Top Quark Form Factors

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Much of the discussion of Beyond the Standard Model physics at the ILC has centered on weakly coupled particles with a new spectroscopy below 1 TeV.

For such scenarios, most especially supersymmetry, the naturalness of the Higgs mass scale pushes the spectrum toward lower energies, with some particles hopefully below 250 GeV.

Now the first results from the LHC are strongly constraining low-mass supersymmetry. So it is interesting to give attention also to other probes of physics beyond the Standard Model that will certainly be accessible at 500 GeV.

The most important of these are the searches for new 4-fermion interactions in $e^+e^- \to f\overline{f}$.

The reaction $e^+e^- \to t\bar{t}$ has a special role.

Recall, e.g., Z' studies at the ILC: What if a Z' is discovered?

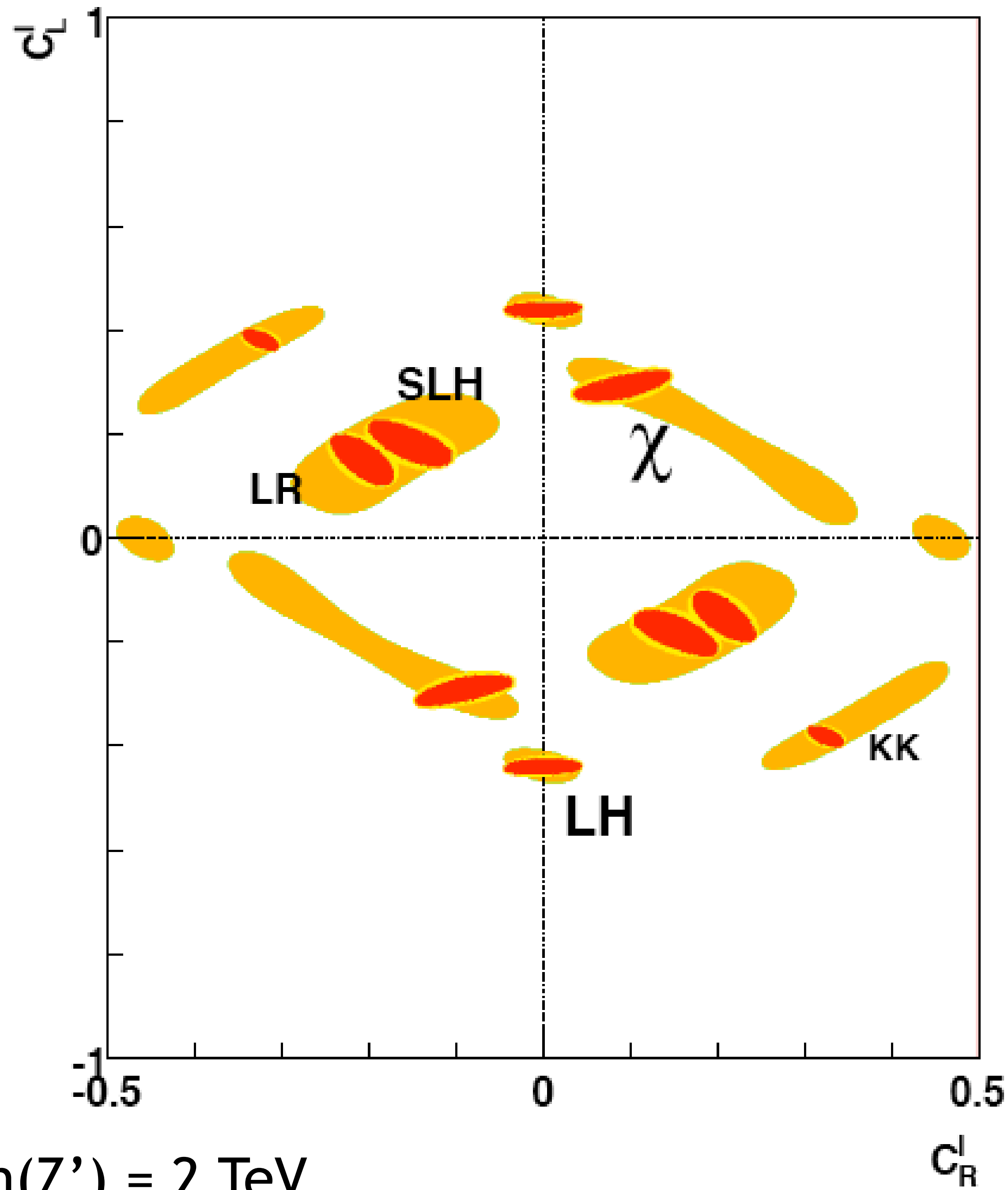
Ideally, we would like to go to the Z' resonance. However, if there is no technology available for this, we can learn a great deal through polarized $e^+e^- \to f\overline{f}$ at the highest available energy.

For example, for $e^-_L e^+_R \to f_L \overline{f}_R$, the Z' adds an amplitude

$$\frac{g_{eL} \cdot g_{fL}}{s - m_Z^2 + i m_Z \Gamma_Z}(1 + \cos\theta)$$

which interferes with the Standard Model pair-production amplitude. Using the mass from the LHC, we can use the polarized forward and backward cross sections to obtain all of the Z' couplings. Many reactions are available:

$$\begin{aligned} e^+e^- &\to e^+e^-, \mu^+\mu^-, \tau^+\tau^- \\ e^+e^- &\to q\overline{q}, c\overline{c}, b\overline{b} \end{aligned}$$



500 GeV, m(Z') = 2 TeV
1 ab-1, $e^+e^- \rightarrow \mu^+\mu^-$

Godfrey, Kalyniak, Tomkins

The program extends to the study of $e^+e^- \to t\bar{t}$.

I should remind you that this reaction has a great deal of structure even within the Standard Model.

(a comprehensive review: C. Schmidt, PRD 1995)

