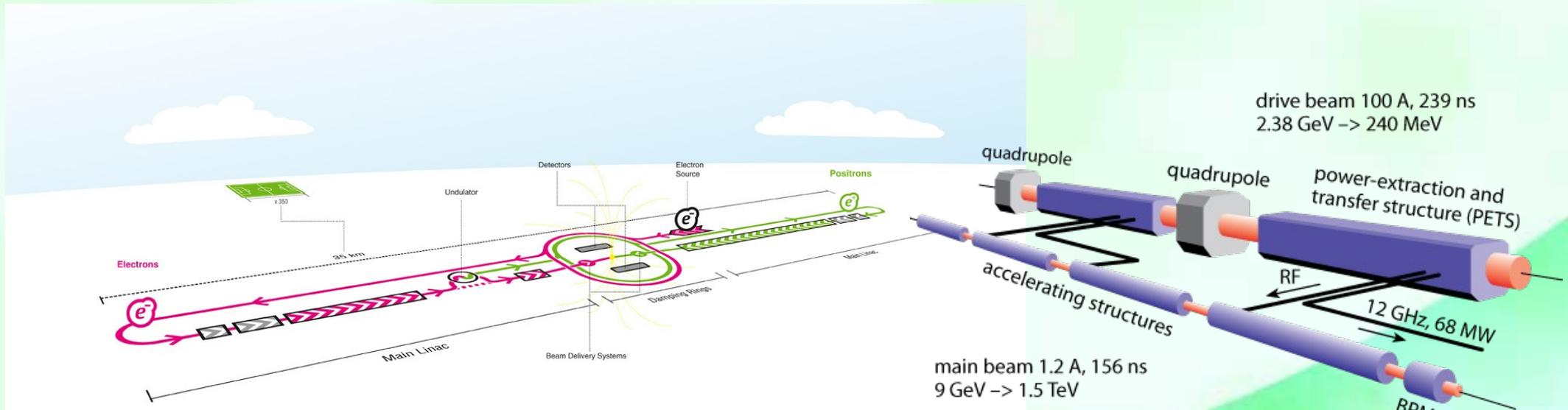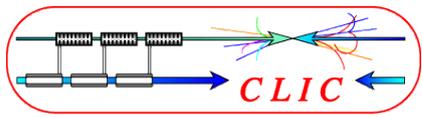# Electron source for Linear Colliders
KURIKI Masao (Hiroshima/KEK)
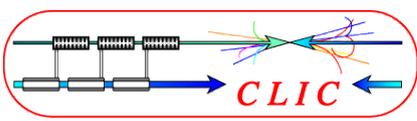
Contents

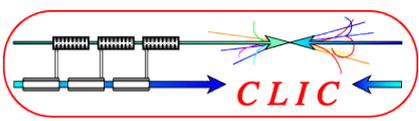►Introduction

►Electron Emission

►Related Physics Process

►Electron Gun

►Electron Source for Linear Colliders

►Laser

►Summary

Introduction

► What is the injector?

– Generate accelerate-able particle beams;

► What is the accelerate-able beams?

– Right amount : Charge

– Right shape : Beam size, emittance, bunch length

– Right direction: beam line

– Right time : timing, phase
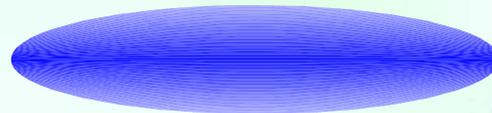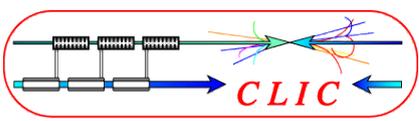
Electron Gun

► **What is electron gun?**

– **Generate electron beam**

- Right amount : Charge
- Right shape : Beam size, emittance, bunch length
- Right direction: beam line
- Right time : timing, phase

Extraction Field

Cathode
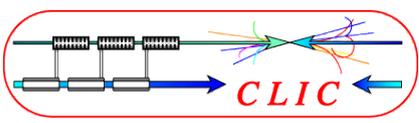
3.2nC Electron Gun

►Bunching : Bunch the beam. What is the bunch?

– Beam : collimated particle flow.

– Bunch : collimated and clustered particle flow. The length should be short enough comparing to the RF period.

Beam

Bunch

Bunching (2)

►Bunching: Shorten the longitudinal length of the beam.

- Right amount : Charge
- Right shape : Beam size, emittance,bunch length
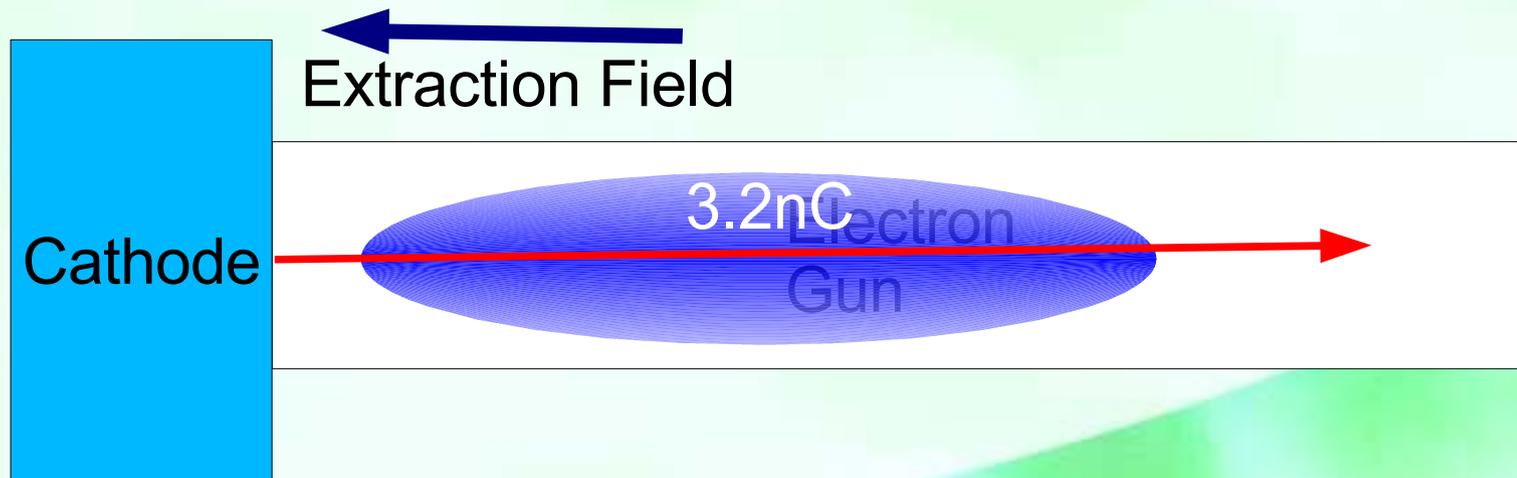- Right direction: beam line
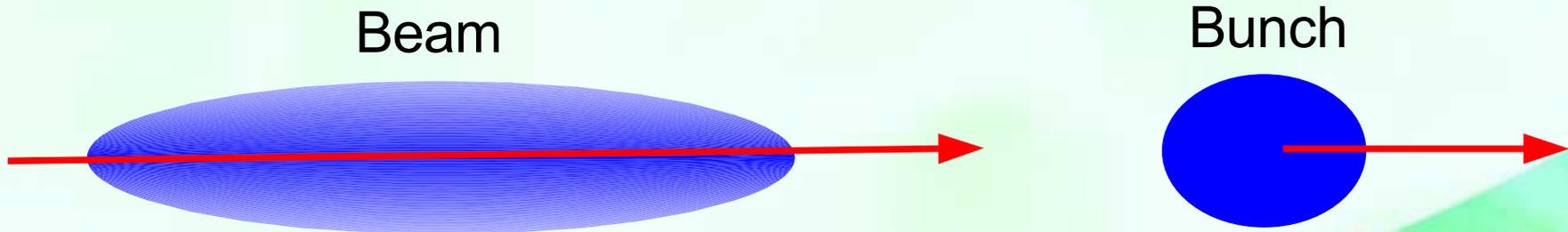- Right time : timing, phase

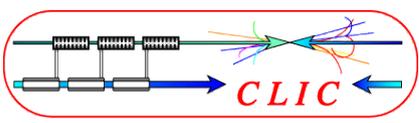3.2nC   Bunching   3.2nC

Injector (2)

Thermal Cathode
Photo-cathode
Field emitter

CW Beam
Long bunch beam
Short bunch beam

DC field
RF field

Pre-buncher
Sub Harmonic Buncher
Buncher

Extraction Field

Cathode

3.2nC

Bunching

3.2nC

To Acc.
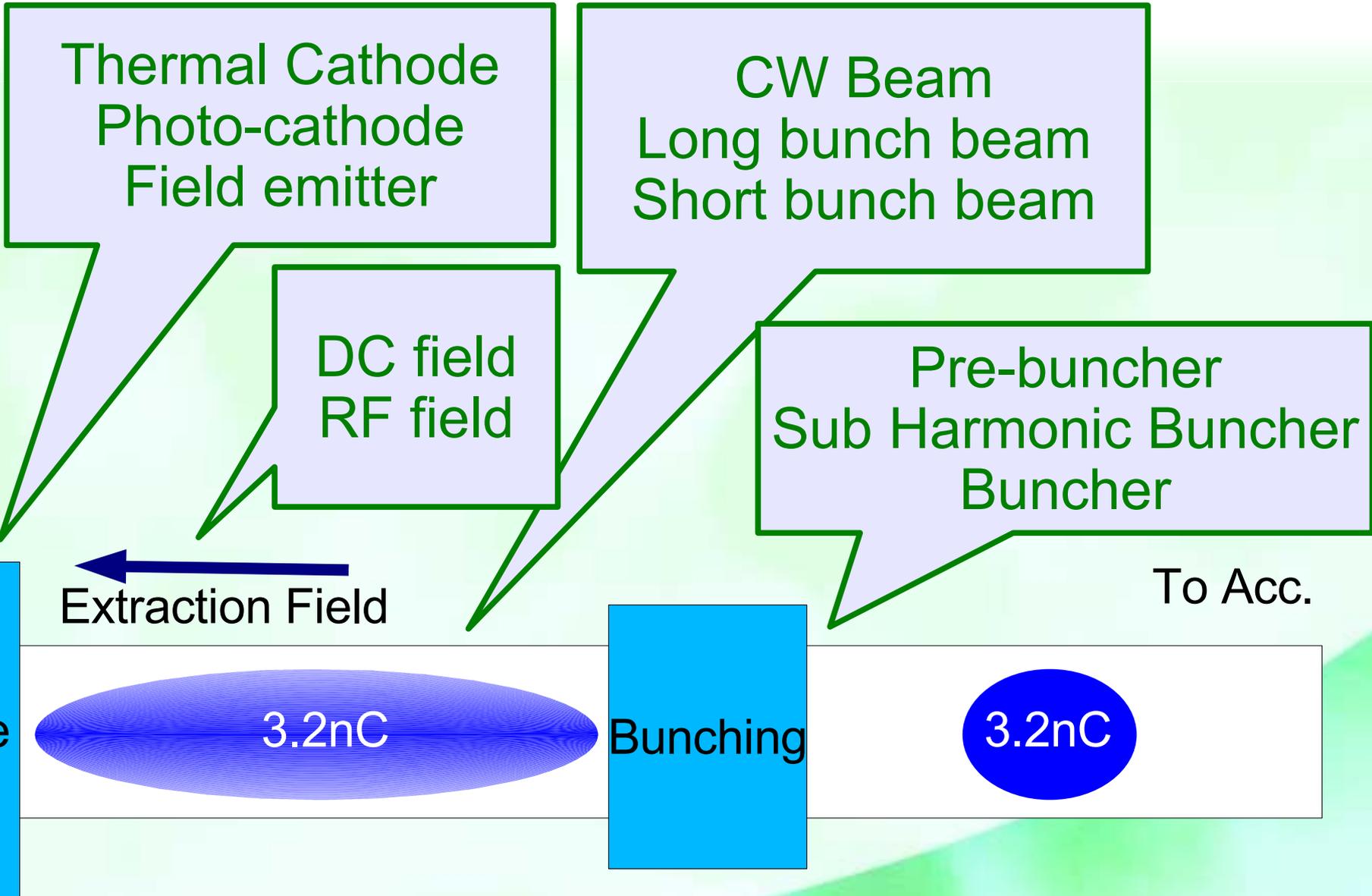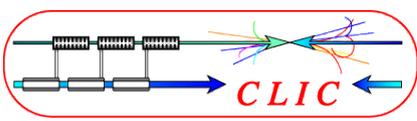
Electron Emission

► **Thermal electron emission** : Electron emission from the heated material (typically 1000 - 3000K).

► **Field emission**: Emission from the high field gradient surface.

► **Photo-electron emission**: Emission by photo-electron effect.

► **Secondary electron emission**: Emission induced by electron absorption.
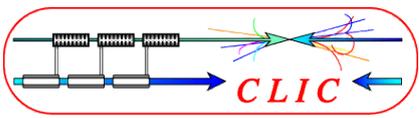
Electronic States

► Electrons in a metal are confined in a well potential and distributed according to Fermi-Dirac Distribution.

► T=0: Electrons occupy the energy states up to Fermi-level (Fermi energy, E_f).

► T>0: Electron distribution extends to higher energy state due to the thermal energy.
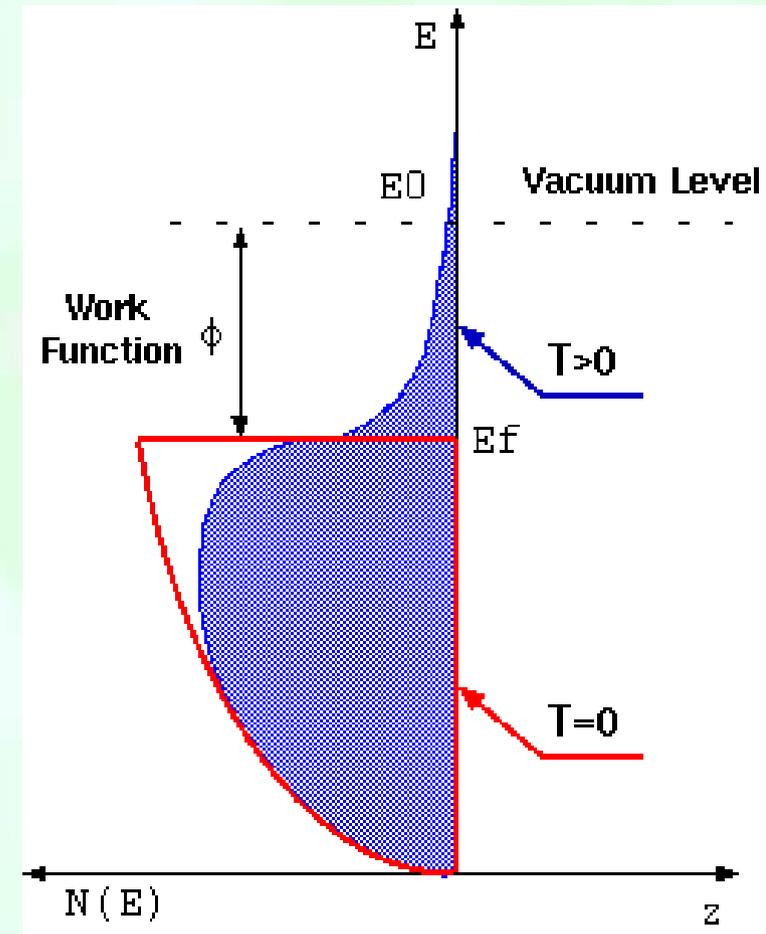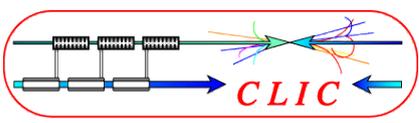
Electron density in a metal is product of state density D(ε) and distribution function f(ε),

$$n(\epsilon) = D(\epsilon)\, f(\epsilon) \qquad (1-1)$$

State density in phase space $(x, v_x) - (x+dx, v_x+dv_x)....$ is

$$D(\epsilon) = \frac{2\mathrm{m}^2}{h^3} dx\, dy\, dz\, dv_x\, dv_y\, dv_z \qquad (1-2)$$

Distribution function f(ε) is given by Fermi-Dirac function

$$f(\epsilon) = \frac{1}{\exp\left|\dfrac{\epsilon - \mu}{kT}\right| + 1} \qquad (1-3)$$

Number of electron with energy ε<E is
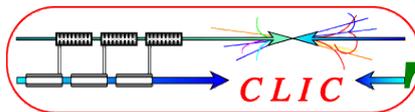
$$N(\epsilon) = \int_0^E f(\epsilon)\, D(\epsilon)\, d\epsilon \qquad (1-4)$$

Thermal Electron Emission

► If the temperature is sufficiently high, so that the electrons are distributed up to more than the vacuum level (E_0), the electrons escape out to the outside.

