Energy variations and emittance growth in the ILC main linac with KCS

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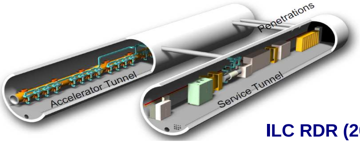
Mar., 2011

ILC main linac tunnel options

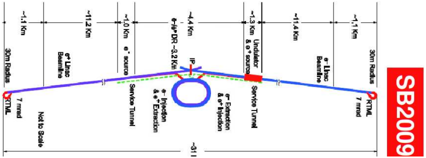
From



ILC RDR (2007)

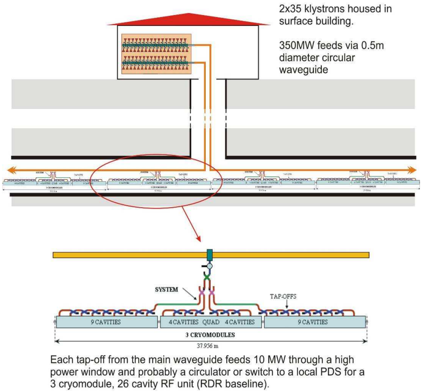
FIGURE 1.3-6. Cutaway view of the linac dual-tunnel configuration.

To



- Distributed RF Scheme (DRFS)
- Klystron Cluster RF Distribution Scheme (KCS)

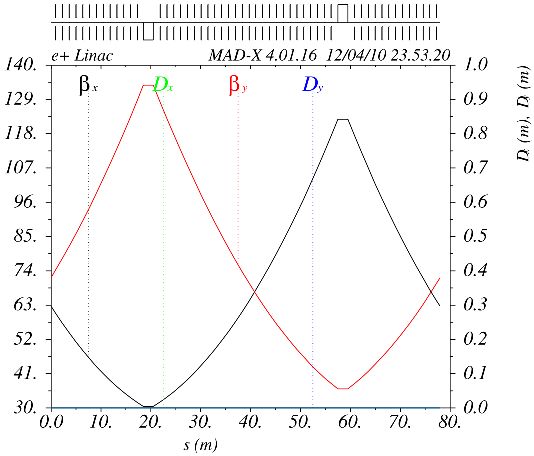
Klystron Cluster RF Distribution Scheme



- Service tunnel eliminated
- Electrical and cooling systems simplified
- Concerns: power handling, LLRF control coarseness

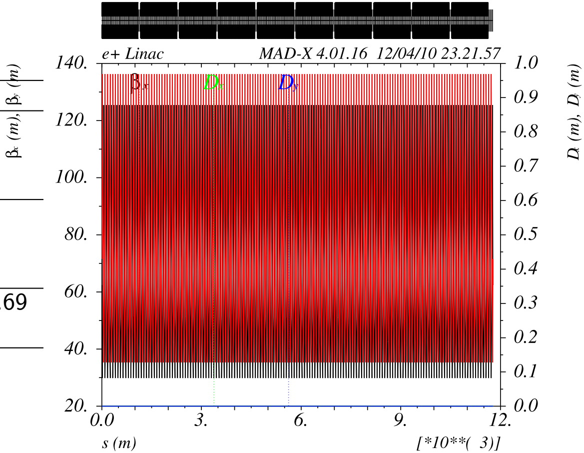
Lattice cell

parameter	value
Cell length	78 m
Length of quad	2 m
Length of 9-cell cavity	1 m
RF frequency	1.3 GHz
Cavity Gradient	31 MV/m
# of cavities	52
# of cryomodule	6
Phase advance	0.22/0.19

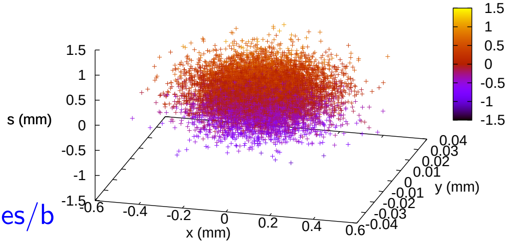


Linac lattice

parameter	value
Length	11778 m
# of super periods	10
# of quad	302
Length of 9-cell cavity	1 m
# of cavities	7280
# of cryomodule	900
Phase advance	33.22/28.69
Chromaticity	-37.5/-35

