Birks' Coefficient of the AHCAL Scintillator

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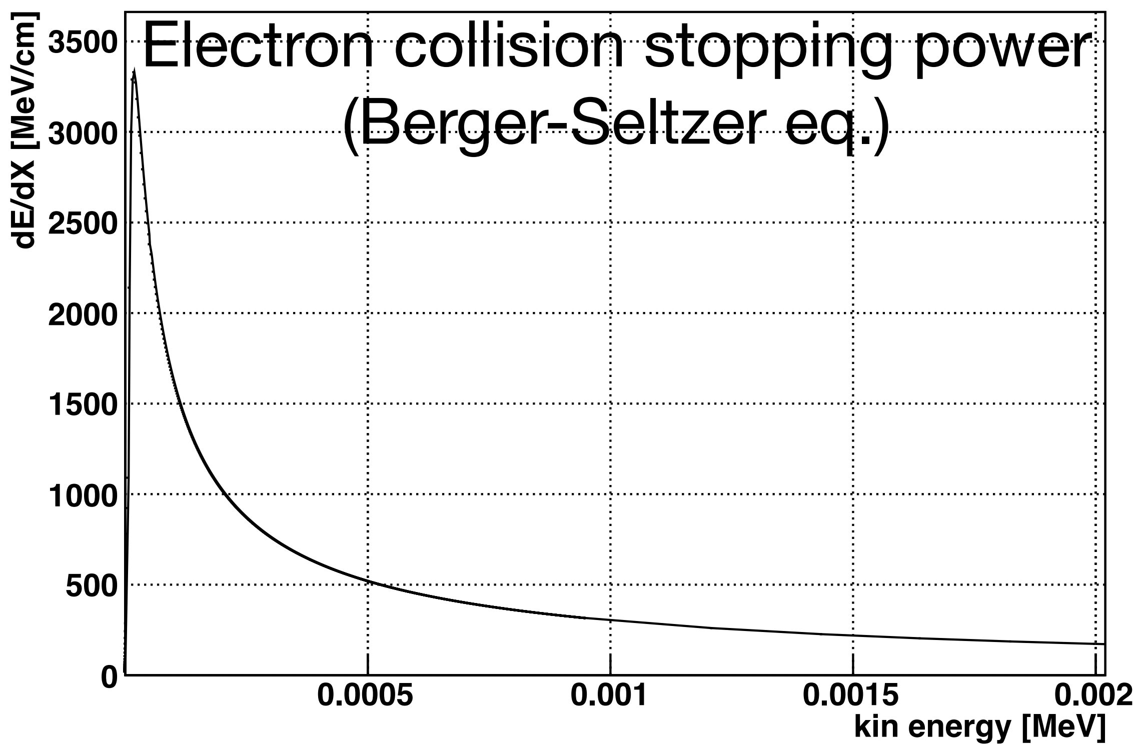
KIRCHHOFF-
INSTITUTE
FOR PHYSICS

CALICE
Calorimeter for ILC

Outline

- Reminder: Birks' law and measurement of Birks' coefficient kB with electrons
- Particle step-size dependence of kB
- Transformation of kB for use in MOKKA
- Birks' coefficient for other particles than electrons
- Conclusion & outlook

Birks' Saturation Formula



- Specific energy loss **dE/dx** is high before particle is stopped
- High ionization density **dI/dx α dE/dx**
- **Quenching:** Excited molecules can interact and may de-excite radiationless
- Light yield per unit length **dL/dx** is reduced for high **dE/dx**
- Non-linearity described by Birks' formula:

$$\frac{dL}{dx} = \frac{S\frac{dE}{dx}}{1 + kB\frac{dE}{dx}}$$

Birks' Saturation Formula

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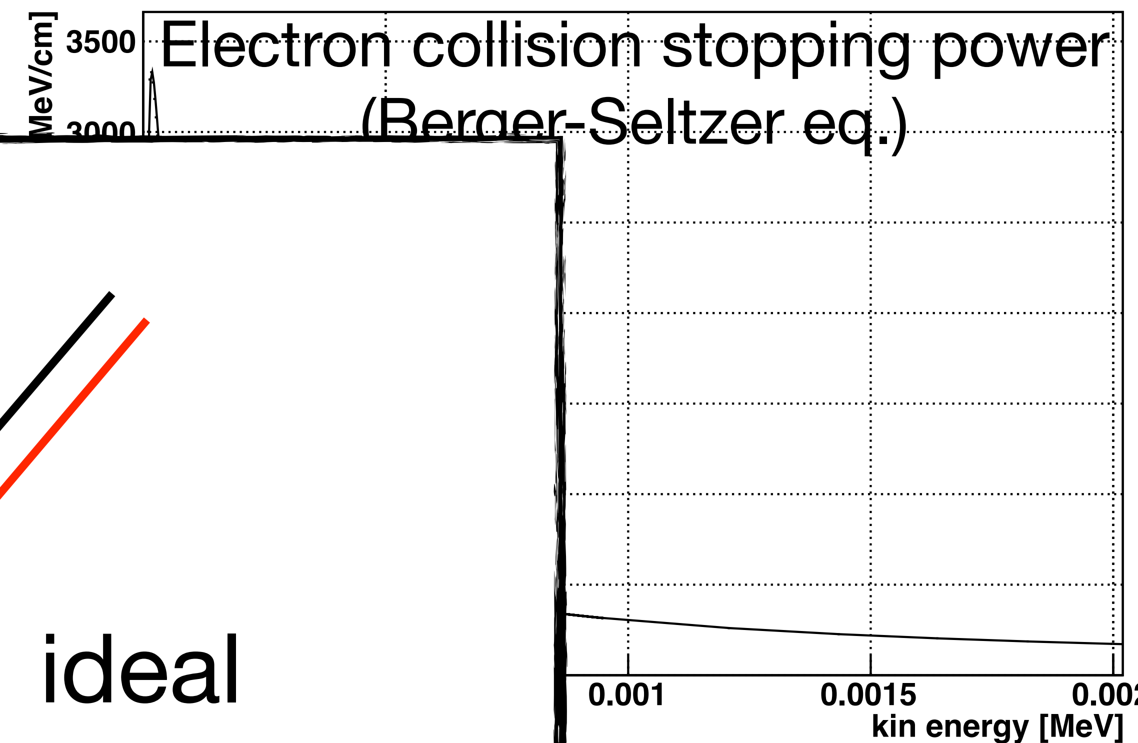
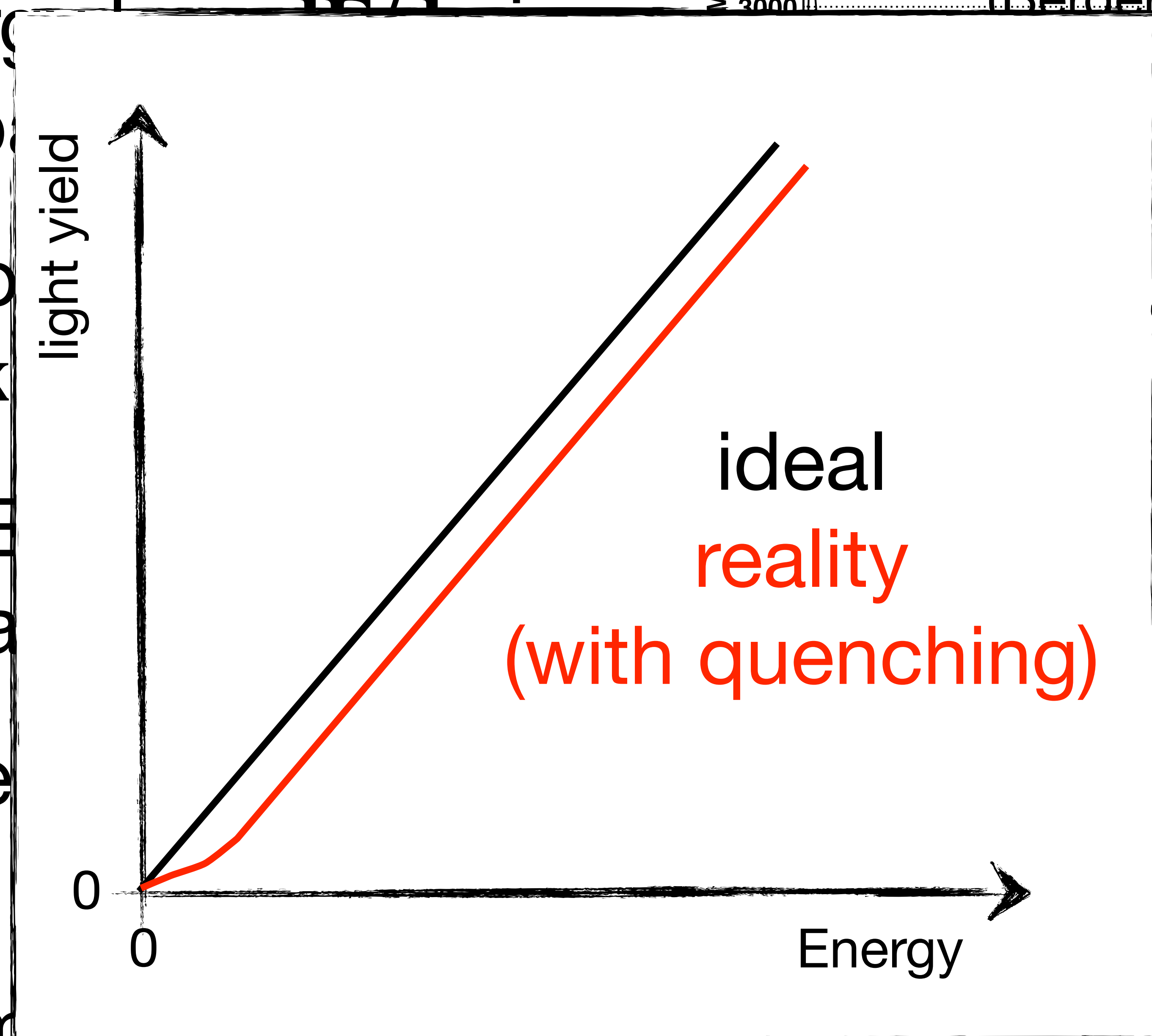
- Non-linearity ... by Birks' form...