For $e^-_L e^+_R$,

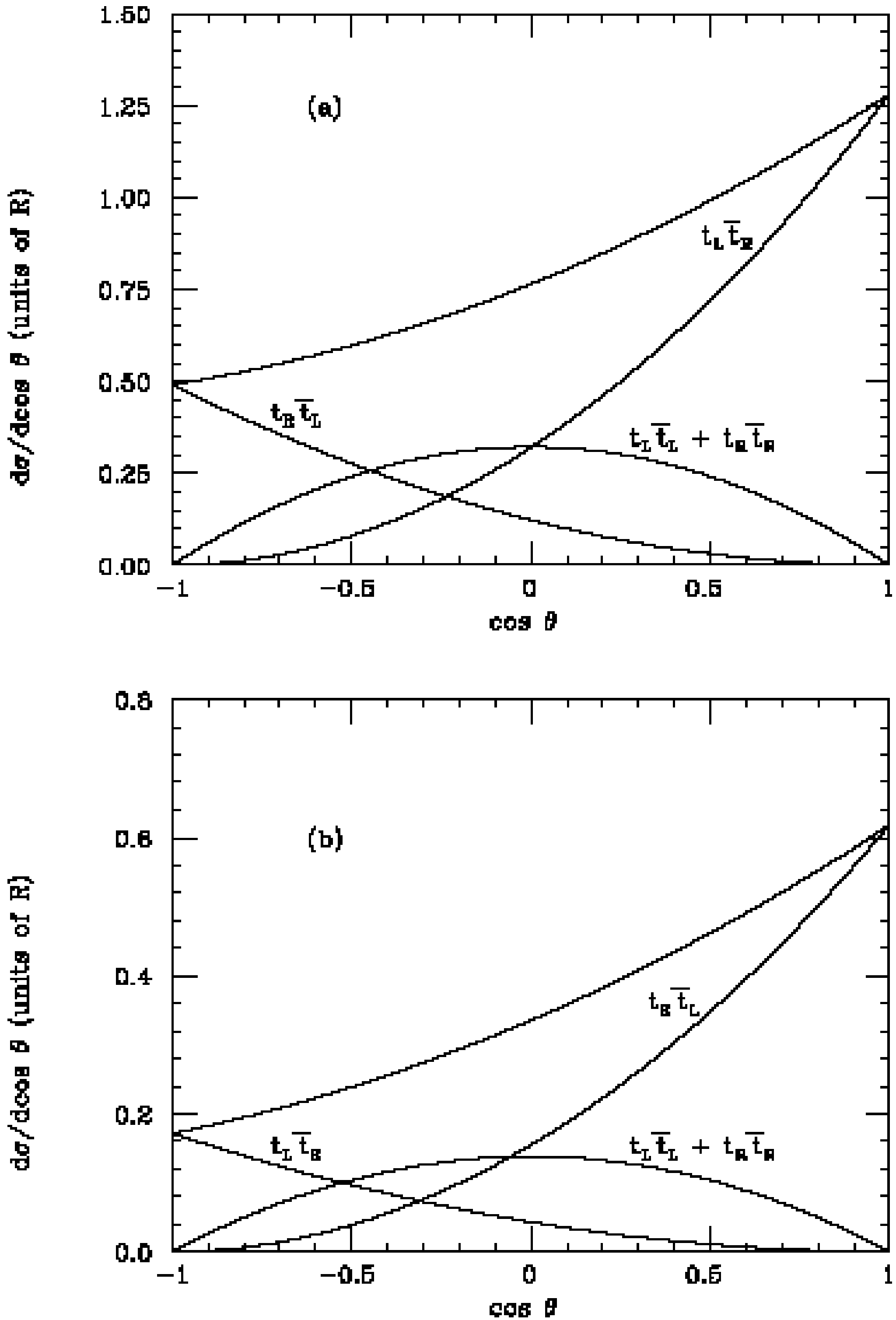
$$\begin{aligned}
i\mathcal{M}(t_L\bar{t}_L) &= -ie^2(\sin\theta)[\frac{m}{\sqrt{s}}(C_{LL}+C_{LR})] \\
i\mathcal{M}(t_L\bar{t}_R) &= +ie^2(1+\cos\theta)[(\frac{1+\beta}{2})C_{LL}+(\frac{1-\beta}{2})C_{LR}] \\
i\mathcal{M}(t_R\bar{t}_L) &= -ie^2(1-\cos\theta)[(\frac{1-\beta}{2})C_{LL}+(\frac{1+\beta}{2})C_{LR}] \\
i\mathcal{M}(t_R\bar{t}_R) &= +ie^2(\sin\theta)[\frac{m}{\sqrt{s}}(C_{LL}+C_{LR})]
\end{aligned}$$

The coefficients in the equations above are:

$$
\begin{aligned}
C_{LL} &= \frac{2}{3} + \frac{\frac{1}{2} - s_w^2}{s_w c_w} \frac{\frac{1}{2} - \frac{2}{3} s_w^2}{s_w c_w} \frac{s}{s - m_Z^2} \\
C_{LR} &= \frac{2}{3} - \frac{\frac{1}{2} - s_w^2}{s_w c_w} \frac{\frac{2}{3} s_w^2}{s_w c_w} \frac{s}{s - m_Z^2}
\end{aligned}
$$

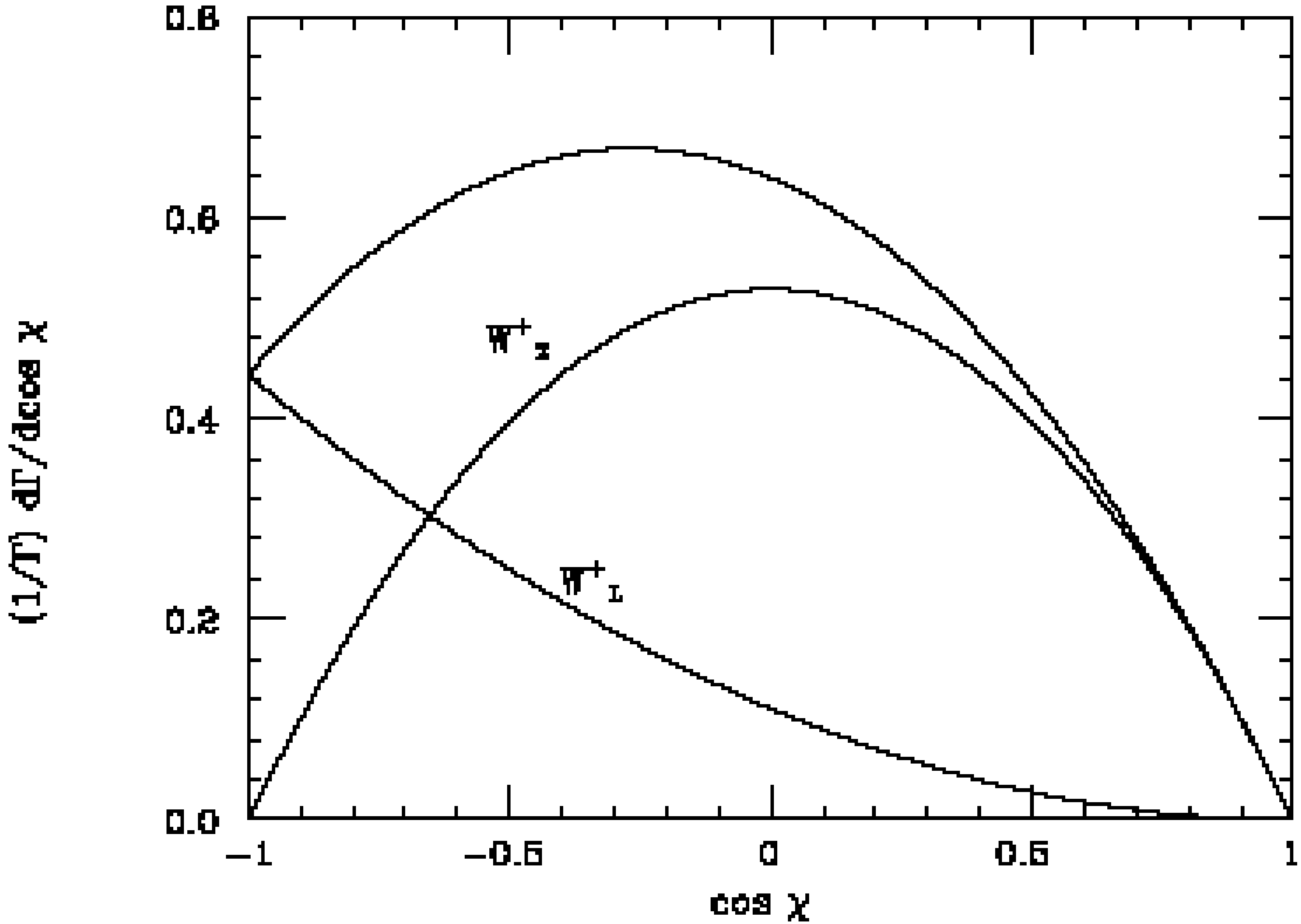
These give constructive interference for $t_L \bar{t}_R$, which peaks forward, and destructive interference for $t_R \bar{t}_L$, which peaks backward.

