

# Kalman filter based processor for track reconstruction in MarlinTPC

– Kalman filter, structure, preliminary results –

Bo LI(Tsinghua U.)

Keisuke FUJII(KEK)

Yuanning GAO(Tsinghua U.)

Katsumasa IKEMATSU(KEK)

Yulan LI(Tsinghua U.)

Yukihiro KATO(Kinki U.)

# Track reconstruction and Kalman filter

Track reconstruction usually includes two parts:

- Track finding
  - Hough transform
  - Neural network
  - Track following(track model, track seed, hit selection criterion)
  - ...
- Track fitting
  - Least squares estimation
  - Maximum likelihood estimation

## Kalman filter

Invented by R.E. Kalman in 1960, and introduced in the HEP community in 1980's.

The motivation of using Kalman filter for TPC tracking:

- Relatively easily deal with inhomogeneous magnetic field(crucial requirement now!), multiple scattering, and energy loss;
- Carry out track finding and track fitting simultaneously;
- The computation time is proportional to site number, which makes computation expense less.

# Kalman filter algorithm for tracking

System equation:

$$\mathbf{x}_{k+1} = \phi_k \mathbf{x}_k + \mathbf{w}_k$$

Meas. equation:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

$\mathbf{x}$ : state vector  
 $\phi$ : transfer matrix  
 $\mathbf{z}$ : measurement  
 $\mathbf{H}$ : projection matrix  
 $\mathbf{K}$ : gain matrix  
 $\mathbf{P}/\mathbf{Q}$ : covariance of  $\mathbf{w}/\mathbf{v}$

$\delta\chi^2$  is reasonable, accept this hit



$\delta\chi^2$  is big, abandon this hit

- Kalman filter accumulates hit information and calculates the track parameters;
- Kalman filter prediction provides a good criterion for hit selection.

$\hat{\mathbf{x}}_0^-, P_0^-$

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

$$\begin{aligned} \hat{\mathbf{x}}_{k+1}^- &= \phi_k \hat{\mathbf{x}}_k \\ \mathbf{P}_{k+1}^- &= \phi_k \mathbf{P}_k \phi_k^T + \mathbf{Q}_k \end{aligned}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-)$$

$$\mathbf{P}_k = (1 - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-$$

Prediction

Update

# Track models

See KalTest reference by K. Fujii

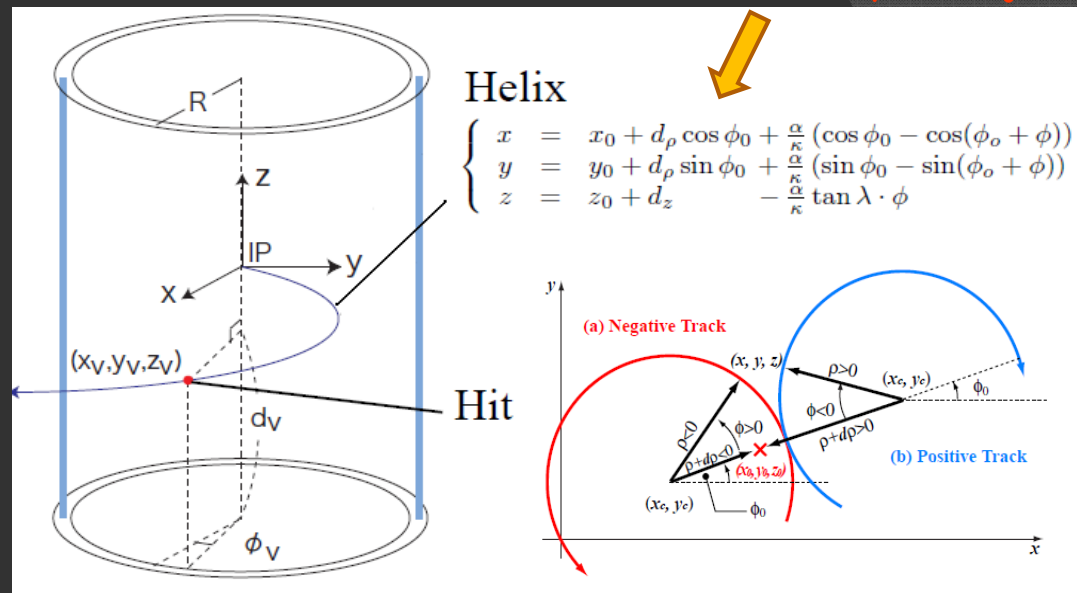
2D straight track  $y = ax + b$

state vector  $X = \begin{bmatrix} a \\ b \end{bmatrix}$

transfer matrix  $\phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

projection matrix  $H = \begin{bmatrix} x & 1 \end{bmatrix}$

- Despite of different track model, the track following method is nearly the same for straight line and helix.
- For helix model, the Kalman filter is not linear, it is usually called as extended Kalman filter.



- $d_\rho$ : the distance between the helix and the hit in x-y plane
- $\phi_0$ : the azimuthal angle of the hit with respect to the helix center
- $\kappa$ :  $Q/P_t$
- $d_z$ : the distance between the helix and the hit in the z direction
- $\tan \lambda$ : the angle of the helix to the x-y plane

