N^3LL Analysis of Thrust Distribution: Determination of $\alpha_s(M_Z)$

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arXiv:1006.3080[hep-ph]

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Outline

Motivation

In the current world average of $\alpha_s(m_Z)$, values coming from event shapes observable have a sizable error. With our theory and the available data, we can obtain a much more precise value for $\alpha_s(m_Z)$

Thrust analysis

We provide, for the first time, a rigorous treatment of power corrections defined in Field Theory which is consistent over the whole thrust distribution.

Application for a Future Linear Collider

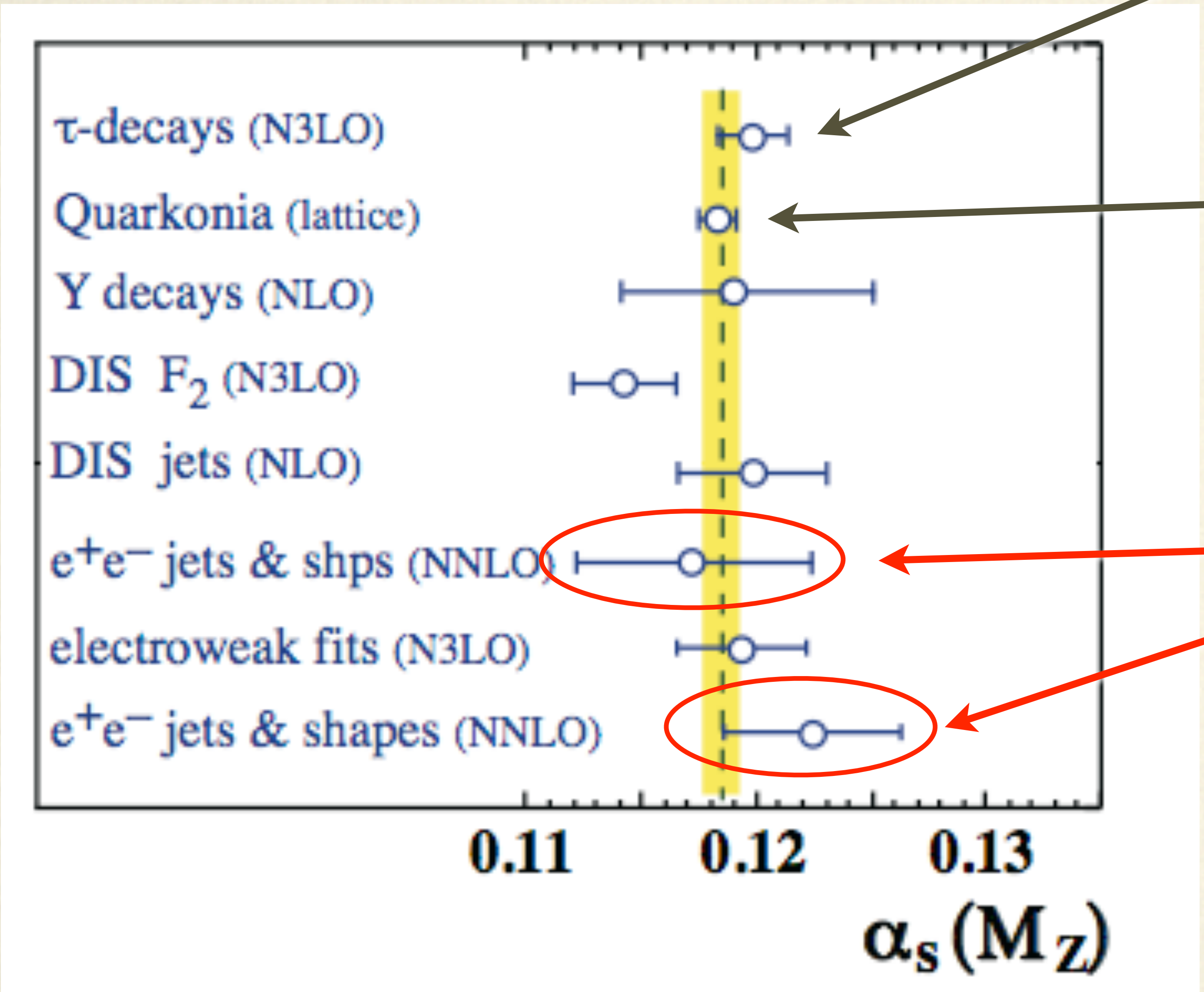
At high center of mass energy, $\alpha_s(m_Z)$ is more sensitive to experimental uncertainties, but non perturbative effects become less important

Motivations

Latest World Average

S. Bethke, arXiv:0908.1135
adopted by PDG for 2010

$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$



errors inflated to account for variation in literature

fit to Υ-splittings, Wilson loops

$$\alpha_s(m_Z) = 0.1183 \pm 0.0008$$

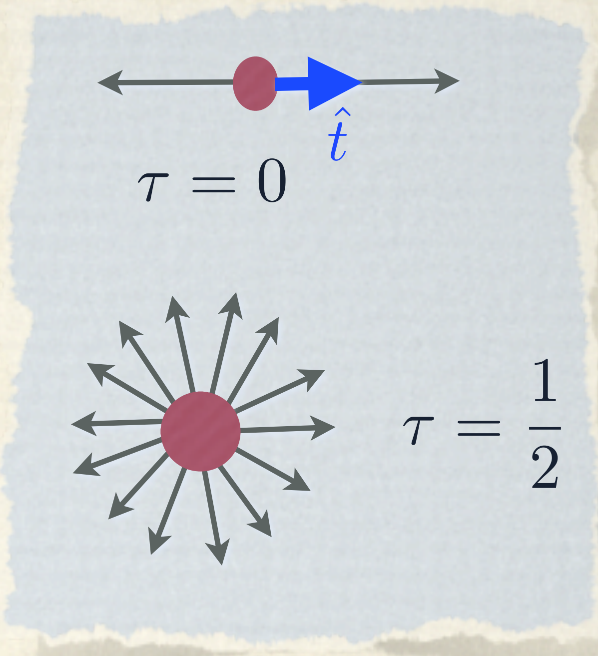
HPQCD 0807.1687

event shape results at $\mathcal{O}(\alpha_s^3)$ (Doesn't include $\mathrm{N}^3\mathrm{LL}$ resummation nor rigorous treatment of power correction)

Thrust

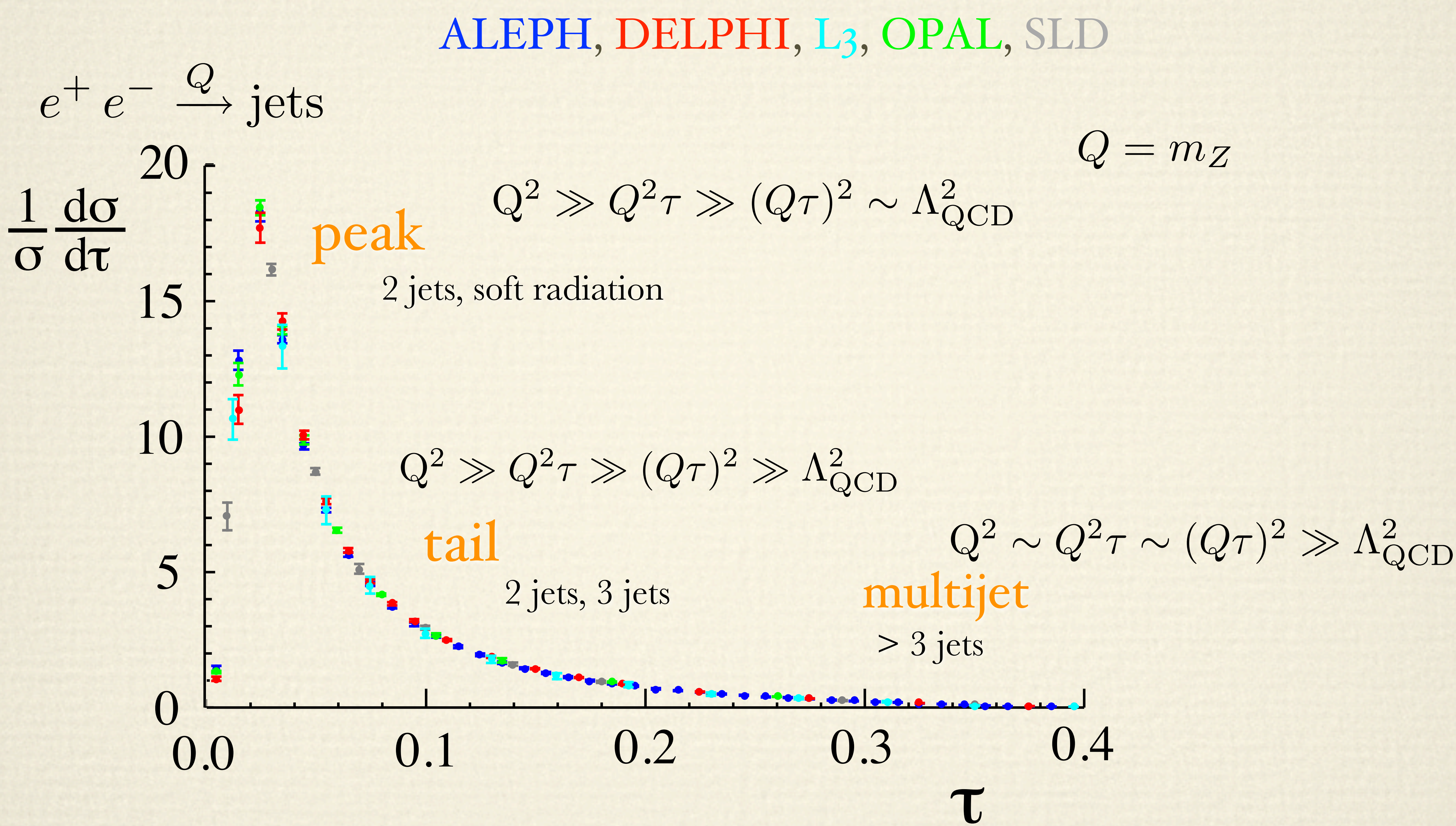
$$e^+ e^- \xrightarrow{Q} \text{jets}$$

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{Q} \qquad \tau = 1 - T$$



	Experiment	Q values (GeV)
LEP	ALEPH	{91.2,133.0,161.0,172.0,183.0,189.0,200.0,206.0}
	DELPHI	{45.0,66.0,76.0,91.2,133.0,161.0,172.0,183.0, 189.0,192.0,196.0,200.0,202.0,205.0,207.0}
	OPAL	{91.0,133.0,161.0,172.0,177.0,183.0,189.0,197.0}
	L3	{41.4,55.3,65.4,75.7,82.3,85.1,91.2,130.1,136.1, 161.3,172.3,182.8,188.6,194.4,200.0,206.2}
SLAC	SLD	{91.2}
DESY	TASSO	{14.0,22.0,35.0,44.0}
	JADE	{35.0,44.0}
KEK	AMY	{55.2}

Thrust



ALEPH, DELPHI, L3, OPAL, SLD
$e^+ e^- \xrightarrow{Q}$ jets
$Q = m_Z$
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
$Q^2 \gg Q^2\tau \gg (Q\tau)^2 \sim \Lambda^2_{\rm QCD}$
peak
2 jets, soft radiation
$Q^2 \gg Q^2\tau \gg (Q\tau)^2 \gg \Lambda^2_{\rm QCD}$
tail
2 jets, 3 jets
$Q^2 \sim Q^2\tau \sim (Q\tau)^2 \gg \Lambda^2_{\rm QCD}$
multijet
> 3 jets
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4
τ

Thrust Analysis

Factorization Theorem for Thrust

$e^+ e^- \xrightarrow{Q} \text{jets}$

AFHMS (arXiv:1006.3080)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\Delta \mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\mathrm{QCD}}}{Q} \right) \right]$$

Factorization Theorem for Thrust

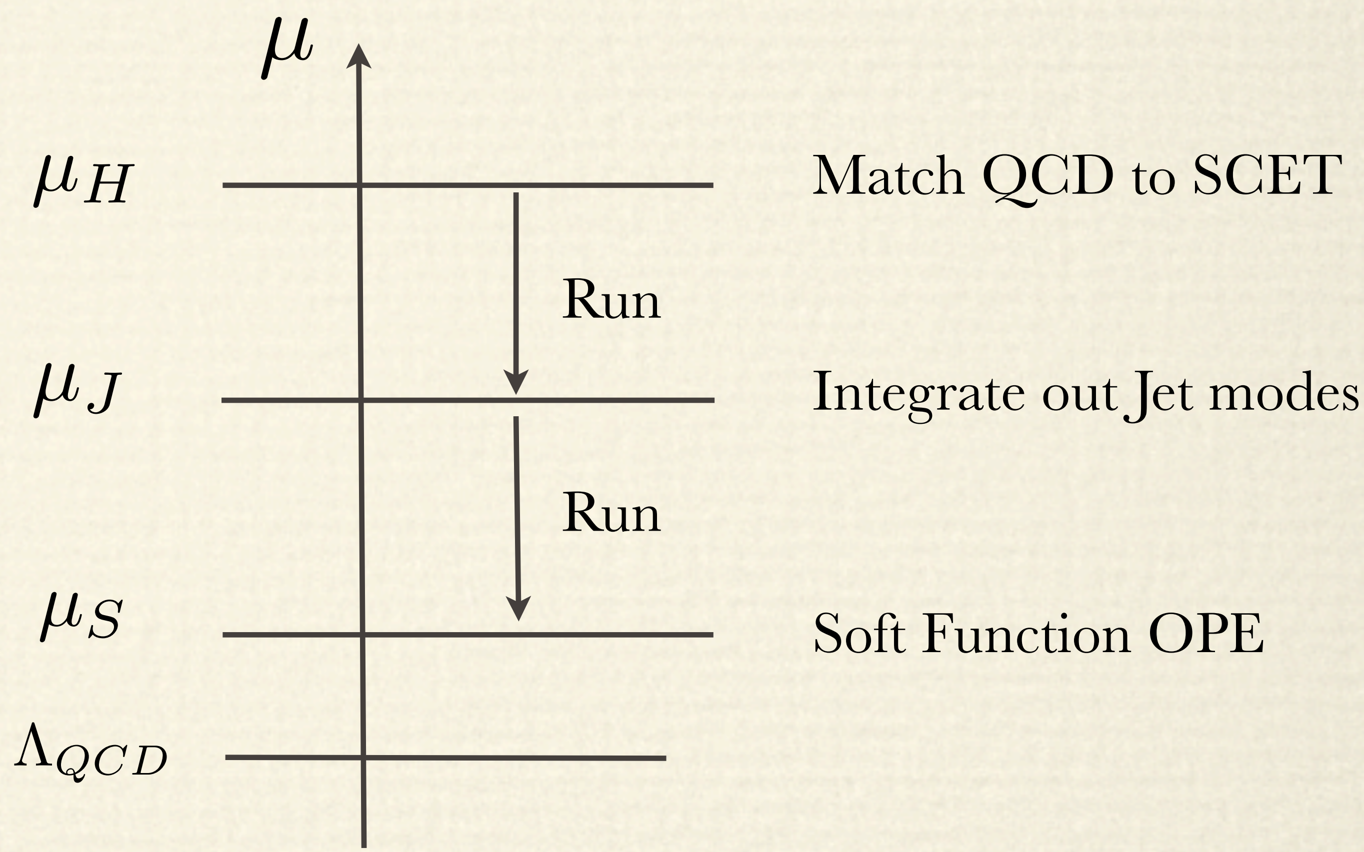
$e^+ e^- \xrightarrow{Q}$ jets

AFHMS (arXiv:1006.3080)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\Delta \mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\mathrm{QCD}}}{Q} \right) \right]$$

$$\begin{aligned} \frac{d\hat{\sigma}_s}{d\tau} &= \sum_n \alpha_s^n \, \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\log^l \tau}{\tau} \right]_+ \\ &= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S) \end{aligned}$$

singular partonic cross for massless quarks, QCD+QED final states



resummation for singular partonic

Becher Schwartz '08

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \ldots$$

$y = FT(\tau)$ LL NLL NNLL $\mathrm{N^3LL}$

Factorization Theorem for Thrust

$e^+ e^- \xrightarrow{Q} \text{jets}$

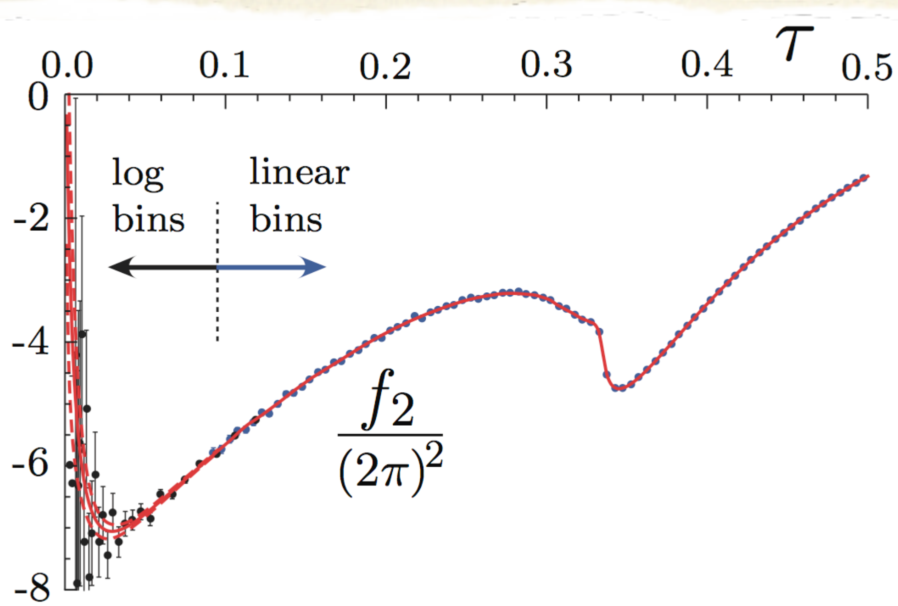
AFHMS (arXiv:1006.3080)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\Delta \mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right)\left(\tau - \frac{k}{Q}\right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)\right]$$

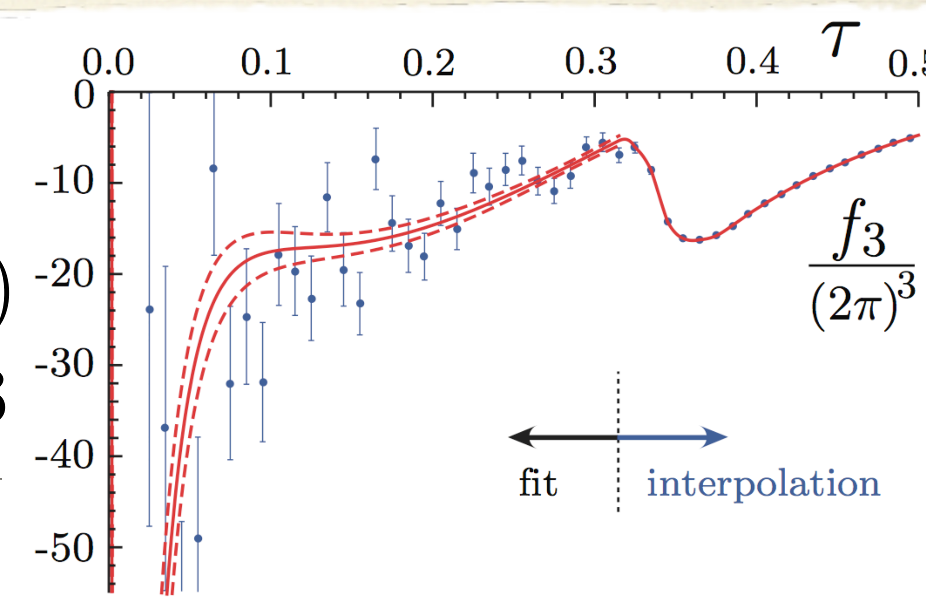
$$\frac{d\hat{\sigma}_{\mathrm{ns}}}{d\tau} = \sum_{n,l} \alpha_s^n \log^l \tau + \sum_n \alpha_s^n f_n(\tau)$$

nonsingular partonic:
fixed order - singular = non singular

$\mathcal{O}(\alpha_s^2)$
EVENT2



$\mathcal{O}(\alpha_s^3)$
EERAD3
Gehrmann et al.