Top quarks decay before they hadronize, and the decay is maximally parity violating. So, there is considerable structure, and many observables (b, ℓ, ν energies and angles) that access this structure.



(a)
1.50
1.25
1.00
0.75
0.50
0.25
0.00
dσ/dcos θ (units of R)
$t_L\bar{t}_R$
$t_R\bar{t}_L$
$t_L\bar{t}_L + t_R\bar{t}_R$
−1
−0.5
0
0.5
1
cos θ
(b)
0.8
0.6
0.4
0.2
0.0
dσ/dcos θ (units of R)
$t_R\bar{t}_L$
$t_L\bar{t}_R$
$t_L\bar{t}_L + t_R\bar{t}_R$
−1
−0.5
0
0.5
1
cos θ

Schmidt



Schmidt

To discuss constraints on these structures, begin with a parametrization.

For the production:

$$\begin{aligned}\mathcal{L} &= eA_\mu \bar{t}[\gamma^\mu (P_L F_{L\gamma} + P_R F_{R\gamma}) + i\frac{\sigma^{\mu\nu} q_\nu}{2m_t} F_{2\gamma}]t \\ &+ eZ_\mu \bar{t}[\gamma^\mu (P_L F_{LZ} + P_R F_{RZ}) + i\frac{\sigma^{\mu\nu} q_\nu}{2m_t} F_{2Z}]t\end{aligned}$$

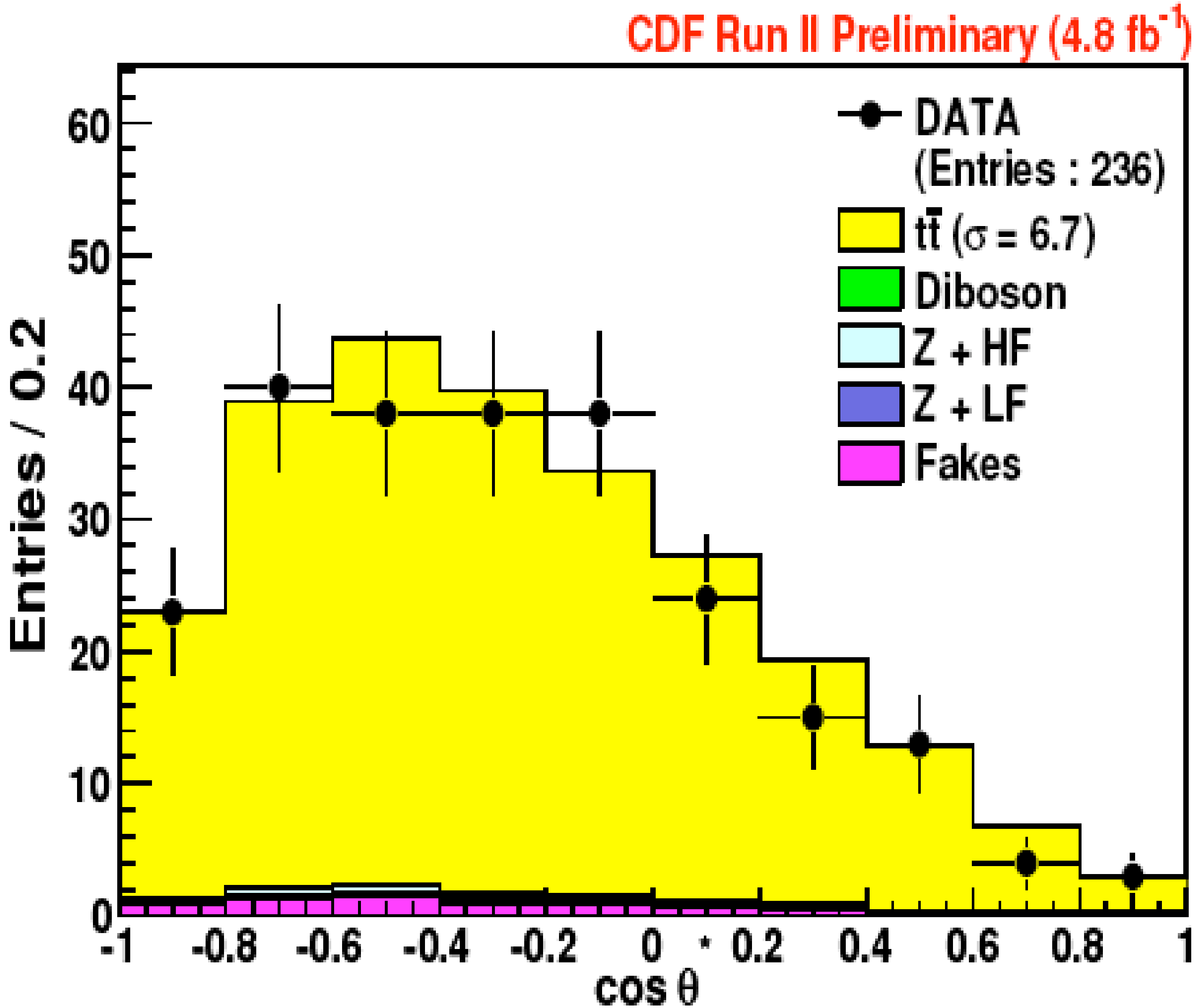
For the decay:

$$\mathcal{L} = \frac{g}{\sqrt{2}} W_\mu \bar{b}[\gamma^\mu (P_L F_{LW} + P_R F_{RW}) + i\frac{\sigma^{\mu\nu} q_\nu}{2m_t} F_{2W}]t$$

The decay form factors are already tested significantly at the Tevatron. From the sample of reconstructable top quark pairs, measure the W decay angle in the W rest frame (helicity angle).