► The gap between the vacuum level and the Fermi energy is Work function, φ, which characterize the thermal emission.
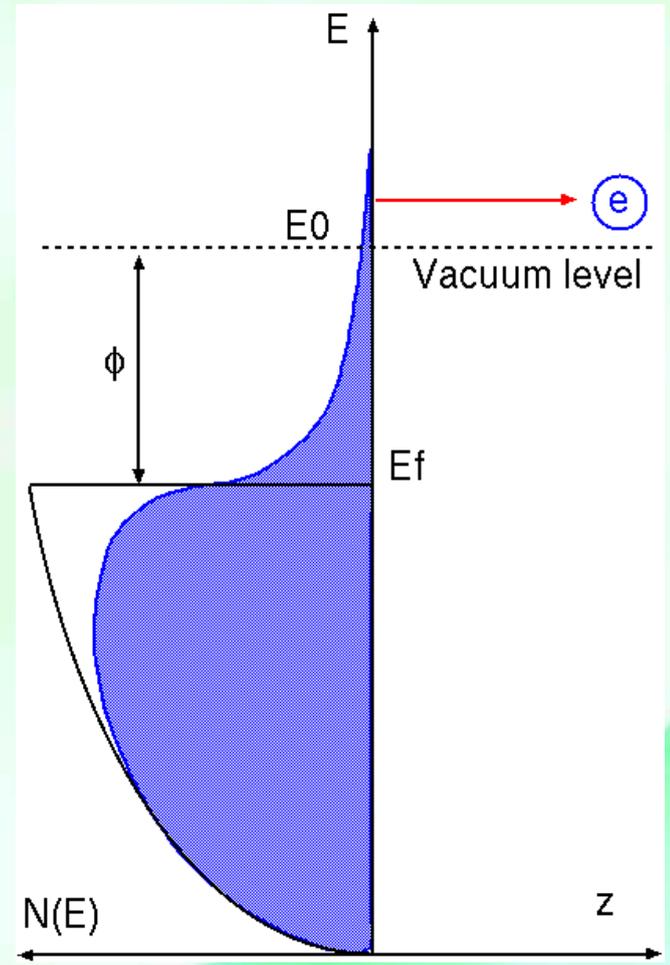
$$E > E_0 = E_f + \phi$$
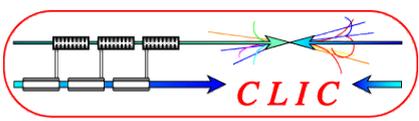
Count number of emitted electron from the surface.

► In depth (z-direction)

$$z \leq v_z \Delta t \qquad (1-5)$$

►Kinetic energy for z-direction must be more than vacuum potential energy, μ+Φ

$$v_z \geq v_{vac} \equiv \sqrt{\frac{2(\mu+\phi)}{m}} \qquad (1-6)$$

►Number of electron emitted from the cathode is give by

material   vacuum

dx

dy

dz=vzdt

$$N = \int dx \int dy \int_0^{v_z \Delta t} dz \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z \, f(\epsilon) D(\epsilon) \quad (1-7)$$
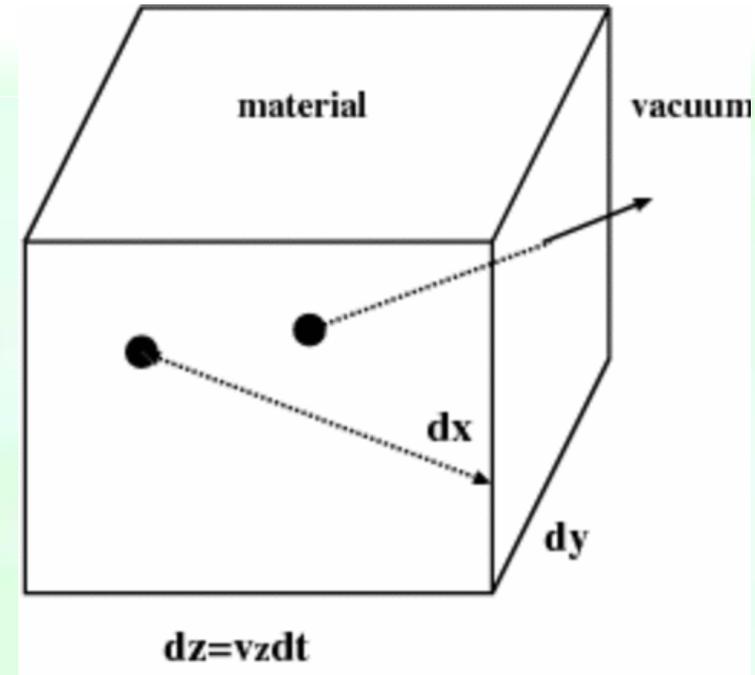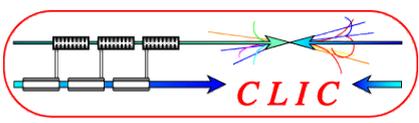
By integrating x, y, z and inserting distribution function,

$$N = \Delta x \, \Delta y \, \Delta t \, \frac{2m^3}{h^3} \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z \, \frac{v_z}{\exp\left(\dfrac{\epsilon - \mu}{kT}\right) + 1} \qquad (1-8)$$

From this equation, emission density per unit time is obtained

$$\sigma \equiv \frac{N}{\Delta x \, \Delta y \, \Delta t} = \frac{2m^3}{h^3} \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z \, \frac{v_z}{\exp\left(\dfrac{\epsilon - \mu}{kT}\right) + 1} \qquad (1-9)$$
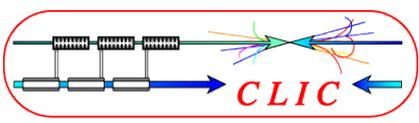
Because ε-μ ≫ kT, f(ε) is approximated to be

$$\frac{1}{\exp\left(\dfrac{\epsilon-\mu}{kT}\right)+1} \sim \exp\left(\dfrac{\mu-\epsilon}{kT}\right) \qquad (1-10)$$

The density is simplified as

$$\sigma = \frac{2m^3}{h^3} \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z\, v_z \exp\left(\frac{\mu-\epsilon}{kT}\right) \qquad (1-11)$$

Replacing the energy with the velocity,

$$\epsilon = \frac{m}{2}\left(v_x^2 + v_y^2 + v_z^2\right)$$

$$\sigma = \frac{2m^3}{h^3} \exp\left(\frac{\mu}{kT}\right) \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z\, v_z \exp\left(\frac{-m\left(v_x^2 + v_y^2 + v_z^2\right)}{2kT}\right) \qquad (1-12)$$
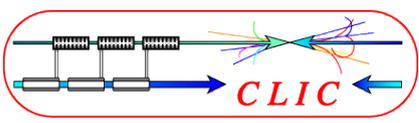
Integral for v_x and v_y can be performed as

$$\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \exp\left(\frac{-m(v_x^2+v_y^2)}{2kT}\right) = \frac{2\pi kT}{m} \qquad (1-13)$$
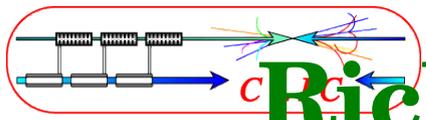
and for v_z as

$$\int_{v_{vac}}^{+\infty} dv_z\, v_z \exp\left(\frac{-mv_z^2}{2kT}\right) = \frac{kT}{m}\exp\left(\frac{-mv_{vac}^2}{2kT}\right) \qquad (1-14)$$

we obtain

$$\sigma = \frac{4\pi m k^2 T^2}{h^3}\exp\left(-\frac{\phi}{kT}\right) \qquad (1-15)$$

Electric current density J is given by

$$J = \frac{4\pi e m k^2 T^2}{h^3}\exp\left(-\frac{\phi}{kT}\right) \qquad (1-16)$$

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Summary

$$J = A T^2 e^{-\frac{\phi}{kT}} \qquad (1-17)$$

$$A = \frac{4\pi emk^2}{h^3} = 1.20 \times 10^6 \, [A/m^2 K^2]$$
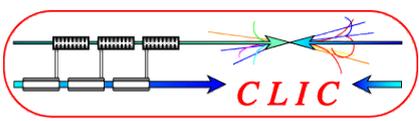
A : thermionic emission constant

T: Temperature (K)

k : Boltzmann constant ; 1.38E-23 (J/K)

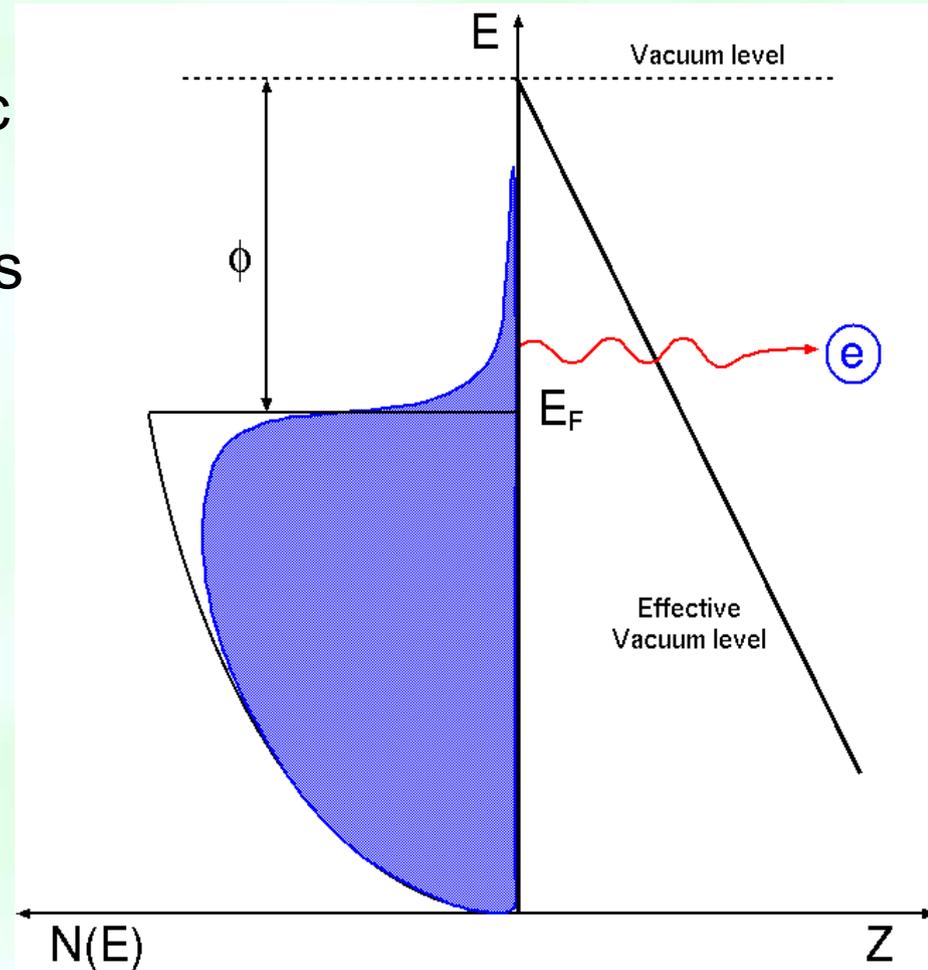e : electronic charge

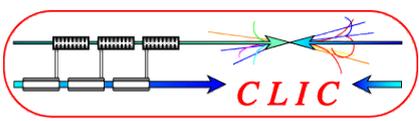m : electron mass

h : Plank constant ; 6.63E-34 (Js)

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► FE is electron emission observed from cold (not hot) material when a high electric field is applied.

– Large surface field makes the potential barrier very thin.

– The tunnel current becomes significant with 1E+8 V/m.

Field Emission (2)

Field emission density is calculated as

$$J = e \int_0^\infty n(\epsilon_z) P(\epsilon_z) d\epsilon_z \qquad (1-18)$$

where P(ε) is transmission probability by tunneling effect. P(ε) by WKB method

$$P(\epsilon_z, F) = \exp\left[-\int_0^w \sqrt{\frac{8m(2\pi)^2}{h^2} \{U(z) - \epsilon_z\}} dz \right]$$

$$= \exp\left[\frac{-8\pi\sqrt{2m}}{3heF}(E_0 - \epsilon_z)^{3/2} \right] \qquad (1-19)$$

$$w = \frac{E_0 - \varepsilon_z}{eF} \qquad U(z) = E_0 - eFz$$
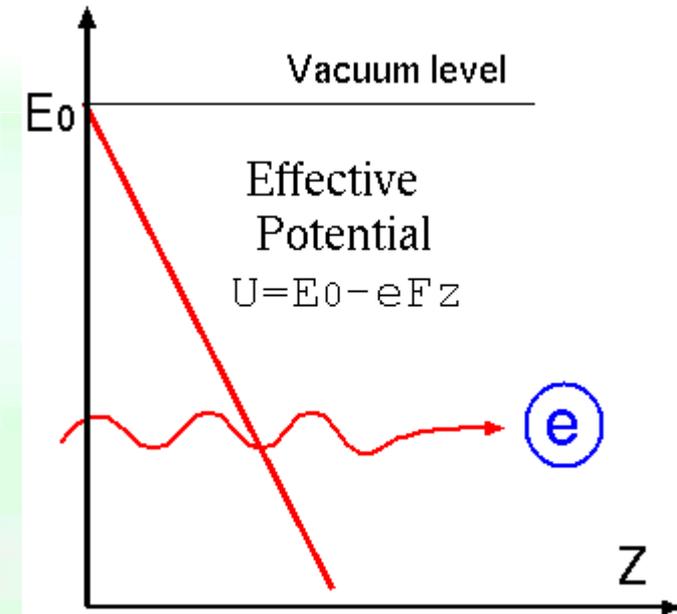
Vacuum level

E_0

Effective Potential

$U = E0 - eFz$

Z

CLIC

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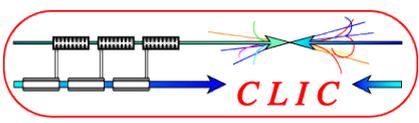Electron Gun
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By Taylor expansion,

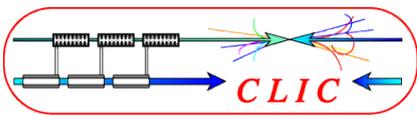$$(E_0 - \epsilon_z)^{3/2} = \left[\phi + (\mu - \epsilon_z)\right]^{3/2} = \phi^{3/2} + \frac{3}{2}\phi^{1/2}(\mu - \epsilon_z) \qquad (1-21)$$

In the low temperature limit, the current density is

$$J(F) = \frac{4\pi em}{h^3} \int_0^\infty d\epsilon_z (\mu - \epsilon_z) \exp\left[-8\pi \frac{\sqrt{2m}}{3heF}(E_0 - \epsilon_z)\right]$$

$$= \frac{4\pi em}{h^3} \exp\left(\frac{-8\pi\sqrt{2m}}{3heF}\phi^{3/2}\right) \int_0^\infty d\acute{\epsilon}\,\acute{\epsilon}\, \exp\left[-4\pi \frac{\sqrt{2m}}{heF}\phi^{1/2}\acute{\epsilon}\right] \qquad (1-22)$$

where ε'=εz-μ. The integral can be performed easily and we get

$$J = \frac{e^3 F^2}{8\pi\phi} \exp\left(-\frac{8\pi\sqrt{2m}}{3heF}\phi^{3/2}\right) \qquad (1-23)$$

(Fowler-Nordheim formula)

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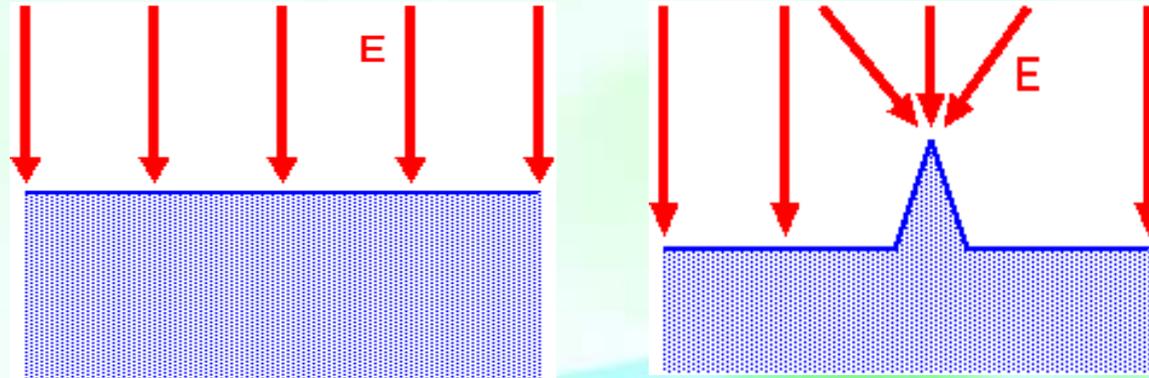
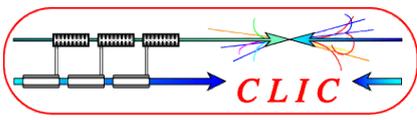Fowler-Nordheim formula with field enhancement factor κ

$$J = \frac{e^3 \kappa^2 F^2}{8 h \pi \phi} \exp\left(-\frac{8\sqrt{2m}}{3 h e \kappa F} \phi^{3/2}\right) \qquad (1-24)$$

κ: local field enhancement by surface condition,

Taking $\ln(J/F^2)$ and plotting as a function of 1/F,

$$\ln\left(J/F^2\right) = \ln\left(\frac{e^3 \kappa^2}{8 h \pi \phi}\right) - \left(\frac{8\sqrt{2m}}{3 h e \kappa} \phi^{3/2}\right)\frac{1}{F} \qquad (1-25)$$

κ is extracted from the gradient of this plot, FN plot.