Factorization Theorem for Thrust

$e^+ e^- \xrightarrow{Q}$ jets

AFHMS (arXiv:1006.3080)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\Delta \mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \tfrac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \tfrac{\Lambda_{\mathrm{QCD}}}{Q} \right) \right]$$

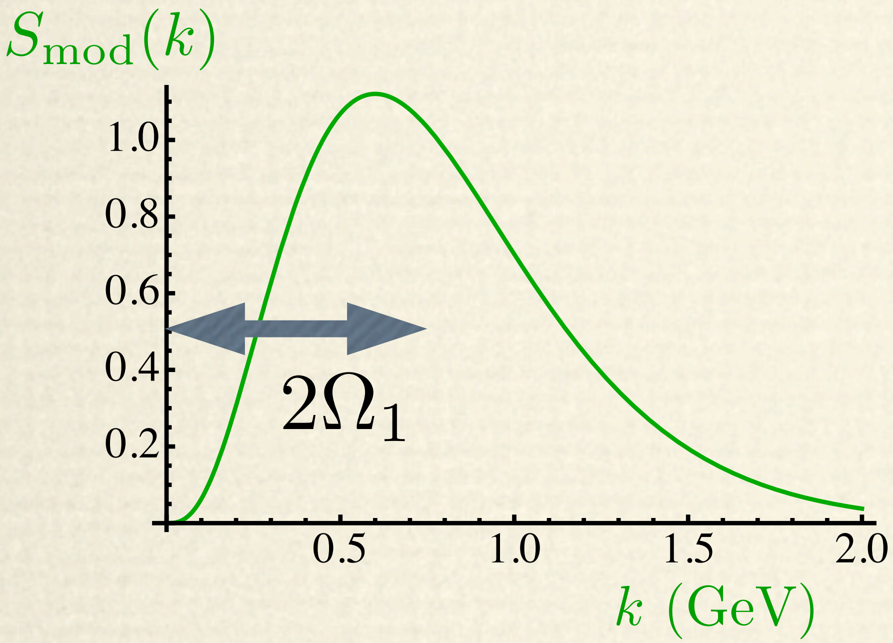
Non Perturbative Effects

In the tail region, where $\ell_{\mathrm{soft}} \sim Q\,\tau \gg \Lambda_{QCD}$

the soft function can be expanded as

$$
\begin{aligned}
S_\tau(Q\tau,\mu) &= \int dk'\; S_{\mathrm{part}}(Q\tau - k',\mu)\; S_\tau^{\mathrm{mod}}(k') = S_{\mathrm{part}}(Q\tau,\mu) - S'_{\mathrm{part}}(Q\tau,\mu)\,2\Omega_1 + \ldots \\
&= S_{\mathrm{part}}(Q\tau - 2\Omega_1,\mu) + \ldots
\end{aligned}
$$

the distribution shifts!



$$
\begin{aligned}
\Omega_1 &= \int dk\,\frac{k}{2}\; S_\tau^{\mathrm{mod}}(k - 2\bar\Delta) \\
&\equiv \frac{1}{2N_c}\langle 0 \big| \mathrm{tr}\; \overline{Y}_{\bar n}(0) Y_n(0)\, i\partial_\tau\, Y_n^\dagger(0) \overline{Y}_{\bar n}^\dagger(0) \big| 0\rangle - \delta \\
&\sim \Lambda_{QCD}
\end{aligned}
$$

R-scheme

$$
S_\tau(k,\mu) = \int dk' \left[e^{-2\delta \frac{\partial}{\partial k}}\; S_{\mathrm{part}}(k-k',\mu) \right] S_\tau^{\mathrm{mod}}(k' - 2\overline{\Delta})
$$

Renormalon free

Factorization Theorem for Thrust $e^+ e^- \xrightarrow{Q} \text{jets}$

AFHMS (arXiv:1006.3080)

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \frac{k}{Q}\right) S_\tau^{\text{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\text{QCD}}}{Q}\right)\right]$$

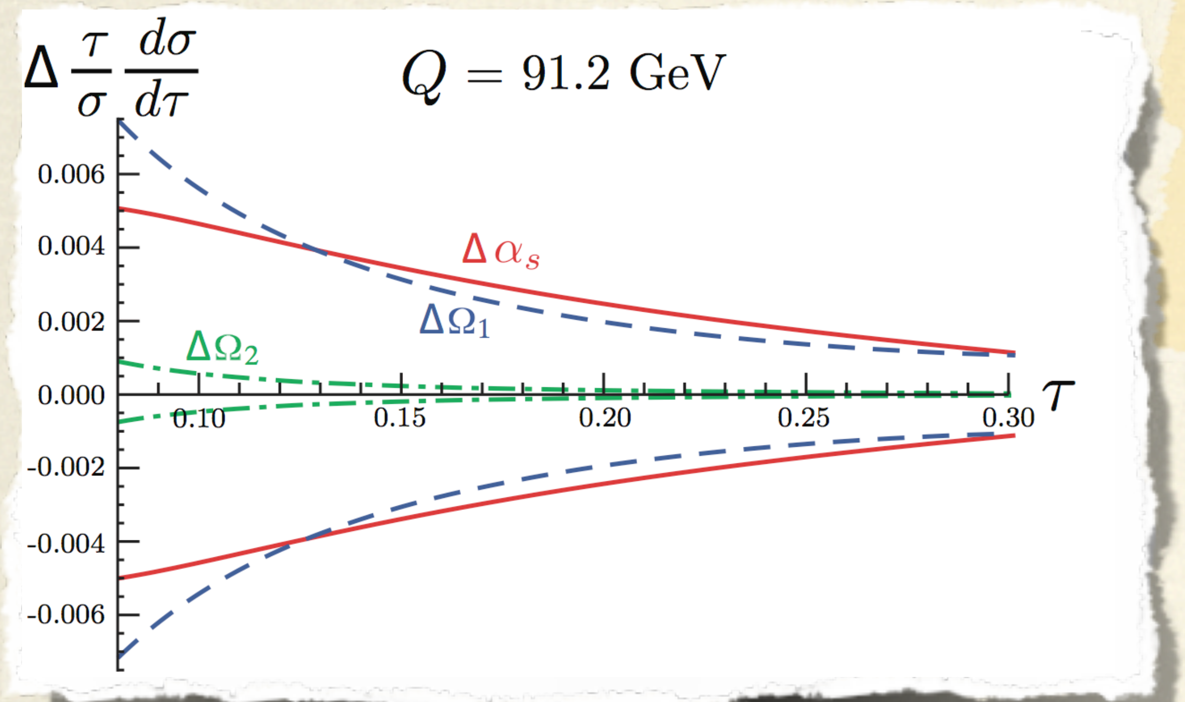
- $\mathcal{O}(\alpha_s^3)$ fixed order (non singular). Event2 $\mathcal{O}(\alpha_s^2)$ and EERAD3 $\mathcal{O}(\alpha_s^3)$
- $\mathcal{O}(\alpha_s^3)$ matrix elements. Axial singlet anomaly. Full hard function at 3 loops
- Resummation at N^3LL . Effective field theory (SCET)

- Correct theory in peak, tail and multijet regions (profile functions)
- Field theory matrix elements for renormalon-free power corrections

- QED effects in Sudakov and FSR at NNLL+$\mathcal{O}(\alpha_s^2)$ with $\alpha \sim \alpha_s^2$
- Bottom mass corrections with factorization theorem
- Computation of bin cumulants in a meaningful way

Why a global fit? (Many Q values)

We fit for $\alpha_s(m_Z)$ and Ω_1 simultaneously.



At a single Q, a variation in $\alpha_s(m_Z)$ can always be compensated be a variation in Ω_1, the two parameters are strongly degenerate

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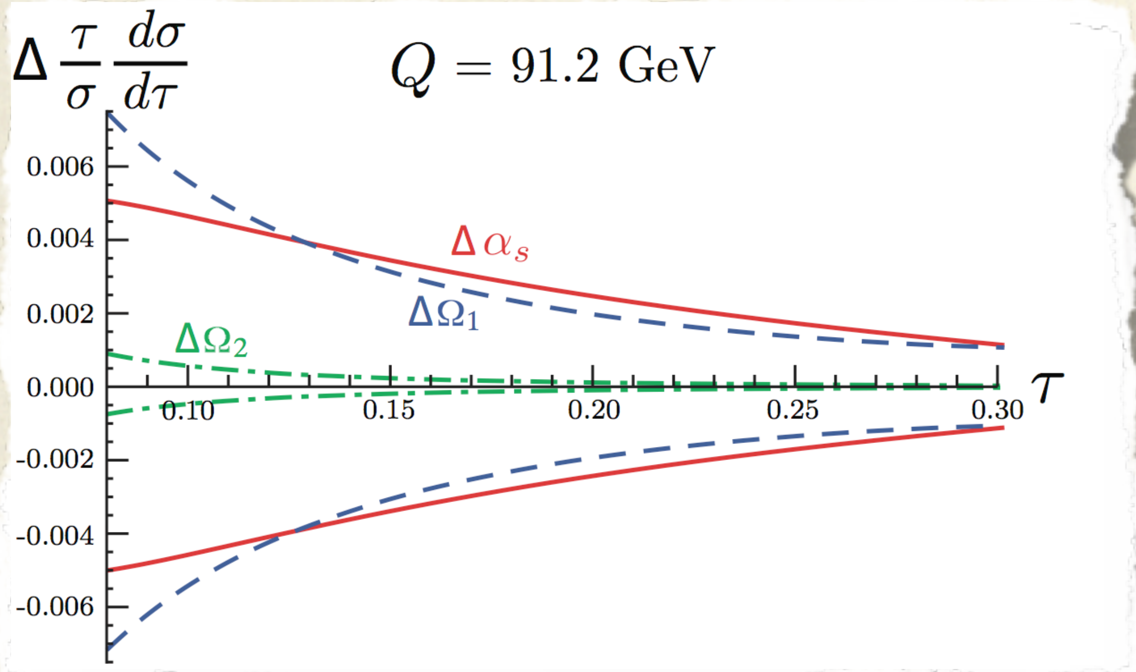
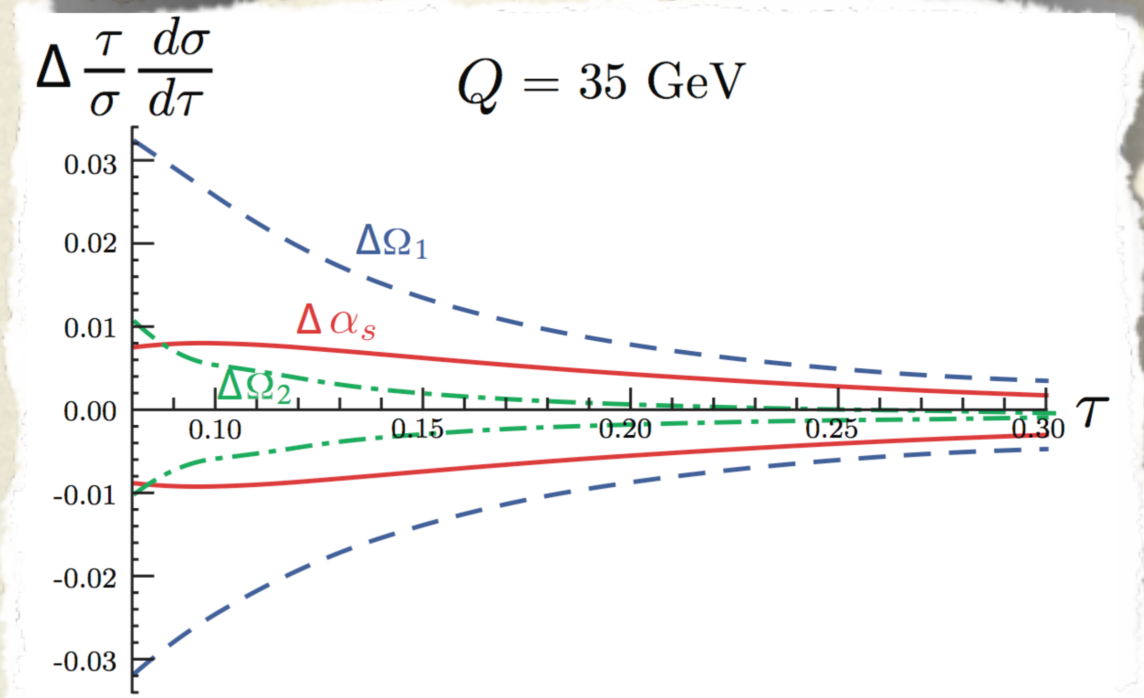
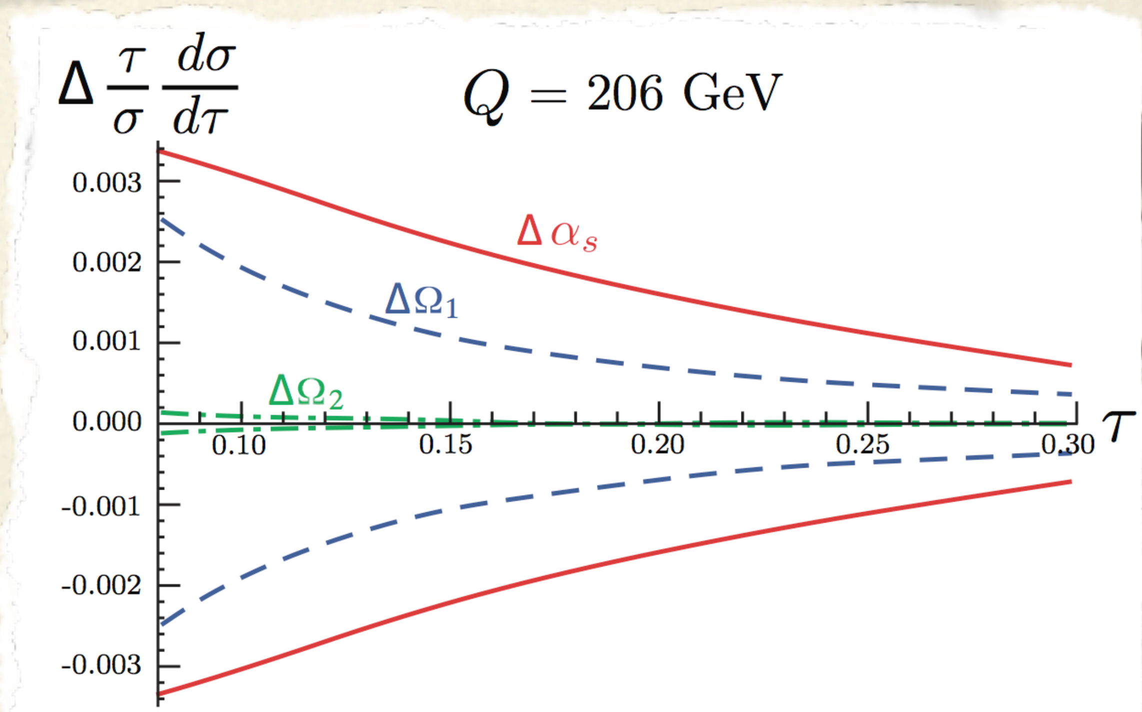


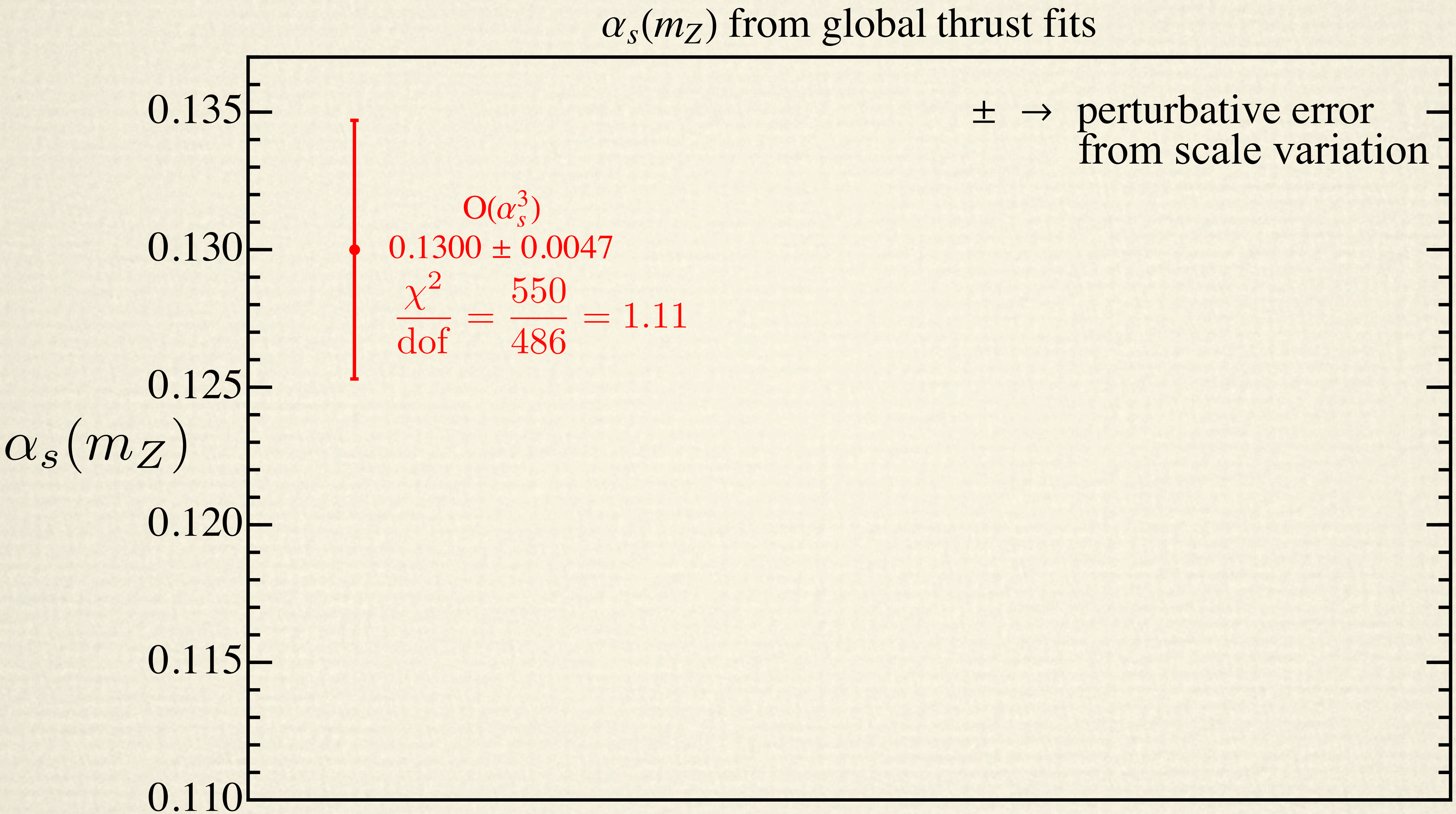
Fitting multiple Q , we can break the degeneracy!

