QCD static potential at three loop

Yuichiro Kiyo@KEK,

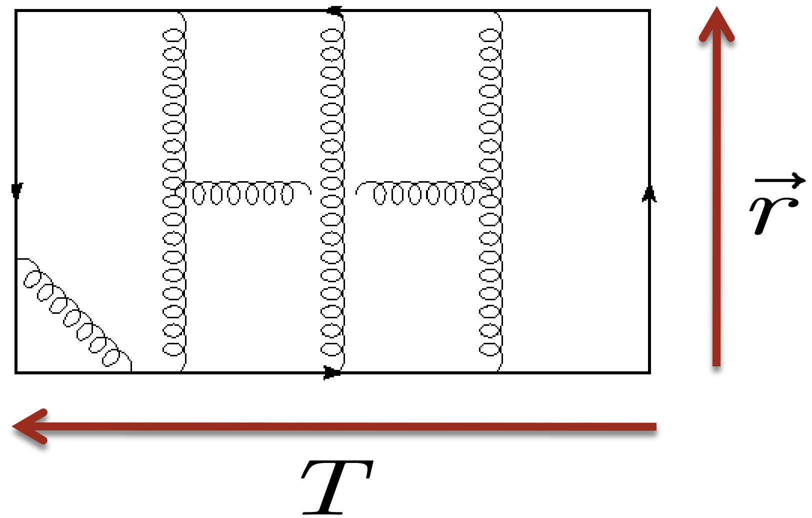
C.Anzai,YK,Y.Sumino, arXiv:0911.4335;
PRL104,112003

LCWS2010Beijing

QCD Static potential V(r)

$$W[C] = \left\langle Tr P e^{ig\oint_C dx \cdot A(x)} \right\rangle$$

$$\xrightarrow{T\to\infty} \; e^{-iT\,V(r)}$$



Wilson loop in large T limit defines the static potential

- W[C] is gauge invariant, and well-defined quantity
  e.g. comparison possible with Lattice QCD
- Important quantity in heavy quark physics, J/ψ,Υ
  Quarkonia energy spectrum

Static potential in pert. QCD

$$\tilde{V}(q) = -\frac{C_F \alpha_s(q)}{\vec{q}^2}\left[1 + \frac{\alpha_s(q)}{4\pi} a_1 + \left(\frac{\alpha_s(q)}{4\pi}\right)^2 a_2 + \left(\frac{\alpha_s(q)}{4\pi}\right)^3 (a_3 + \delta a_3)\right]$$

$$a_1 = \frac{31}{9} C_A - \frac{20}{9} T_F n_l$$

W. Fischler '77, A. Billoire '80

$$a_2 = \left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^4}{4} + \frac{22\varsigma_3}{3}\right) C_A^2 - \left(\frac{1798}{81} + \frac{56\varsigma_3}{3}\right) C_A T_F n_l - \left(\frac{55}{3} - 16\varsigma_3\right) C_F T_F n_l + \left(\frac{20}{9} T_F n_l\right)^2$$

M.Peter '97, Y. Schroder '99

$$a_3 = 502.22(12) C_A^3 - 136.8(14) \frac{d_F^{abcd} d_A^{abcd}}{N_A}$$

Anzai-YK-Sumino (2009),
Smirnov-Smirnov-Steinhauser (2009)

$$-709.717 C_A^2 T_F n_l - 56.83(1) \frac{d_F^{abcd} d_F^{abcd}}{N_A} + \left(-\frac{71281}{162} + 264\varsigma_3 + 80\varsigma_5\right) C_A C_F T_F n_l$$

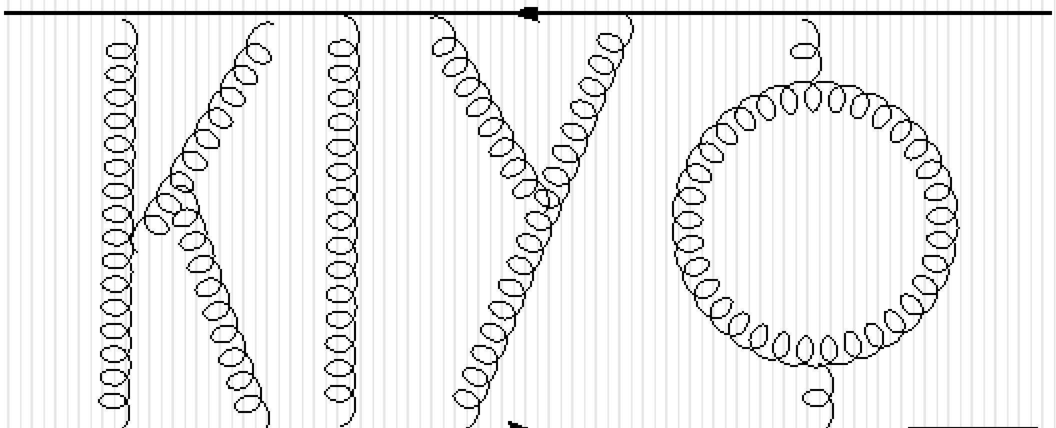
$$+\left(\frac{286}{9} + \frac{296\varsigma_3}{3} - 160\varsigma_5\right) C_F^2 T_F n_l + \left(\frac{12541}{243} + \frac{368\varsigma_3}{3} + \frac{64\pi^2}{135}\right) C_A (T_F n_l)^2$$

