}'' \lambda^a \gamma_L d'') - (\bar{t} \lambda^a \gamma_L d) (\bar{d}'' \lambda^a \gamma_L u'')] + \text{h.c.}$$

4a. Total non-4F operators from electroweak sector

The equations of motion relate these operators with 4F ones (in $\gamma\gamma$ world they do not have a say!)

$$\mathcal{O}_{tB} = i \frac{\alpha_{it}^{B,R}}{\Lambda^2} \bar{u}_R^i \gamma_\mu D_\nu t_R B^{\mu\nu}$$

$$\mathcal{O}_{qB} = i \frac{\alpha_{it}^{B,L}}{\Lambda^2} \bar{q}_L^i \gamma_\mu D_\nu q_L^3 B^{\mu\nu} \quad , \quad \mathcal{O}_{qW} = i \frac{\alpha_{it}^{W,L}}{\Lambda^2} (\bar{q}_L^i \tau_I \gamma_\mu D_\nu q_L^3) W_{\mu\nu}^I$$

$$\mathcal{O}_{tB\phi}^{RL} = \frac{\beta_{it}^B}{\Lambda^2} (\bar{q}_L^i \sigma^{\mu\nu} t_R) \tilde{\phi} B_{\mu\nu} \quad , \quad \mathcal{O}_{tW\phi}^{RL} = \frac{\beta_{it}^W}{\Lambda^2} (\bar{q}_L^i \tau_I \sigma^{\mu\nu} t_R) \tilde{\phi} W_{\mu\nu}^I$$

$$\mathcal{O}_{tB\phi}^{LR} = \frac{\beta_{ti}^B}{\Lambda^2} (\bar{q}_L^3 \sigma^{\mu\nu} u_R^i) \tilde{\phi} B_{\mu\nu} \quad , \quad \mathcal{O}_{tW\phi}^{LR} = \frac{\beta_{ti}^W}{\Lambda^2} (\bar{q}_L^3 \tau_I \sigma^{\mu\nu} u_R^i) \tilde{\phi} W_{\mu\nu}^I$$

**2 + 2
constants**

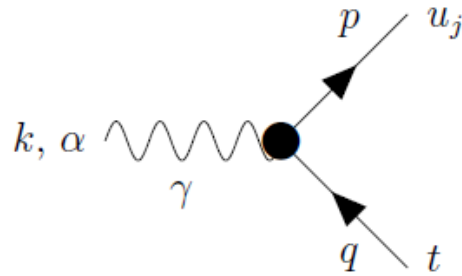
$$\mathcal{O}_{D_t} = \frac{\eta_{it}}{\Lambda^2} (\bar{q}_L^i D^\mu t_R) D_\mu \tilde{\phi} \quad , \quad \mathcal{O}_{\bar{D}_t} = \frac{\bar{\eta}_{it}}{\Lambda^2} (D^\mu \bar{q}_L^i t_R) D_\mu \tilde{\phi}$$

$$\mathcal{O}_{\phi_t}^R = i \theta_{it}^R (\phi^\dagger D_\mu \phi) (u_R^i \gamma^\mu t_R)$$

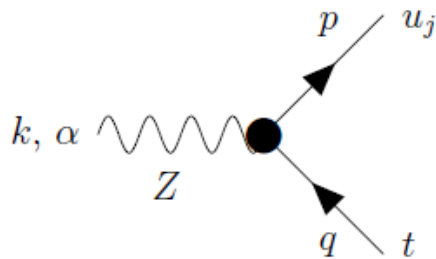
$$\mathcal{O}_{\phi_{(1)}}^L = i \theta_{it(1)}^L (\phi^\dagger D_\mu \phi) (\bar{q}_L^i \gamma^\mu q_L^3) \quad , \quad \mathcal{O}_{\phi_{(3)}}^L = i \theta_{it(3)}^L (\phi^\dagger D_\mu \tau_I \phi) (\bar{q}_L^i \gamma^\mu \tau_I q_L^3)$$

**3 → 2
constants**

4b. Therefore for photon colliders the FCNC top interactions can safely be reduced to the following form



$$\frac{v}{\Lambda^2} \sigma_{\mu\nu} (\beta_{jt}^{\gamma} \gamma_R + \beta_{tj}^{\gamma*} \gamma_L) (k^{\mu} g^{\nu\alpha} - k^{\nu} g^{\mu\alpha})$$



$$\frac{v}{\Lambda^2} \left[i v \gamma_{\alpha} (\theta^R \gamma_R + \theta^L \gamma_L) + \sigma_{\mu\nu} (\beta_{jt}^Z \gamma_R + \beta_{tj}^{Z*} \gamma_L) (k^{\mu} g^{\nu\alpha} - k^{\nu} g^{\mu\alpha}) \right]$$

5. ... while for lepton colliders we add the 4F operators

$$\mathcal{M}_{4F} = \frac{1}{\Lambda^2} \sum_{i,j=L,R} V_{ij} (\bar{e} \gamma_{\mu} P_i e) (\bar{t} \gamma^{\mu} P_j c) + S_{ij} (\bar{e} P_i e) (\bar{t} P_j c) + T_{ij} (\bar{e} \sigma_{\mu\nu} P_i e) (\bar{t} \sigma^{\mu\nu} P_j c)$$

$$S_{LL} = S_{LR} = S_{RL} = T_{LL} = T_{LR} = T_{RL} = 0$$

Bar-Shalom and Wudka, PRD60, 1999.

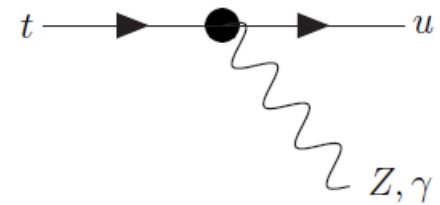
Operators and redundancy

$$O_1 + O_2 + O_{4F} = 0$$

$$O_{4F} = \bar{u} \gamma_\mu t (\bar{u} \gamma^\mu u + \bar{e} \gamma^\mu e)$$

If processes do not include 4F operators,

$$O_1 + O_2 = 0$$



Otherwise we can use

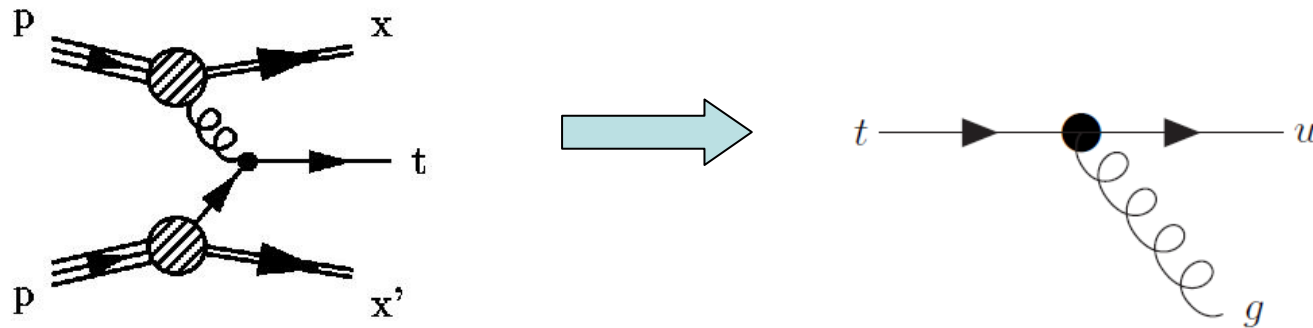
$$O_2 \text{ and } O_{4F}$$

3. The story so far I - LEP, Hera and the Tevatron

	LEP	HERA	Tevatron
$Br(t \rightarrow q Z)$	$< 7.8\%$	$< 49\%$	$< 3.7\%d$
$Br(t \rightarrow q \gamma)$	$< 2.4\%$	$< 0.64\%(u)$	$< 3.2\%d$
$Br(t \rightarrow q g)$	$< 17\%$	$< 13\%$	$< 0.045\%$

- Indirect limits - cross sections converted to branching ratios
- Direct limits - from top decays (Tevatron only)