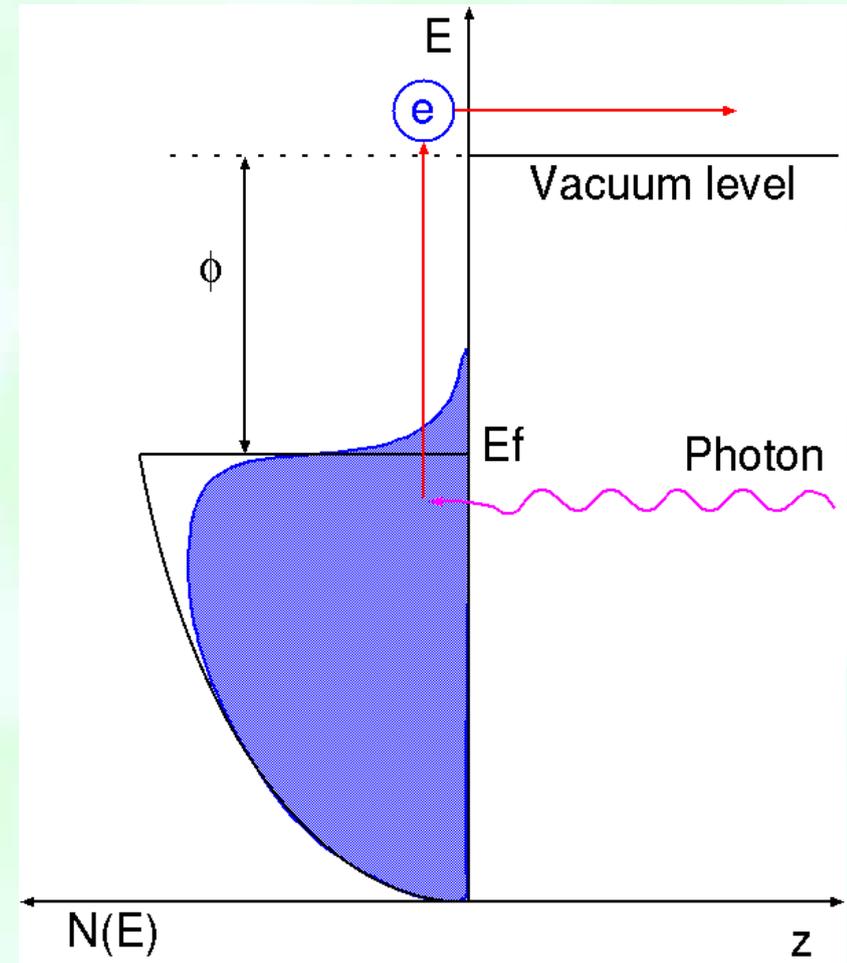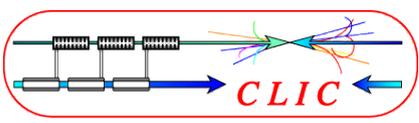► Electron emission by photo-electron effect.

– Photons excite electrons into higher energy states.

– If the states are higher than the vacuum level, the electrons goes to vacuum.

– Condition for photo-emission is

$$h\nu > \phi \qquad (1-26)$$

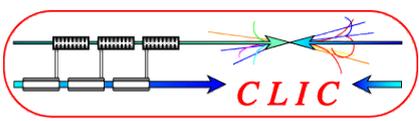Photo-electron current density is given by

$$J = \frac{4\pi\, emkT}{h^3} P \int_{E_0 - h\nu}^{\infty} d\,\epsilon_z \ln\left[1 + \exp\frac{(\mu - \epsilon_z)}{kT}\right] \qquad (1-27)$$

where P is transition probability by photon excitation. For further manipulation, replacing y=(ϵ_z+hv-E$_0$)/kT and δ=h(v-v$_0$)/kT,

$$J = \frac{4\pi\, emkT}{h^3} P \int_{E_0 - h\nu}^{\infty} d\,\epsilon_z \ln\left[1 + \exp\frac{(\mu - \epsilon_z)}{kT}\right]$$

$$= \frac{4\pi\, em\, k^2 T^2}{h^3} P \int_{0}^{\infty} dy\, \ln\left[1 + \exp(\delta - y)\right] \qquad (1-29)$$

$$f(\delta)=\int_0^\infty dy\, \ln\left[1+e^{\delta-y}\right] \qquad (1-30)$$
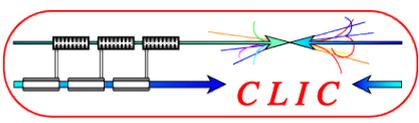
(a) $\delta=h(v-v_0)/kT <0$ (photon energy is less than the work function), the formula is expanded as

$$f(\delta)=\sum_{n=1}^\infty (-1)^{n-1}\frac{e^{n\delta}}{n}\int_0^\infty dy\, e^{-ny}$$

$$=\sum_{n=1}^\infty (-1)^{n-1}\frac{e^{n\delta}}{n^2} \qquad (1-31)$$

since

$$\ln(1+x)=\sum_{n=1}^\infty (-1)^{n-1}\frac{x^n}{n} \qquad (1-32)$$

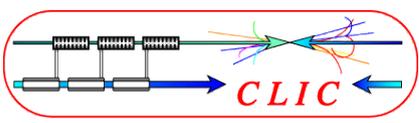$$\ln(1+e^{\delta-y})=\sum_{n=1}^\infty (-1)^{n-1}\frac{e^{n(\delta-y)}}{n} \qquad (1-33)$$

(b) $\delta = h(v-v_0)/kT > 0$ (photon energy is more than the work function),

$$f(\delta) = \left(\int_0^\delta dy + \int_\delta^\infty dy \right) \left[\ln(1 + e^{\delta - y}) \right] \qquad (1-34)$$

(b-1) first integral, $w = \delta - y$

$$\int_0^\delta dy \ln(1 + e^{\delta - y}) = \int_0^\delta dw \ln(1 + e^w)$$

$$= \int_0^\delta dw \left\{ w + \ln(1 + e^{-w}) \right\}$$

$$= \left[\frac{w^2}{2} \right]_0^\delta + \sum_{n=1}^\infty (-1)^n \frac{1}{n^2} \left[e^{-nw} \right]_0^\delta$$

$$= \frac{\delta^2}{2} + \frac{\pi^2}{12} + \sum_{n=1}^\infty (-1)^n \frac{e^{-n\delta}}{n^2} \qquad (1-35)$$

Electron Source
Masao Kuriki (Hiroshima/KEK)

(b-2) second integral, $w = y - \delta$

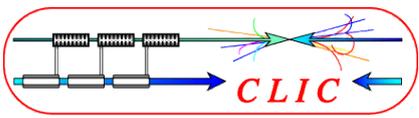$$\int_\delta^\infty dy \ln(1 + e^{\delta - y}) = \int_0^\infty dw \ln(1 + e^{-w})$$

$$= \left[w \ln(1 + e^{-w}) \right]_0^\infty + \int_0^\infty dw \frac{w}{1 + e^w} \quad (1-36)$$

the first term of rhs is 0 and the second term is

$$\int_0^\infty dw \frac{w}{1 + e^w} = \frac{\pi^2}{12} \quad (1-37)$$

Finally, sum of (b-1) + (b-2) gives f(δ)

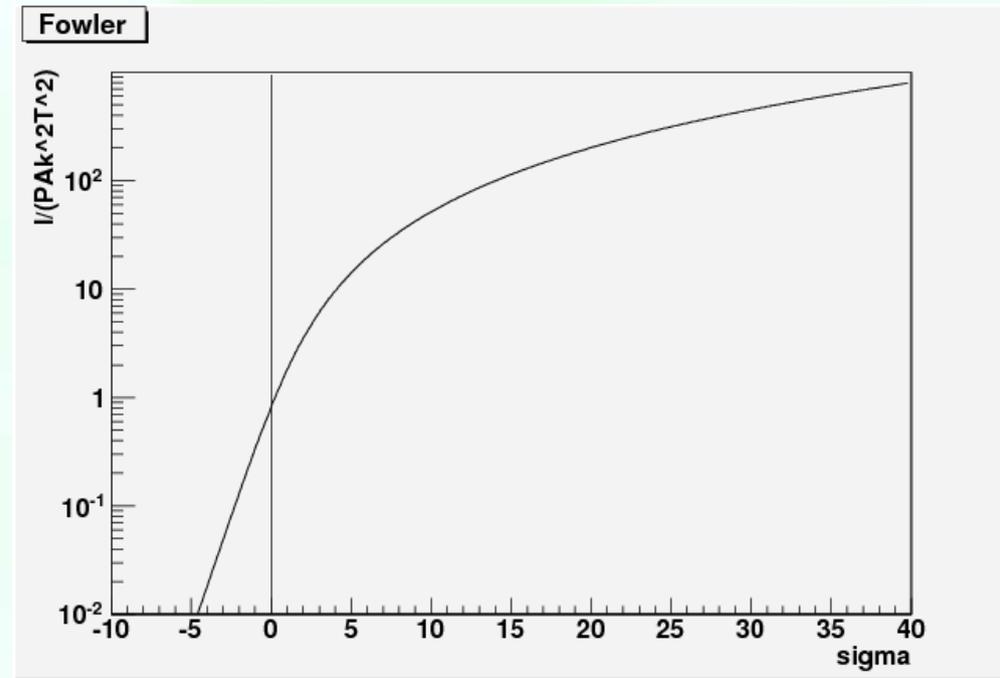$$f(\delta) = \frac{\delta^2}{2} + \frac{\pi^2}{6} + \sum_{n=1}^\infty (-1)^n \frac{e^{-n\delta}}{n^2} \quad (1-38)$$

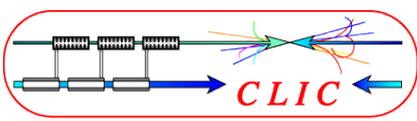$$J = AT^2 P \left| \begin{array}{ll} \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{e^{n\delta}}{n^2} & \delta < 0 \\ \dfrac{\delta^2}{2} + \dfrac{\pi^2}{12} + \sum_{n=1}^{\infty} (-1)^n \dfrac{e^{-n\delta}}{n^2} & \delta > 0 \end{array} \right. \qquad (1-39)$$

$$A = \frac{4\pi e m k^2}{h^3}$$

► Fowler equation gives photo-current spectrum. By taking the spectrum and fit to the curve, work function can be extracted.

► The absolute density is hard to estimate because *P* depends on the surface condition.
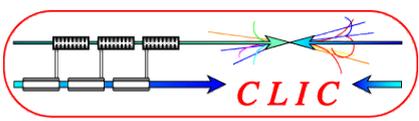
Quantum Efficiency

Quantum Efficiency, η, is practically used to qualify the photo-electron emission

$$\eta = \frac{number\ of\ photo\ electrons}{number\ of\ photons} \qquad (1-40)$$

With practical units,

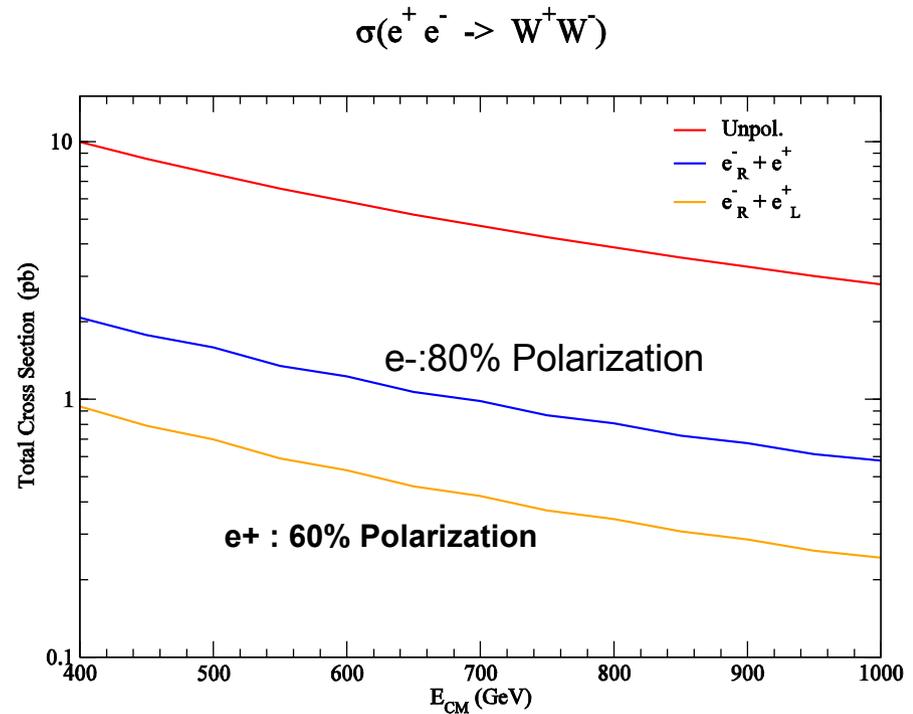$$\eta[\%] = 124 \frac{J[nA]}{P[\mu W]\lambda[nm]} \qquad (1-41)$$
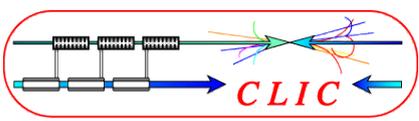
Polarized Electron (1)

▶ Electron has spin ½ and two states.

  – Right handed electron e_R (spin ½)

  – Left handed electron e_L (spin - ½)

▶ In e+e- collider, WW-scattering is the biggest background.

▶ Polarized electron (and also positron) can compensate this background very small.

▶ Polarization is defined as

$\sigma(e^+ e^- \rightarrow W^+ W^-)$

e-:80% Polarization

e+ : 60% Polarization

with GRACE System Developed by Computational Physics Group in KEK
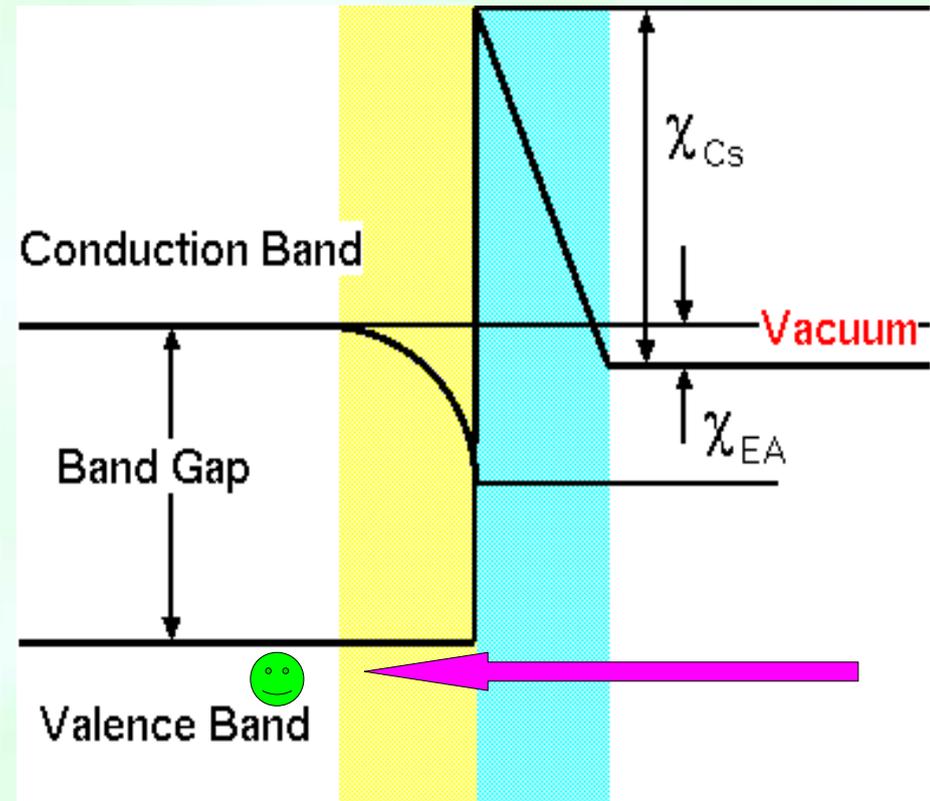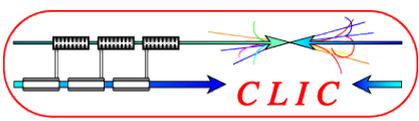
$$P = \frac{N_R - N_L}{N_R + N_L} \qquad (1-42)$$

CLIC

Introduction
**Electron
Emission**
Related
Physics
Electron
Gun
e- Source
for LCs
Laser
Summary

► **Polarized Electron generation in 3 steps**

  – Selective excitation from the valence band to conduction band.

  – Transported to the surface.

  – Extracted to the vacuum.

