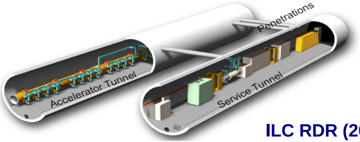
Energy variations and emittance growth in the ILC main linac with KCS

Yipeng Sun and Chris Adolphsen
ARD Division, SLAC National Accelerator Lab

Sep., 2010

ILC main linac tunnel options

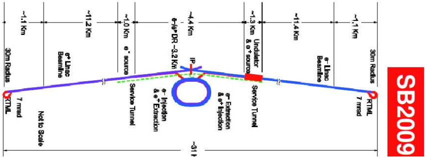
From



ILC RDR (2007)

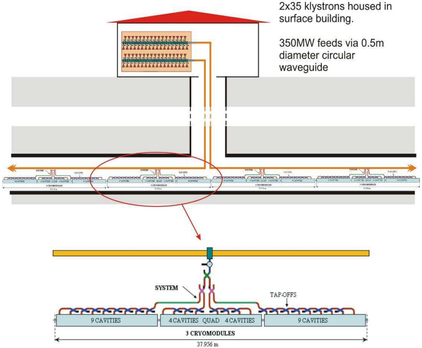
FIGURE 1.3-6. Cutaway view of the linac dual-tunnel configuration.

To



- Distributed RF Scheme (DRFS)
- Klystron Cluster RF Distribution Scheme (KCS)

Klystron Cluster RF Distribution Scheme

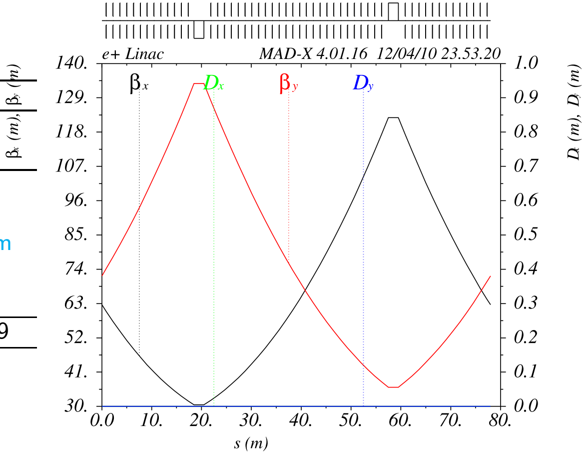


Each tap-off from the main waveguide feeds 10 MW through a high power window and probably a circulator or switch to a local PDS for a 3 cryomodule, 26 cavity RF unit (RDR baseline).

- Service tunnel eliminated
- Electrical and cooling systems simplified
- Concerns: power handling, LLRF control coarseness

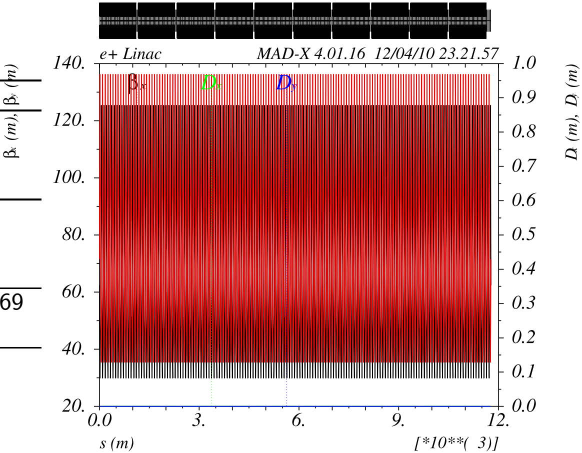
Lattice cell

parameter	value
Cell length	78 m
Length of quad	2 m
Length of 9-cell cavity	1 m
RF frequency	1.3 GHz
Cavity Gradient	31 MV/m
# of cavities	52
# of cryomodule	6
Phase advance	0.22/0.19