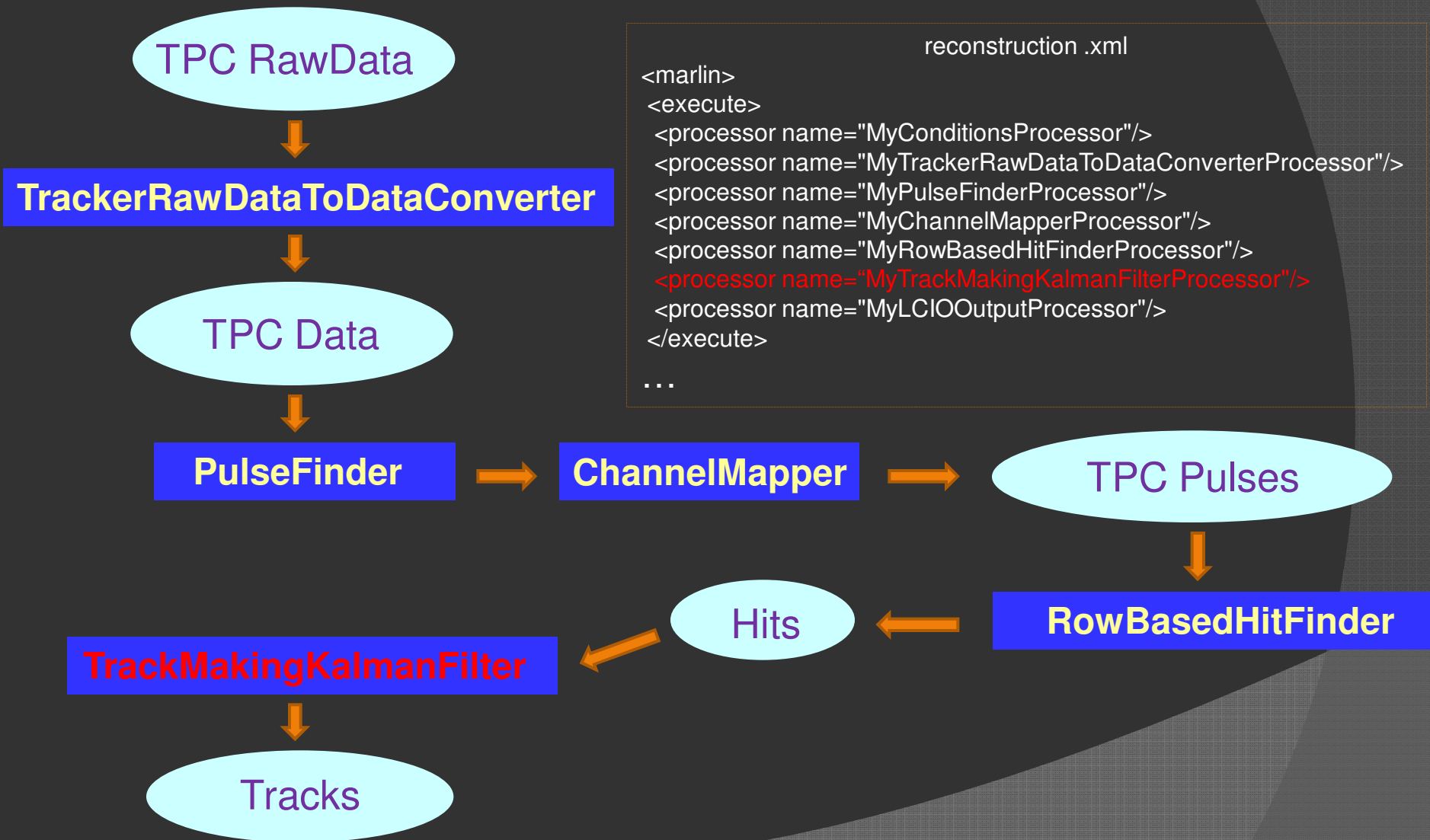
state vector  $X_k = (d_\rho, \phi_0, \kappa, d_z, \tan \lambda)^T$