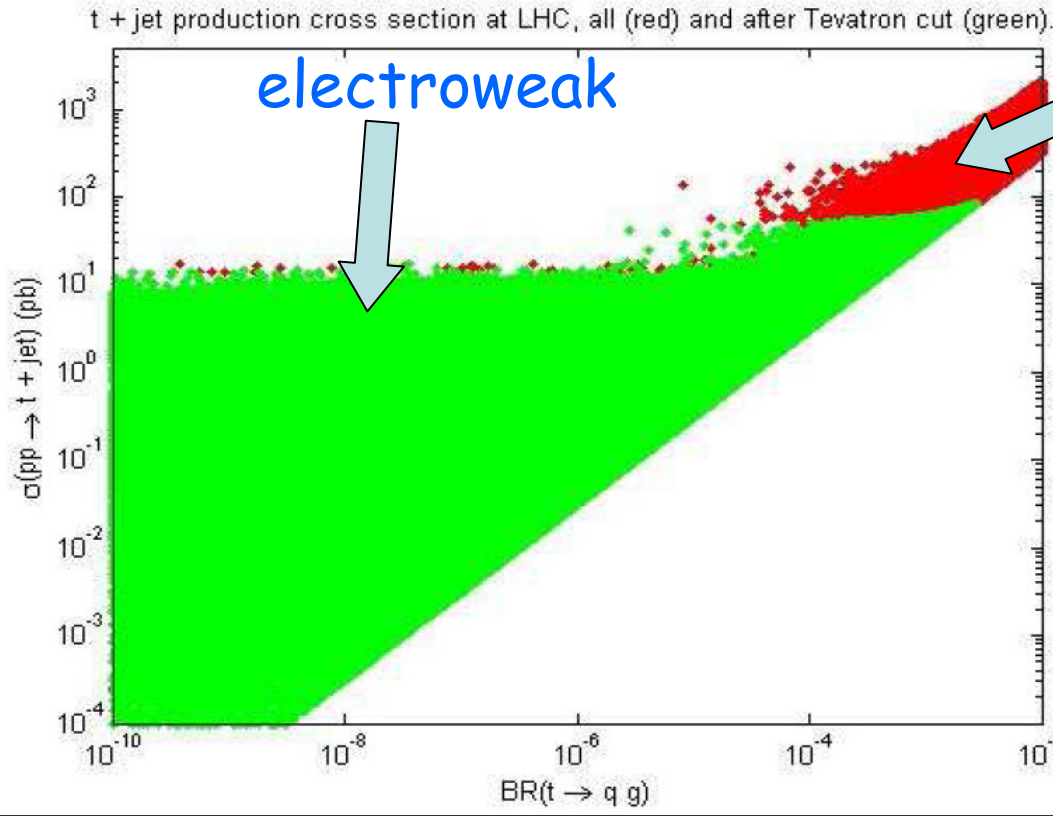
Do indirect limits have any meaning?



In the case of direct top production we can consider only one operator.

Indirect limits

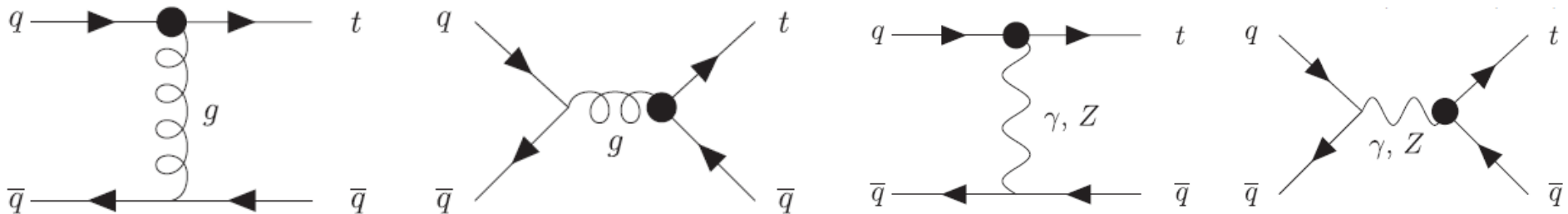
It can be strong or electroweak



The electroweak sector can be probed even if the strong sector is negligible.

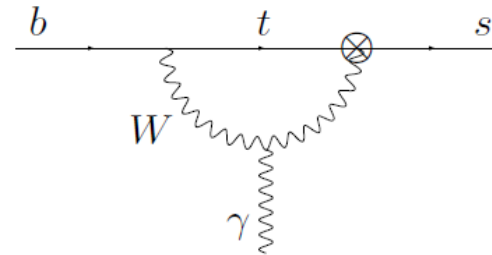
Excess in the cross section could be wrongly interpreted as coming from FCNC in the strong sector.

The numbers have to be added to the SM cross section.



4. The story so far II - limits from B physics

B physics constraints from
Fox et al, PRD78 054008, 2008.

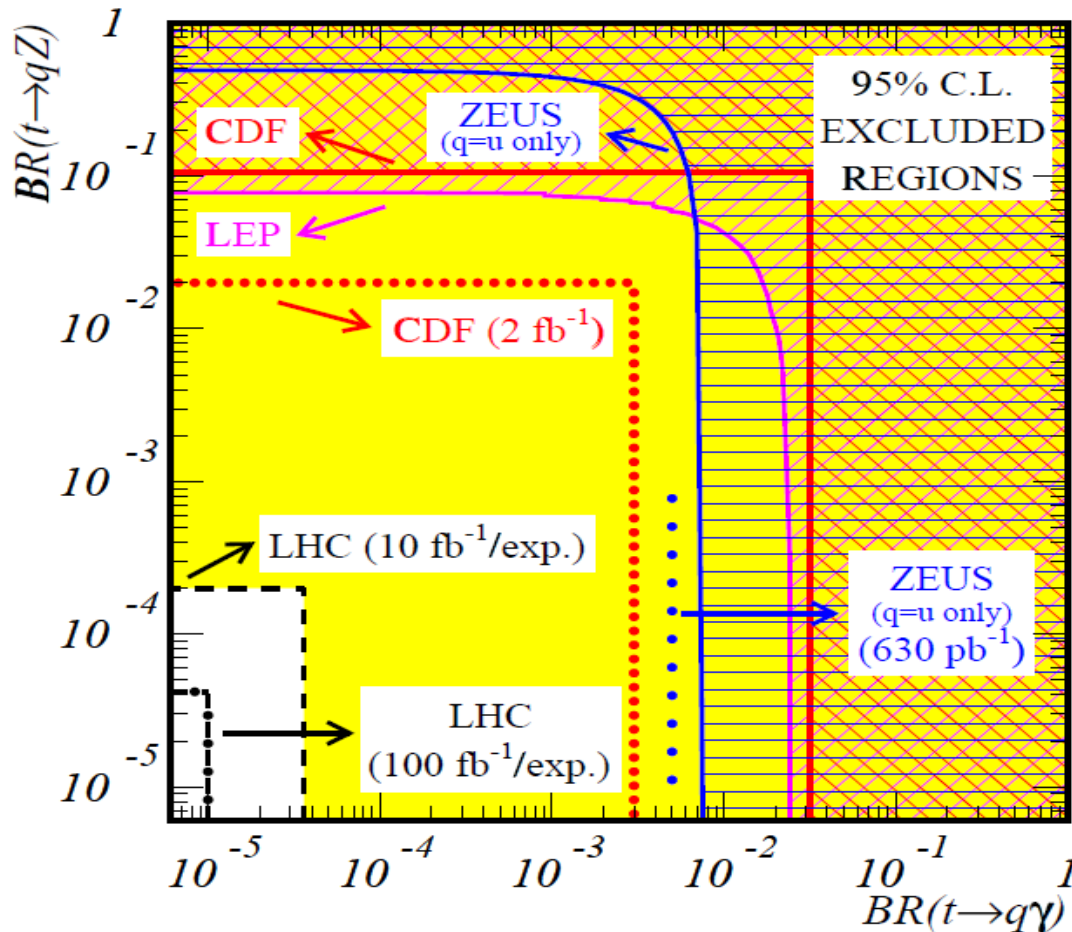


	\mathcal{O}_ϕ^L	$\mathcal{O}_{tW\phi}^{RL}$	$\mathcal{O}_{tB\phi}^{RL}$	$\mathcal{O}_{tW\phi}^{LR}$	$\mathcal{O}_{tB\phi}^{LR}$	$\mathcal{O}_{\phi t}^R$
$Br(t \rightarrow c Z)$	$\mathcal{O}(10^{-6})$	3.4×10^{-5}	8.4×10^{-6}	4.5×10^{-3}	d	d
$Br(t \rightarrow c \gamma)$	—	1.8×10^{-5}	4.8×10^{-5}	2.3×10^{-3}	d	d
$Br(t \rightarrow u Z)$	$\mathcal{O}(10^{-5})$	4.1×10^{-5}	1.2×10^{-4}	3.2×10^{-3}	d	d
$Br(t \rightarrow u \gamma)$	—	2.1×10^{-5}	6.7×10^{-4}	1.6×10^{-3}	d	d

Limits can only improve but each bound in the table just takes one operator at a time - all remaining couplings vanish.

