Full Simulation and Reconstruction of Multiple π^0 's using Mass-Constrained Fits

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Outline

- Previous Single π^0 Study
- Multiple π^0's Using Truth Information
  - Ideal Conditions
  - Z^0 Study
- Reconstruction without Truth Information
  - Procedure
  - Matching Algorithms
- Conclusion and Future Work

Previous Single π^0 Work

- Mass Constrained Fit

Given process $\pi^0 \rightarrow \gamma_1 + \gamma_2$

We apply mass of π^0 as constraint C. Then minimize S by adjusting x^f subject to C.

$$C = (p_{\gamma 1} + p_{\gamma 2})^2 - m_{\pi^0}^2 = 0 \qquad S = \sum \left(\frac{x_i^{(m)} - x_i^{(f)}}{\sigma_i} \right)^2$$

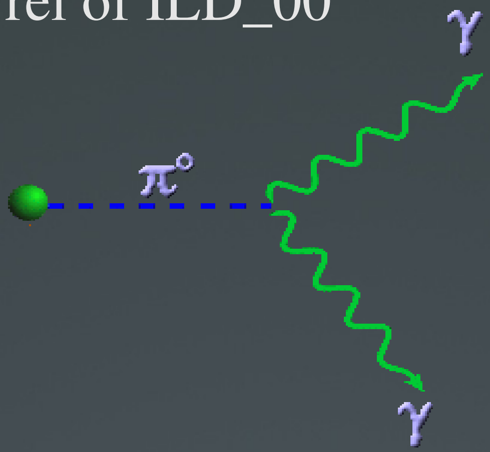
Our case using E, θ, ϕ

$$S = \left(\frac{E_1^{(m)} - E_1^{(f)}}{\sigma_{E1}} \right)^2 + \left(\frac{\theta_1^{(m)} - \theta_1^{(f)}}{\sigma_{\theta 1}} \right)^2 + \left(\frac{\phi_1^{(m)} - \phi_1^{(f)}}{\sigma_{\phi 1}} \right)^2 + \left(\frac{E_2^{(m)} - E_2^{(f)}}{\sigma_{E2}} \right)^2 + \left(\frac{\theta_2^{(m)} - \theta_2^{(f)}}{\sigma_{\theta 2}} \right)^2 + \left(\frac{\phi_2^{(m)} - \phi_2^{(f)}}{\sigma_{\phi 2}} \right)^2$$

$$C = (p_{\gamma 1}^{(0)} + p_{\gamma 2}^{(0)})^2 - (p_{\gamma 1}^{(1)} + p_{\gamma 2}^{(1)})^2 - (p_{\gamma 1}^{(2)} + p_{\gamma 2}^{(2)})^2 - (p_{\gamma 1}^{(3)} + p_{\gamma 2}^{(3)})^2 - m_{\pi^0}^2 = 0$$

Previous Single π^0 Work

- Generation: π^0 4-Vectors towards the barrel of ILD_00
  $45° < \theta < 135°$



- Simulation: MOKKA – Geant4
  ilcsoft v01-09
- Reconstruction: Marlin framework
  - 1) Pandora Particle Flow Analysis
    - Reconstruction of 4-vectors of all visible particles
    - Identification of particle (photon, electron, neutron, etc...)
  - 2) pi^0 mass constrained fitting using MarlinKinFit
    - Implemented as a Pandora algorithm

First we must characterize how this detector and reconstruction software responds to photons

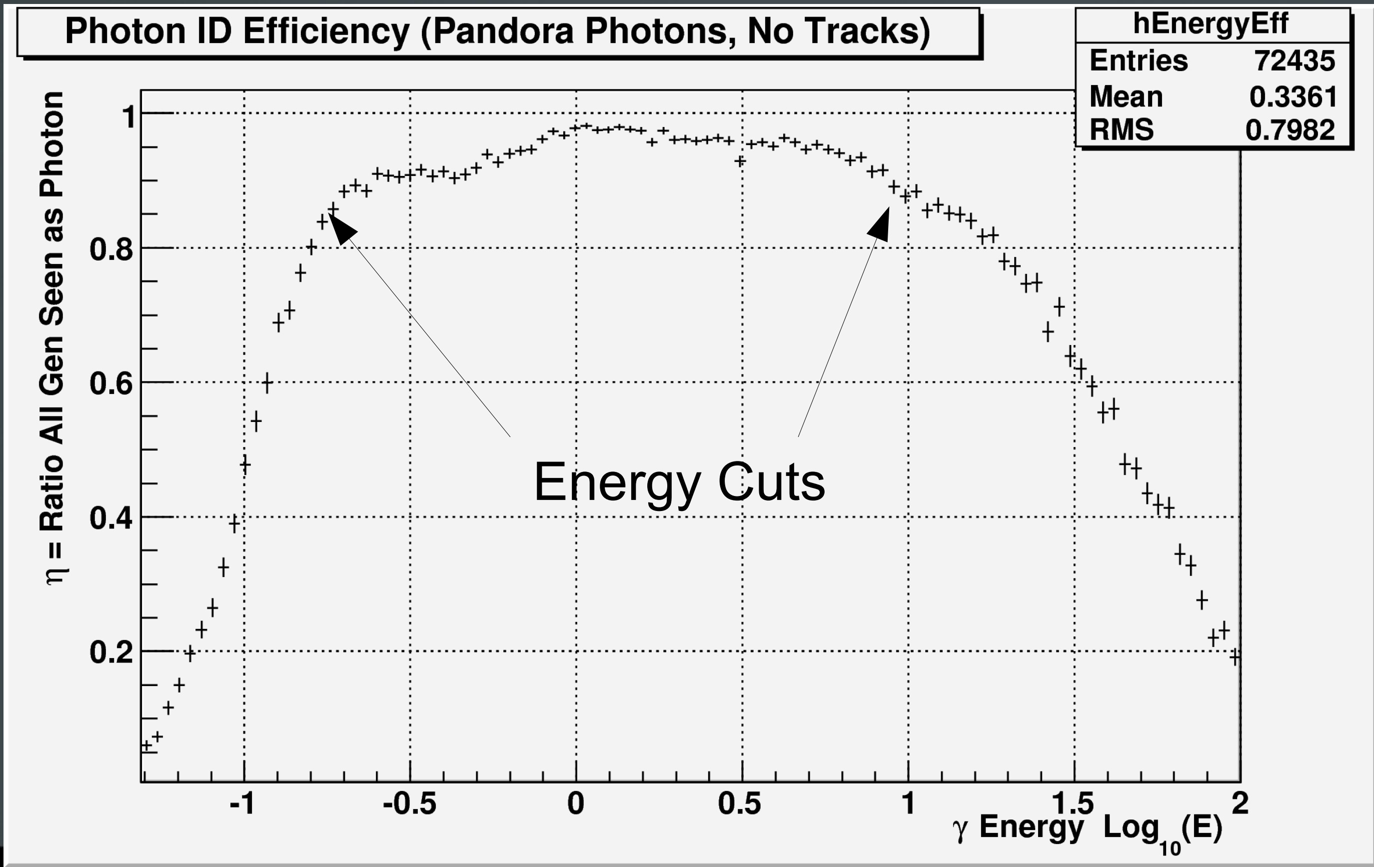
Previous Single π^0 Work

- Overall efficiency of correctly detecting photons

$$\eta = \frac{\text{single PFO identified as photon}}{\text{all photon events with no tracks}}$$