transfer matrix  $F_k = \left( \frac{\partial X_k}{\partial X_{k-1}} \right)$

# KalTest

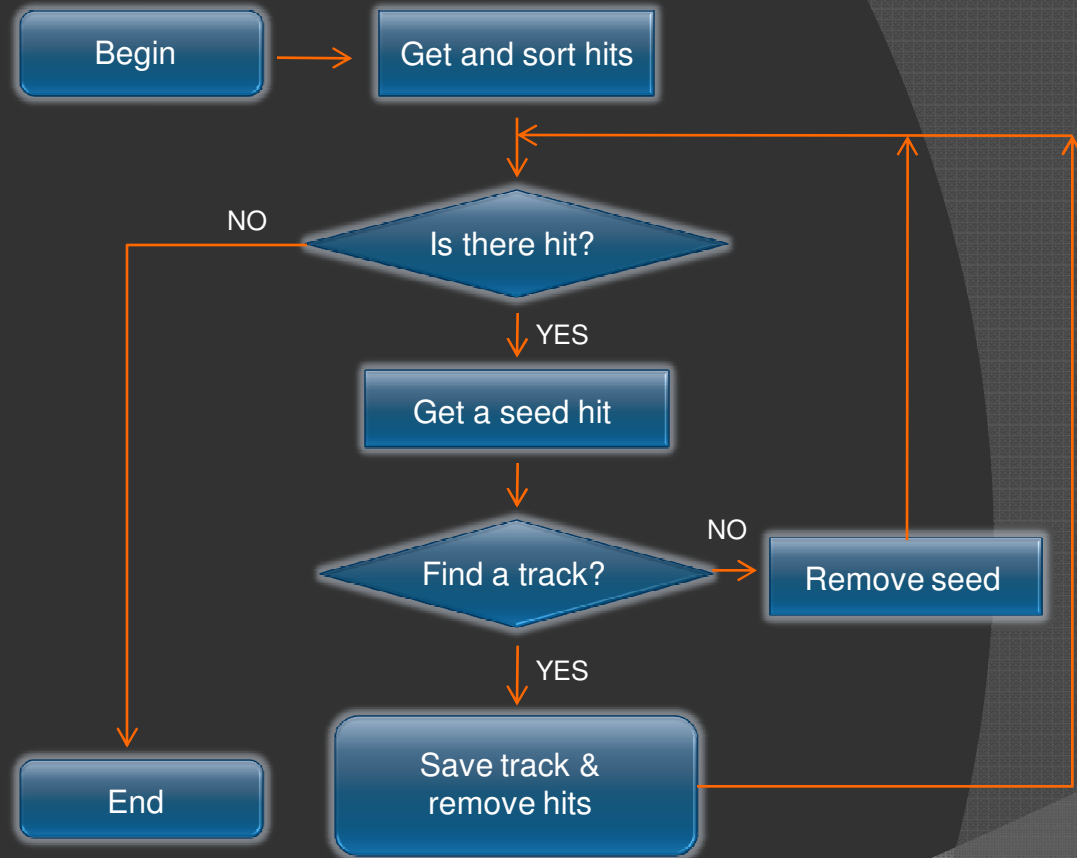
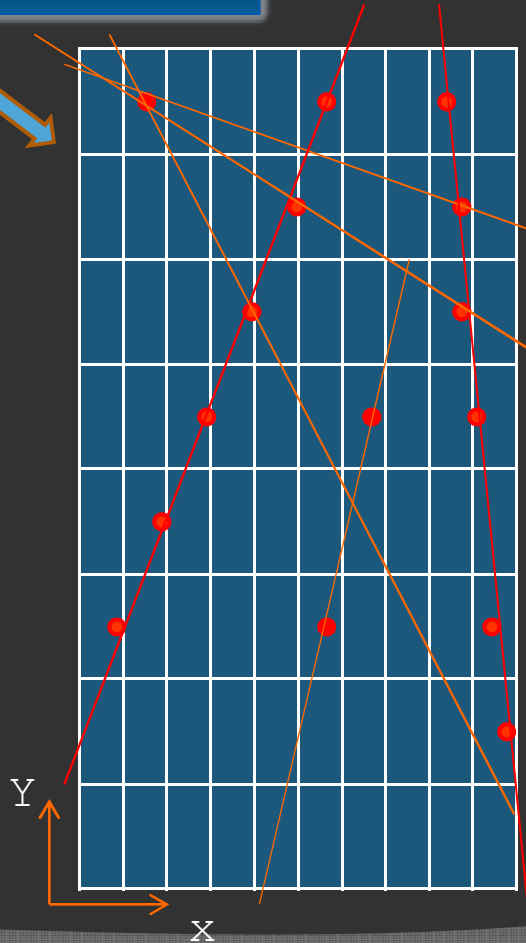
- KalTest is a Kalman filter package written in C++. It provides basic libraries for track fitting with Kalman filter technique.
- KalTest contains 3 subpackages:
  - Kallib: implements the generic algorithm of the Kalman filter
  - KalTrackLib: inherits from the generic base classes in KalLib, and implements their pure virtual methods for track fitting
  - GeomLib: provides track model and detector surfaces
- To get KalTest package and more information about helix track model, please visit KalTest homepage:  
<http://www-jlc.kek.jp/subg/offl/kaltest/>