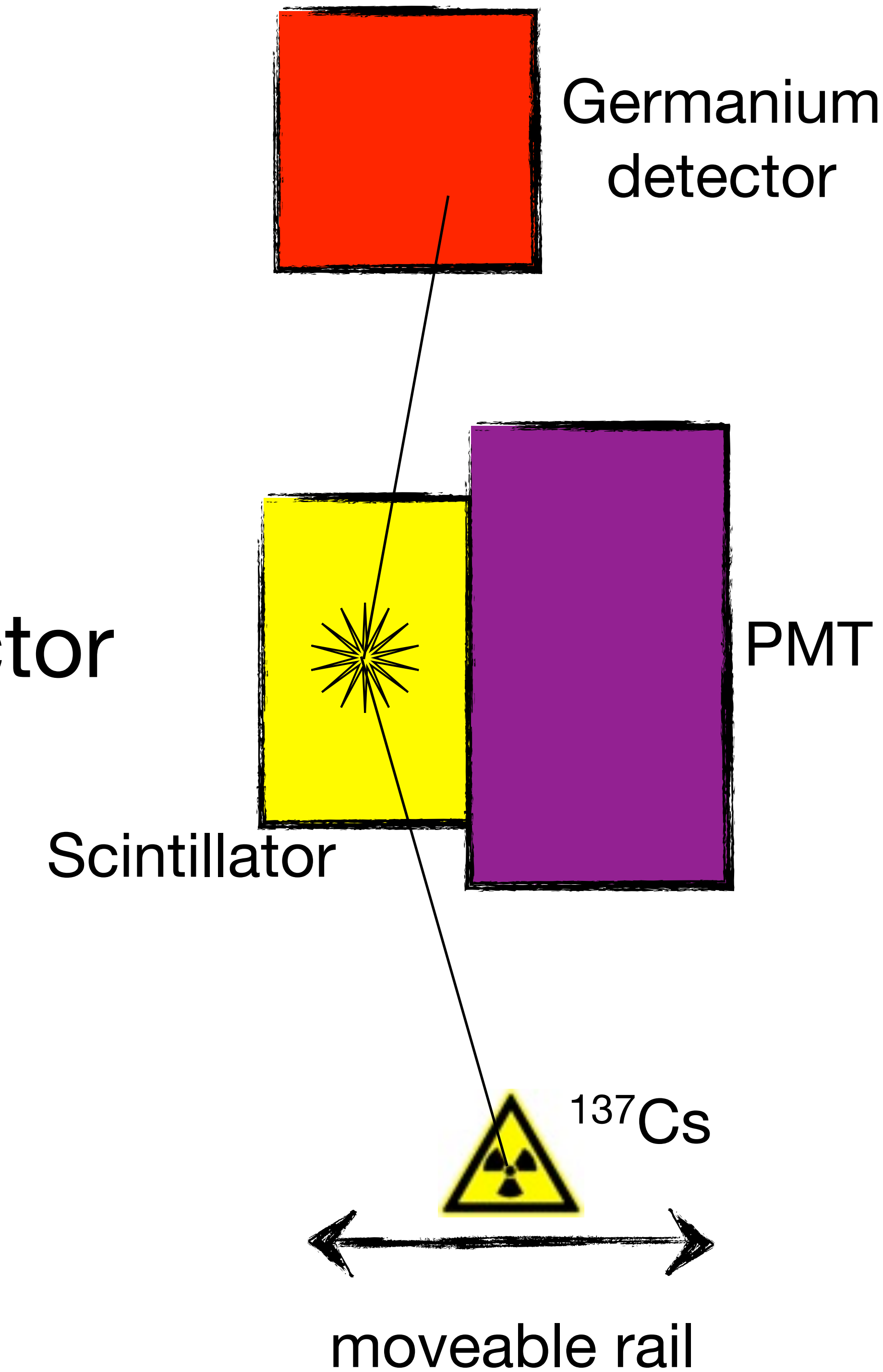
$$\ldots\frac{dE}{dx} \ldots dx \ldots \frac{\ldots}{1 + kB\frac{dE}{dx}}$$

Experimental Setup (MPIK Heidelberg)

- PMT measures light yield
- Germanium detector measures Energy of Compton scattered photon E_{Ge}

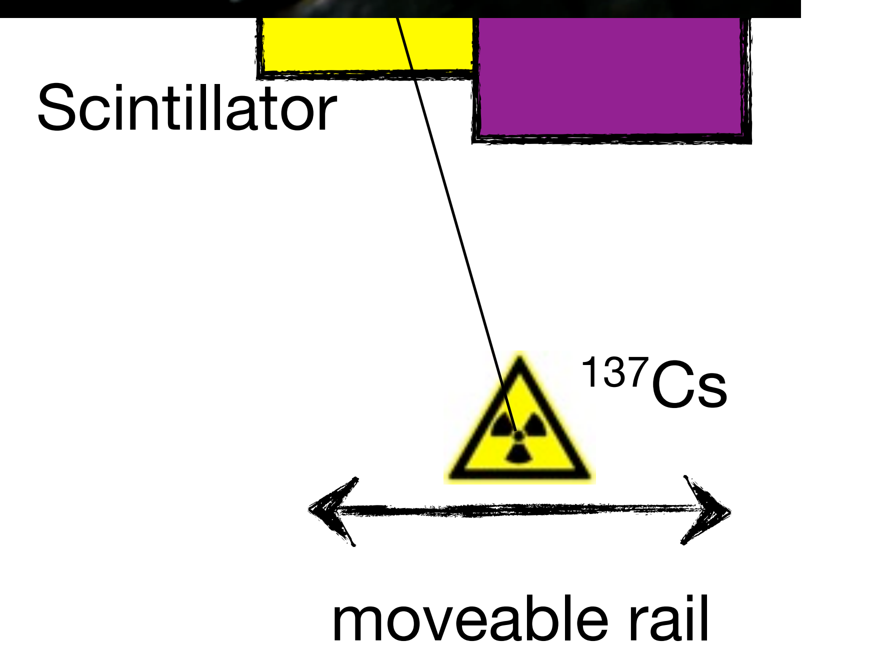
$$E_{e^-} = 662\,keV - E_{Ge}$$

- Coincidence trigger PMT and Ge-detector
- Measured energy range of electrons ~ 30 - 140 keV
- Thanks to Christoph Aberle and Stefan Wagner for the ability to use the setup
- Detailed setup description in [1]



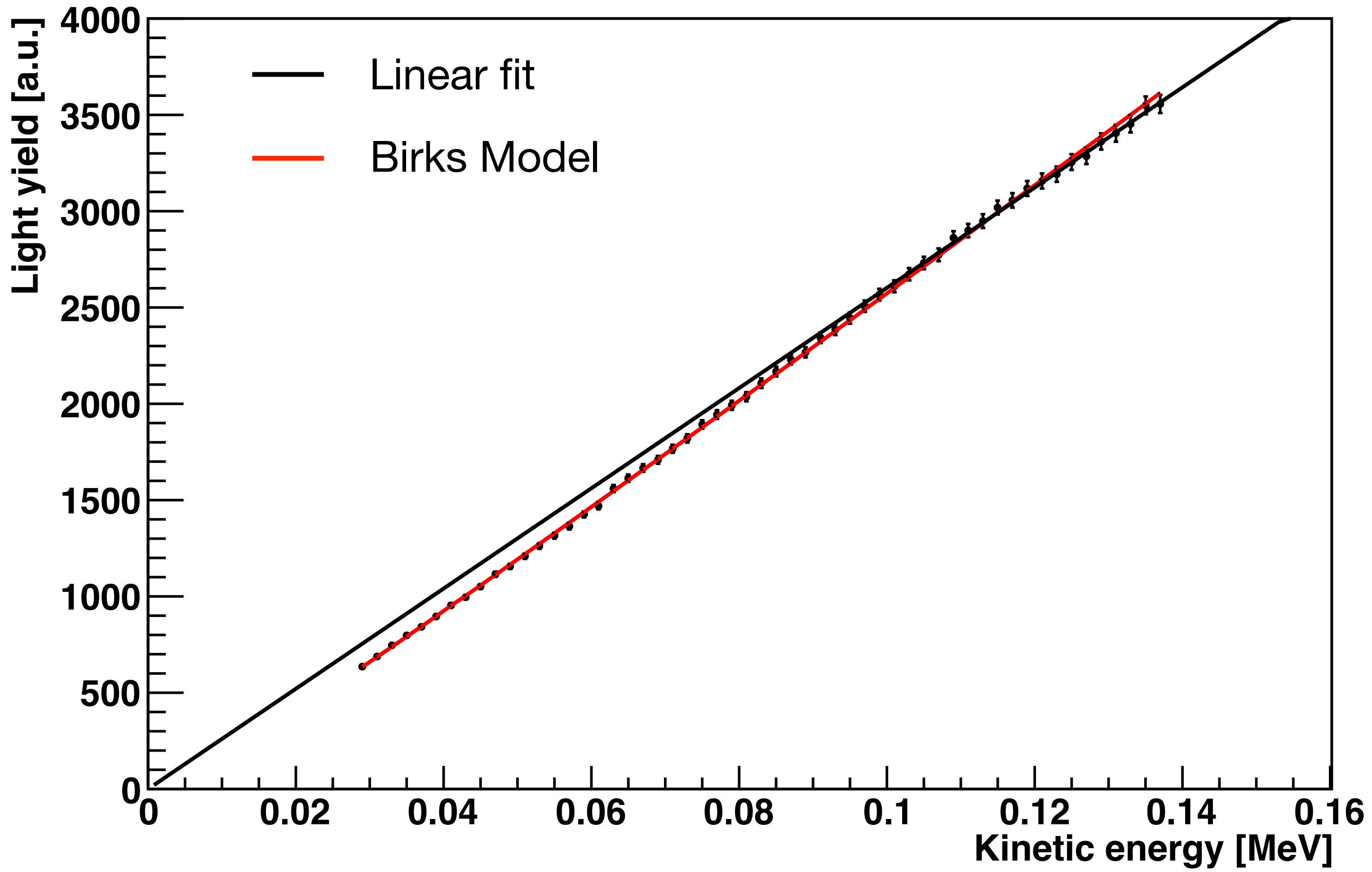
Experimental Setup (MPIK Heidelberg)