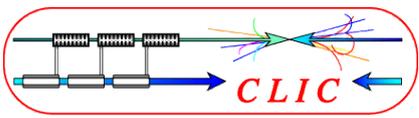
Conduction Band

χ_{Cs}

Vacuum

χ_{EA}

Band Gap

Valence Band

- **Transition probability ~ Fermi's golden rule**

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} M^2 D(\hbar\omega) f(E)$$

  - **M**: Matrix element
  - **D**: joint density of states of ℏω photon
  - **f**: Fermi distribution function

- **Considering only near the band gap, the transition probability is proportional to M.**

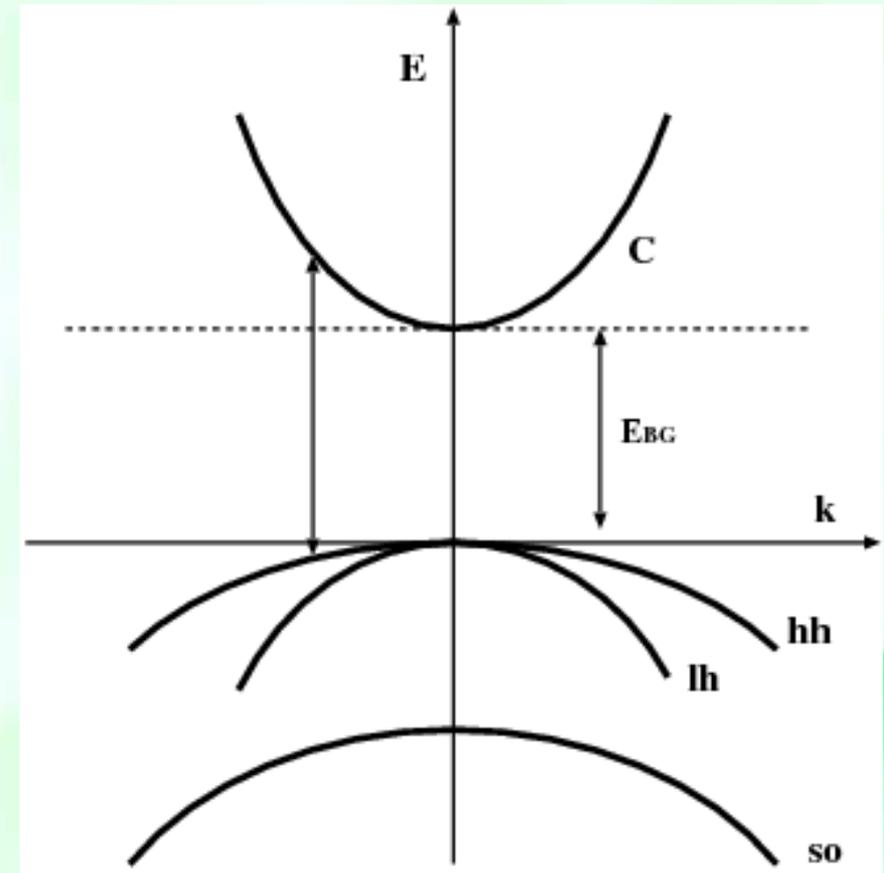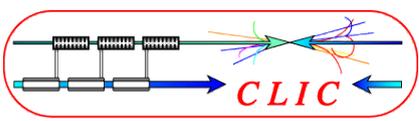Introduction
Electron Emission
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Summary

- *Band gap of GaAs is Γ point (k=0).*
  - *Valence Band:*
    - *J=|3/2,±3/2> (heavy hole)*
    - *J=|3/2, ±1/2> (light hole).*
  - *Conduction band:*
    - *J=|1/2,±1/2>*
- *Matrix Element of transition (Clebsh-Gordon coef.）*
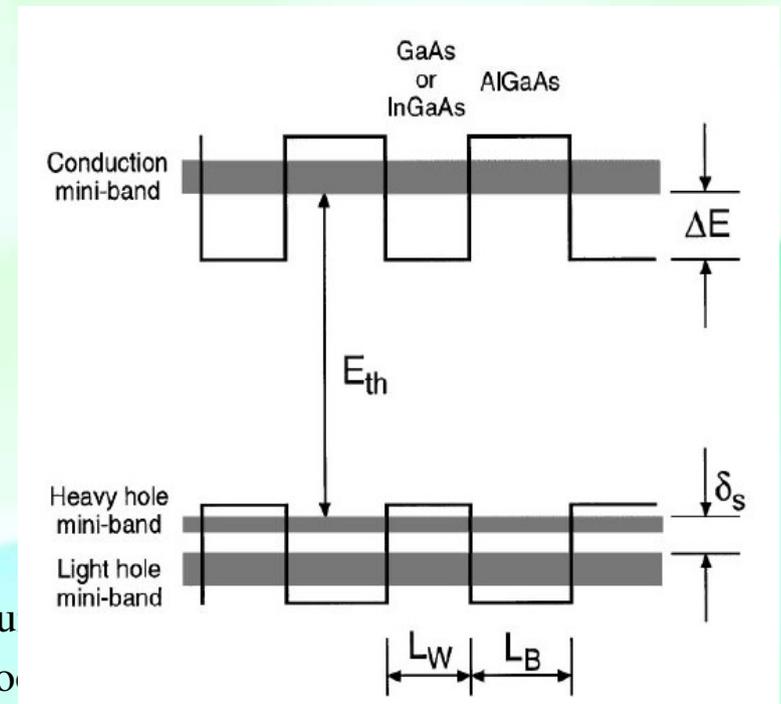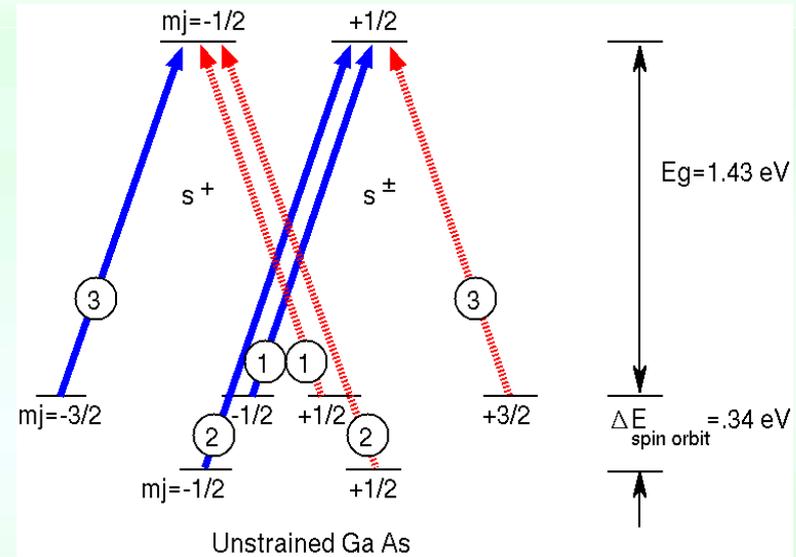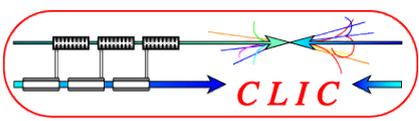 - *Heavy hole:$\sqrt{3}/2$*
  - *Light hole: 1/2*

► Electron from photo-excitation by circularly polarized photon in near BG of GaAs is 50% polarized.

► The polarization is enhanced by introducing energy selection.

– Untied the degeneration by strained or super-lattice structure.

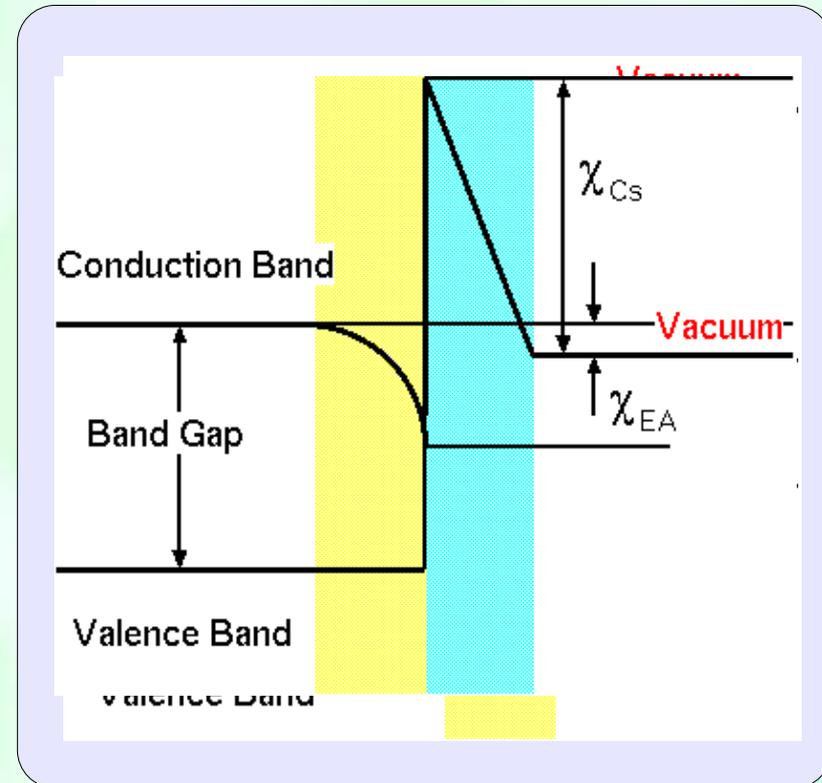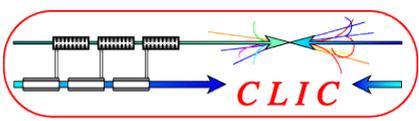– One of the transition is suppressed and the polarization can be up to 90%.

►NEA surface has an important role.

– Nominal material has positive electron affinity; Electrons are confined in the well potential.

– The electron affinity can be negative (NEA surface).

– With NEA surface, polarized electron in the conduction band can be extracted to the vacuum.
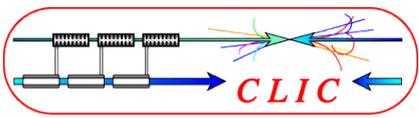
► NEA surface is made by evaporation of Cs and O_2.

► At first, the surface is conditioned by chemical etching by H_2SO_4 and treatment by HCl-Isopropanol solution followed by heat cleaning.

► Alternating deposition of Cs and O_2 make NEA surface on GaAs.
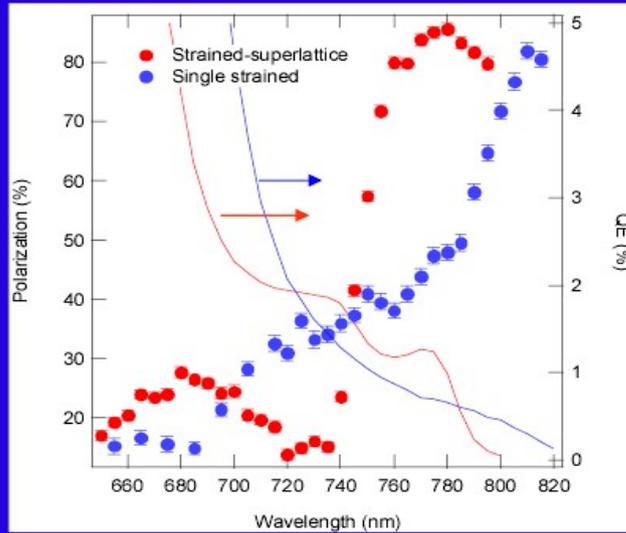
► The process should be made extremely high vacuum, <5.0E-9Pa.
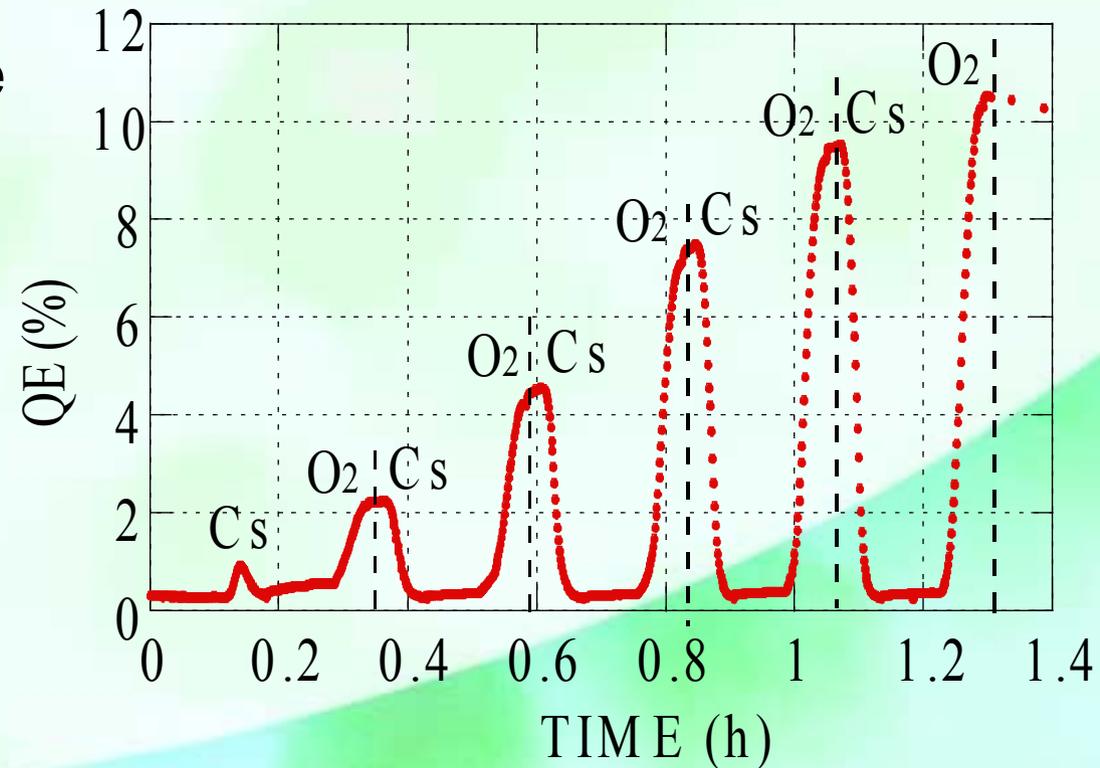
C. Shonaka

QE (%) vs TIME (h)
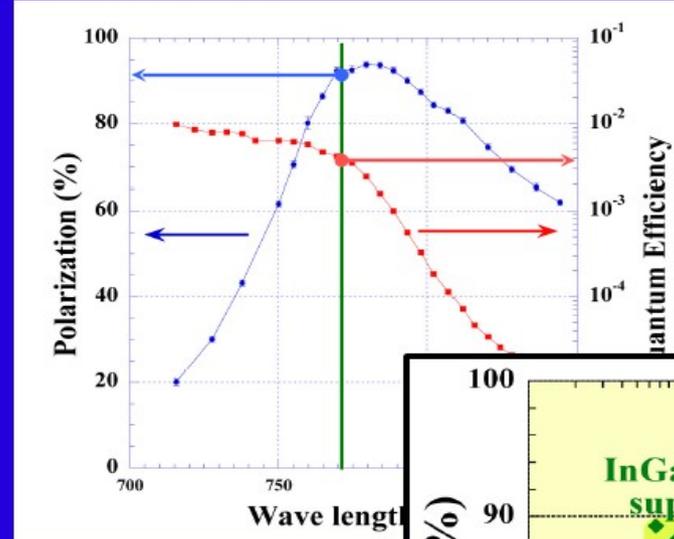
Performance of GaAs/GaAsP superlattice

SLAC

NAGOYA

T. Nishitani, M. Yamamoto

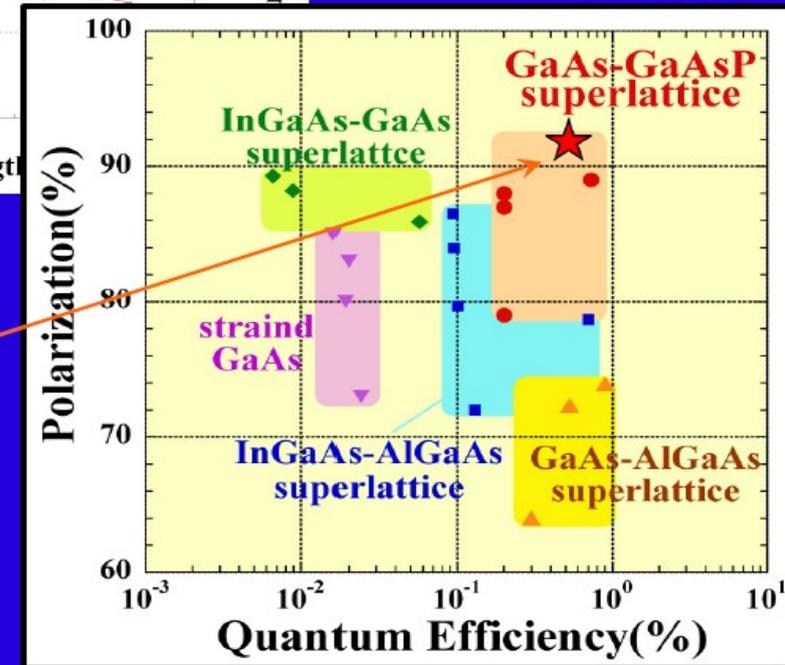### NAGOYA

GaAs-GaAsP superlattice shows the best performance !

@778nm

Polarization ~ 90%

Q.E.   ~ 0.5%
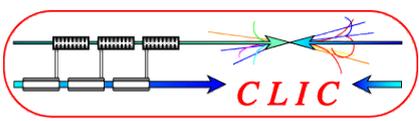
► There is no established model for NEA surface. There are two main candidates.

► **Cs-O electric dipole model**

  – Composition of Cs-Ox forms electric dipole on the surface.