# Reconstruction chain in MarlinTPC



# Tracking process

Track criteria  
MaxSkipRows = 1  
MinTrackHits = 5

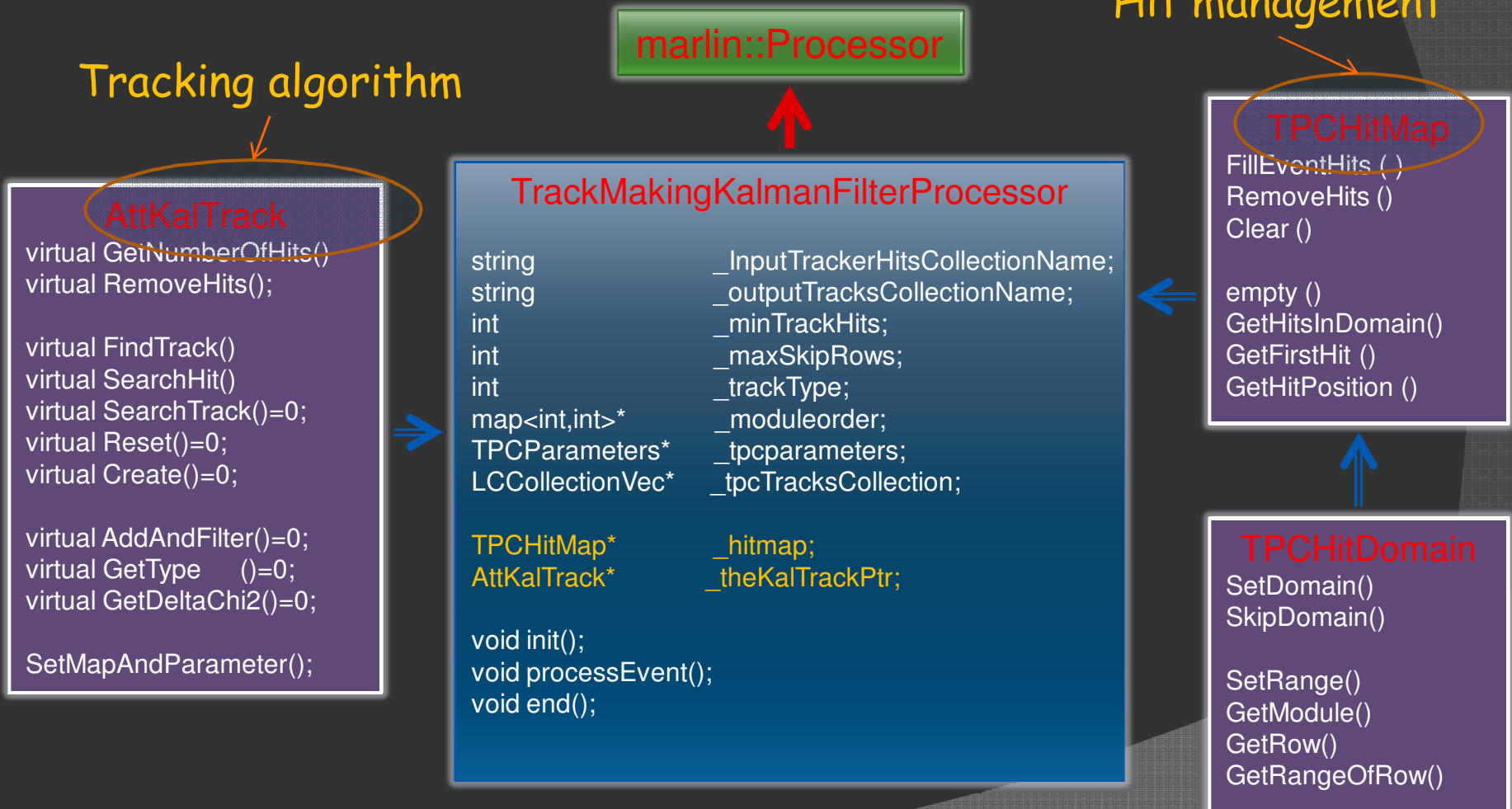


If Process Oriented → Object Oriented,  
the whole work will be distributed to different  
objects, making the code flexible.