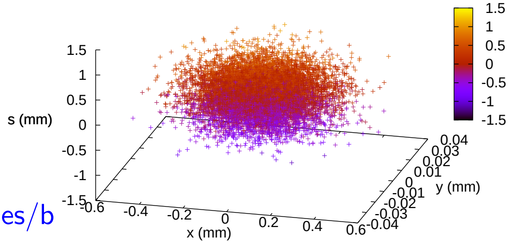


Linac lattice

parameter	value
Length	11778 m
# of super periods	10
# of quad	302
Length of 9-cell cavity	1 m
# of cavities	7280
# of cryomodule	900
Phase advance	33.22/28.69
Chromaticity	-37.5/-35

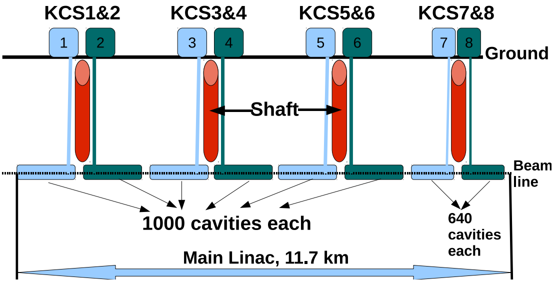


Simulation code



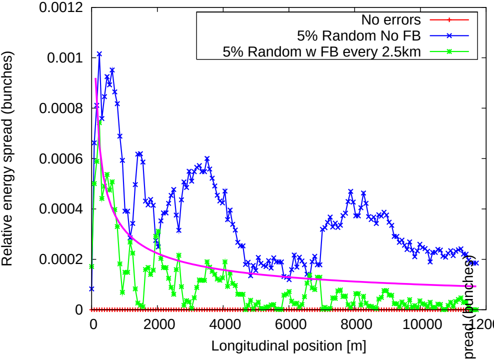
- Fortran90 code
- Self-Defined lattice
- Up to 10M macro-particles/b
- Multi bunch (pulse)
- 6-D Gaussian distribution (correlated)
- Longitudinal cut at 4 sigma
- Horizontal emittance (nor.): 10$mm.mrad$; Vertical emittance (nor.): 0.02$mm.mrad$
- RMS Bunch length 0 μm (no Wake field); RMS energy spread 1.5×10^{-2}

KCS model and error type



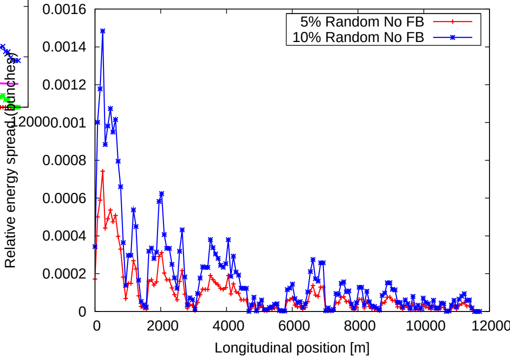
- Random error ($\sigma = 0.05$) on each cavity's gradient (freq. error)
- Correlated errors: RF power and beam current (loading). Assumed perfectly corrected.
- Feedback works within each KCS; Systematic error: gradient $\sigma = 0.01$, phase $\sigma = 1$ degree
- Linear correlation for bunches in same pulse

Energy spread between 2625 bunches in one pulse

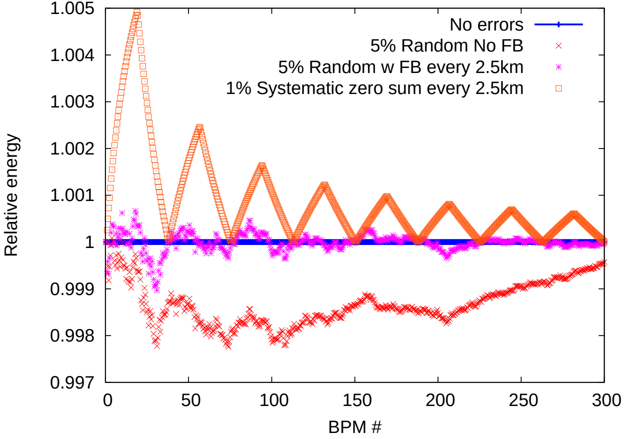


Linearly proportional to gradient error's amplitude

With feedback, energy spread decreases to zero ($\propto \sigma_g \sqrt{N}/E$)