~90% Efficiency between -0.75 < Log(E) < 1.0

Use 180 MeV as low energy cut



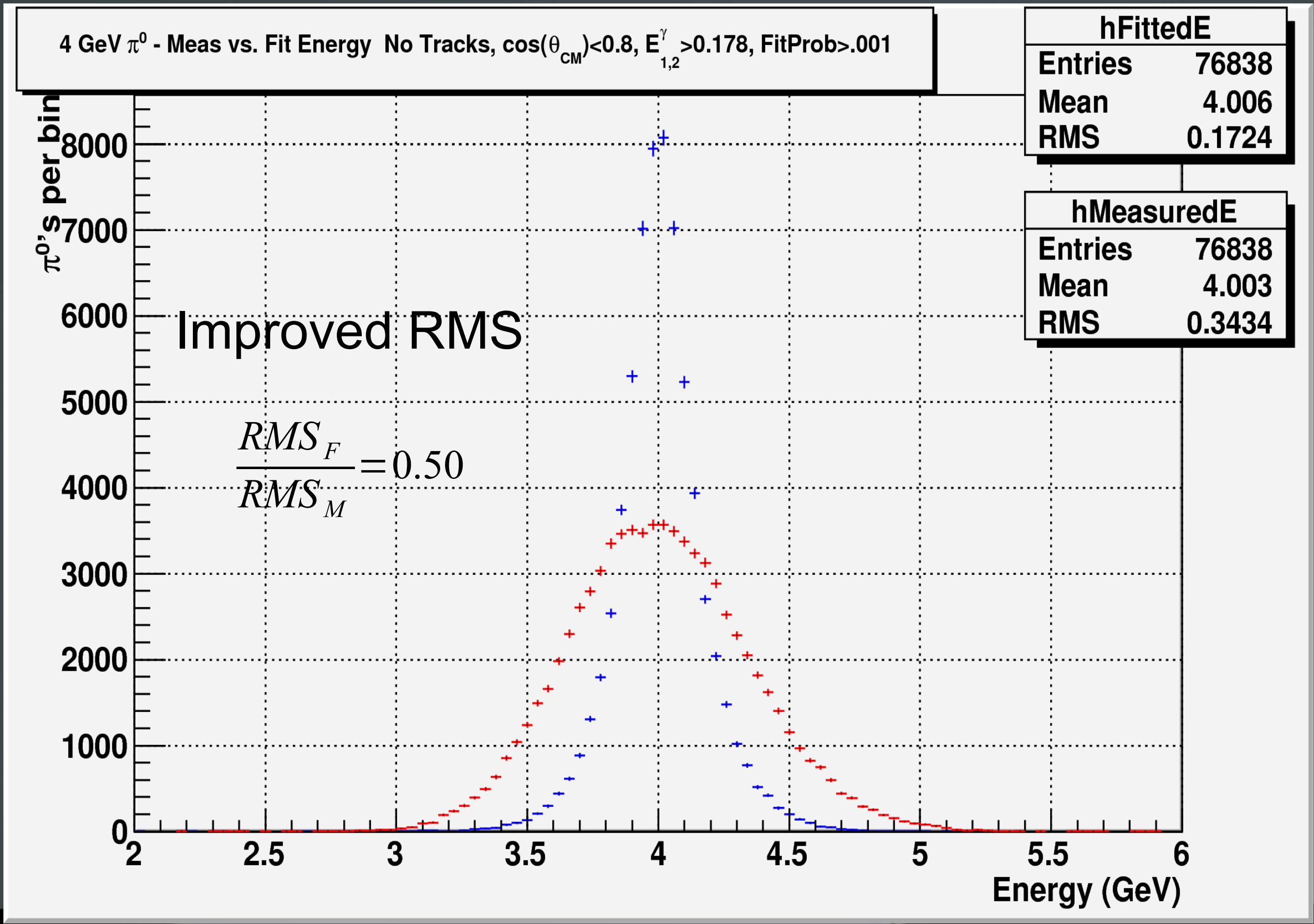
4.0 GeV π^0 Mass Constrained Fits

- After all cuts, results are comparable with Toy Monte Carlo
- Efficiency of $\cos(\theta_{CM})$ cut: 84%

Relative to $\cos(\theta_{CM})$ cut

No Tracks	92%
Fit Prob >.001	98%
Low E Cut	99%
Combined	91%

Overall efficiency is 77%



Fitting Multiple π^0's

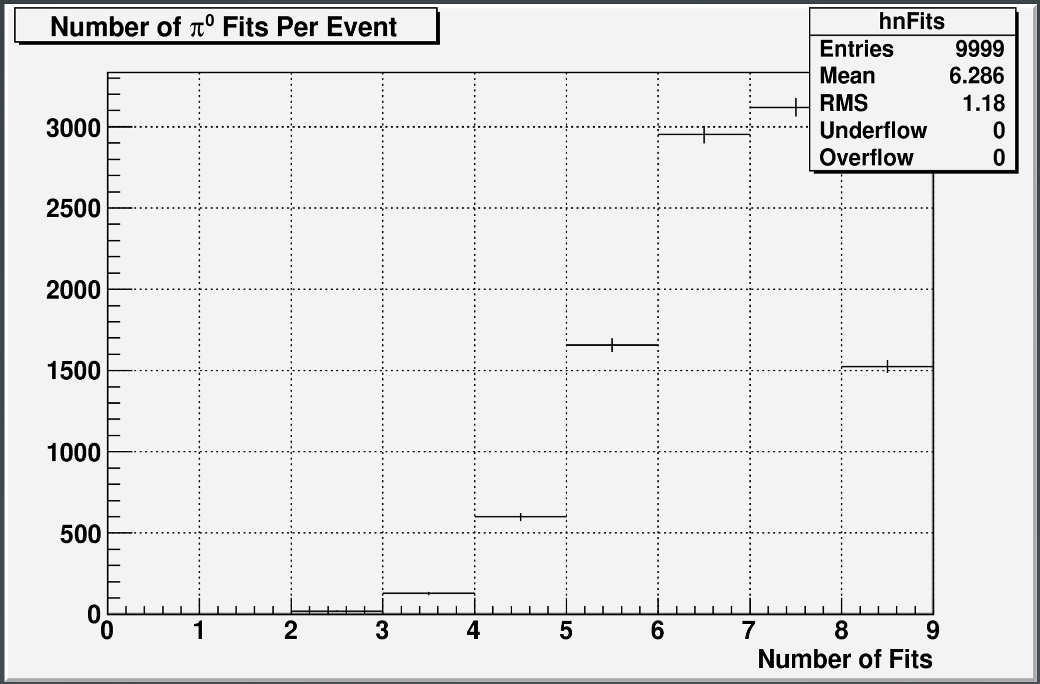
- When faced with reconstructing multiple π^0's we want to know: How well **can** we do?
- Consider an idealized event consisting of 8 π^0's, each 4 GeV directed towards the barrel.
- Cheat with pairing by using truth information to match photons with their parent π^0

Fitting Multiple π^0's

Using truth information, matching is about 80% efficient for 8 pi0's at 4 GeV

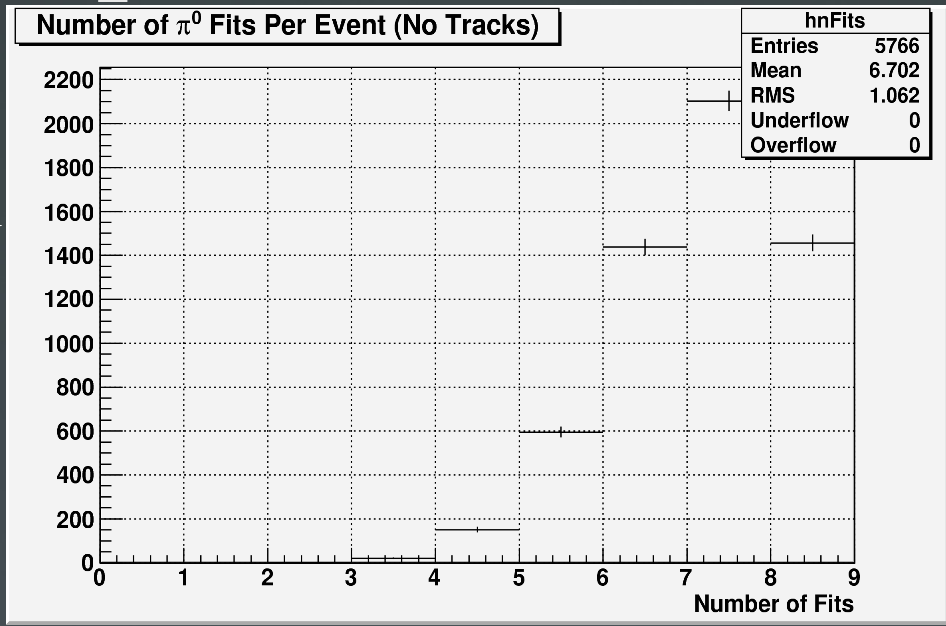
Why is it not 100%?

- e^+e^- pair production
- low energy photon cut (180 MeV)
- Base 1% fit probability cut

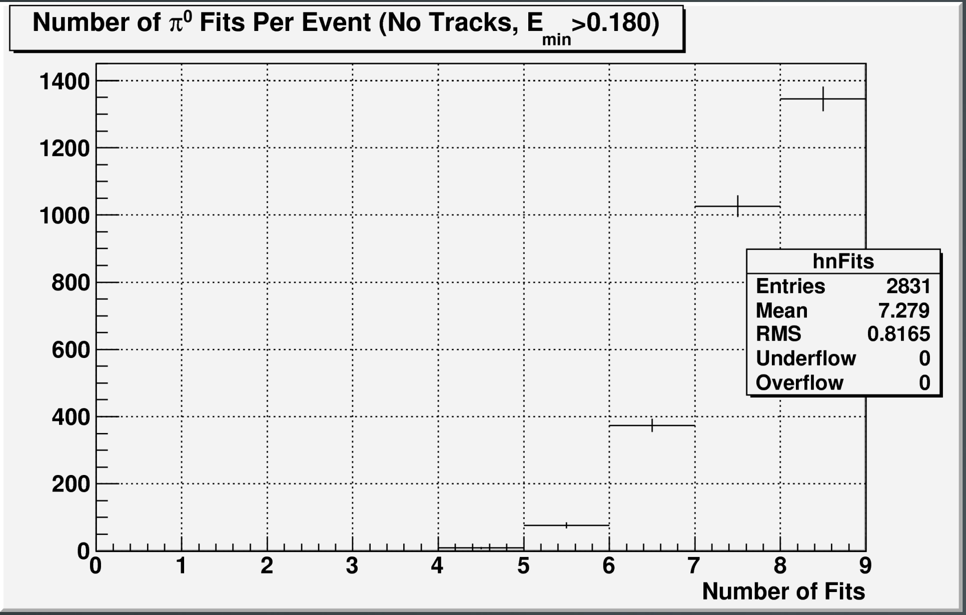


Fitting Multiple π^0's

Removing events with tracks increases efficiency to ~84%



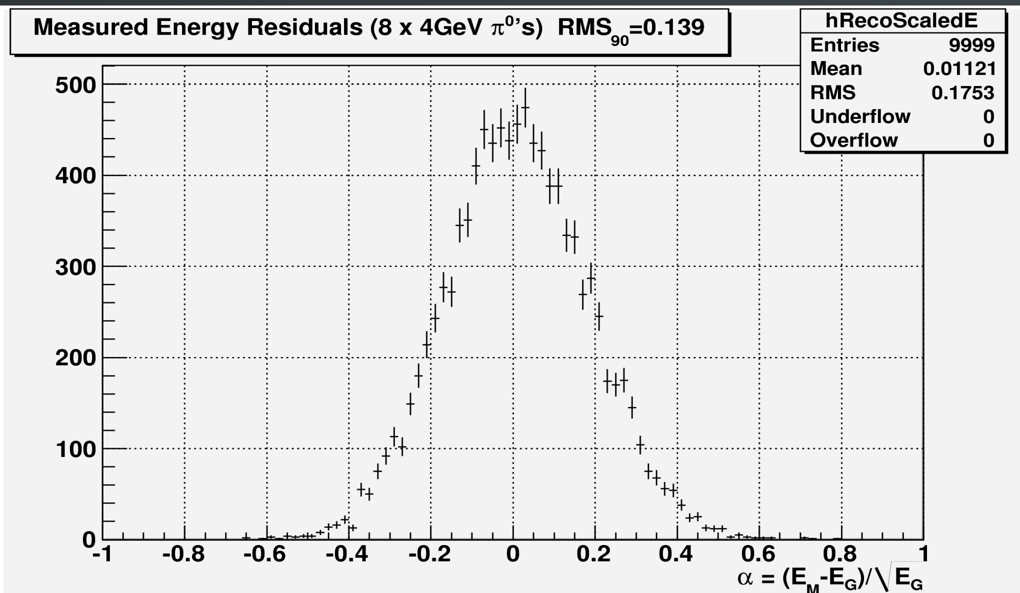
Additionally, removing events with photons below 180 MeV results in ~91% efficiency