-
- res E
  n E_G
- d Ge
- ectrons
- and Stefan
  the setup
- [1]



kB Determination (standalone)

- **Exp. data:**
  Light-yield as function of electron energy
- **Fit calc. Light yield (Birks' formula)**
  to exp. data
  ➜ Best kB, S



Fit function:

$$LY = \int_0^R \frac{dL}{dx}(E)dx \approx \sum_{i=1}^{R/\delta x} \frac{dL}{dx}(E_i)\delta x$$

Range R: Total distance a particle can travel before it stops

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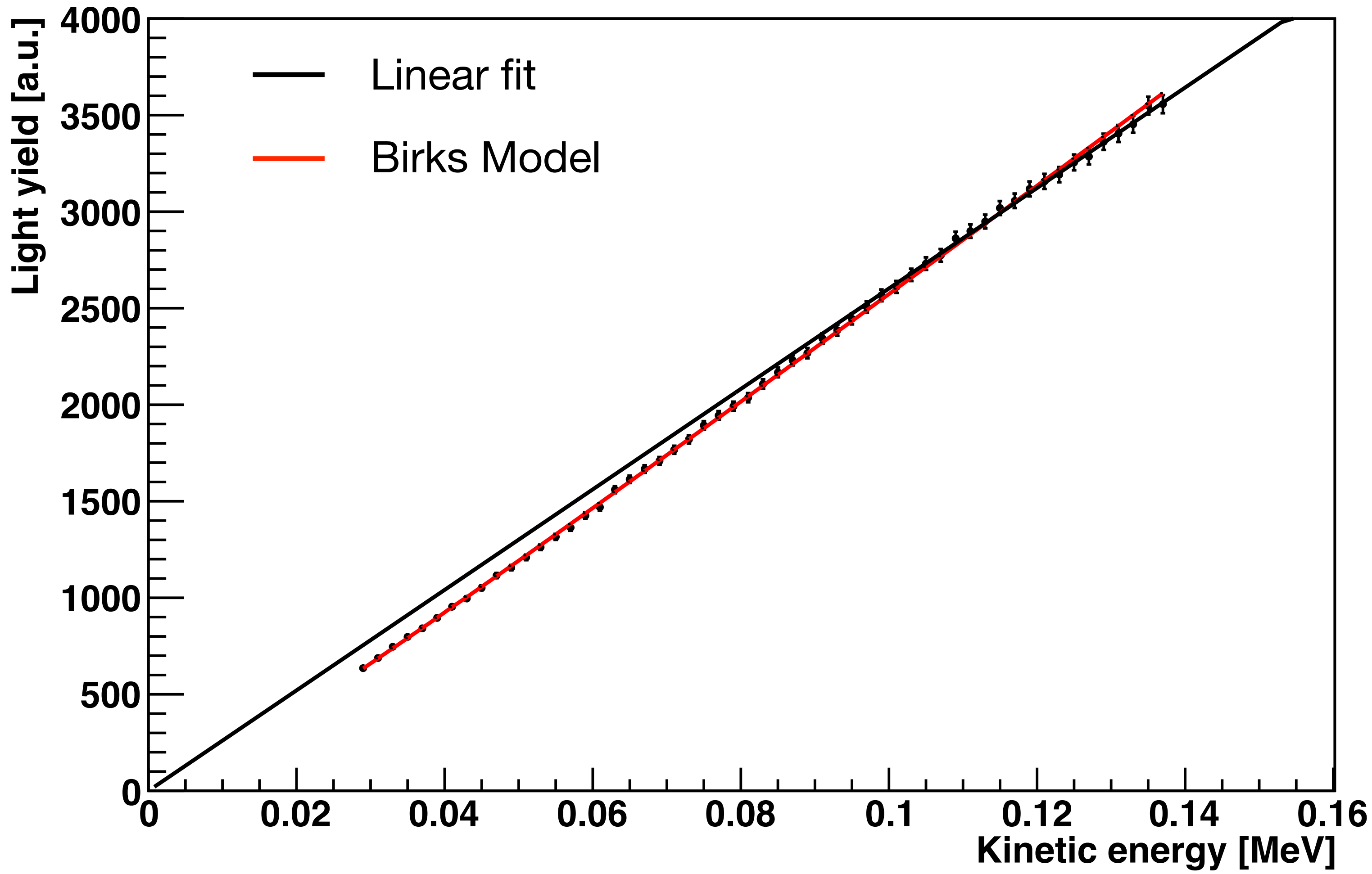

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size of δx important!

Range R: Total distance a particle can travel before it stops

Step-Size Influence

Fit function:

$$LY \approx \sum_{i=1}^{R/\delta x} \frac{S \cdot \frac{dE}{dx}(E_i)}{1 + kB \cdot \frac{dE}{dx}(E_i)} \delta x$$

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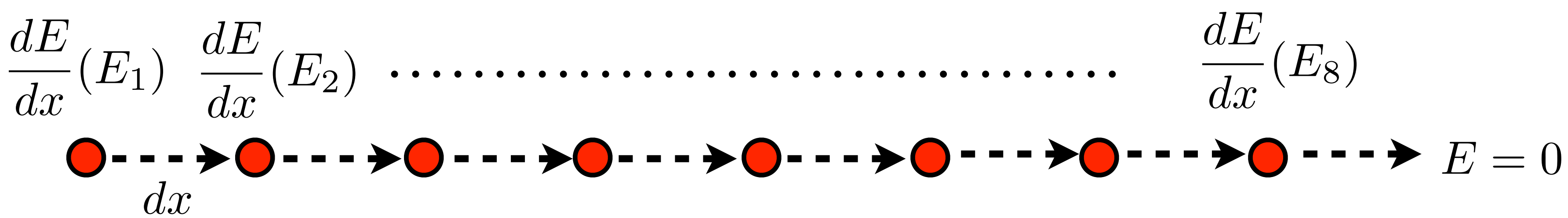

$\longrightarrow LY_1$

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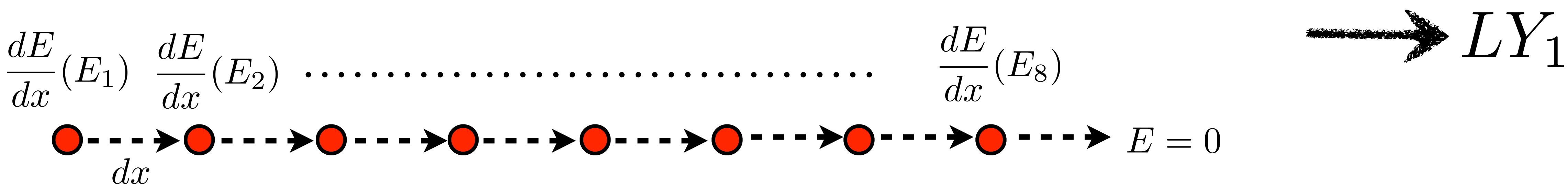
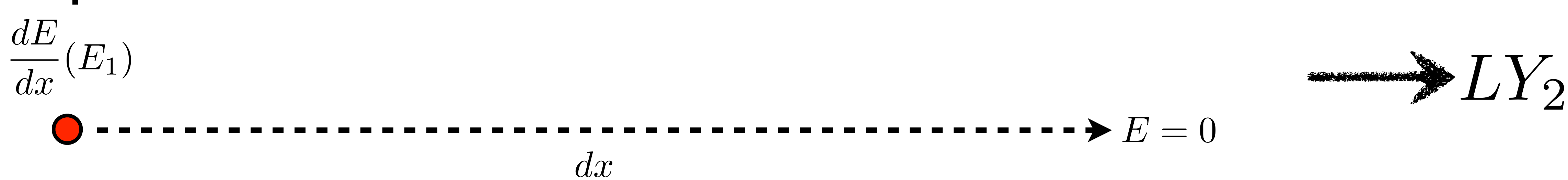


Long step-size:



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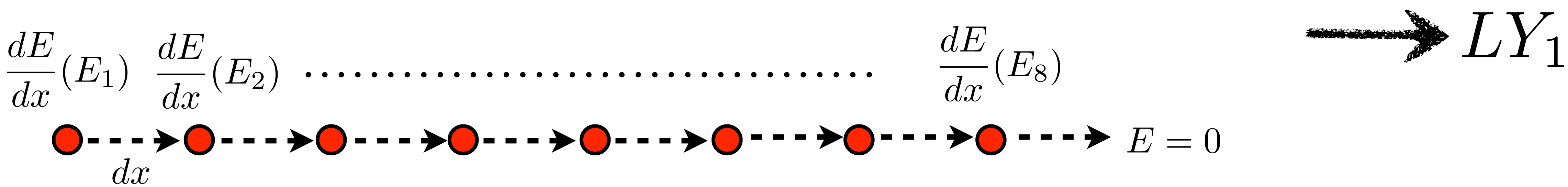

