Room temperature RF
Part 2.1: *Strong beam-cavity coupling* (beam loading)

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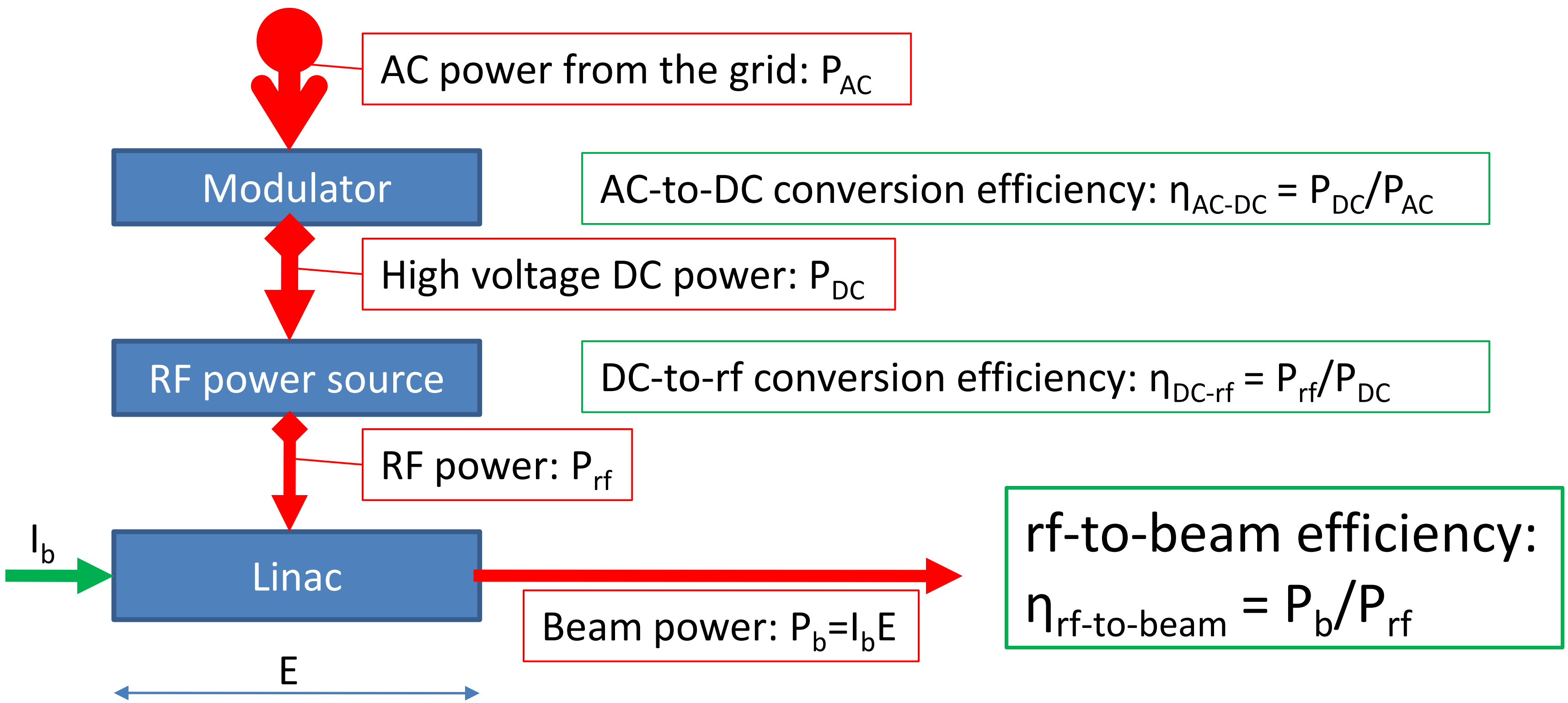
A.Grudiev

5th IASLC, Villars-sur-Ollon, CH

Outline

1. Linear collider and rf-to-beam efficiency
2. Beam loading effect: what is good and what is bad about it
    a. Beam-cavity coupling
        i. Steady-state power flow
        ii. Efficiency in steady-state and in pulsed regime
        iii. Compensation of the transient effect on the acceleration
    b. Standing wave structures (SWS)
    c. Travelling wave structure (TWS)
        i. Steady-state power flow
        ii. Efficiency in steady-state and in pulsed regime
        iii. Compensation of the transient effect on the acceleration
3. Examples:
    a. CLIC main linac accelerating structure
    b. CTF3 drive beam accelerating structure

Linear collider and rf-to-beam efficiency

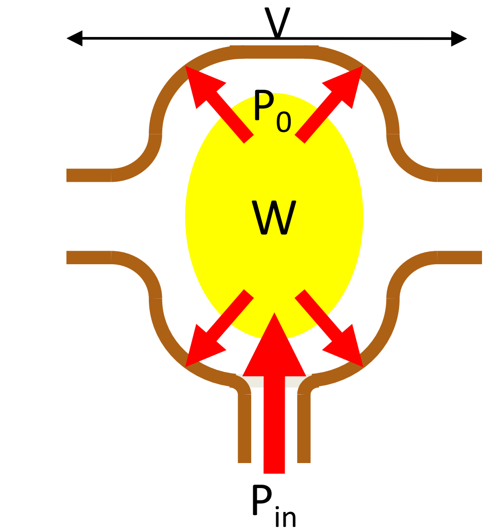


Two main parameters of a collider are center of mass collision energy E and luminosity L

For a fixed E: $L \sim I_b \sim P_b \sim P_{AC}\eta_{AC\text{-}DC}\eta_{DC\text{-}rf}\eta_{rf\text{-}to\text{-}beam}$

All efficiencies are equally important. In this lecture, we will focus on the $\eta_{rf\text{-}to\text{-}beam}$

Beam loading in steady-state (power flow)



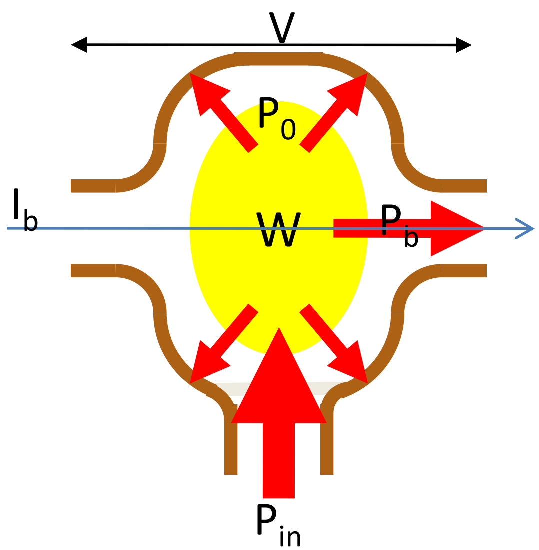
Cavity parameters w/o beam (reminder):

$$P_0 = \frac{V^2}{R}; \quad Q_0 = \frac{W\omega}{P_0}; \quad Q_{ext} = \frac{W\omega}{P_{in}};$$

$$\text{Matching condition: } P_{in} = P_0;$$

$$Q_{ext} = Q_0; \quad \beta = \frac{Q_0}{Q_{ext}} = 1$$

satisfied at any V



Cavity parameters with beam:

$$P_b = V\,I_b; \quad Q_b = \frac{W\omega}{P_b} = \frac{V\,Q_0}{I_b\,R} \neq const$$

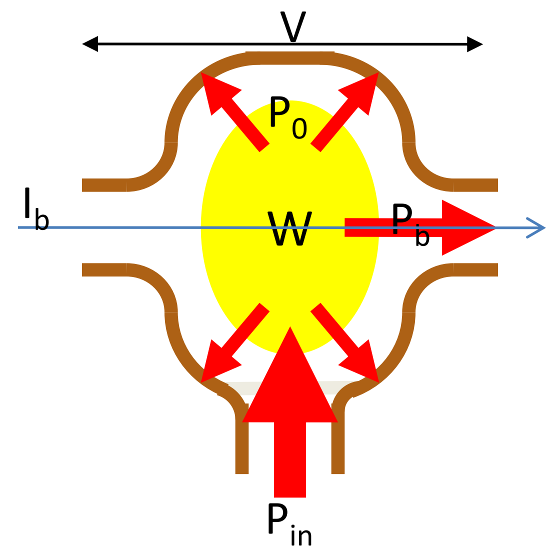
$$\text{Matching condition: } P_{in} = P_0 + P_b$$

$$\frac{1}{Q_{ext}} = \frac{1}{Q_0} + \frac{1}{Q_b}; \quad \beta = 1 + \frac{Q_0}{Q_b} = 1 + Y_b > 1$$

$$\text{where } Y_b = \frac{I_b R}{V} - \text{relative beam loading}$$

satisfied only if $V = I_b R$

Beam loading and efficiency

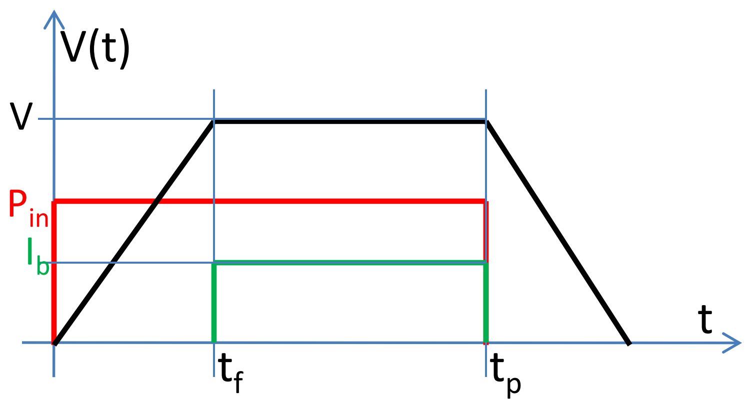


In steady-state regime which correspond to CW (Continues Wave) operation V = const:

$$\eta_{rf-to-beam}^{SWS} = \frac{P_b}{P_{in}} = \frac{Q_{ext}}{Q_b} = \frac{Y_b}{1+Y_b}$$