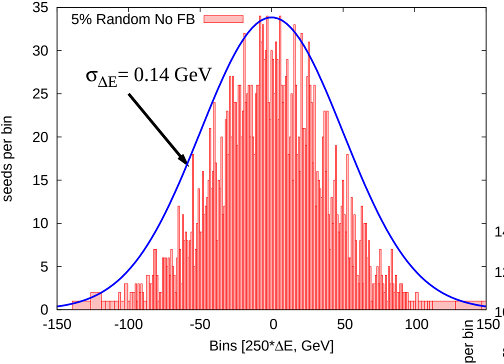


Energy variations along Linac (one seed)

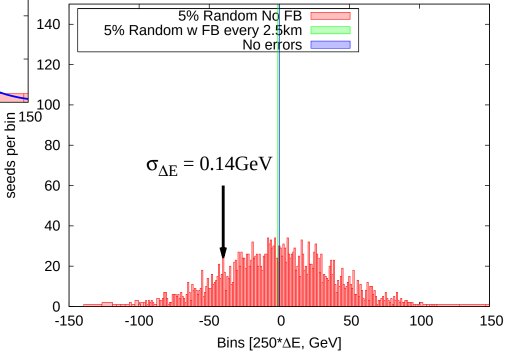


Worst case: 1% Systematic zero sum every 2.5km

Energy distribution at linac end w 2600 pulses(seeds)

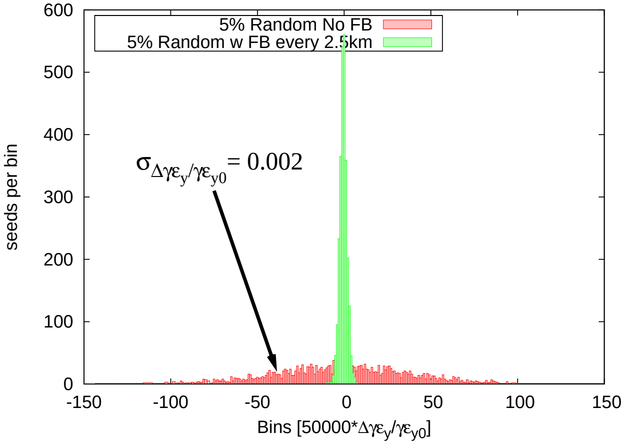


5% Random w FB every 2.5km, or 1% Systematic zero sum every 2.5km, 250GeV at linac end



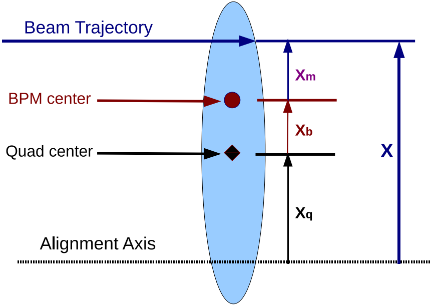
5% Random No FB, Gaussian distribution, RMS= $\sqrt{N_{cav}} \cdot \sigma_g \cdot V_{rf}$

$\gamma\epsilon_y$ distribution at linac end w 2600 pulses(seeds)