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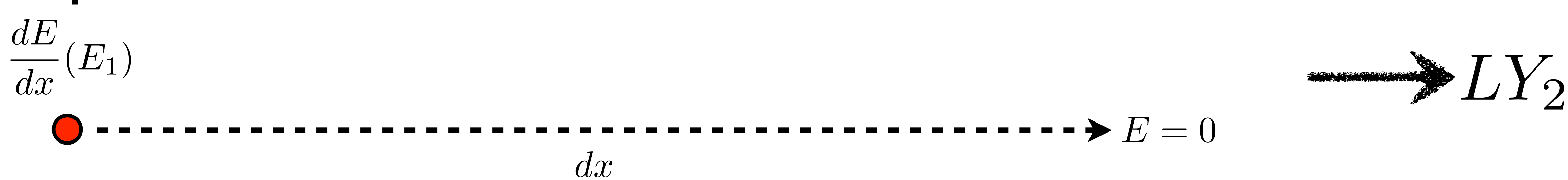

$$S_1 = S_2,\ kB_1 = kB_2$$

$$\longrightarrow LY_1 \neq LY_2$$

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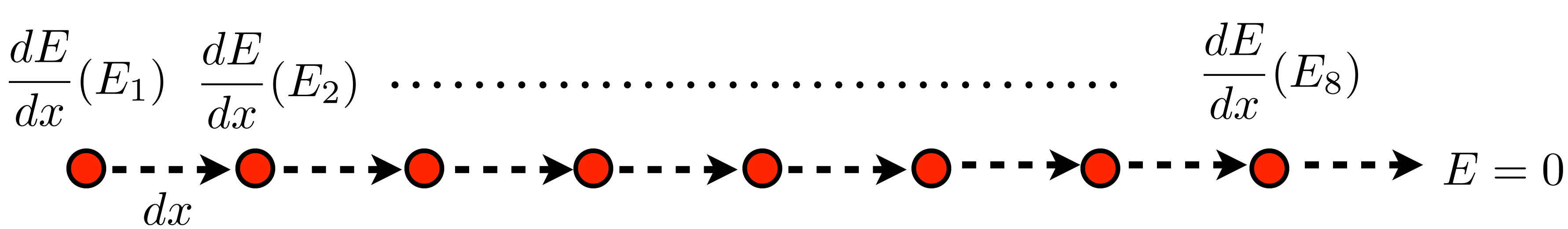

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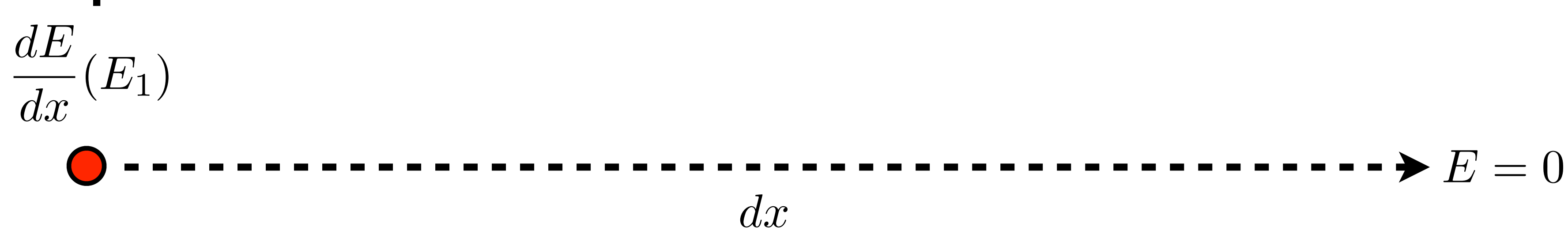

$\longrightarrow LY_2$

$$S_1 = S_2,\ kB_1 = kB_2$$

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$$\longrightarrow$$

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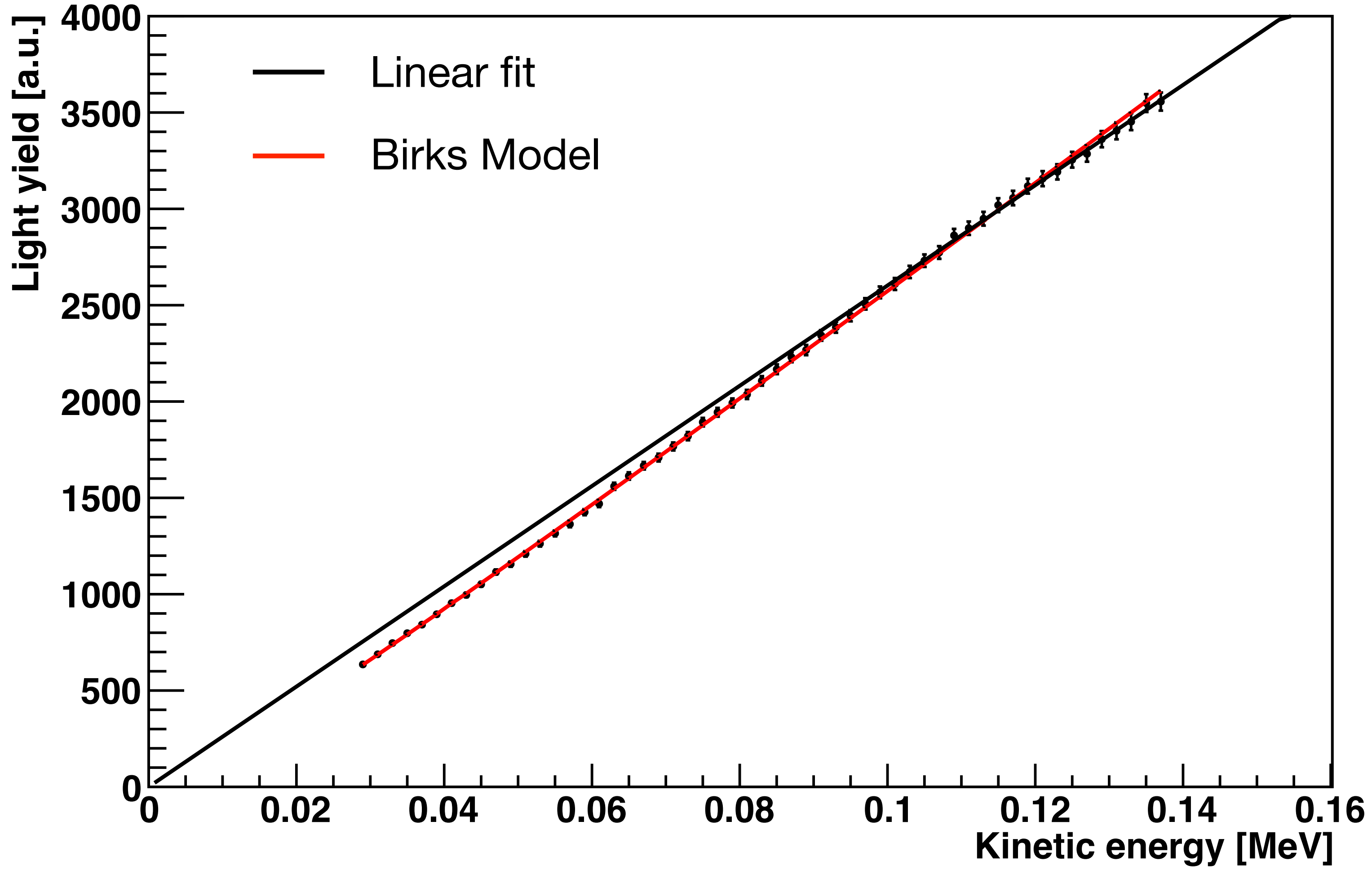
Measurement of kB

- Exp. data:
  Light-yield as function of electron energy
- Fit calc. Light yield (Birks' formula) to exp. data
  ➜ Best kB, S



$$LY_{exp} \stackrel{!}{=} LY \approx \sum_{i=1}^{R/\delta x} \frac{S \cdot \frac{dE}{dx}(E_i)}{1 + kB \cdot \frac{dE}{dx}(E_i)} \delta x$$

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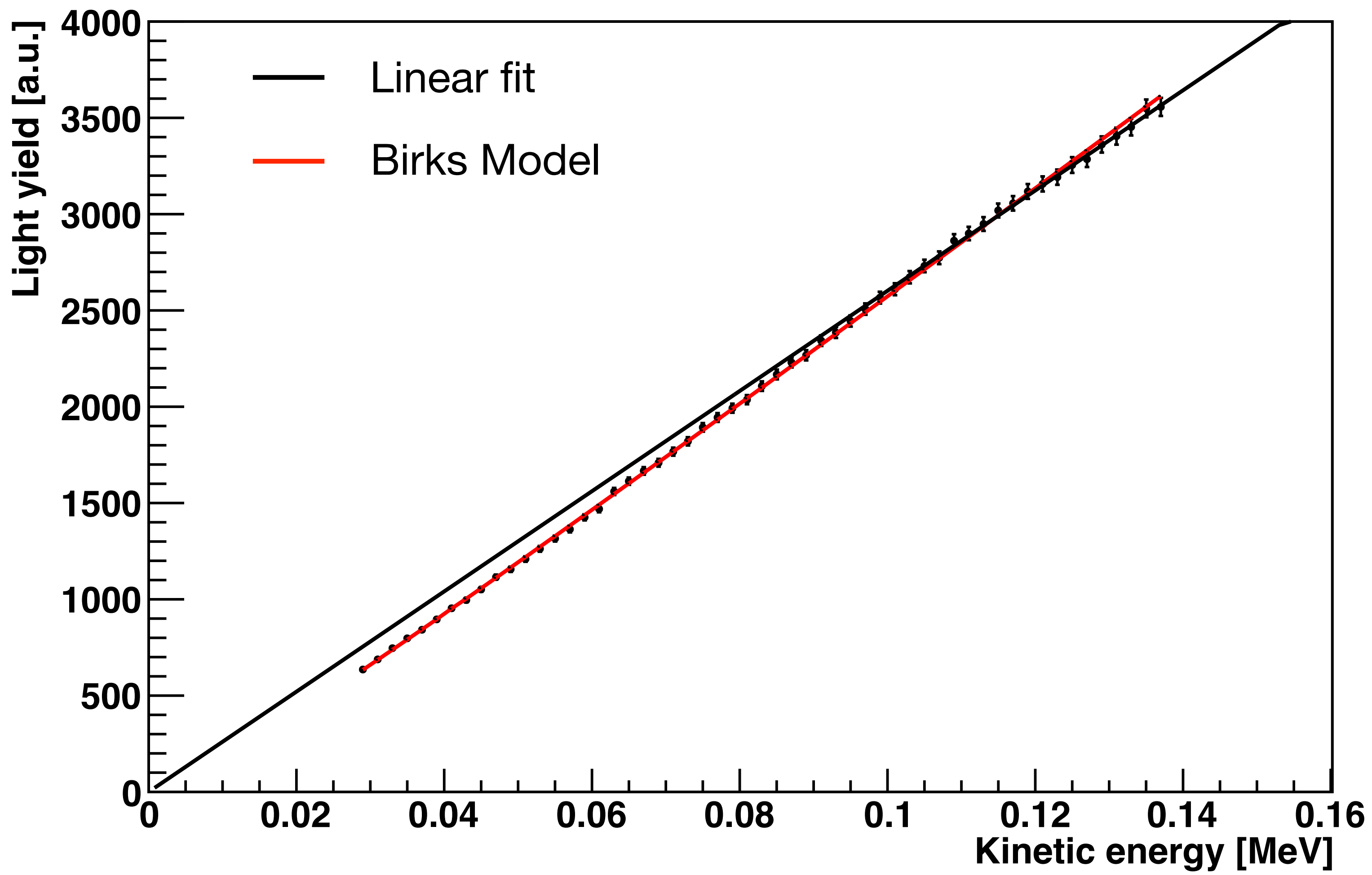