$$
\begin{aligned}
i\mathcal{M}(W_L^+ b_L) &\sim \sqrt{2}(F_{LW} - \frac{m_t}{m_W} F_{2W})(1+\cos\theta)^{1/2} \\
i\mathcal{M}(W_0^+ b_L) &\sim (\frac{m_t}{m_W} F_{LW} - F_{2W})(1-\cos\theta)^{1/2} \\
i\mathcal{M}(W_R^+ b_L) &\sim 0 \\
i\mathcal{M}(W_L^+ b_R) &\sim 0 \\
i\mathcal{M}(W_0^+ b_R) &\sim (\frac{m_t}{m_W} F_{RW} - F_{2W})(1-\cos\theta)^{1/2} \\
i\mathcal{M}(W_R^+ b_R) &\sim \sqrt{2}(F_{RW} - \frac{m_t}{m_W} F_{2W})(1+\cos\theta)^{1/2}
\end{aligned}
$$

SM prediction: 30% L 70% 0 0% R



$$f_0 = 0.62 \pm 0.11(stat.) \pm 0.06(syst.)$$

$$f_0 = 0.78^{+0.19}_{-0.20}(stat.) \pm 0.06(syst.)$$

$$f_+ = -0.12^{+0.11}_{-0.10}(stat.) \pm 0.04(syst.)$$

CDF 2011

There is no evidence for a $t \to b_R$ vertex. Fitting for F_{2W} only, we find

$$F_{2W} = 0.23 \pm 0.35$$

These constraints should become much stronger at the LHC - to the percent level - or definite nonzero values should be measured.

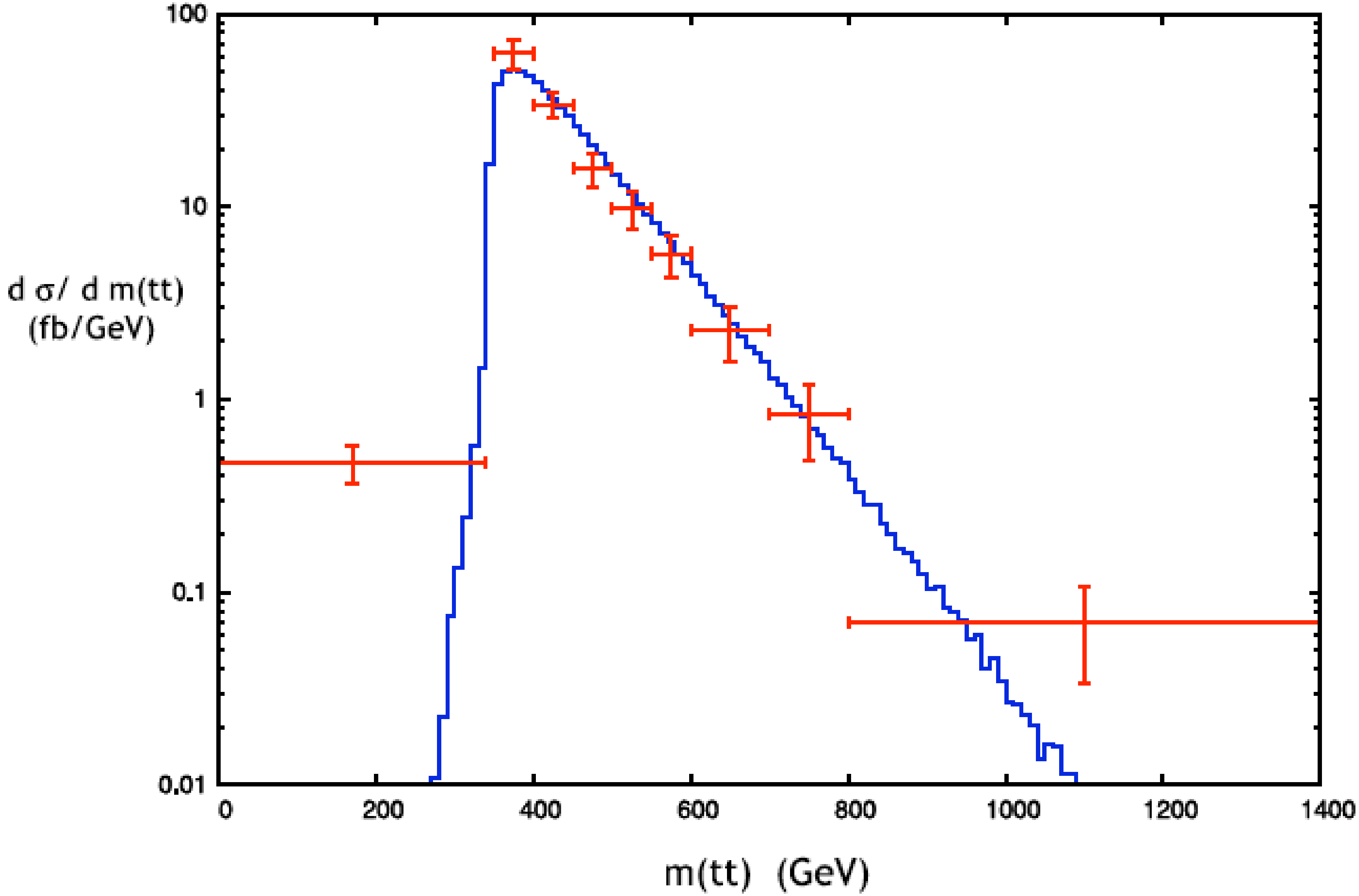
Hadron colliders will also give constraints on the form factors in hadronic $t\bar{t}$ production. These are QCD or gluon form factors. A typical current set of predictions (Kidonakis) is

$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV}, 1.96\,{\rm TeV}) \;=\; 7.08^{+0.00+0.36}_{-0.32-0.27}\,{\rm pb}\,,$$