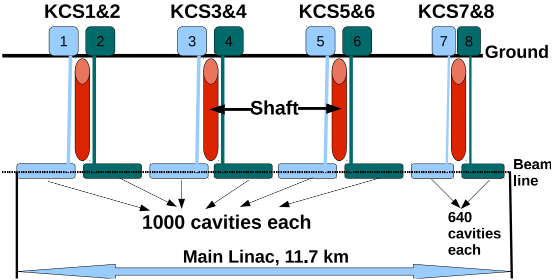


Simulation code



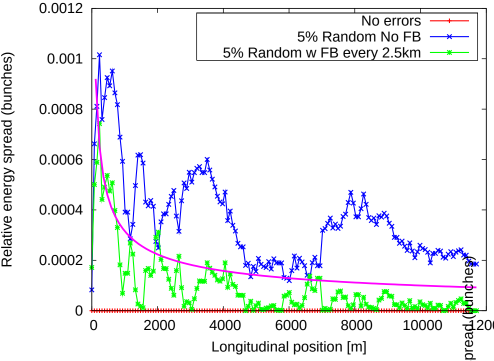
- Fortran90 code
- Self-Defined lattice
- Up to 10M macro-particles/b
- Multi bunch (pulse)
- 6-D Gaussian distribution (correlated)
- Longitudinal cut at 4 sigma
- Horizontal emittance (nor.): $10mm.mrad$; Vertical emittance (nor.): $0.02mm.mrad$
- RMS Bunch length 0 μm (no Wake field); RMS energy spread 1.5×10^{-2}

KCS model and error type



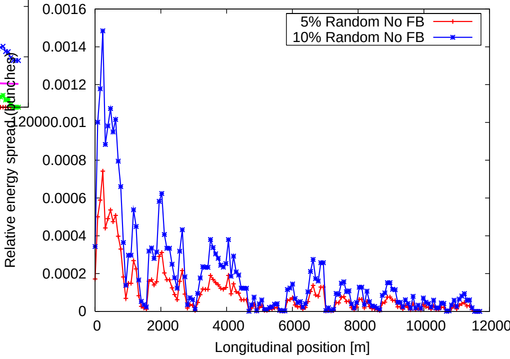
- Random error ($\sigma = 0.05$) on each cavity's gradient (freq. error)
- Correlated errors: RF power and beam current (loading). Assumed perfectly corrected.
- Feedback works within each KCS; Systematic error: gradient $\sigma = 0.01$, phase $\sigma = 1$ degree
- Linear correlation for bunches in same pulse

Energy spread between 2625 bunches in one pulse

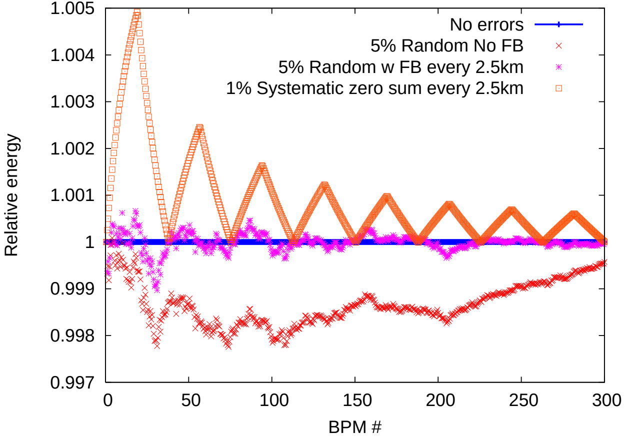


Linearly proportional to gradient error's amplitude

With feedback, energy spread decreases to zero ($\propto \sigma_g \sqrt{N}/E$)

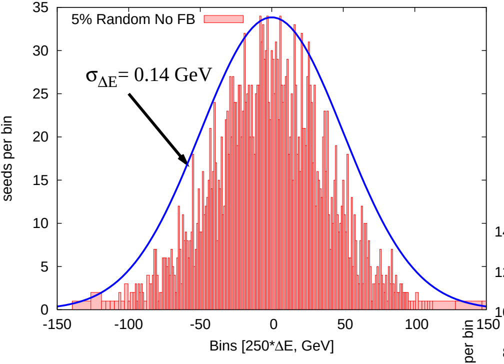


Energy variations along Linac (one seed)

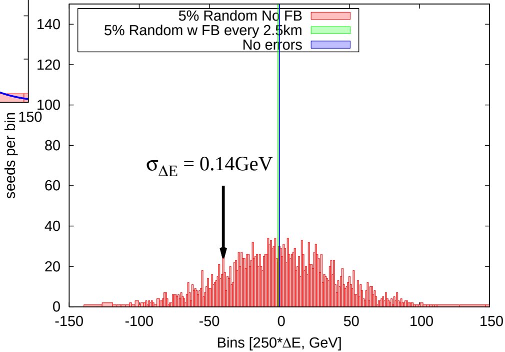


Worst case: 1% Systematic zero sum every 2.5km

Energy distribution at linac end w 2600 pulses(seeds)