5. The future (at the LHC)

	ATLAS & CMS (10 fb^{-1})	ATLAS & CMS (100 fb^{-1})
$Br(t \rightarrow qZ)$	2.0×10^{-4}	4.2×10^{-5}
$Br(t \rightarrow q\gamma)$	3.6×10^{-5}	1.0×10^{-5}
$Br(t \rightarrow qg)$ (ATLAS)	1.3×10^{-3}	4.2×10^{-4}



**LHC:
the (100 fb^{-1})
future**

Report of Working Group 1
of the CERN Workshop
Flavor in the Era of the
LHC.
EPJC57 (183) 2008.

$$BR^{10 \text{ fb}^{-1}}(t \rightarrow qg) < 9 \times 10^{-5}$$

Cheng and Teixeira-Dias,
ATL-PHYS-PUB-2006-029.

100 fb^{-1} used as benchmark for the "future"!

6. Is there life for anomalous top FCNC after the LHC?

- $e^+e^- \rightarrow t\bar{q} + q\bar{t}$
- $e^+e^- \rightarrow t\bar{q}g + q\bar{t}g$
- $\gamma\gamma \rightarrow t\bar{q} + q\bar{t}$

Our approach is to use physical observables whenever possible - starting with cross sections involving the top and top branching ratios. Other observables will be considered in the future.

A generator based on CalHEP to use in FCNC top physics is being tested.

Direct limits on the anomalous FCNC top BR

$$e^+e^- \rightarrow t\bar{t} \rightarrow bW \bar{q}X$$

Aguilar-Saavedra and Riemann, hep-ph/0102197

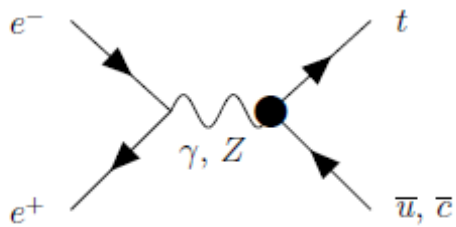
Aguilar-Saavedra, PLB502, 2001.

$\sqrt{s} = 500 \text{ GeV} (300 \text{ fb}^{-1})$	
$Br(t \rightarrow qZ)$	$\mathcal{O}(10^{-3})$
$Br(t \rightarrow q\gamma)$	$\mathcal{O}(10^{-4})$

Limits do not improve with energy. The scenario for the direct bounds does not look very promising.

It seems the LHC can do better.

Single FCNC top production

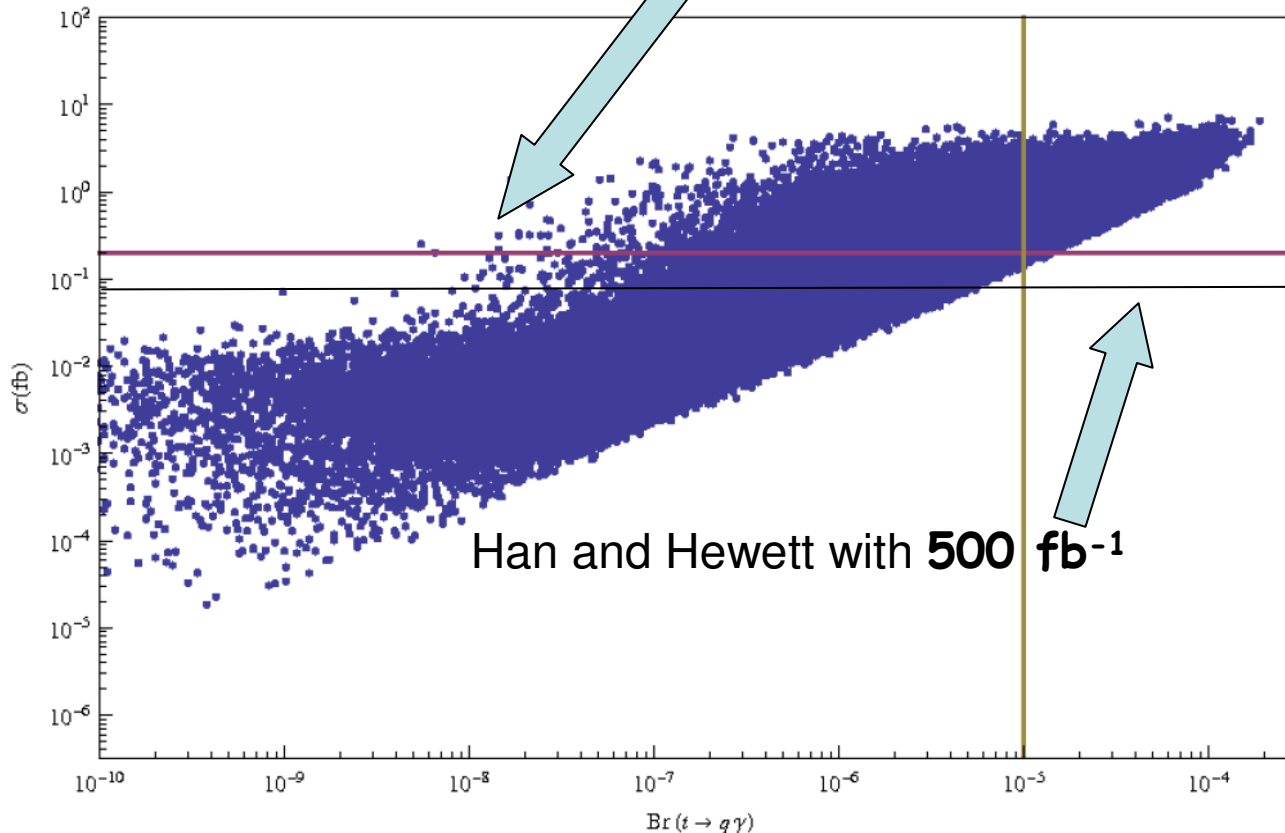


Han and Hewett, PRD60, 1999.

Aguilar-Saavedra and Riemann, hep-ph/0102197

Aguilar-Saavedra, PLB502, 2001.

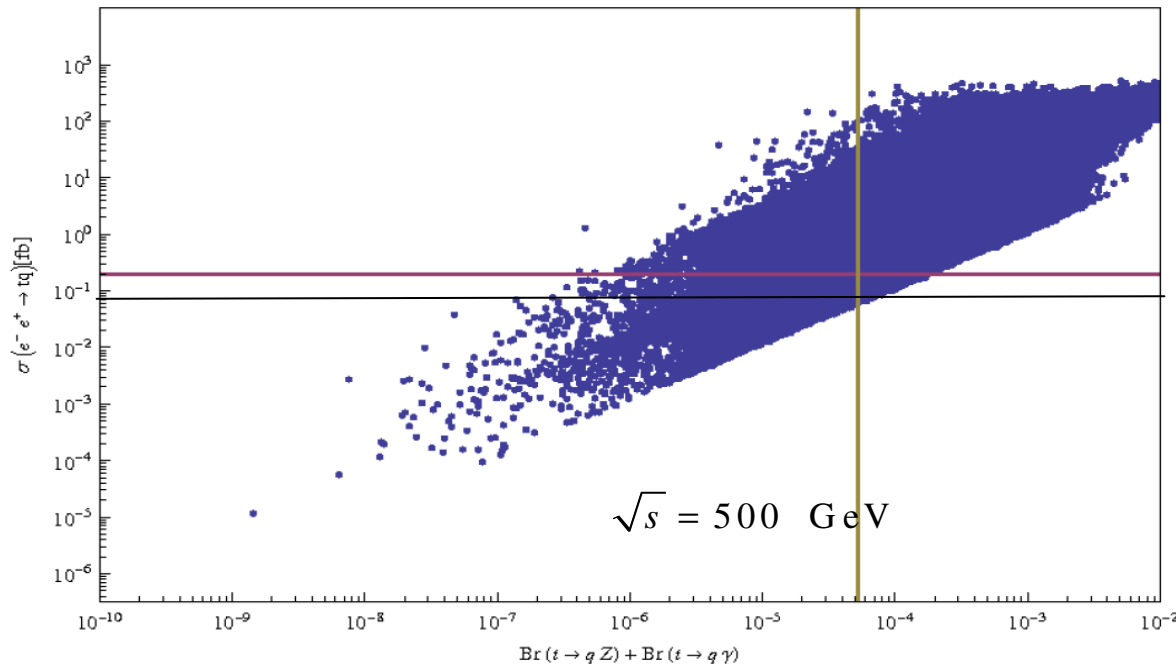
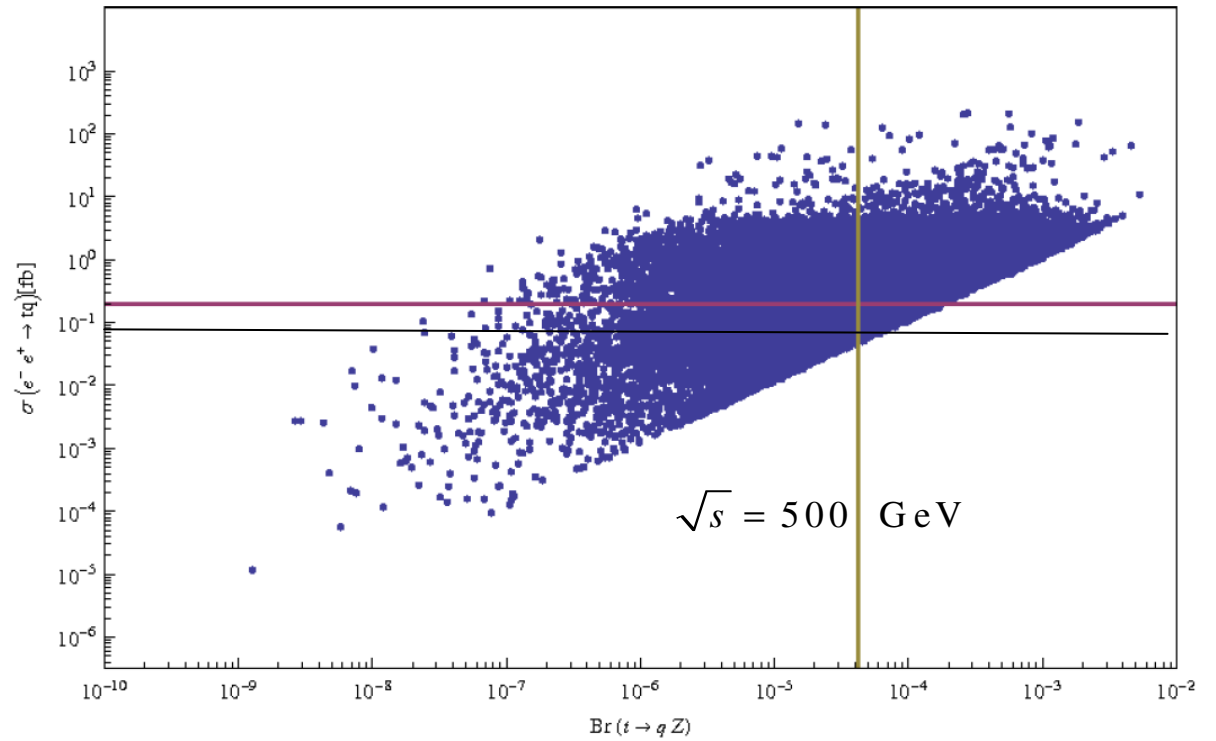
**All analysis reach similar conclusions -
a cross section of roughly 0.2 fb can be probed with 300 fb⁻¹ of integrated
luminosity.**



