

LITHIUM LENS (II)

Lithium Flow Magneto-Hydrodynamics

Progress report on Cornell Activity in Positron Production Scheme for ILC

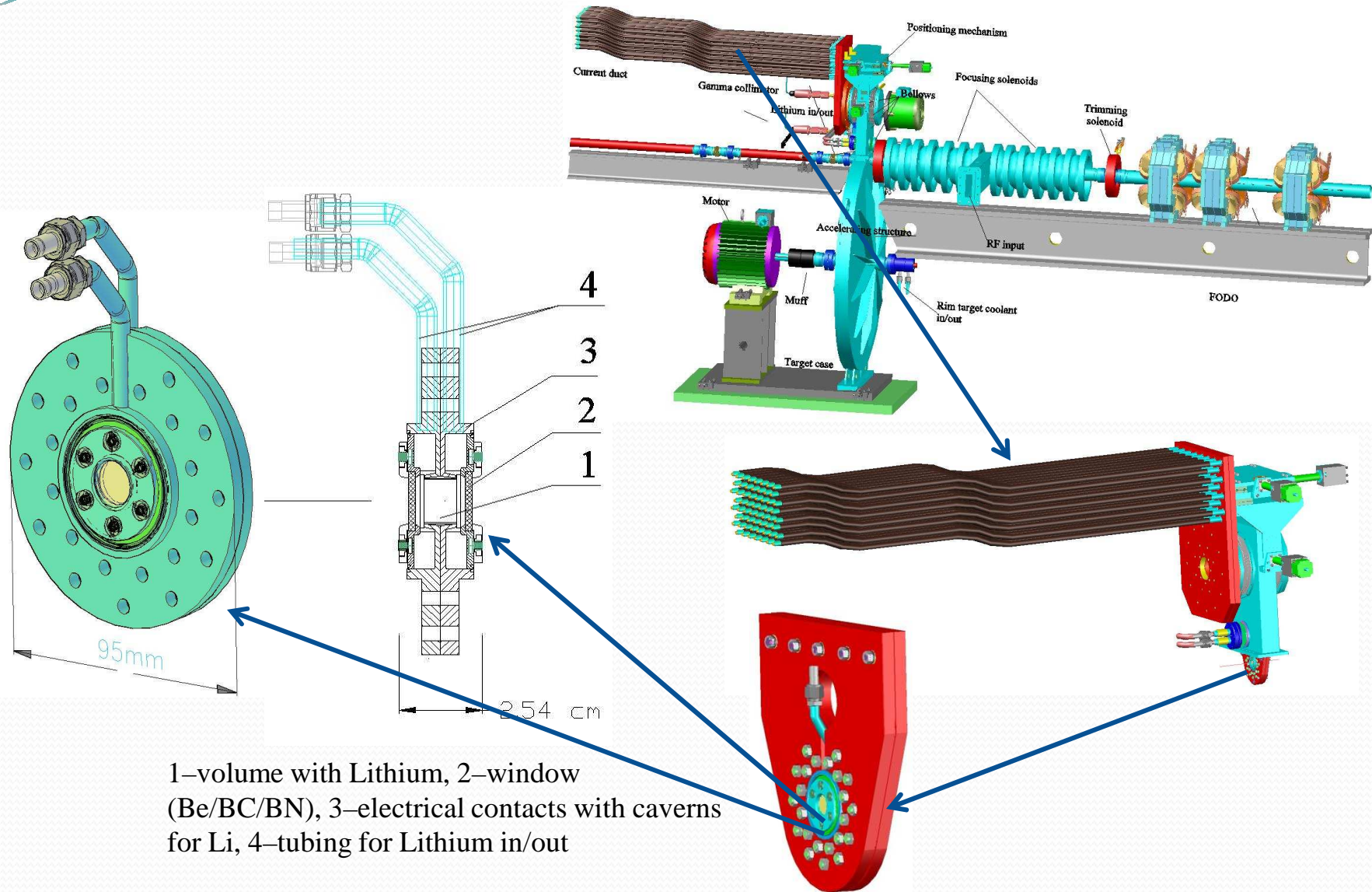
Alexander Mikhailichenko

Cornell U., LEPP, Ithaca, NY 14853

International Linear Collider Workshop 2010 (LCWS10 and ILC10)

Institute of High Energy Physics, Beijing, China

28 March 2010



Activities

Was considered:

Conceptual design

Windows attachment

Energy Deposition in lens

Magnetic field in surroundings

Lithium Flow

Pressure dynamics

Some engineering for prevention of current flow through lithium duct

More to do:

Stress-strain in windows

Shock waves due to single bunch

Radiation damage conclusion (+remote handling etc)

Drawings for critical components: (feed-through, Lithium duct)

} Support required

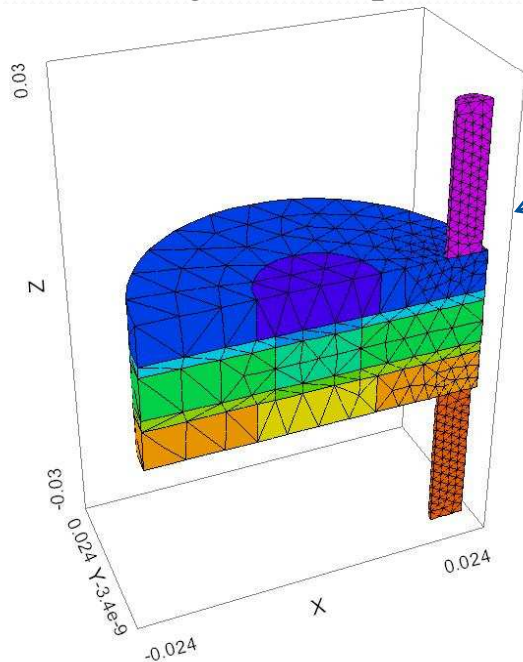
See for latest in:

A.Mikhailichenko, "Lithium lens (II). Lithium Flow Magneto-Hydrodynamics", CBN 10-3,
http://www.lns.cornell.edu/public/CBN/2010/CBN10-3/CBN_10-3.pdf

Set of equations used for modeling could be grouped into three categories: **Electromagnetic, Hydrodynamic and Thermodynamic**.

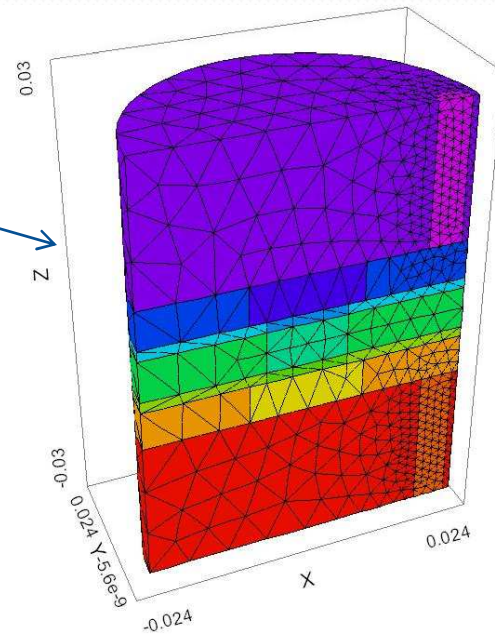
All three have cross terms linking electric current, fluid flow and temperature relaxation due to thermo-conductivity, Ohmic heating, fluid motion and frictional heat generation due to viscose phenomena.