  – The vacuum potential is effectively decreased by the dipole potential.
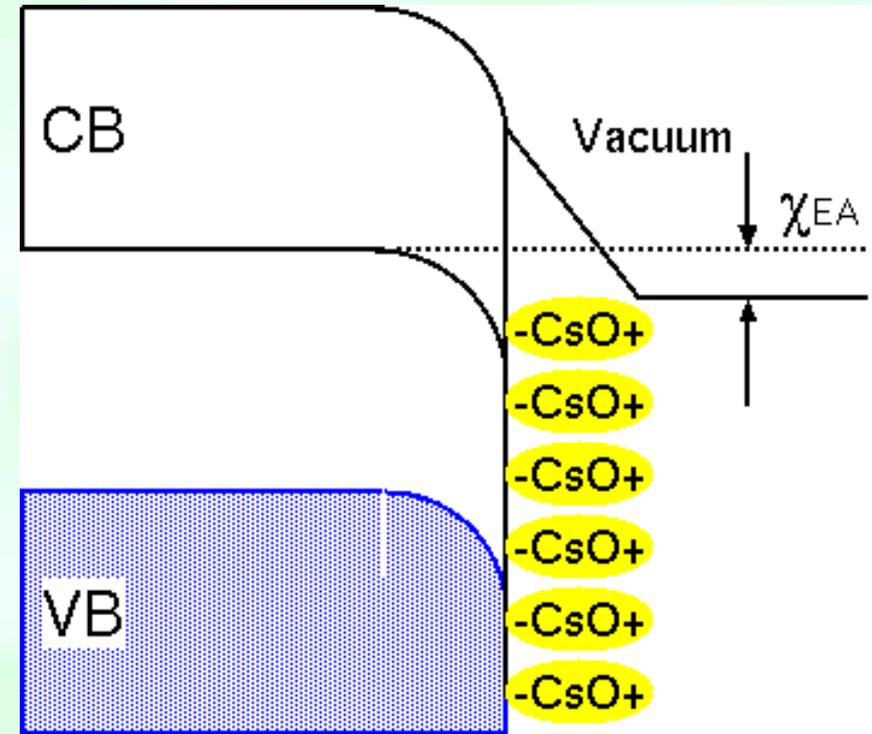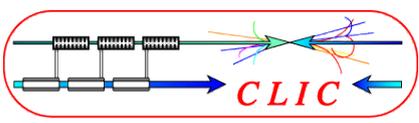
CB

Vacuum

χ_{EA}

-CsO+
-CsO+
-CsO+
-CsO+
-CsO+
-CsO+

VB

► **Hetero-junction model**

– III-V semiconductor + Cs_xO_{1-x} hetero-junction is made at the surface of GaAs.

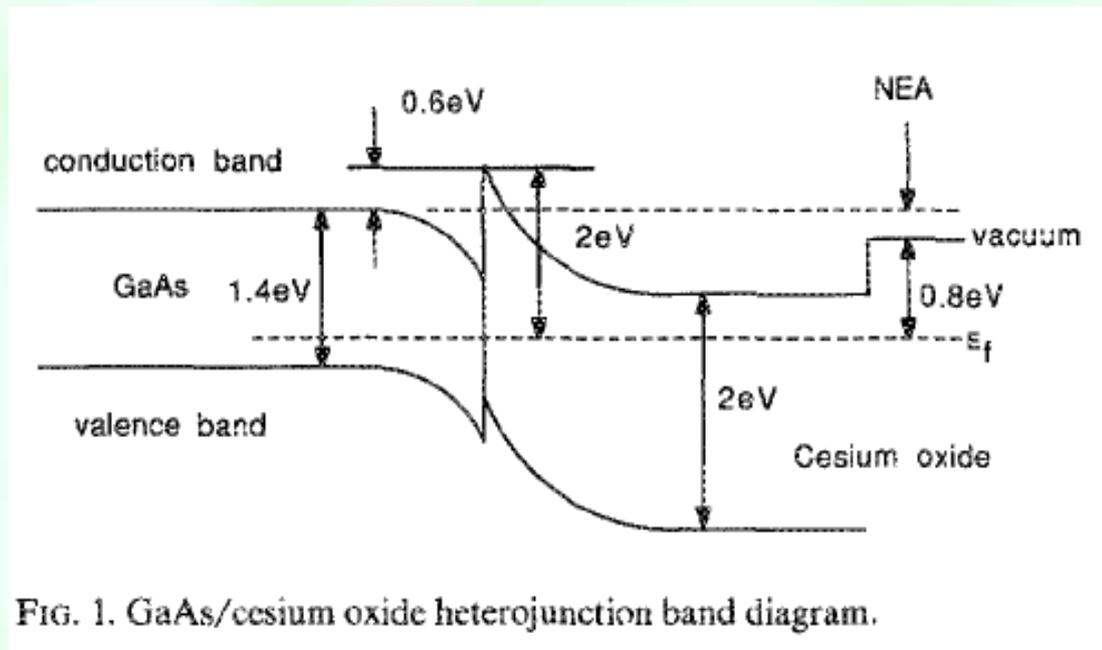– Bulk Cs_2O is n-type semi-conductor, φ=0.8eV and electron affinity χ=0.55 eV.

– In GaAs and Cs_2O hetero-junction, the vacuum level becomes below the conduction band in GaAs.

C. A. Sanford, J. Vac. Sci. Tech. B7(6), 1989
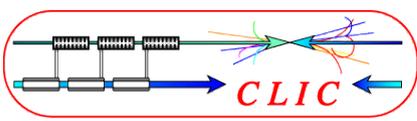
FIG. 1. GaAs/cesium oxide heterojunction band diagram.

Related Physics Process

CLIC
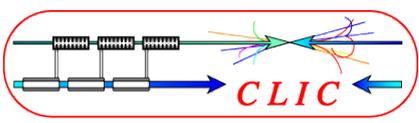
Effective work function is decreased by surface field (Schottky effect)

Cathode surface

Mirror charge force

Force by mirror charge

$$F_m(z) = -\frac{1}{4\pi\epsilon}\frac{e^2}{(2z)^2} \quad (2-1)$$

Potential of the mirror charge

$$V_m(z) = -\frac{1}{4\pi\epsilon}\int_z^\infty \frac{e}{4z'^2}dz' = -\frac{e^2}{16\pi\epsilon z} \quad (2-2)$$
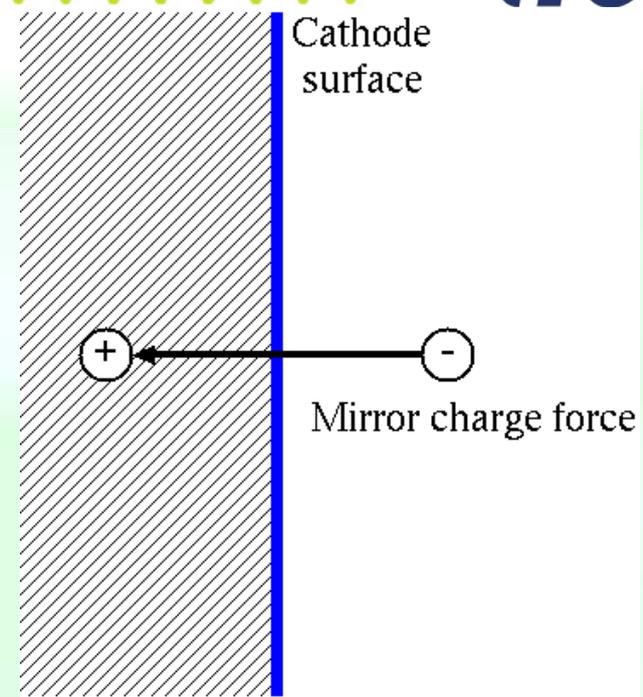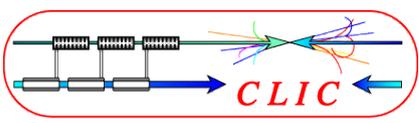
Vacuum potential

$$V(z) = \phi_0 - \frac{e^2}{16\pi\epsilon z} - eEz \quad (2-3)$$

Maximum

$$V_{max} = V_0 - e\sqrt{\frac{eE}{4\pi\epsilon}} \quad (2-4)$$

$$z_{max} = \frac{1}{4}\sqrt{\frac{e}{\pi\epsilon E}} \quad (2-5)$$

Effective work function is

$$\phi(F) = V_{max} - \mu = \phi_0 - e\sqrt{\frac{eE}{4\pi\epsilon}} \quad (2-6)$$
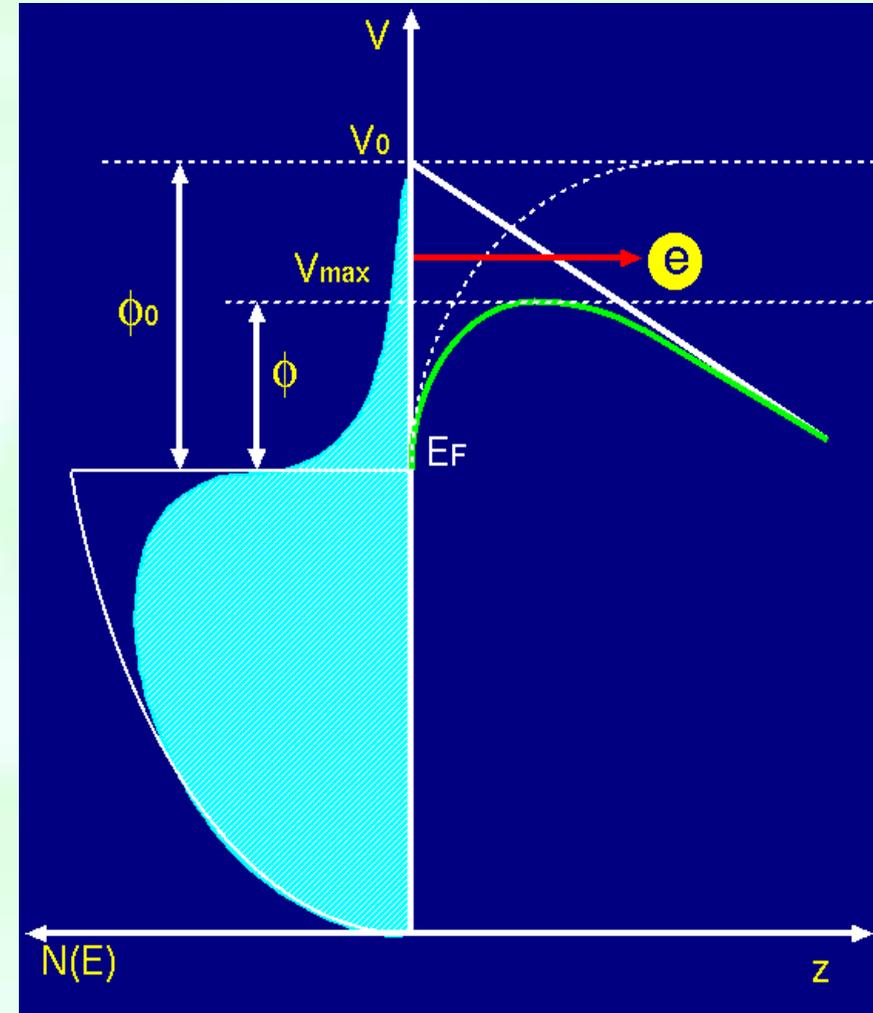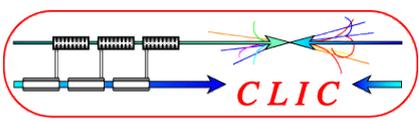
Space Charge Limit

► **Electron current density has a limit by space charge effect (Space charge limit).**

– Electron terminates the electric flux (remember Gauss's law).

– Electric field is weakened by the space charge.

– At some limit, the field at the cathode surface is disappeared and no electrons extracted further; the space charge limit.
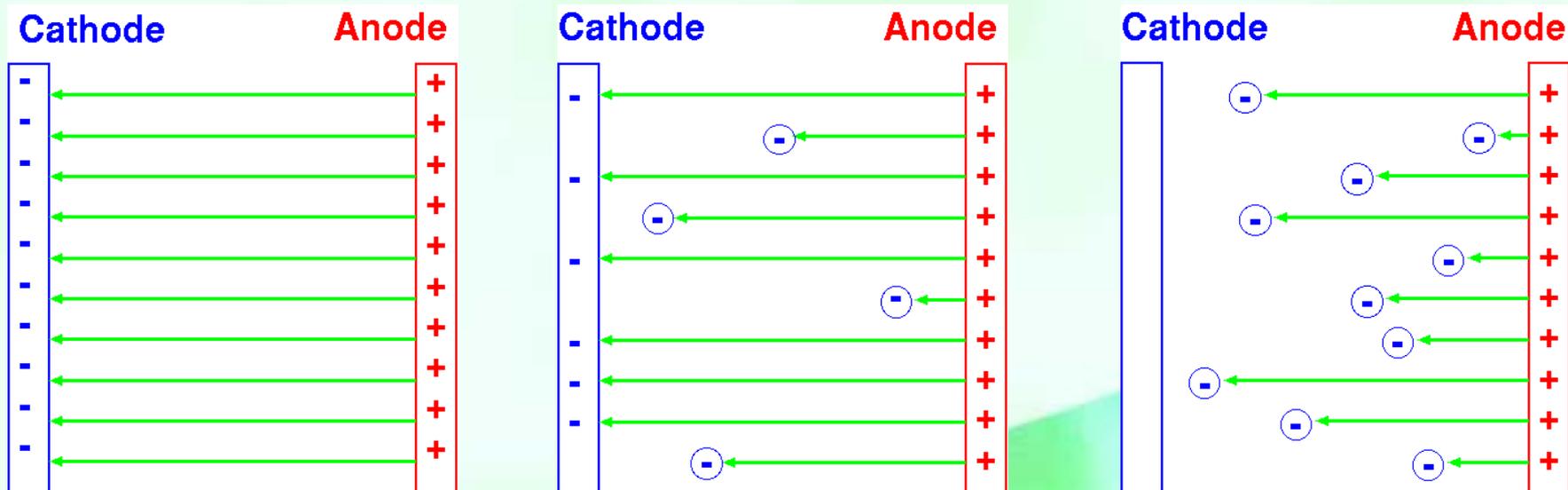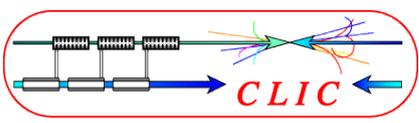
Space charge limited current in 1D case.

Cathode is V=0, z=0, and anode is V=V_A and z=d.

Poisson equation is

$$\frac{d^2 V(z)}{dz^2} = -\frac{\rho(z)}{\epsilon_0} \quad (2-7)$$

The current density J is given by the charge density ρ and velocity v,

$$J = -\rho(z) v(z) \quad (2-8)$$

According energy conservation,

$$\frac{1}{2} m v(z)^2 = e V(z) \quad (2-9)$$

Integrating these three formulas,

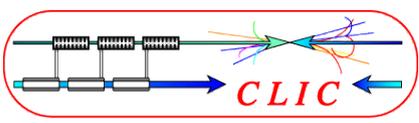$$\frac{d^2 V(z)}{dz^2} = \frac{J}{\epsilon_0} \sqrt{\frac{m}{2e}} V(z)^{-1/2} \quad (2-10)$$

Multiplying *2(dV/dz)* and integrating both sides,

$$\left|\frac{d V(z)}{dz}\right|^2 = \frac{4J}{\epsilon_0}\sqrt{\frac{m}{2e}}\,V(z)^{1/2} \qquad (2-11)$$

Taking square root of both sides and integrate it again,

$$\frac{4}{3}V^{3/4} = \sqrt{\frac{4J}{\epsilon_0}}\sqrt[4]{\frac{m}{2e}}\,z \qquad (2-12)$$

Extract *J*, we got

$$J = \frac{4\,\epsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V(z)^{3/2}}{z^2}$$

$$= 2.33\times10^{-6}\frac{V(z)^{3/2}}{z^2} \qquad (2-13)$$
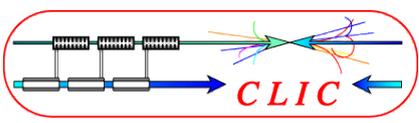
Electron Source
Masao Kuriki (Hiroshima/KEK)

Substituting the anode conditions, the space charge limited current density is obtained as
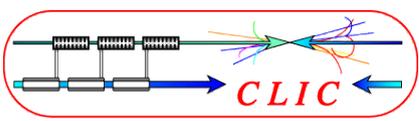
$$J(V_A, d) = 2.33 \times 10^{-6} \frac{V_A^{3/2}}{d^2} \qquad (2-14)$$

In this case, V(z), E(z) , ρ(z) are expressed as a function of z

$$V(z) = V_A \left(\frac{z}{d}\right)^{3/4} \qquad (2-15)$$

$$E(z) = -\frac{dV(z)}{dz} = -\frac{4}{3} \frac{V_A}{d^{4/3}} z^{1/3} \qquad (2-16)$$

$$\rho(z) = -\frac{4\epsilon_0}{9} \frac{V_A}{d^{4/3}} z^{-2/3} \qquad (2-17)$$

Child-Langmuir Law

If the electron source is operated in space charge limited regime, the current is given by C-L law
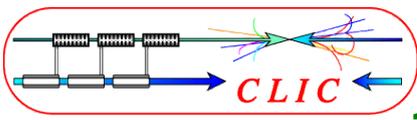
$$I = 2.33 \times 10^{-6} \frac{S V^{3/2}}{d^2} = P V^{3/2} (A) \qquad (2-18)$$

V and d : voltage and distance between two electrodes.

S : cathode area

P : perveance defined as;

$$P = 2.33 \times 10^{-6} \frac{S}{d^2} (A V^{-3/2}) \qquad (2-19)$$

► When the surface field is not sufficiently high, the actual current is determined by the space charge limit.

► When the surface field is sufficiently high, the actual current is determined by that from the cathode.

► Then, the actual emission current form a cathode is

$$I_E = min(I_C, I_{SC}) \qquad (2-20)$$

  – I_C: Emission current of the fundamental process (thermal emission, etc.)
  – I_{SC}: Space charge limit
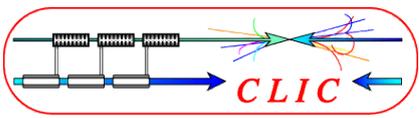
SC Limited Flow

Space charge limited flow can be made in a realistic geometry.