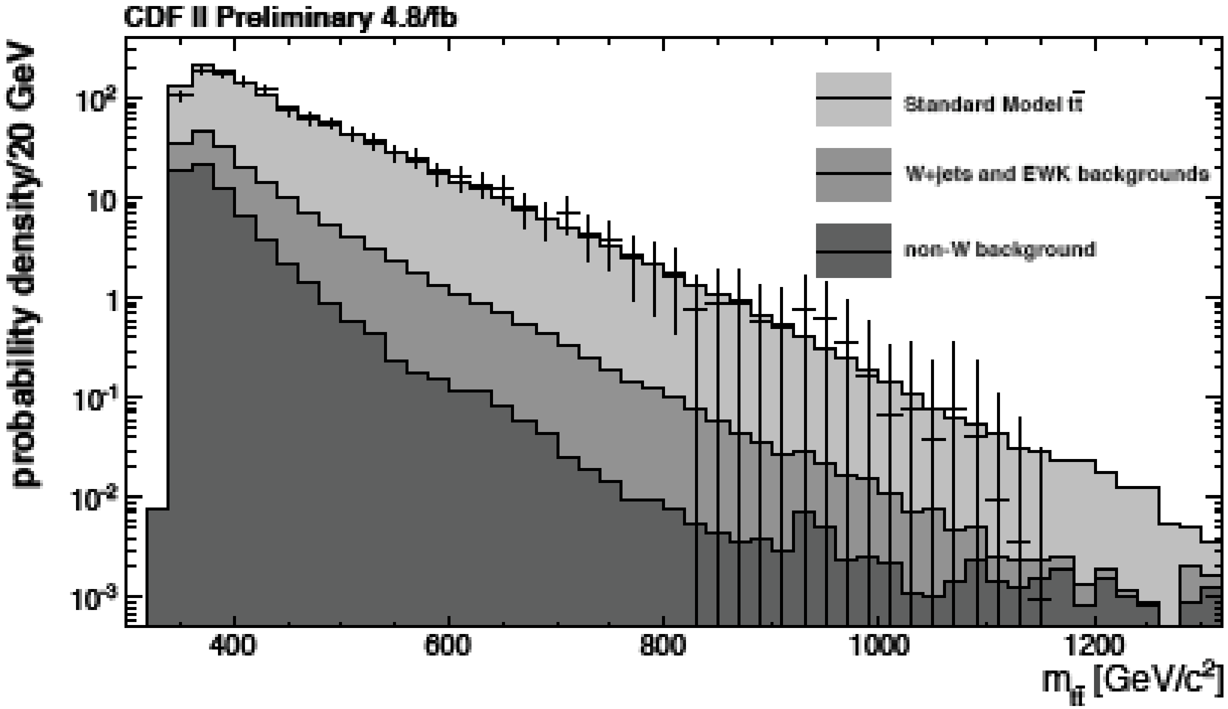
$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV}, 7\,{\rm TeV}) \;=\; 163^{+4+9}_{-8-9}\,{\rm pb}\,.$$

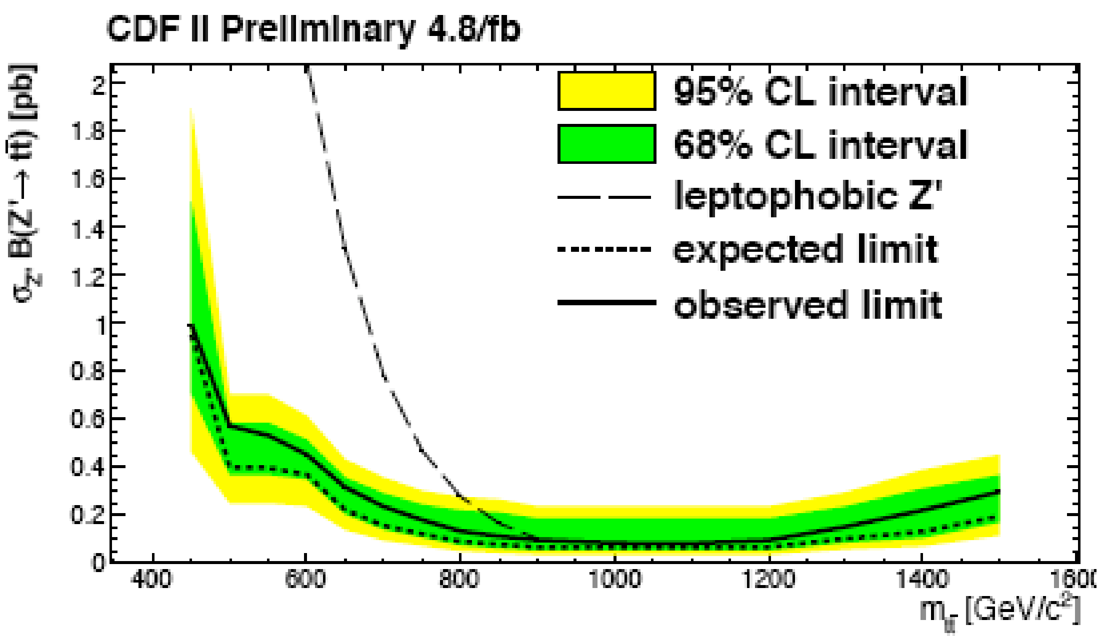
i.e. a 5% error from QCD theory + 5% error from PDFs. The current CDF and D0 cross section measurements are within this band. ILC will need to compete with this. Of course, the theoretical error on the SM electroweak cross section is at the part per mil level.

At the moment, the $t\bar{t}$ total cross section and the mass spectrum are in good agreement with Standard Model expectations.