For solving set of equation we used Partial Differential Equation Solver FlexPDE

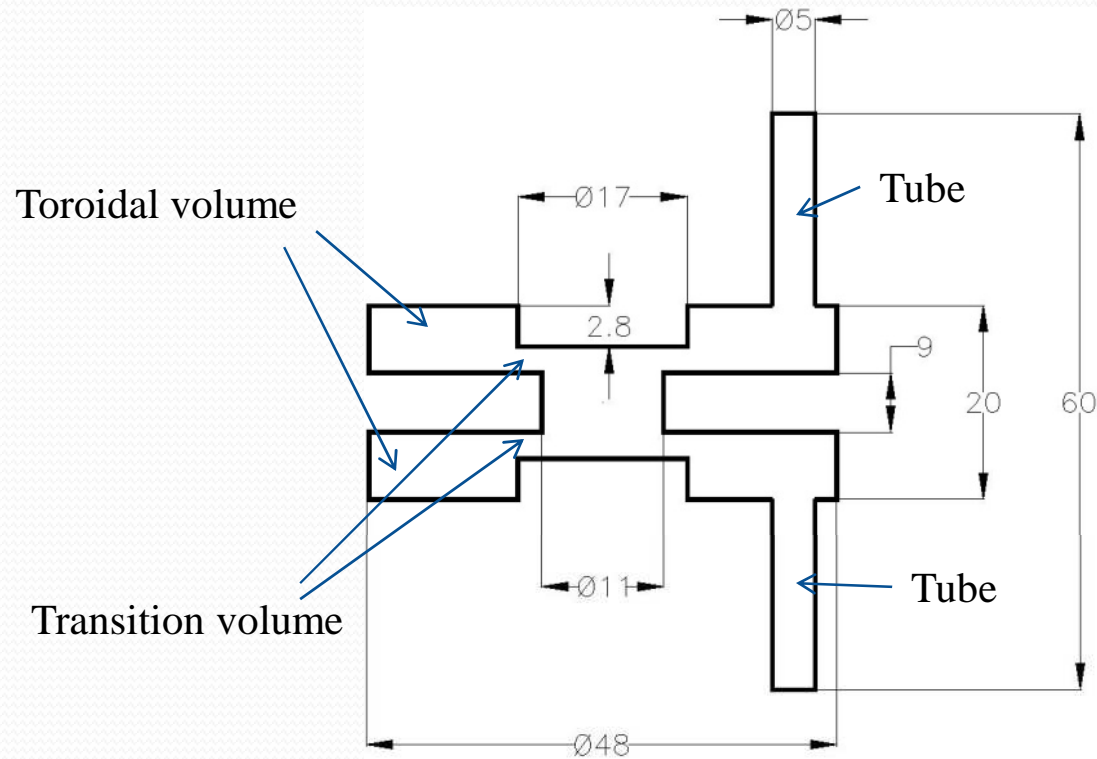


Model 1 and Model 2 use the same equations; model 2 has no mesh in upper and lower layers, sometimes we excluded even volume occupied by windows.

Boundary conditions applied on each surface



Dimensions of model



Dimensions of model, *mm*. All dimensions are subject of optimization process.

In volume without material artificial values for electric conductivity, viscosity, heat conductivity and density were applied:

layer 'down' sigma=1e-15 visc=1e10 k=1e-15 density=1e10
layer 'up' sigma=1e-15 visc=1e10 k=1e-15 density=1e10

Electromagnetism:

We used *voltage* as given parameter (<5V).

Set of equations:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \text{grad}(U) \quad \vec{B} = \text{rot}(\vec{A}) \quad \vec{B} = \mu \cdot \vec{H} \quad \vec{j} = \sigma \cdot (\vec{E} + \vec{v} \times \vec{B}) \quad \text{div}(\vec{j}) = 0$$

$$\text{div}(\text{grad}(A_x)) + \mu \cdot j_x = 0 \quad \text{div}(\text{grad}(A_y)) + \mu \cdot j_y = 0 \quad \text{div}(\text{grad}(A_z)) + \mu \cdot j_z = 0$$

Total energy deposition into the lens by feeding current during time ΔT can be calculated as

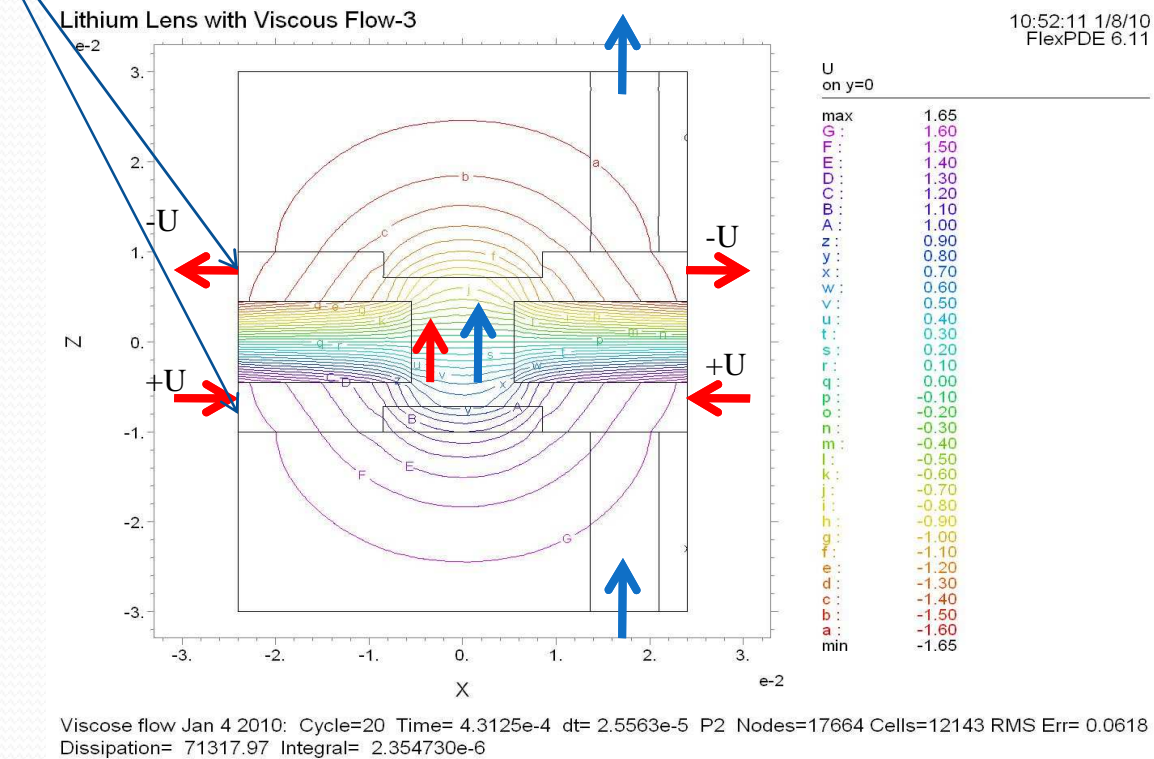
$$\Delta Q_{\text{tot}} = \int_0^{\Delta T} dt \int_V (\vec{j} \cdot \vec{E}) dV = \int_0^{\Delta T} dt \int_V \frac{j^2}{\sigma} dV \quad (\approx 680 \text{ Joules})$$

$$\text{div}(\vec{j}) = - \left(\text{div} \left(\sigma \cdot \frac{\partial \vec{A}}{\partial t} \right) + \text{div}(\sigma \cdot \text{grad}(U)) + \text{div}(\sigma \cdot (\vec{v} \times \vec{B})) \right) = 0$$

$$\text{div}(\text{grad}(A_x)) + \mu \cdot j_x = \text{div}(\text{grad}(A_x)) - \mu \cdot \sigma \frac{\partial A_x}{\partial t} - \mu \cdot \sigma \cdot \frac{\partial U}{\partial x} + \mu \cdot \sigma \cdot (\vec{v} \times \vec{B})_x = 0$$

Parabolic (diffusion) equation: skin layer etc

Voltage applied to cylindrical surfaces



Potential. By blue arrows it is shown the Lithium flow, by red-current.

Different combinations of directions (current/flow) were used.

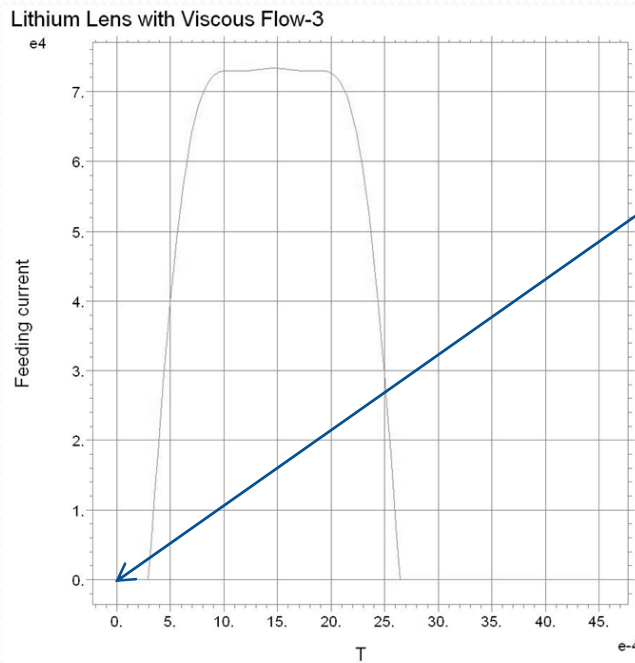
Voltage applied to the lens is a combination of three harmonics:

$$U(t) = U_0 \cdot \left[-4.5 \cdot \sin\left(\frac{\pi \cdot (t - \tau/10)}{\tau}\right) - 0.9 \cdot \sin\left(\frac{3\pi \cdot (t - \tau/10)}{\tau}\right) - 0.17 \cdot \sin\left(\frac{5\pi \cdot (t - \tau/10)}{\tau}\right) \right] \quad [\text{Volts}]$$