**Anomalous coupling
constants are
generated randomly
and cross section and
BR are calculated
(500000 points
generated).**

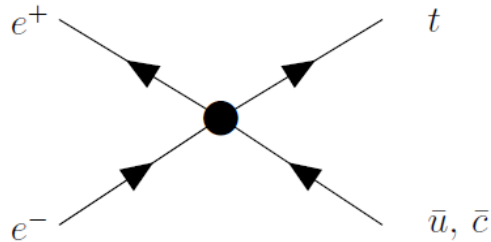
**All predicted bounds
for the LHC 100 fb⁻¹
are used.**

Same as before but
now plotted against
 $\text{BR}(t \rightarrow qZ)$.



And for the sum of
BR's.

The 4F contribution



$$\Gamma_{4F}^{t \rightarrow q e^+ e^-} = \frac{m_t^5}{2048\pi^3\Lambda^4} (S_{RR}^2 + 48T_{RR}^2 + 4V^2)$$

$$\sigma_{4F}^{e^+ e^- \rightarrow qt} = \frac{(m_t^2 - s)^2}{32\pi s^2\Lambda^4} (2m_t^2 (16T_{RR}^2 + V^2) + s (3S_{RR}^2 + 4(4T_{RR}^2 + V^2)))$$

\sqrt{s}	L	$V_{ij} = 1$	$S_{RR} = 1$	$T_{RR} = 1$
200 GeV	2.5 fb^{-1}	1.5 TeV	1.3 TeV	2.5 TeV
500 GeV	50 fb^{-1}	9.3 TeV	8.5 TeV	13.6 TeV
1000 GeV	200 fb^{-1}	19.3 TeV	17.9 TeV	27.5 TeV

Bar-Shalom and Wudka
PRD60, 1999.

Andringa et al
DELPHI 2006-003 CONF749 2006.

TRR=1 (560 pb⁻¹) → Λ=1.2 TeV

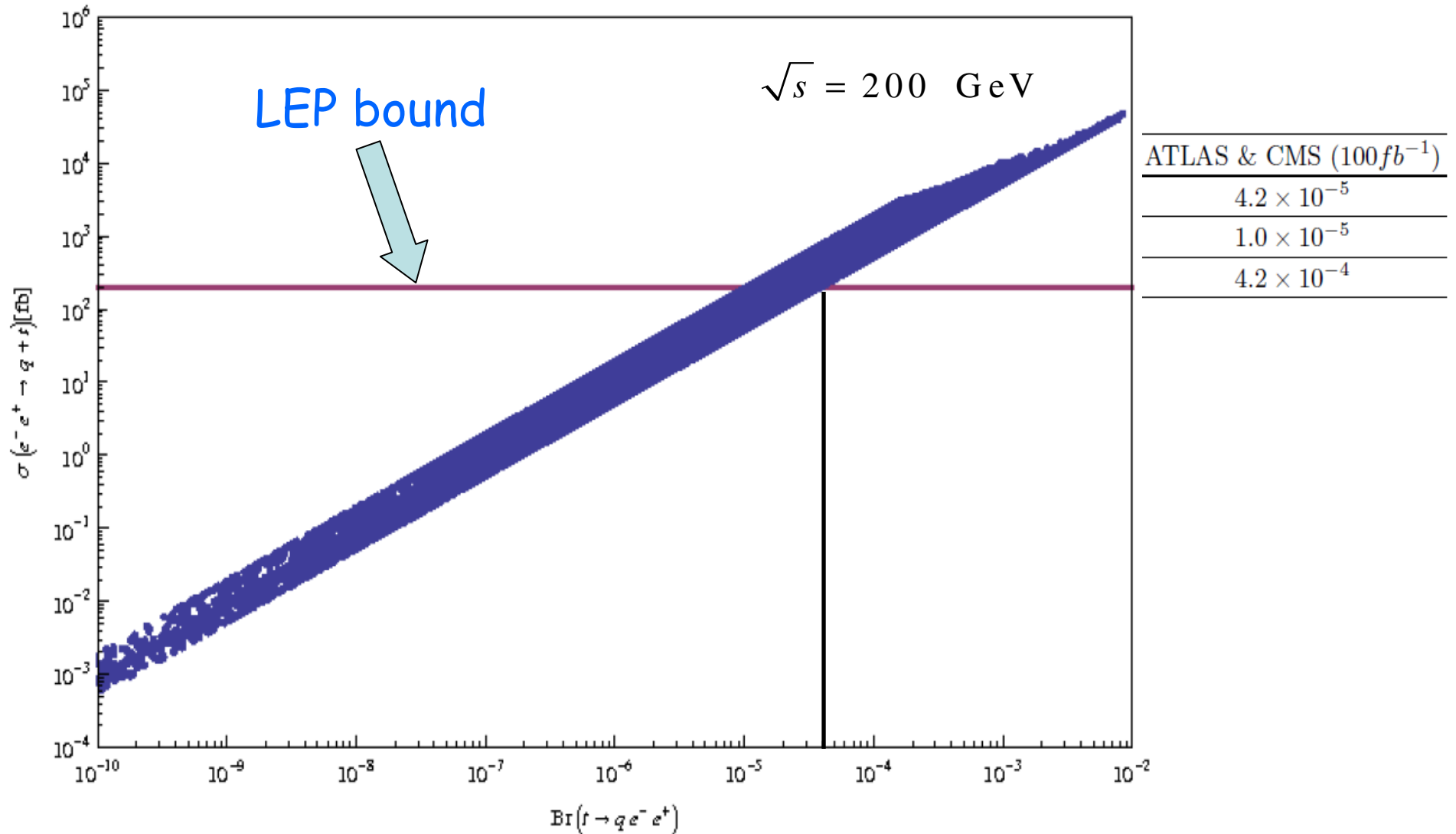
L3 Coll. PLB549, 2002.

Last DELPHI paper on FCNC limits to appear soon.