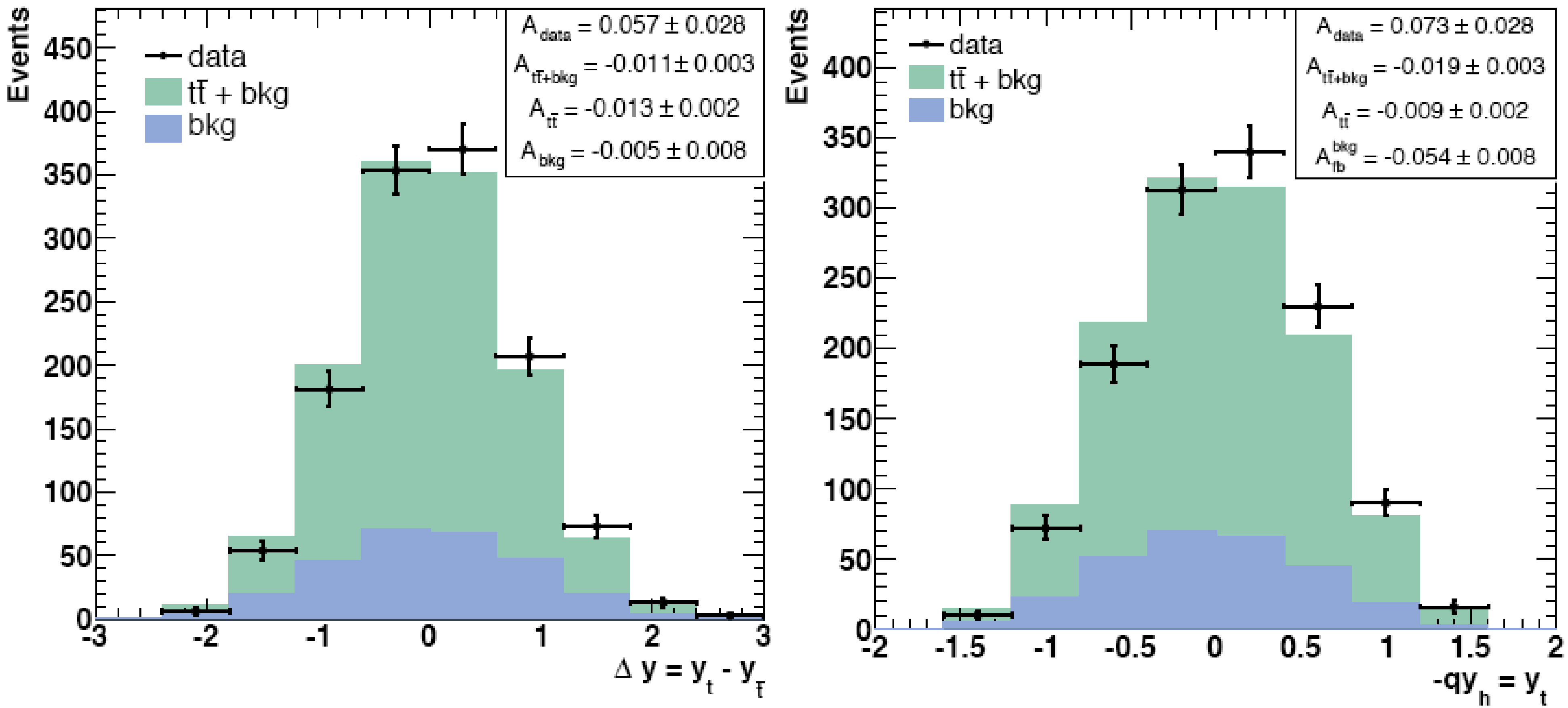
CDF unfolded m(tt) distribution


CDF II Preliminary 4.8/fb
probability density/20 GeV
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
Standard Model $t\bar{t}$
W+jets and EWK backgrounds
non-W background
400
600
800
1000
1200
$m_{t\bar{t}}$ [GeV/c^2]


CDF II Preliminary 4.8/fb
$\sigma_{Z'} \cdot B(Z' \rightarrow t\bar{t})$ [pb]
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
95% CL interval
68% CL interval
leptophobic Z'
expected limit
observed limit
400
600
800
1000
1200
1400
1600
$m_{t\bar{t}}$ [GeV/c^2]

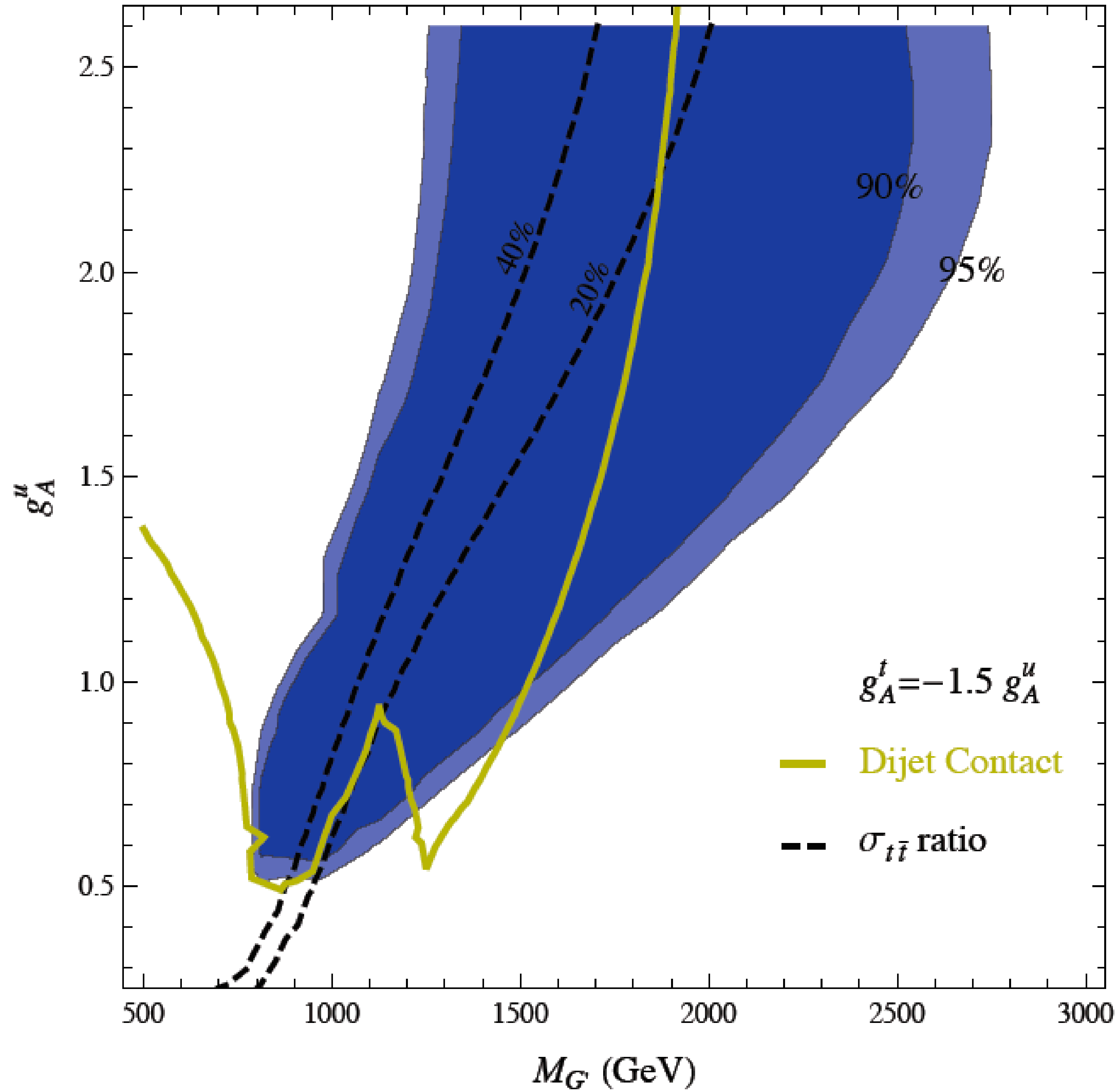
However, not all aspects agree with the Standard Model

CDF 2011



$$A_{FB}(t\bar{t}) \quad = \quad 0.158 \pm 0.074$$

$$MCFM \qquad 0.058 \pm 0.009$$



Bai, Hewett, Kaplan, Rizzo

Is it possible that a signal of new physics can be visible in the electroweak form factors while it is hidden in the QCD form factors ?

Easily!

The top could couple to a color-singlet resonance that is obscured by color-octet dynamics in strong production.

There could be a color-octet resonance discoverable at the LHC, and a color-singlet resonance at a lower energy that is difficult to observe at the LHC.