$$+\left(\frac{14002}{81} - \frac{416\varsigma_3}{3}\right) C_F (T_F n_l)^2 - \left(\frac{20}{9} T_F n_l\right)^3$$

Smirnov-Smirnov-Steinhauser (2008)

Higher loop computation

Here we surmarize steps for higher loop calculation in general with keyword.



Reduction to Master Integrals

- Generation of amplitudes by

  Grace (Ishikawa-Kaneko-Kato-Kawabata-Shimizu-Tanaka, 1992)

  Q-graph: P. Nogueira (1993)

  Color.h with FORM (Vermaseren, Ritbergen-Schellekens-Vermaseren, 1999)

- IntegrationByParts Identity (IBP) (Chtyrkin –Tkachov, 1981)

$$\int d^D k \, f(k) = \int d^D k \, f(k+a)$$

- Laport algorithm (S.Laporta 2000, see also)

solve system of equations by Laporta algorithm and rewrite

all the Feynman integrals with few number of Master integrals(MI)

Taming Master Integrals

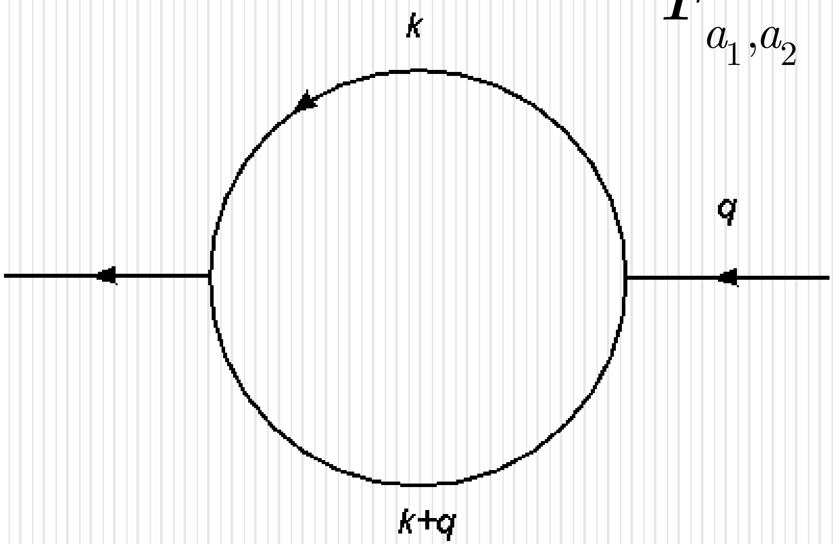
We evaluated most of MIs twice with independent codes by means of MB/SD, and obtained agreement within estimated error.

- Mellin-Barnes representaion (…,V. Smirnov 1999,…)
- Sector decomposition (…, Binoth-Heinrich 2000,…)

There are several codes available in market which is quite powerful for traditional Feynman integrals:

MB.m(M.Czakon), AMBRE.m(Gluza-Kajda-Rieman), Sector_decomposition(Bogner-Weinzierl), FIESTA.m(Smirnov-Tentyukov), HypExp.m(Huber-Maitre)

IBP and reduction: Simplest example

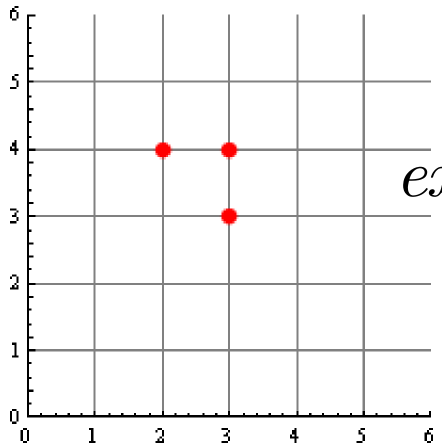


$$F_{a_1,a_2} = \int d^d k \frac{1}{[k^2]^{a_1}[(k+q)^2]^{a_2}}$$

IBP:

$$0 = \int_k \frac{\partial}{\partial k^i} \frac{\{k^i, q^i\}}{[k^2]^{a_1} [(k+q)^2]^{a_2}}$$

$$0 = -a_2 F_{a_1-1, a_2+1} + (d - a_2 - 2a_1) F_{a_1, a_2} + a_2 q^2 F_{a_1, a_2+1}$$



$$ex : F_{10,10} \longrightarrow F_{10,9}, F_{9,10} \longrightarrow F_{10,8}, F_{9,9}, \ldots \cdots \longrightarrow F_{1,1}$$