Very small change in $\gamma\epsilon_y$ due to chromatic effects

Linac alignment problem

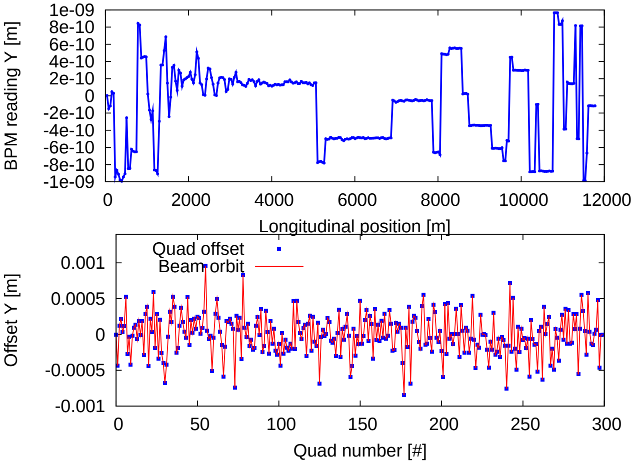


- $X_m = X - X_q + X_B + X_r$
- From X_m: BPM readings to calculate X_q

Alignment algorithms

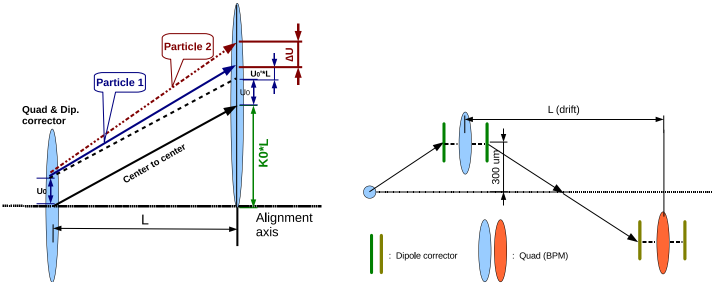
- One to One
- Global correction

One-to-one correction



Iterate to zero all BPM readings

Model for Analytical approach



- Dispersive emittance growth = local effect
- Only Quad at dispersion region contributes

Analytical approach (1)

Definition of projected emittance:

$$\gamma\epsilon_y = \gamma\sqrt{(<y^2> - <y>^2)\cdot(<y'^2> - <y'>^2) - (<yy'> - <y><y'>)^2}$$

With respect to the centroid trajectory:

$$(\gamma\epsilon_y)^2 = \gamma^2\left[\left({\sigma_y}^2 + <\Delta y^2>\right)\cdot\left(\sigma_{y'}^2 - <\Delta y'^2>\right) - \left(\sigma_{yy'}^2 - <\Delta y\Delta y'>\right)^2\right]$$

With:

$$(\gamma\epsilon_y)^2 = \gamma^2\left({\sigma_y}^2\cdot\sigma_{y'}^2 - \sigma_{yy'}^2\right)$$

$${\sigma_y}^2 = \epsilon_y\beta_y$$

$${\sigma'_y}^2 = \frac{\epsilon_y}{\beta_y}$$

$$\sigma_{yy'} = 2\epsilon_y\alpha_y$$

$$\gamma\epsilon_y = \gamma\epsilon_{y0}\sqrt{1 + 2\Delta\gamma\epsilon/\gamma\epsilon_{y0}}$$

$$\Delta\gamma\epsilon = \frac{\gamma}{2}\left(\frac{1+\alpha^2}{\beta}\Delta y^2 + 2\alpha\Delta y\Delta y' + \beta\Delta y'^2\right)$$

$$\Delta y = \frac{k_0 L}{1+\delta} - k_0 L = k_0 L(-\delta + \delta^2 - \delta^3 + \ldots)$$

$$\Delta y' = K\Delta y$$

Analytical approach (2)

Square of 2-D projected emittance = determinant of matrix

$$\epsilon^{\mathbf{2}} = \begin{pmatrix} \sigma_{y_0}^2 + \sum < \Delta {y_i}^2 > & 0 \\ 0 & \sigma_{y_0'}^2 + \sum < \Delta {y_i'}^2 > \end{pmatrix}$$

at the n^{th} quad

$< \Delta {y_i}^2 >= \sum_{j=1}^{i-1} R_{12_{j,i}} < \Delta {y_j'}^2 >$

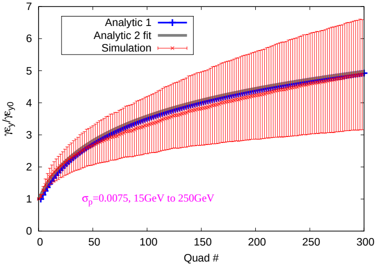
$< \Delta {y_i'}^2 >= \sum_{j=1}^{i-1} R_{22_{j,i}} < \Delta {y_j'}^2 >$