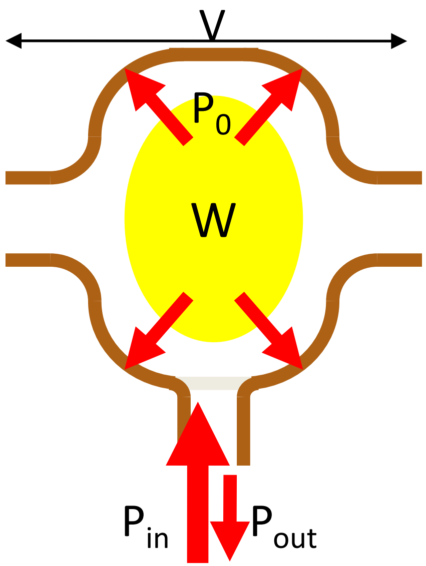
In pulse regime, V ≠ const is a function of time:

$$\eta_{rf-to-beam}^{pulsed} = \frac{P_b t_b}{P_{in} t_p} = \eta_{rf-to-beam} \frac{t_p - t_f}{t_p}$$

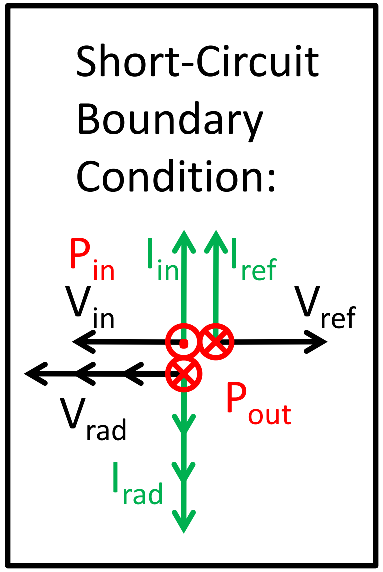


The higher is the beam loading the higher is the rf-to-beam efficiency ☺

Transient beam loading effect



Energy conservation in a cavity without beam (reminder):



$$P_{in} = P_{out} + P_0 + \frac{dW}{dt}$$

where

$$P_{in} = \frac{\hat{V}_{in}^2}{Z}; P_{out} = \frac{(\hat{V}_{rad} + \hat{V}_{ref})^2}{Z}; \hat{V}_{in} = -\hat{V}_{ref}$$

$$P_0 = \frac{\hat{V}^2}{R} = \frac{\omega W}{Q_0}; \quad \frac{\omega W}{Q_{ext}} = \frac{\hat{V}_{rad}^2}{Z}$$

what results in

$$2\hat{V}_{in}\sqrt{\frac{1}{\beta}\frac{R}{Z}} = \left(1 + \frac{1}{\beta}\right)\hat{V} + \frac{2Q_0}{\beta\omega}\frac{d\hat{V}}{dt}$$

for a step-function excitation

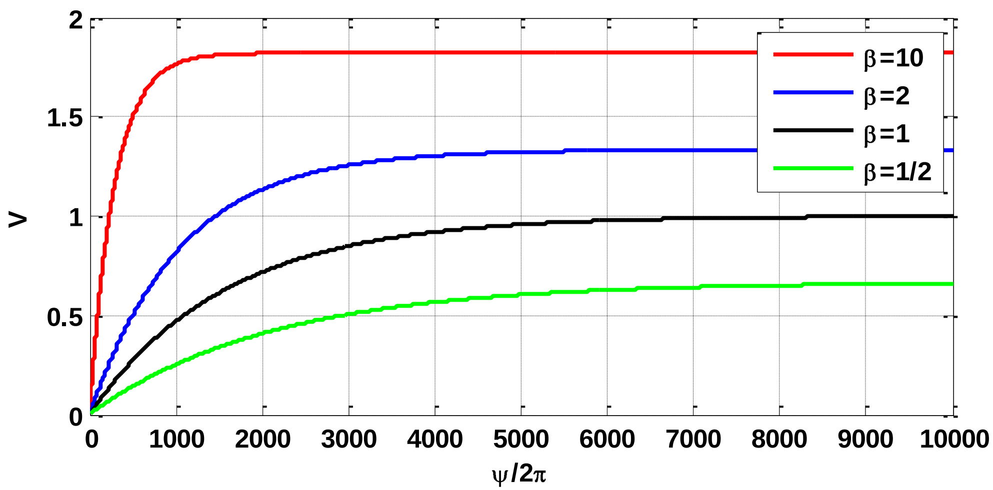
$$\hat{V}_{in}(t)\sqrt{\frac{1}{\beta}\frac{R}{Z}} = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases};$$

$$\hat{V}(t) = \frac{2\beta}{1+\beta}\left(1 - e^{-\frac{\omega t}{2Q_L}}\right);$$

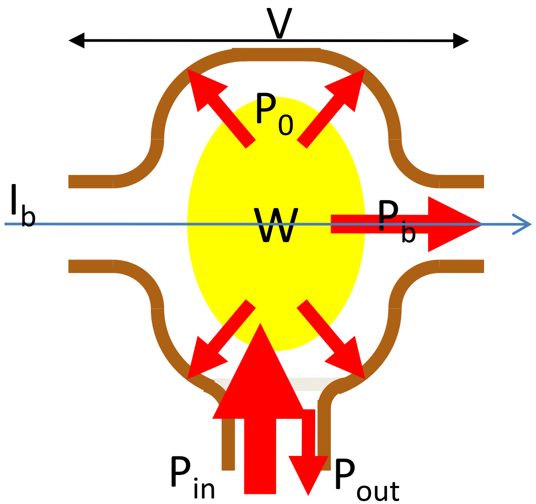
where:

$$Q_L = \frac{Q_0}{1+\beta} - \text{loaded } Q - \text{factor}$$

Matching condition: $V_{rad} = V_{in}$, only if $\beta=1$



Transient beam loading effect



Energy conservation in a cavity with beam:

$$P_{in} = P_b + P_{out} + P_0 + \frac{dW}{dt}$$

where $P_b = \hat{V}\,\hat{I}_b$

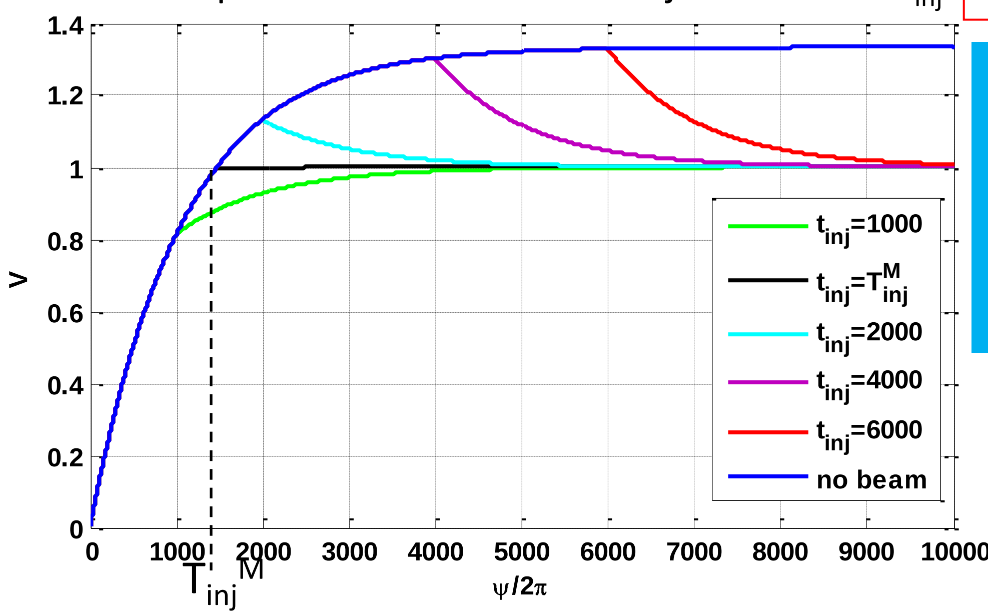
what results in

$$2\hat{V}_{in}\sqrt{\frac{1}{\beta}\frac{R}{Z}} - \hat{I}_b\frac{R}{\beta} = \left(1+\frac{1}{\beta}\right)\hat{V} + \frac{2Q_0}{\beta\omega}\frac{d\hat{V}}{dt}$$

for a step-function excitation

$$\hat{V}_{in}(t)\cdot\sqrt{\frac{1}{\beta}\frac{R}{Z}} = \begin{cases}0, t<0\\ 1, t\geq 0\end{cases};\quad \hat{I}_b(t)\cdot\frac{R}{\beta} = \begin{cases}0, t<t_{inj}\\ 1, t\geq t_{inj}\end{cases}$$

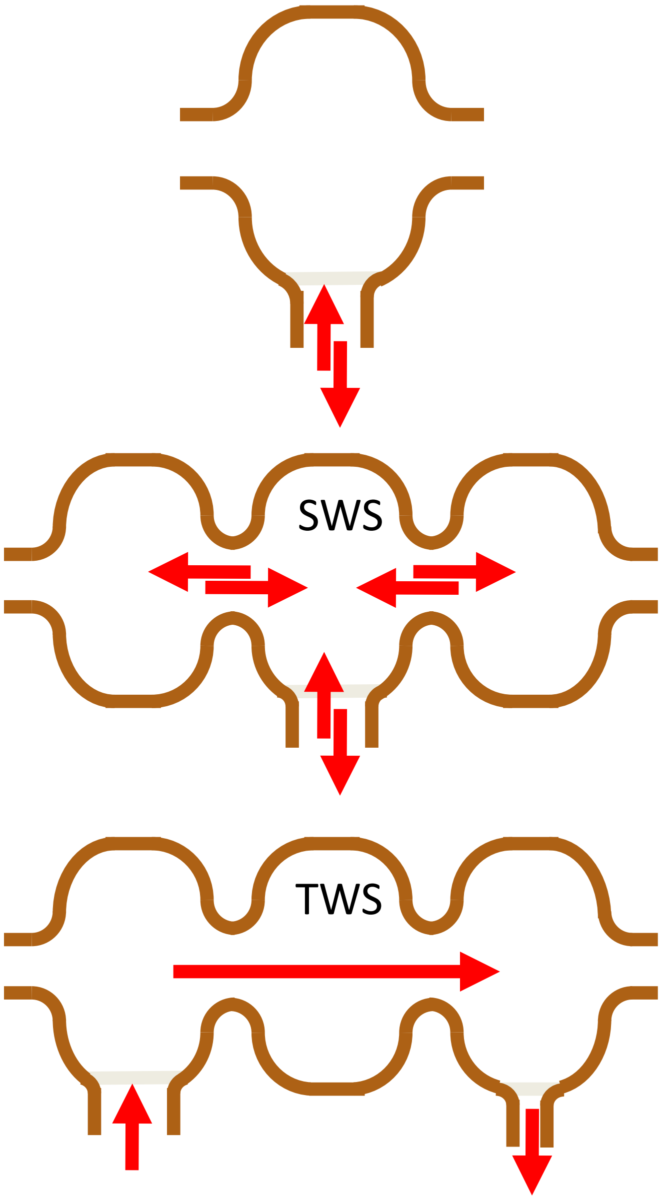
Solution for β=2 and different beam injection times t_{inj}