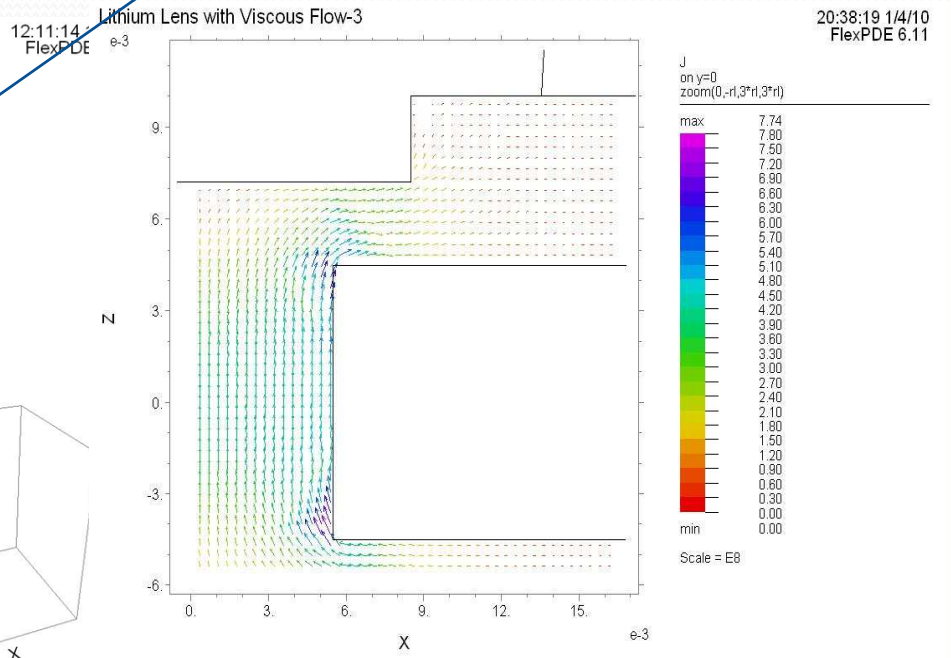
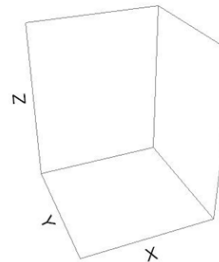
$U_0 = 0.8$

$\tau = 2.3 \text{ msec}$

Pulse delayed by $\tau/10$



HISTORY
1: Current_tot



Viscose flow Jan 4 2010: Cvcle=160 Time= 4.5531e-3 dt= 2.6118e-5 P2 Nodes=17741 Cells=12198 RMS Err=

Viscose flow Jan 4 2010: Cycle=88 Time= 2.4381e-3 dt= 2.6118e-5 P2 Nodes=17741 Cells=12198 RMS Err= 0.0609 Power of dissipation, Watts= 126634.3

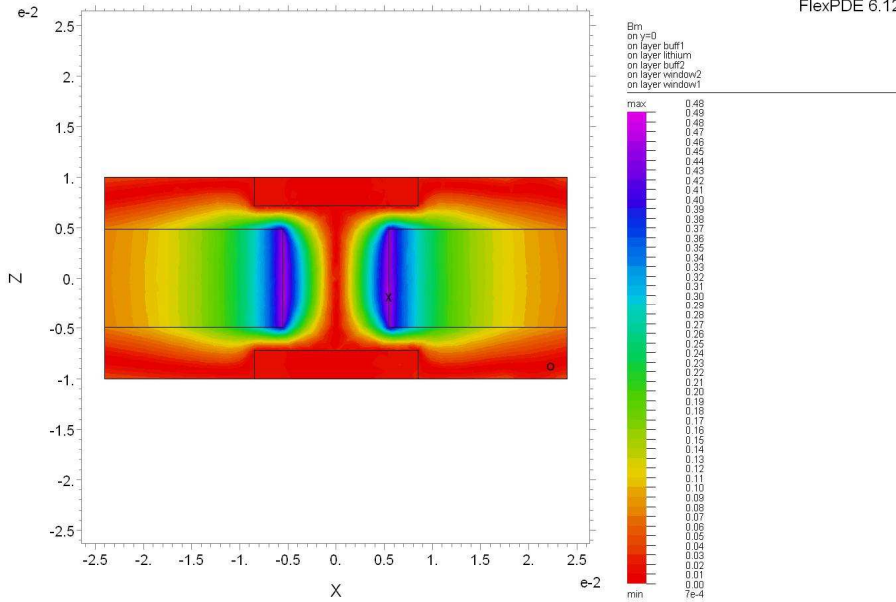
Electric current, *Ampers*.

Vectors of current density. Value of current density represented by the length of arrows and theirs color.

Magnetic field

Lithium Lens with Viscous Flow-3

12:57:33 3/3/10
FlexPDE 6.12

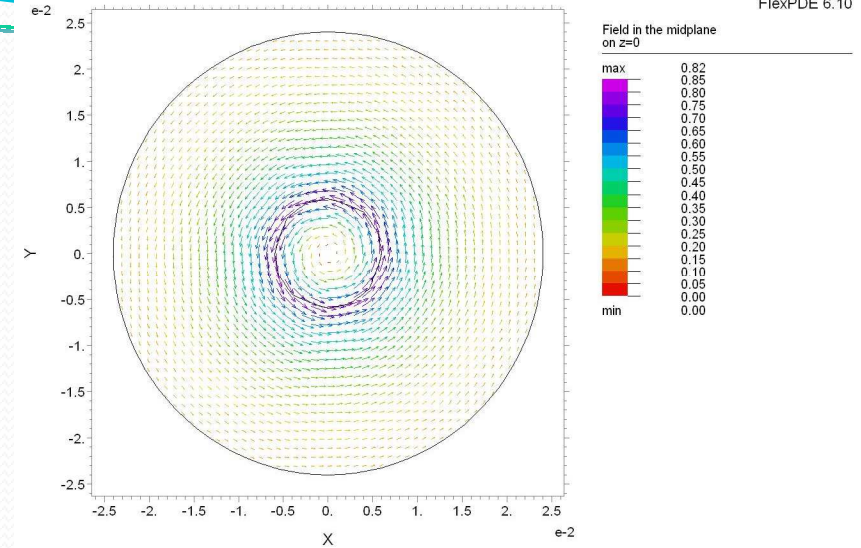


Viscose flow Feb 20 2010 electricity only: Cycle=10 Time= 6.9000e-5 dt= 9.1131e-6 P2 Nodes=98387 Cells=69850 RMS Integral= 1.163397e-4

Equal B -values counter plot

Lithium Lens with Viscous Flow-3

11:16:41 12/14/09
FlexPDE 6.10

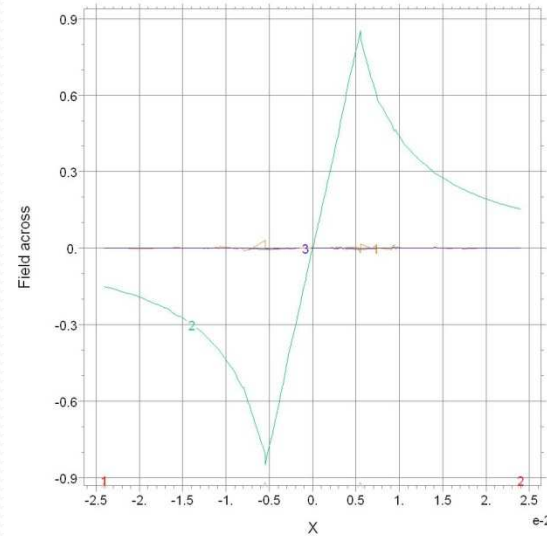


Viscose flow Dec 12 2009: Grid#4 P2 Nodes=74816 Cells=53615 RMS Err= 2.8e-4

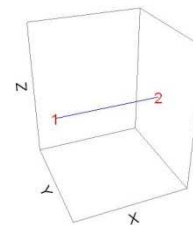
Vector plot of B in midplane of model.