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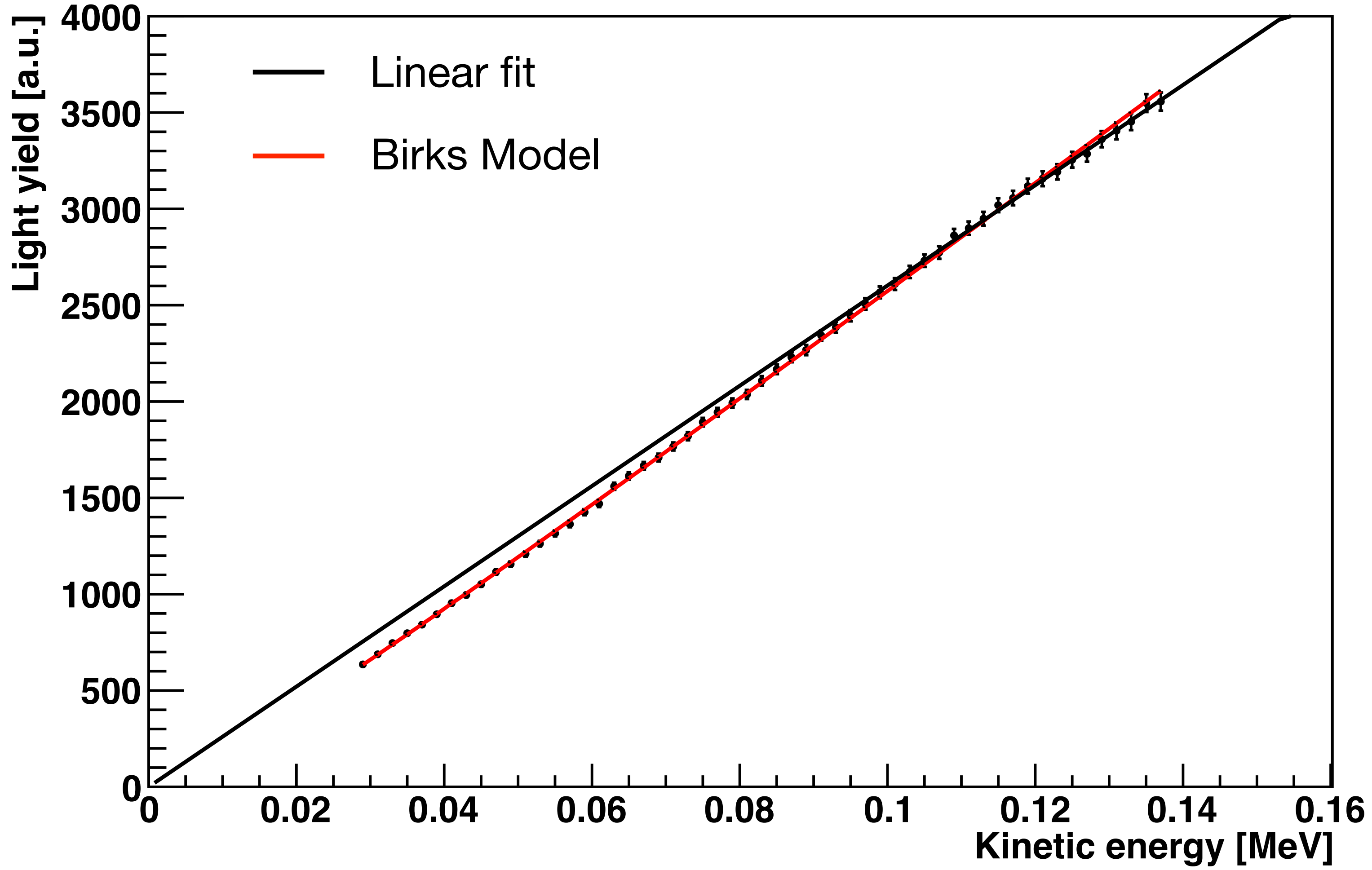
➜ Birks' coefficient kB and S depend on the step-size!

Measurement of kB

small step-size *(~1nm)*:
kB = 0.0151cm/MeV

Currently in GEANT4 [2]:
kB = 0.007943cm/MeV
Scintillator of ZEUS calorimeter



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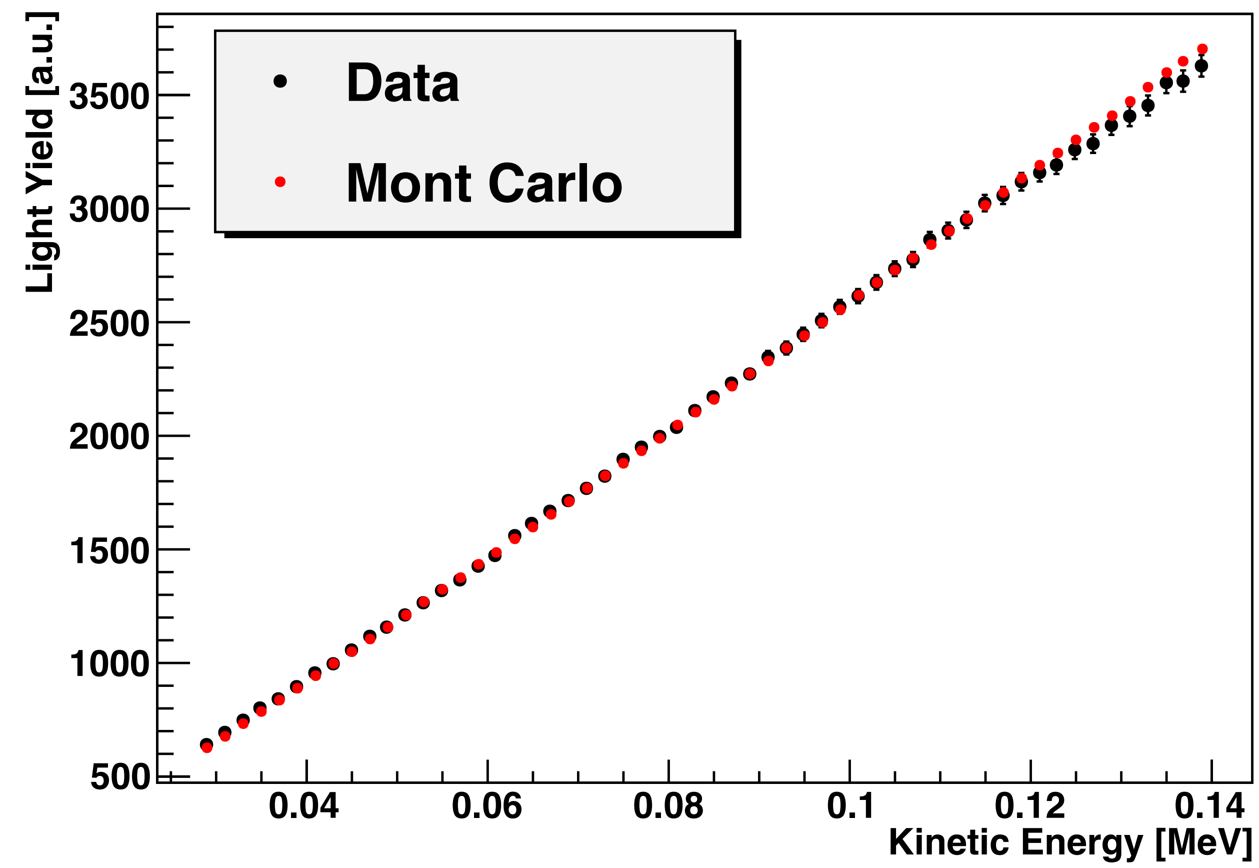
kB in GEANT4

$$LY \approx \underbrace{\sum_{i=0}^{R/\delta x} \frac{S \cdot \frac{dE}{dx}(E_i)}{1 + kB \cdot \frac{dE}{dx}(E_i)} \delta x}_{}$$