# The class organization in processor

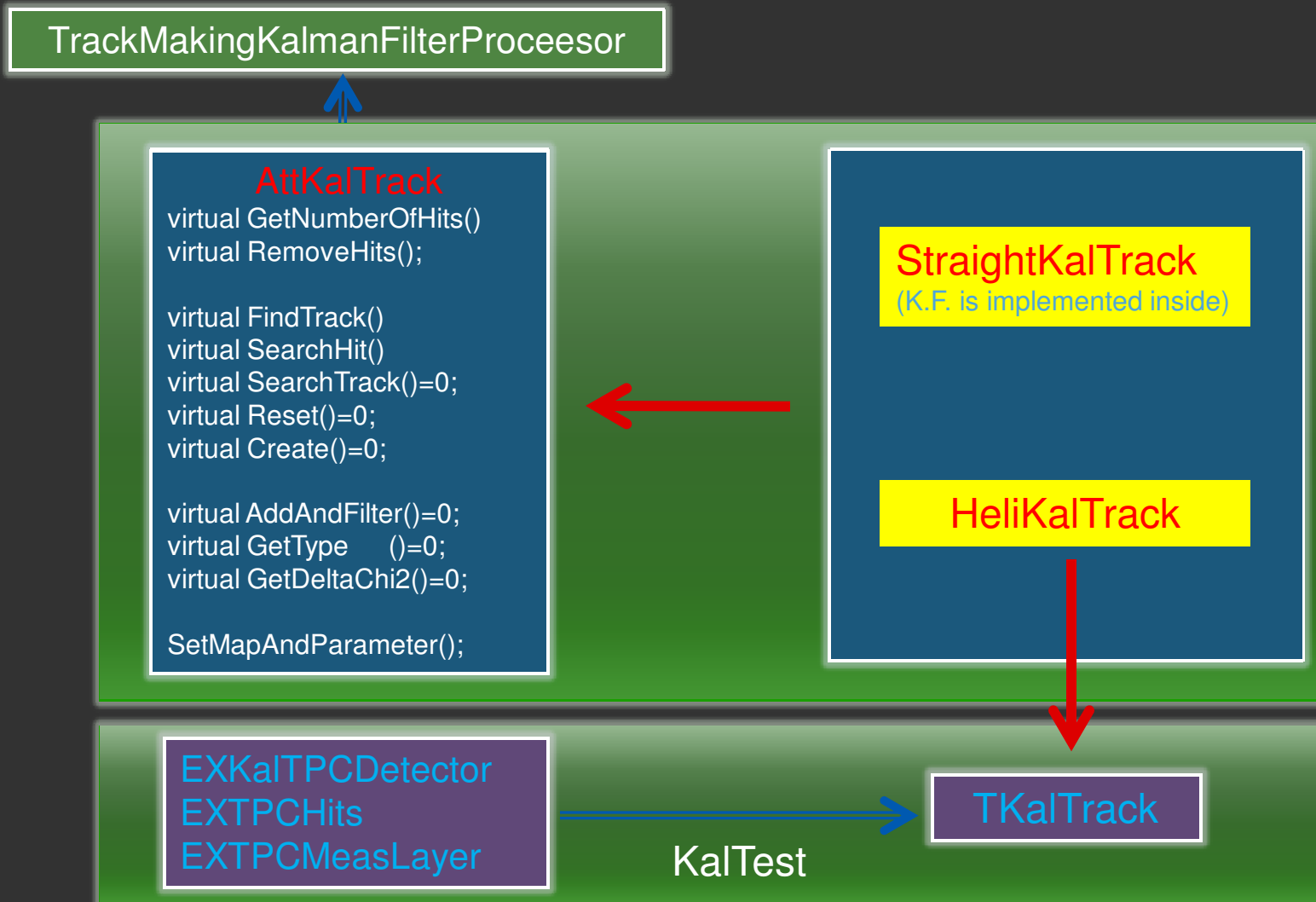
## Tracking algorithm

## Hit management





# Track classes



# Implementation in the processor

Then, the code in the processor becomes very neat:

```
void TrackMakingKalmanFilterProcessor::init()
{
    const TPCParameters* tpcparameters = & Global::GEAR->getTPCParameters();
    _moduleorder=new map<int,int>;

    _hitmap=new TPCHitMap(tpcparameters, _InputTrackerHitsCollectionName, _moduleorder);
    _theKalTrackPtr = new StraightKalTrack;
    _theKalTrackPtr->SetMapAndParameter(_tpcparameters, _hitmap);
}

void TrackMakingKalmanFilterProcessor::processEvent(LCEvent *evt)
{
    _tpcTracksCollection = new LCCollectionVec(LCIO::TRACK);

    _hitmap->FillEventHits(evt);

    while(!_hitmap->empty())
        if( TrackImpl* atrack=_theKalTrackPtr->FindTrack(_hitmap->GetFirstHit()) )
            _tpcTracksCollection->addElement(atrack);

    if(int tracknum = _tpcTracksCollection->getNumberOfElements())
        evt->addCollection( _tpcTracksCollection, _outputTracksCollectionName );
}

void TrackMakingKalmanFilterProcessor::end()
{
    delete _theKalTrackPtr;
    delete _hitmap;

    cout << "End of TrackMakingKalmanFilterProcessor" << endl;
}
```

# Configuration of processor

This is an configuration file of TrackMakingKalmanFilterProcessor:

```
<processor name="MyTrackMakingKalmanFilterProcessor" type="TrackMakingKalmanFilterProcessor">
<!--For simple track finding and fitting by Kalman filter!-->
  <!--Name of the Input TrackerHits collection-->
  <parameter name="InputTrackerHits" type="string" lcioInType="TrackerHit"> TPCHits </parameter>
  <!--Name of the output Tracks collection-->
  <parameter name="OutputTracks" type="string" lcioOutType="Track"> TPCTracks </parameter>
  <!--Maximum number of subsequently missing hits (default: 1)-->
  <parameter name="MaxSkipRows" type="int" value="1" />
  <!--Minimum number of hits on track (default: 15)-->
  <parameter name="MinTrackHits" type="int" value="15" />
  <!--if not 0 the output hits collection is set transient (default: 0)-->
  <parameter name="SetOutputHitsTransient" type="int" value="0" />
  <!--if not 0 the output trackscollection is set transient (default: 0)-->
  <parameter name="SetOutputTrackCandidatesTransient" type="int" value="0" />
  <!--Track type (default: 1 0=straight line 1=helix)-->
  <parameter name="TrackType" type="int" value="0" />
</processor>
```

# Straight line reconstruction

```
<PadRowLayout2D
```

```
  type="RectangularPadRowLayout"  
  xMin="-32." xMax="32." yMin="-48.">  
    <row repeat="24" nPad="64" padHeight="3.8"  
    padWidth="0.8" rowHeight="4." />
```

```
</PadRowLayout2D>
```