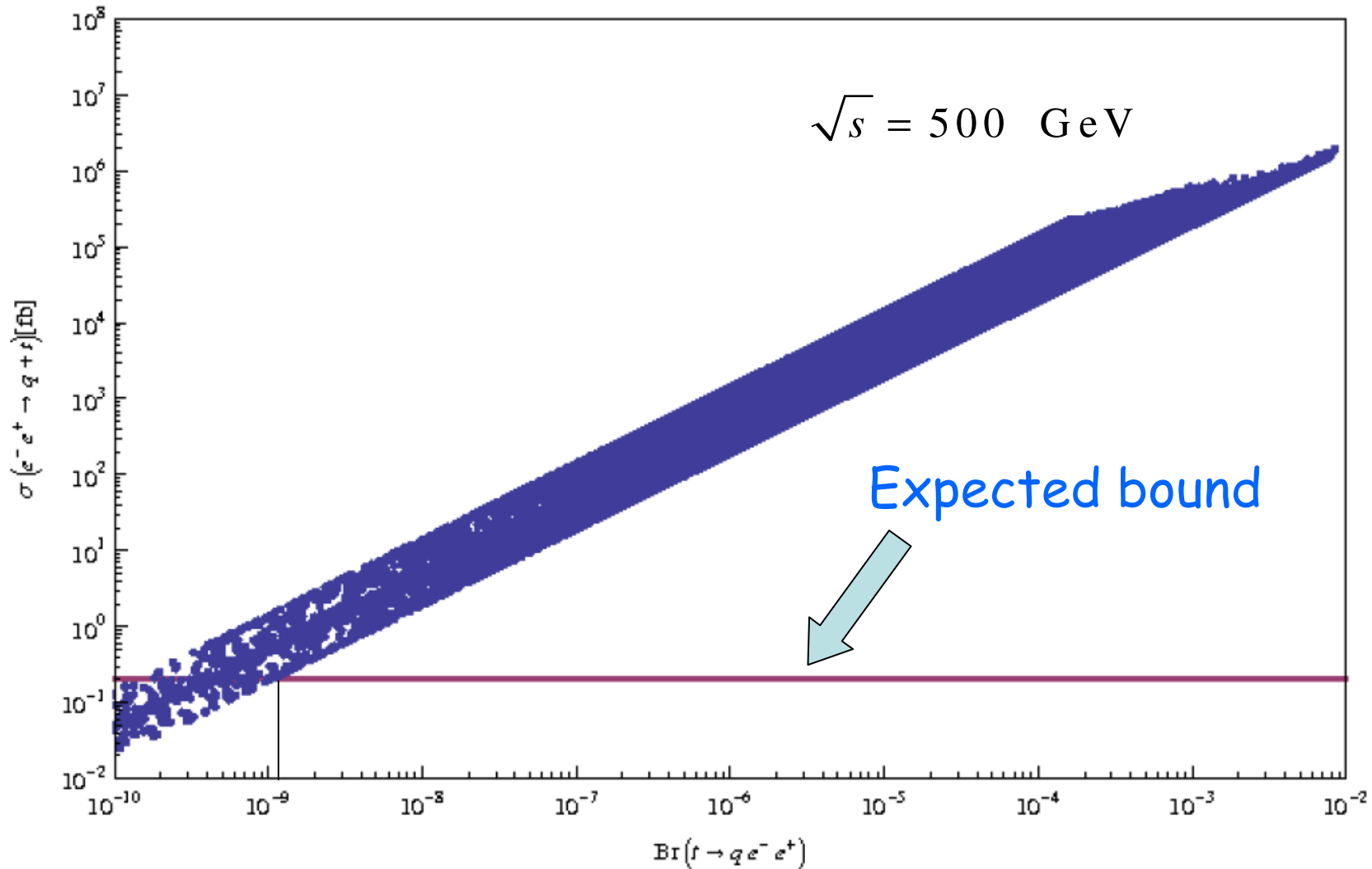
Using LEP data - bound for the FCNC BR of the three body decay of the top

$$t \rightarrow q e^+ e^-$$

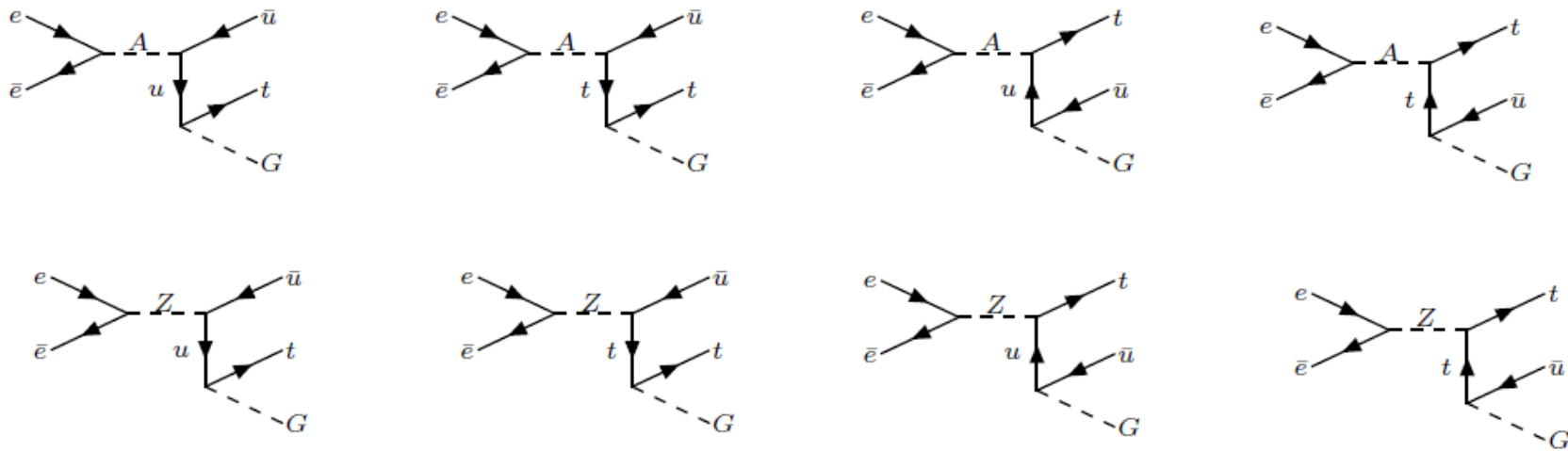
There is still no prediction available for the LHC.



For a 500 GeV collider we again use 0.2 fb (Han and Hewett; Aguilar-Saavedra and Riemann) as the cross that can be probed with 300 fb^{-1} of integrated luminosity. This is slightly more pessimistic than the cross section value predicted in Bar-Shalom and Wudka.