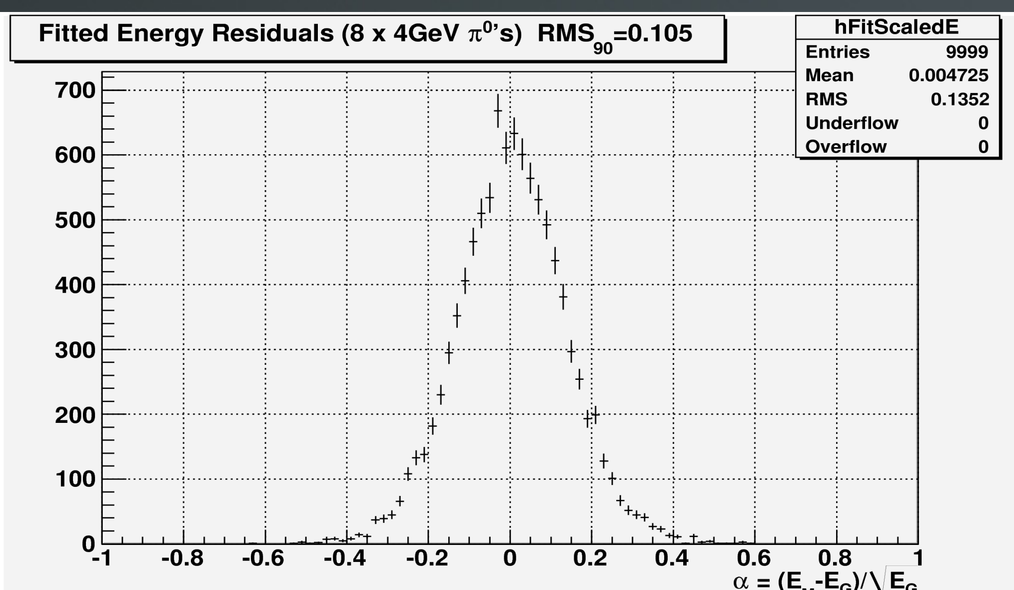


$$\binom{n}{k} p^k (1-p)^{n-k}$$

Consistent with binomial distribution where $p = .99^8$ suggesting 1% cut responsible for remainder

Fitting Multiple π^0's





At 80% matching efficiency the energy uncertainty improves

$\alpha=17.5\%$

to

$\alpha=13.5\%$

$$\frac{\alpha}{\sqrt{E}}=\frac{\Delta E}{E}$$

Fitting Multiple π^0's

- How do these efficiencies vary with multiplicity and energy?

4 GeV π^0's

# of π^0's per event	2	4	8	16	32
% π^0's Fit	79	79	80	78	77.7
Unfitted α	.179	.176	.175	.180	.175
Fitted α	.137	.137	.135	.137	.139

8 π^0's per event

Angular resolution limits high energy fits

Energy (GeV)	4	8	16	32
% π^0's Fit	80	78.3	63.6	46.3
Unfitted α	.175	.179	.189	20.8
Fitted α	.135	.162	.197	20.8

$$\frac{\alpha}{\sqrt{E}} = \frac{\Delta E}{E}$$

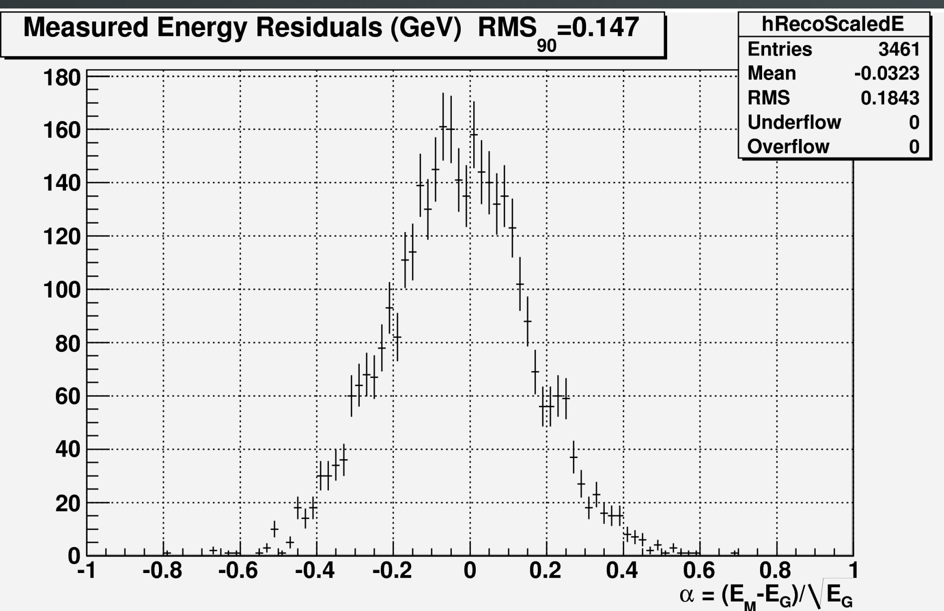
Fitting Multiple π^0's

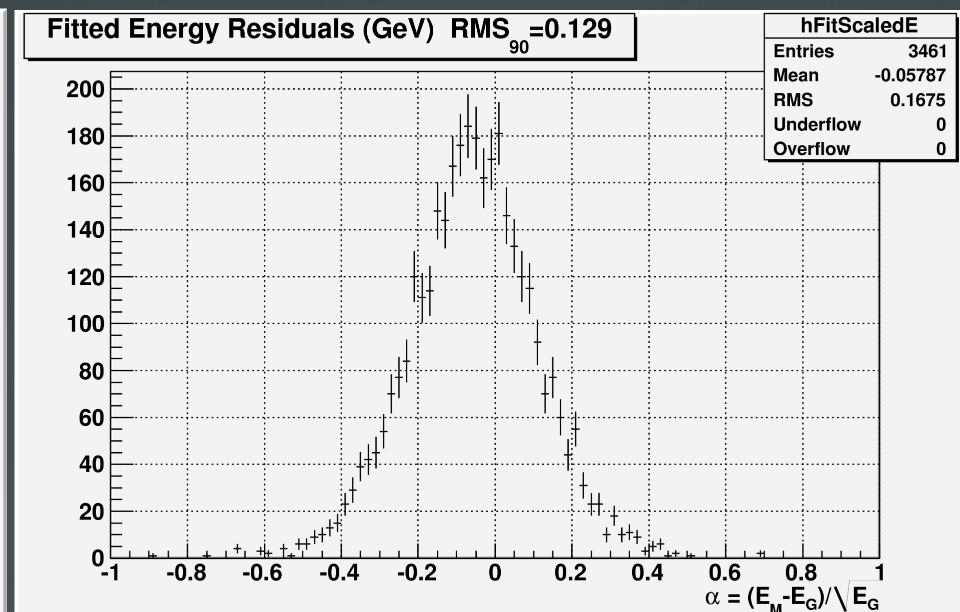
- What happens if we apply this to a more realistic situation?
- Consider 91.2 GeV Z^0 -> q q-bar, q = uds
- Extract the π^0 information and apply fitting procedure using truth information
- Require 95% of energy deposited in barrel

Fitting Multiple π^0's

Results of procedure on 91.2 GeV Z^0 -> q q-bar

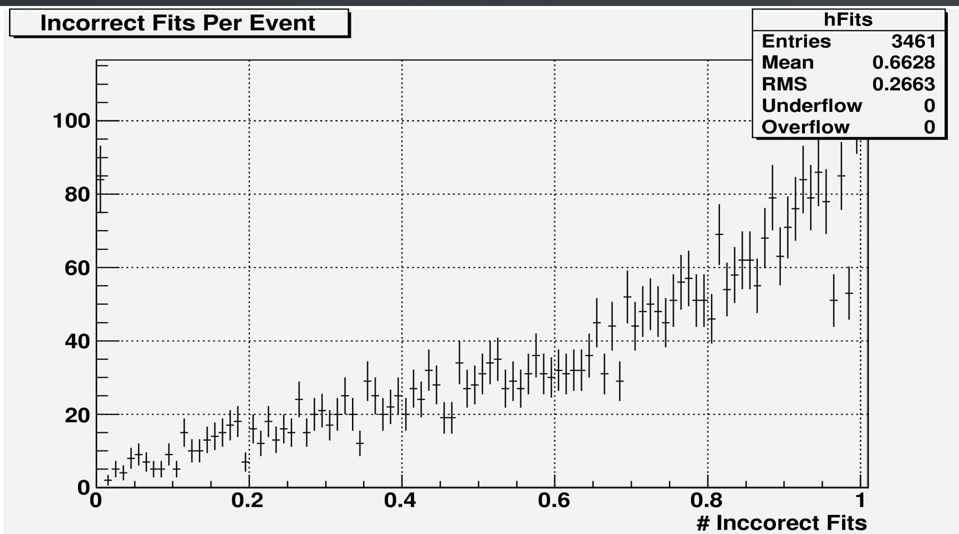
(π^0 contribution only, 95% energy in barrel)