Single module

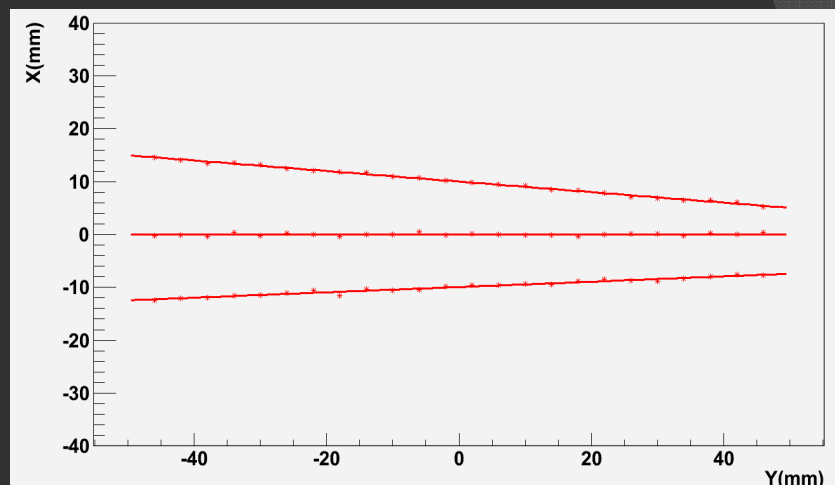
Pad Width: 0.8mm

Pad height: 3.8mm

Pad Pitch: 1mm

Row Height: 4mm

Hit position error: 0.2mm



Real

Slope	Intercept
0.05	-10.00
0.00	0.00
-0.01	10.00

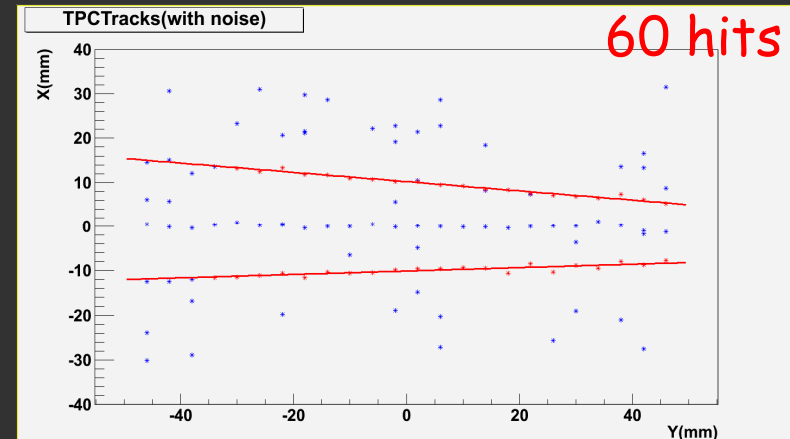
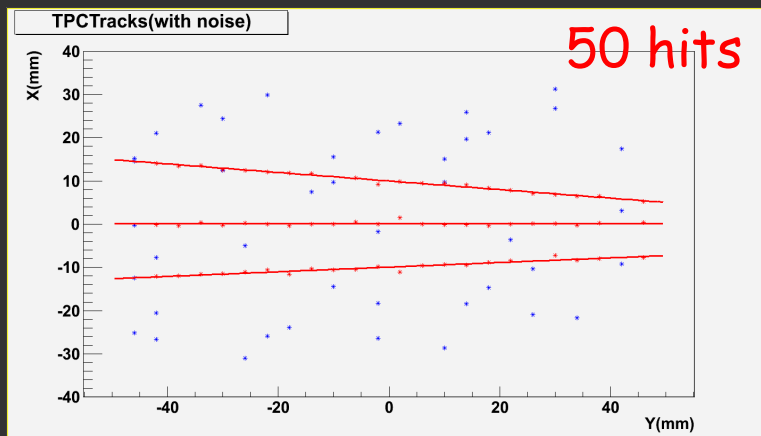
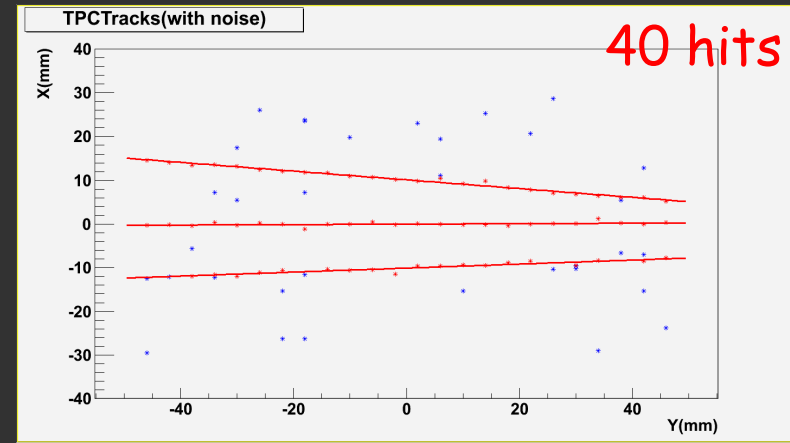
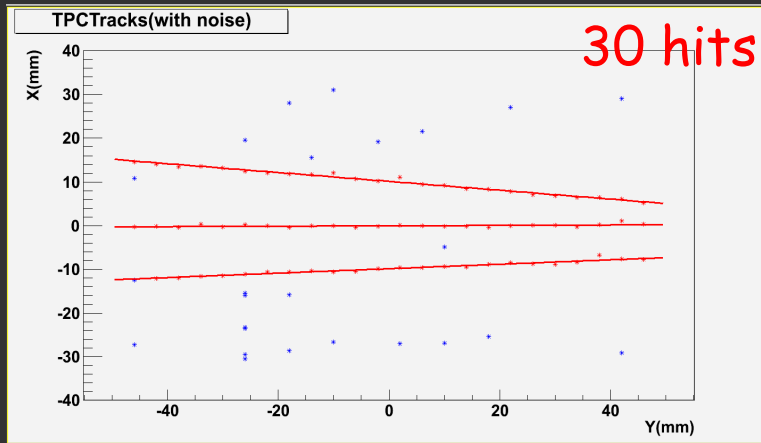


Reconstructed

Slope	Intercept
0.0502	-9.9354
0.0008	-0.0010
-0.0997	10.0189

# Straight line reconstruction(Cont.)

Noise hits are added into...

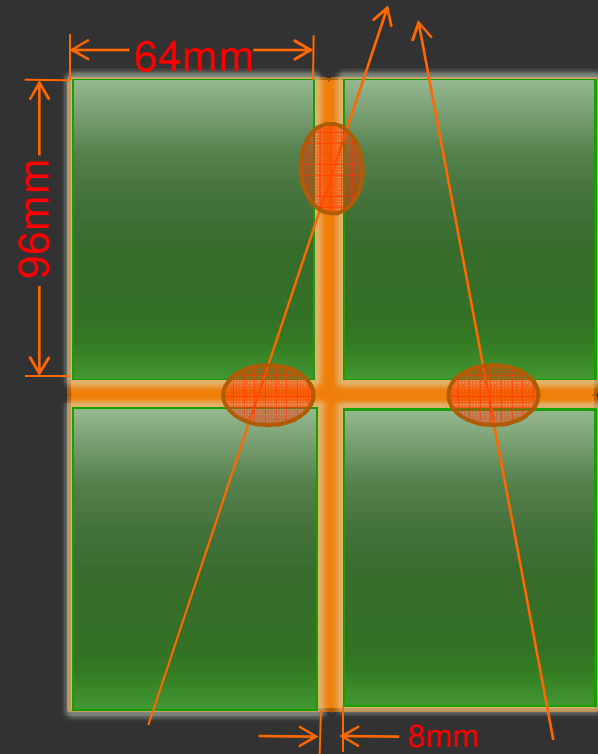


# Multi-module

```
<modules moduleIDStartCount="0">  
  <default>  
    <PadRowLayout2D type="RectangularPadRowLayout"  
      xMin="-32." xMax="32." yMin="-48.">  
      <row repeat="24" nPad="64" padHeight="3.8"  
        padWidth="0.8" rowHeight="4." />  
    </PadRowLayout2D>  
  </default>  
</modules>
```

