2010 LINEAR COLLIDER SCHOOL
SUPERCONDUCTIN RF
HOMEWORK

Problem 1

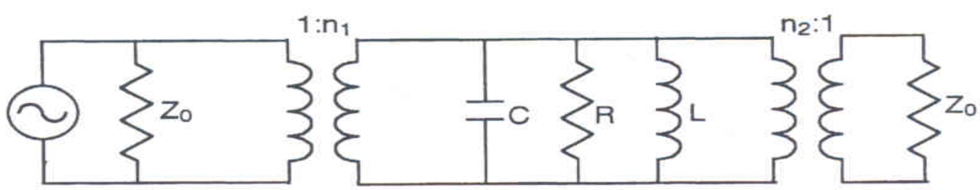
A film of thickness $2d$ made of a London superconductor (of penetration depth λ) is placed in a parallel magnetic field H_0.

Calculate the magnetic field and supercurrent in the film.

Use the coordinate system where the film extends from $x = -d$ to $x = +d$, and the magnetic field is in the $+z$ direction.

Plot both for $\frac{d}{\lambda} = 0.5, 1,$ and $5.$

Problem 2



Assume a 2-port cavity with coupling coefficients β_1 (input) and β_2 (output)
Calculate the dissipated, transmitted, and reflected power for a given incident power.
Note: With a little bit of thinking you should be able to write down the answers directly with almost no calculations.
What happens if we interchange input and output?
What happens when $\beta_1=\beta_2$?

Problem 3

A superconducting cavity has 2 sources of power dissipation

-- Resistive losses with constant surface resistance

-- Field emission losses

What is the functional dependence of the quality factor (or its reciprocal) of the cavity on the accelerating gradient?

Problem 4

The power required to operate a cavity in the presence of beam loading is given by

$$P_g = \frac{V_c^2}{R_{sh}} \frac{1}{4\beta} \left\{ (1+\beta+b)^2 + \left[(1+\beta)\tan\psi - b\tan\phi\right]^2 \right\}$$

where $b = \frac{R_{sh} i_0 \cos\phi}{V_c}$

Derive that expression and show that the required rf power is minimal when

$(1+\beta_{opt})\tan\psi_{opt} = b\ \tan\phi,$ and $\beta_{opt} = |1+b|$

and is given by $P_g^{opt} = \frac{V_c^2}{R_{sh}} \frac{|1+b| + (1+b)}{2}$

If additionally we need to control microphonics, then show that the optimal coupling and rf power are given by

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \text{ and } P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[(b+1) + \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

Problem 5

Assume that we have a cavity providing a voltage V_0 to a very high current I_0. If the detuning and coupling are optimized, show that the required rf power is $V_0 I_0 \cos\phi$ where ϕ is the phase between the beam and the cavity voltage.

We now want the cavity to provide a voltage V using the same amount of rf power, and we want to determine what is the maximum current I we will be able to accelerate.

Define $v = \frac{V}{V_0}$ and $i = \frac{I}{I_0}$

Show that, if we can reoptimize the coupling, then the relationship between v and i is $vi = 1$.

For example, this means that if we want to increase the gradient to 1.5 times its original value then we must decrease the current to 2/3 and still provide the same beam power.

On the other hand, if the power coupler is fixed and we cannot reoptimize the coupling, then show that the relationship between v and i is $v + i = 2$.

In this case we would have to decrease the current to 1/2 and only 2/3 of the power will be transferred to the beam and 1/3 will be reflected.

Problem 6

-- Calculate the radius and length of a 1.3 GHz TM_{010} pill box cavity

-- What are the geometrical factor and R/Q for a single cell

-- Calculate the BCS surface resistance at 2K.

Use the following formula to calculate R_{BCS}:

$$R_{BCS} = 9\times10^{-5}\frac{f^2(\text{GHz})}{T}\exp\left(-1.83\frac{T_c}{T}\right)$$ where T_c = 9.2 K

-- If a real cavity has an R/Q which is 60% of that of the pill box cavity, but has the same geometrical factor, what is the total power dissipation for a 1 TeV accelerator if the cavities are operating at 30 MV/m. Assume a residual surface resistance of 5 nΩ

-- If the cost of a helium refrigerator per watt of capacity is

$C_{ref} = 1240 + 4\times10^4\left(\frac{1}{b_p} - \frac{1}{760}\right)$ where $b_p = 0.394T^{5.798}$, what would be the refrigerator cost for 2K cw operation?

-- If the cavities operate at 30 MV/m, and the cost of cryomodules is $50k per meter of active length, what would be the cryomodule cost for a 1 TeV accelerator?