Power correction needed at 20% accuracy to get $\alpha_s(m_Z)$ at the 1% level





Global tail fit for $\alpha_s(m_Z)$



Pure Fixed Order

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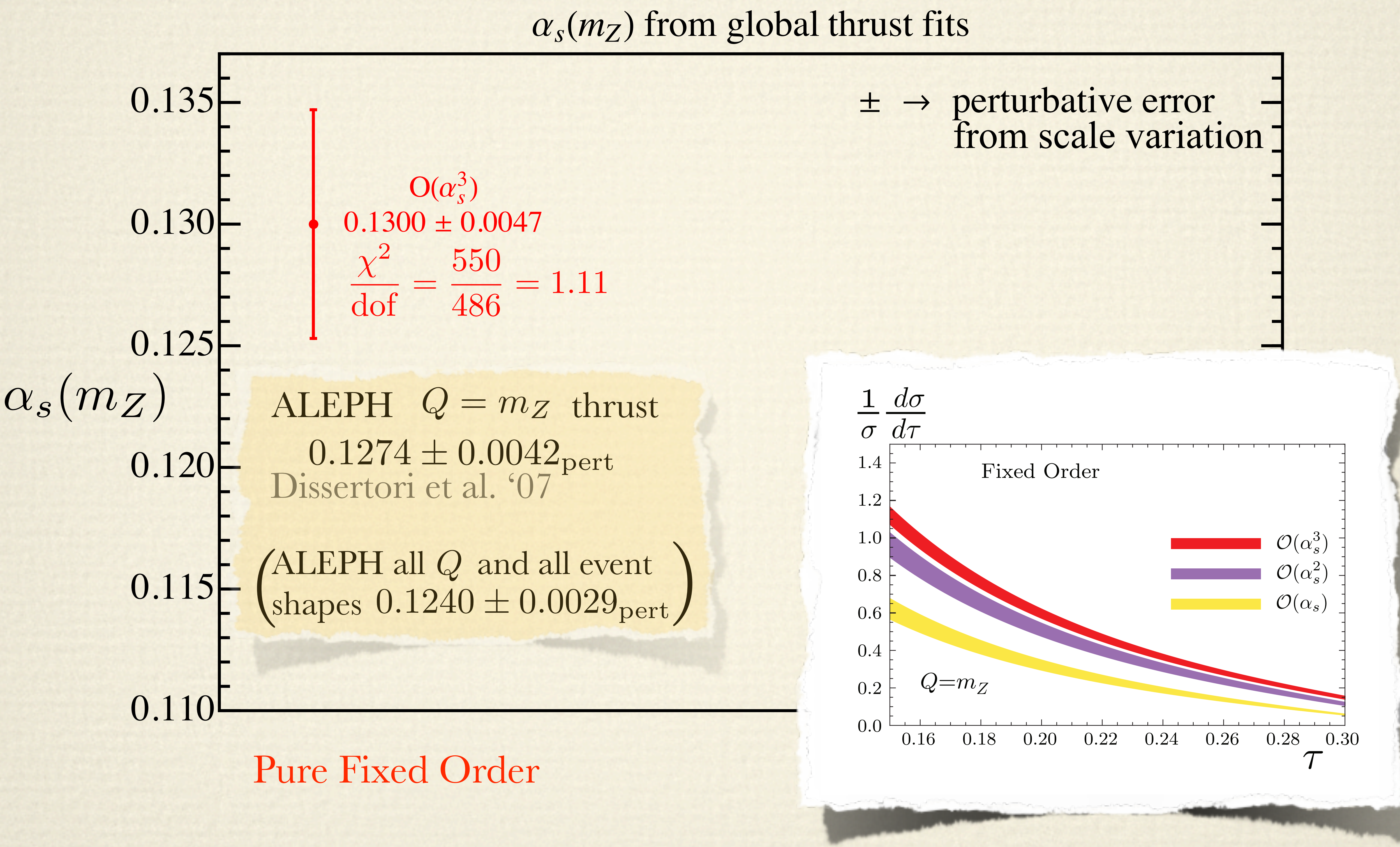

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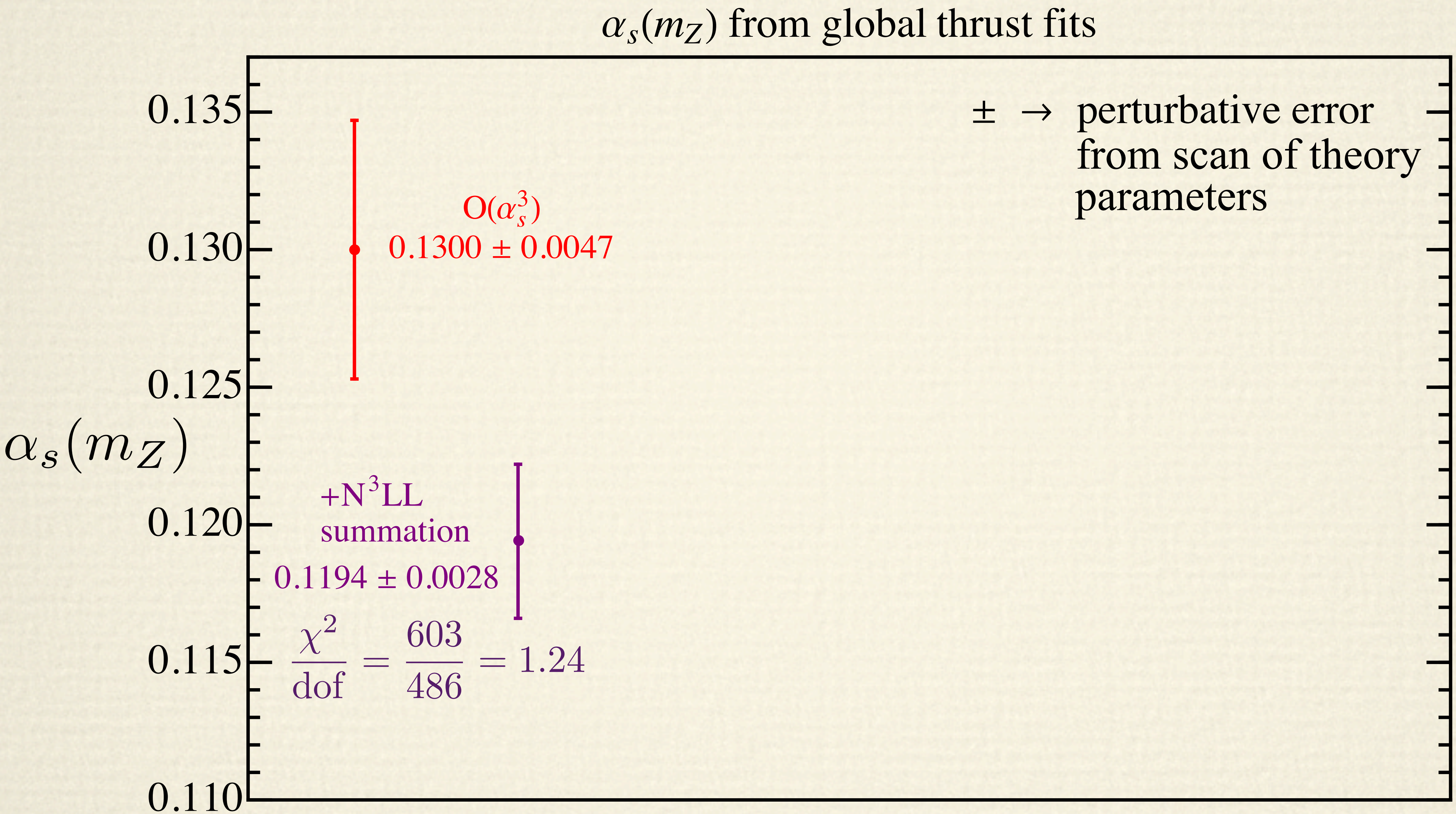

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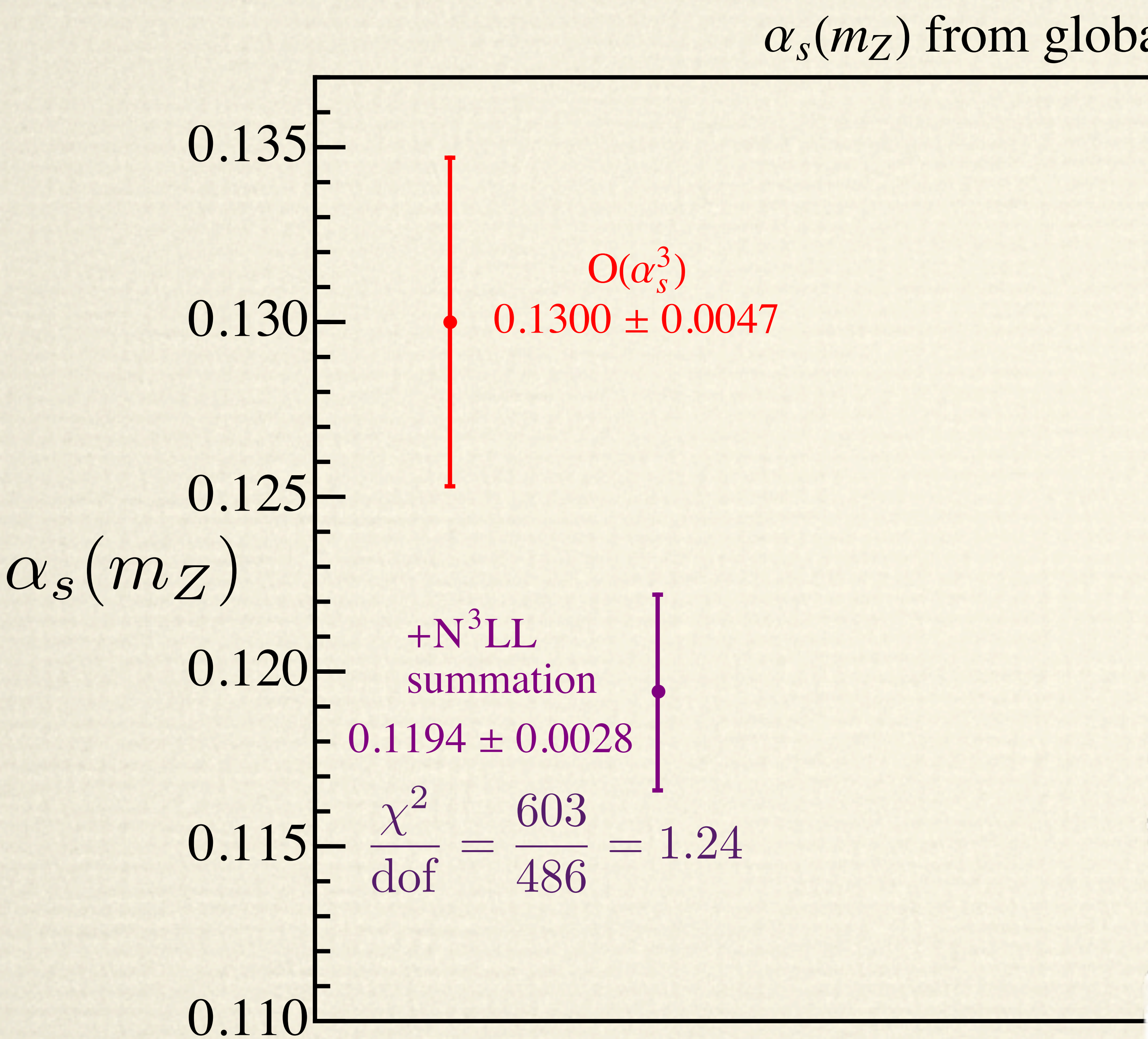
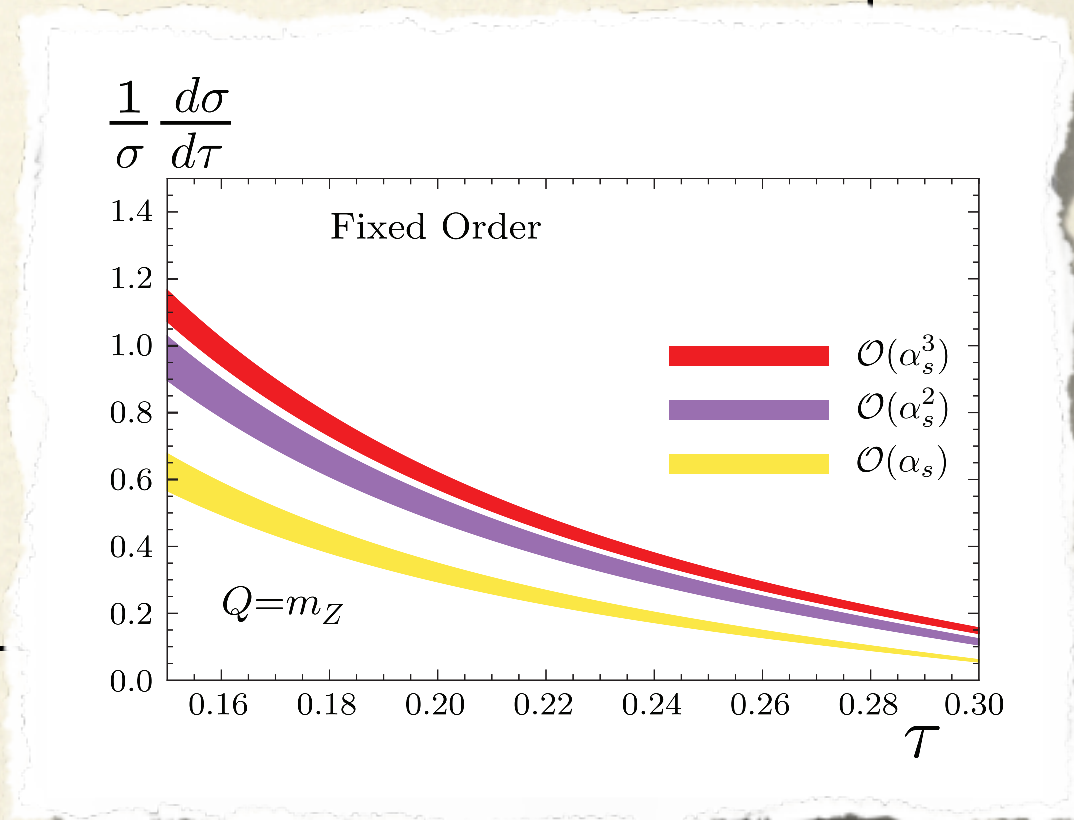
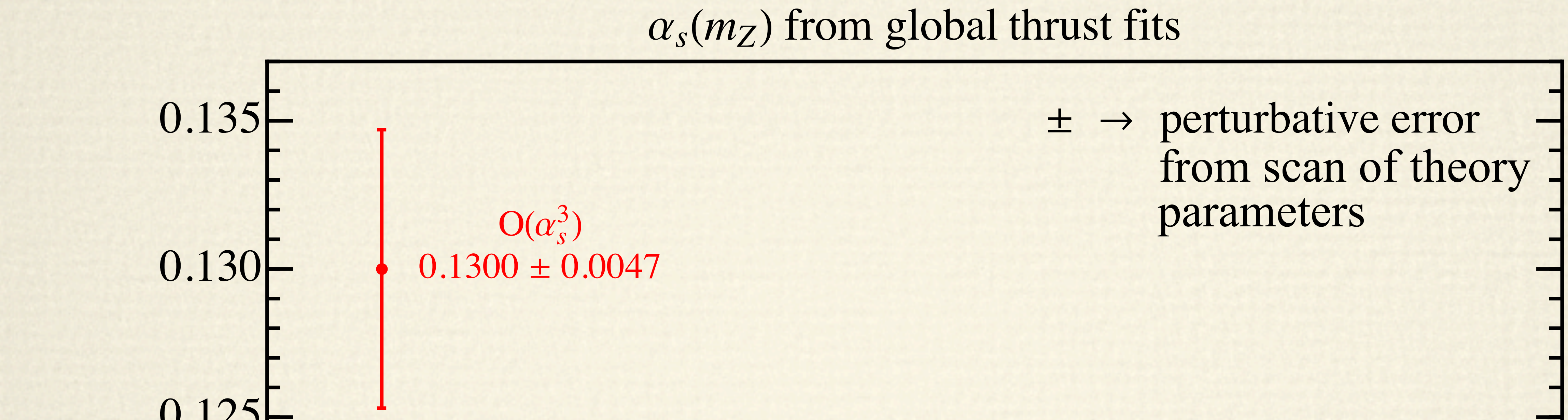
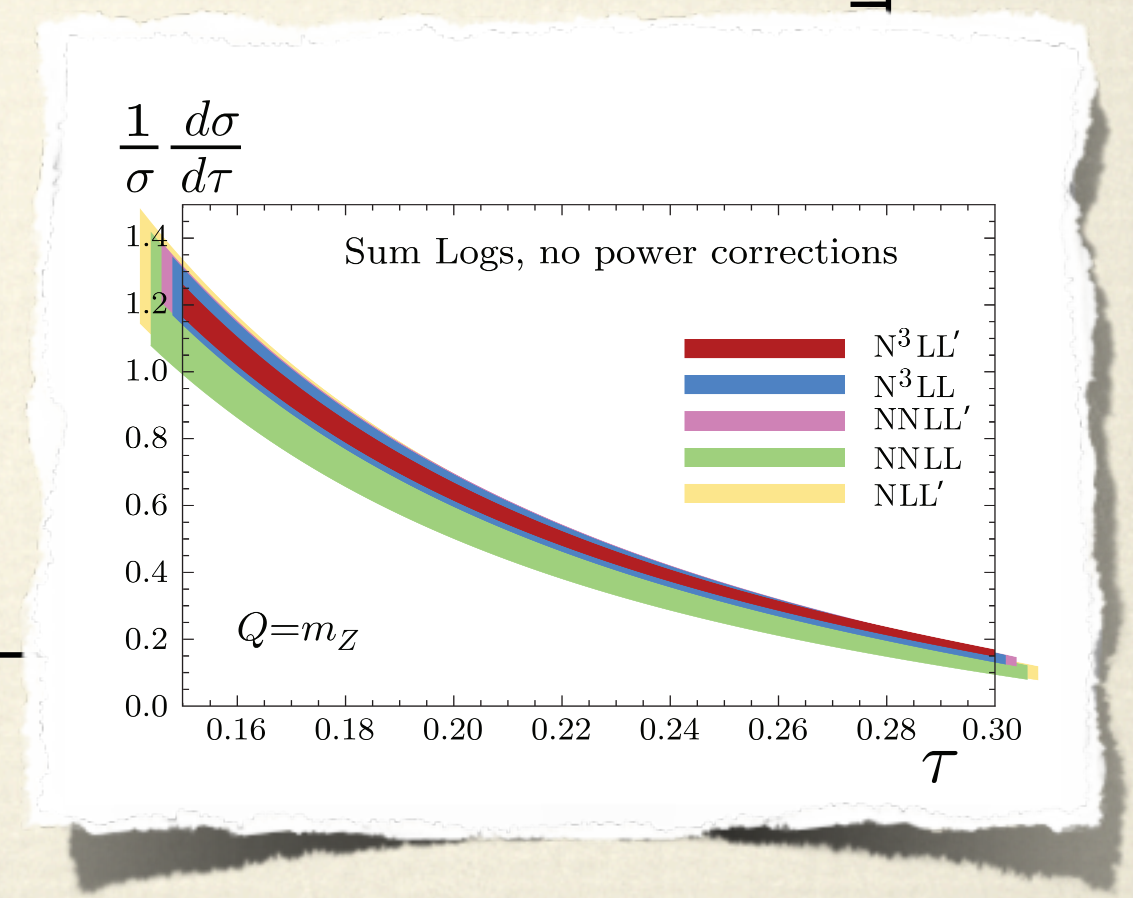