2D case: solution
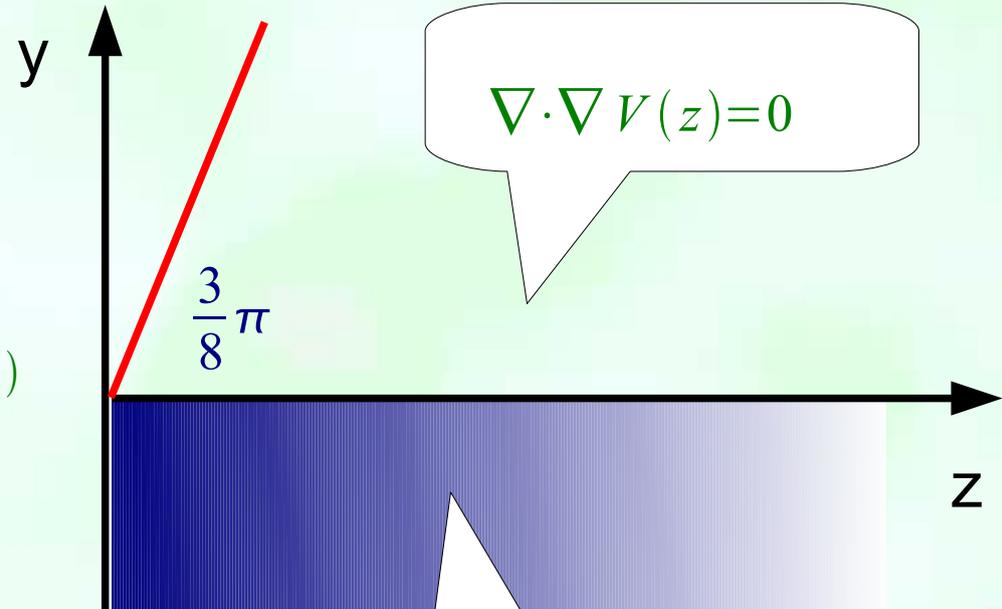
$$V(z,y) = V_A \frac{\Re\left[(z+iy)^{4/3}\right]}{d^{4/3}}$$

$$= V_A(z^2+y^2)^{2/3}\cos\frac{4}{3}\theta \quad (2-22)$$
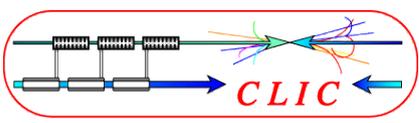
V=0

$$\cos\frac{4}{3}\theta = 0 \rightarrow \theta = \frac{3}{8}\pi$$

By setting an electrode (Wehnelt) with this angle, SCL flow is produced.

y

$\frac{3}{8}\pi$

z

$$\nabla\cdot\nabla V(z) = 0$$

$$V(z) = V_A\left(\frac{z}{d}\right)^{4/3}$$

Charged particle beam has repulsion force due to the own electronic charge: Space Charge Force. The space charge force causes various beam quality degradations, e.g. bunch lengthening, emittance growth, tune shift, etc. The effect is suppressed by acceleration because it scaled as $1/\gamma^2$.
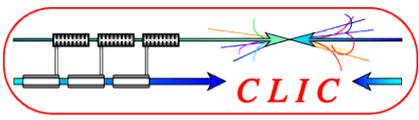
Consider a cylindrical beam (radius a) with a constant density (N electron per unit length) with infinite length. The space charge force is

$$E_r = \frac{N e}{2 \pi a^2 \epsilon_0} r \qquad (2-23)$$

The electron beam is electronic current, which induces magnetic flux density

$$B(r) = \frac{\mu_0}{r} \int_0^r r' J(r') dr' \qquad (2-24)$$
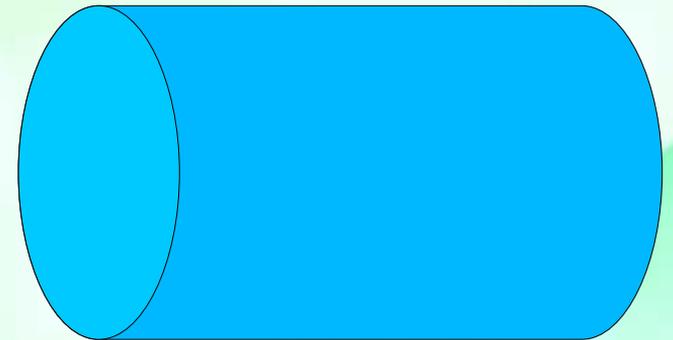
Current density is

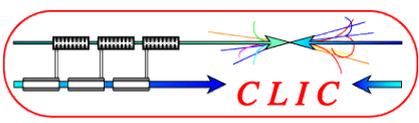$$J(r) = \frac{Ne}{\pi a^2} \beta c \qquad (2-25)$$

Then, the magnetic flux is

$$B(r) = \frac{\mu_0 N e \beta c}{2 \pi a^2} r \qquad (2-26)$$

The direction is azimuthal. The force to electron beam itself is

$$F = e\,\boldsymbol{E} + e\,\beta\,c\,\boldsymbol{B} = \frac{Ne^2 r}{2\pi a^2 \epsilon_0}(1-\beta^2)\vec{e}_r$$

$$= \frac{Ne^2 r}{2\pi a^2 \epsilon_0 \gamma^2}\vec{e}_r \qquad (2-27)$$

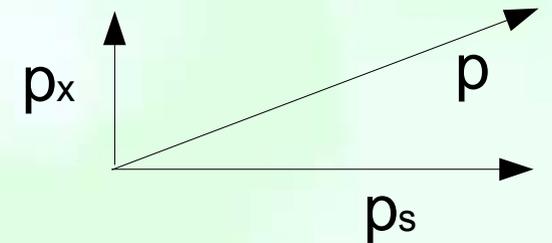which is scaled as $1/\gamma^2$. The space charge force is suppressed greatly by acceleration.

Emittance is defined as area in the phase space, where particles occupy. The phase space is defined x and x'=dx/ds

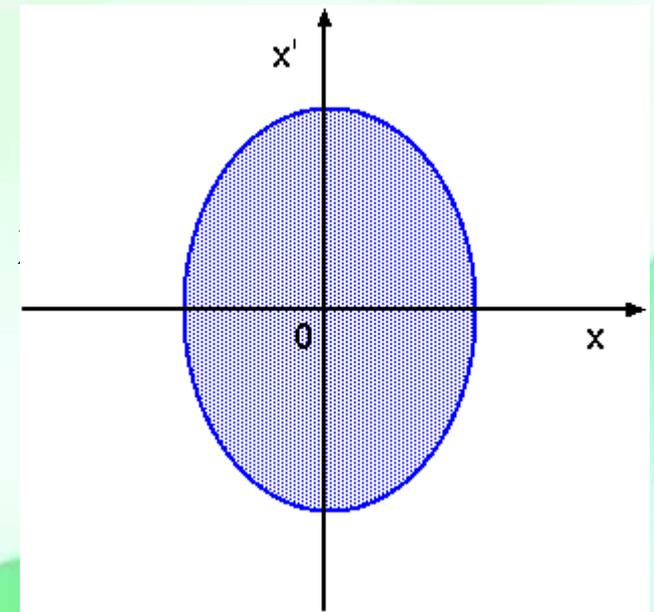$$\acute{X} = \frac{dx}{ds} = \frac{v_x}{v_s} = \frac{p_x}{p_s} \sim \frac{p_x}{p} \qquad (2-28)$$
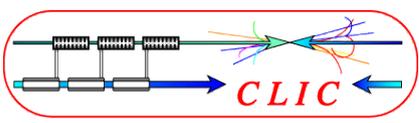
p_x $\qquad$ p

p_s

In general, RMS emittance is given as

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle \acute{x}^2 \rangle - \langle x\acute{x} \rangle^2} \qquad (2-29)$$

If there is no correlation between x and

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle \acute{x}^2 \rangle} \qquad (2-30)$$

In acceleration, transverse momentum p_x is conserved, but p is scaled as

$$p_s = \gamma \beta \, mc \qquad (2-31)$$

The emittance depend on the energy. To avoid the energy dependence $(\gamma\beta)$ on the emittance, the normalized emittance is defined
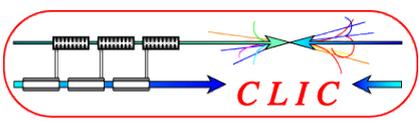
$$\epsilon_{nx} = \gamma \beta \epsilon_x \qquad (2-32)$$
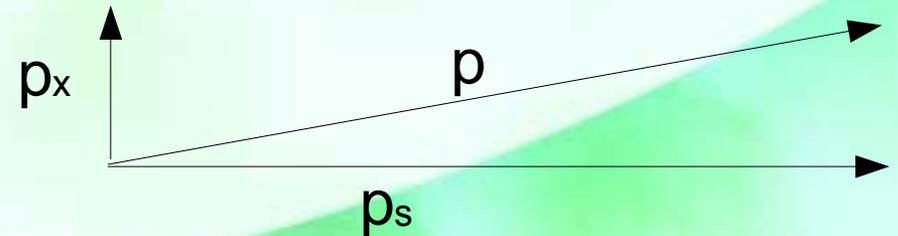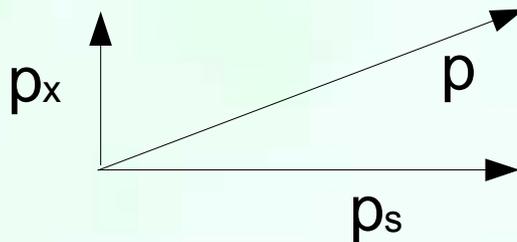
Introduction
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Thermionic electron emission density is already obtained

$$N = \frac{4\pi m}{h^3} k^2 T^2 \exp\left(-\frac{\phi}{kT}\right) \quad (2-33)$$
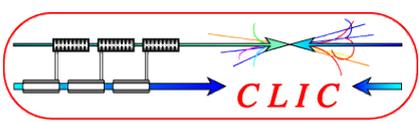
Total transverse energy of emitted electron is obtained with a similar calculation as

$$E_t = \frac{4\pi m}{h^3} \int_{\mu+\phi}^{\infty} d\epsilon_z \int_0^{\infty} d\epsilon_t \epsilon_t \exp\left(-\frac{\epsilon_z + \epsilon_t - \mu}{kT}\right)$$

$$= \frac{4\pi m}{h^3} k^3 T^3 \exp\left(-\frac{\phi}{kT}\right) \quad (2-34)$$

The average transverse energy per electron is

$$\langle \epsilon_t \rangle = \frac{E_t}{N} = kT \quad (2-35)$$

X component is,

$$\langle \epsilon_x \rangle = \frac{kT}{2} \qquad (2-36)$$

The transverse emittance is

$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle} = \frac{1}{\gamma \beta m c} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle} \qquad (2-37)$$
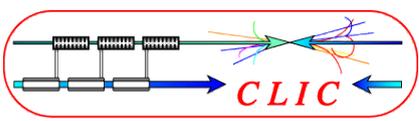
Substituting the thermal energy,

$$\frac{\langle p_x^2 \rangle}{2m} = \langle \epsilon_x \rangle = \frac{kT}{2}$$

$$\varepsilon_x = \frac{1}{\gamma \beta} \sqrt{\langle x^2 \rangle \frac{kT}{mc^2}} = \frac{1}{\gamma \beta} \frac{R}{2} \sqrt{\frac{kT}{mc^2}} \qquad (2-38)$$

normalized emittance is

$$\varepsilon_{nx} = \gamma \beta \varepsilon_x = \frac{R}{2} \sqrt{\frac{kT}{mc^2}} \qquad (2-39)$$

Transverse energy of Photo-emission is

$$E_t = \frac{4\pi m}{h^3} \int_{\mu+\phi-h\nu}^{\infty} d\epsilon_z \int_0^{\infty} d\epsilon_t \, \epsilon_t \left[\exp\left(\frac{\epsilon_z + \epsilon_t - \mu}{kT}\right) + 1 \right]^{-1} \qquad (2-40)$$

With T=0 approximation,

$$E_t = \frac{4\pi m}{h^3} \int_{\mu+\phi-h\nu}^{\mu} d\epsilon_z \int_0^{\mu-\epsilon_z} d\epsilon_t \, \epsilon_t$$

$$= \frac{4\pi m}{h^3} \frac{(h\nu-\phi)^3}{6} \qquad (2-41)$$
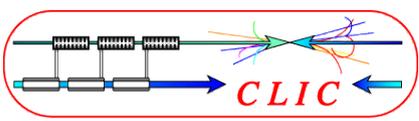
Number of emitted electron

$$N = \frac{4\pi m}{h^3} \int_{\mu+\phi-h\nu}^{\mu} d\epsilon_z \int_0^{\mu-\epsilon_z} d\epsilon_t = \frac{4\pi m}{h^3} \frac{(h\nu-\phi)^2}{2} \qquad (2-42)$$

The average of the transverse energy is

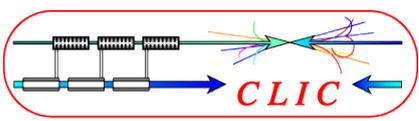$$\epsilon_t = \frac{E_t}{N} = \frac{h\nu-\phi}{3} \qquad (2-43)$$

Average momentum is obtained

$$\langle p_x^2 \rangle = 2m\frac{\epsilon_t}{2} = m\frac{h\nu-\phi}{3} \qquad (2-44)$$

Emittance is

$$\varepsilon_x = \frac{1}{\gamma\beta}\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3mc^2}} \qquad (2-45) \qquad\qquad \varepsilon_{nx} = \frac{R}{2}\sqrt{\frac{h\nu-\phi}{3mc^2}} \qquad (2-46)$$
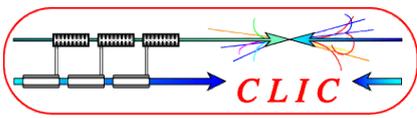
Thermal energy is not included at all. If the additional energy is accounted, the transverse energy becomes

$$\epsilon_t = \frac{E_t}{N} = \frac{h\nu - \phi}{3} + kT \qquad (2-47)$$

The transverse emittance is extracted as

$$\varepsilon_x = \frac{1}{\gamma\beta} \frac{R}{2} \sqrt{\frac{h\nu - \phi}{3mc^2} + \frac{kT}{mc^2}} \qquad (2-48)$$

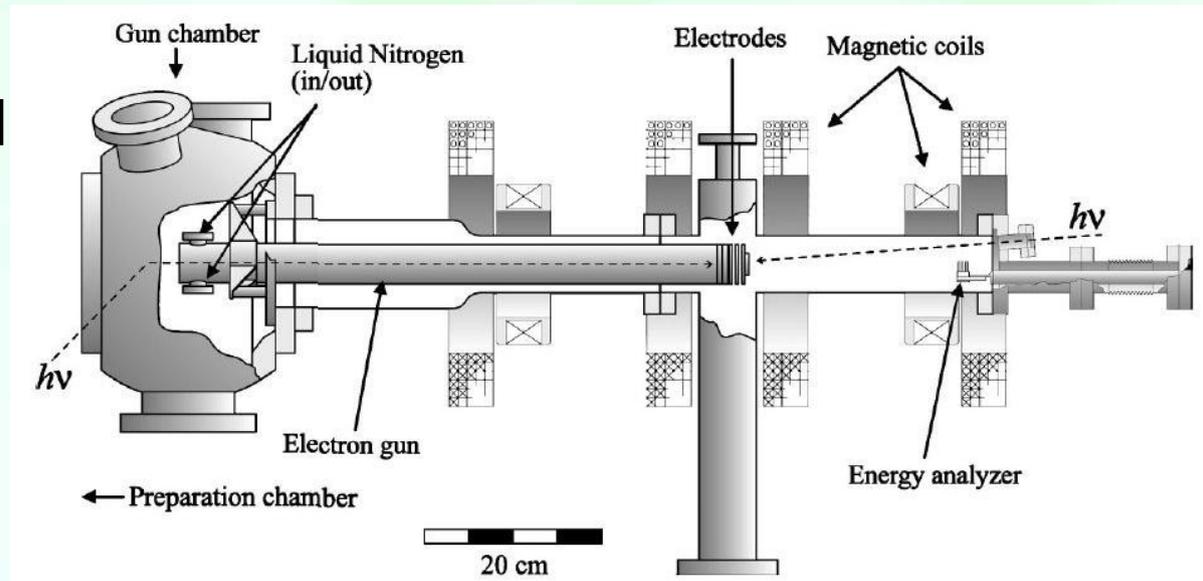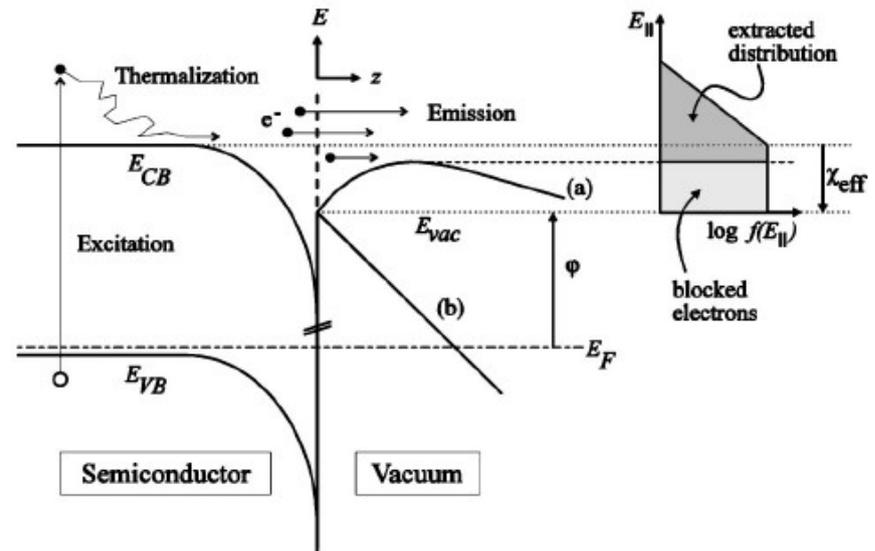$$\varepsilon_{nx} = \frac{R}{2} \sqrt{\frac{h\nu - \phi}{3mc^2} \frac{kT}{mc^2}} \qquad (2-49)$$

Introduction
Electron
Emission
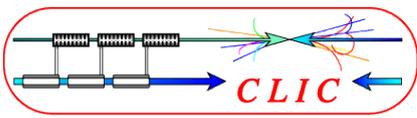**Related
Physics**
Electron
Gun
e- Source
for LCs
Laser
Summary

► Energy spread from GaAs photo-cathode is directly measured by blocked electrode.