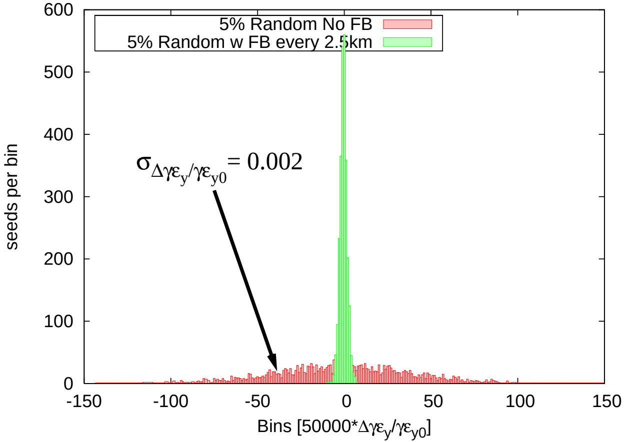


5% Random w FB every 2.5km, or 1% Systematic zero sum every 2.5km, 250 GeV at linac end



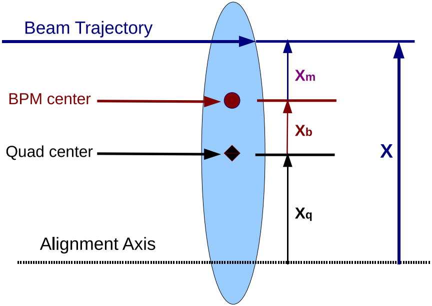
5% Random No FB, Gaussian distribution, RMS= $\sqrt{N_{cav}} \cdot \sigma_g \cdot V_{rf}$

$\gamma\epsilon_y$ distribution at linac end w 2600 pulses(seeds)



Very small change in $\gamma\epsilon_y$ due to chromatic effects

Linac alignment problem

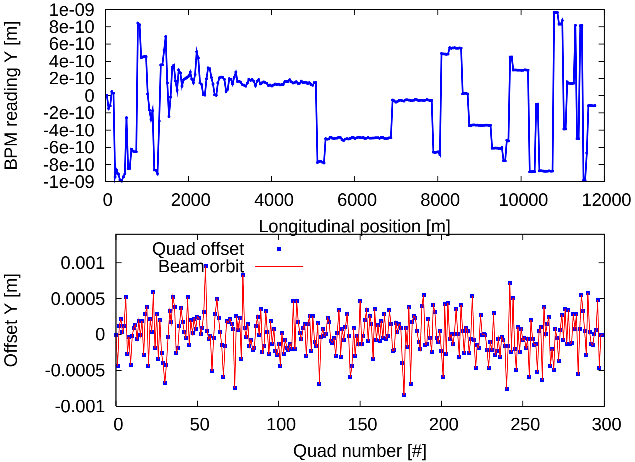


- $X_m = X - X_q + X_B + X_r$
- From X_m: BPM readings to calculate X_q

Alignment algorithms

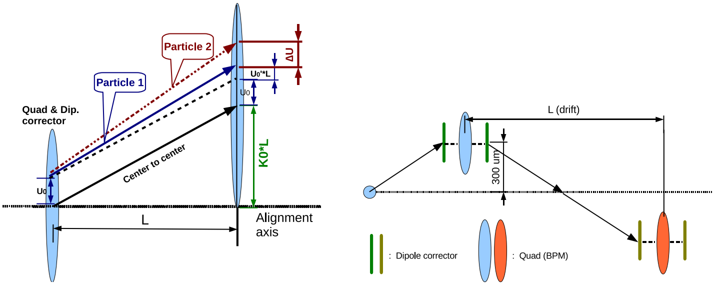
- One to One
- Global correction

One-to-one correction



Iterate to zero all BPM readings

Model for Analytical approach



- Dispersive emittance growth = local effect
- Only Quad at dispersion region contributes

Analytical approach (1)

Definition of projected emittance:

$$\gamma\epsilon_y = \gamma\sqrt{(<y^2> - <y>^2)\cdot(<y'^2> - <y'>^2) - (<yy'> - <y><y'>)^2}$$

With respect to the centroid trajectory:

$$(\gamma\epsilon_y)^2 = \gamma^2\left[\left({\sigma_y}^2 + <\Delta y^2>\right)\cdot\left(\sigma_{y'}^2 - <\Delta y'^2>\right) - \left(\sigma_{yy'}^2 - <\Delta y\Delta y'>\right)^2\right]$$