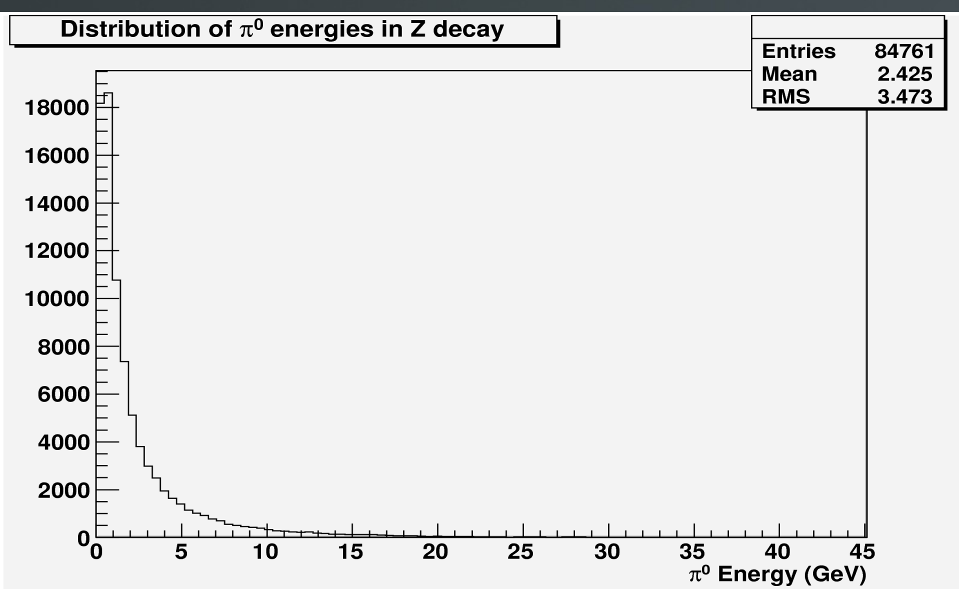




Improvement in α: .184 -> .168

Fitting Multiple π^0's





Fraction of overall energy that is fitted is significantly less than idealized events (66% vs 80%)

This may be due to high concentration of low energy π^0's generating many photons with $E < 180$ MeV

Fitting Multiple π^0's

- Exploration of matching procedures that do **not** use truth information

- The challenge: Enumerate over all potential event solutions and determine the "best"

- Some basic restrictions:

    Minimum photon energy 180 MeV

    95% of energy deposited in barrel

    Accept potential fits with greater than 1% fit probability

Fitting Multiple π^0's

- Photon Matching Procedure

1 Perform kinematic fits on all photon pairs

2 Remove fits where fit probability is less than 1%

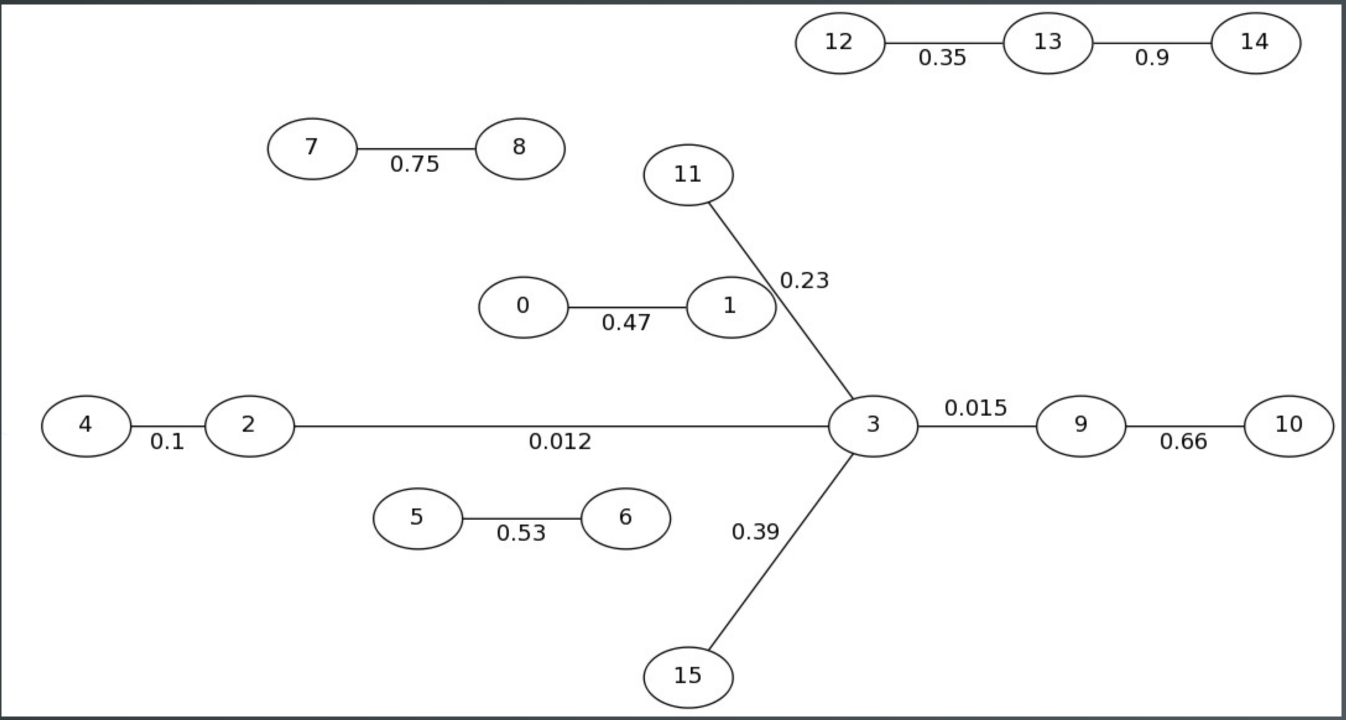
3 Generate all potential solutions by combining remaining pairs such that each photon is used at most once

4 "Score" each solution and pick the best

Fitting Multiple π^0's

- Photon Matching Procedure

The collection of all >1% pairs can be modeled as a graph with vertexes and edges

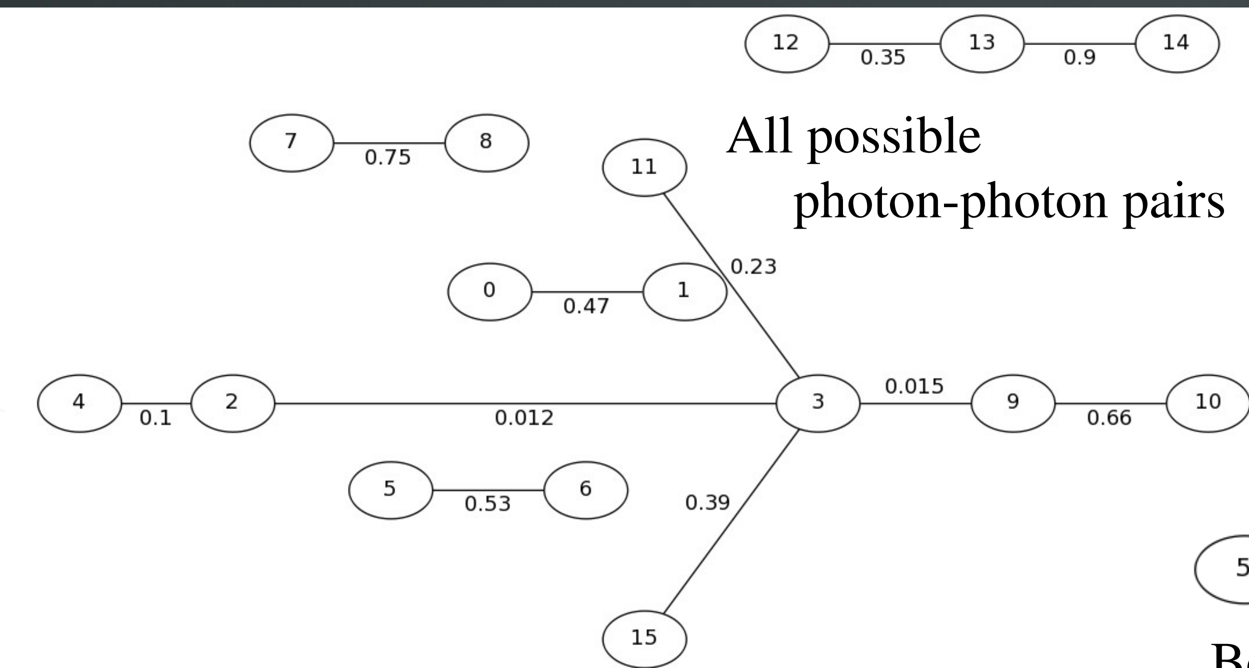


Vertexes are photons

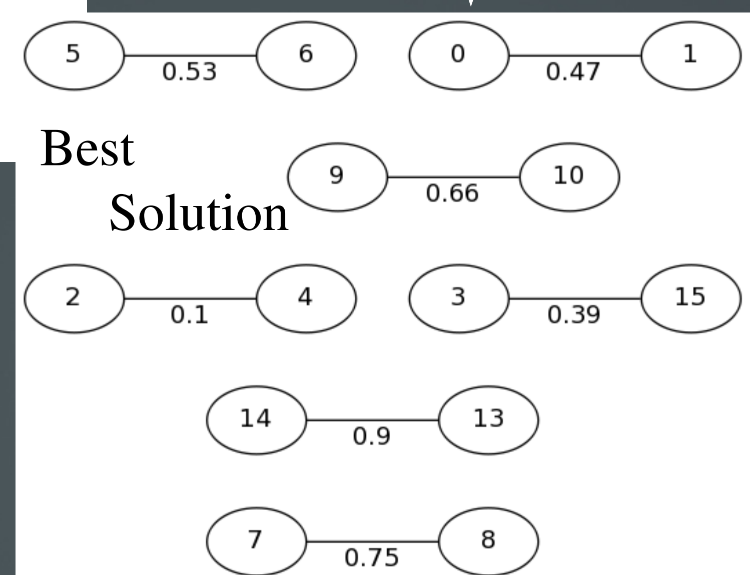
Edges represent fit probability between the photons