Fit to ALEPH and OPAL
$0.1172 \pm 0.0012_{\text{pert}}$
Becher Schwartz '08

Global tail fit for $\alpha_s(m_Z)$





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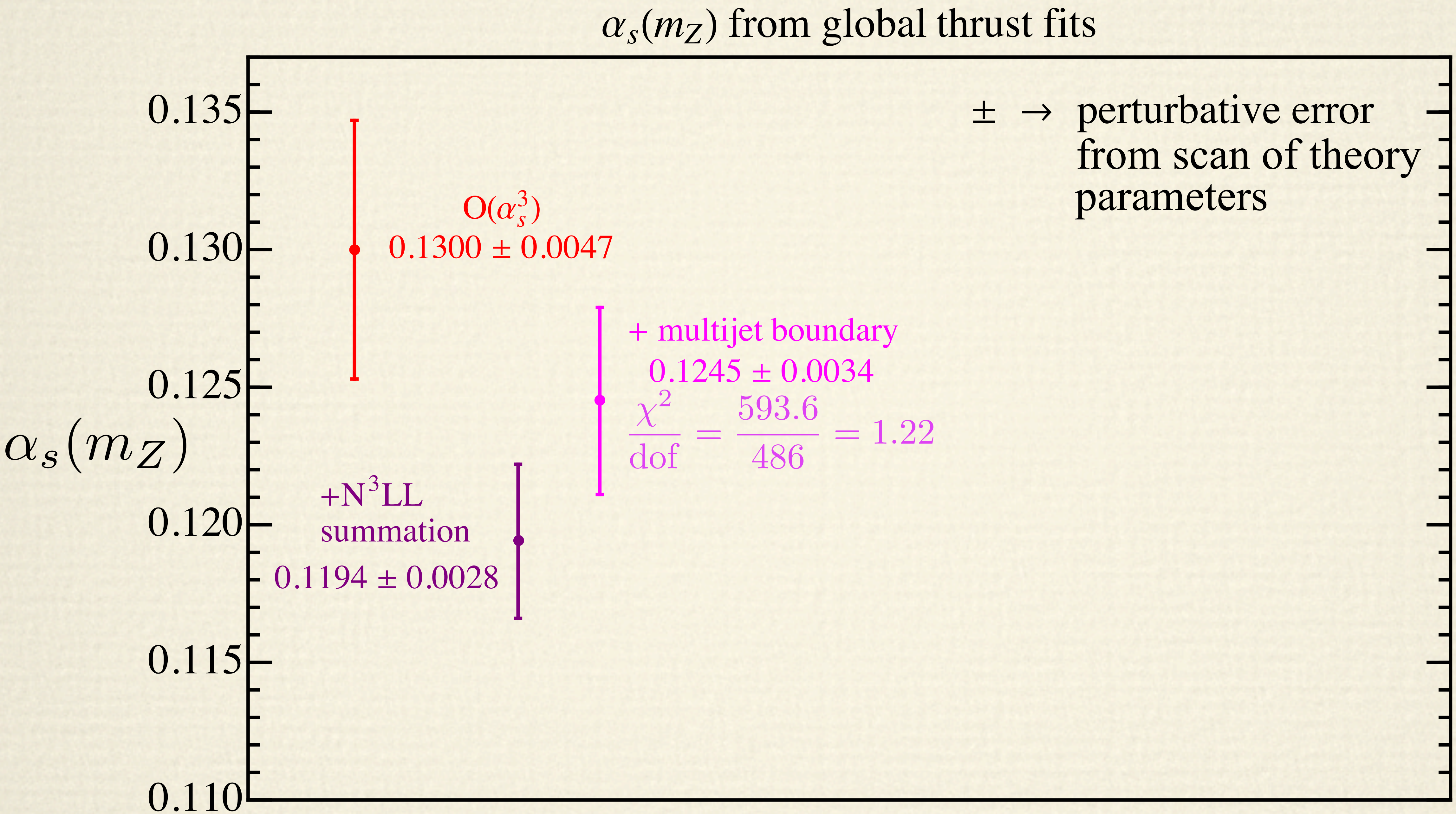

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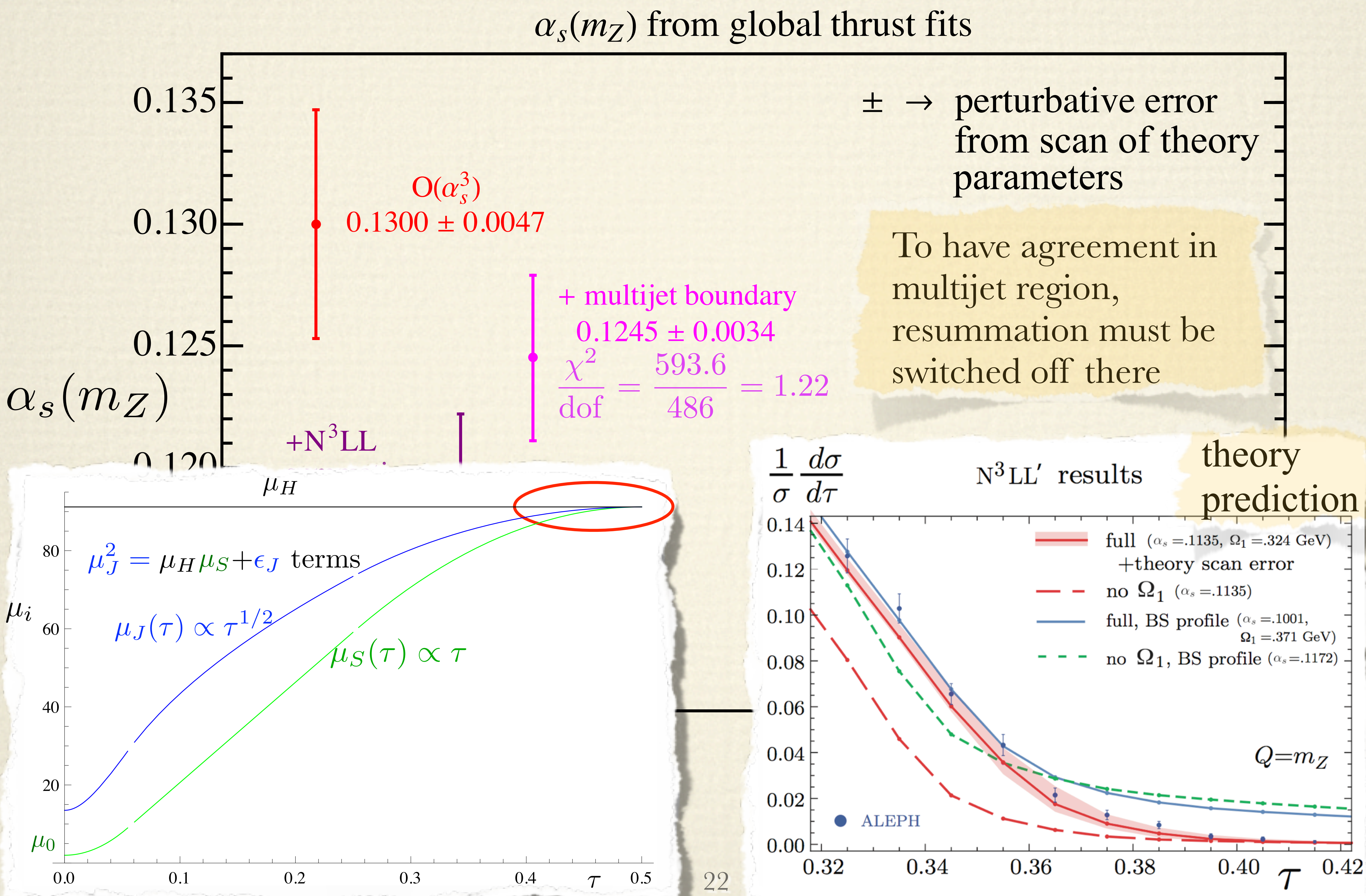
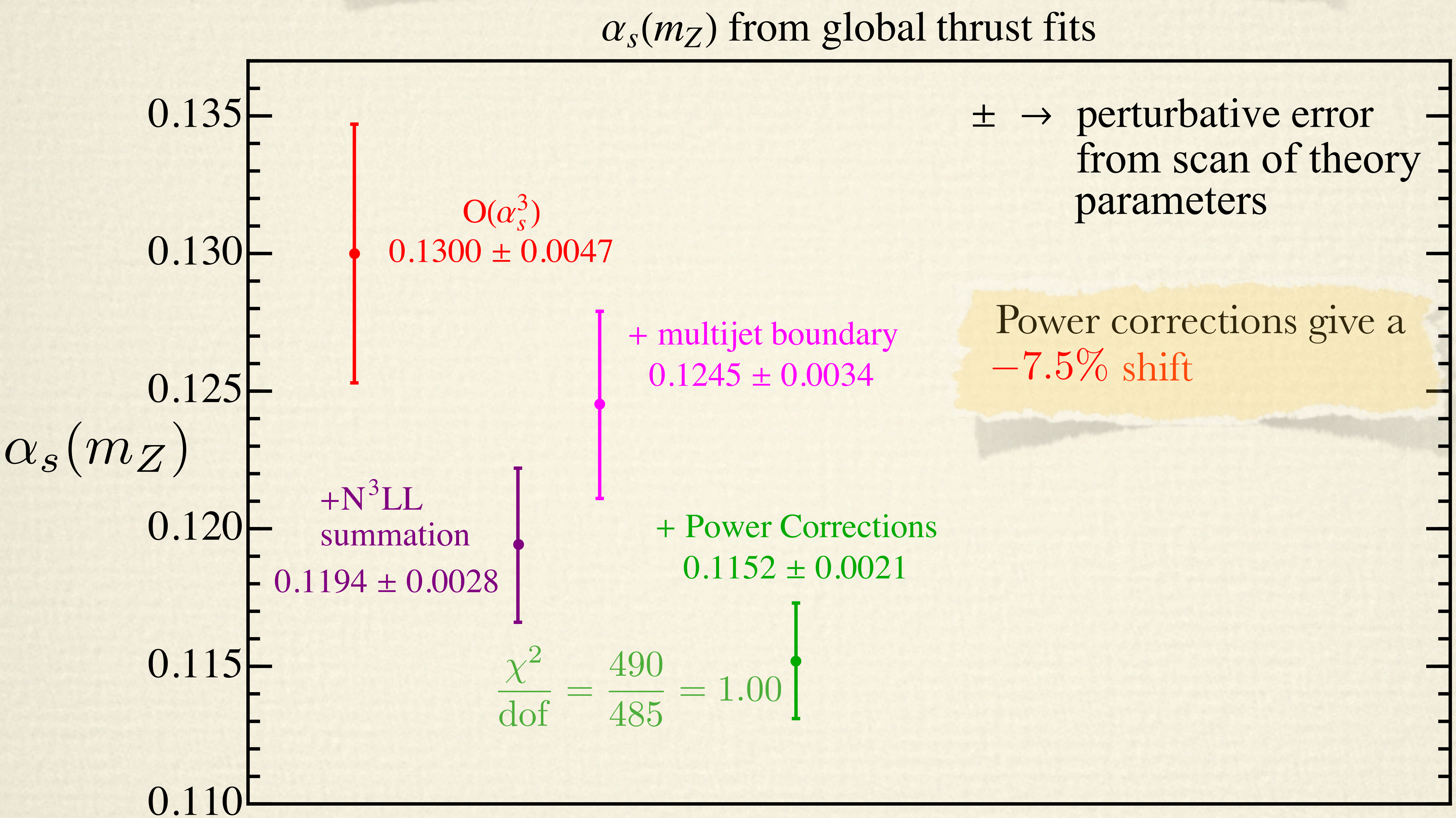


Global tail fit for $\alpha_s(m_Z)$ and Ω_1



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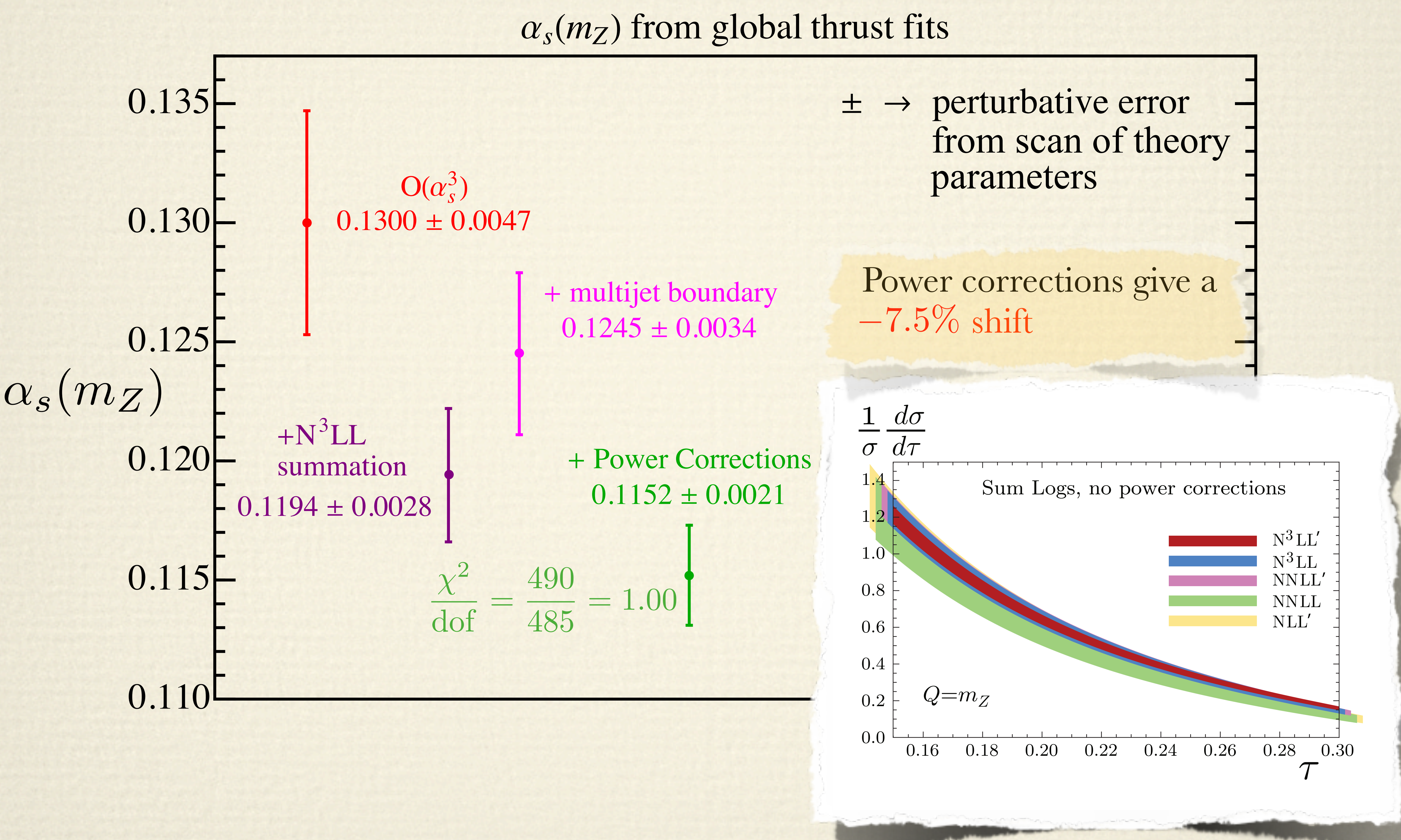

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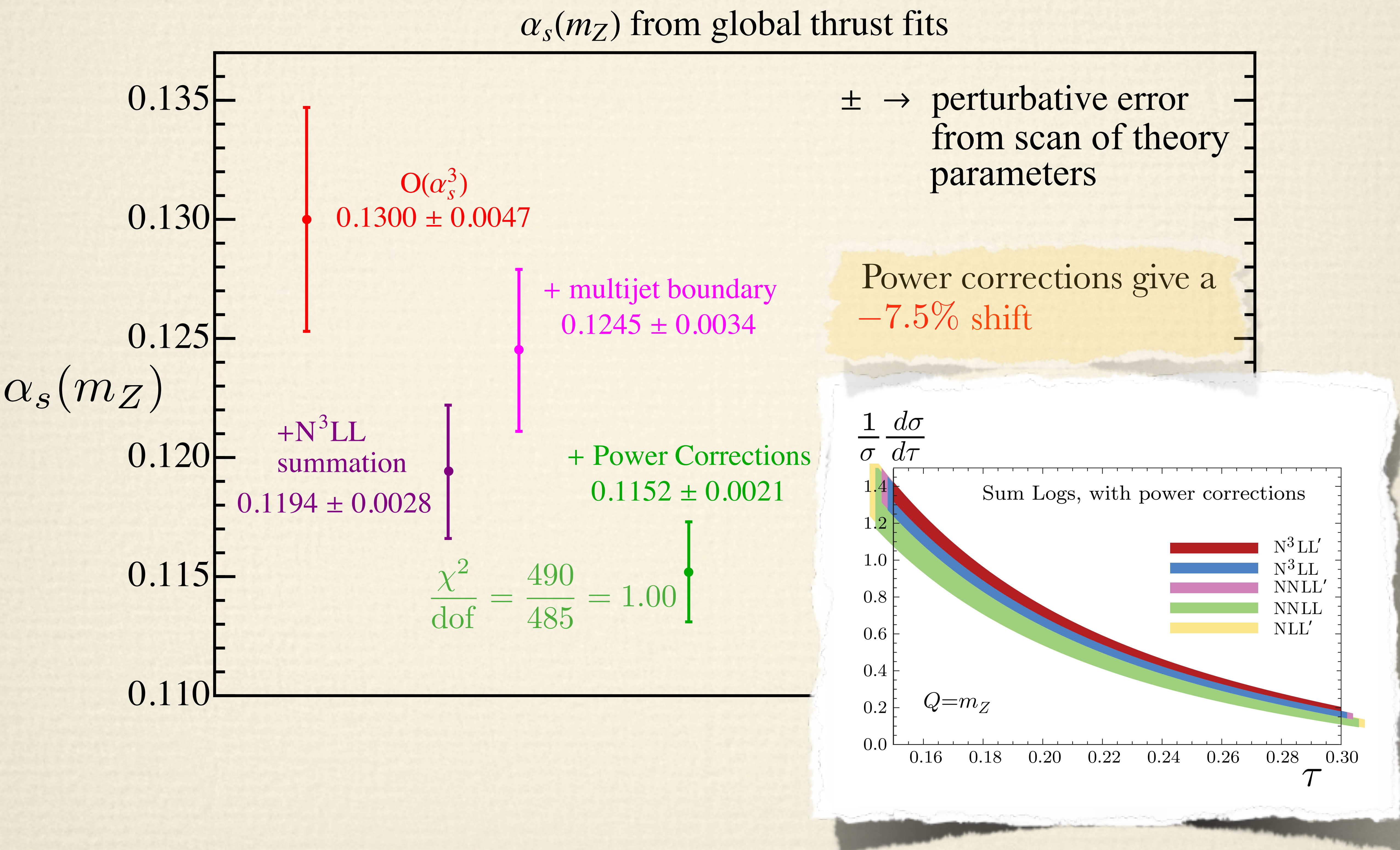