With:

$$(\gamma\epsilon_y)^2 = \gamma^2\left({\sigma_y}^2\cdot\sigma_{y'}^2 - \sigma_{yy'}^2\right)$$

$${\sigma_y}^2 = \epsilon_y\beta_y$$

$${\sigma'_y}^2 = \frac{\epsilon_y}{\beta_y}$$

$$\sigma_{yy'} = 2\epsilon_y\alpha_y$$

$$\gamma\epsilon_y = \gamma\epsilon_{y0}\sqrt{1 + 2\Delta\gamma\epsilon/\gamma\epsilon_{y0}}$$

$$\Delta\gamma\epsilon = \frac{\gamma}{2}\left(\frac{1+\alpha^2}{\beta}\Delta y^2 + 2\alpha\Delta y\Delta y' + \beta\Delta y'^2\right)$$

$$\Delta y = \frac{k_0 L}{1+\delta} - k_0 L = k_0 L(-\delta + \delta^2 - \delta^3 + \ldots)$$

$$\Delta y' = K\Delta y$$

Analytical approach (2)

Square of 2-D projected emittance = determinant of matrix

$$\epsilon^2 = \begin{pmatrix} \sigma_{y_0}^2 + \sum < \Delta {y_i}^2 > & 0 \\ 0 & \sigma_{y_0'}^2 + \sum < \Delta {y_i'}^2 > \end{pmatrix}$$

at the n^{th} quad

$< \Delta {y_i}^2 >= \sum_{j=1}^{i-1} R_{12_{j,i}} < \Delta {y_j'}^2 >$

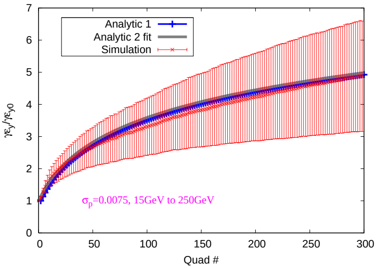
$< \Delta {y_i'}^2 >= \sum_{j=1}^{i-1} R_{22_{j,i}} < \Delta {y_j'}^2 >$

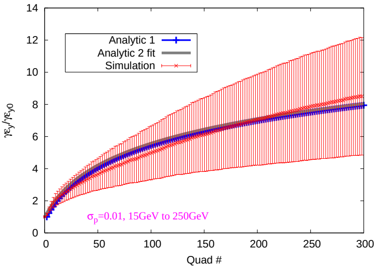
Simply and we have

$< \Delta {y_i}^2 >= \sum_{j=1}^{i-1} (\frac{1}{2}) \beta^2 < \Delta {y_j'}^2 >$

$< \Delta {y_i'}^2 >= \sum_{j=1}^{i-1} (\frac{1}{2}) < \Delta {y_j'}^2 >$

Benchmark: with acceleration ($\sigma_z = 0$)





Use same fitting parameter 1k seeds

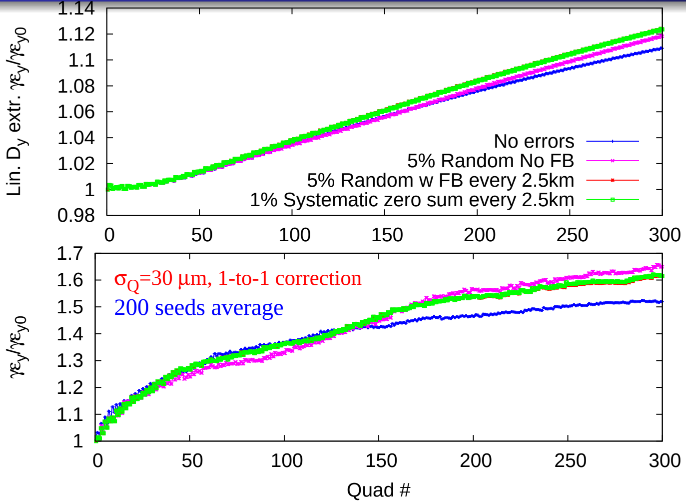
Left: $\sigma_{p0} = 0.0075$; Right: $\sigma_{p0} = 0.01$

With acceleration, integrate over cell numbers, assuming that the energy gain is the same in each cell.

$$\gamma\epsilon_n = \gamma_0 \sqrt{{\sigma_{x_0}}^2 + A \cdot log_e\left(\frac{E_n}{E_0}\right) \cdot \beta^2 \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2}$$

$$\cdot \sqrt{{\sigma_{x_0'}}^2 + A \cdot log_e\left(\frac{E_n}{E_0}\right) \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2}$$