Depending on the t_{inj}, there is **transient beam loading** which is variation of the voltage gained in the cavity along the upstream part of the beam. It is compensated only when:

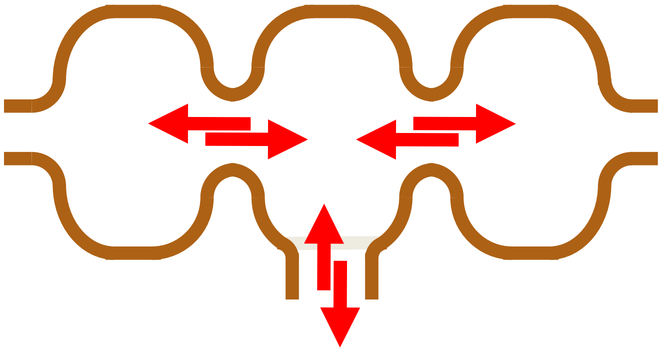
$$t_{inj} = T_{inj}^{M} = \frac{2Q_L}{\omega}\ln\left(\frac{2\beta}{\beta-1}\right) \cong t_f$$

From a cavity to a linac



- For obvious reasons multi cell accelerating structures are used in linacs instead of single cell cavities
- We will distinguish two types of accelerating structures:
  - Standing Wave Structures (SWS)
  - Travelling Wave Structures (TWS)
- In linacs, it useful to use some parameters normalized to a unit of length:
  - Voltage V [V] -> Gradient G [V/m] = V/L_c
  - Shunt impedance: R [Ω] -> R' [Ω/m] = R/L_c
  - Stored energy: W [J] -> W' [J/m] = W/L_c
  - Power loss: P [W] -> P' [W/m] = P/L_c
  - where L_c – cell length
-

Beam loading in SWS



Here it is assumed that the **coupling between cells** is much stronger than the **input coupling** and the **coupling to the beam** then the beam loading behavior in SWS is identical to the beam loading behavior in a single cell cavity.

SWS parameters w/o beam (reminder):

$$P'_0 = \frac{G^2}{R'}; \quad Q_0 = \frac{W'\omega}{P'_0}; \quad Q_{ext} = \frac{W\omega}{P_{in}};$$

$$\text{Matching condition: } P_{in} = P_0 = \int_0^{L_s} P'_0 dl;$$

satisfied at any G

$$Q_{ext} = Q_0; \quad \beta = \frac{Q_0}{Q_{ext}} = 1$$

SWS parameters with beam:

$$P'_b = G\, I_b; \quad Q_b = \frac{W'\omega}{P'_b} = \frac{G\,Q_0}{I_b\, R'} \neq const$$

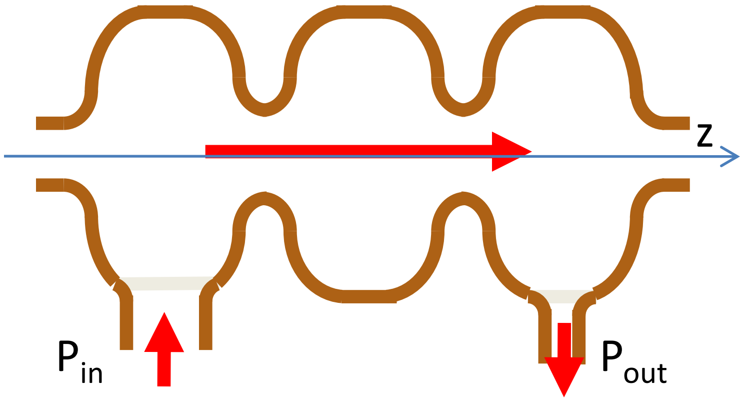
$$\text{Matching condition: } P_{in} = P_0 + P_b$$

satisfied only if G = $I_b R'$

$$\frac{1}{Q_{ext}} = \frac{1}{Q_0} + \frac{1}{Q_b}; \quad \beta = 1 + \frac{Q_0}{Q_b} = 1 + Y_b > 1$$

$$\text{where } Y_b = \frac{I_b R'}{G} - \text{relative beam loading}$$

Beam loading in TWS in steady-state



TWS parameters are function of z:

$$P_0' = \frac{G^2}{R'}; \quad Q_0 = \frac{W'\omega}{P_0'}; \quad - \text{ wall losses}$$

$$P = v_g \frac{G^2}{\omega R'/Q_0}; - \text{power flow along TWS}$$

$$\text{where: } v_g = \frac{P}{W'}; - \text{group velocity}$$

Efficiency in steady-state:

$$\eta_{rf-to-beam}^{CTWS} = \frac{P_b}{P_{in}} = \frac{I_b}{P_{in}} \int_0^{L_s} G(z)dz$$

In a cavity or SWS field amplitude is function of time only f(t).
In TWS, it is function of both time and longitudinal coordinate f(z,t).
Let's consider steady-state f(z).

Energy conservation law in steady-state yields:
(it is assumed hereafter that power flow is matched and there are no reflections)

$$\frac{dP}{dz} = -P_0' - P_b' = -\frac{G^2}{R'} - GI_b$$

$$\frac{dG}{dz} = -\frac{G}{2}\left[\frac{1}{v_g}\frac{dv_g}{dz} + \frac{1}{Q_0}\frac{dQ_0}{dz} - \frac{1}{R'}\frac{dR'}{dz} + \frac{\omega}{v_g Q_0}\right] - \frac{I_b R'}{2}\frac{\omega}{v_g Q_0}$$

$$\text{with } G\big|_{z=0} = G_0 = \sqrt{\frac{P_{in} R'\omega}{v_g Q_0}}; - \text{input boundary condition}$$

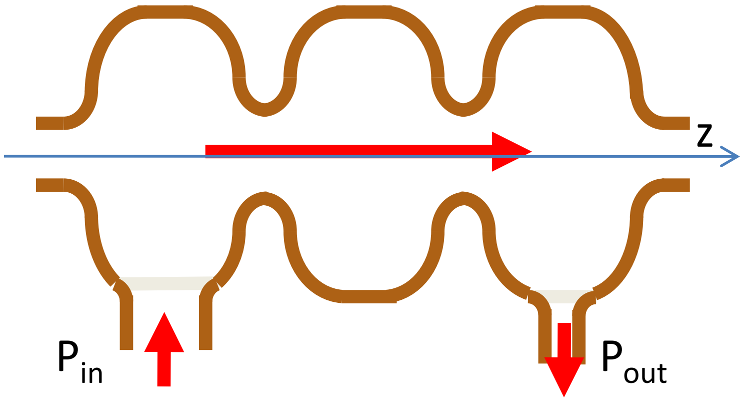
Solution is obtained in a closed form[*]:

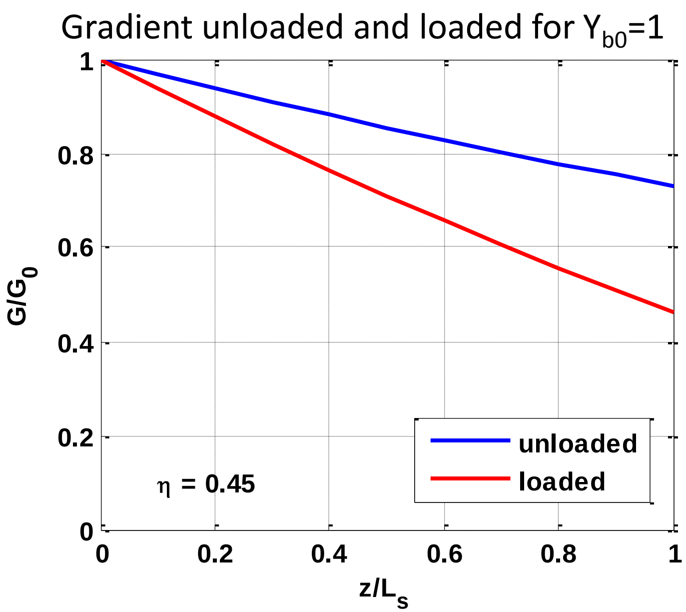
$$G(z) = G_0 \sqrt{\frac{v_g(0)}{v_g(z)}} \sqrt{\frac{Q_0(0)}{Q_0(z)}} \sqrt{\frac{R'(z)}{R'(0)}} \cdot e^{-\frac{1}{2}\int_0^z \frac{\omega}{Q_0(z) v_g(z)} dz}$$

$$G_l(z) = G(z)\left[1 - \int_0^z \frac{I_b}{G(z)} \frac{\omega R'(z)}{2 v_g(z) Q_0(z)} dz\right]$$

[*] – A. Lunin, V. Yakovlev, unpublished

TWS example 1: constant impedance TWS





In constant impedance TWS, geometry of the cells is identical and group velocity, shunt impedance and Q-factor are constant along the structure.