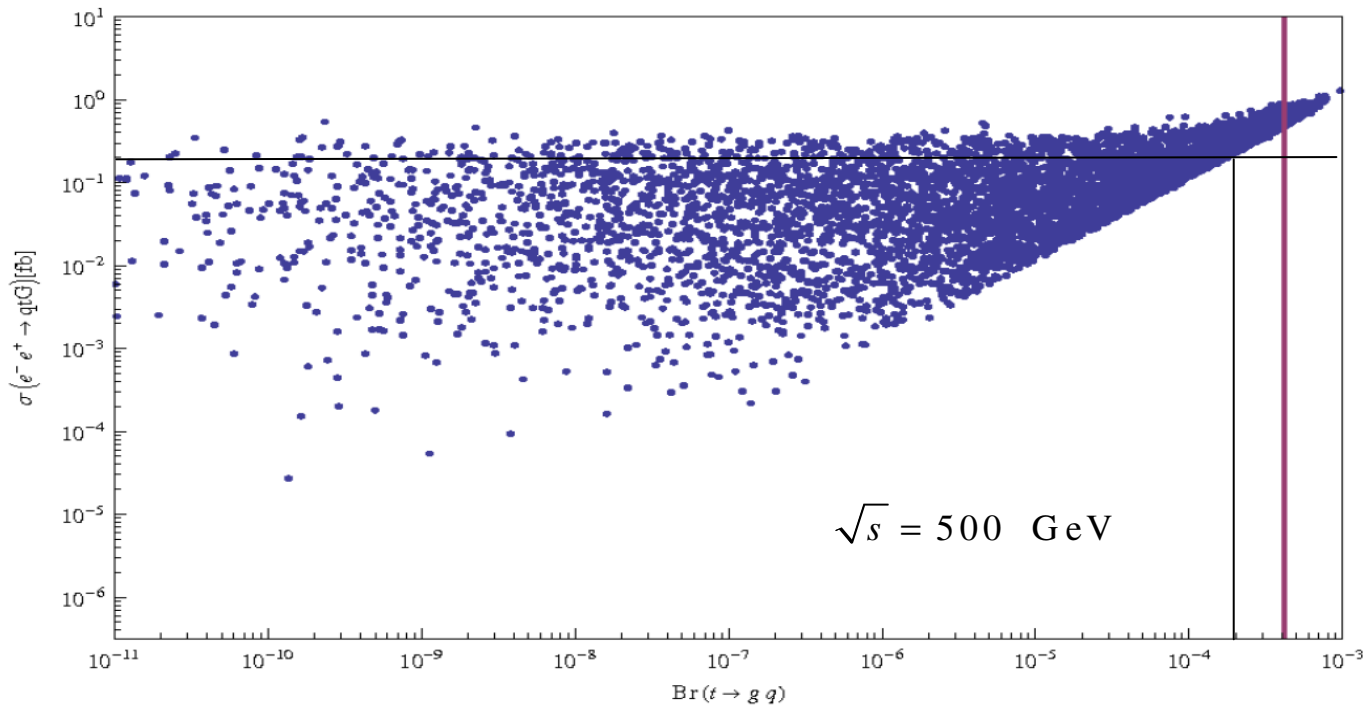
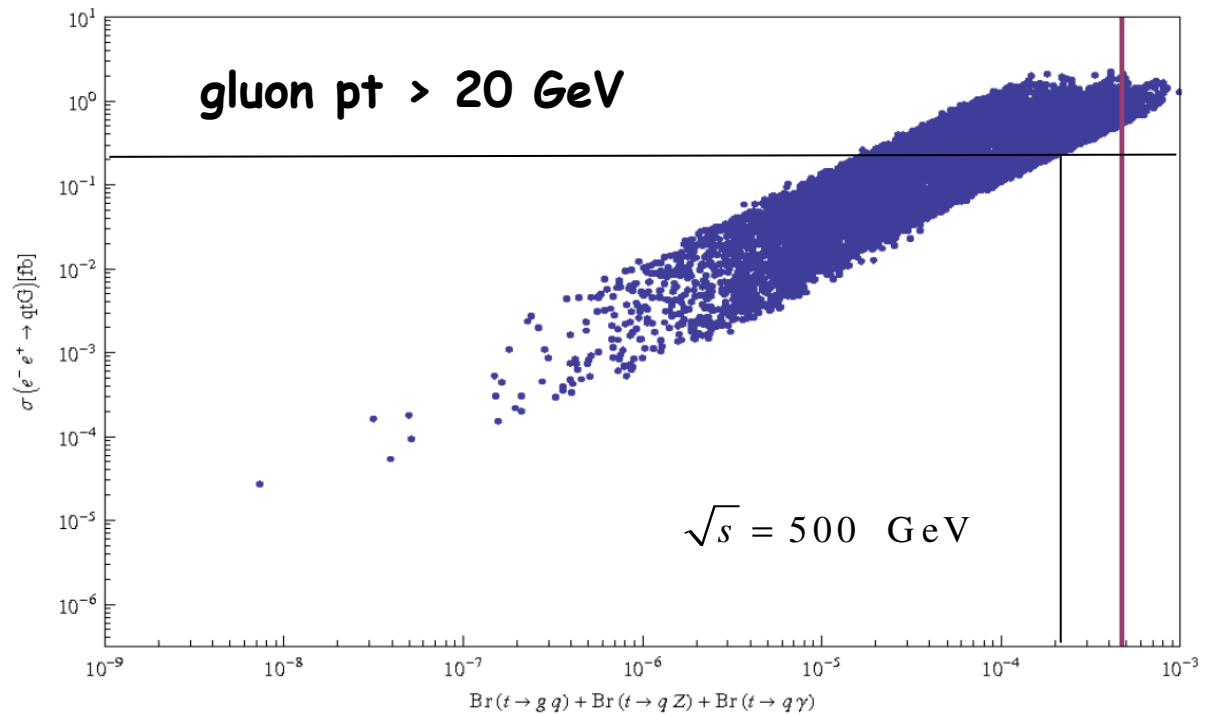
Single FCNC top+jet production



This is one of the simplest way to test anomalous strong FCNC top couplings. So far, no analysis was performed for this final state. The top decays to bW and therefore we will be looking for a final state $bl\nu jj$. The gluon has $p_t > 20 \text{ GeV}$.

Anomalous coupling constants are generated randomly and cross section and BR are calculated (10000 points generated).

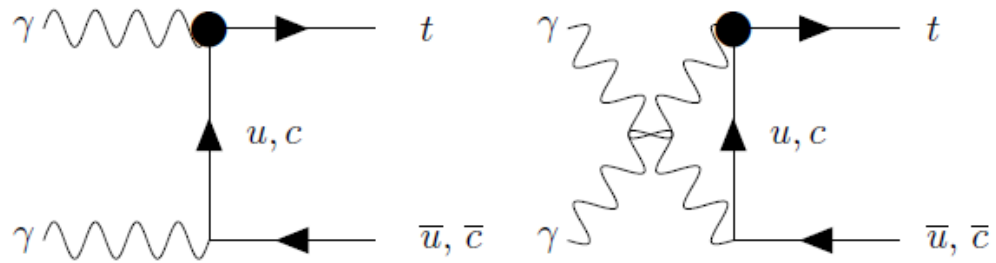
All predicted bounds for the LHC 100 fb⁻¹ are used.



No improvement in the Z and photon BR.

At a $\gamma\gamma$ collider

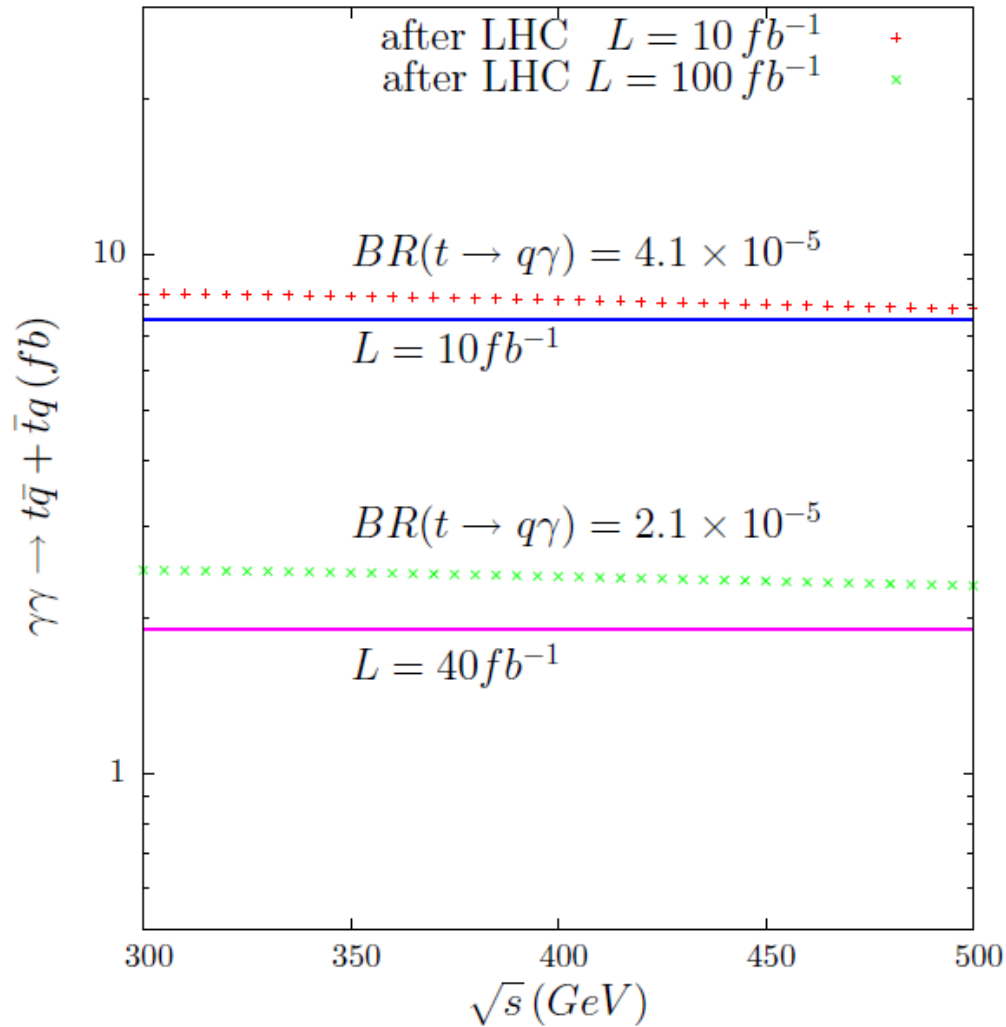
Very clean process where a measurement of the cross section is truly equivalent to a measurement of the branching ratio



The vertex has the simple form

$$\frac{v}{\Lambda^2} \sigma_{\mu\nu} (\beta_{jt}^R \gamma_R + \beta_{jt}^{L*} \gamma_L) (k^\mu g^{\nu\alpha} - k^\nu g^{\mu\alpha})$$

At a $\gamma\gamma$ collider



Detailed study for 400 and 500 GeV c.o.m. energy:

Abraham, Whisnant, Young,
*PLB*419 (1998) 381.

$$BR(\tau \rightarrow q\gamma) \approx 10^{-6}$$

QCD corrections

QCD corrections based on our chosen set of operators show little difference in the BR's to Z and photon but a significant difference in the decay to gluon. QCD corrections for tq and tqg production in an electron-positron collider are not available.