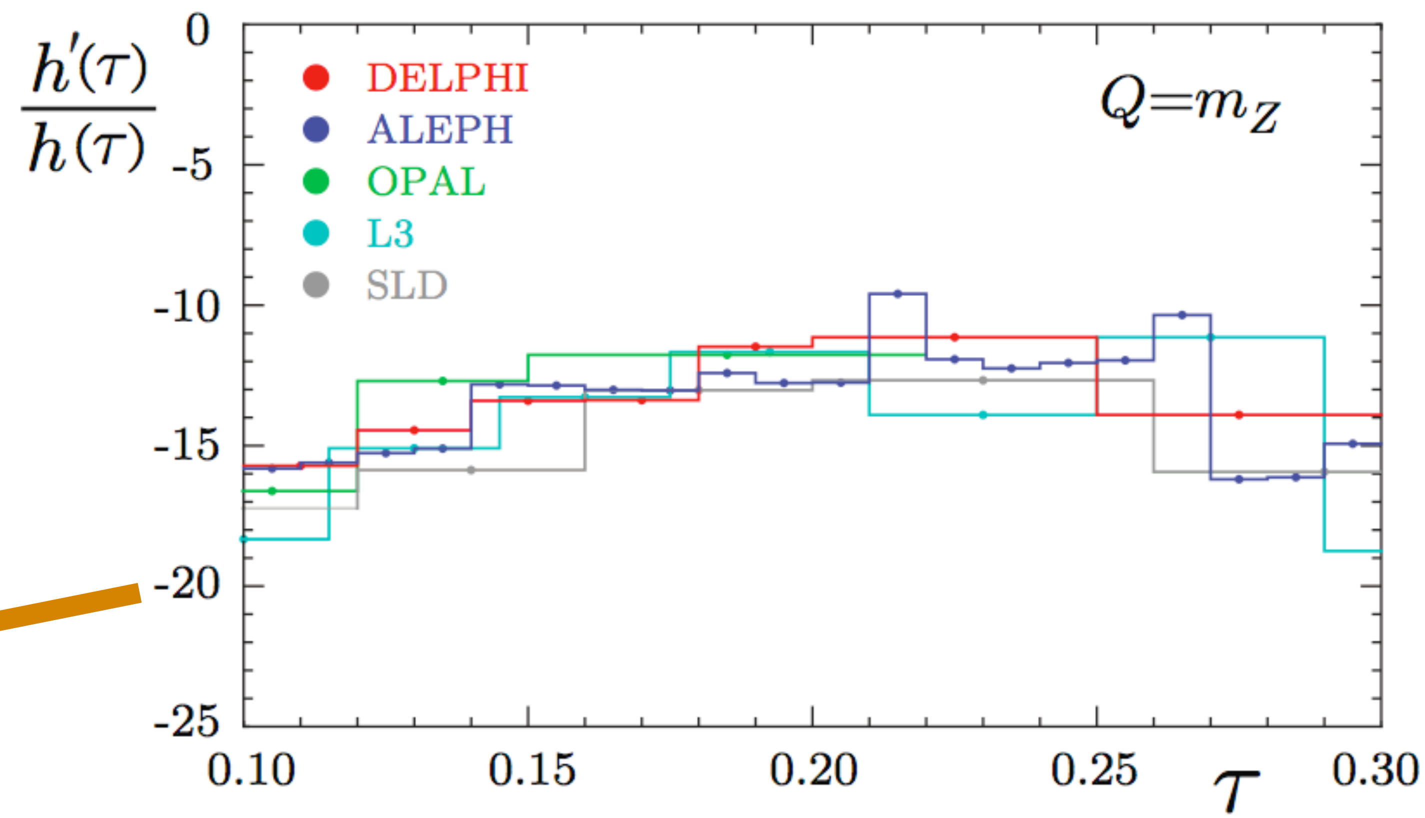
Leading power correction estimation

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - \frac{2\Lambda}{Q}\right)$$

Assuming $h \sim \alpha_s$

$$\frac{\delta\alpha_s}{\alpha_s} = \frac{2\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$$

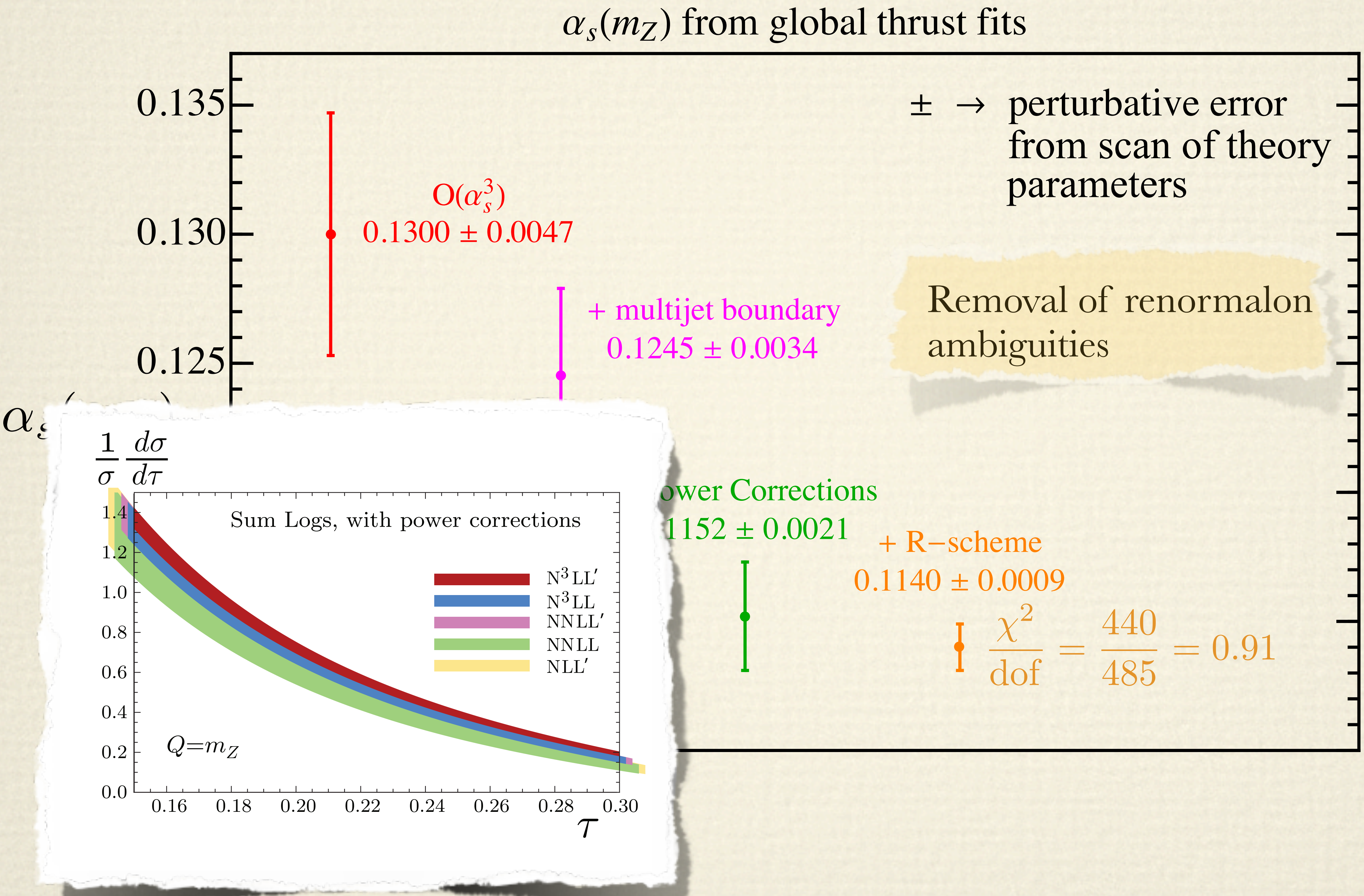
$$\frac{h'(\tau)}{h(\tau)} = -14 \pm 4$$



Assuming $\Lambda = 0.3\,\text{GeV}$

$$\frac{\delta\alpha_s}{\alpha_s} = -(9 \pm 3)\%$$

Global tail fit for $\alpha_s(m_Z)$ and Ω_1



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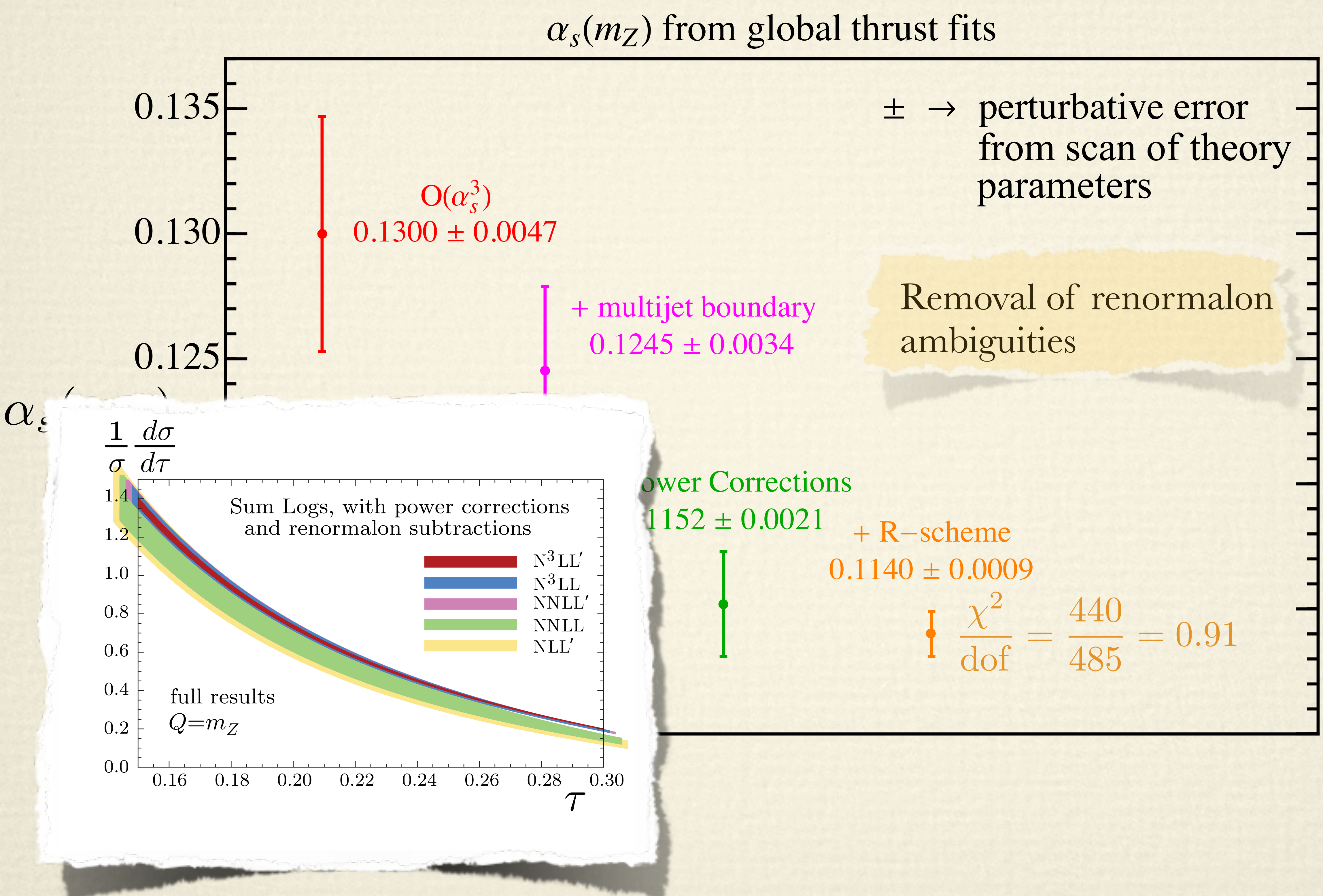

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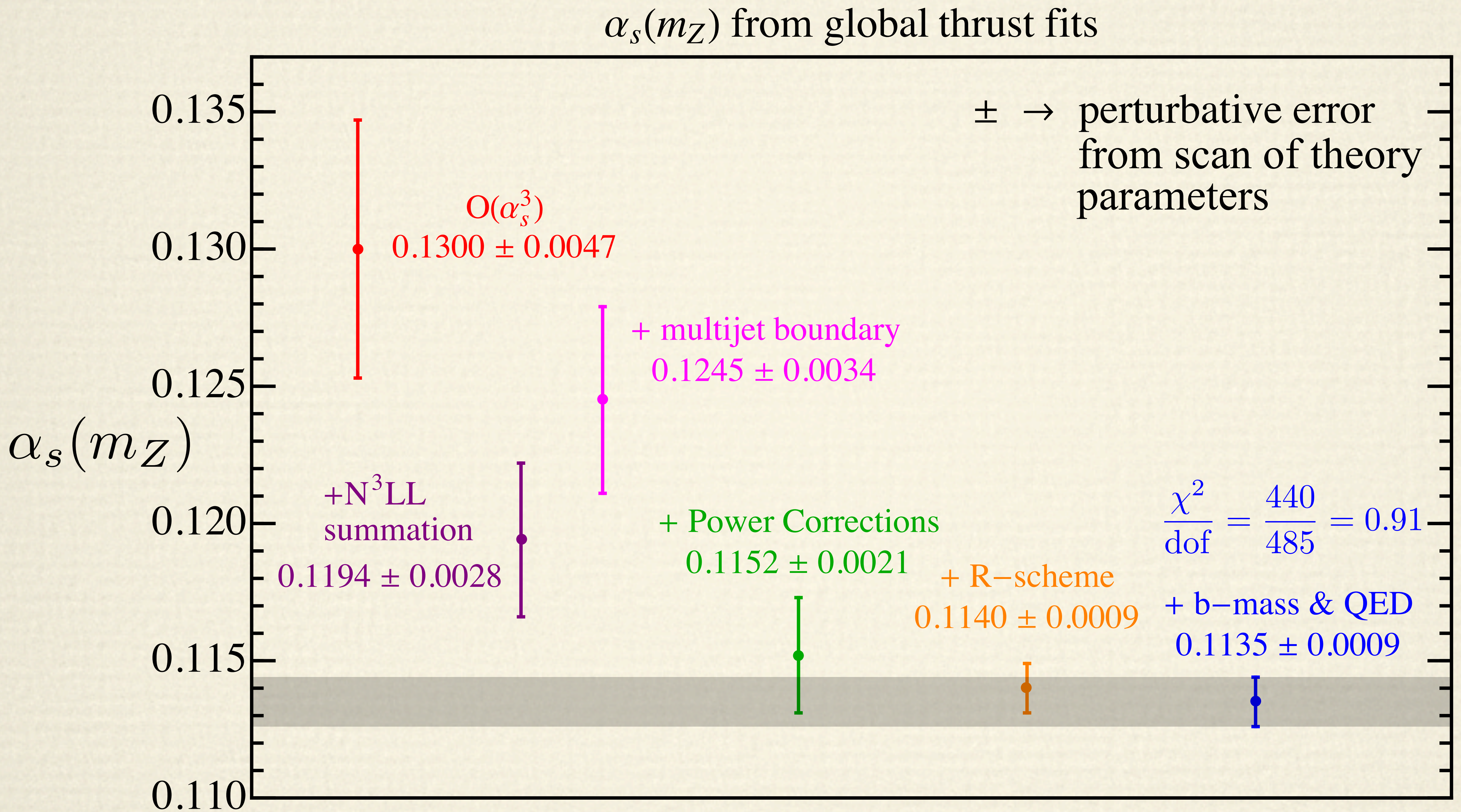

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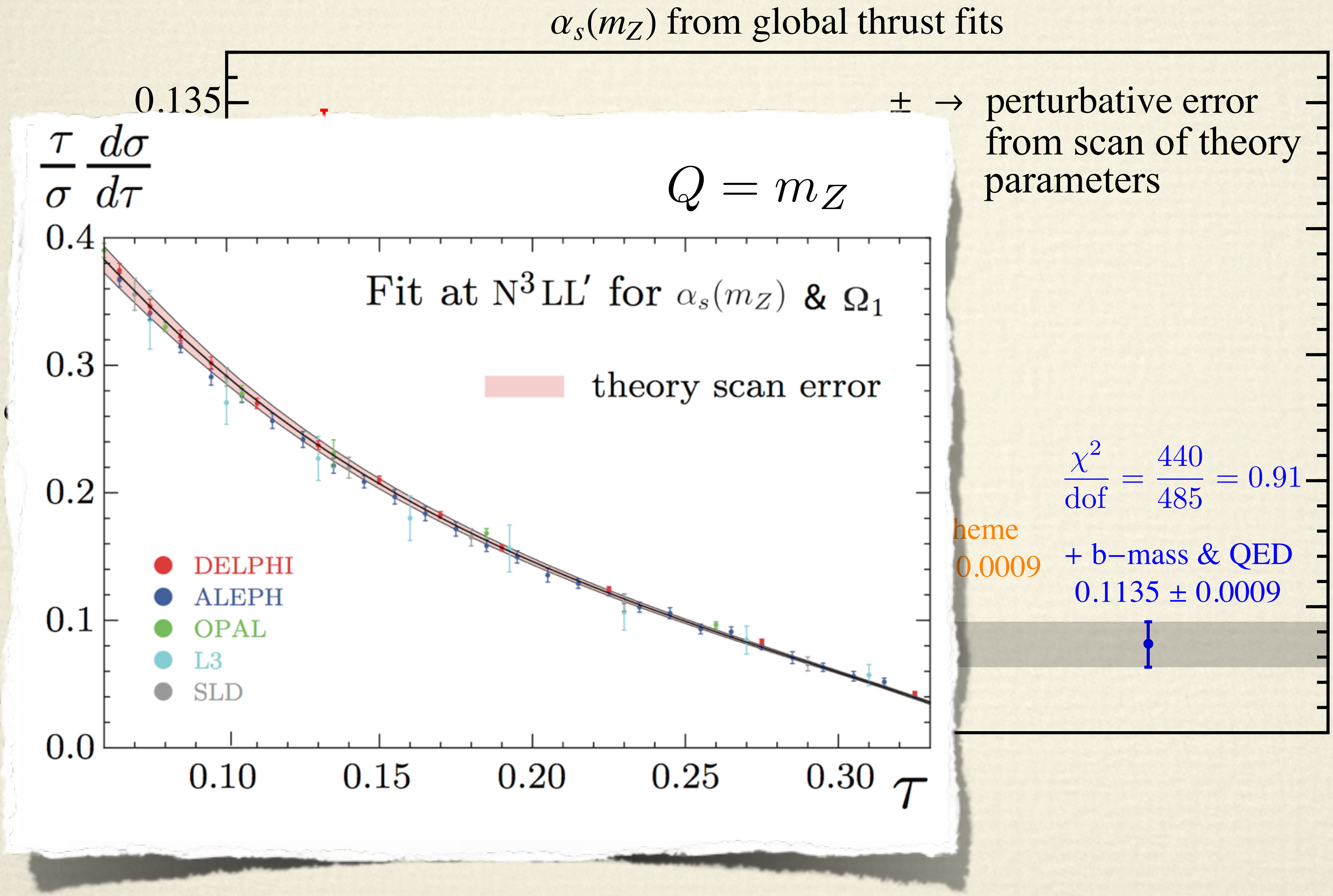
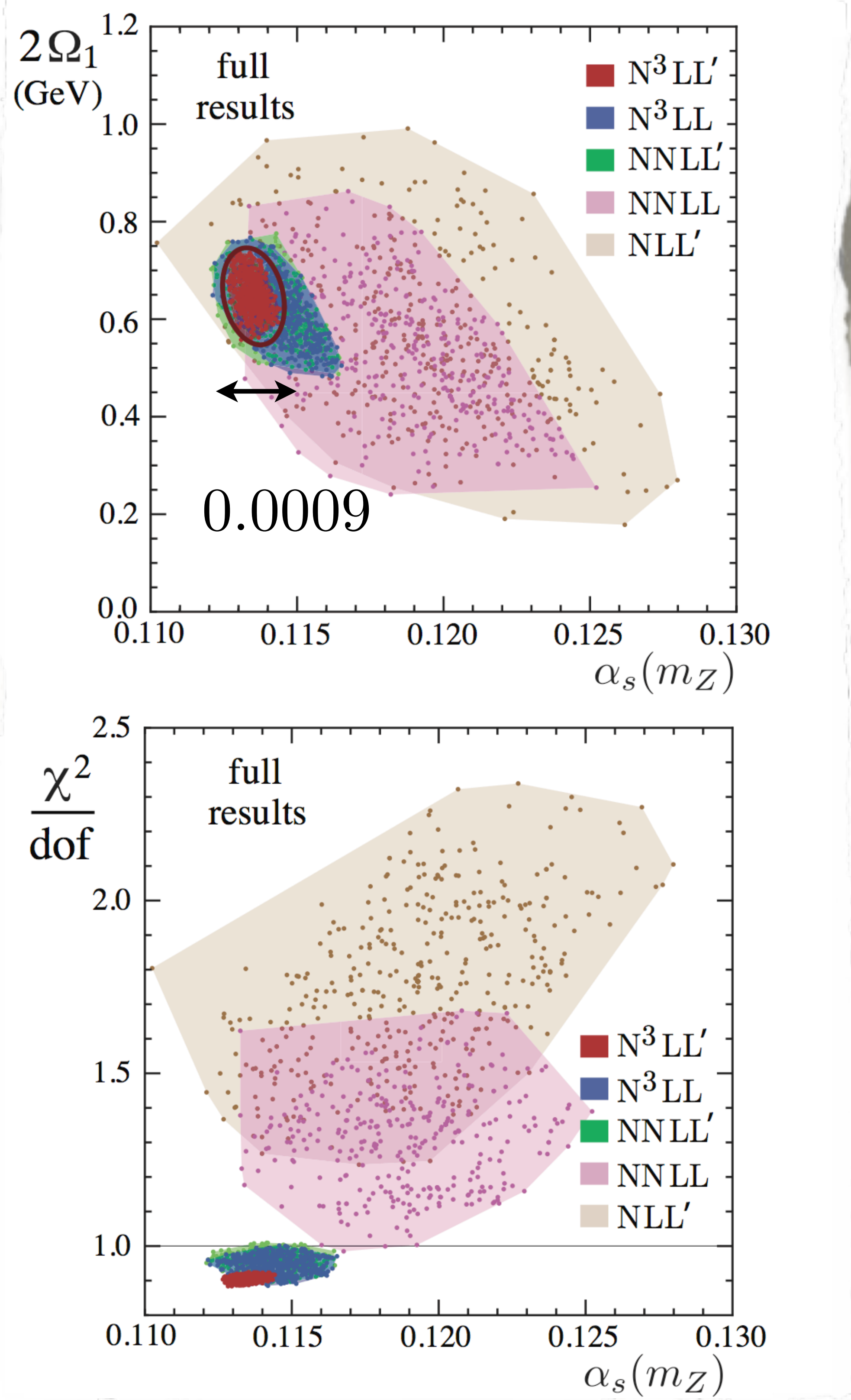