This simplifies a lot the equations:

$$G_l(z) = G_0 e^{-\alpha z} - I_b R' \cdot \left(1 - e^{-\alpha z}\right)$$

where

$$\alpha = \frac{\omega}{2Q_0 v_g}; \text{– TWS attenuation constant}$$

Efficiency in steady-state:

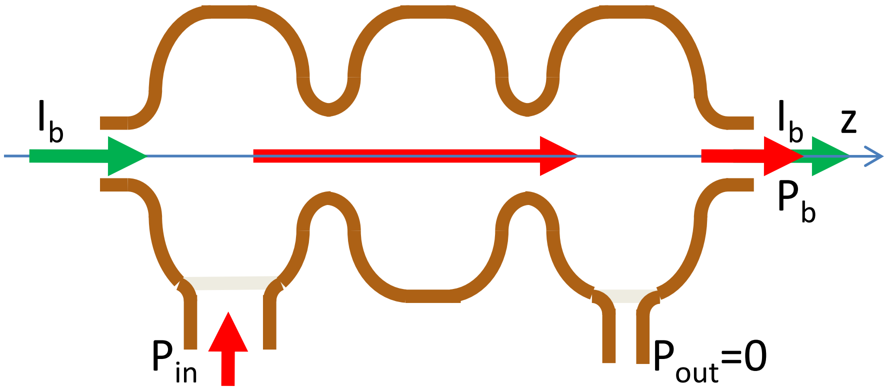
$$\eta_{rf-to-beam}^{TWS-CI} = 2Y_{b0}\left(1 - e^{-\alpha L_s}\right) + 2Y_{b0}^2\left(1 - \alpha L_s - e^{-\alpha L_s}\right)$$

where

$$Y_{b0} = \frac{I_b R'}{G_0}; \text{– TWS relative beam loading}$$

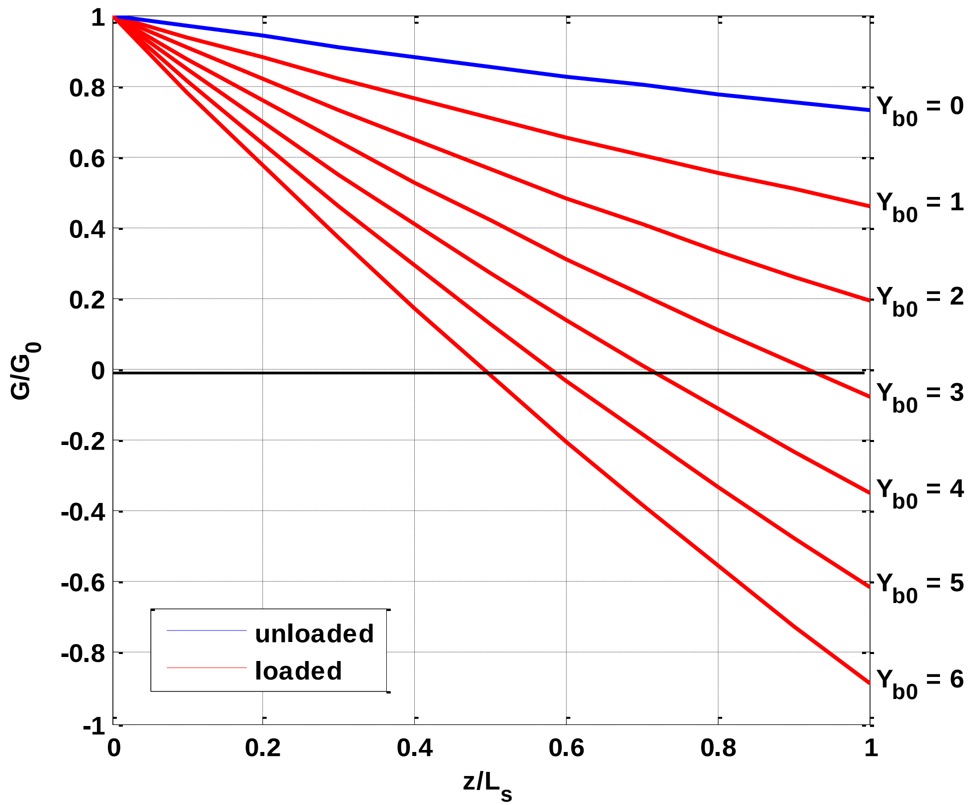
- The higher is the beam loading the higher is the rf-to-beam efficiency
- Beam loading reduces the loaded gradient compared to unloaded

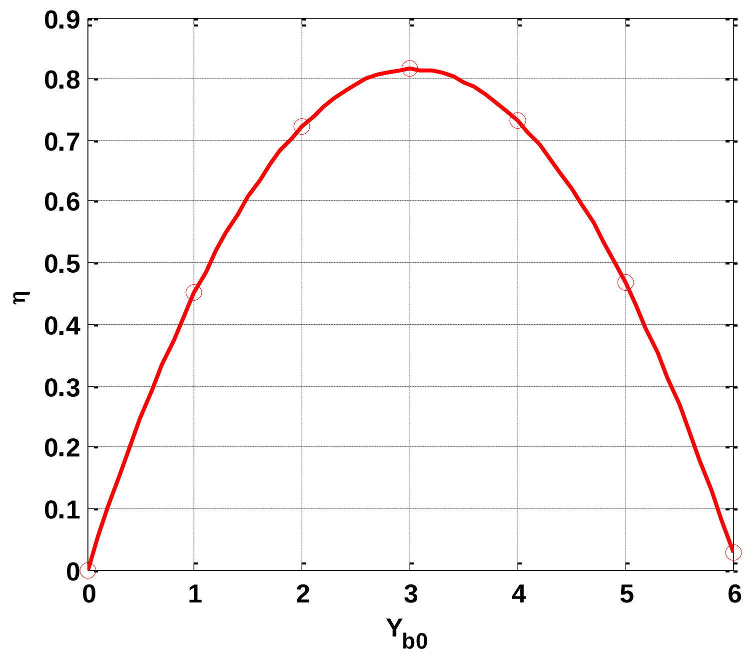
Full beam loading



In TWS, depending on the beam current, beam can absorb all available rf power so that $\mathbf{P_{out} = P_{in} - P_0 - P_b = 0}$

This is the case of **full beam loading** where rf-to-beam efficiency is closed to maximum.

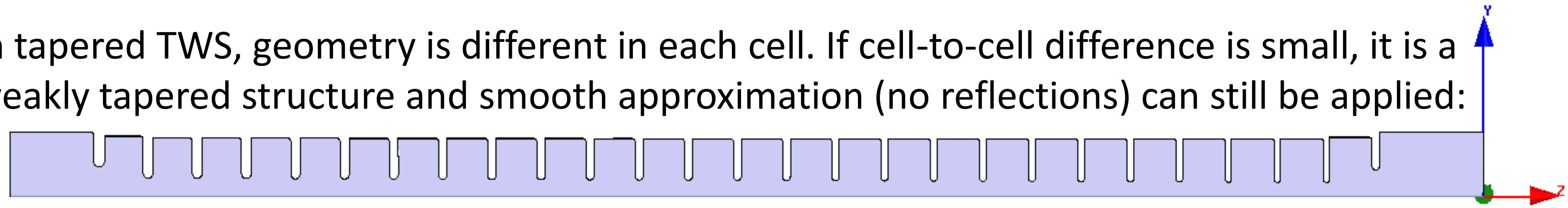




Structure is overloaded if the beam current is even higher

TWS example 2: tapered TWS

In tapered TWS, geometry is different in each cell. If cell-to-cell difference is small, it is a weakly tapered structure and smooth approximation (no reflections) can still be applied:

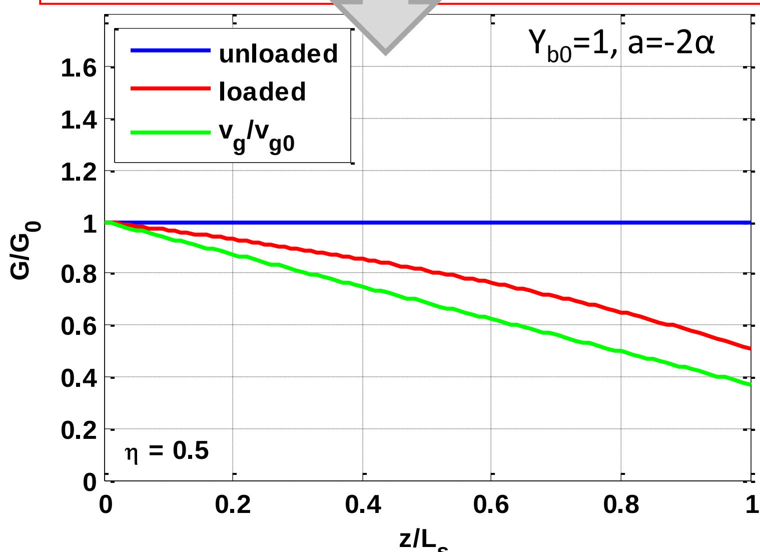


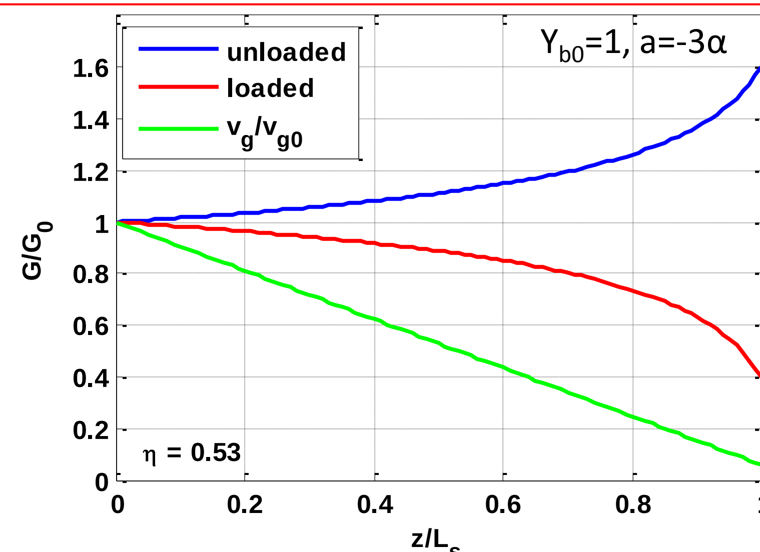
Let's consider a case of linear v_g-tapering such that Q_0=const, R'=const but $v_g = v_{g0}(1+az)$:

$$G(z) = G_0 (1+az)^{-\left(\frac{\alpha_0}{a}+\frac{1}{2}\right)}; \text{– unloaded; } G_l(z) = G(z) - I_b R' \frac{\alpha_0}{a} \frac{1}{\left(\frac{\alpha_0}{a}+\frac{1}{2}\right)} \left[1 - (1+az)^{-\left(\frac{\alpha_0}{a}+\frac{1}{2}\right)}\right]; \text{– loaded,}$$

$$\text{Const gradient TWS: } G(z) = G_0;\ a = -2\alpha_0; \quad G_l(z) = G_0 - I_b R' \frac{\alpha_0}{a} \ln(1+az); \text{ where: } \alpha_0 = \alpha(0)$$