Replace by Monte Carlo

modified G4EmSaturation class



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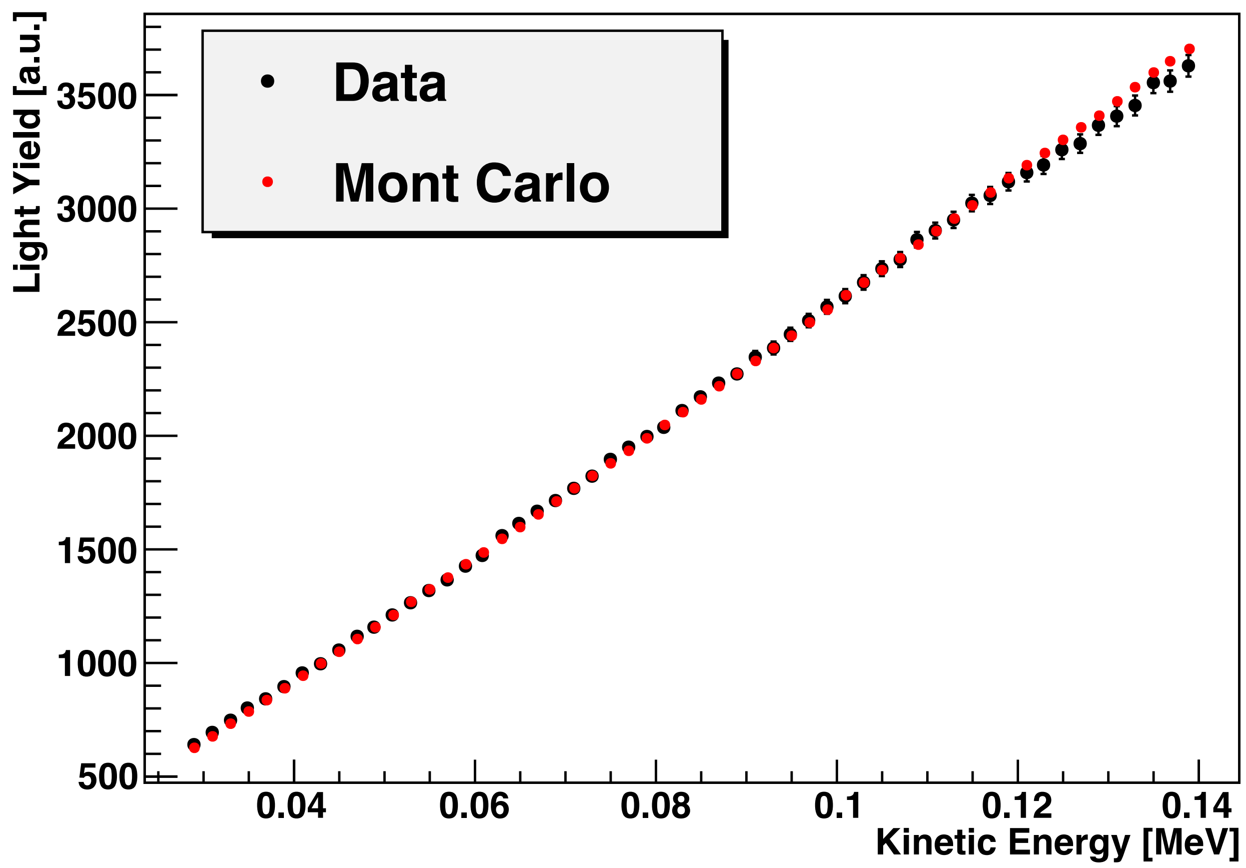
Best match between data and Monte Carlo ➜ S, kB



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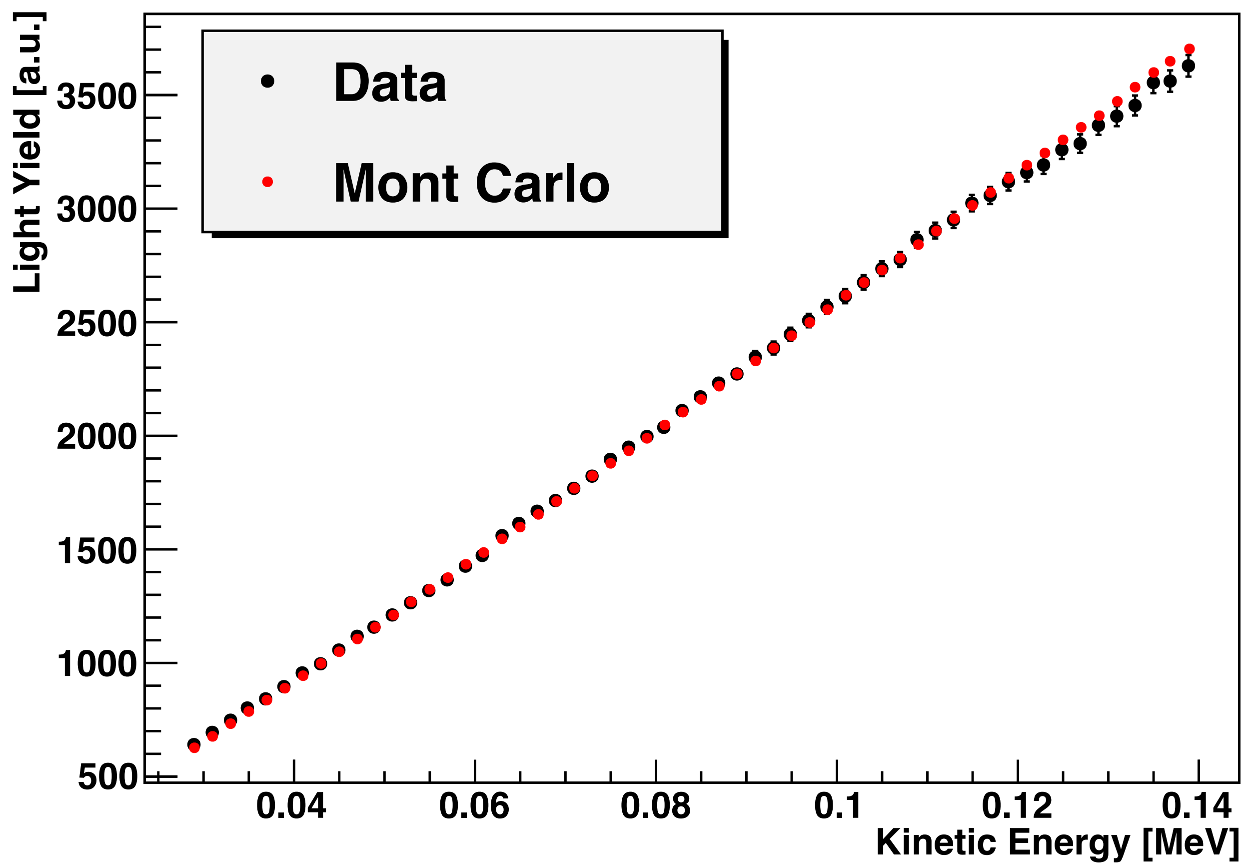


- Two things need special attention compared to “standalone” calculation
  - Particle step-size
  - Secondary particle production cut

Step-size in GEANT4

- Reduce computational effort
  ➜ Tradeoff between computation time and precision

- For ionization process, **Stepping function** determines the maximum step-size allowed Δx_{max}

$$\Delta x_{max} = \begin{cases} \alpha R + f(\rho, \alpha), & R > \rho \quad \text{(high energy)} \\ R, & R < \rho \quad \text{(low energy)} \end{cases}$$

  - ρ: Final range *(Default value: 1mm)*
  - α: dR/R *(Default value: 0.2)*

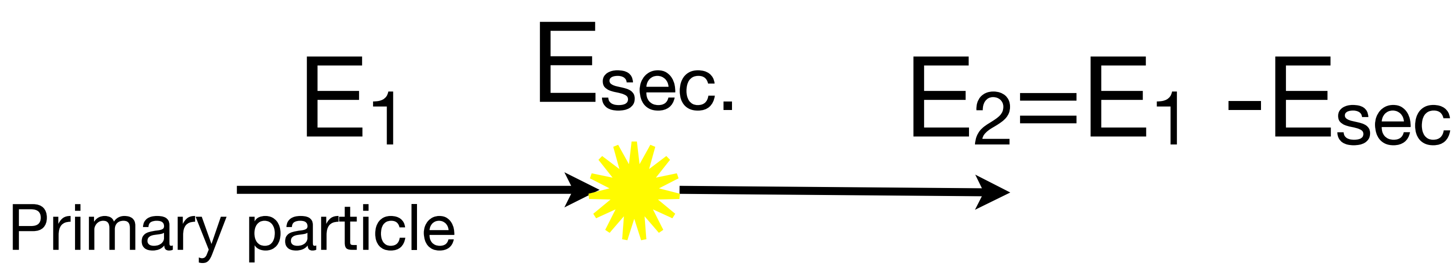
➜ Small values of ρ and α result in a small step size

Secondary production cut P_{cut}

- Secondary particles (delta-electrons) are simulated differently, according to their range at production $R_{sec.}$

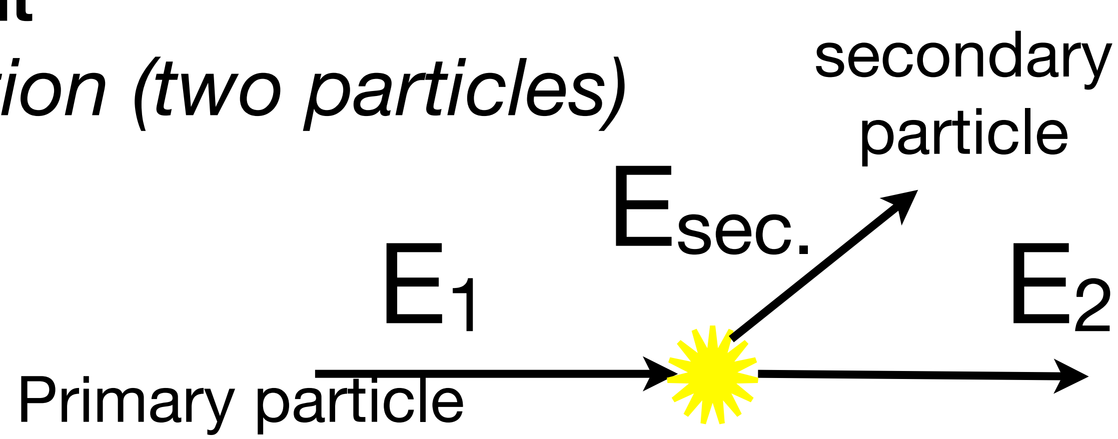
1. $R_{sec.} < P_{cut}$

<u>no</u> explicit generation (still one particle)