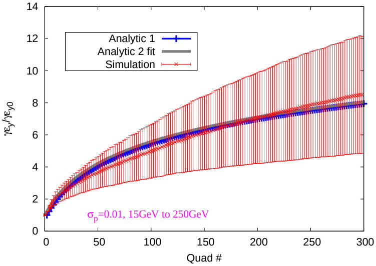
Simply and we have

$< \Delta {y_i}^2 >= \sum_{j=1}^{i-1} (\frac{1}{2}) \beta^2 < \Delta {y_j'}^2 >$

$< \Delta {y_i'}^2 >= \sum_{j=1}^{i-1} (\frac{1}{2}) < \Delta {y_j'}^2 >$

Benchmark: with acceleration ($\sigma_z = 0$)





Use same fitting parameter 1k seeds

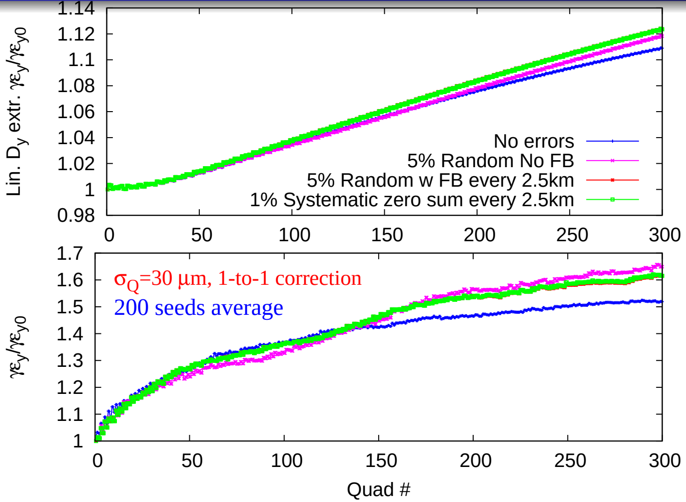
Left: $\sigma_{p0} = 0.0075$; Right: $\sigma_{p0} = 0.01$

With acceleration, integrate over cell numbers, assuming that the energy gain is the same in each cell.

$$\gamma\epsilon_n = \gamma_0\sqrt{{\sigma_{x_0}}^2 + A \cdot log_e\left(\frac{E_n}{E_0}\right) \cdot \beta^2 \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2}$$

$$\cdot\sqrt{{\sigma_{x_0'}}^2 + A \cdot log_e\left(\frac{E_n}{E_0}\right) \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2}$$

Comparison: emittance growth (1-to-1)



- Small difference w or w/o energy error
- 1-to-1 correction algorithm

Solving method (Global correction algorithm)

$$x_i = R_{12_{i,1}}\theta_1 - \sum_{j=1}^{i-1} R_{12_{i,j}} K_j x_{q,j}$$

$$x_i{}' = R_{22_{i,1}}\theta_1 - \sum_{j=1}^{i-1} R_{22_{i,j}} K_j x_{q,j} - x_{q,i} K_i / 2$$