$$\text{Stronger tapered TWS: } G(z>0) > G_0;\ -1/L_s < a < -2\alpha_0$$





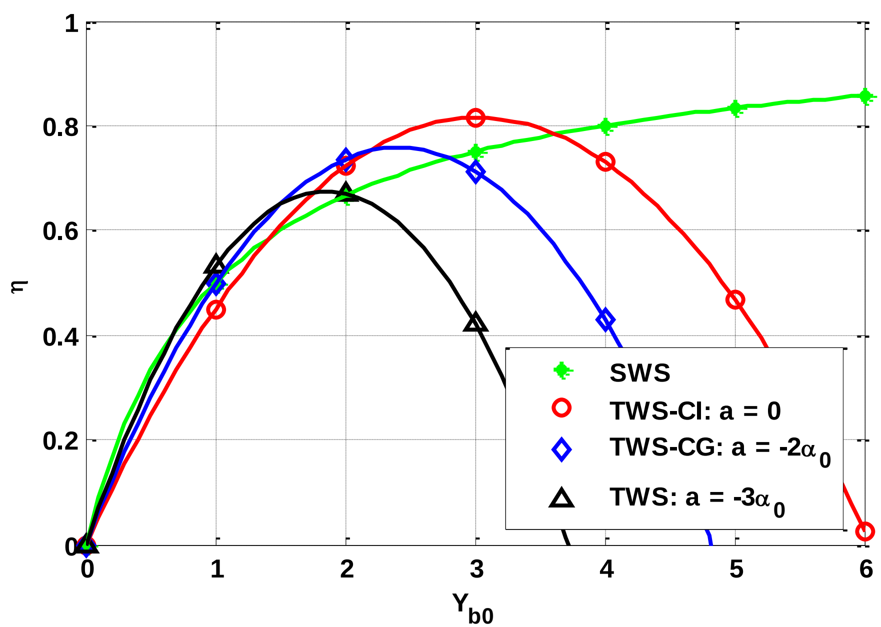
$W' = P/v_g \approx$ const

Tapering and rf-to-beam efficiency

For the case of linear v_g-tapering such that Q_0=const, R'=const but $v_g = v_{g0}(1+az)$
Efficiency in steady-state:

$$\eta^{TWS}_{rf-to-beam} = 2Y_{b0}\frac{\alpha_0}{a}\frac{1}{\frac{1}{2}-\frac{\alpha_0}{a}}\left[\left(1+aL_s\right)^{\frac{1}{2}-\frac{\alpha_0}{a}}-1\right] - 2Y_{b0}^2\frac{\alpha_0^2}{a}\frac{1}{\frac{1}{2}+\frac{\alpha_0}{a}}\left[1-\frac{1}{a}\frac{1}{\frac{1}{2}-\frac{\alpha_0}{a}}\left[\left(1+aL_s\right)^{\frac{1}{2}-\frac{\alpha_0}{a}}-1\right]\right]$$

$$\eta^{TWS-CG}_{rf-to-beam} = 2Y_{b0}\alpha_0 L_s - \frac{1}{2}Y_{b0}^2\left[\left(1+aL_s\right)\left(\ln\left(1+aL_s\right)-1\right)+1\right] \text{ for: } a=-2\alpha_0;$$



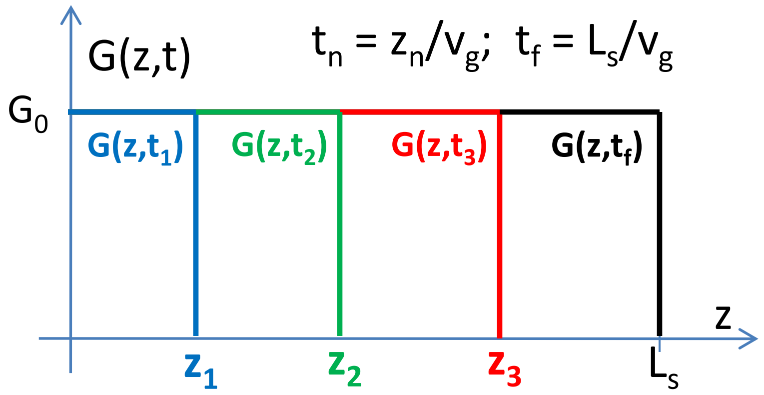
- The stronger is the tapering the higher is the efficiency at low beam loading
- The stronger is the tapering the low is the Y_{b0} for the maximum efficiency (full beam loading)
- SWS has higher efficiency than TWS at low beam loading BUT lower efficiency than fully beam loaded TWS

For low beam loading tapering helps to increase rf-to-beam efficiency

TWS efficiency in pulsed regime

In pulse regime, V ≠ const is a function of time.

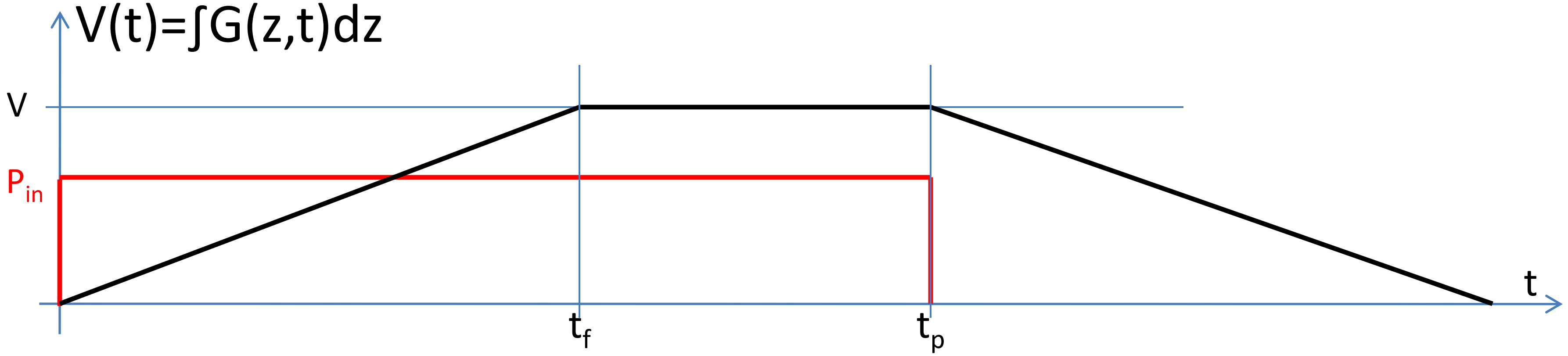
In simplest case of v_g=const, R'/Q_0=const, Q_0=∞



In general, in TWS:

$$\eta_{rf-to-beam}^{pulsed} = \frac{P_b t_b}{P_{in} t_p} = \eta_{rf-to-beam} \frac{t_p - t_f}{t_p}$$

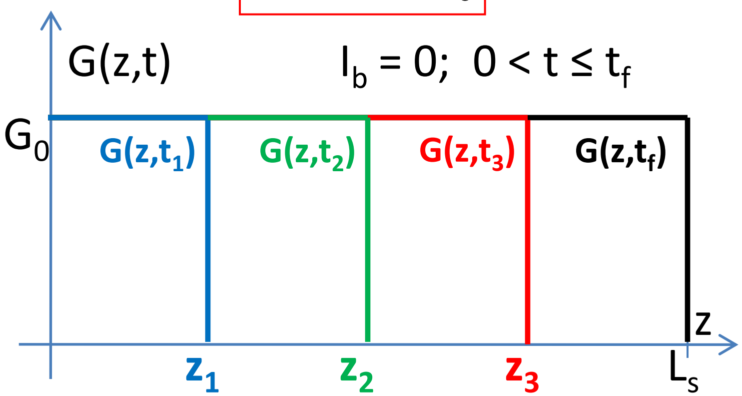
$$\text{where: } t_f = \int_0^{L_s} \frac{dz}{v_g(z)}; \text{–filling time}$$



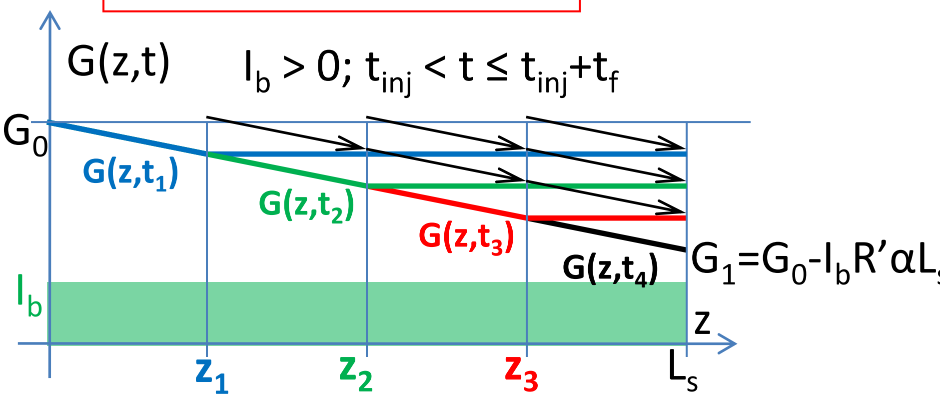
Transient beam loading in TWS

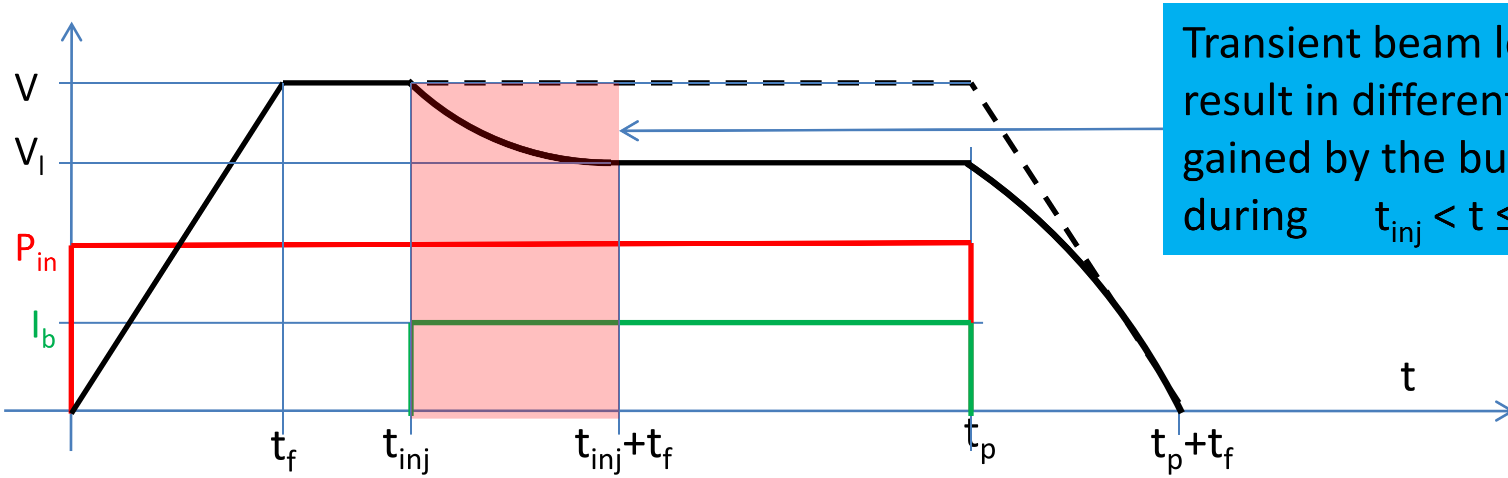
Let's continue with the simplest case of v_g=const, R'/Q_0=const, $Q_0=\infty$