Lithium Lens with Viscous Flow-3

16:42:01 12/14/09
FlexPDE 6.10



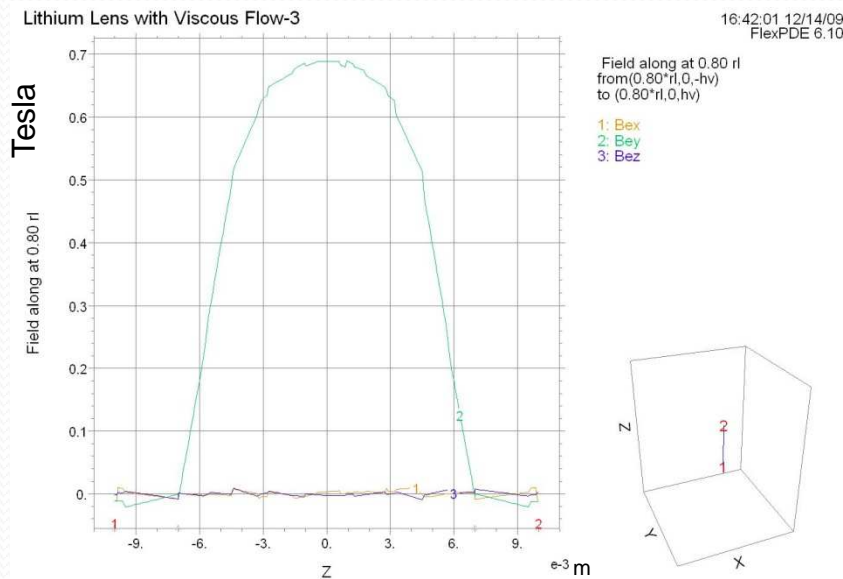
Field across through center



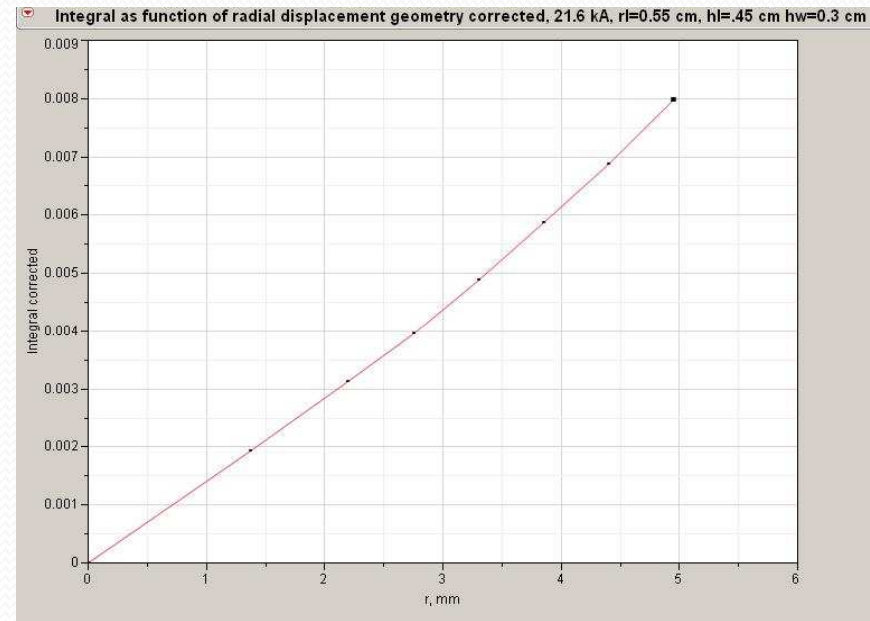
Some optimization of dimensions were done:

Increased length of Lithium cylinder from 0.6 to 1 cm;

Transition volume between cylinder and toroidal chamber squeezed;

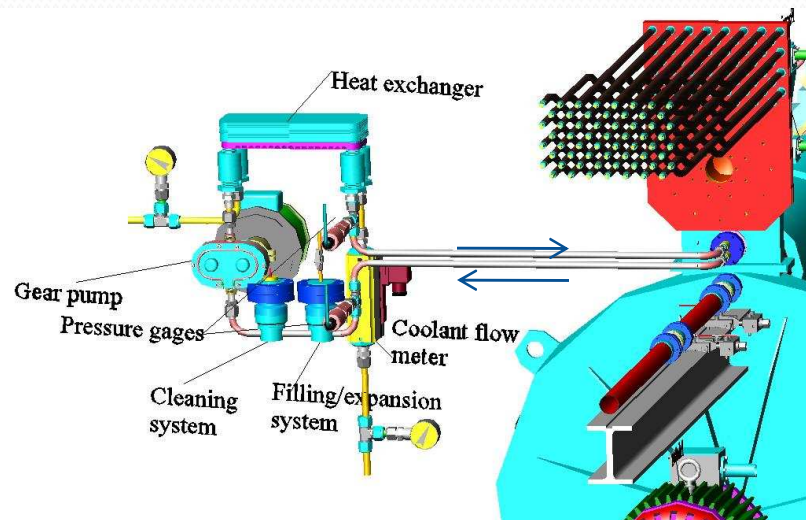


Longitudinal field dependence at 0.8 radius of lithium rod (which is $0.8 \times 5.5 \text{ mm} = 4.4 \text{ mm}$)



Field integral as function of radial displacement

One comment



As the Lithium duct runs from inlet to outlet where the voltage applied, the current runs through the body of lens, but some fraction is running through the duct

Current runs not through the body of lens only, but through the Lithium duct (tubes, pump) also.

Length of duct $\sim 2\text{m}$, diameter $\sim 7\text{mm}$, so resistance is higher in $200/0.7^2 \sim 400$ times

So this back current is $\sim 0.25\%$ of the one running through the lens i.e. $70\text{kA}/400 \sim 175\text{ A}$ - not big, but it could be eliminated by insertion of non conducting interrupter, which is device similar to gear pump, but with non-conducting case and gears .

Lithium flow:

Flow of Lithium is governed by vector equation

$$\rho \cdot \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) + \text{grad}(P) - \overset{\text{viscosity}}{\eta} \cdot \nabla^2 \vec{v} - \left(\overset{\text{Second viscosity; not used}}{\zeta + \frac{1}{3} \eta} \right) \text{grad}(\text{div}(\vec{v})) = (\vec{j} \times \vec{B})$$

In FlexPDE Grammatik components of velocity look like the following

$$\text{vx: density}*(\text{dt}(\text{vx})+\text{vx}*\text{dx}(\text{vx})+\text{vy}*\text{dy}(\text{vx})+\text{vz}*\text{dz}(\text{vx}))+\text{dx}(\text{p})-\text{visc}*\text{div}(\text{grad}(\text{vx}))=\text{xcomp}(\text{cross}(\text{J},\text{B}))$$

$$\text{vy: density}*(\text{dt}(\text{vy})+\text{vx}*\text{dx}(\text{vy})+\text{vy}*\text{dy}(\text{vy})+\text{vz}*\text{dz}(\text{vy}))+\text{dy}(\text{p})-\text{visc}*\text{div}(\text{grad}(\text{vy}))=\text{ycomp}(\text{cross}(\text{J},\text{B}))$$

$$\text{vz: density}*(\text{dt}(\text{vz})+\text{vx}*\text{dx}(\text{vz})+\text{vy}*\text{dy}(\text{vz})+\text{vz}*\text{dz}(\text{vz}))+\text{dz}(\text{p})-\text{visc}*\text{div}(\text{grad}(\text{vz}))=\text{zcomp}(\text{cross}(\text{J},\text{B}))$$

FlexPDE-version 6 allows writing this system as single vector equation

$$\text{density}*(\text{dt}(\text{v})+\text{dot}(\text{v},\text{grad}(\text{v}))) + \text{grad}(\text{p}) - \text{visc}*\text{div}(\text{grad}(\text{v})) - \frac{1}{3}*\text{visc}*\text{grad}(\text{div}(\text{v})) = \text{cross}(\text{J},\text{B})$$

For pressure

$$\text{div}(\text{grad}(P)) \left\{ -\frac{1}{c_B^2} \frac{\partial^2 P}{\partial t^2} \right\} = \text{div}(\vec{j} \times \vec{B}) - \rho \cdot \text{div} \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) + C \eta \cdot \text{div}(\vec{v}) \left\{ -\Gamma \cdot \ddot{Q}(\vec{r}, t) \right\}$$

Not used in this modeling

FlexPDE:

$$\text{p: div}(\text{grad}(\text{p})) = \text{density}*\text{div}(\text{dt}(\text{v})+\text{dot}(\text{v},\text{grad}(\text{v}))) + \text{div}(\text{cross}(\text{J},\text{B})) + 1\text{e}5*\text{visc}/\text{r}^2*\text{div}(\text{v}) + \text{visc}/3*\text{div}(\text{grad}(\text{div}(\text{v}))) \{ +1/\text{c}02*\text{dt}(\text{dpt}) - \text{G}/\text{c}02*\text{Qdott} \}$$

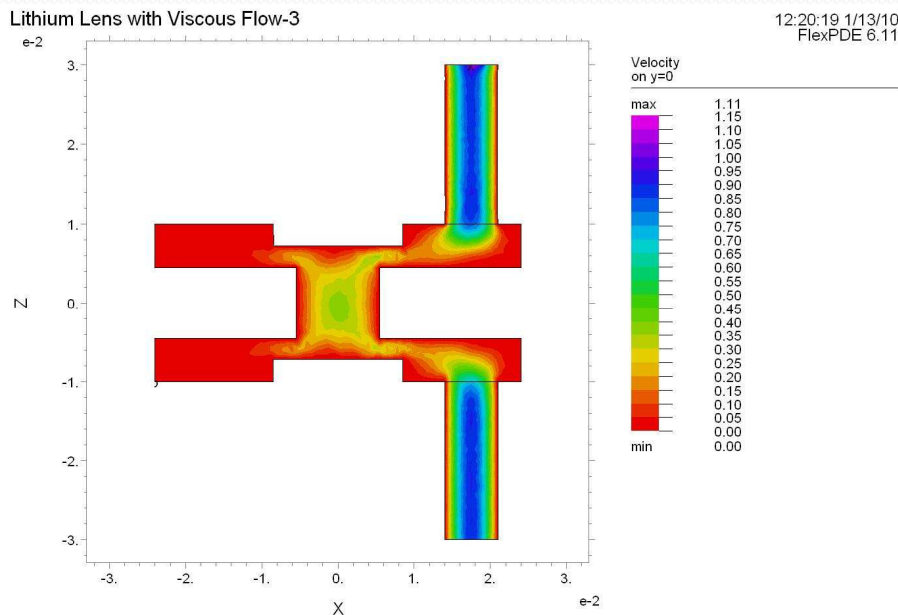
Velocity

Pressure rises smoothly:

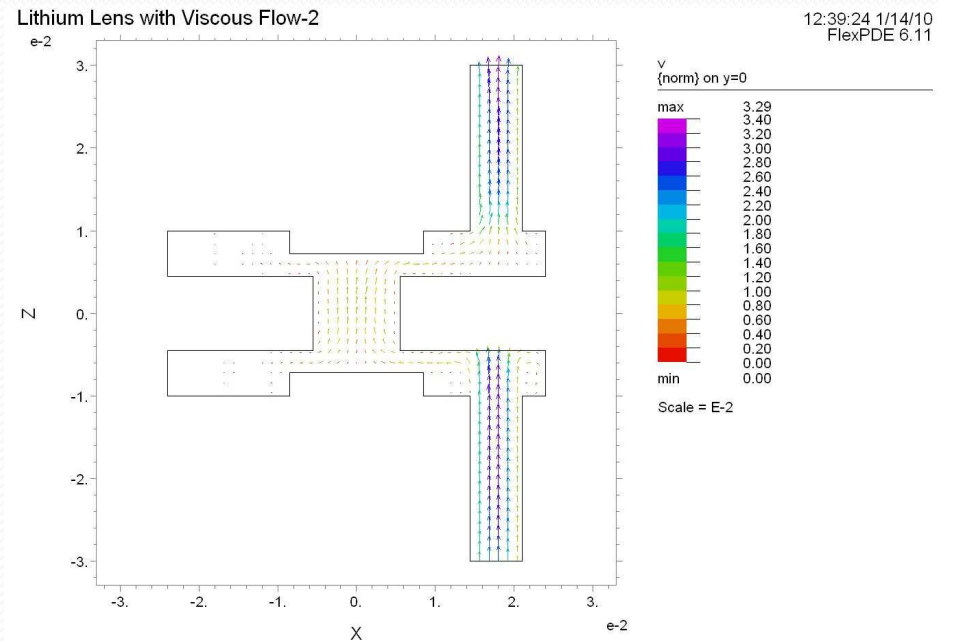
$$p_{out} = \text{IF } t < \tau/10 \text{ THEN } p_{in} - \text{delp} * 10 * (t/\tau) \text{ else } p_{in} - \text{delp}$$

$$\tau \equiv \tau = 2.3 \text{ms}$$

Pressure drop



Viscose flow Jan 12 2010: Cycle=13 Time= 2.3000e-4 dt= 2.5563e-5 P2 Nodes=8170 Cells=4645 RMS Err= 0.0551
Reynolds number max= 12445.39 Integral= 2.309626e-4



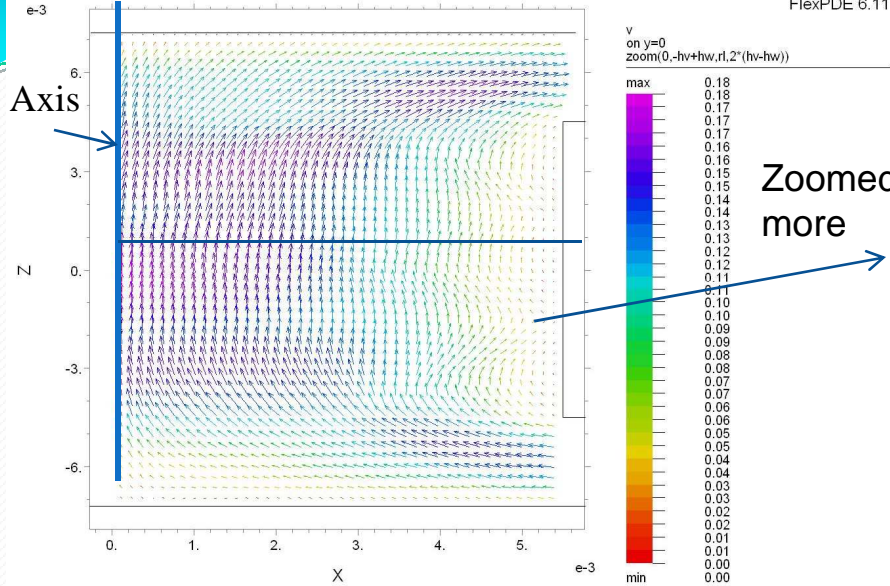
just Viscose flow 2: Grid#2 P2 Nodes=7723 Cells=4357 RMS Err= 0.3994
Stage 2

Contour of Lithium velocity module;
no electrical current yet.

Vectors of velocity; no electrical
current yet.

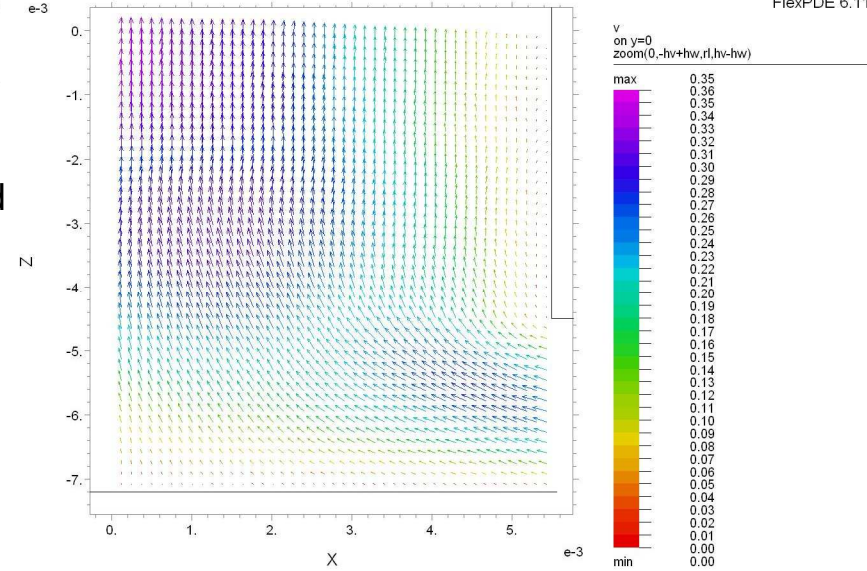
Without current

Lithium Lens with Viscous Flow-3



Viscose flow Jan 12 2010: Cycle=2 Time= 5.7500e-5 dt= 2.8750e-5 P2 Nodes=18469 Cells=12705 RMS Err= 0.0427
Reynolds number max= 34173.52

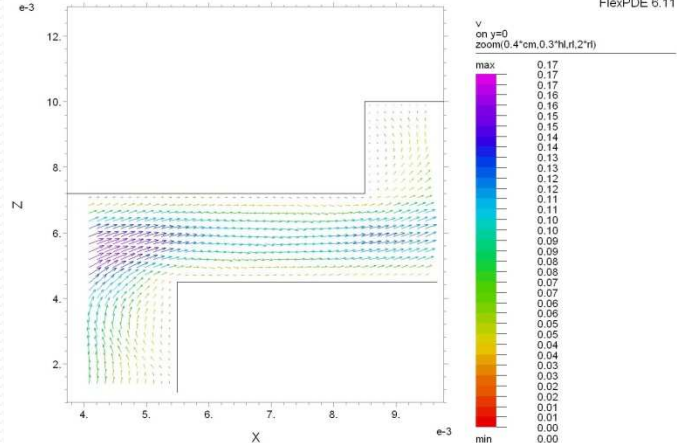
Lithium Lens with Viscous Flow-3



Viscose flow Jan 4 2010: Cycle=2640 Time= 0.0758 dt= 2.5563e-5 P2 Nodes=17741 Cells=12198 RMS Err= 0.0615
Reynolds number max= 0.053353

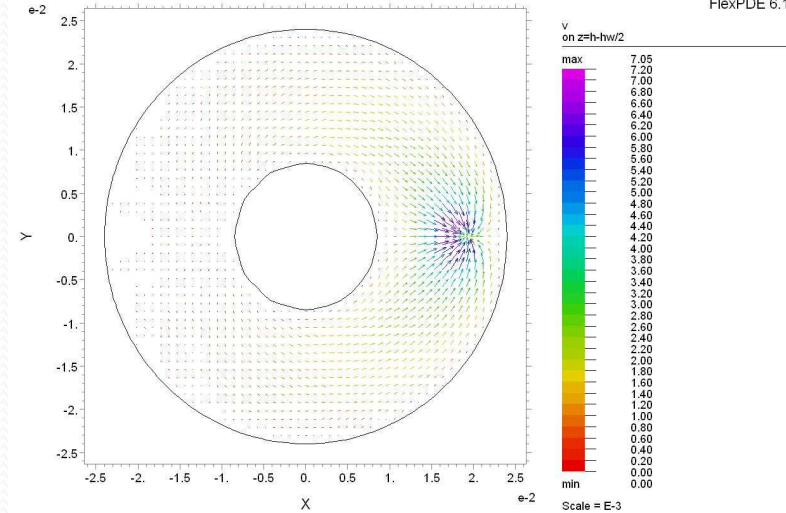
Vectors of velocity zoomed

Lithium Lens with Viscous Flow-3



Viscose flow Jan 12 2010: Cycle=2 Time= 5.7500e-5 dt= 2.8750e-5 P2 Nodes=18469 Cells=12705 RMS Err= 0.0427
Reynolds number max= 34173.52