Comparison: emittance growth (1-to-1)



- Small difference w or w/o energy error
- 1-to-1 correction algorithm

Solving method (Global correction algorithm)

$$x_i = R_{12_{i,1}}\theta_1 - \sum_{j=1}^{i-1} R_{12_{i,j}} K_j x_{q,j}$$

$$x_i{}' = R_{22_{i,1}}\theta_1 - \sum_{j=1}^{i-1} R_{22_{i,j}} K_j x_{q,j} - x_{q,i} K_i/2$$

$$\mathbf{R} \cdot \mathbf{Q} = \mathbf{B}$$

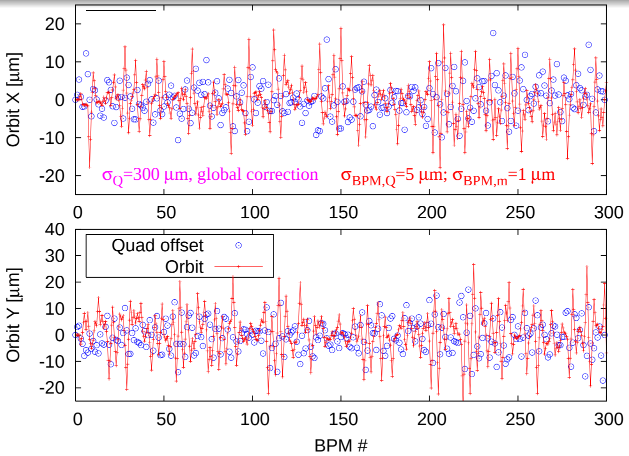
R combined R matrix, $n \times n$ sparse matrix
Q quad offsets+initial beam offset, $n \times 1$
B BPM reading, $n \times 1$
B includes BPM to Q offset error, and BPM measurement error

Use Row reduction (Gaussian elimination) to solve the system
For ILC linac, 300×300 matrix

Error Table

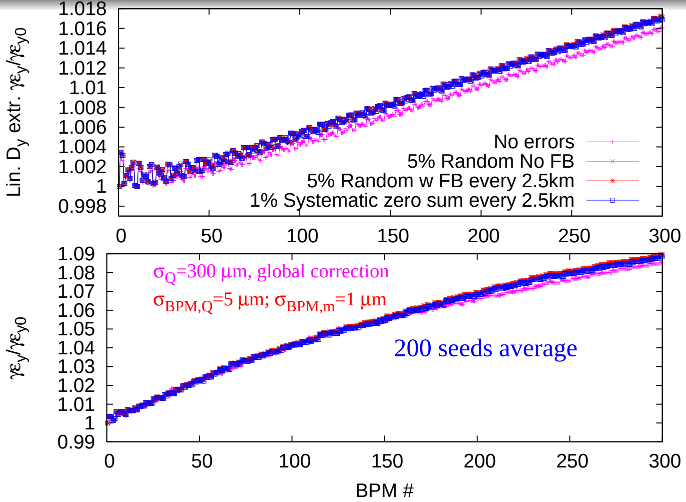
parameter	value
Quad offset	$\sigma = 300\mu m$
BPM to Quad offset	$\sigma = 5\mu m$
BPM measurement	$\sigma = 1\mu m$
Initial beam offset	$\sigma = 1\mu m$
Initial beam angle	$\sigma = 1\mu rad$

New Quadrupole offset (Moving Quad option)



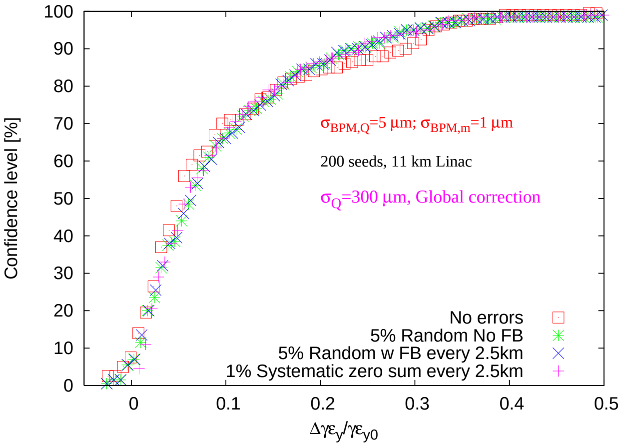
- Global correction algorithm , One seed
- New quad offset Globally correlated; roughly $6\mu m$