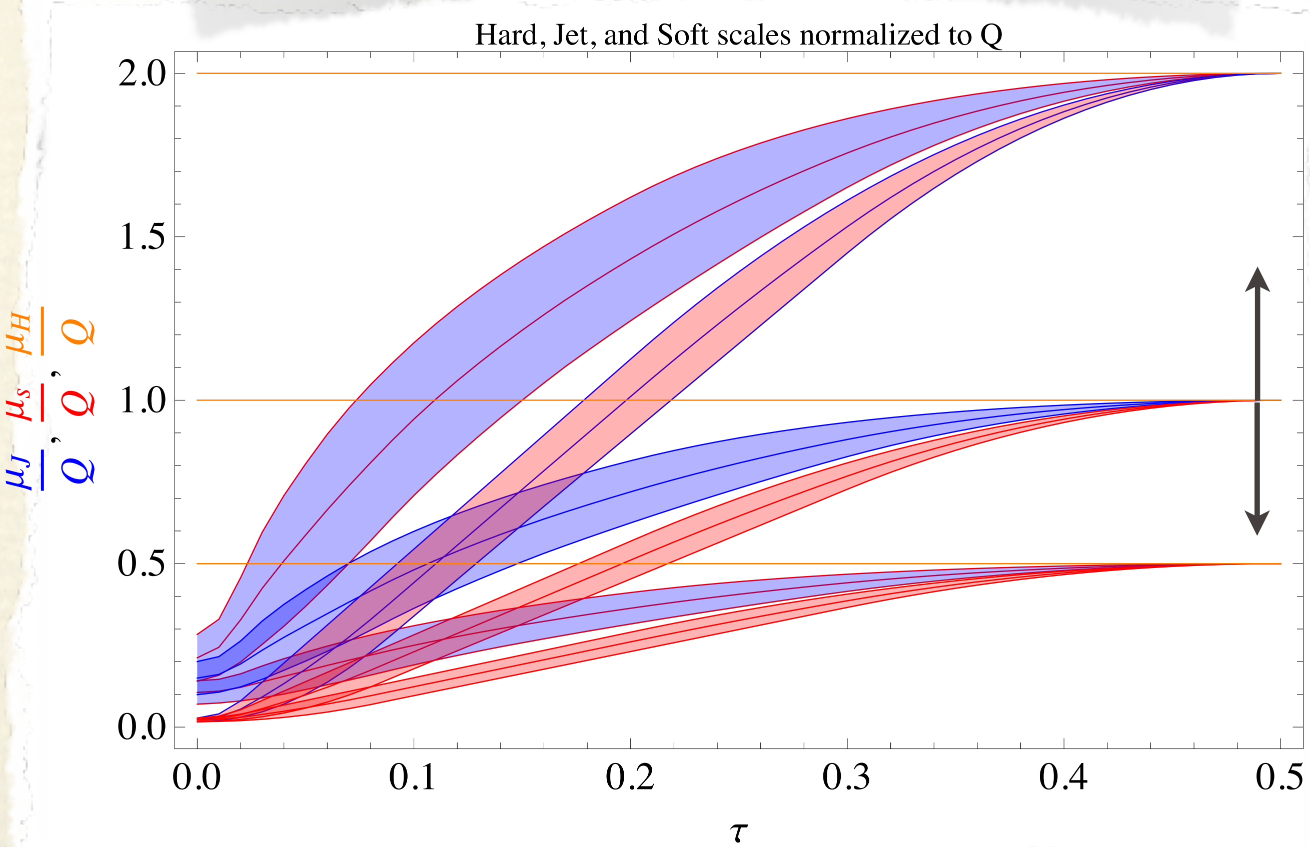
Convergence of results



Ω_1 determined to 16% accuracy

500-points random scan per order

Largest contribution to perturbative uncertainty comes from variation of profile parameters



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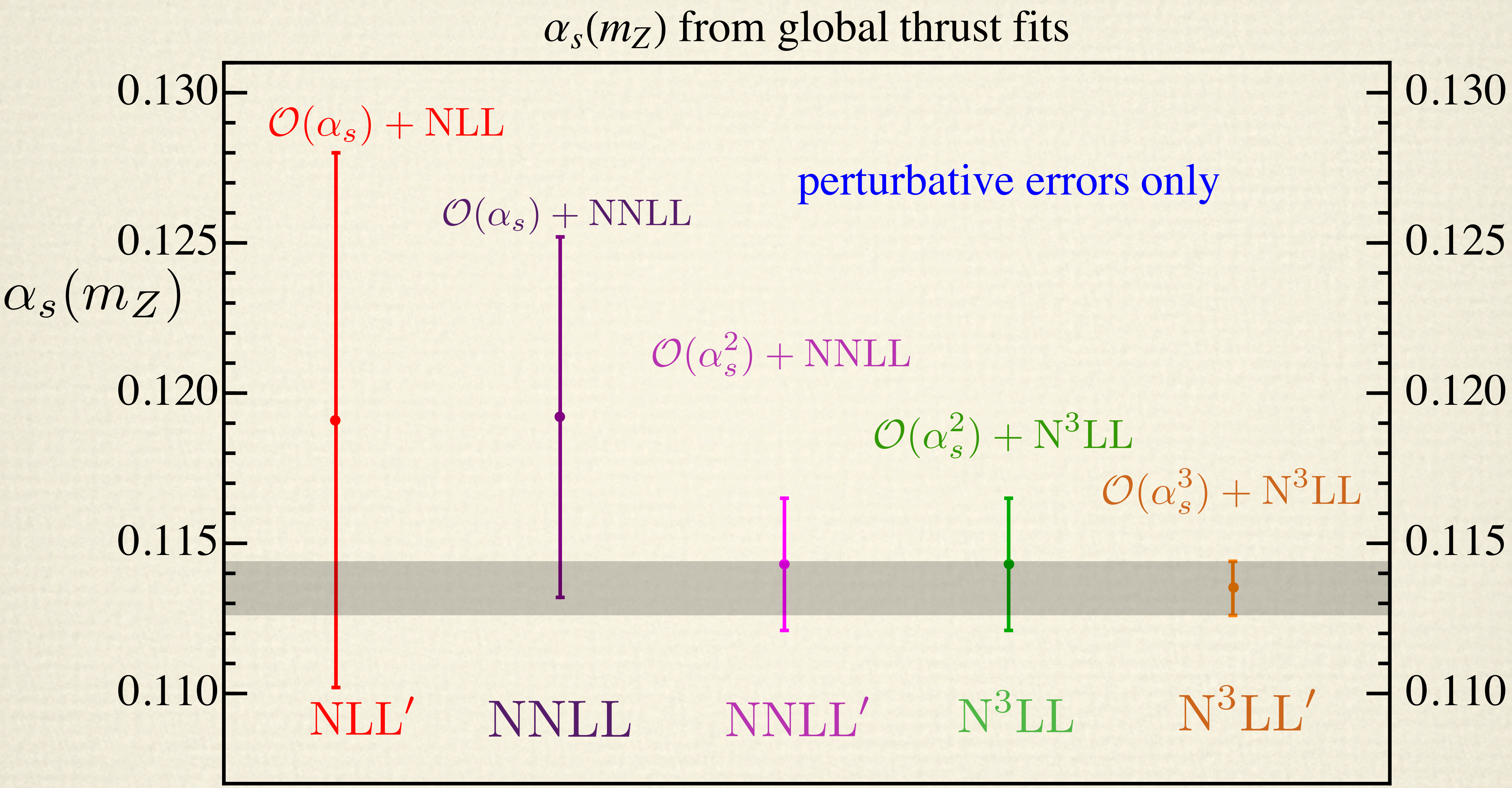
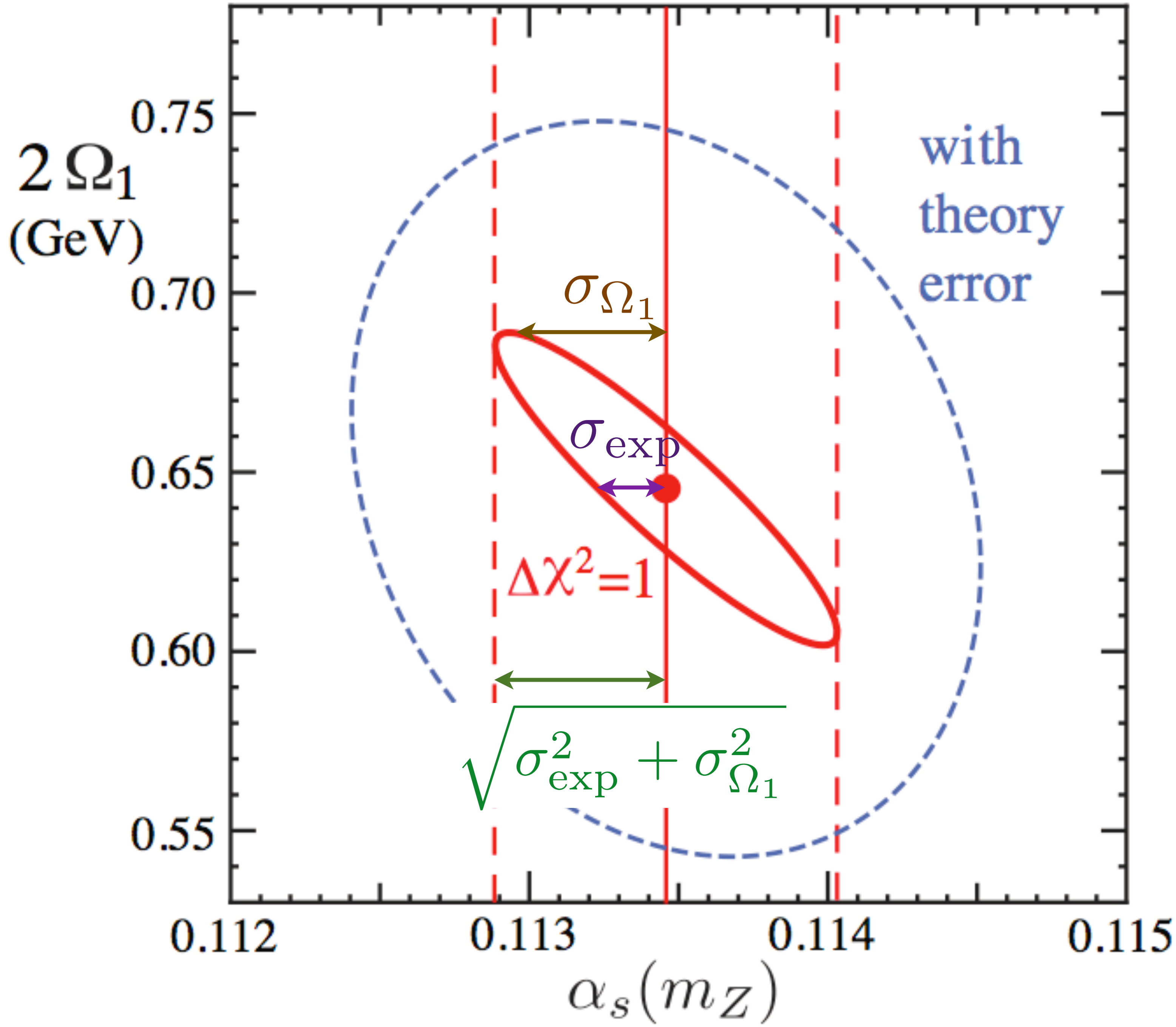




$$\frac{\chi^2}{\text{dof}} = \frac{440}{485} = 0.91$$

"Standard" dataset

$$Q \geq 35\,\text{GeV}$$

$$\frac{6\,\text{GeV}}{Q} \leq \tau \leq 0.33$$

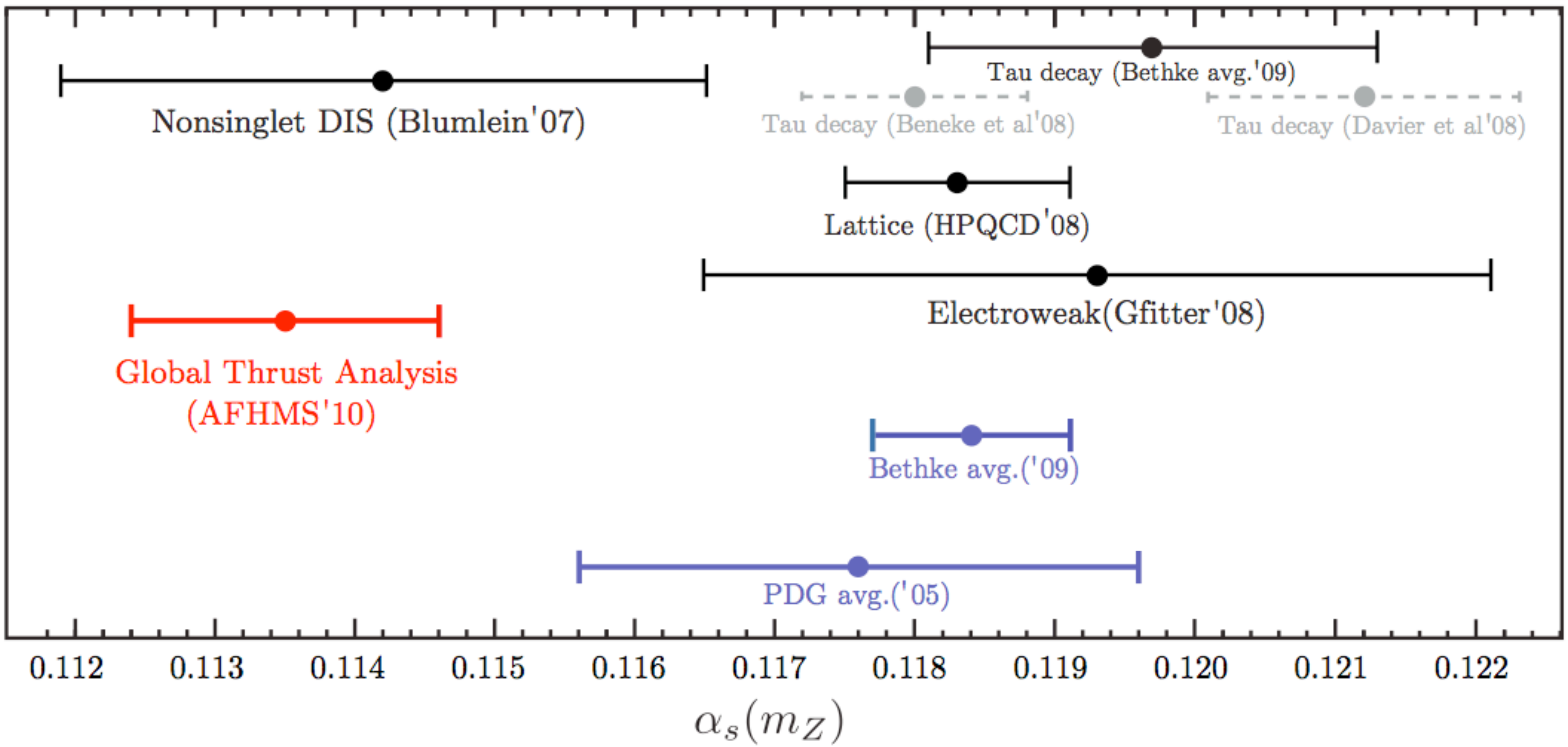
487 bins

Correlations treated with the minimal overlap model

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

Final thrust results

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\rm exp} \pm 0.0005_{\rm had} \pm 0.0009_{\rm pert}$$