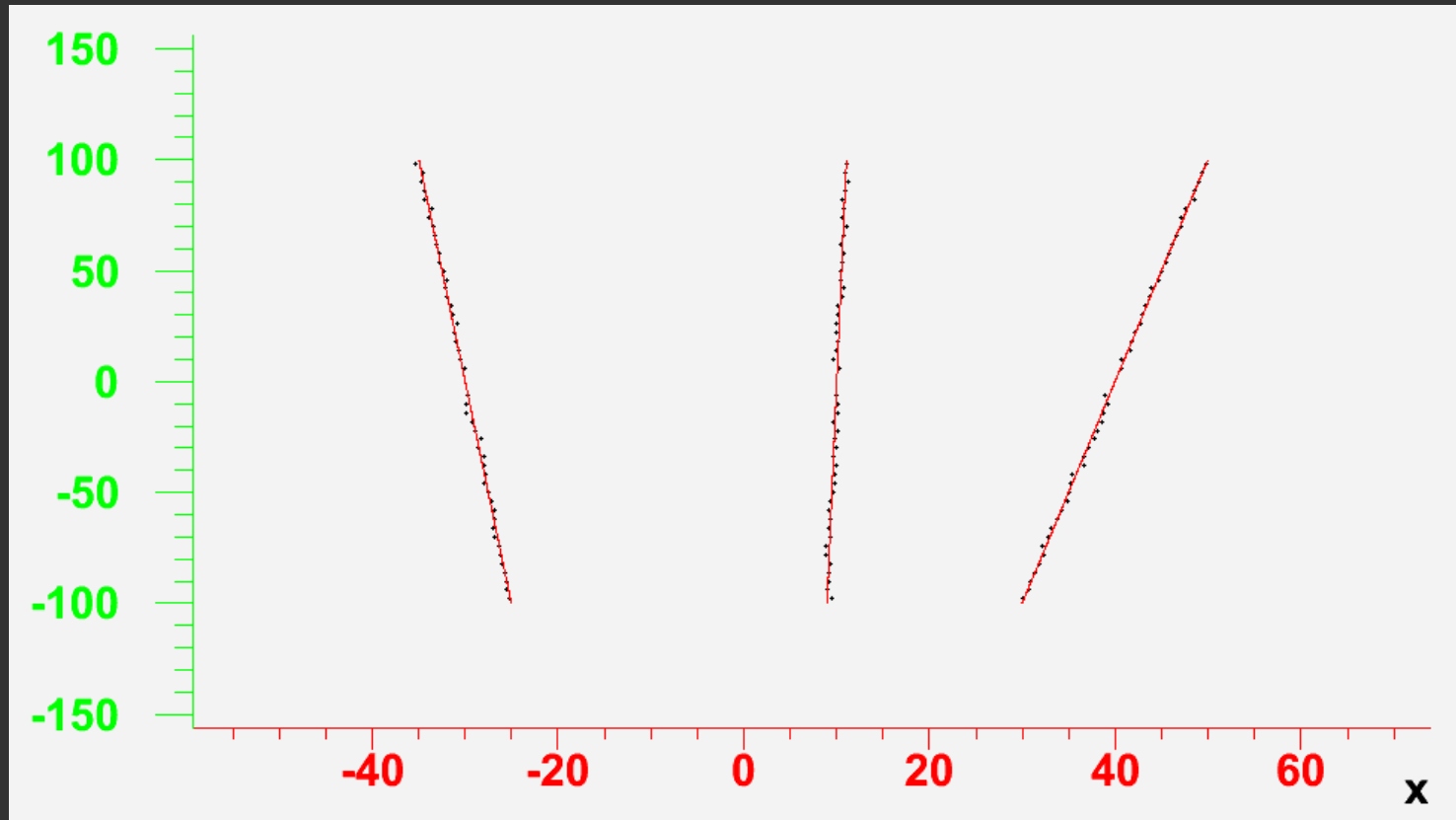
```
<module>  
  <offset x_r="36." y_phi="-52." />  
</module>  
  
<module>  
  <offset x_r="-36." y_phi="-52." />  
</module>  
  
<module>  
  <offset x_r="-36." y_phi="52." />  
</module>  
  
<module>  
  <offset x_r="36." y_phi="52." />  
</module>  
</modules>
```