Fitting Multiple π^0's

- Photon Matching Procedure



Evaluate all solutions



- Final solution uses each photon at most once

Fitting Multiple π^0's

- Several ways to approach scoring of the solutions:

  Evaluated functions involving: fit probability, number of fits, overall χ^2.

  Best scoring method so far is to consider solutions with maximal fits and the lowest total χ^2

  Example:

  Solution a: 6 Fits, $\chi^2/6 = 5/6$

  Solution b: 7 Fits, $\chi^2/7= 8.2/7$

  Solution c: 7 Fits, $\chi^2/7 = 14/7$

  Best solution is "b"

Fitting Multiple π^0's

- How does this method compare to using truth information?

4 GeV π^0's

# of π^0's	2	4	8	16
% π^0's Fit	79	79.5	79.3	74.7
% Correct	79	79	78.0	72.9
Cheating	79	79	80	78

8 π^0's per event

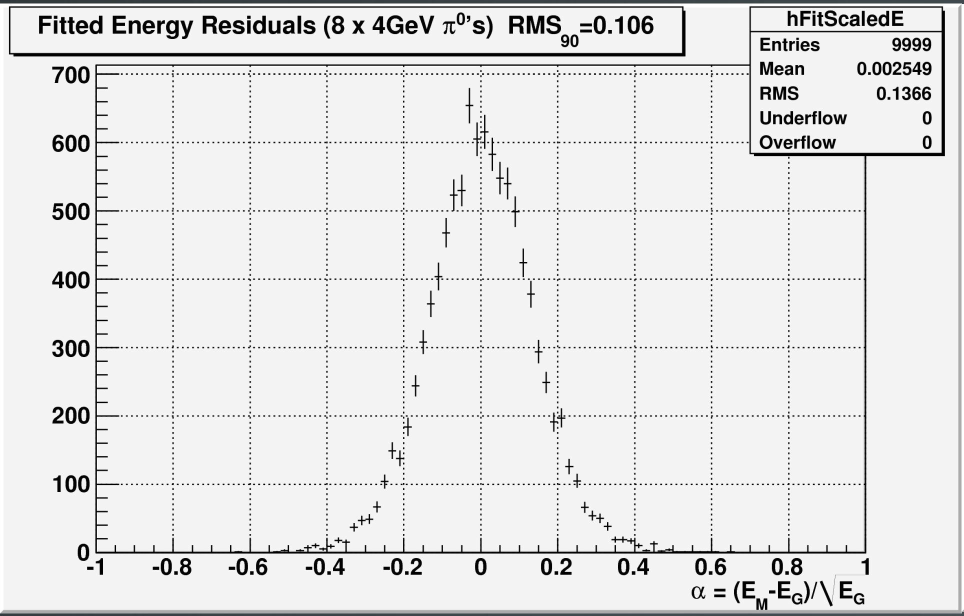
Energy (GeV)	4	8	16
% π^0's Fit	79.3	79.0	66.3
% Correct	78	77.9	63.6
Cheating	.80	78.3	63.6

Pretty good!

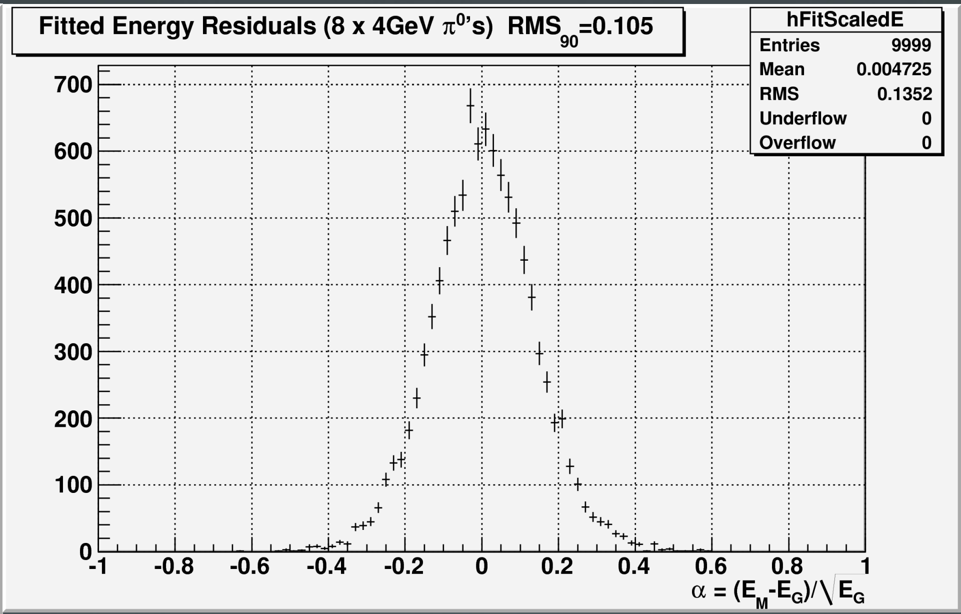
Fitting Multiple π^0's

- Comparison to truth information (8 x 4 GeV π^0's)

Max Fits, Min χ^2



Truth Information



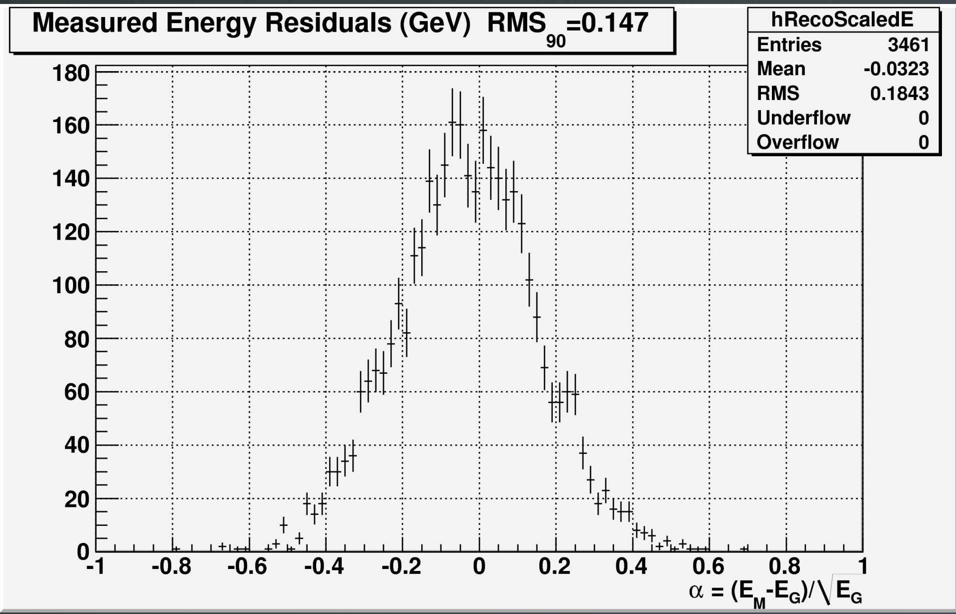
Performance is nearly identical
(for this situation)

$\alpha = .137$ vs. $\alpha = .135$

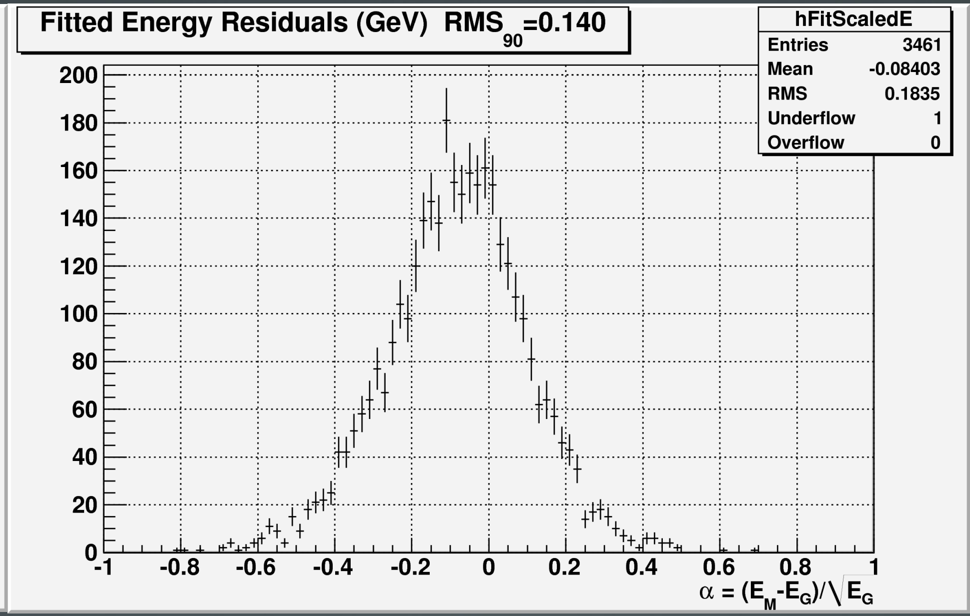
Fitting Multiple π^0's

- Comparison to unfitted (91.2 GeV Z^0)