ALEPH $Q = m_Z$ thrust

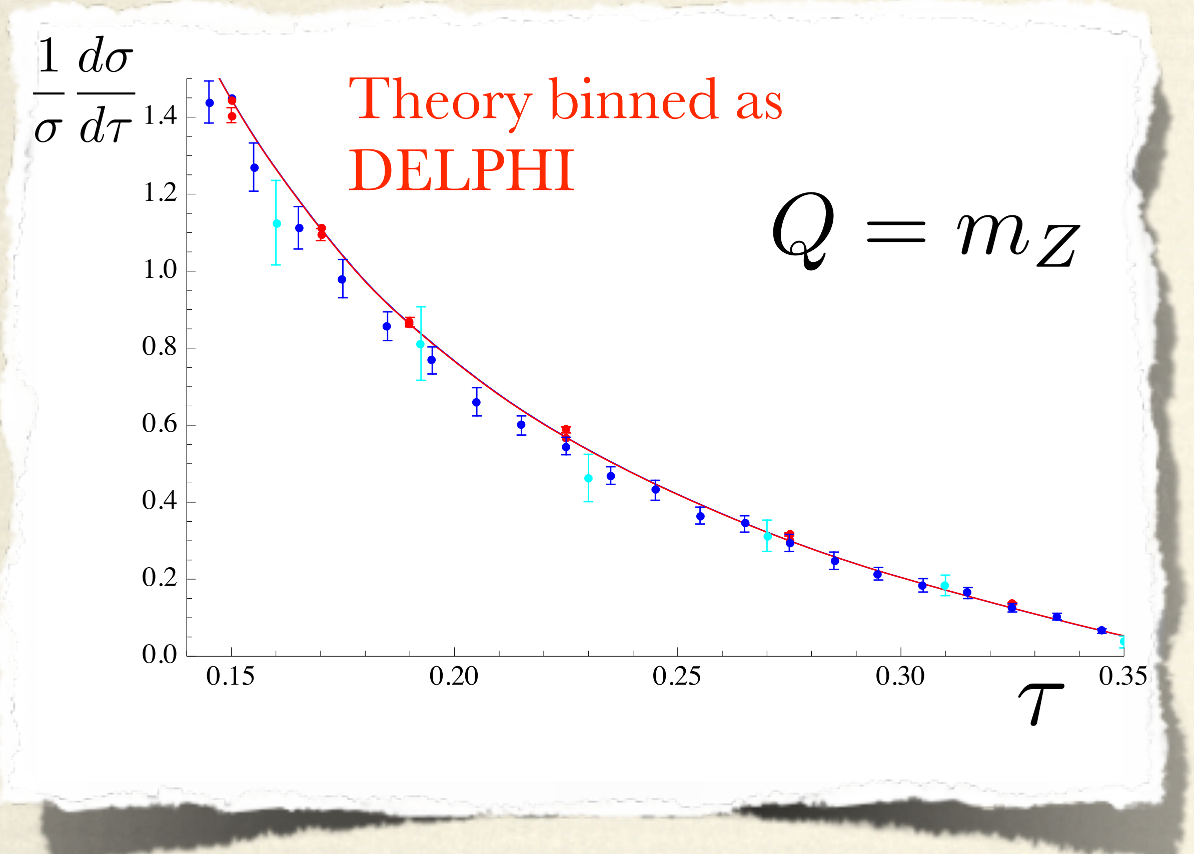
$0.1274 \pm 0.0042_{\rm pert}$

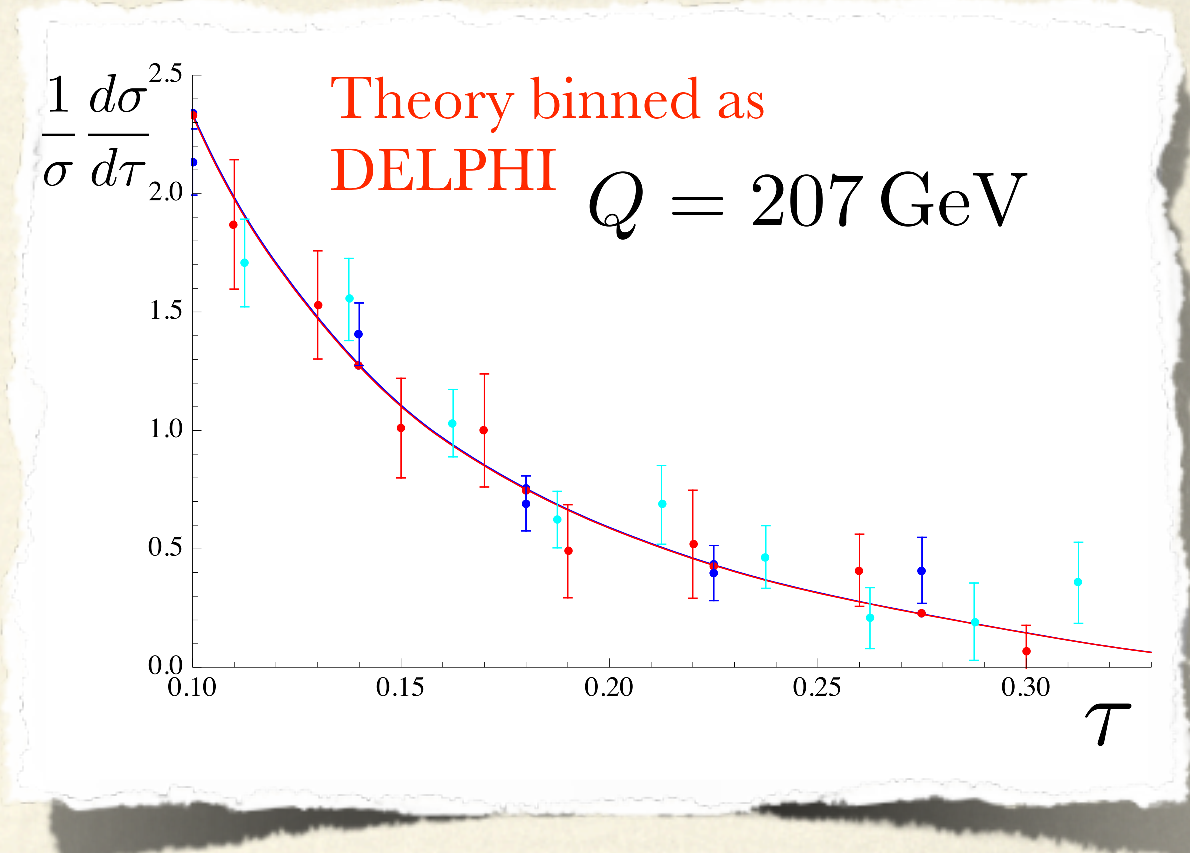
Dissertori et al. '07

Applications for a Future Linear Collider

LEP accuracy

ALEPH, DELPHI, L3





Experimental accuracy $\sim 1\%$

$\sigma_{\rm syst}$ dominates over $\sigma_{\rm stat}$

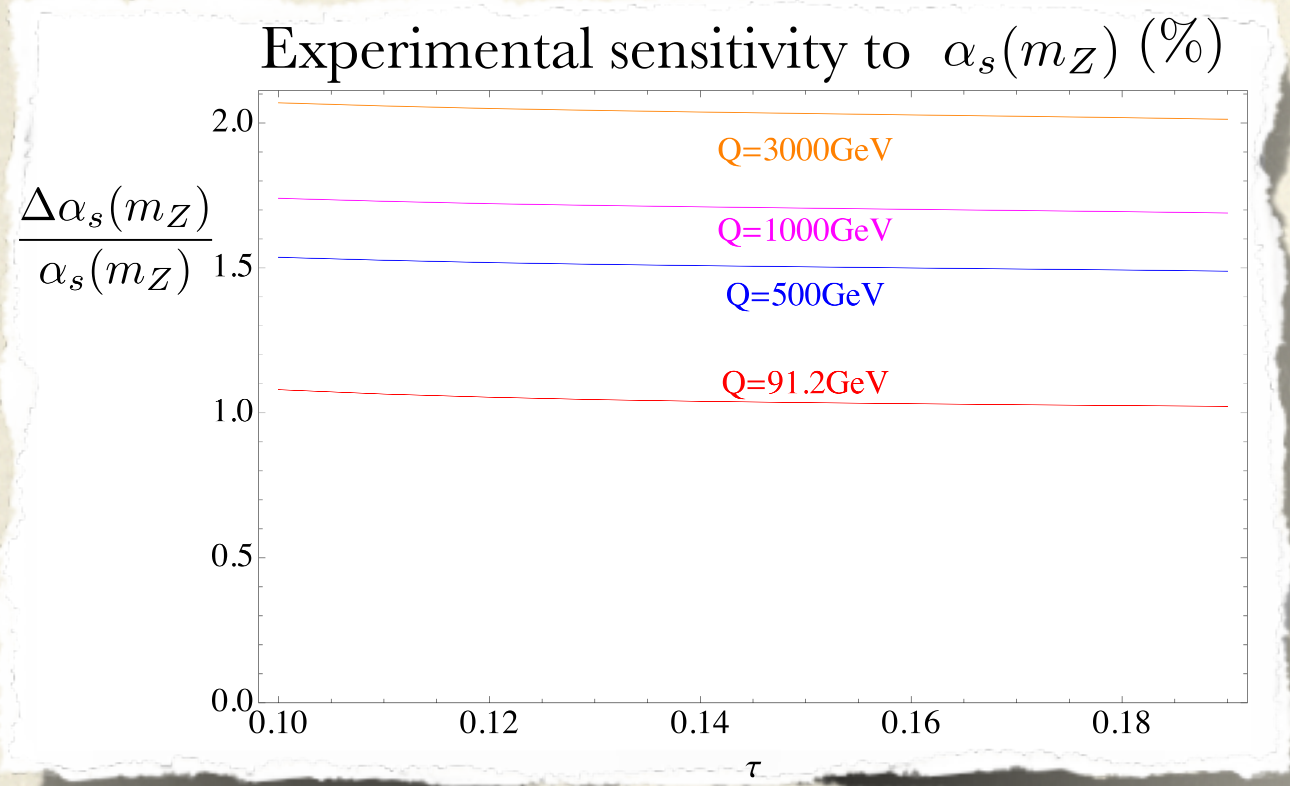
Experimental accuracy $\sim 10\%$

$\sigma_{\rm stat}$ dominates over $\sigma_{\rm syst}$

At high energy, LEP data are not very accurate.
ILC will provide precise data at $Q = 500\,\mathrm{GeV}\,(1000\,\mathrm{GeV})$

Sensitivity to α_s

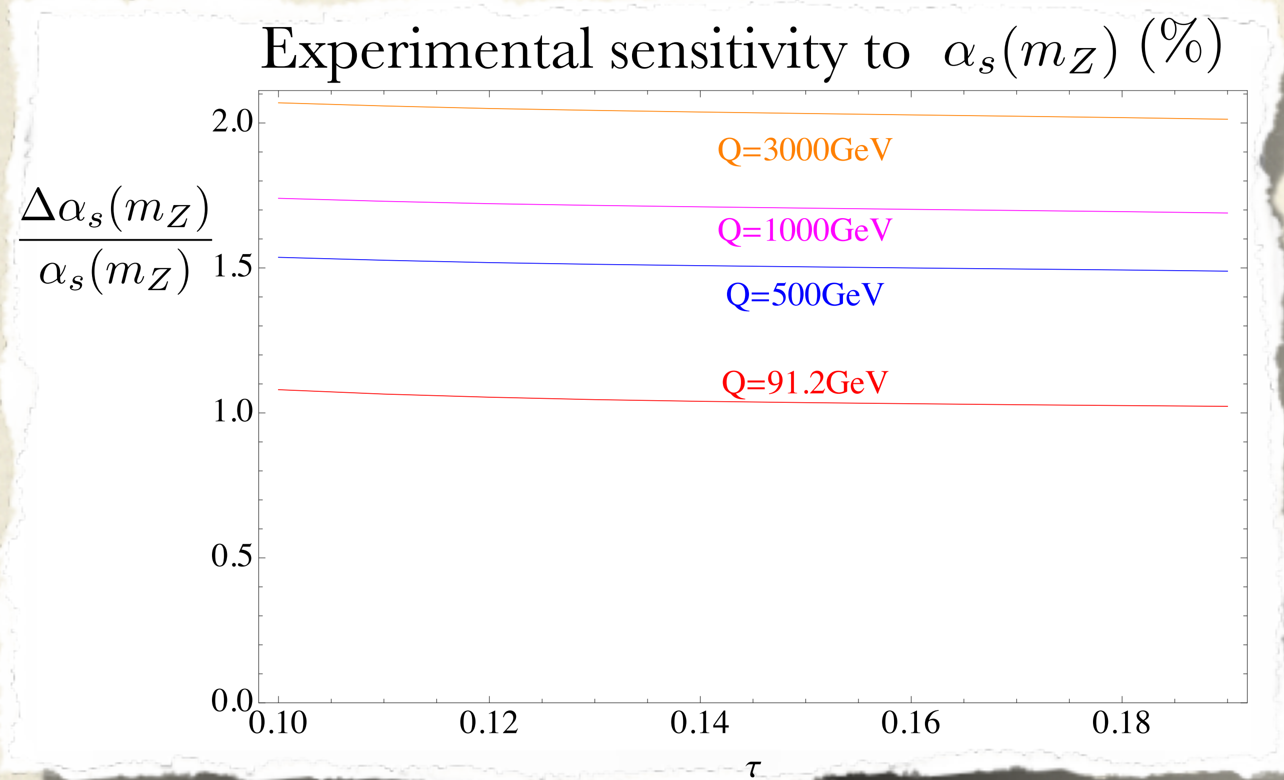
As an exercise, let's assume we can achieve a 2% experimental accuracy at every Q

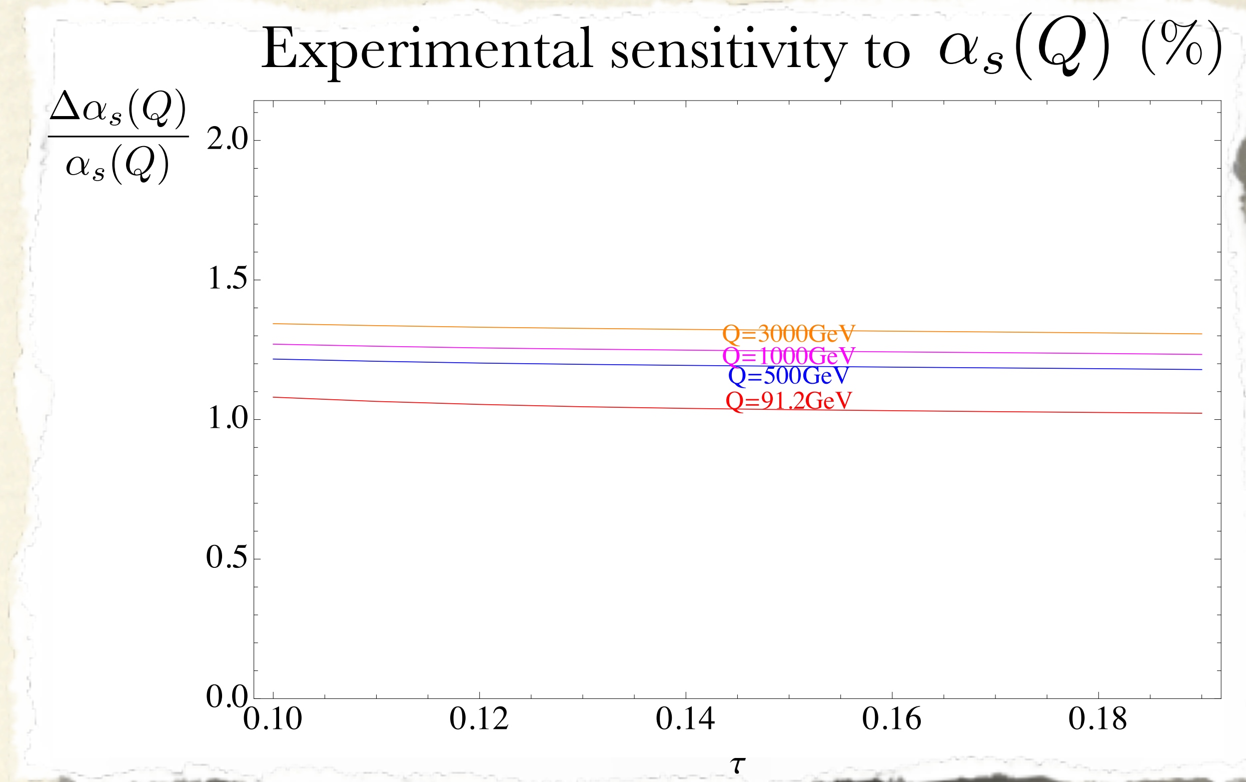


Experimental uncertainty on $\alpha_s(m_Z)$ given a 2% experimental accuracy

Sensitivity to α_s

As an exercise, let's assume we can achieve a 2% experimental accuracy at every Q



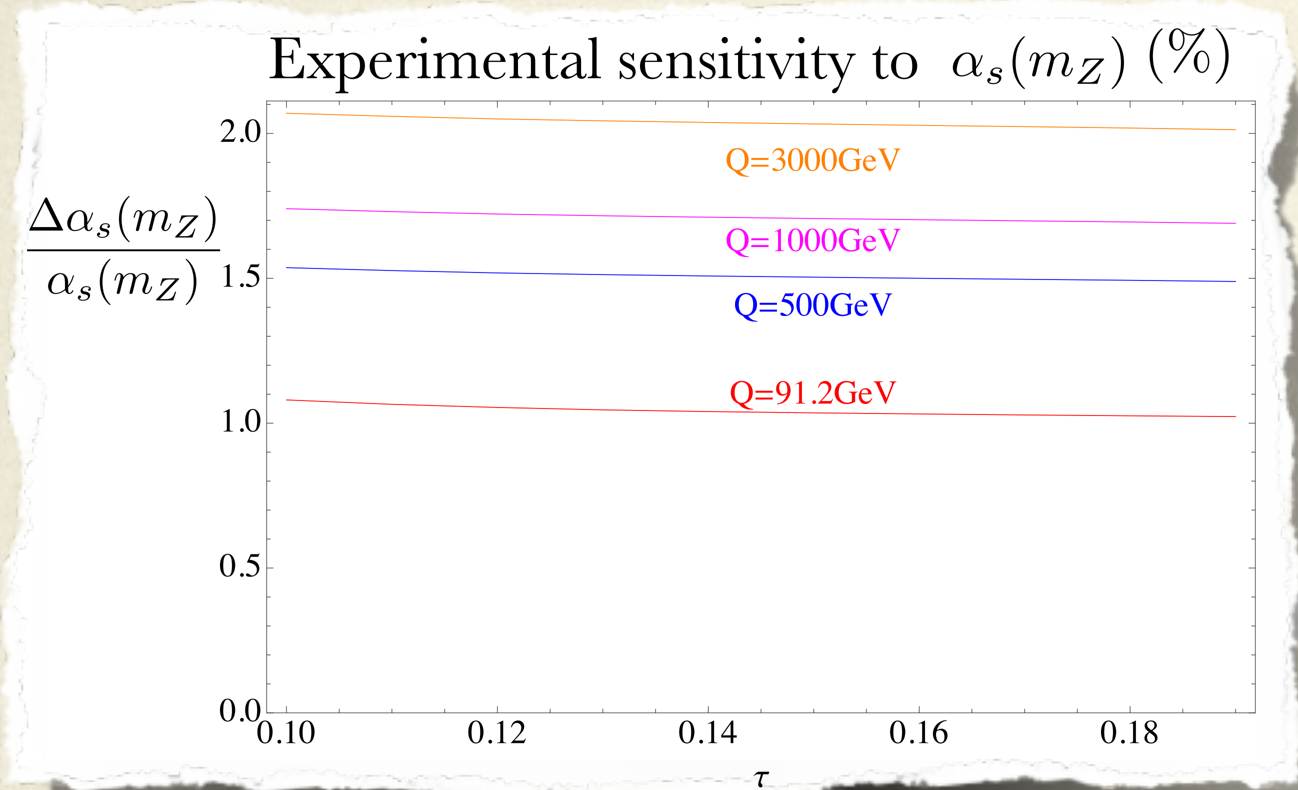


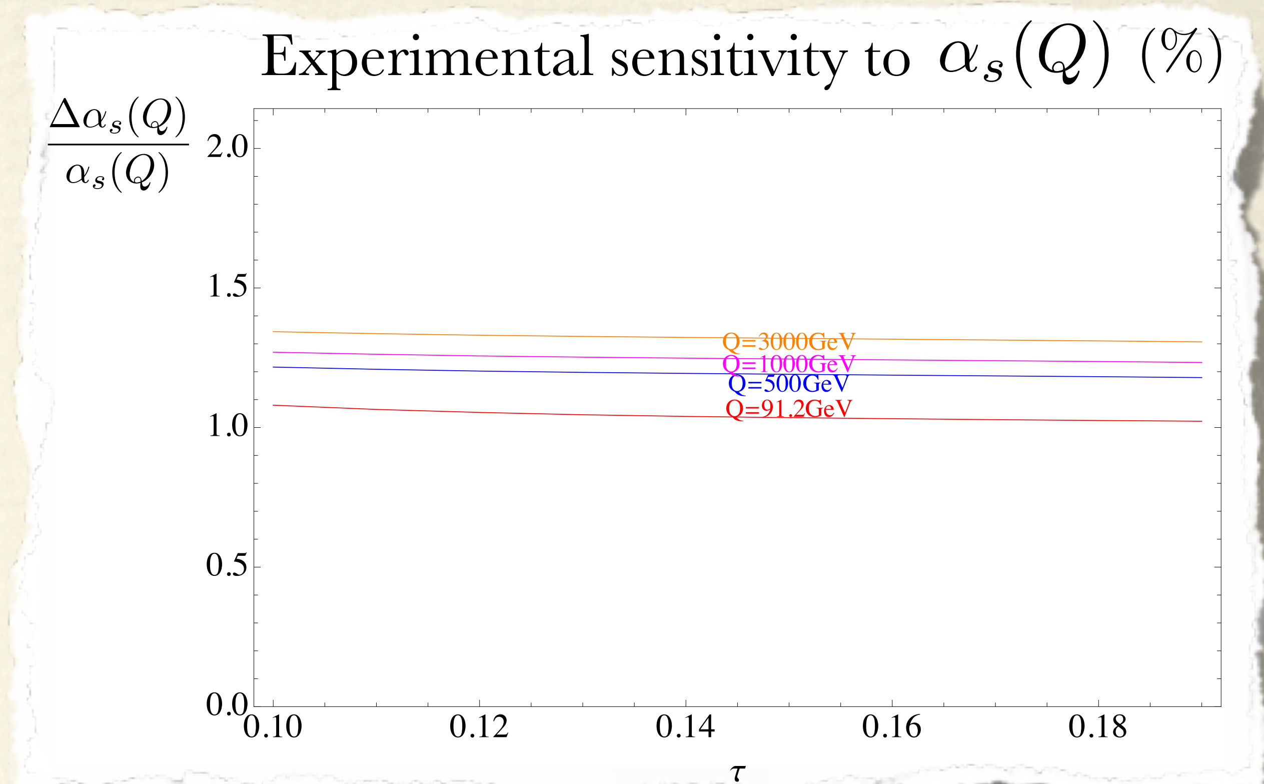
Most of the effect is due to running of α_s
Running from ILC energies to m_Z slightly decreases the accuracy on the determination of $\alpha_s(m_Z)$