$$\mathbf{R} \cdot \mathbf{Q} = \mathbf{B}$$

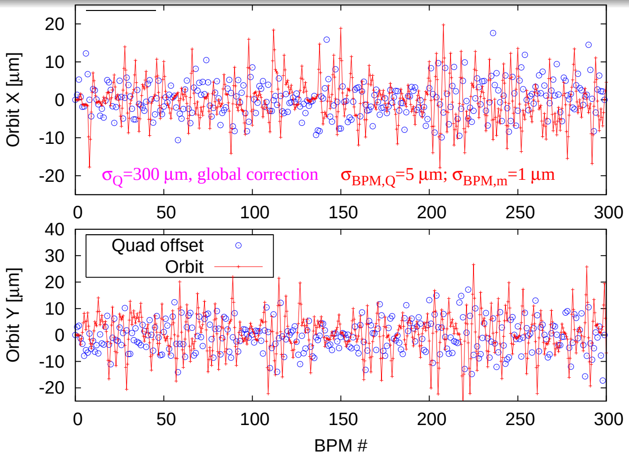
R combined R matrix, $n \times n$ sparse matrix
Q quad offsets+initial beam offset, $n \times 1$
B BPM reading, $n \times 1$
B includes BPM to Q offset error, and BPM measurement error

Use Row reduction (Gaussian elimination) to solve the system
For ILC linac, 300×300 matrix

Error Table

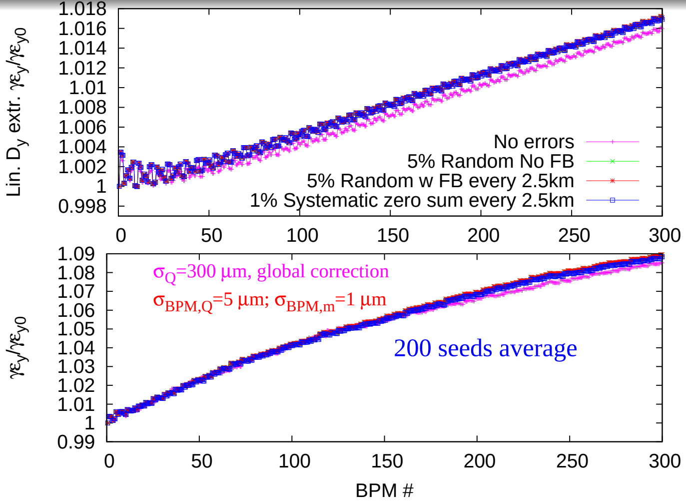
parameter	value
Quad offset	$\sigma = 300 \mu m$
BPM to Quad offset	$\sigma = 5 \mu m$
BPM measurement	$\sigma = 1 \mu m$
Initial beam offset	$\sigma = 1 \mu m$
Initial beam angle	$\sigma = 1 \mu rad$

New Quadrupole offset (Moving Quad option)



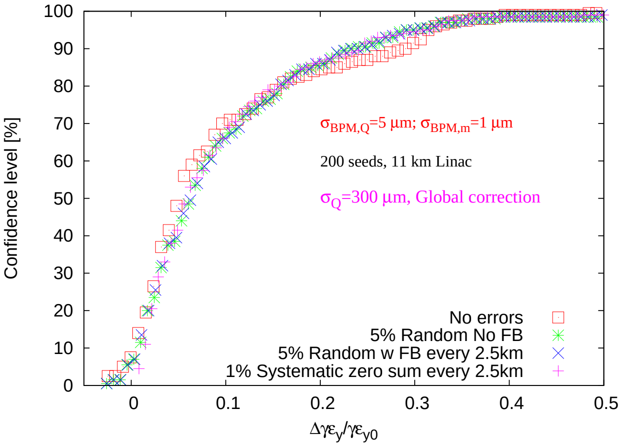
- Global correction algorithm , One seed
- New quad offset Globally correlated; roughly $6\mu m$

Comparison: emittance growth (global c. algorithm)



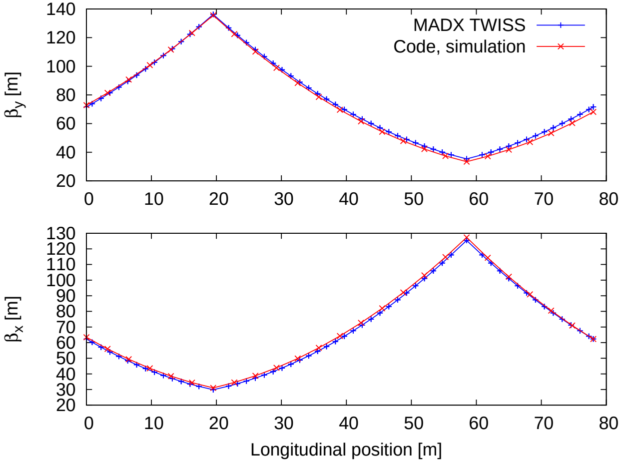
- Very small difference; with global correction algorithm