Reconstructed



Max Fits, Min χ^2

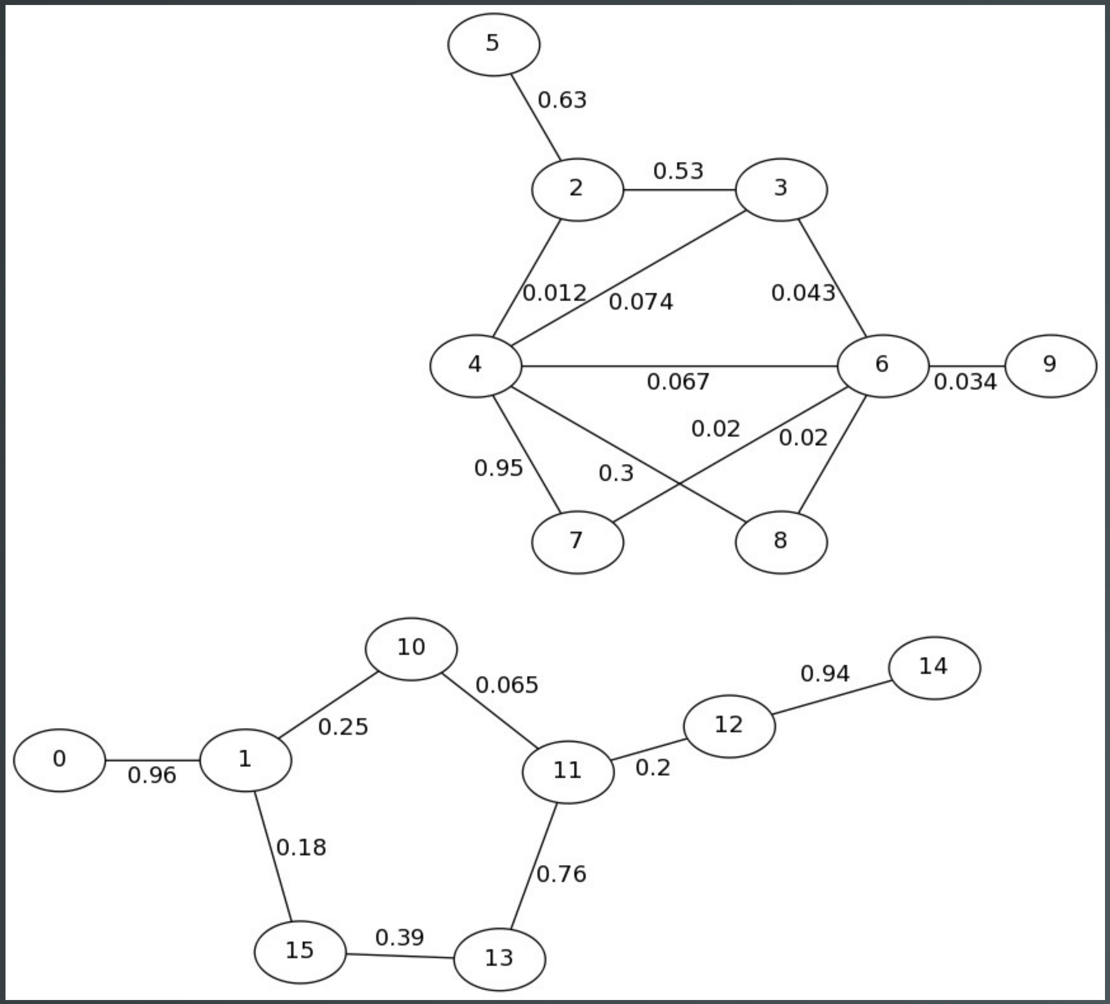


$\alpha = .184$ -> $\alpha = .183$

No real improvement... what's going on?

Fitting Multiple π^0's

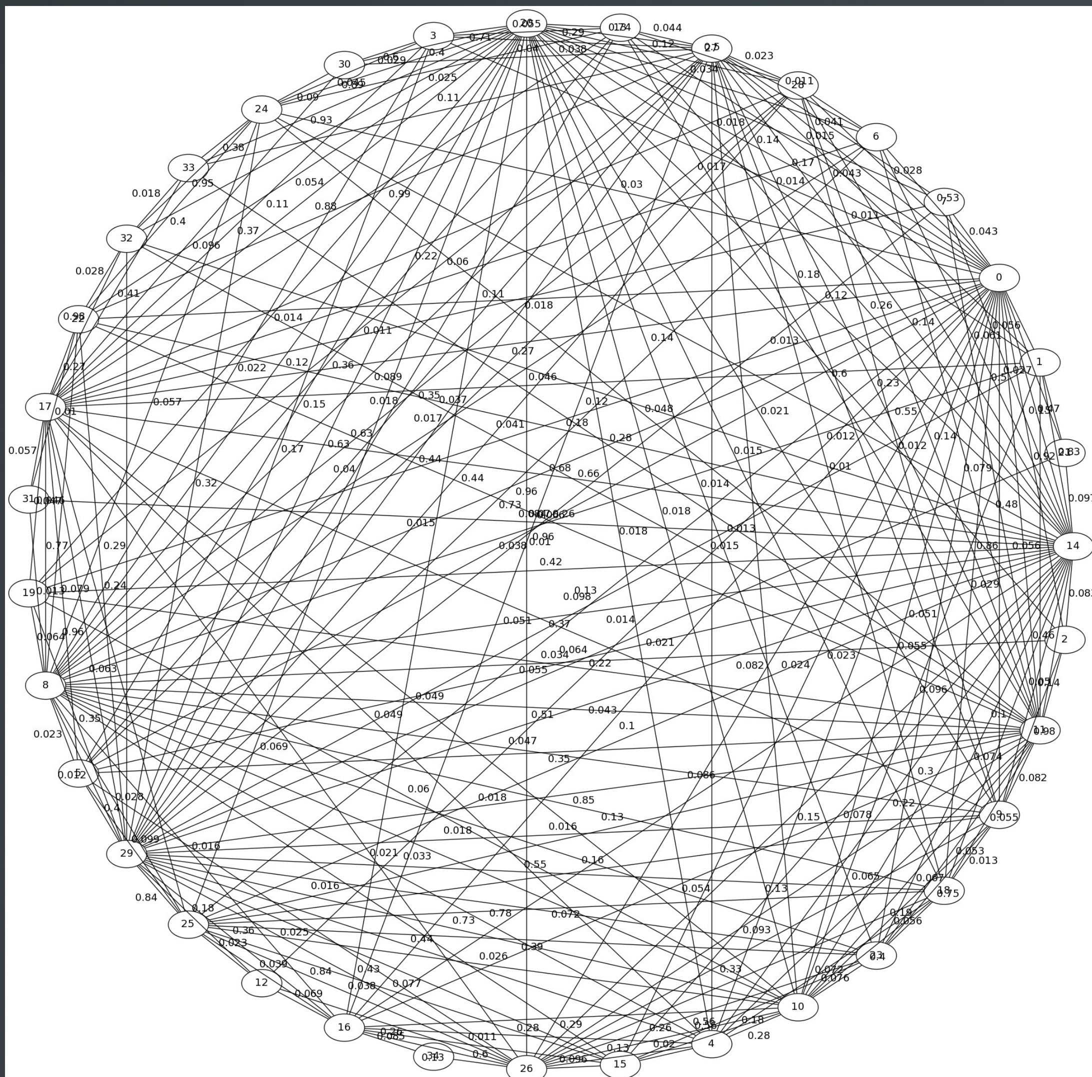
- What does the problem look like for 91.2 GeV Z^0?



It can be easy...

Fitting Multiple π^0's

- What does the problem look like for 91.2 GeV Z^0?