Comparison: emittance growth (global c. algorithm)



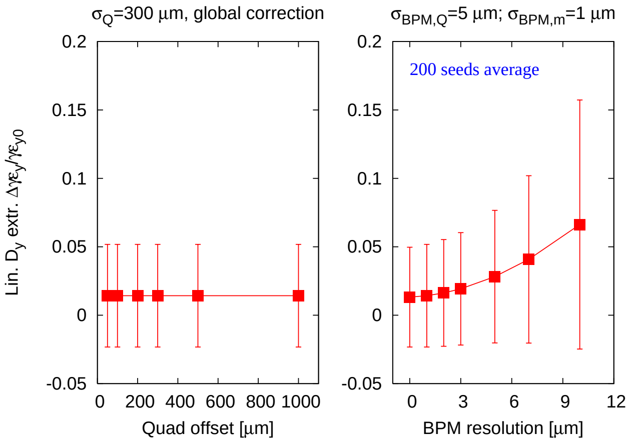
- Very small difference; with global correction algorithm

Confidence level (projected $\gamma\epsilon_y$)



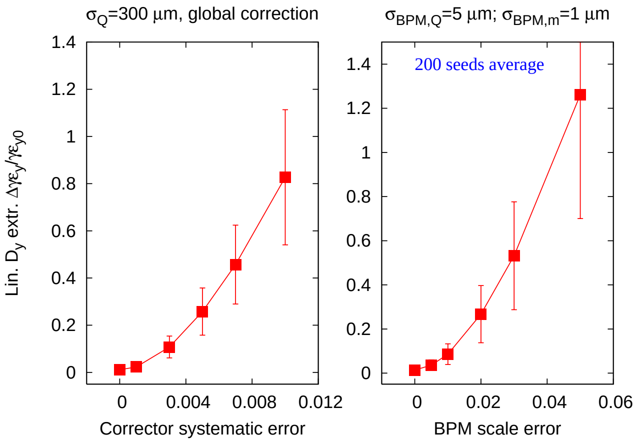
- At 90% confidence level, less than $4nm$ (20% of $20nm$) growth in projected $\gamma\epsilon_y$, global correction algorithm

Sensitivity, Error scan (1)



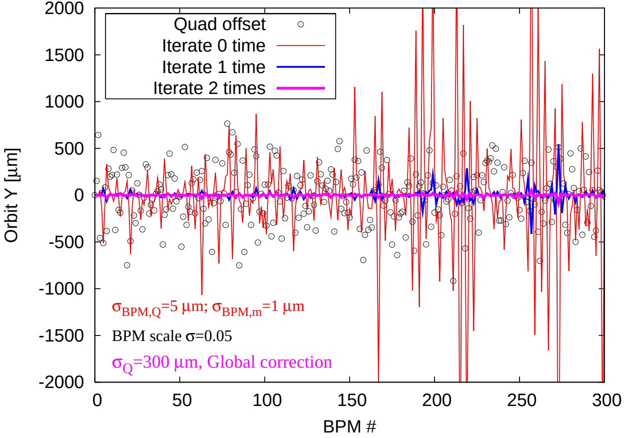
- BPM resolution $10\mu m$, less than 10% growth

Sensitivity, Error scan (2)



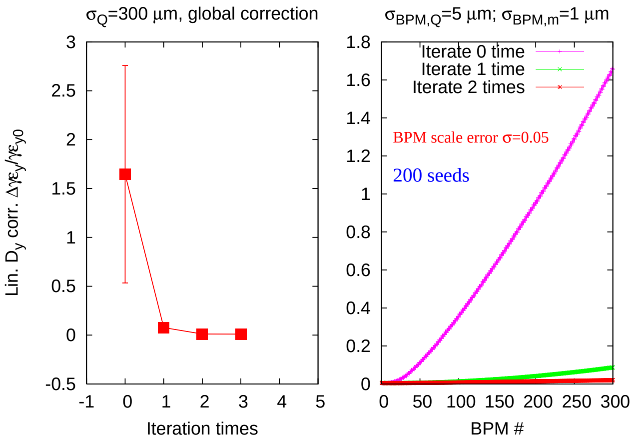
- BPM scale error $\sigma = 0.04$, around 100% growth
- Corrector systematic error=0.01, around 100% growth