$$G(z) = G_0$$



$$G_l(z) = G_0 - I_b R' \alpha z$$

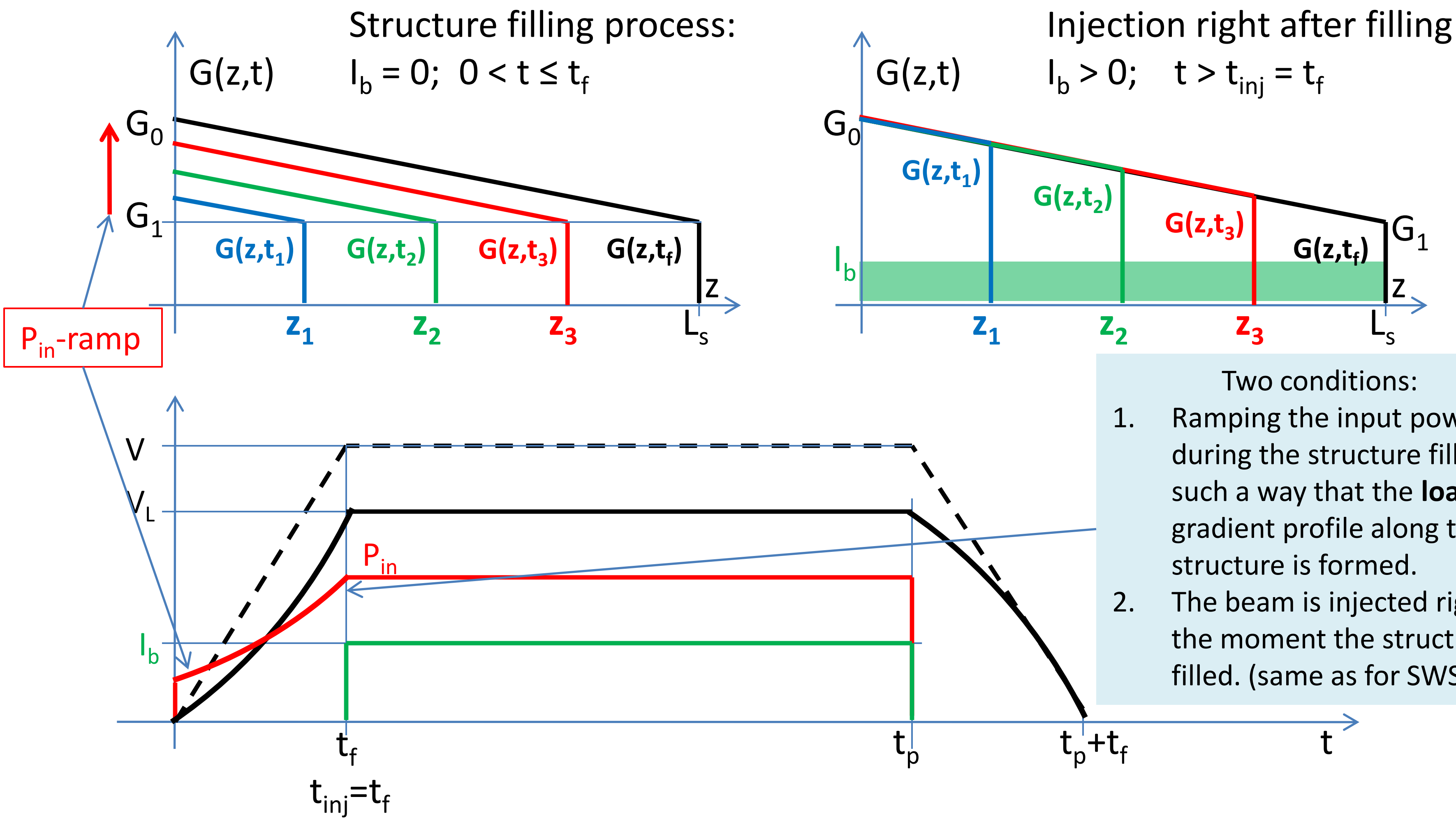




Transient beam loading result in different voltage gained by the bunches during $t_{inj} < t \leq t_{inj}+t_f$

Compensation of the transient beam loading in TWS

Let's continue with the simplest case of v_g=const, R'/Q_0=const, $Q_0=\infty$



Two conditions:

1. Ramping the input power during the structure filling in such a way that the **loaded** gradient profile along the structure is formed.
2. The beam is injected right at the moment the structure is filled. (same as for SWS)

Transient beam loading in TWS

Let's consider general case of $v_g(z)$, $R'(z)$, $Q_0(z)$

$$1.\ \text{Steady-State solution: } G_0 = const;\ I_b = const$$

$$G(z) = G_0 g(z)$$

$$G_l(z) = G(z) - G_b(z) = G_0 g(z) - g(z)\int_0^z I_b \frac{f(z)}{g(z)} dz$$

$$2.\ \text{Time-dependent model: } G_0 = G_0(t);\ I_b = I_b(t)$$

$$G(z,t) = G_0\left(t - \tau(z)\right) g(z)$$

$$G_b(z,t) = g(z)\int_0^z I_b\left(t - \tau(z)\right)\frac{f(z)}{g(z)} dz$$

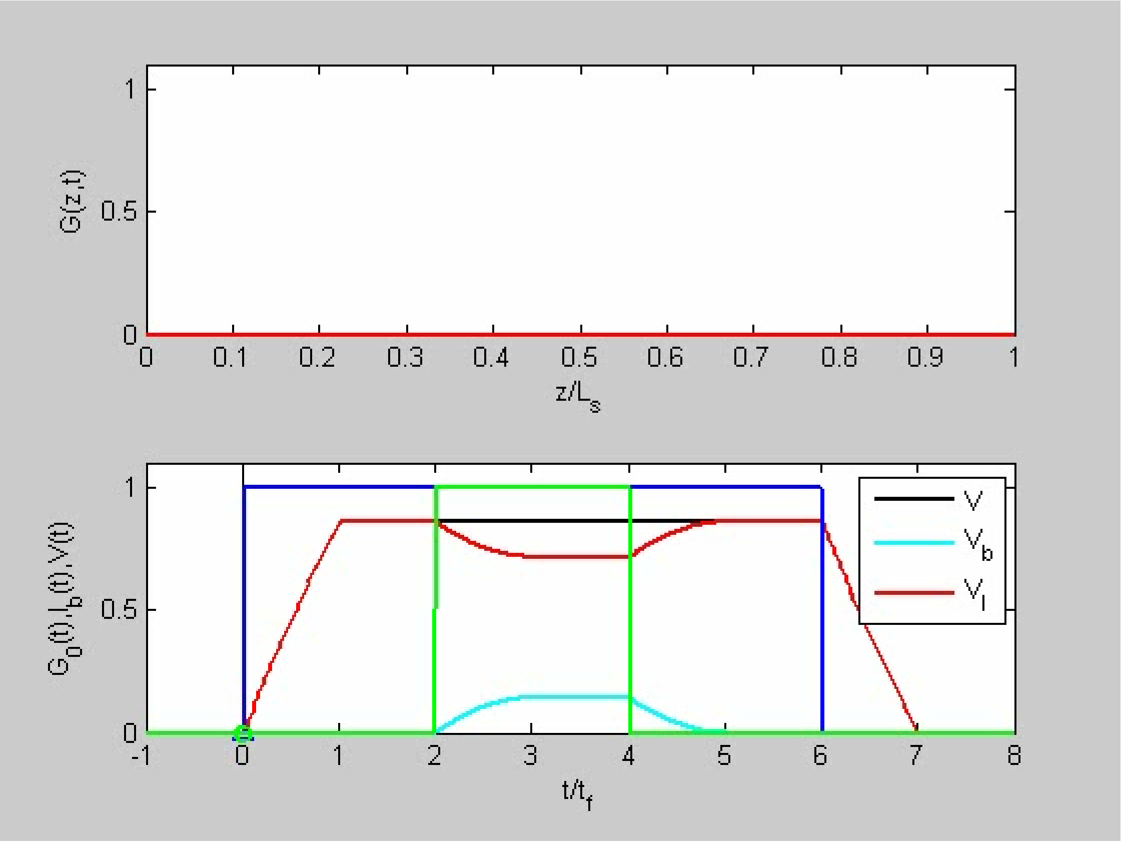
$$where: \tau(z) = \int_0^z \frac{dz}{v_g(z)};\ -\ \text{time to arrive to } z$$

$$G_l(z,t) = G(z,t) - G_b(z,t)$$

Model approximations:

1. Smooth propagation of energy along the structure (no reflections)
2. No effects related to the finite bandwidth of the signal and the structure (TWS bandwidth >> signal bandwidth)
3. Time of flight of the beam through the structure is much **shorter** than the filling time ($L_s/c \ll t_f$)

Transient beam loading in TWS (cont.)



where:

$$V_l(t) = V(t) - V_b(t); \quad \text{– loaded voltage}$$

$$V(t) = \int_0^z G(z,t)\,dz; \quad \text{– unloaded voltage}$$

$$V_b(t) = \int_0^z G_b(z,t)\,dz; \quad \text{– beam voltage}$$

Compensation of the transient beam loading in TWS

In general case of v_g=const, R'=const, Q_0=const

If in steady-state:

$G(z) = G_0 g(z)$ is unloaded solution and

$G_l(z)$ is loaded solution

Then input gradient during filling time:

$$G_0 = G_0(t);\ I_b = 0;\quad 0 < t < t_f$$

$$G_0(t) = \frac{G_l\left(z(t_f - t)\right)}{g\left(z(t_f - t)\right)}$$

where: $z(t)$ is the solution of $t(z) = \int_0^z \frac{dz}{v_g(z)};$

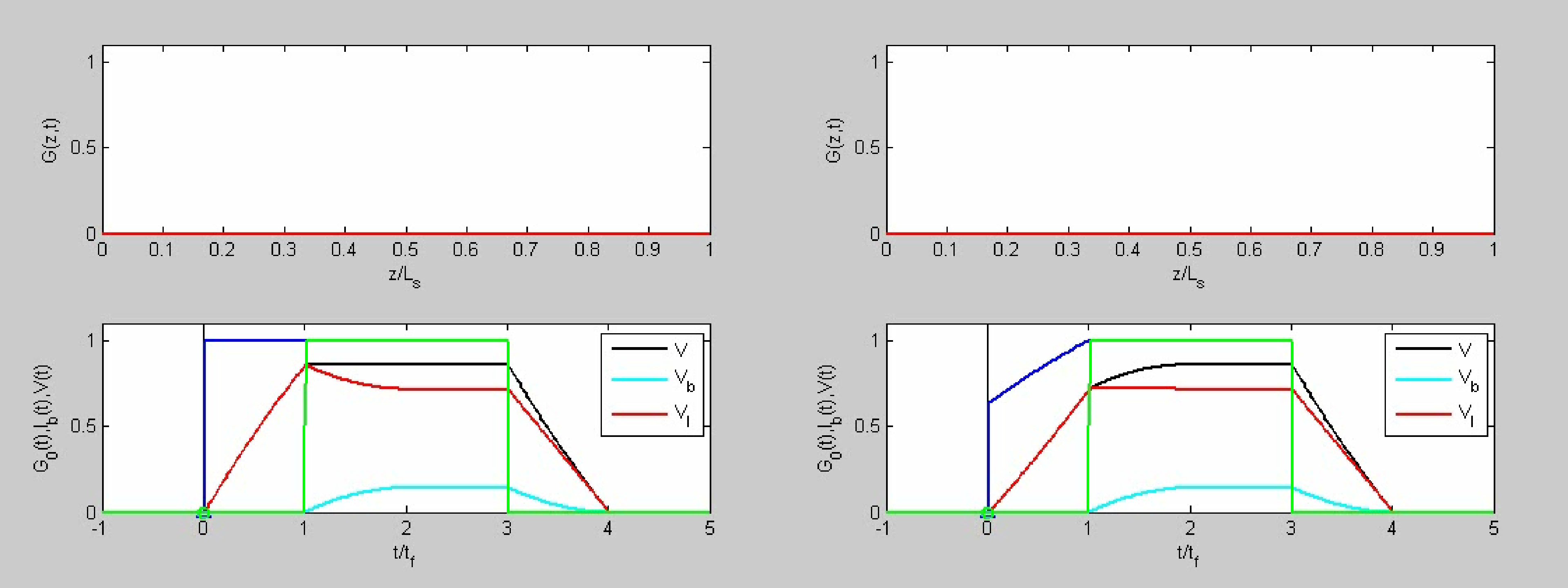
Compensation of the transient beam loading in constant impedance TWS

Let's consider the case of constant impedance TWS: v_g=const, R'=const, Q_0=const

$$G(z) = G_0 e^{-\alpha z}; \quad G_l(z) = G_0 e^{-\alpha z} - I_b R' \cdot \left(1 - e^{-\alpha z}\right); \quad \text{steady-state solution and} \quad t(z) = \int_0^z \frac{dz}{v_g(z)} = \frac{z}{v_g}$$

$$\text{when the ramp-function for } 0 < t < t_f = \frac{L_s}{v_g}: \; G_0(t) = G_0(t_f) - I_b R' \cdot \left(e^{\alpha v_g (t_f - t)} - 1\right)$$

Uncompensated case | Compensated case

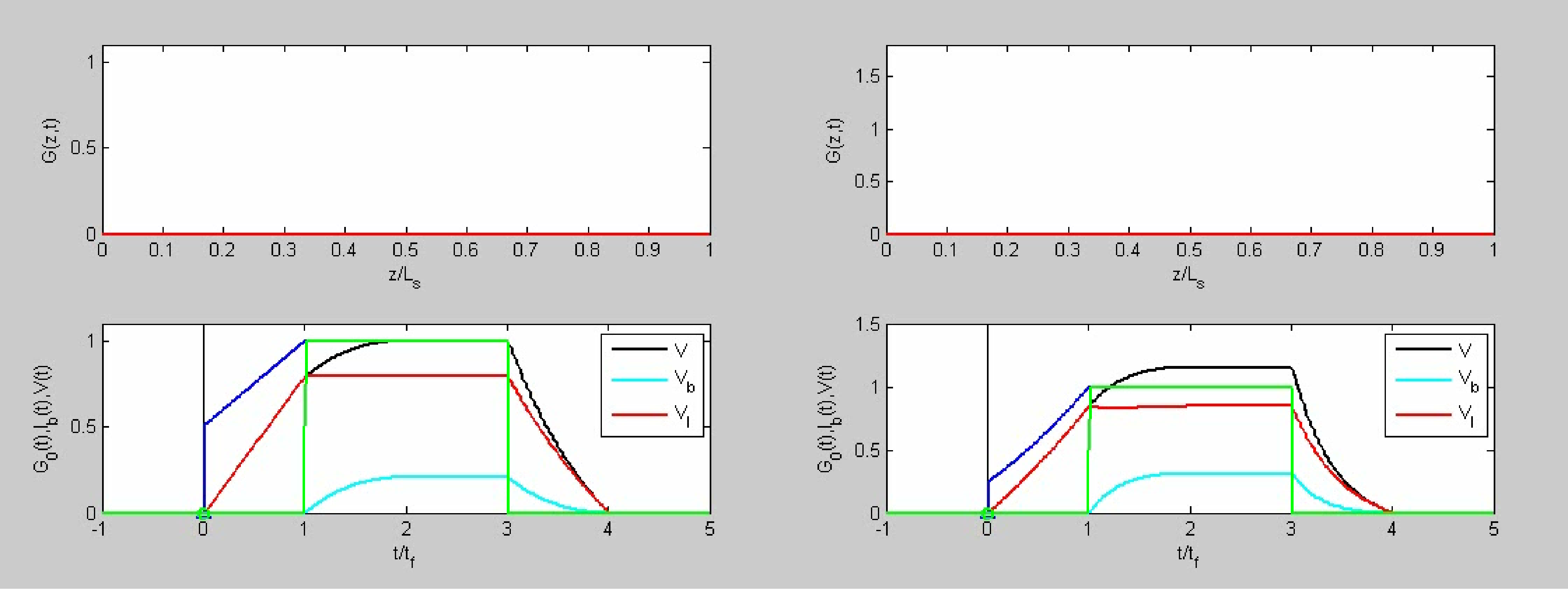


Compensation of the transient beam loading in constant gradient TWS

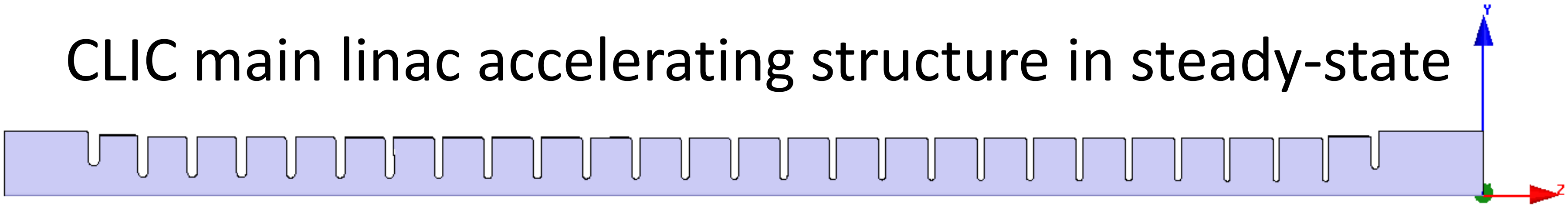
Let's consider a case of linear v_g-tapering such that Q_0=const, R'=const but $v_g = v_{g0}(1+az)$:

Constant gradient: $a=-2\alpha_0$

$a=-3\alpha_0$



CLIC main linac accelerating structure in steady-state

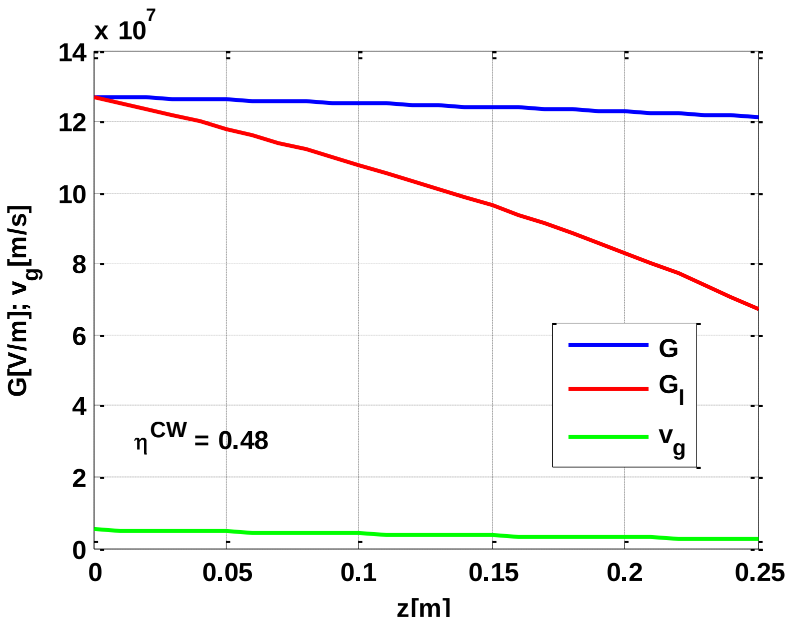


Parameters: input – output	
f [GHz]	12
L_s [m]	0.25
v_g/c [%]	1.7 – 0.8
Q_0	6100 – 6300
R’ [MΩ/m]	90 – 110
I_b [A]	1.3
t_b [ns]	156
<G> [MV/m]	100