Lithium Lens with Viscous Flow-2



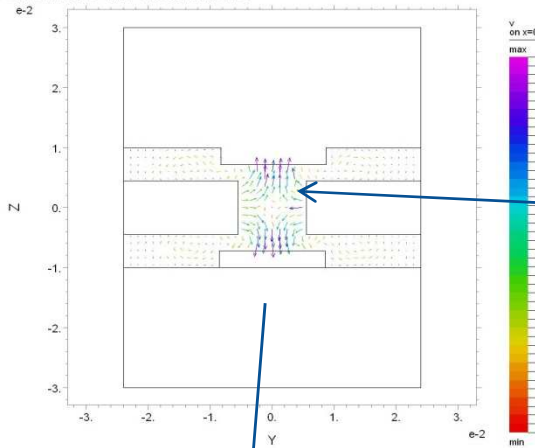
just Viscose flow 2: Cycle=30 Time= 1.1500e-3 dt= 4.0900e-5 P2 Nodes=8745 Cells=4959 RMS Err= 5.8e-4
Re= 2353.704

Vectors of velocity in transition volume

Vectors of velocity just below outlet tube

With current

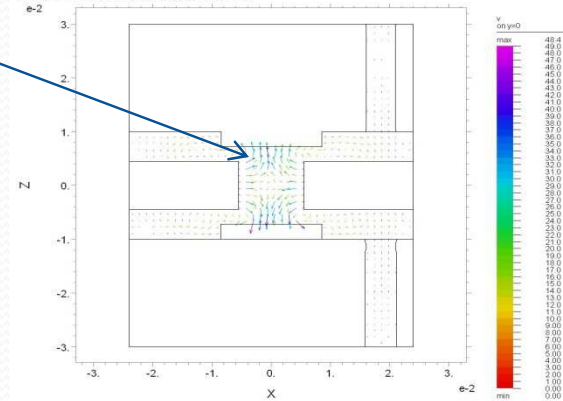
Lithium Lens with Viscous Flow-3



12:10:50 1/25/10
FlexPDE 6.11

Vortex motion is clearly seen here

Lithium Lens with Viscous Flow-3

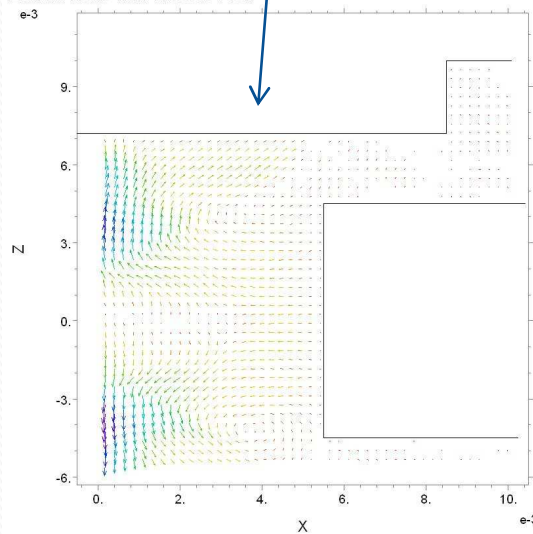


12:10:50 1/25/10
FlexPDE 6.11

Viscose flow Jan 28 2010: Cycle=205 Time= 2.3000e-3 dt= 1.0225e-5 P2 Nodes=39225 Cells=27733 RMS Err= 0.0609
Reynolds number max= 829.9611

Viscose flow Jan 28 2010: Cycle=205 Time= 2.3000e-3 dt= 1.0225e-5 P2 Nodes=39225 Cells=27733 RMS Err= 0.0609
Reynolds number max= 829.9611

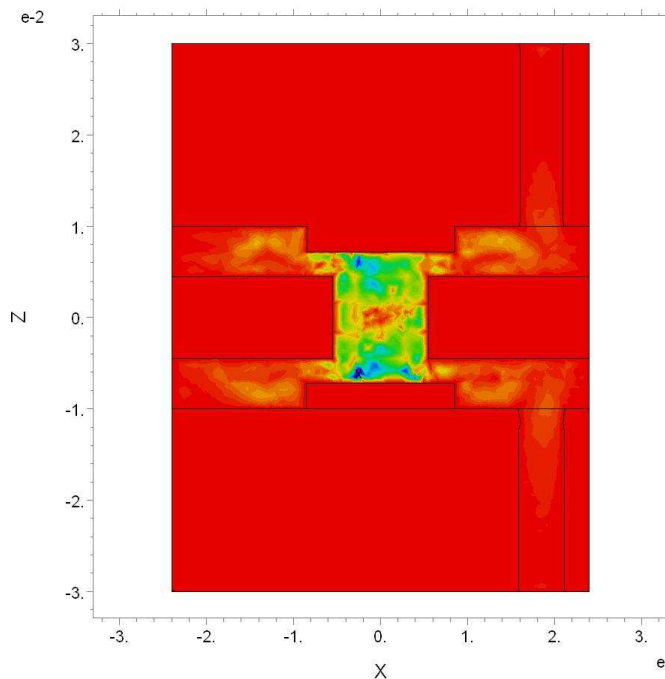
Lithium Lens with Viscous Flow-3



11:22:56
FlexPI

Viscose flow Jan 15 2010: Cycle=14 Time= 2.5875e-4 dt= 2.5563e-5 P2 Nodes=19316 Cells=13339 RMS Err
Reynolds number max= 6655.362

Lithium Lens with Viscous Flow-3



12:10:50 1/25/10
FlexPDE 6.11

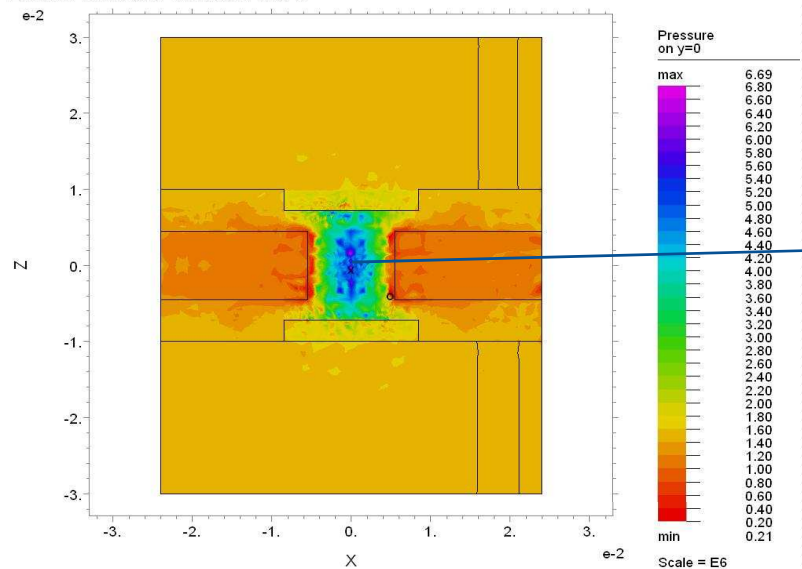
Velocity module painted

Viscose flow Jan 28 2010: Cycle=205 Time= 2.3000e-3 dt= 1.0225e-5 P2 Nodes=39225 Cells=27733 RMS Err= 0.0609
Reynolds number max= 829.9611 Integral= 5.512972e-3