Iterate to remove errors



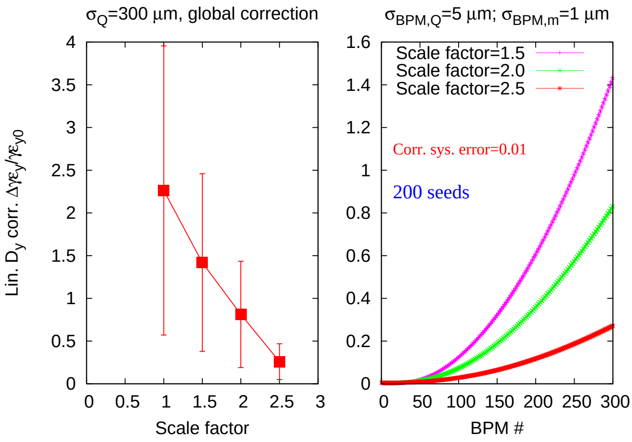
- BPM scale error $\sigma = 0.05$, 2 times iteration enough

Emittance, BPM scale error



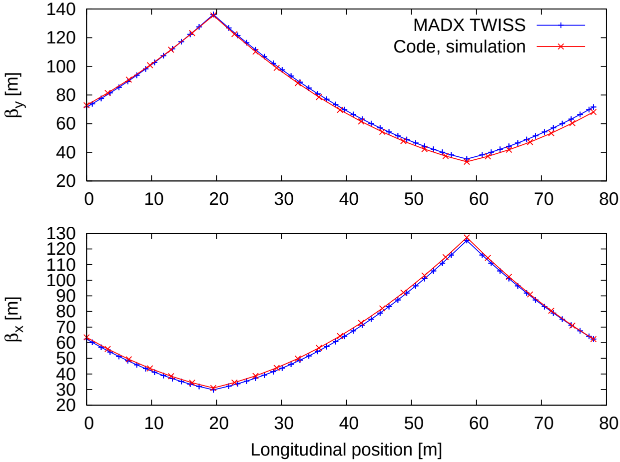
- BPM scale error $\sigma = 0.05$

Emittance, Corrector systematic error



- Corrector systematic error=0.01

Backup(0): Benchmark of code (β-function)



Benchmark of the beta function in one FODO cell, between MADX TWISS output and the simulation results of this code.

Backup(1): Dispersion and emittance

Projected emittance

$\epsilon = \sqrt{(< x^2 > - < x >^2) \cdot (< x'^2 > - < x' >^2) - (< xx' > - < x >< x' >)^2}$

Linear dispersion corrected emittance

$\epsilon = (< (x - D_x\delta)^2 > - < x - D_x\delta >^2) \cdot (< (x' - D_x'\delta)^2 > - < x' - D_x'\delta >^2)$

$-(< (x - D_x\delta)(x' - D_x'\delta) > - < x - D_x\delta >< x' - D_x'\delta >)^{20.5}$

Dispersion

$D_x = (< x\delta > - < x >< \delta >) / \left(< \delta^2 > - < \delta >^2\right)$

$D_x' = (< x'\delta > - < x' >< \delta >) / \left(< \delta^2 > - < \delta >^2\right)$

Backup(2): Example 5 quads

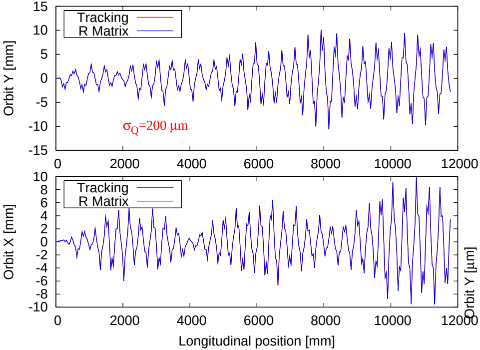
$$\mathbf{R} = \begin{pmatrix} R_{12}(2,1) & 1 & 0 & 0 \\ R_{12}(3,1) & R_{12}(3,2)K_2 & 1 & 0 \\ R_{12}(4,1) & R_{12}(4,2)K_2 & R_{12}(4,3)K_3 & 1 \\ R_{12}(5,1) & R_{12}(5,2)K_2 & R_{12}(5,3)K_3 & R_{12}(5,4)K_4 \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} \theta_1 \\ x_{q,2} \\ x_{q,3} \\ x_{q,4} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_{BPM,2} \\ x_{BPM,3} \\ x_{BPM,4} \\ x_{BPM,5} \end{pmatrix}$$