2. $R_{sec.} > P_{cut}$

explicit generation (two particles)



 secondary production point

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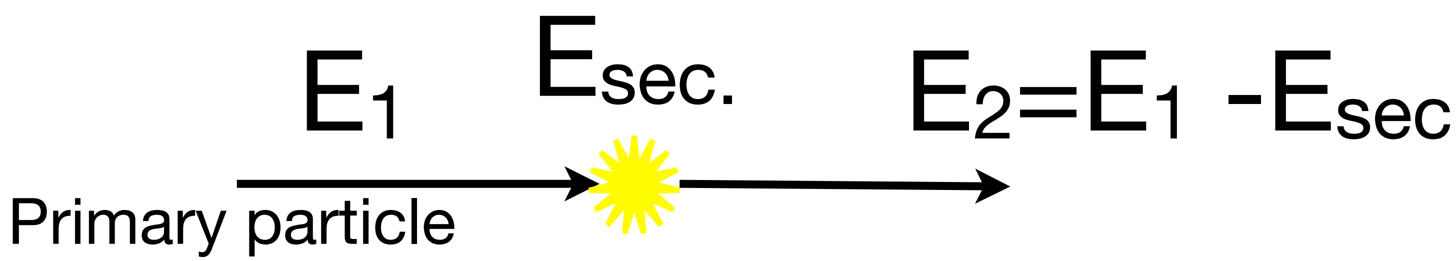

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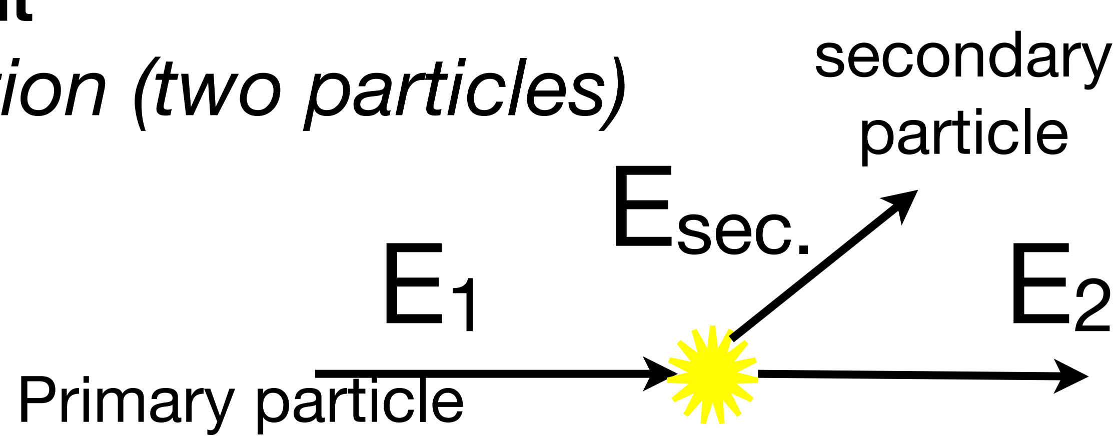


secondary production point

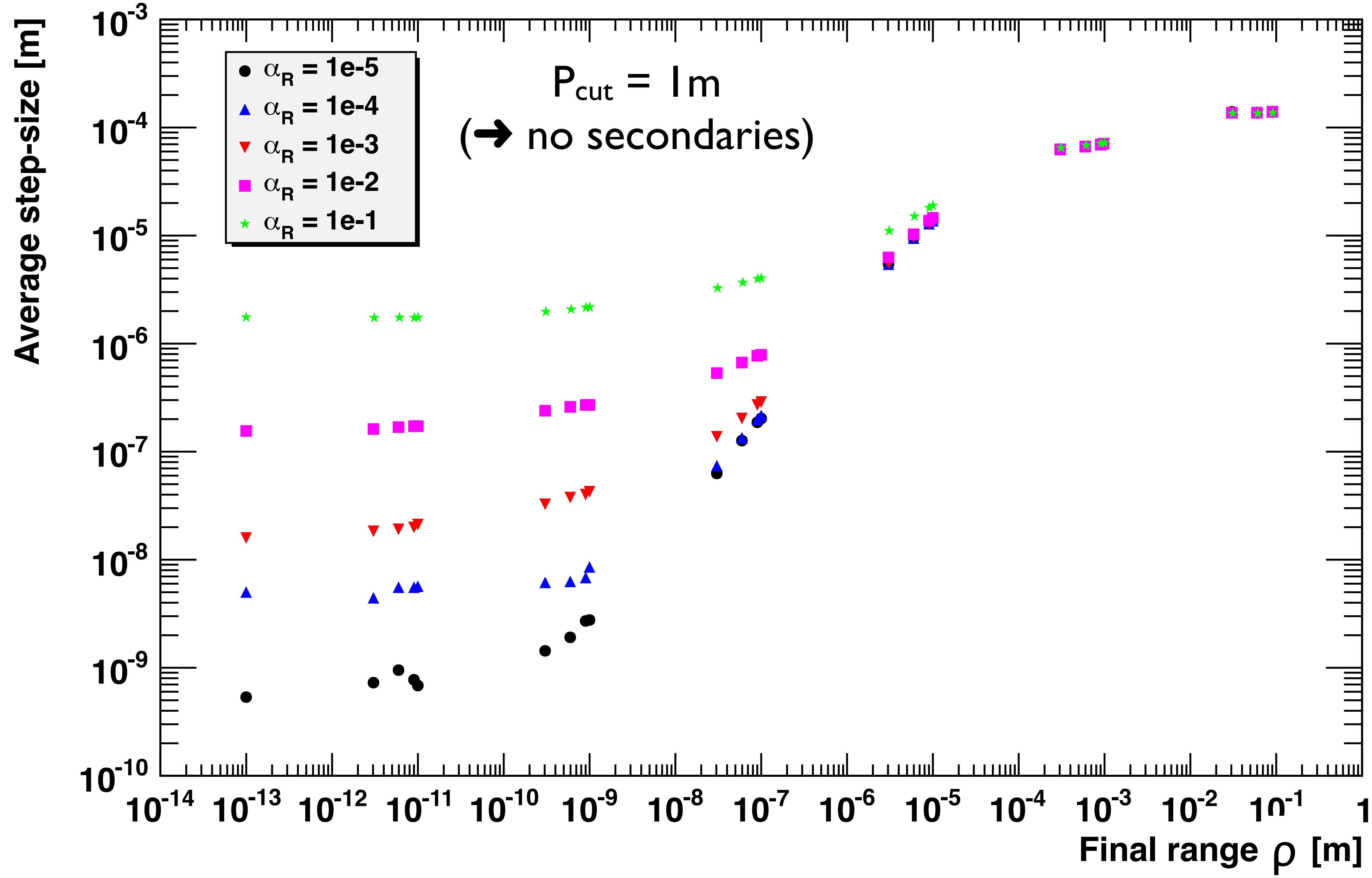
Large P_{cut} ➜ mostly <u>no</u> explicit secondary generation (single particle)

Small P_{cut} ➜ explicit secondary generation possible (multi particle)

➜ kB depends on P_{cut}!