It can be... challenging

Current algorithm has trouble with high multiplicity

Evaluating a solution from Computer Science (blossom V)

Summary

- In a controlled environment, significant improvement in energy resolution
  - from17.5% down to 13.5%
- Application to Z^0 decay only small improvement
  - Overall resolution of Z^0 is worse than more controlled π^0's suggesting additional source of uncertainty is competing with fit improvements (low energy photons?)
- Matching algorithms that do not cheat perform similarly to using truth information in a wide range of situations
- Further Study:

  Identify source of extra uncertainty in Z^0

  Evaluate alternative matching algorithms

Back up slides

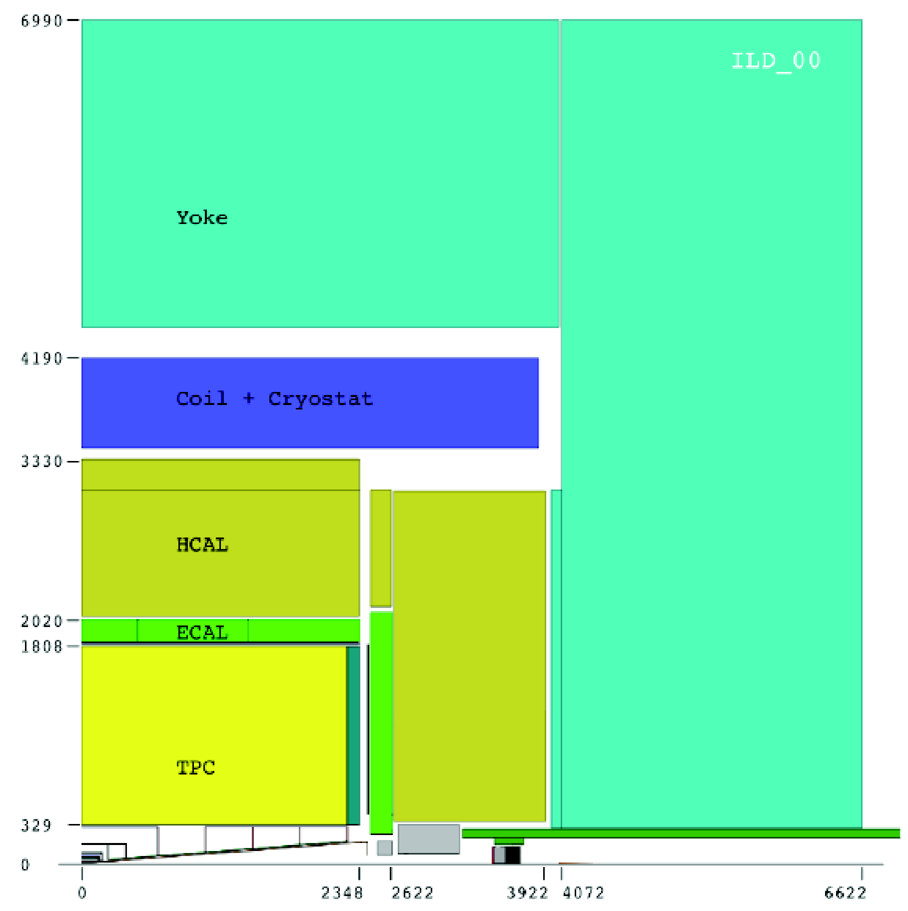
International Large Detector (ILD)

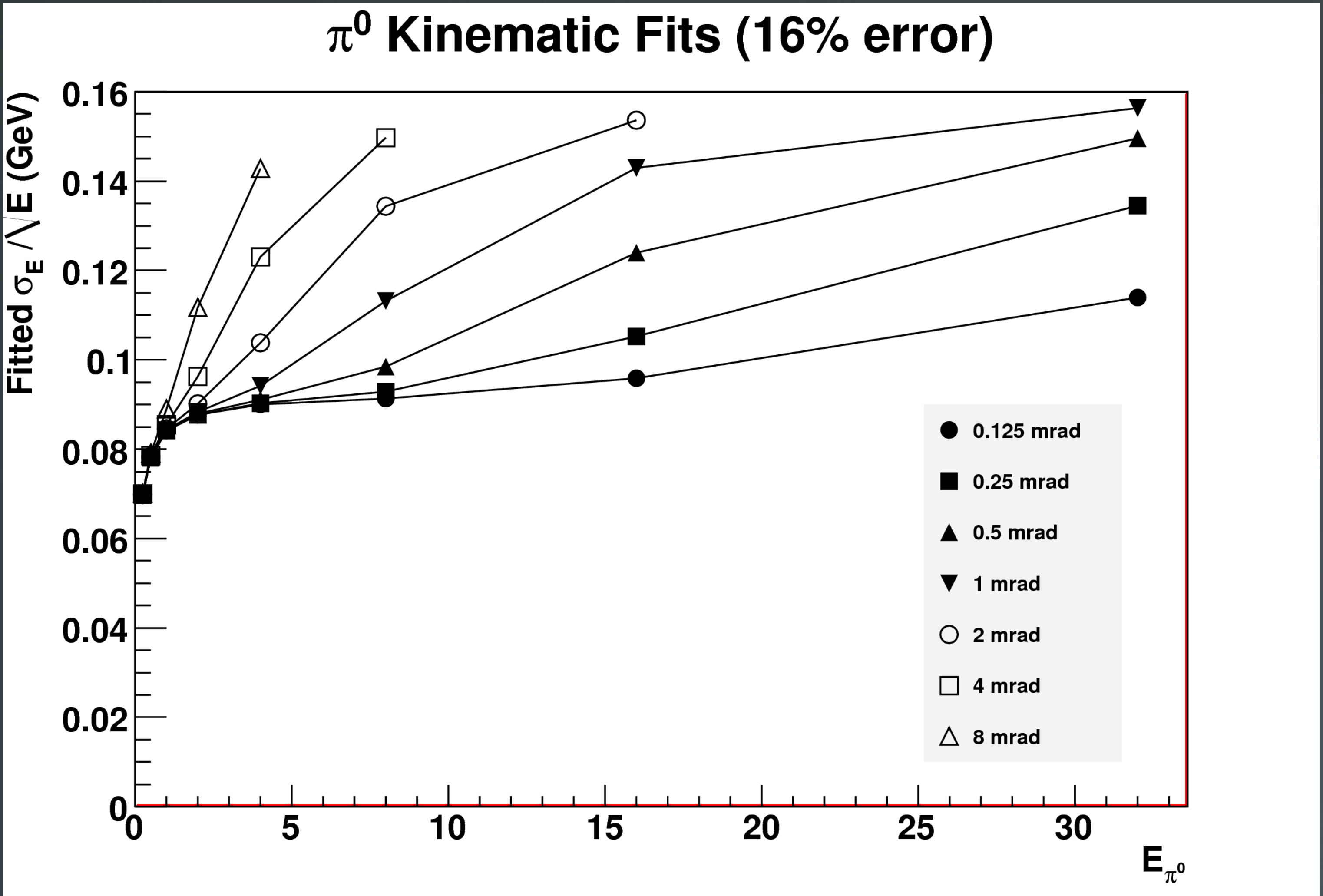
- Detector concept being studied for the International Linear Collider (electron-positron).

ECAL

- 20+9 Layers Si-W
- Active layer segmented into 5mm x 5mm "*highly granular*"
- Typical photon uncertainties

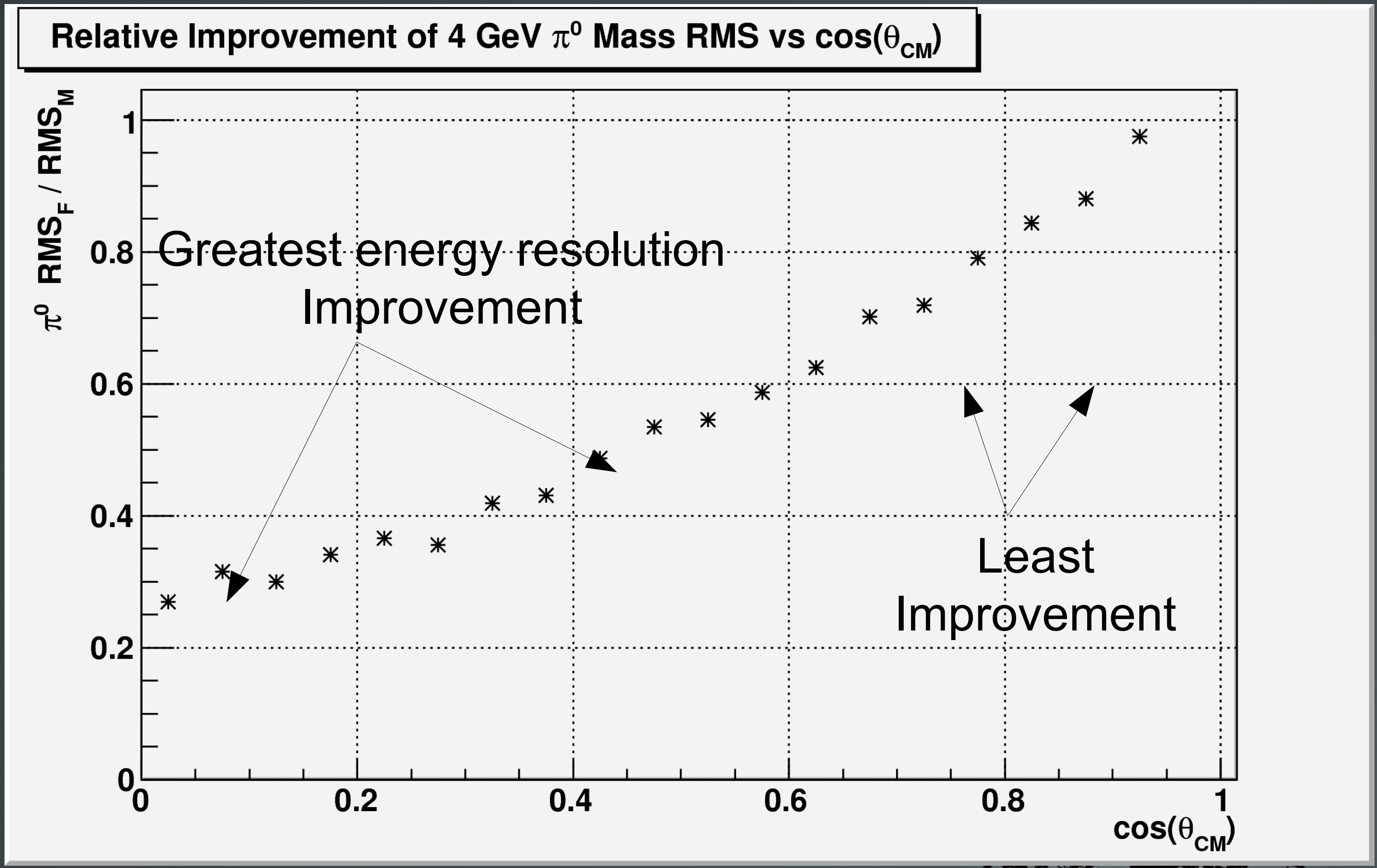
$$\sigma_E = 16\% \sqrt{E}$$
$$\sigma_\phi = 1.2\, mrad @ 1\, GeV$$
$$\sigma_\theta = \text{similar, but } \theta \text{ dependence}$$