Zoomed

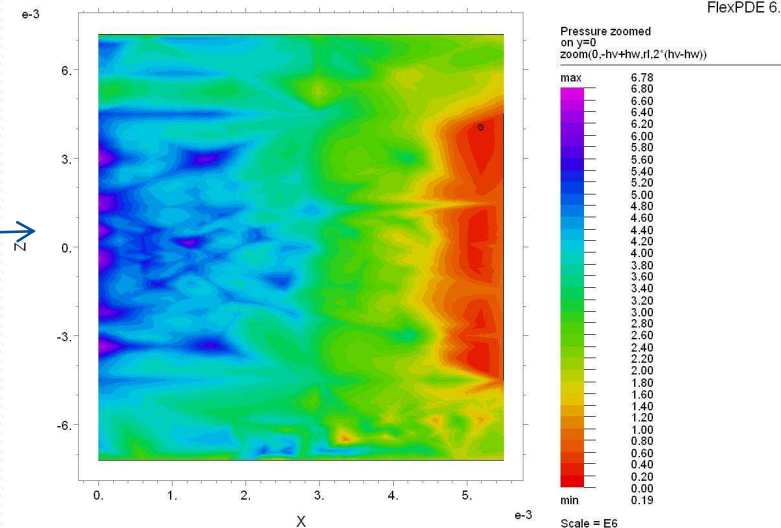
Pressure

Lithium Lens with Viscous Flow-3



Zoomed

Lithium Lens with Viscous Flow-3

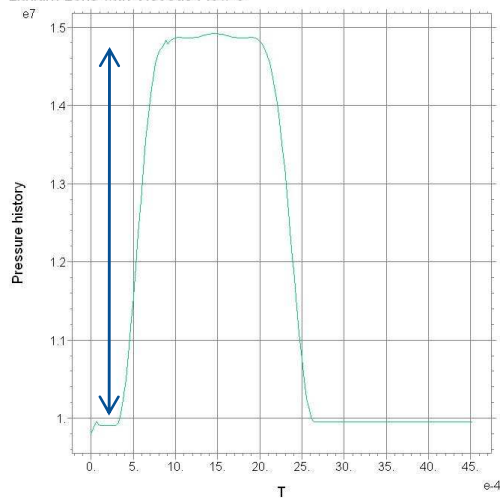


18:43:47 2/1/10
FlexPDE 6.11

Viscose flow Jan 28 2010: Cycle=135 Time= 1.4950e-3 dt= 1.0225e-5 P2 Nodes=55609 Cells=39554
Integral= 4451.796

Viscose flow Feb 2 2010: Cycle=165 Time= 1.8400e-3 dt= 1.0780e-5 P2 Nodes=73398 Cells=52442 RMS Err= 0.0304
Reynolds number max= 1635.005 Integral= 278.4650

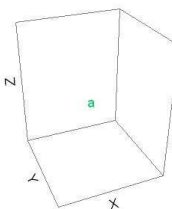
Lithium Lens with Viscous Flow-3



20:38:19 1/4/10
FlexPDE 6.11

HISTORY

1: p



History of pressure at central point.

Viscose flow Jan 4 2010: Cycle=159 Time= 4.5237e-3 dt= 2.6118e-5 P2 Nodes=17741 Cells=12198 RMS Err= 0.061

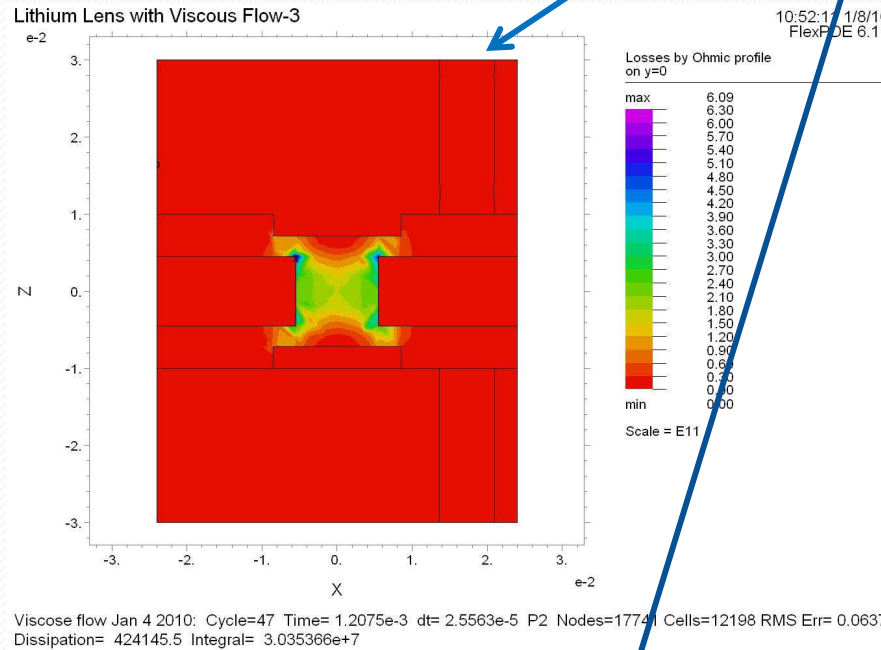
Jump $\sim 5 \times 10^6 \text{ Pa}$ $\sim 50 \text{ atm}$

Temperature:

$$\rho \cdot C_p \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \text{grad}(T) \right) - \text{div}(k \cdot \text{grad}(T)) + P \cdot \text{div}(\vec{v}) = (\vec{j} \cdot \vec{E}) + \sigma'_{ik} \frac{\partial v_i}{\partial x_k} \{ + \dot{Q}(\vec{r}, t) \}$$

Beam; not used here

Ohmic losses.



$$\text{phi} = \sigma'_{ij} \frac{\partial v_i}{\partial x_j} = \eta \frac{\partial v_i}{\partial x_j} \left[\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial v_k}{\partial x_k} \right) + \zeta \cdot \delta_{ij} \frac{\partial v_k}{\partial x_k} \right] =$$

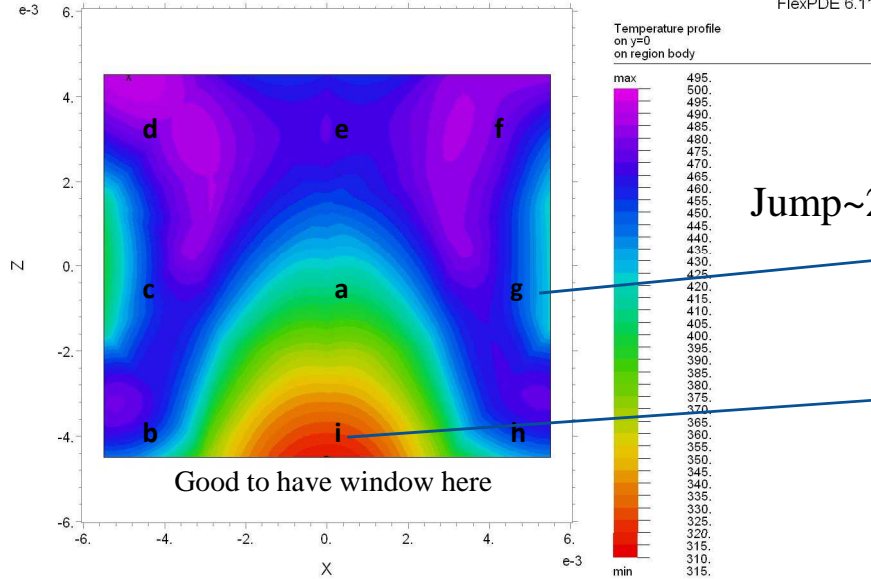
Not used in this run

$$2\eta \left[\left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 + \frac{1}{2} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 + \frac{1}{2} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)^2 + \frac{1}{2} \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right)^2 + \{ \zeta \cdot (\text{div}(\vec{v}))^2 \} \right]$$

Temperature history and profile, when only inlet/outlet temperature is fixed

Lithium Lens with Viscous Flow-3

10:15:09 1/7/10
FlexPDE 6.11

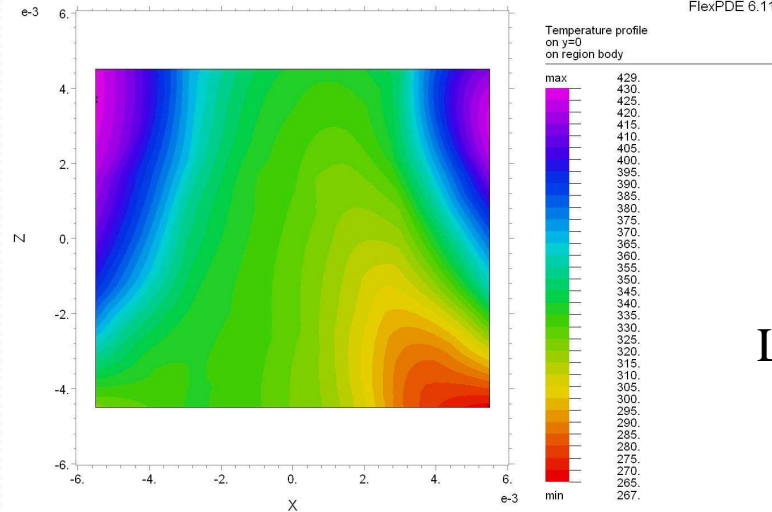


Viscose flow Jan 4 2010: Cycle=405 Time= 0.0115 dt= 2.5563e-5 P2 Nodes=17741 Cells=12198 RMS Err= 0.0615
Dissipation= 1.009784e-19 Integral= 0.043638