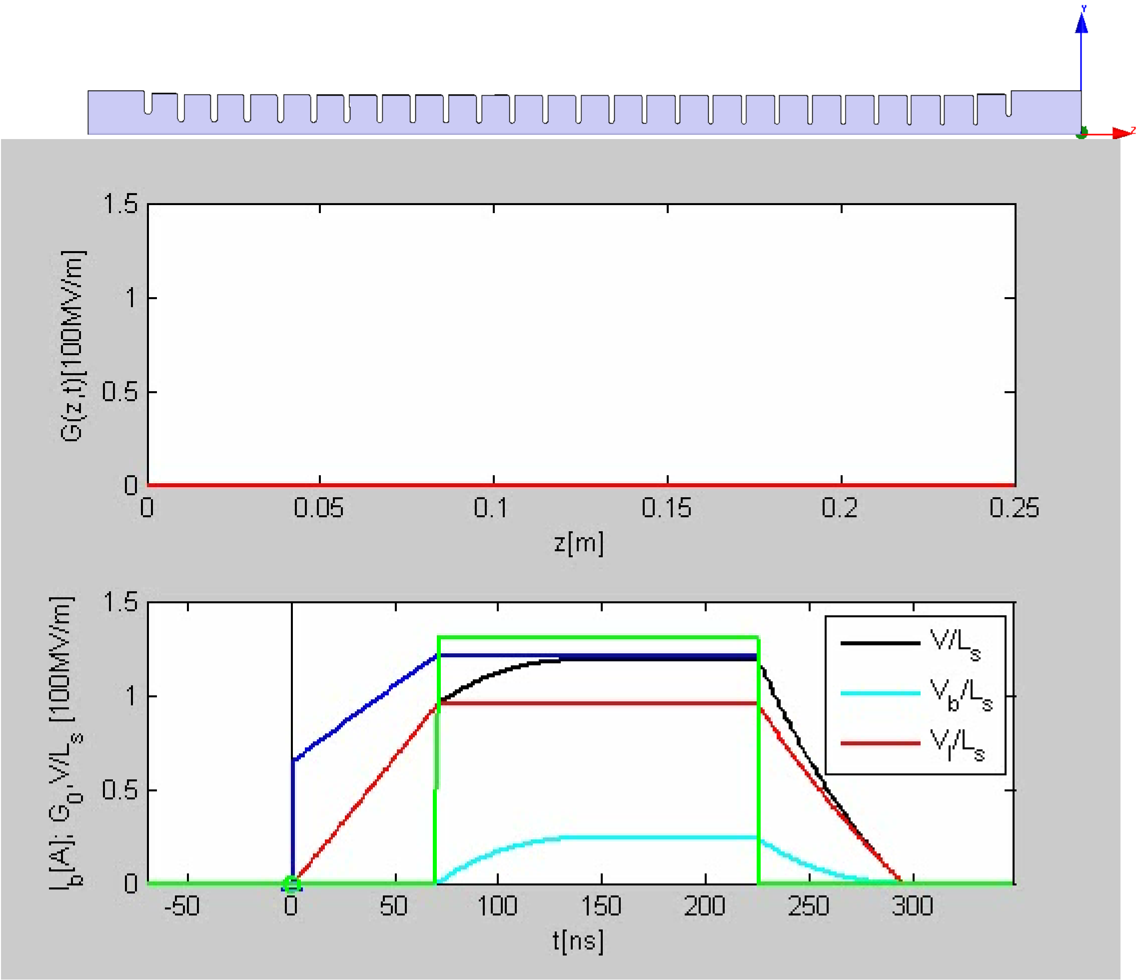
Parameters calculated	
P_{in} [MW]	67.6
η^{CW} [%]	48
t_f [ns]	70
η [%]	33

In reality, $v_g \neq$ const, R’ ≠ const, $Q_0 \neq$ const and general expressions must be applied but often a good estimate can be done by averaging R’ and Q_0 and assuming linear variation of v_g

In this case: $\langle Q_0 \rangle$=6200; <R’>=100 MΩ/m; and $v_g/c = 1.7 - 0.9/0.25 \cdot z$



CLIC main linac accelerating structure during transient

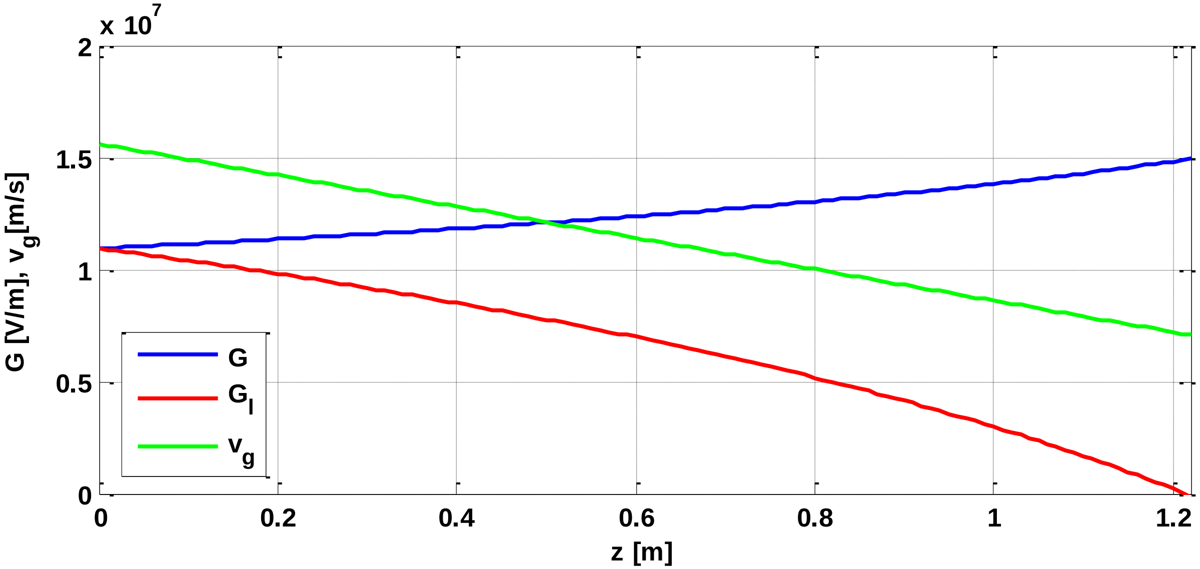


SICA - CTF3 3GHz accelerating structure in steady-state

Parameters: input – output	
f [GHz]	3
L_s [m]	1.22
v_g/c [%]	5.2 – 2.3
Q_0	14000 – 11000
R' [MΩ/m]	42 – 33
I_b [A]	4
Pin [MW]	33

Parameters calculated	
η^{CW} [%]	95

This structure is designed to operate in full beam loading regime



Experimental demonstration of full beam loading

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN - AB Department

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CLIC Note 604
CTF3 Note 66

FIRST FULL BEAM LOADING OPERATION WITH THE CTF3 LINAC

M. Bernard[2], G. Bienvenu[2], H. Braun, G. Carron, R. Corsini, A. Ferrari[5],
O. Forstner, T. Garvey[2], G. Geschonke, L. Groening[1], E. Jensen, R. Koontz[4],
T. Lefevre[3], R. Miller[4], L. Rinolfi, R. Roux[2], R. Ruth[4], D. Schulte, F. Tecker,
L. Thorndahl, D. Yeremian[4]
CERN, Geneva, Switzerland

Abstract

The aim of the CLIC (Compact Linear Collider) Study is to investigate the feasibility of a high luminosity, multi-TeV linear e+e- collider. CLIC is based on a two-beam method, in which a high current drive beam is decelerated to produce 30 GHz RF power needed for high-gradient acceleration of the main beam running parallel to it. To demonstrate the outstanding feasibility issues of the scheme a new CLIC Test Facility, CTF3, is being constructed at CERN by an international collaboration. In its final configuration CTF3 will consist of a 150 MeV drive beam linac followed by a 42 m long delay loop and an 84 m combiner ring. The installation will include a 30 GHz high power test stand, a representative CLIC module and a test decelerator. The first part of the linac was installed and commissioned with beam in 2003. The first issue addressed was the generation and acceleration of a high-current drive beam in the "full beam loading" condition where RF power is converted into beam power with an efficiency of more than 90 %. The full beam loading operation was successfully demonstrated with the nominal beam current of 3.5 A. A variety of beam measurements have been performed, showing good agreement with expectations.

[1] GSI, Darmstadt, Germany
[2] LAL, Orsay, France
[3] North Western University, Evanston, Illinois, USA
[4] SLAC, Menlo Park, California, USA
[5] Uppsala University, Uppsala, Sweden

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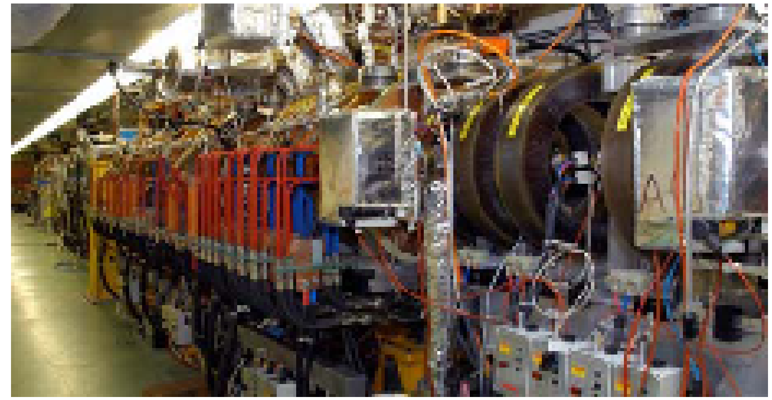

Figure 3: View of the CTF3 injector from the gun.

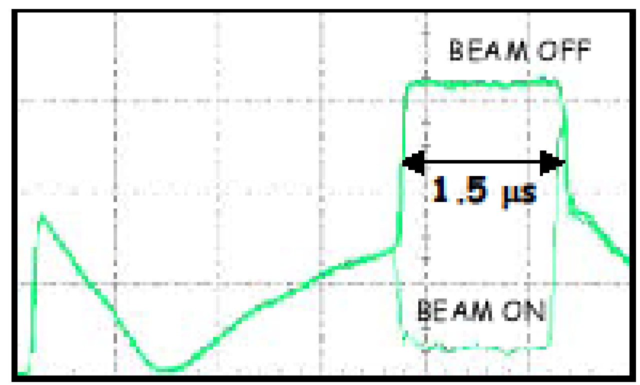

Figure 4: Scope trace showing the RF pulse at the output coupler of a structure. When the beam is on, it extracts more than 90 % of the energy contained in the useful part of the RF pulse (1.5 µs). Virtually no power goes to the load.