Symmetry F(i,j)=F(j,i)

Master Integral

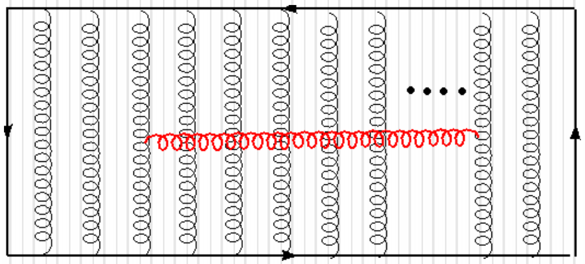
Go down/left to reduce the indices

$$F_{10,10} = F_{1,1} \times \left(85\,336\,948\,840 - \frac{31\,023\,871\,448\,999\, d}{252} + \frac{10\,088\,011\,232\,182\,595\, d^2}{127\,008} - \frac{4\,633\,826\,882\,477\,423\, d^3}{151\,200} + \frac{3\,641\,862\,782\,690\,152\,973\, d^4}{457\,228\,800} - \frac{1\,359\,646\,658\,698\,095\,169\, d^5}{914\,457\,600} + \frac{853\,044\,554\,700\,900\,767\, d^6}{4\,115\,059\,200} - \frac{5\,783\,538\,232\,181\,713\, d^7}{261\,273\,600} + \frac{40\,378\,041\,628\,264\,283\, d^8}{21\,946\,982\,400} - \frac{251\,167\,627\,189\,919\, d^9}{2\,090\,188\,800} + \frac{271\,876\,669\,208\,771\, d^{10}}{43\,893\,964\,800} - \frac{102\,331\,700\,207\, d^{11}}{406\,425\,600} + \frac{132\,071\,490\,479\, d^{12}}{16\,460\,236\,800} - \frac{29\,560\,139\, d^{13}}{149\,299\,200} + \frac{81\,301\,357\, d^{14}}{21\,946\,982\,400} - \frac{26\,527\, d^{15}}{522\,547\,200} + \frac{10\,541\, d^{16}}{21\,946\,982\,400} - \frac{41\, d^{17}}{14\,631\,321\,600} + \frac{d^{18}}{131\,681\,894\,400} \right)$$

Result and Summary

There are non-perturbative corrections for Wilson loop, which at leading order is known as ultra-soft correction.

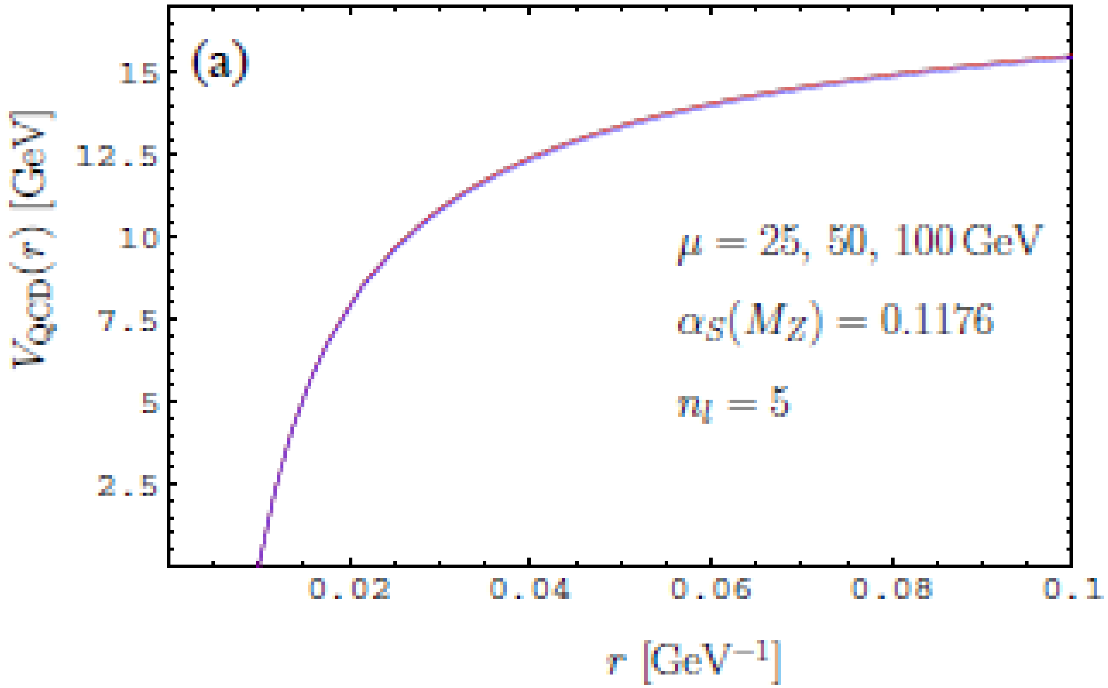
In the following we will combine this with our perturbative corrections.