Vector-dominated form factor:

$$F_{1L,R} = \frac{1}{1 - s/M^2}$$

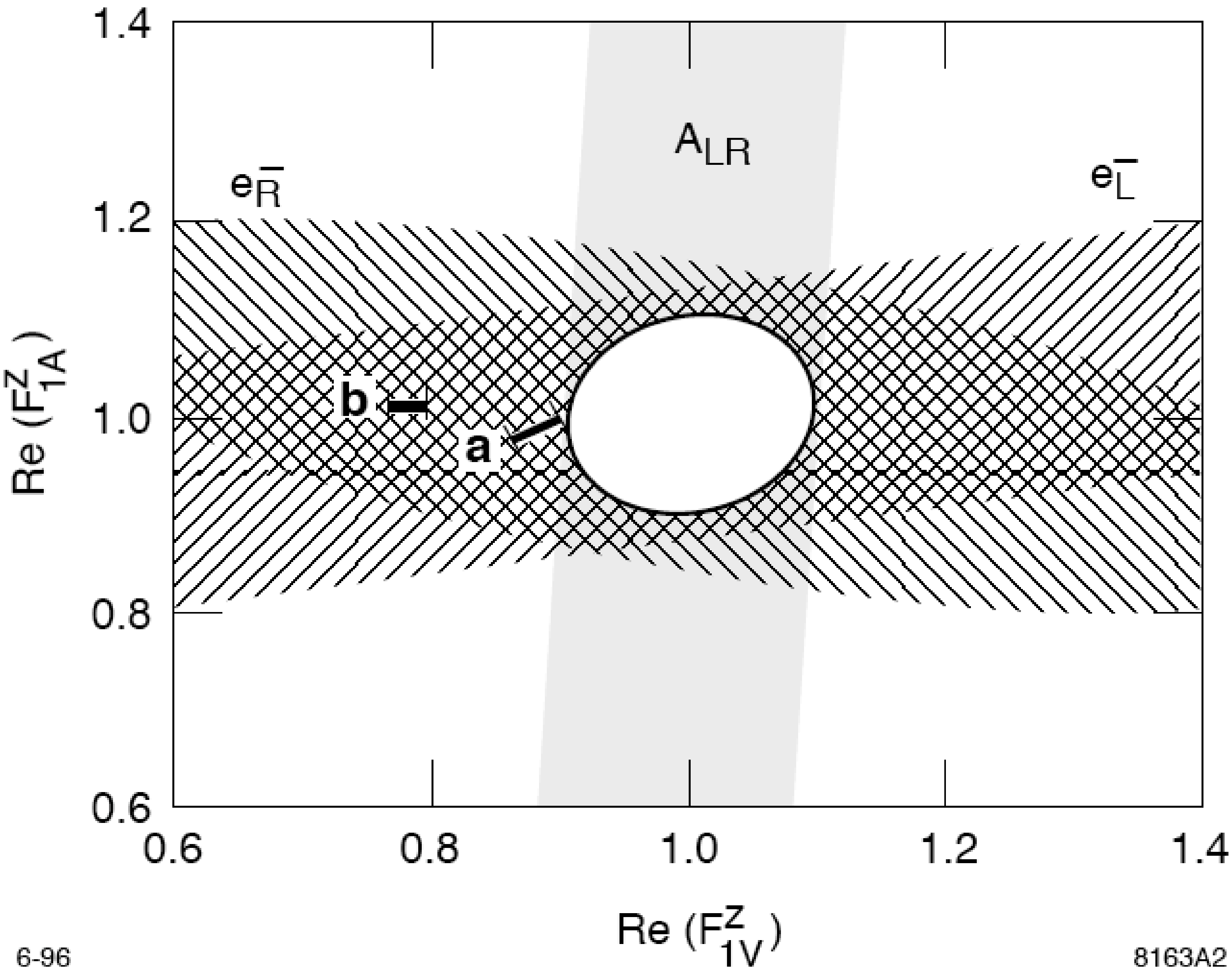
For example, for

$$\sqrt{s} = 500 \text{ GeV} , \quad M = 2 \text{ TeV}$$

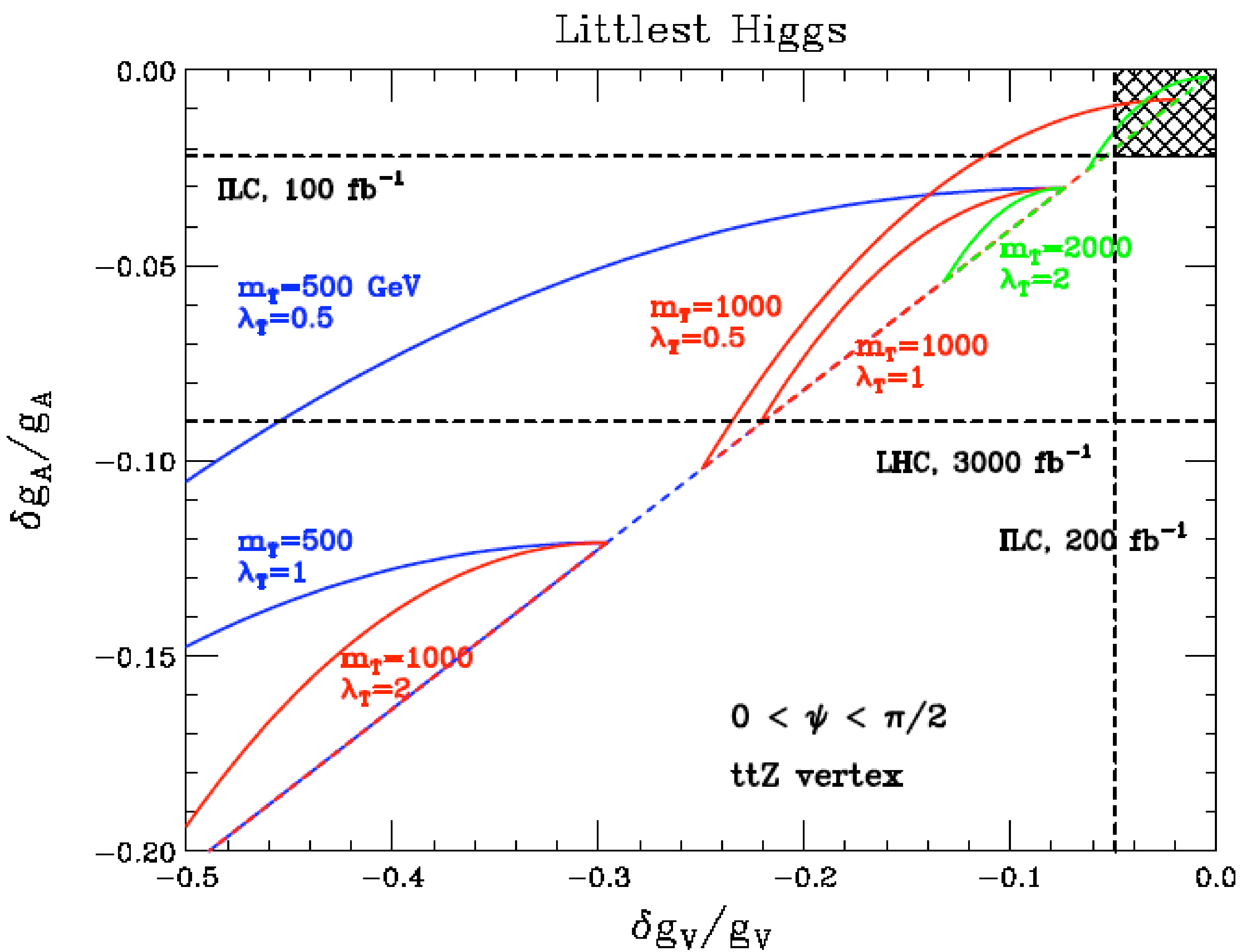
this is a 10 % perturbation of the Standard Model dynamics.