The accuracy of the determination of $\alpha_s(Q)$ is roughly constant

Sensitivity to α_s

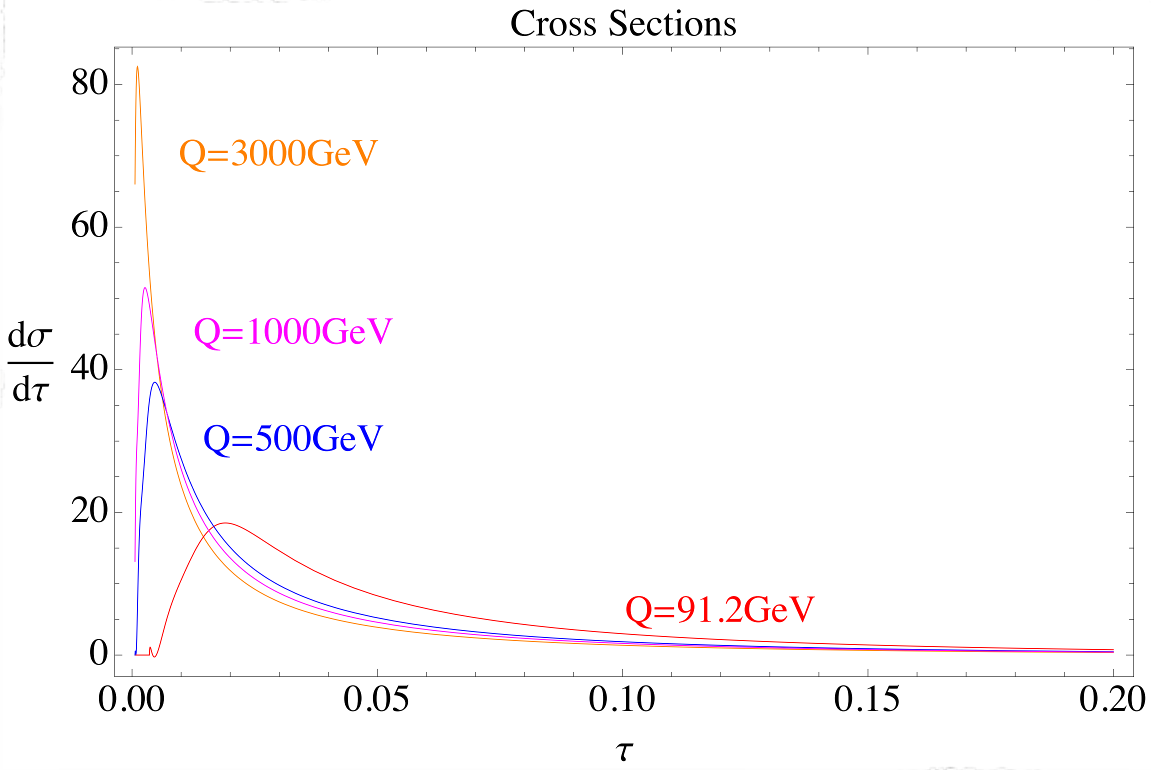
As an exercise, let's assume we can achieve a 2% experimental accuracy at every Q





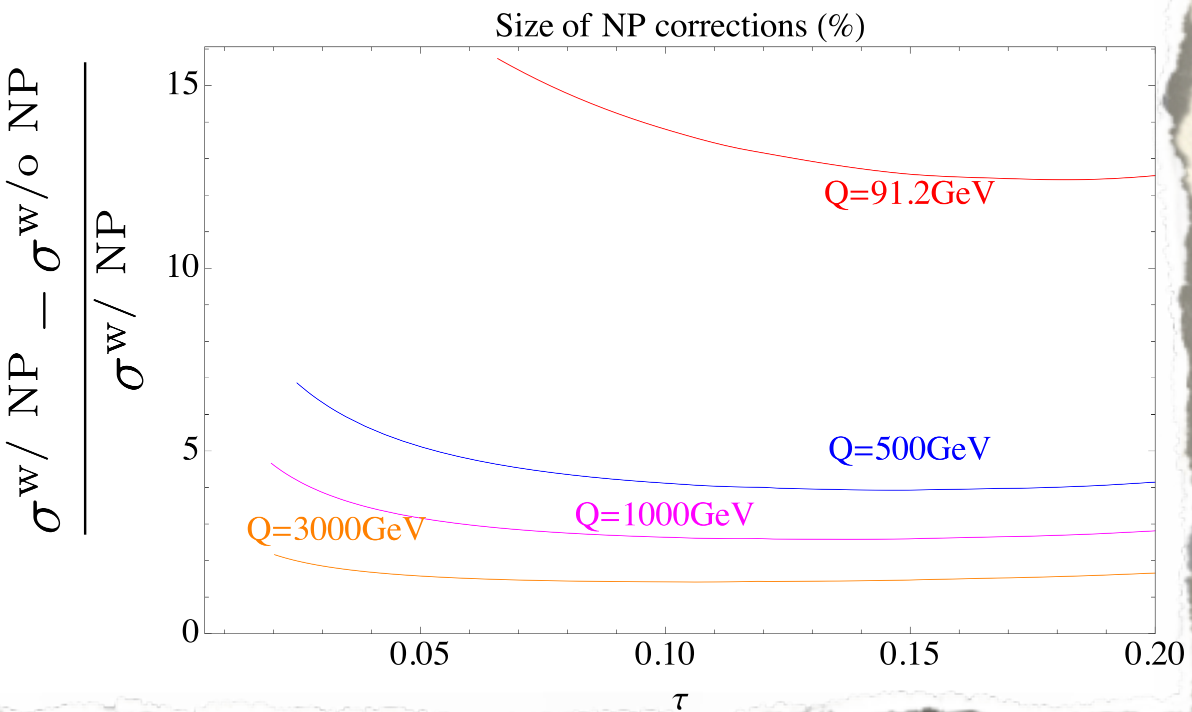
Measurement will test the matter content of the QCD β function

Size of nonperturbative effects



for increasing Q

- the peak moves toward smaller τ
- events tend to accumulate in the region with small τ
- tail region becomes larger but less populated



- Size of non perturb. corrections decreases for high values, scaling as $\frac{\Lambda_{QCD}}{Q}$

ILC outlook

- ILC could provide data with precision comparable to LEP
- Observed strongly interacting new physics particles will be tested in virtual graphs through the evolution of α_s
- At ILC energies, non perturbative effects become less important:
    - possibility to fit higher moments of nonperturbative soft function
    - possibility to expand fit region
- Global fit for $\alpha_s(m_Z)$ and Ω_1 will become more precise

Summary of Global Fits Discussed today

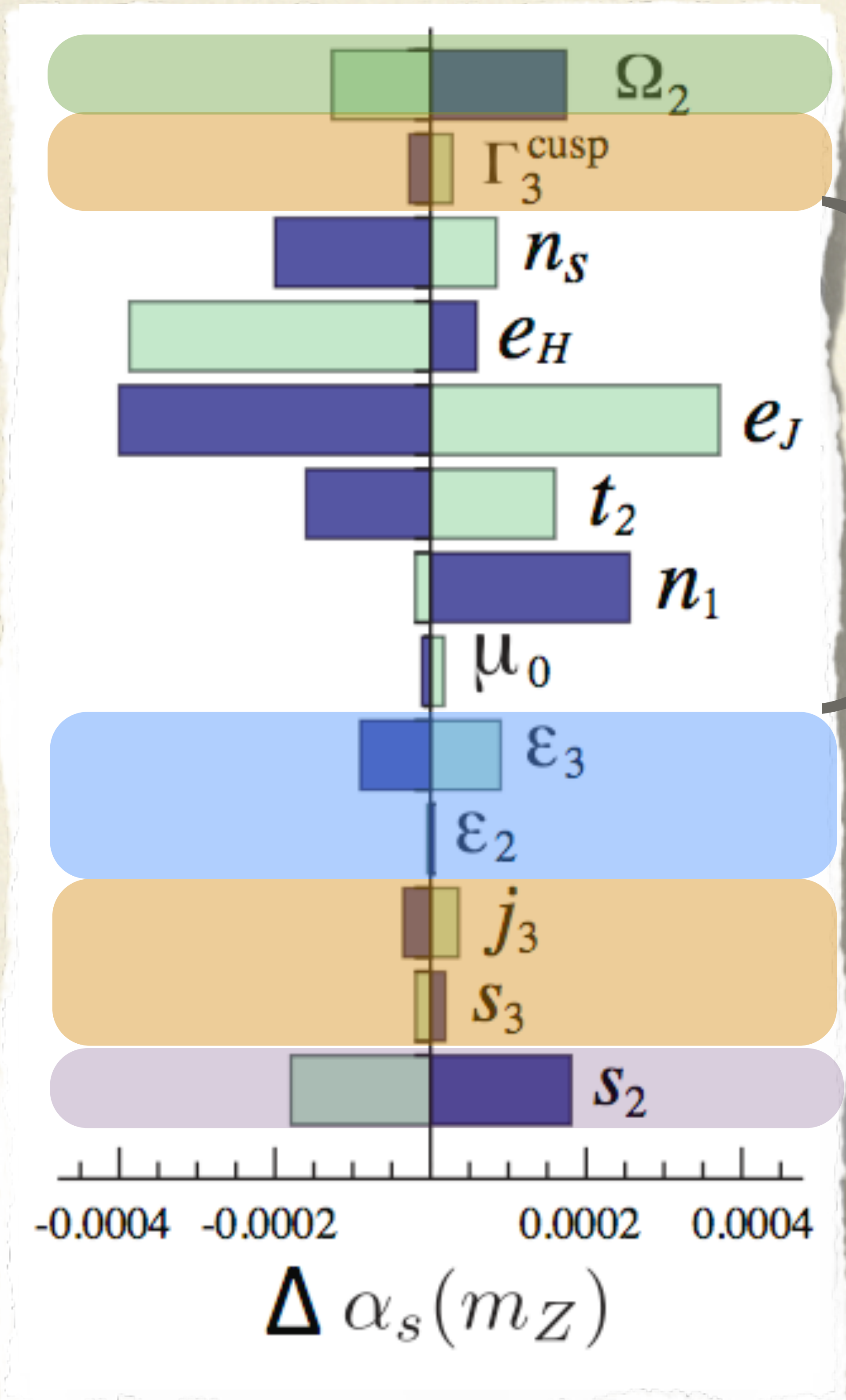
- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets
- SCET has finally provided theorists with a means to catch up with the experimental precision of LEP. The result of our global fit is

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

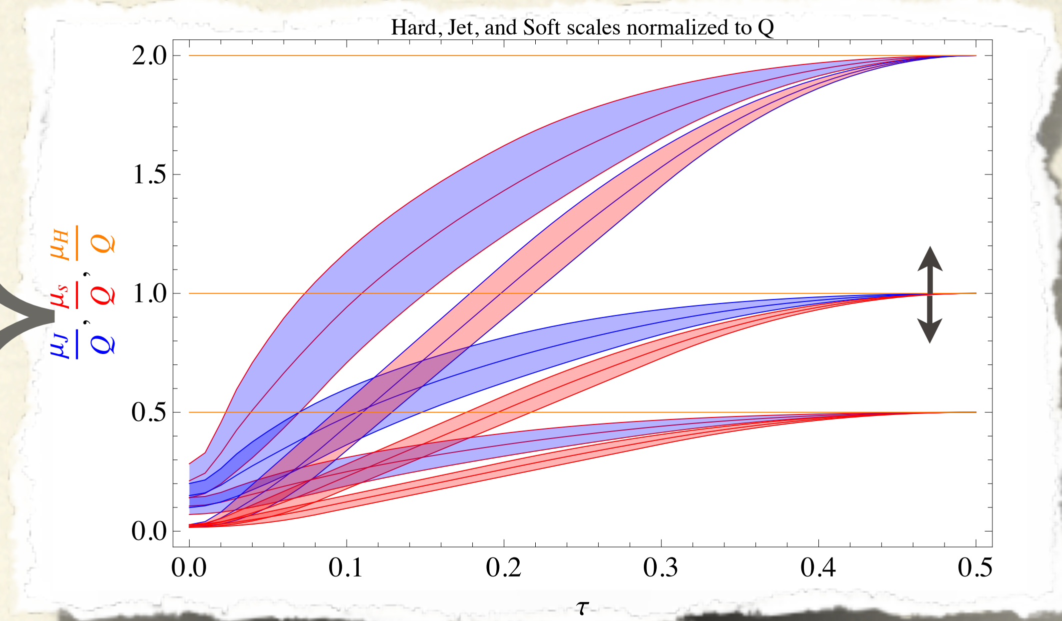
- First rigorous field theoretical treatment of nonperturbative effects
- Analysis of other event shape observables is under way! (Heavy Jet Mass)

Backup slides

Uncertainty budget from scan



Uncertainty from higher moments



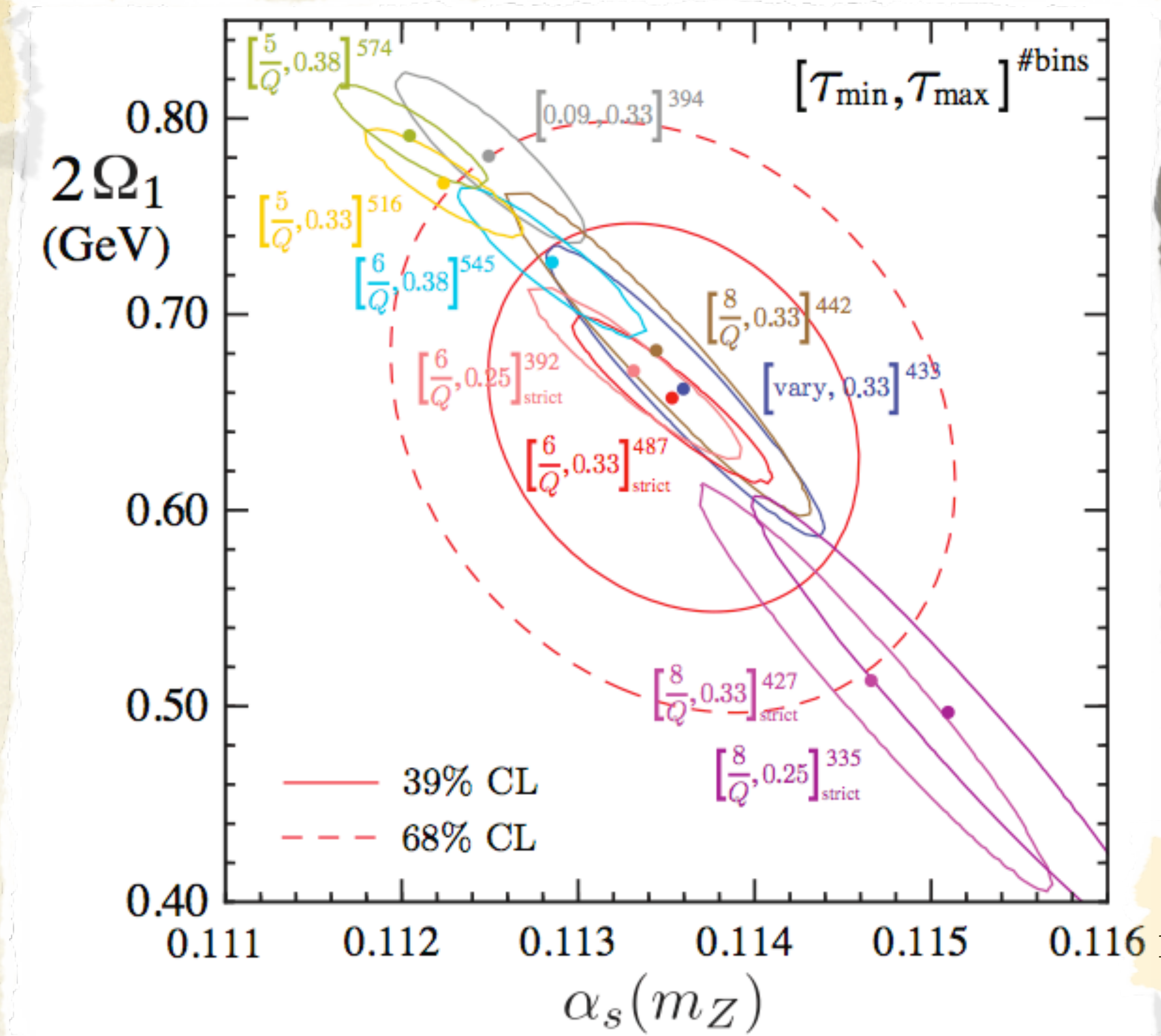
Statistical error from nonsingular extraction

$\mathcal{O}(\alpha_s^3)$ unknown. Estimated with Padè

$\mathcal{O}(\alpha_s^2)$ extracted numerically from EVENT2 with an accuracy of 6.4%

Different datasets

Ω_2 effect increases



Statistical errors increase