► Only electrons above the block potential barrier, is observed.

► Cathode is placed longitudinal B field (immerse).

S. Pastuszka, JAP, 88(11), 6788-6800 (2000)

Adiabatic condition

$$\frac{\lambda}{B}\left|\frac{dB}{dz}\right| \leqslant 1 \qquad (2-50)$$

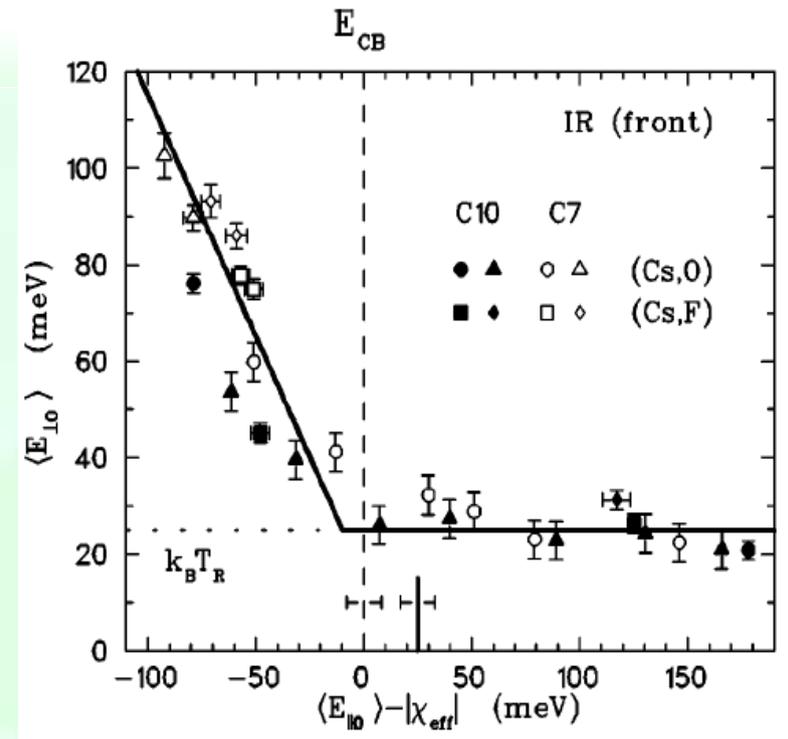ratio of transverse energy $E_\perp$ and magnetic flux B is an adiabatic constant,

$$\frac{E_\perp}{B} = const \qquad (2-51)$$

From the energy conservation

$$E_{\parallel f} = E_{\parallel i} + \left(1 - \frac{B_f}{B_i}\right) E_{\perp i} \qquad (2-52)$$

The initial transverse energy is obtained as

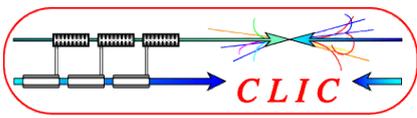$$\langle E_{\perp i} \rangle = -\frac{d\langle E_{\parallel f} \rangle}{d\alpha} \qquad (2-53)$$

$E_{\parallel i}$ = 25 meV is confirmed.

S. Pastuszka, JAP, 88(11), 6788-6800 (2000)

► Beam emittance from SL GaAs photocathode is measured by pepper-pot method.

► The beam image passing small holes (pepper-pot) are observed.

► The phase-space distribution is reconstructed from the image.
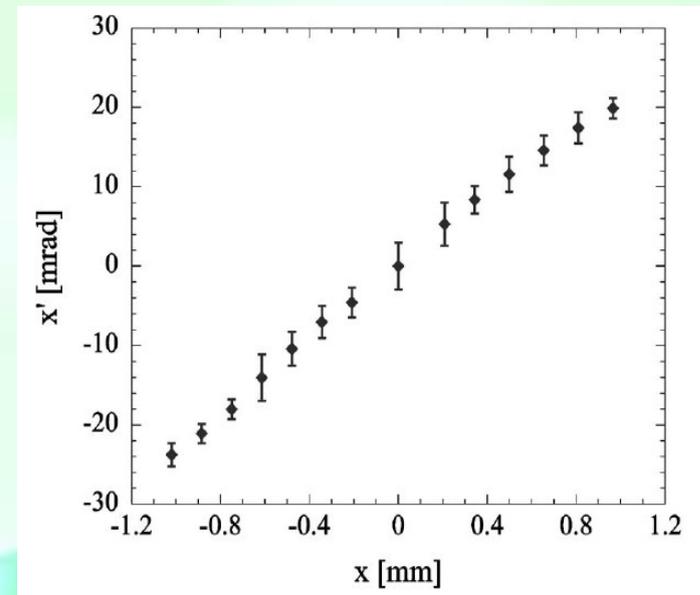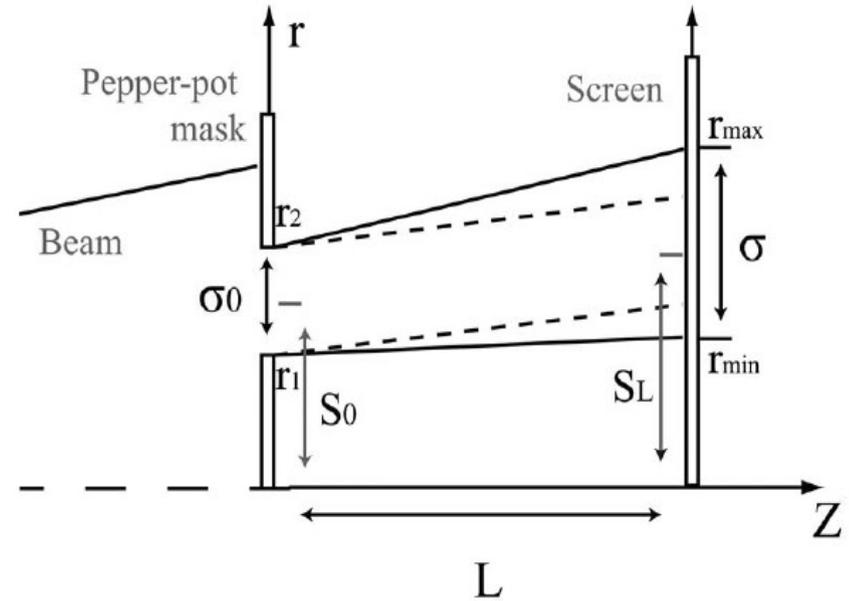
N. Yamamoto, JAP(102) 024904(2007)
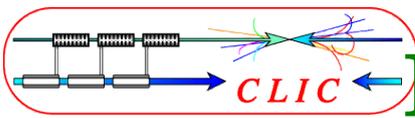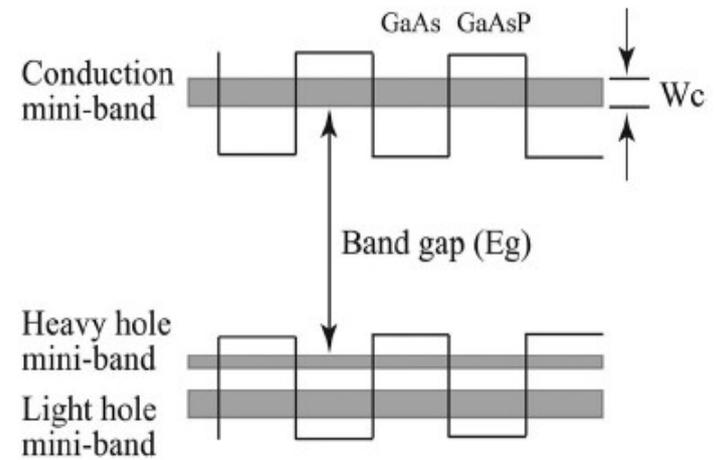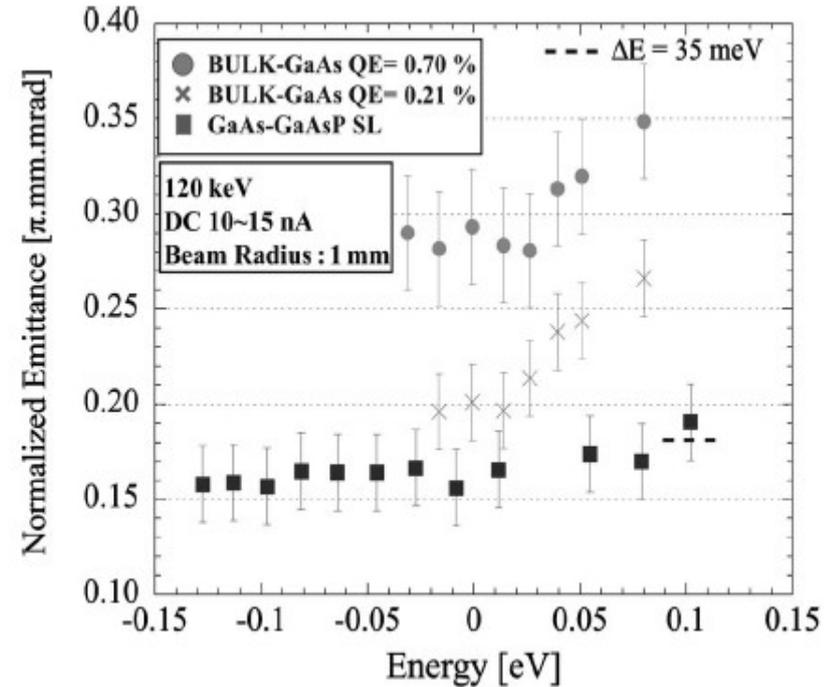
► Emittance is measure as a function of laser wave length.

► Comparing Super-lattice GaAs and bulk GaAs, SL has smaller emittance, especially for shorter wave length.

► It can be considered due to confinement of the excited electrons in the conduction mini-band.

► ε_x~0.16 mm.mrad is confirmed.

N. Yamamoto, JAP(102) 024904(2007)
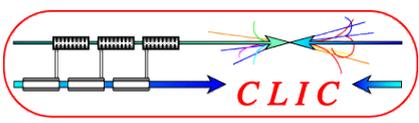
Bunch Compression (1)

▶ In any accelerator with RF field, the beam should be concentrated in a short period of longitudinal space for small energy spread;

– E=E_0cos(ωt-ks), wt-ks=0 for efficient acceleration.

▶ Bunch compressor and buncher shorten the bunch length down to an adequate size for acceleration.
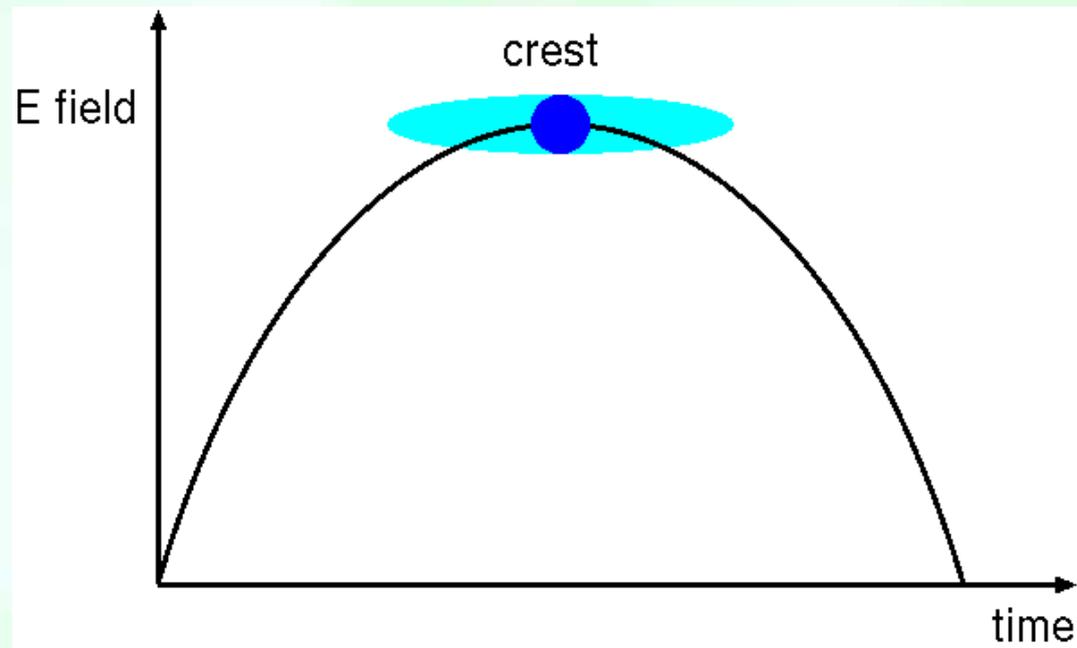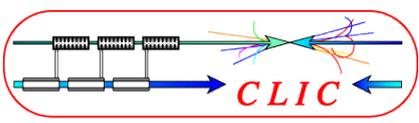
► There are two ways for bunching (bunch compression):
  – Velocity Bunching
  – Magnetic Bunching

► Velocity bunching is effective only for low energy region.
  – It is used for bunching after the source.
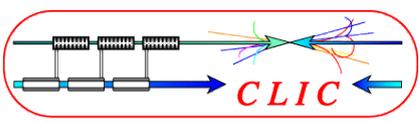 – Particle source can generate only long bunch or continuous beam.
  – It should be bunched for RF acceleration.

► Magnetic bunching is effective for all energy region.
  – It is used for bunching after the storage ring
  – Long bunch length tend to reduce beam instabilities in DR.
  – Thus, the bunch length is compressed after DR.

► Bunch compression is performed by velocity modulation within a bunch;

– Bunch head is decelerated.

– Bunch tail is accelerated.

► Velocity is modulated by this energy modulation according to

$$c\beta = c\sqrt{1 - \frac{1}{\gamma^2}} \qquad (2-54)$$

► Velocity is saturated to c at γ>>1. Then, it works only for low energy particle (β<1).

– Bunch compression at the injector.
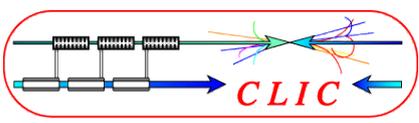
RF cavity voltage varies as a function of relative position (*t*) in a bunch
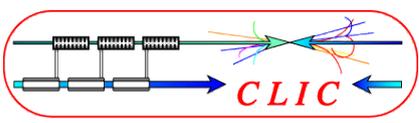
$$V = -V_0 \sin(\omega t) \qquad (2-55)$$

In linear approximation, energy modulation by the RF field depends on *t* as

$$\frac{dE}{E_0} \sim \frac{-eV_0}{E_0} \omega t \qquad (2-56)$$
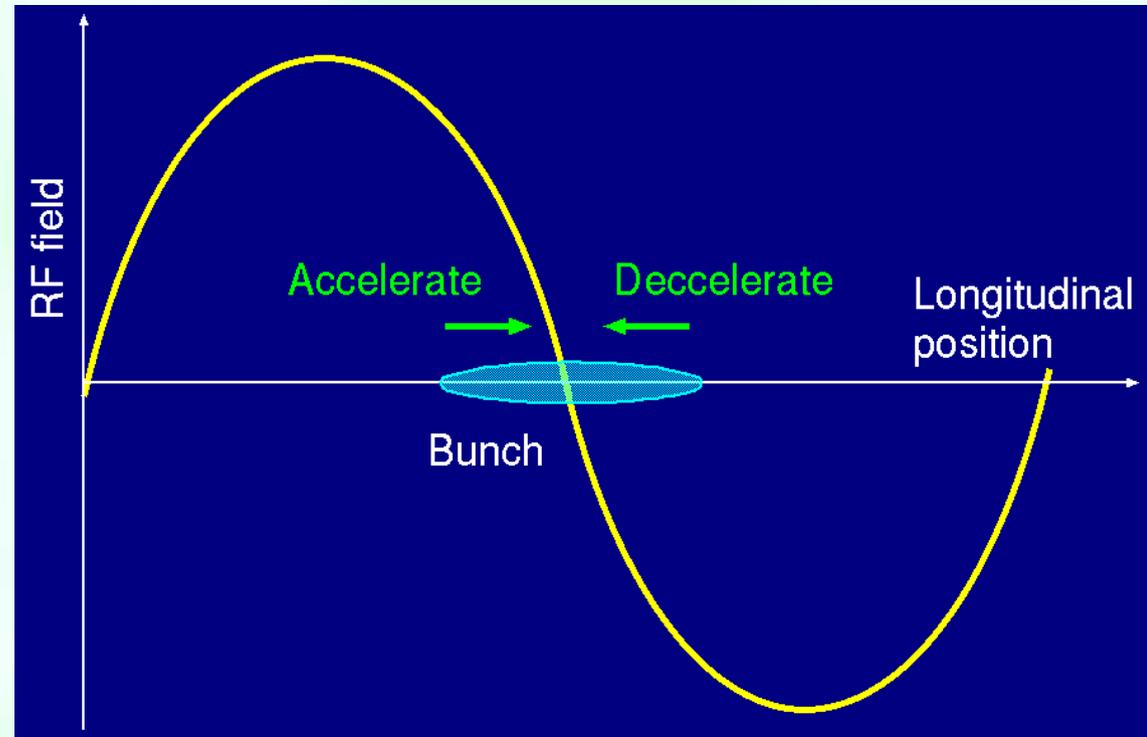
► Time to travel distance L is

► Relative particle position in a bunch to the bunch center is modulated as

► If dτ equals to -t, all particles are gathered at the bunch center, bunched.