4 modules

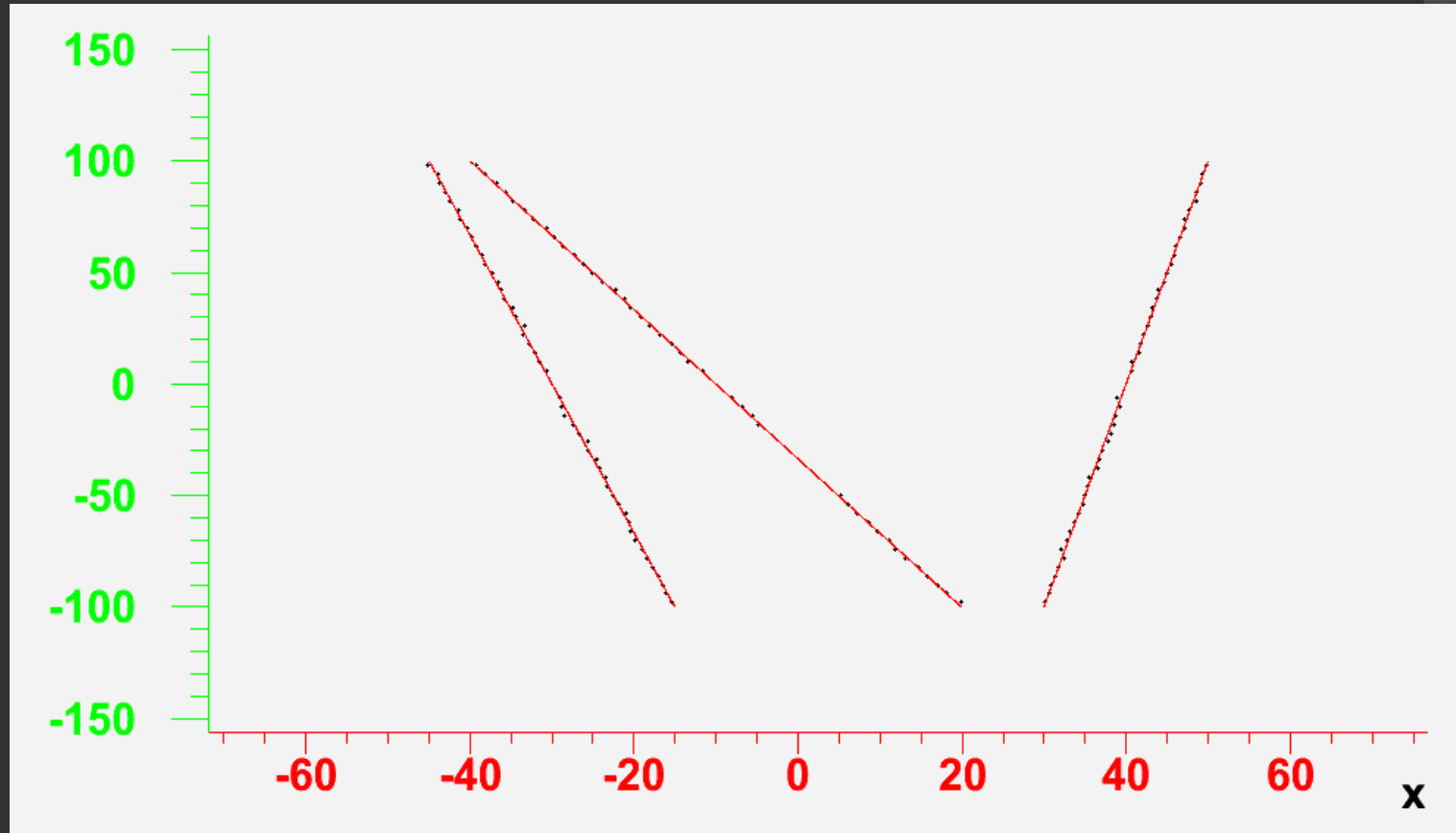


The main problem is how to cross boundaries and transfer modules. This work is done by TPCHitDomain.

# Multi-module result



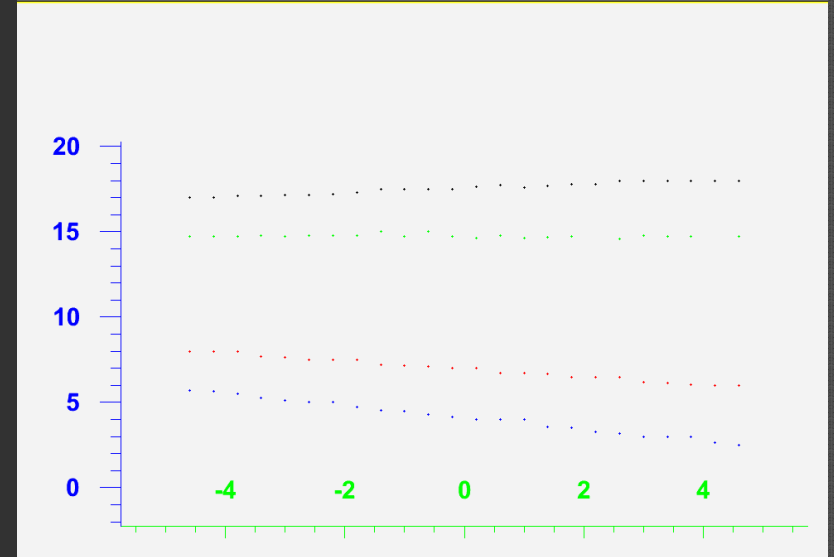
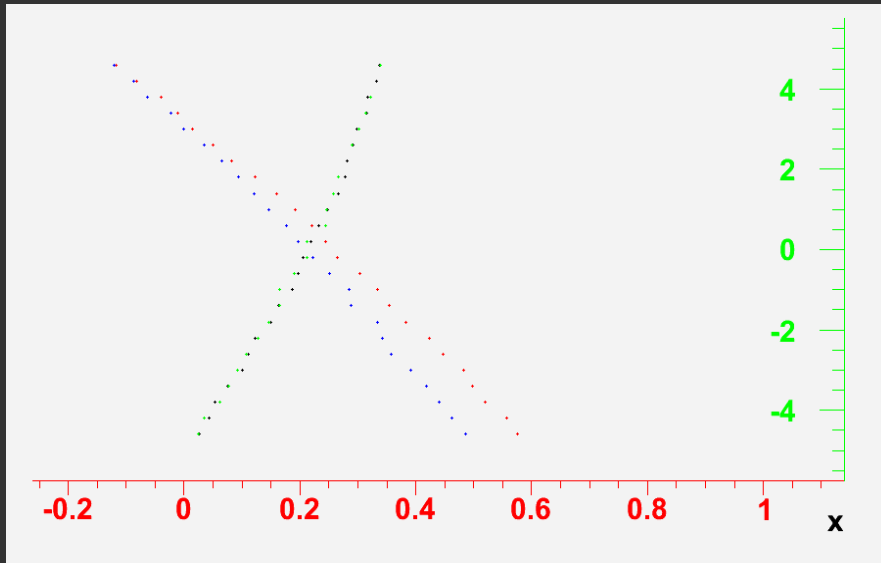
# Multi-module result(cont.)



- The result for straight line tracking is proper. 😊
- Framework for multi-module tracking also works. 😊



# Helix result



- Track finding result for helix is reasonable 😊
- But at present, the fitting result seems to be dependent on the track seed 😞, which need to be improved.

# Summary and plan

- ◉ Kalman filter based processor has been developed in MarlinTPC;
  - ◉ Processor was used to reconstruct straight line and helix with toy MC data;
  - ◉ Multi-module framework of processor has been setup.
- 
- ◉ Check and improve the helix track reconstruction result;
  - ◉ Implement multi-module helix reconstruction code, and apply it to test beam data;
  - ◉ In the long run, we can feed back to MarlinReco.

**Thanks for your attention!**