Zhang et al, PRL102, 2009.

BR [in unit of $(\frac{\kappa_{tq}^V}{\Lambda} \text{TeV})^2$]	LO	NLO	NLO/LO
$t \rightarrow q + g$	1.0010	1.1964	1.195
$t \rightarrow q + \gamma$	0.0544	0.0542	0.996
$t \rightarrow q + Z$	0.0448	0.0458	1.022

Both corrections have to be taken into account in future analysis.

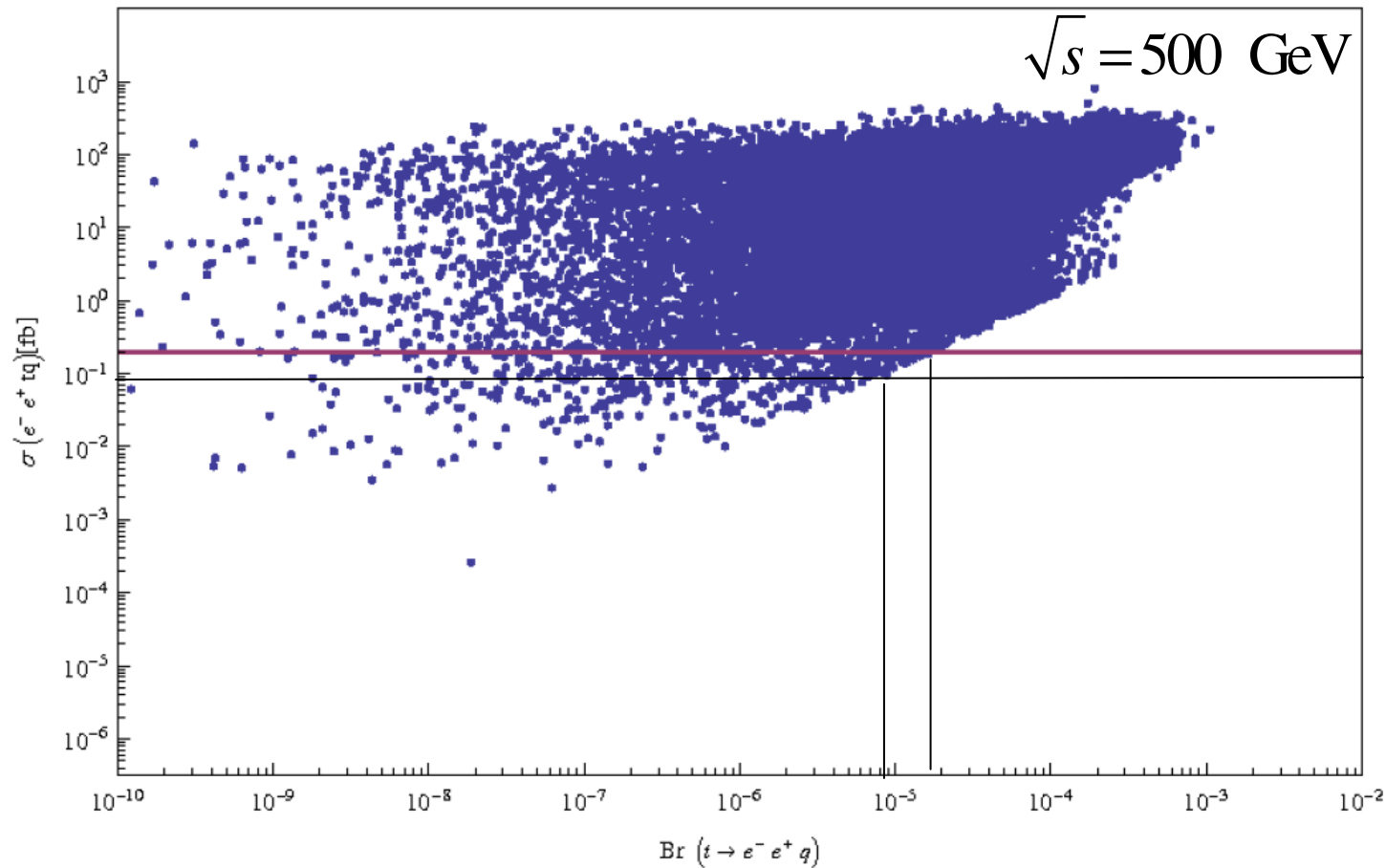
Conclusions

- We have considered a minimal set of operators for top anomalous FCNC production at a future linear collider.
- The future was set to LHC's 100 fb^{-1} .
- In this scenario, improving the LHC bounds depend on the energy and especially on the luminosity of the future collider. If new physics is found, particular operators can be probed with definite observables.
- Better sensitivity to the FCNC $\text{BR}(t \rightarrow qee)$.
- "The" photon-photon collider will most certainly improve the bound on the top to photon FCNC branching ratio.
- Generator will "soon" be available.

The end

Extra slides

Contribution from the non 4F operators - cross section is smaller - bound is not as good as for the 4F operators.

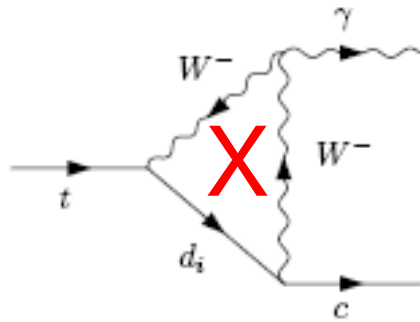


A framework for top FCNC physics - the effective operator approach

$$L = L_{SM} + \frac{1}{\Lambda} L^{(5)} + \frac{1}{\Lambda^2} L^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

Λ is the scale of new physics.

X is from a new untested model (the same SM particles in the external lines). It can have the same or a different Lorentz structure.



$\bar{t}c\gamma$

electroweak

strong

$$\mathcal{O}_{tW\phi} = \frac{\beta_{it}^W}{\Lambda^2} (\bar{q}_L^i \tau_I \sigma^{\mu\nu} t_R) \phi W_{\mu\nu}^I$$

$$\mathcal{O}_{tB\phi} = \frac{\beta_{it}^B}{\Lambda^2} (\bar{q}_L^i \sigma^{\mu\nu} t_R) \phi B_{\mu\nu}$$

$$\mathcal{O}_{tG\phi} = \frac{\beta_{it}^S}{\Lambda^2} (\bar{q}_L^i \lambda^a \sigma^{\mu\nu} t_R) \phi G^{a\mu\nu}$$

All terms are invariant under the SM gauge group.

Operators and observables

In a simple world, in the top FCNC case suppose we had only three observables and three operators! However...

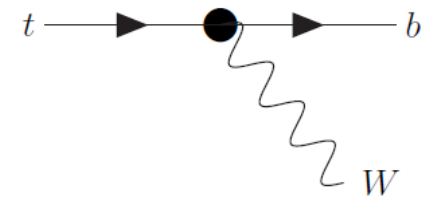
$$\left. \begin{aligned}
 \mathcal{O}_{tW\phi} &= \frac{\beta_{it}^W}{\Lambda^2} (\bar{q}_L^i \tau_I \sigma^{\mu\nu} t_R) \phi W_{\mu\nu}^I \\
 \mathcal{O}_{tB\phi} &= \frac{\beta_{it}^B}{\Lambda^2} (\bar{q}_L^i \sigma^{\mu\nu} t_R) \phi B_{\mu\nu} \\
 \mathcal{O}_{tG\phi} &= \frac{\beta_{it}^S}{\Lambda^2} (\bar{q}_L^i \lambda^a \sigma^{\mu\nu} t_R) \phi G^{a\mu\nu}
 \end{aligned} \right\} \begin{array}{c} \xrightarrow{\hspace{1cm}} \\ \xleftarrow{\hspace{1cm}} \end{array} \left\{ \begin{array}{l} \sigma_{pp \rightarrow tj} \\ \sigma_{pp \rightarrow t\bar{t}} \\ A_{FB} \end{array} \right.$$

...this is not the case. Therefore we have to find as many physical observables as possible. The number of operators we already know is huge.