π0 Kinematic Fits (16% error)
Fitted σE /√E (GeV)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
Eπ0
0.125 mrad
0.25 mrad
0.5 mrad
1 mrad
2 mrad
4 mrad
8 mrad

4.0 GeV π^0 Mass Constrained Fits

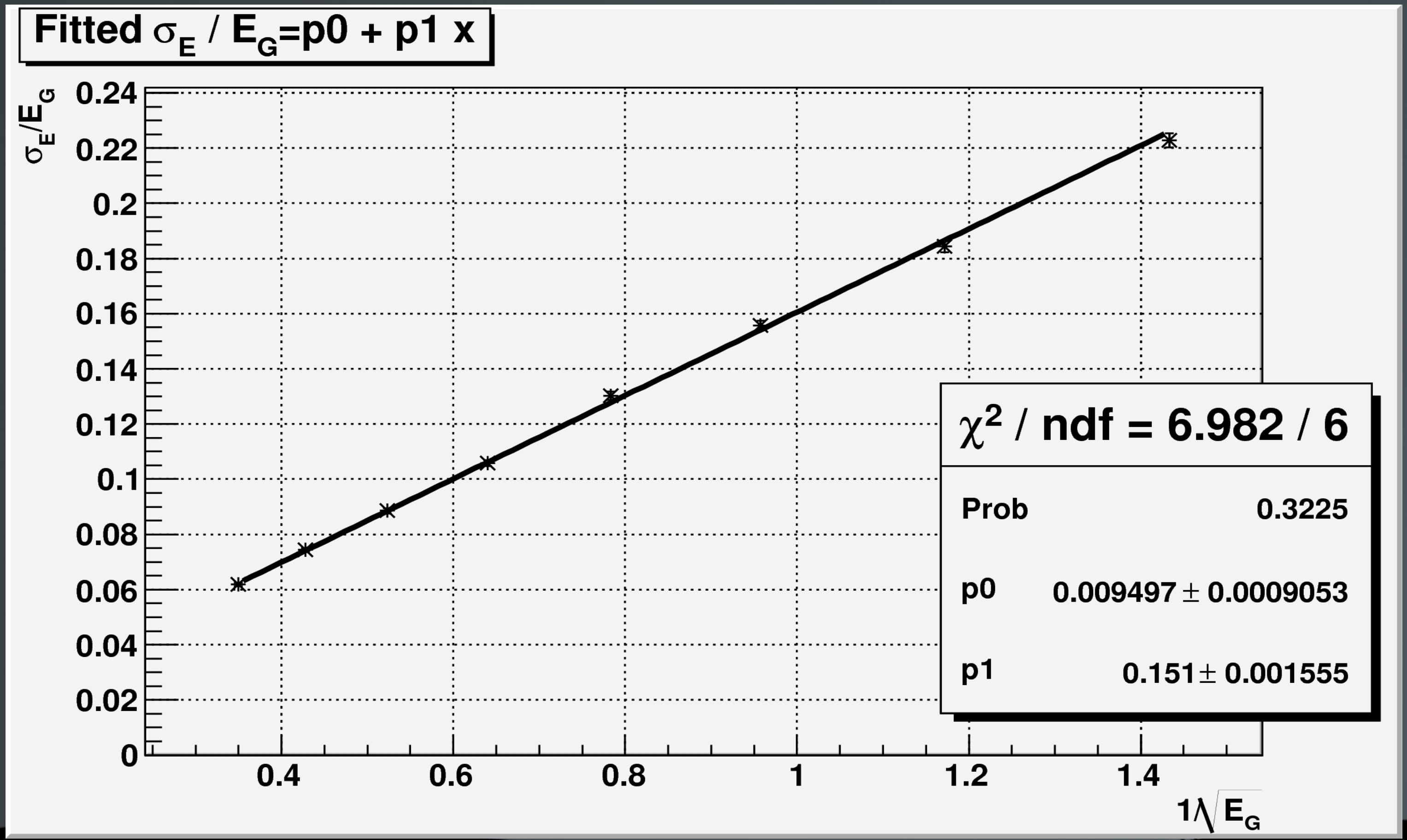
- Greatest improvement with symmetric decays.



Software: Simulation and Reconstruction

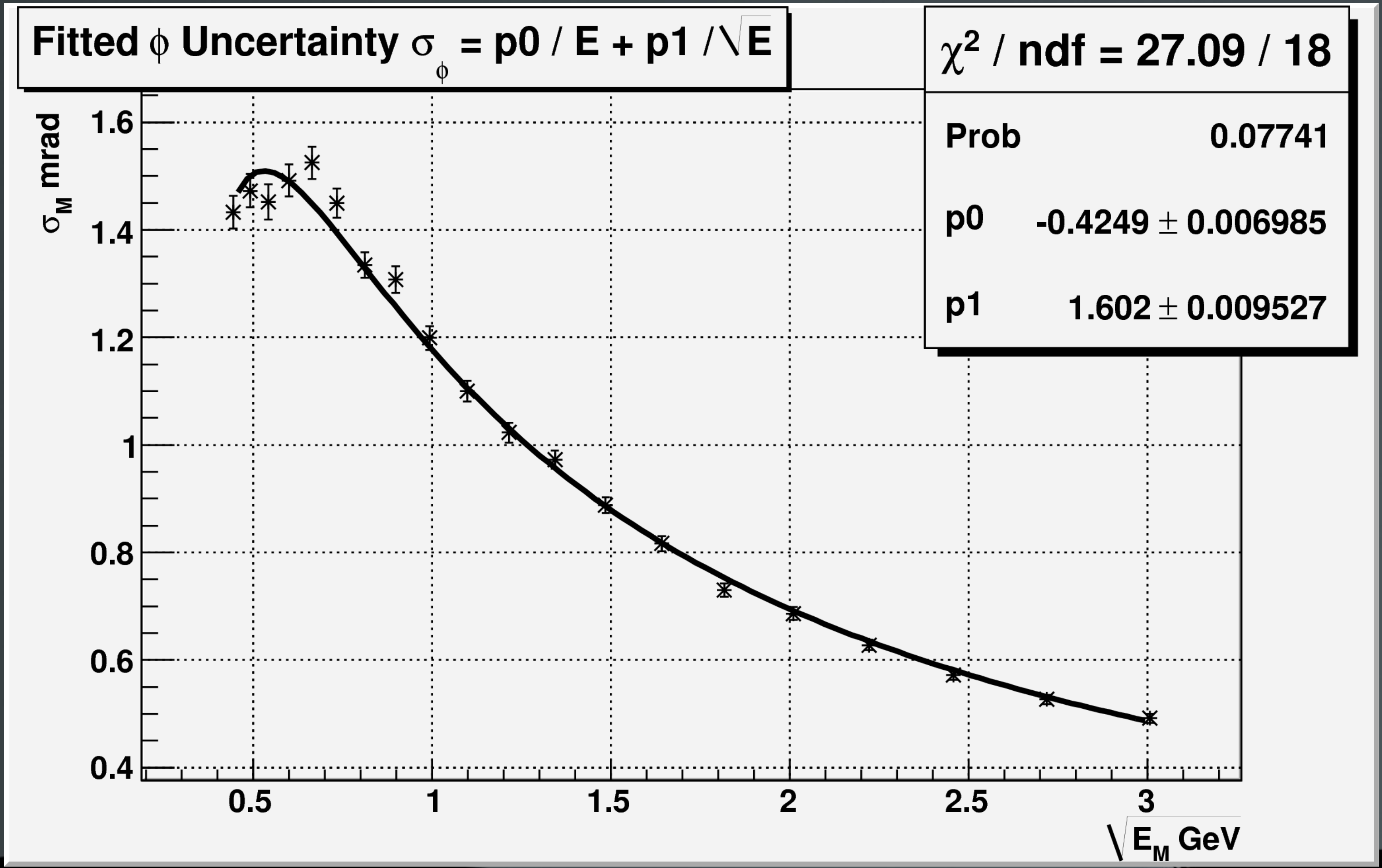
- Uncertainty Modeling: Accuracy important for kinematic fits.

- Energy Uncertainty as function of Energy $\frac{\sigma_E}{E} = \frac{.151}{\sqrt{E}} + 0.0095$



Software: Simulation and Reconstruction

- Uncertainty Modeling: Phi
  - "Turns over" or "flattens out" at low energies



Software: Simulation and Reconstruction

- Uncertainty Modeling: Theta

Want smooth function

Hypothesis: $\sigma_\theta \rightarrow \sigma_\phi$ as $\theta \rightarrow \pi/2$

$\sigma_\theta \rightarrow 0$ as $\theta \rightarrow 0$

Try: $\sigma_\theta^2 = 0.91^2[(\sigma_\phi^* \sin(\theta))^2 + 0.4^2]$

$$\sigma_\phi^* = \sqrt{\sigma_\phi^2 - 0.4^2}$$