Confidence level (projected $\gamma\epsilon_y$)



- At 90% confidence level, less than $4nm$ (20% of $20nm$) growth in projected $\gamma\epsilon_y$, global correction algorithm

Backup(0): Benchmark of code (β-function)



Benchmark of the beta function in one FODO cell, between MADX TWISS output and the simulation results of this code.

Backup(1): Dispersion and emittance

Projected emittance

$\epsilon = \sqrt{(< x^2 > - < x >^2) \cdot (< x'^2 > - < x' >^2) - (< xx' > - < x >< x' >)^2}$

Linear dispersion corrected emittance

$\epsilon = (< (x - D_x\delta)^2 > - < x - D_x\delta >^2) \cdot (< (x' - D_x'\delta)^2 > - < x' - D_x'\delta >^2)$

$-(< (x - D_x\delta)(x' - D_x'\delta) > - < x - D_x\delta >< x' - D_x'\delta >)^{20.5}$

Dispersion

$D_x = (< x\delta > - < x >< \delta >) \,/\, \left(< \delta^2 > - < \delta >^2\right)$

$D_x' = (< x'\delta > - < x' >< \delta >) \,/\, \left(< \delta^2 > - < \delta >^2\right)$

Backup(2): Example 5 quads

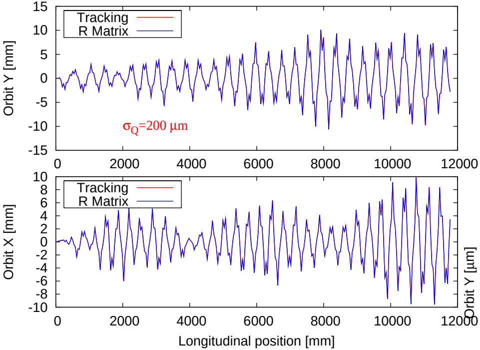
$$\mathbf{R} = \begin{pmatrix} R_{12}(2,1) & 1 & 0 & 0 \\ R_{12}(3,1) & R_{12}(3,2)K_2 & 1 & 0 \\ R_{12}(4,1) & R_{12}(4,2)K_2 & R_{12}(4,3)K_3 & 1 \\ R_{12}(5,1) & R_{12}(5,2)K_2 & R_{12}(5,3)K_3 & R_{12}(5,4)K_4 \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} \theta_1 \\ x_{q,2} \\ x_{q,3} \\ x_{q,4} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_{BPM,2} \\ x_{BPM,3} \\ x_{BPM,4} \\ x_{BPM,5} \end{pmatrix}$$

Backup(3): Global Alignment algorithm

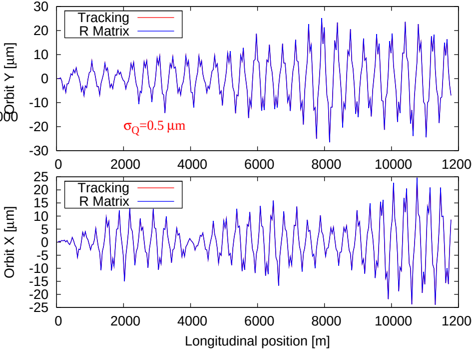
- Define error: Quad offset; BPM to Quad offset (do not change for one specified seed); BPM measurement error; Initial beam offset; Initial beam angle (change from pulse to pulse)
- Track single particle, get BPM readings
- Algorithm to calculate Quad offset (Row reduction, Gaussian elimination)
- Move Quad (or Use steering correctors)
- Track bunch (10,000 macro-particles) to calculate emittance etc. statistically
- Another seed