Even in this simple case we need

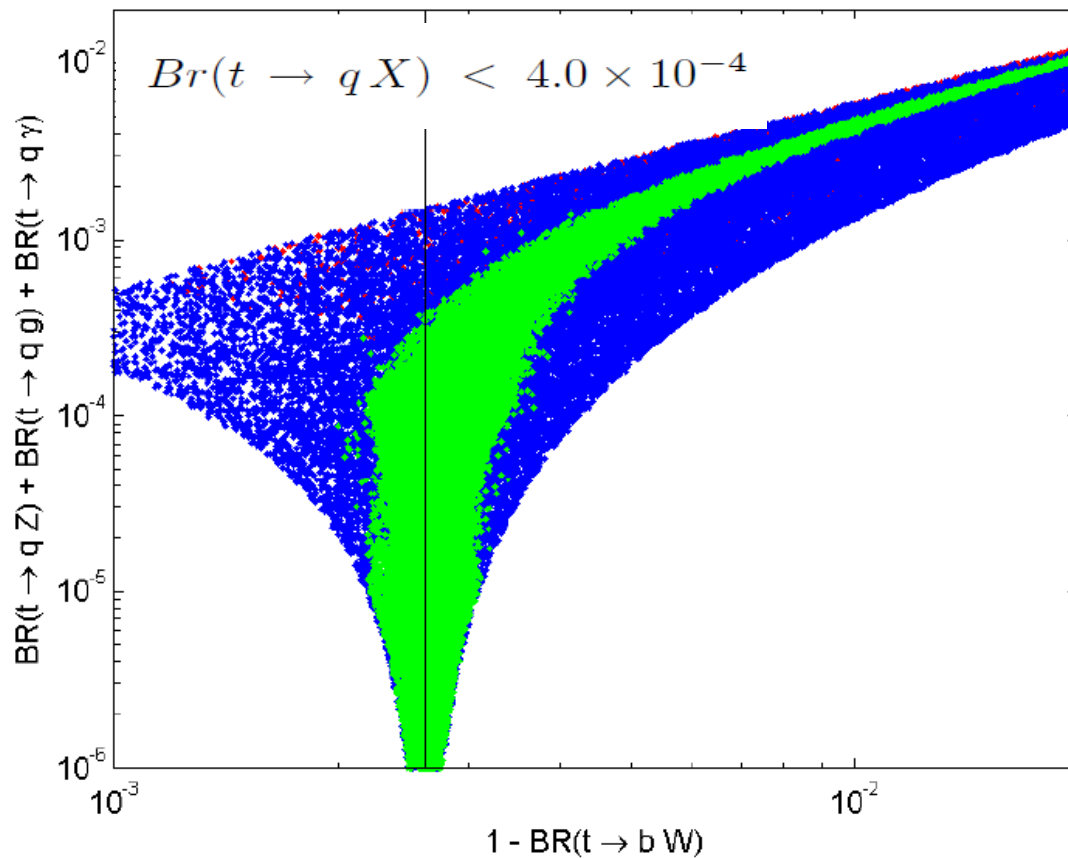
- Great precision from the theoretical side
 - Great precision from the experimental side
- to look for the small deviations from the SM.

Relating observables



$$\Gamma(t \rightarrow dW) = 1.42 |V_{td}|^2 - \frac{3g}{8\pi\Lambda^2} \frac{m_t^4 - m_W^4}{m_t^2} v \operatorname{Re}(V_{td}\beta_{ut}^W)$$

$$BR(t \rightarrow bW) = \frac{\Gamma(t \rightarrow bW)}{\sum_{q=u,c} [\Gamma(t \rightarrow qg) + \Gamma(t \rightarrow q\gamma) + \Gamma(t \rightarrow qZ)] + \sum_{q=d,s,b} \Gamma(t \rightarrow qW)}$$



1. Use the Tevatron results for single top production.

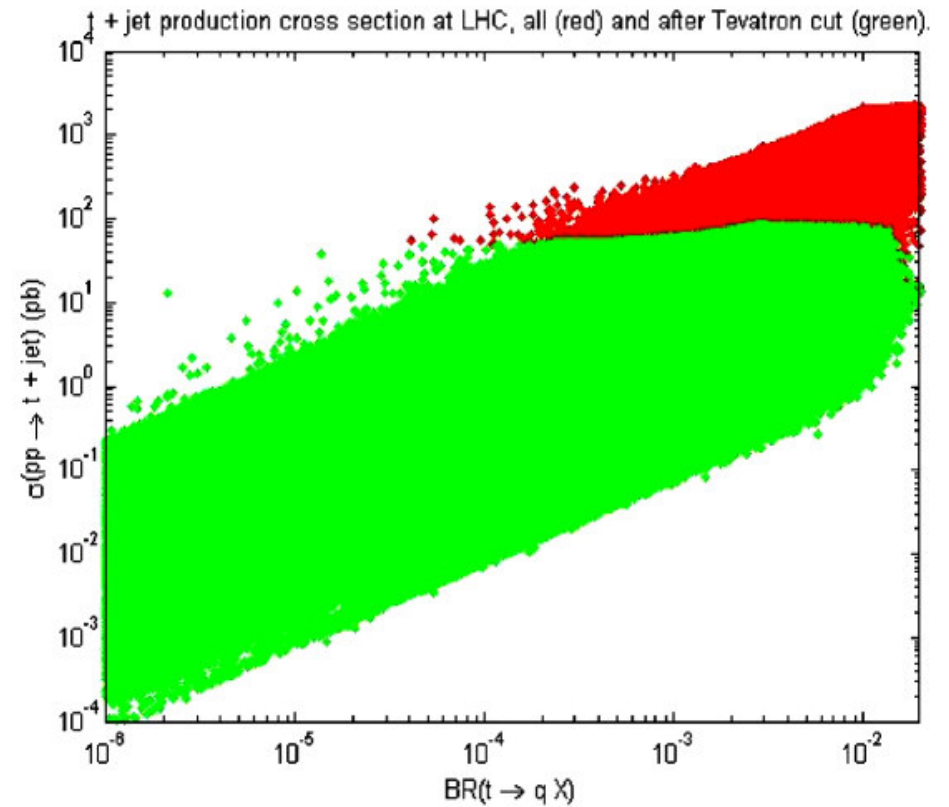
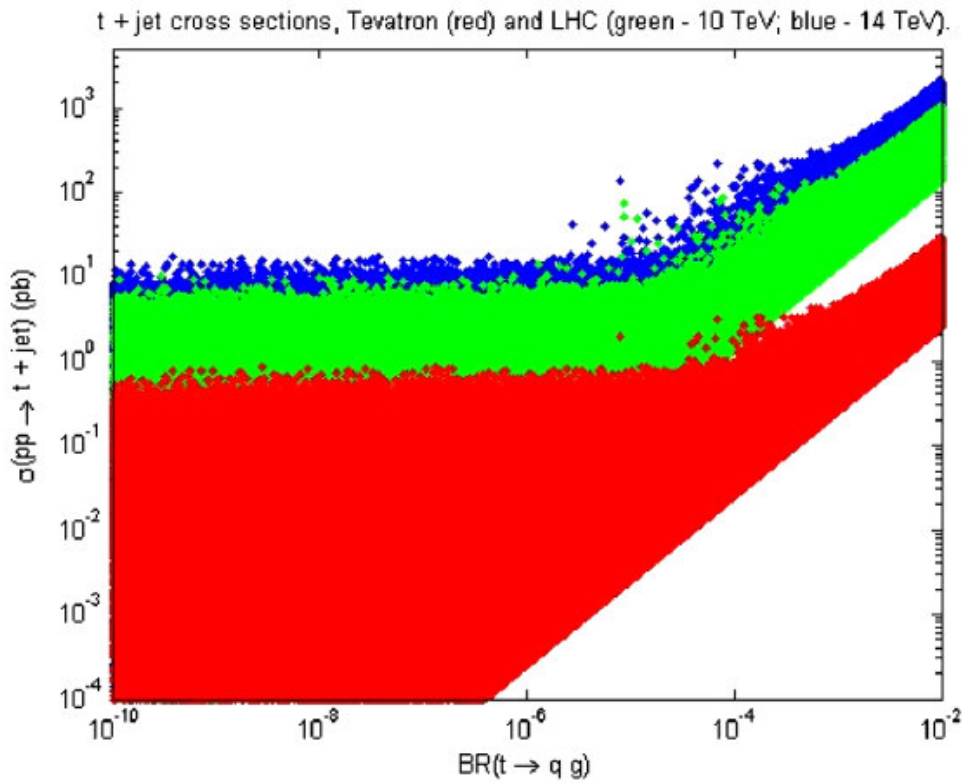
2. B physics constraints from Fox et al, PRD78 054008, 2008.

Tevatron bound

$$1.0 \times 10^{-3}$$

Results

3a. Tevatron LHC10 and LHC14



When we increase the energy from 500 GeV to 1 TeV the cross section increases by a factor 4.

Depending on the luminosity extremely small values of the 4F BR could be reached.