Central region of lens

Lithium Lens with Viscous Flow-3

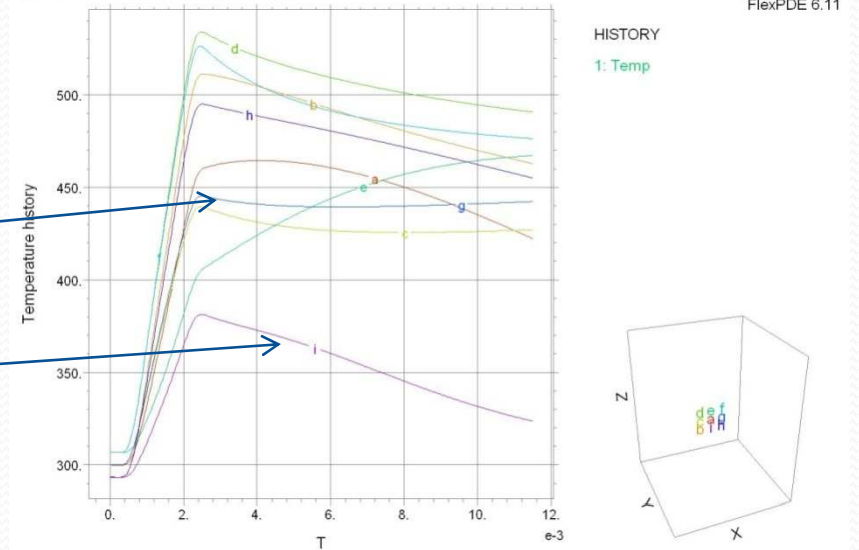
18:54:40 1/9/10
FlexPDE 6.11



Viscose flow Jan 4 2010: Cycle=2664 Time= 0.0764 dt= 2.5563e-5 P2 Nodes=17741 Cells=12198 RMS Err= 0.0615
Dissipation= 1.079542e-151 Integral= 0.033990

Lithium Lens with Viscous Flow-3

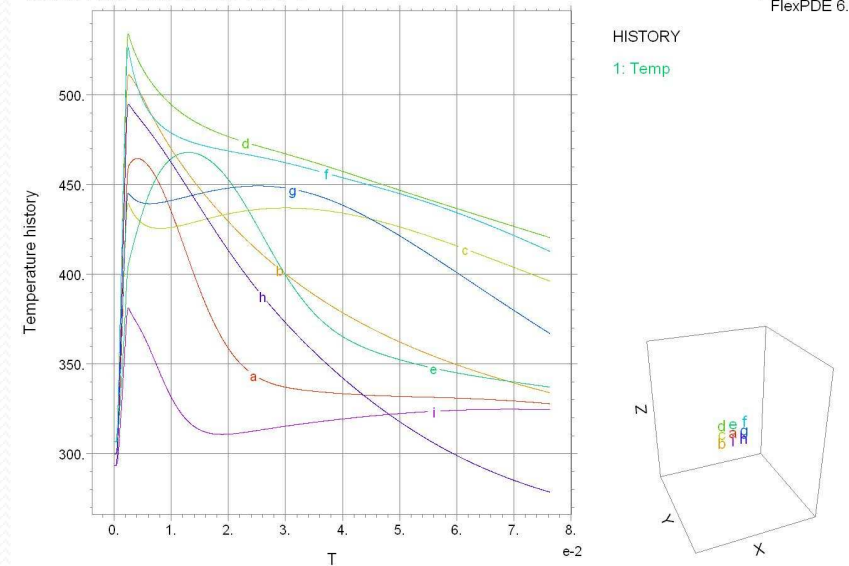
10:15:09 1/7/10
FlexPDE 6.11



Viscose flow Jan 4 2010: Cycle=405 Time= 0.0115 dt= 2.5563e-5 P2 Nodes=17741 Cells=12198 RMS Err= 0.0615

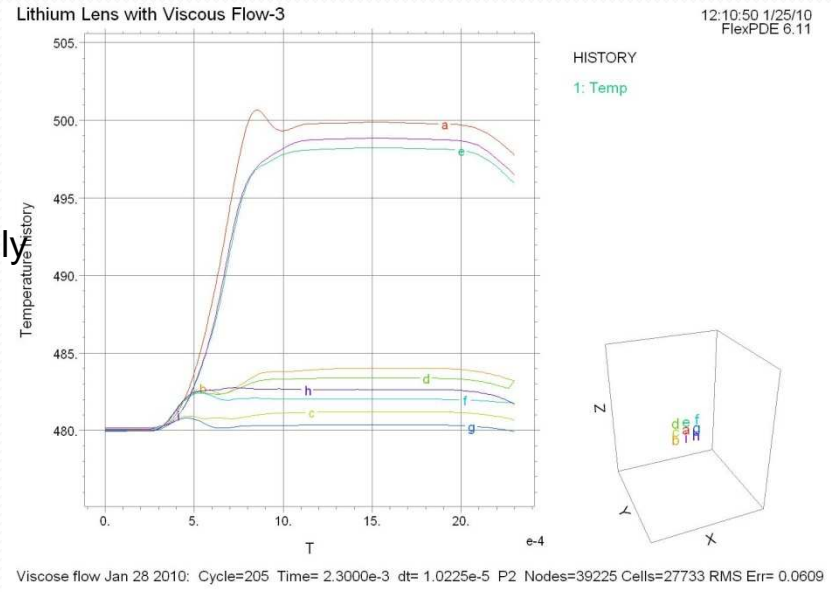
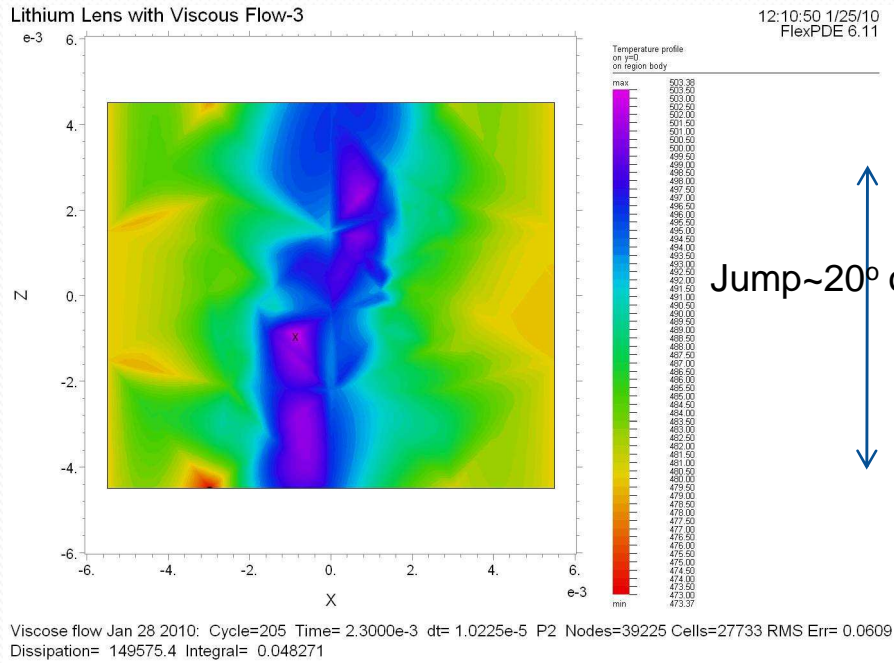
Lithium Lens with Viscous Flow-3

18:54:40 1/9/10
FlexPDE 6.11



Viscose flow Jan 4 2010: Cycle=2662 Time= 0.0764 dt= 2.5563e-5 P2 Nodes=17741 Cells=12198 RMS Err= 0.0615

Temperature history and profile, when **all** walls kept at the same temperature 480°K



Hot fractions located in central region

Temperature history

So all Lithium case is keeping at fixed temperature, the temperature jump of Lithium is minimal.

SUMMARY

Vortex circulation of liquid Lithium due to magnetic force helps in reduction of temperature in central region of windows, thanks to forced mixing.

Calculated pressure and temperature profiles confirmed that parameters of lens are technically reasonable. Generally, the temperature rise is below 50°C coming to a total of ~100 °C with beam as the beam duty of train is ~1 msec. So far there is no indication of cavitation.

We recommend increase of length of space occupied by Lithium (in direction along the beam pass) to ~1 *cm* (from ~0.6cm). This brings reduction of current to ~70 kA.

We also slightly modified the configuration of junction between Lithium cylindrical container and the buffer toroid-like volume. This improved linearity of integrated field as function of transverse coordinate.

Numerical code developed allows further investigation of Lithium lens dynamics and easy modification of geometry.

Stress-strain in windows, Shock waves and Cavitations introduced by the beam will be investigated in future, while funds allow.

Usage of FlexPDE computer code looks adequate here.

Best test for liquid Lithium lens is fabrication of full scale prototype:

Tests with full scale prototype if is were to built:

1. Test of Lithium pump (electromagnetic vs mechanical pump)
2. Best conditions for Lithium flow (pressure, pressure drop)
3. Thermal conditions (insulation, cooling, temperature control)
4. Starting procedure (warming, pumping start)
5. Full scale Power Supply (single thyristor, duct; *this PS is good for Flux concentrator*)
6. Operation without beam (full current, repetition rate)
7. Test with beam (electron/positron/proton beam)

Just this one



Reverse Switched Diodes (RSD) for peak current from 200 kA to 500 kA and blocking voltage of 2400 V, encapsulated in hermetic metal-ceramic housing and without housing (RSD sizes of 64, 76 and 100 mm)