► Because all electrons concentrate at t=0 position, RF phase of bunching determines the bunch longitudinal position.

$$\tau = \frac{L}{c\beta} \qquad (2-57)$$

$$d\tau = -\frac{L}{c\gamma^2\beta^3}\frac{dE}{E}$$

$$\sim -\frac{L}{c\gamma^2\beta^3}\frac{eV_0\omega}{E}t \qquad (2-58)$$
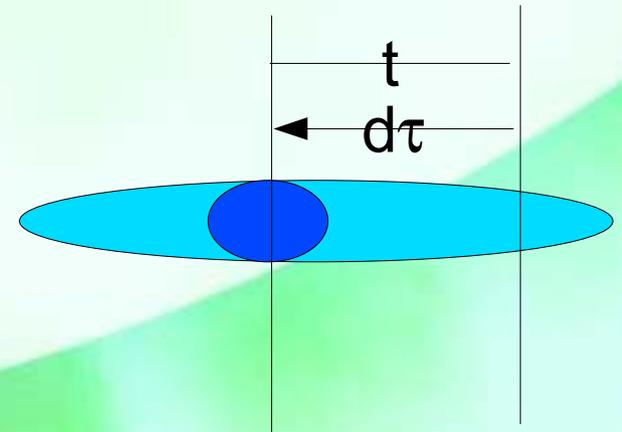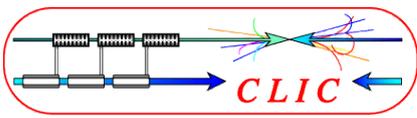
Magnetic Bunching (1)

▶ Bunch compression is performed by energy modulation with dispersive path length difference.

– Chicane, Wiggler, Arc, etc.

▶ A path length difference by a dispersive section, Δz is

$$\Delta z = \eta_l \frac{\Delta P}{P} \qquad (2-59)$$

$$\eta_l = \int_L ds \frac{\eta}{\rho} \qquad (2-60)$$

▶ It works well for any energy particle.
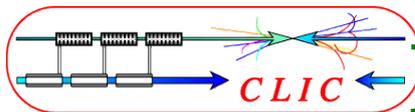
► Energy Modulation : RF cavity.

► Dispersive section : Chicane, Wiggler, Bend,..

    – E.g. four bending magnets compose a chicane

► Bunch head (tail) travels longer (shorter) path and bunch length becomes shorter.
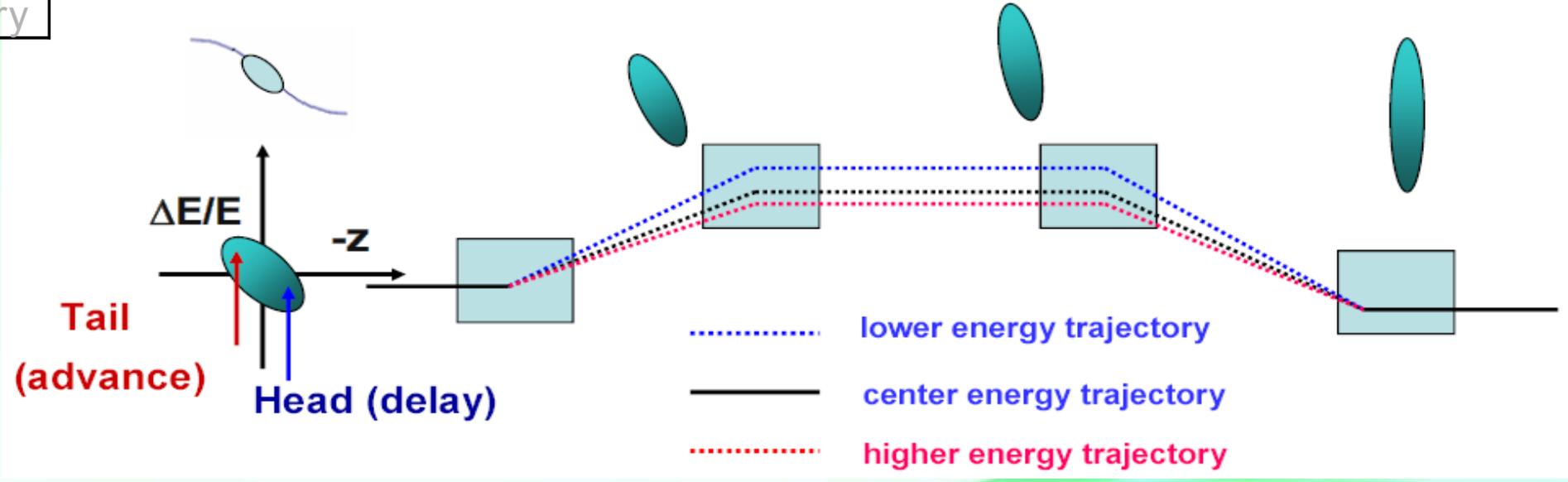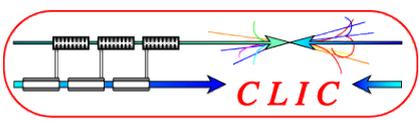

By E.S. Kim

Common formalism (1)

► For both bunching method, it can be formalized with transfer matrix in linear approximation.

► Energy modulation is made by RF (acc- and deceleration).

► Simple drift causes the rotation in phase space in case of velocity bunching.

► Drift through a dispersive section rotates the beam in case of magnetic bunching.

► For both cases, the bunch rotates 90 deg.
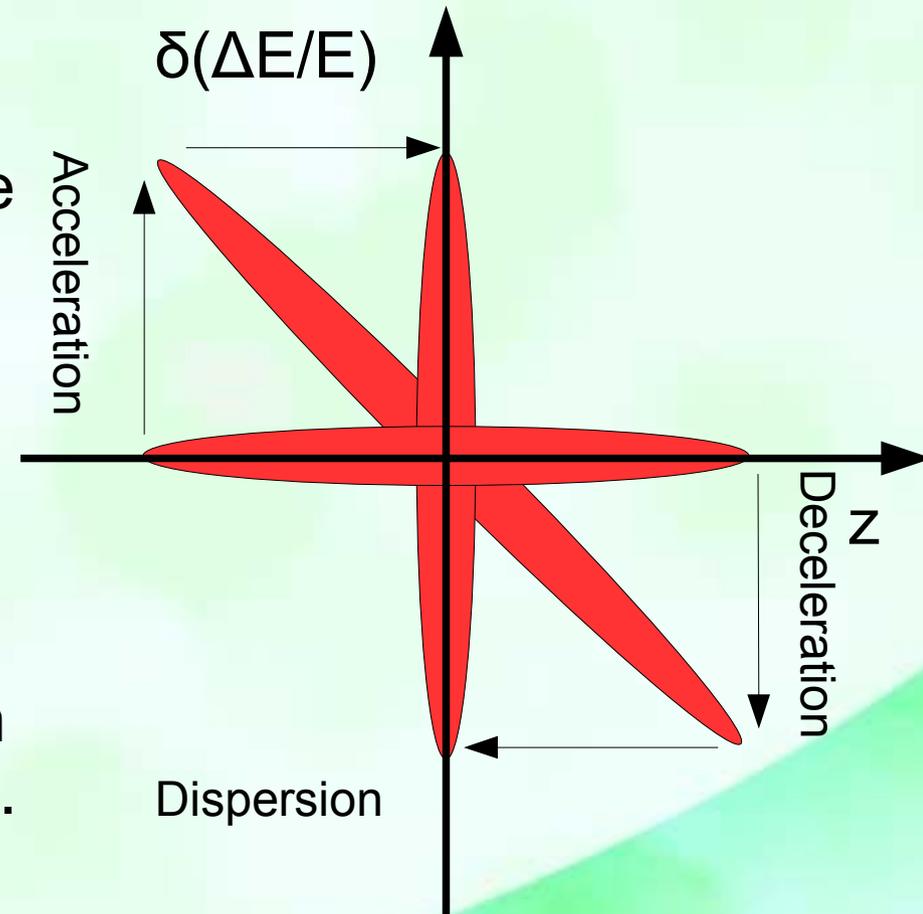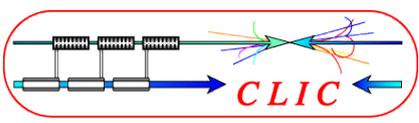
δ(ΔE/E)

Acceleration

Z Deceleration

Dispersion

R-matrices

Drift space (velocity bunching)

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-60)$$

$$R_{56} = -\frac{L}{\gamma^2 \beta^2} \quad (2-61)$$

Dispersive section (magnetic bunching)

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-62)$$

$$R_{56} = \eta_l = \int ds \frac{\eta}{\rho} \quad (2-63)$$
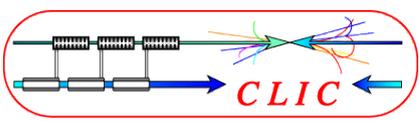
RF modulation

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-64)$$

$$R_{65} = \frac{1}{z} \frac{\Delta E}{E} \sim \pm \frac{eV_0}{E} \frac{\omega}{\beta c} \quad (2-65)$$

Total Transfer Matrix of BC section.

$$\begin{bmatrix} z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + R_{56} R_{65} & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} \qquad (2-66)$$
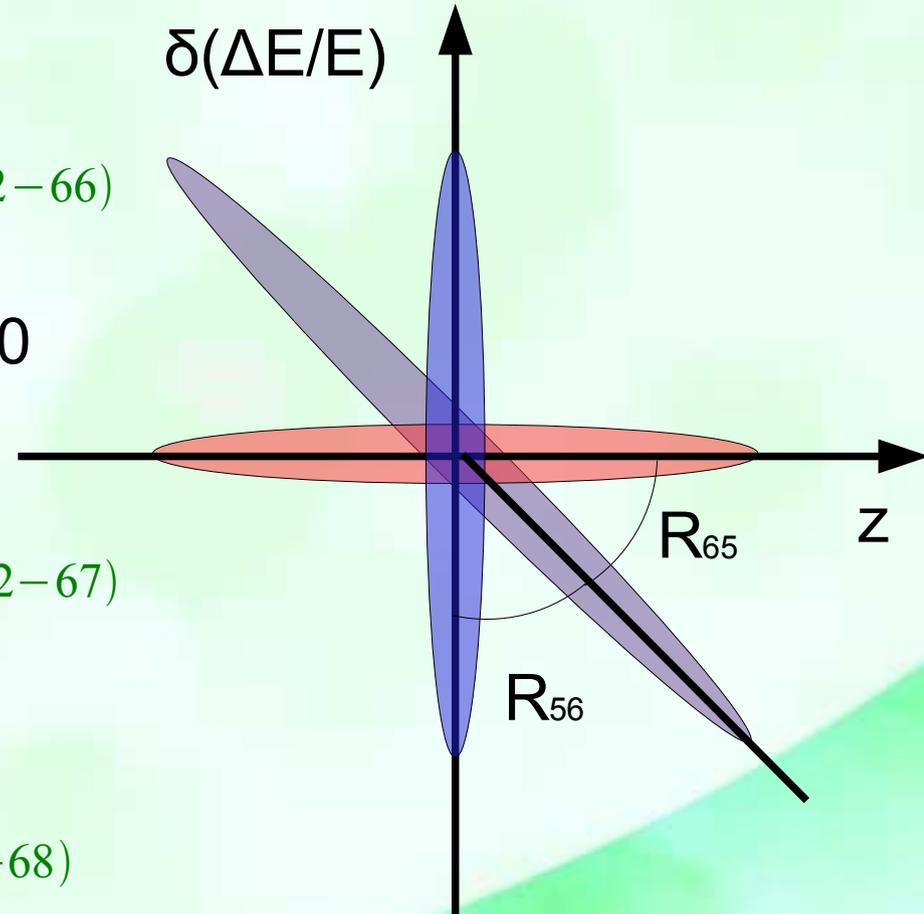
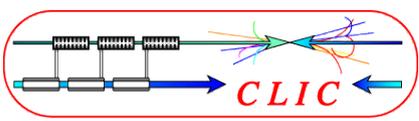Bunching condition : $1 + R_{56} R_{65} = 0$

Velocity bunching

$$1 + R_{56} R_{65} = 1 - \frac{L}{\gamma^2 \beta^2} \frac{eV_0}{E} \frac{\omega}{\beta c} = 0 \qquad (2-67)$$

Magnetic bunching

$$1 + R_{56} R_{65} = 1 + \eta_l \frac{eV_0}{E} \frac{\omega}{\beta c} = 0 \qquad (2-68)$$

δ(ΔE/E)

R_{65}
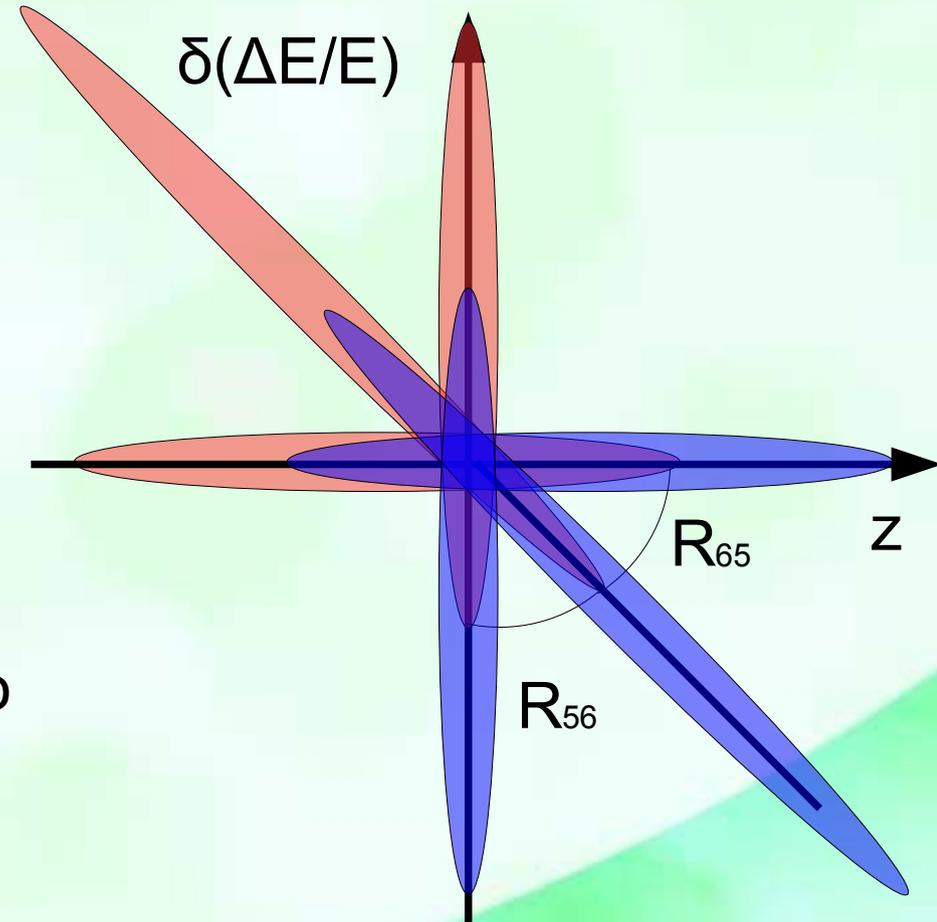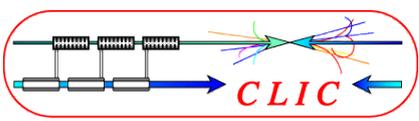
z

R_{56}

CLIC

When the bunching condition is satisfied,

$$\begin{bmatrix} z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} 0 & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} \qquad (2-68)$$

► The phase in the linac, $z(s_2)$, does not depends on $z(s_0)$. It is insensitive to fluctuation on $z(s_0)$.

► This is a good mechanism to stabilize the bunch phase.

$\delta(\Delta E/E)$

R_{65}

z

R_{56}

Final bunch length after an optimized BC section $(1+R_{56}R_{65}=0)$ is determined by the initial energy spread;

$$z_2 = R_{56}\,\delta_0 \qquad (2-69)$$

It can be understood by considering the transport of a reference point.

$$\begin{bmatrix} R_{56}\,\delta_0 \\ \delta_0 \end{bmatrix} = \begin{bmatrix} 0 & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \delta_0 \end{bmatrix} \qquad (2-70)$$

The actual bunch length is also limited by non-linearity on RF modulation and transfer matrix. To prevent such effects, several compressor sections are employed.

$\delta(\Delta E/E)$

δ_0

R_{56}

z