Backup(3): Global Alignment algorithm

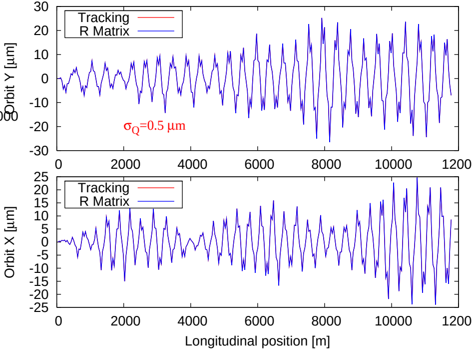
- Define error: Quad offset; BPM to Quad offset (do not change for one specified seed); BPM measurement error; Initial beam offset; Initial beam angle (change from pulse to pulse)
- Track single particle, get BPM readings
- Algorithm to calculate Quad offset (Row reduction, Gaussian elimination)
- Move Quad (or Use steering correctors)
- Track bunch (10,000 macro-particles) to calculate emittance etc. statistically
- Another seed

Backup(4): Reproduce orbit from R matrix

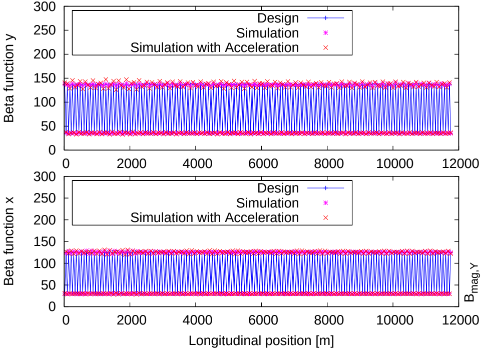


200 μm, 10 mm orbit

0.5 μm, 20 μm orbit

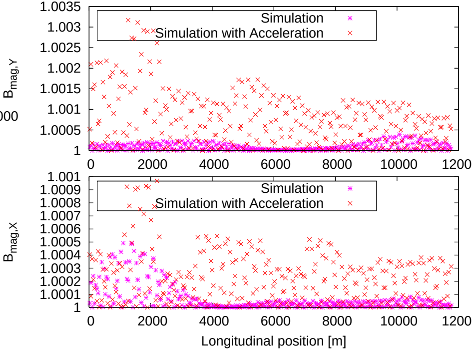


Backup(5): Beta function from tracking



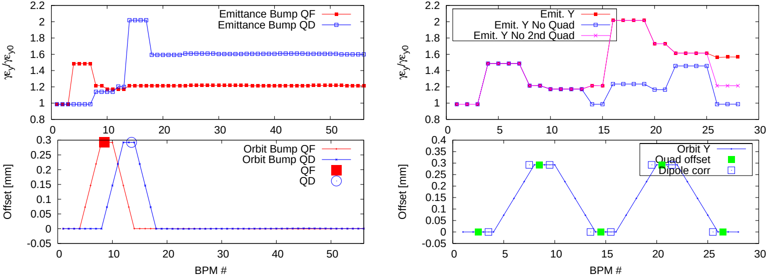
$$B_{mag} = \frac{1}{2}[(\frac{\beta}{\beta^*} + \frac{\beta^*}{\beta}) + (\alpha^* \sqrt{\frac{\beta}{\beta^*}} - \alpha\sqrt{\frac{\beta^*}{\beta}})^2]$$

$$\frac{\Delta\gamma\epsilon}{\gamma\epsilon} \approx B_{mag} - 1$$



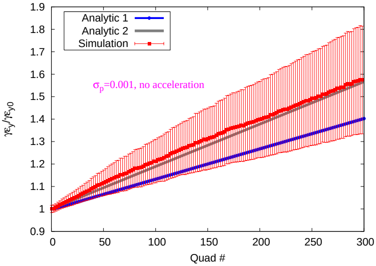
Good agreement

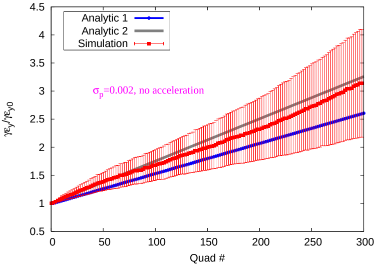
Backup(6): Model for Analytical approach



- Dispersive emittance growth = local effect
- Only Quad at dispersion region contributes

Backup(7): Comparison: no acceleration





Left: $\sigma_{p0} = 0.001$; Right: $\sigma_{p0} = 0.002$

Without acceleration, the physical vertical emittance at the n^{th} cell is

$$\epsilon_n = \sqrt{\left({\sigma_{y_0}}^2 + 0.5 \cdot n \cdot \beta^2 \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2\right)\left({\sigma_{y_0'}}^2 + 0.5 \cdot n \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2\right)}$$