Dynamics of a strongly-interacting Higgs sector could uniquely affect the $Zt\bar{t}$ form factors:

This actually occurs in technicolor models, in Randall-Sundrum models (giving order-1 perturbations of F_{RZ}), and in Little Higgs models.



Barklow and Schmidt



Berger, Perelstein, Petriello

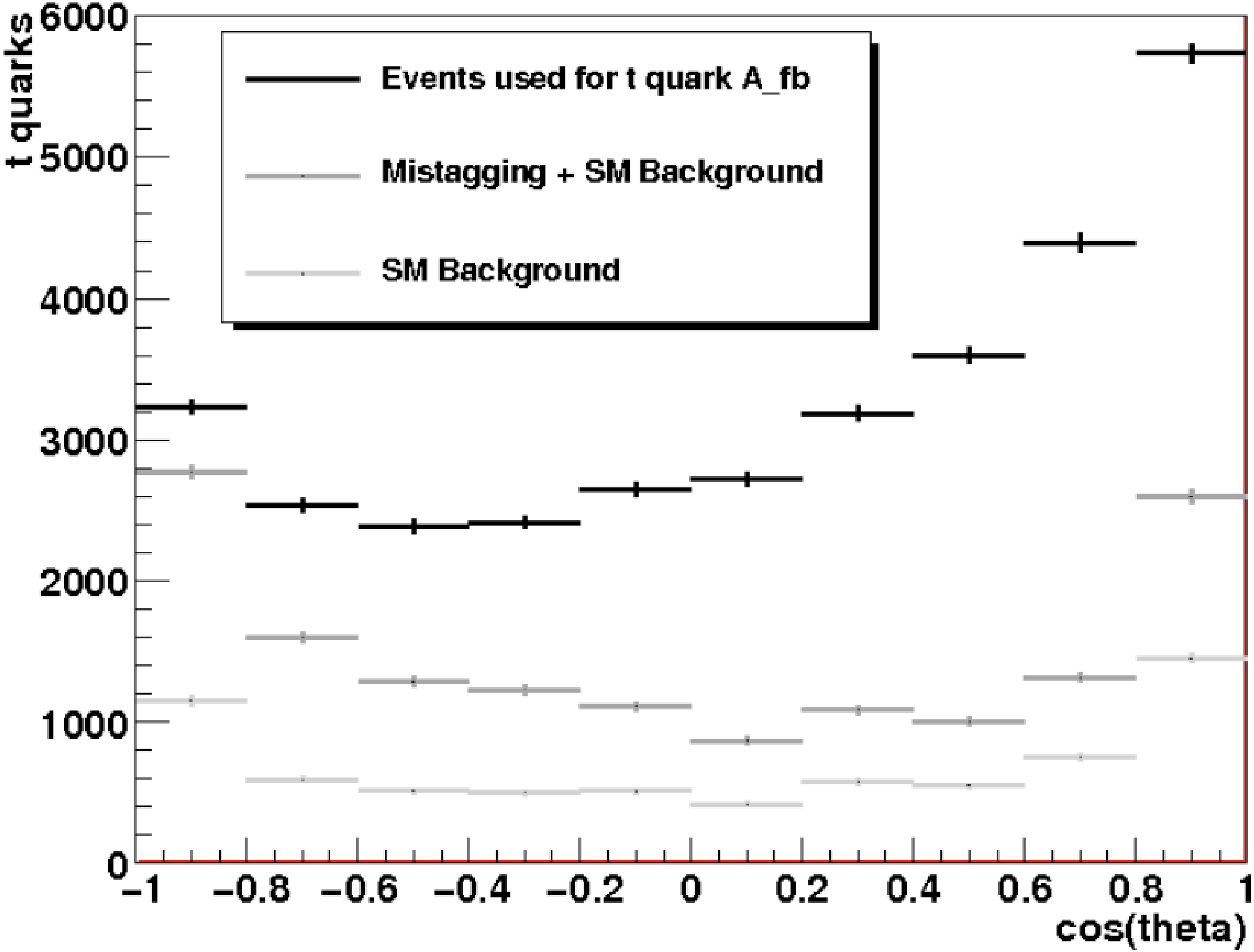
It would be interesting to know the actual power of the ILC to detect these effects.

In the LOI study for SiD, Devetak and Nomerotski studied the measurement of the top quark forward-backward asymmetry at the ILC at 500 GeV using a full-simulation study. They considered only full-hadronic top quark events. In principle, one could carry out a parallel analysis with semileptonic top events that would double the statistics.

They found agreement with the Standard Model predictions within errors (500 fb^{-1})

$$\sigma(A_{FB}(e^-_L e^+_R)) = 0.010 \qquad \sigma(A_{FB}(e^-_R e^+_L)) = 0.012$$

where the electron polarization states denote realistic conditions with 80%/30% beam polarization.



Devetak and Nomerotski - SiD

In principle, the study of $e^+e^- \to t\bar{t}$ contains enough information to determine all 6 production form factors independently:

For each of 2 initial beam polarization states, the cross section has the form

$$\frac{d\sigma}{d\cos\theta} = A(1+\cos\theta)^2 + B(1-\cos\theta)^2 + C\sin^2\theta$$

The magnetic moment form factors mainly affect the size of the C term. With this constrain, the absolute cross sections and forward-backward asymmetries determine the L and R form factors for A and Z.

The decay form factors can be constrained by separate study of decay variables in the W rest frames. In addition, these will already have been strongly constrained at the LHC.

As an illustration, we could fit for only the two parameters F_{LZ} and F_{RZ} using the two measurements analyzed by Devetak and Nomerotski.

I find

$$\begin{pmatrix} \delta A_{FB}(LR) \\ \delta A_{FB}(RL) \end{pmatrix} = \begin{pmatrix} 0.164 & -0.375 \\ 0.367 & -0.238 \end{pmatrix} \begin{pmatrix} \delta F_{LZ} \\ \delta F_{RZ} \end{pmatrix}$$

Using this relation to propagate the errors, I find

$$\sigma(\delta F_{LZ}) = 0.051 \quad \sigma(\delta F_{RZ}) = 0.042$$

This would incisively test the models of strong Higgs dynamics discussed earlier.

The study of $e^+e^- \longrightarrow t\bar{t}$ at threshold and above contains many potential observables and a rich suite of measurements. The most important of these are unavailable at the LHC.

This reaction will have a particularly important role if there is no weakly-coupled new spectroscopy such as supersymmetry at the TeV scale. In that case, it might provide crucial clues toward the understanding of electroweak symmetry breaking.