$$\left[V(r)\right]_{US} = \frac{C_F C_A^3 \alpha_s^4}{24\pi r}\left[\frac{1}{\varepsilon} + 8\ln(\mu r) - 2\ln(C_A\alpha_s) + \frac{5}{3} + 6\gamma_E\right]$$

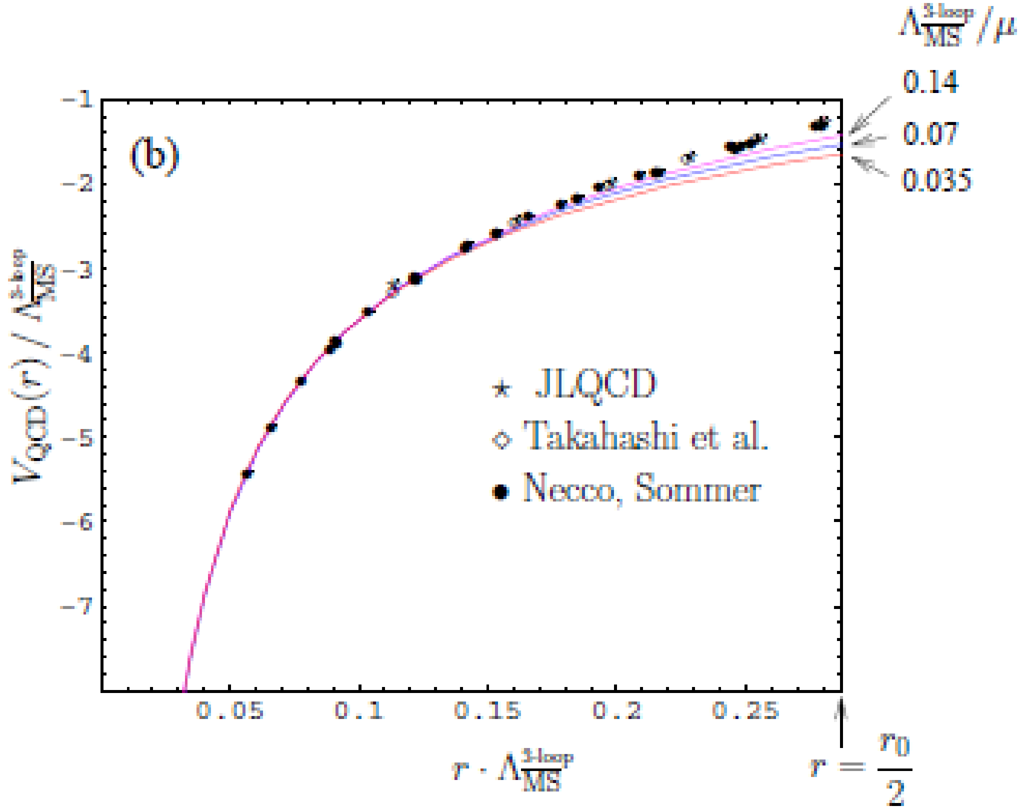
Appelquist-Dine-Muznich (1977),Brambilla-Pineda-Soto-Vairo(1999),

QCD potential at three loop



- QCD potential (PT+US) at three loop for μ=25,50,100 [GeV] for top quark. Three lines coincide and one sees no difference.

QCD potential at three loop



- Comparison with Lattice data for 0-flavour QCD, region roughly corresponds to Upsilon 1S.

Summary

- QCD static potential at three loop is completed after 32 (10) years from 1-loop (2-loop) computation.

(Anzai-YK-Sumino, Smirnov-Smirnov-Steinhauser 2009)

- Scale dependence is tiny for top, convergence to quench Lattice data is also good, which encourage us for phenomenological applications.

a. Precision determination of QCD coupling with Lattice data
b. Quarkonium phenomenology with obaitned potential
c. Another applications of technology to three loop?