Backup(4): Reproduce orbit from R matrix

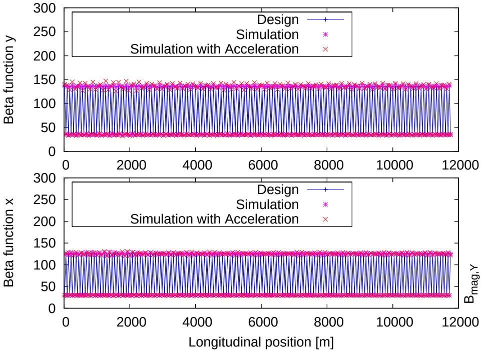


200 μm, 10 mm orbit

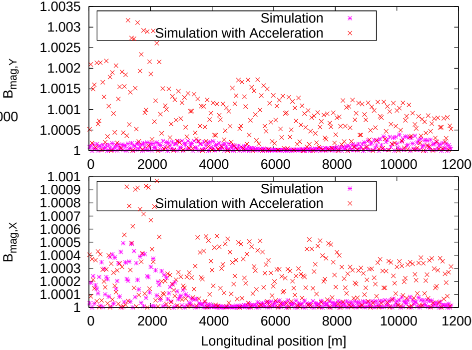
0.5 μm, 20 μm orbit



Backup(5): Beta function from tracking

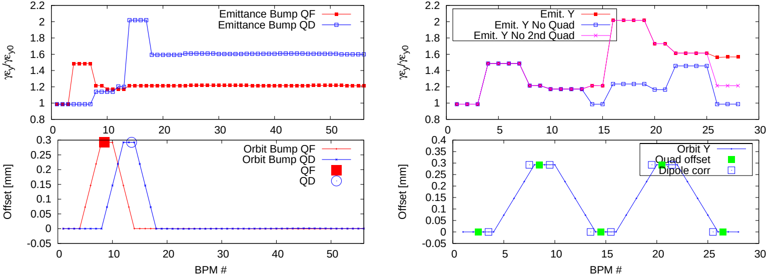


$$B_{mag} =$$
$$\frac{1}{2}[(\frac{\beta}{\beta^*}+\frac{\beta^*}{\beta})+(\alpha^*\sqrt{\frac{\beta}{\beta^*}}-\alpha\sqrt{\frac{\beta^*}{\beta}})^2]$$
$$\frac{\Delta\gamma\epsilon}{\gamma\epsilon} \approx B_{mag}-1$$



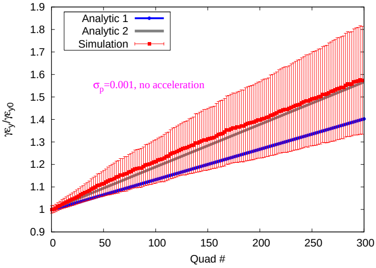
Good agreement

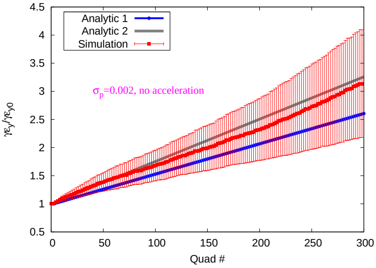
Backup(6): Model for Analytical approach



- Dispersive emittance growth = local effect
- Only Quad at dispersion region contributes

Backup(7): Comparison: no acceleration





Left: $\sigma_{p0} = 0.001$; Right: $\sigma_{p0} = 0.002$

Without acceleration, the physical vertical emittance at the n^{th} cell is

$$\epsilon_n = \sqrt{\left({\sigma_{y_0}}^2 + 0.5 \cdot n \cdot \beta^2 \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2\right)\left({\sigma_{y'_0}}^2 + 0.5 \cdot n \cdot (K_1 \cdot \sigma_Q \cdot \sigma_{p0})^2\right)}$$