Average step-size

Electron step-size averaged over simulation runs with specific ρ and α values



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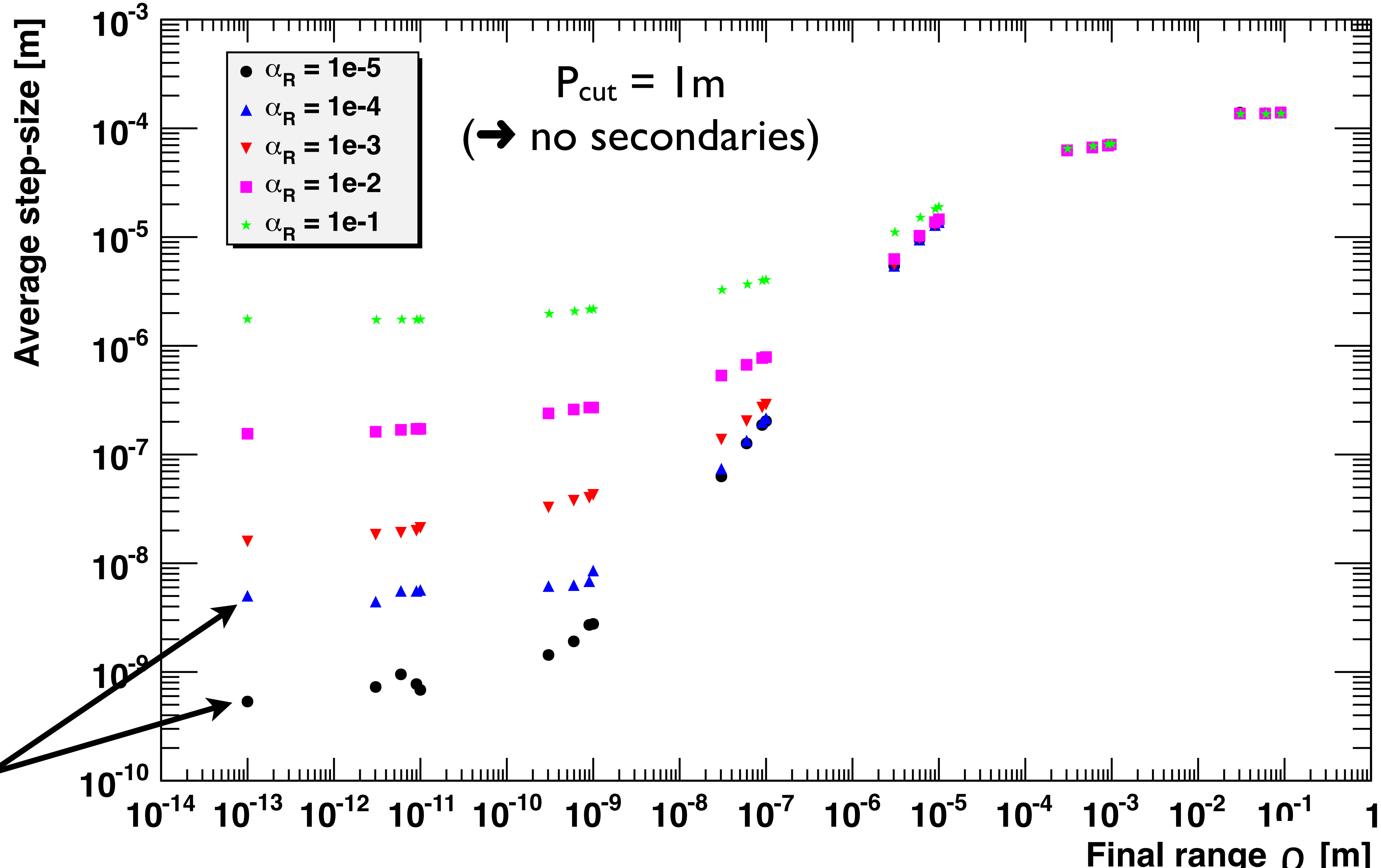
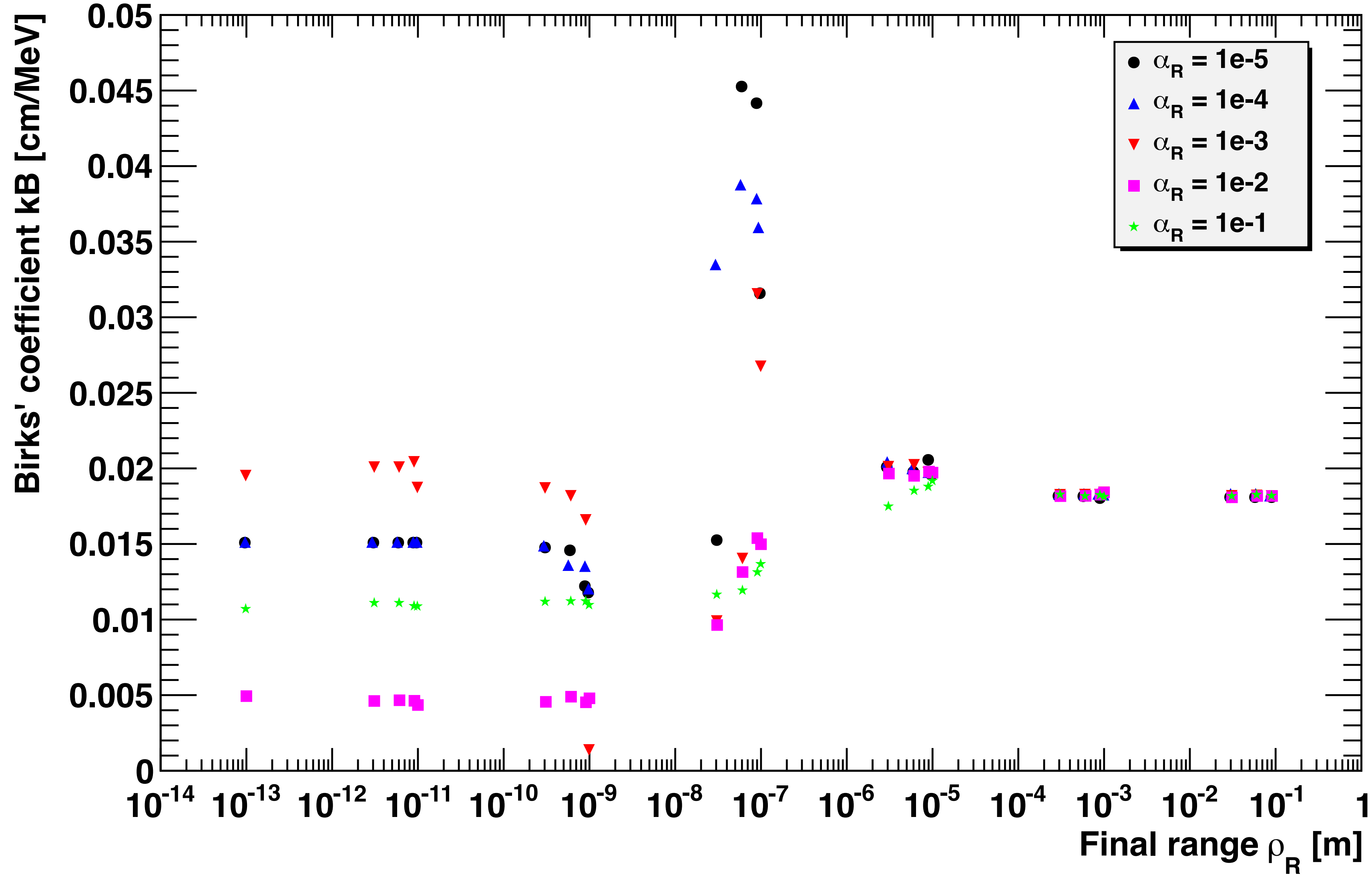


Comparable average step-size as in "standalone" calculation *(~1nm)*

kB with GEANT4

Secondary production cut p_{cut} = 1m

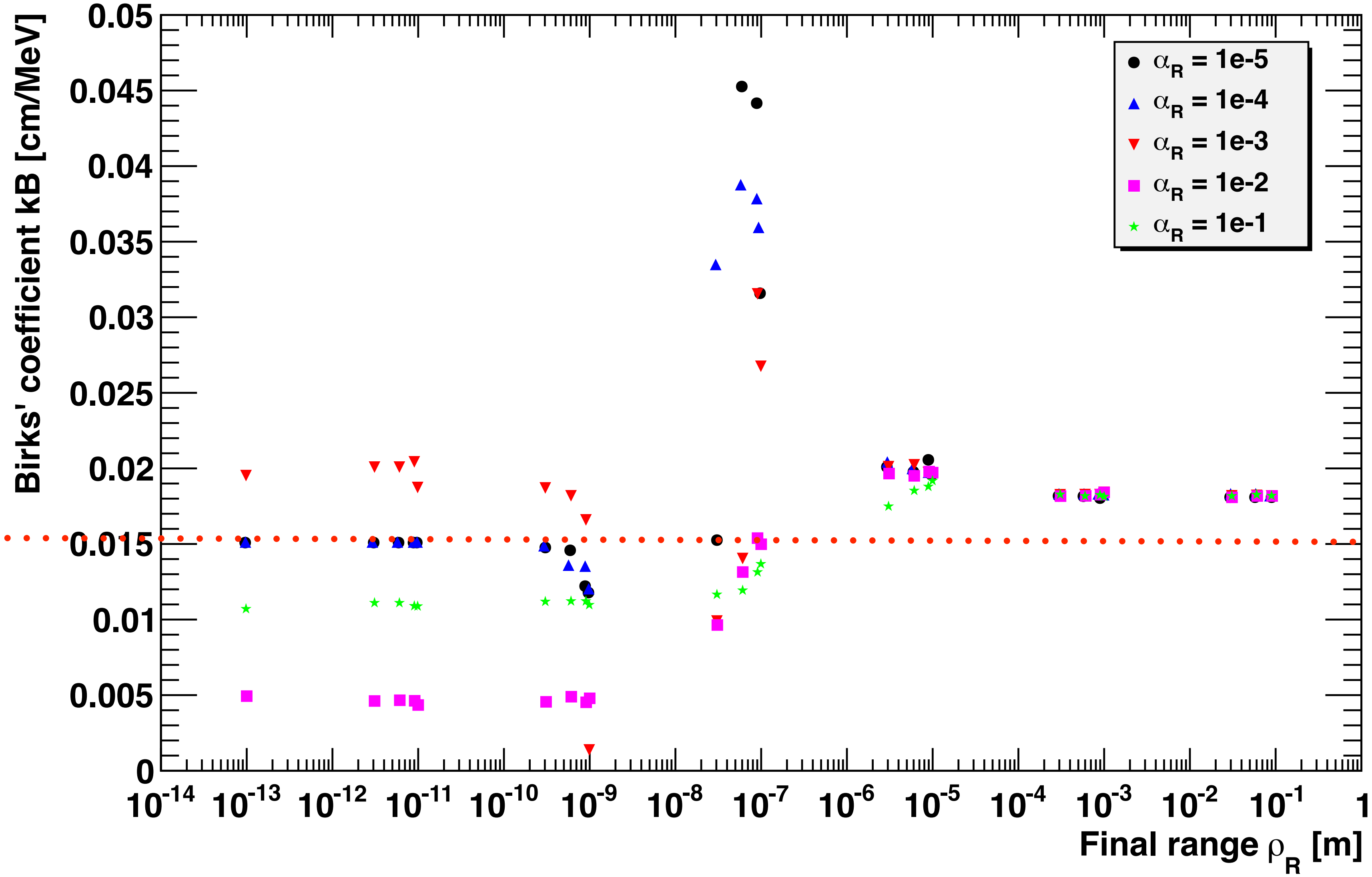


kB with GEANT4

Secondary production cut p_{cut} = 1m

Same result as "standalone" calculation for small values of ρ and α

kB = 0.0151 cm/MeV



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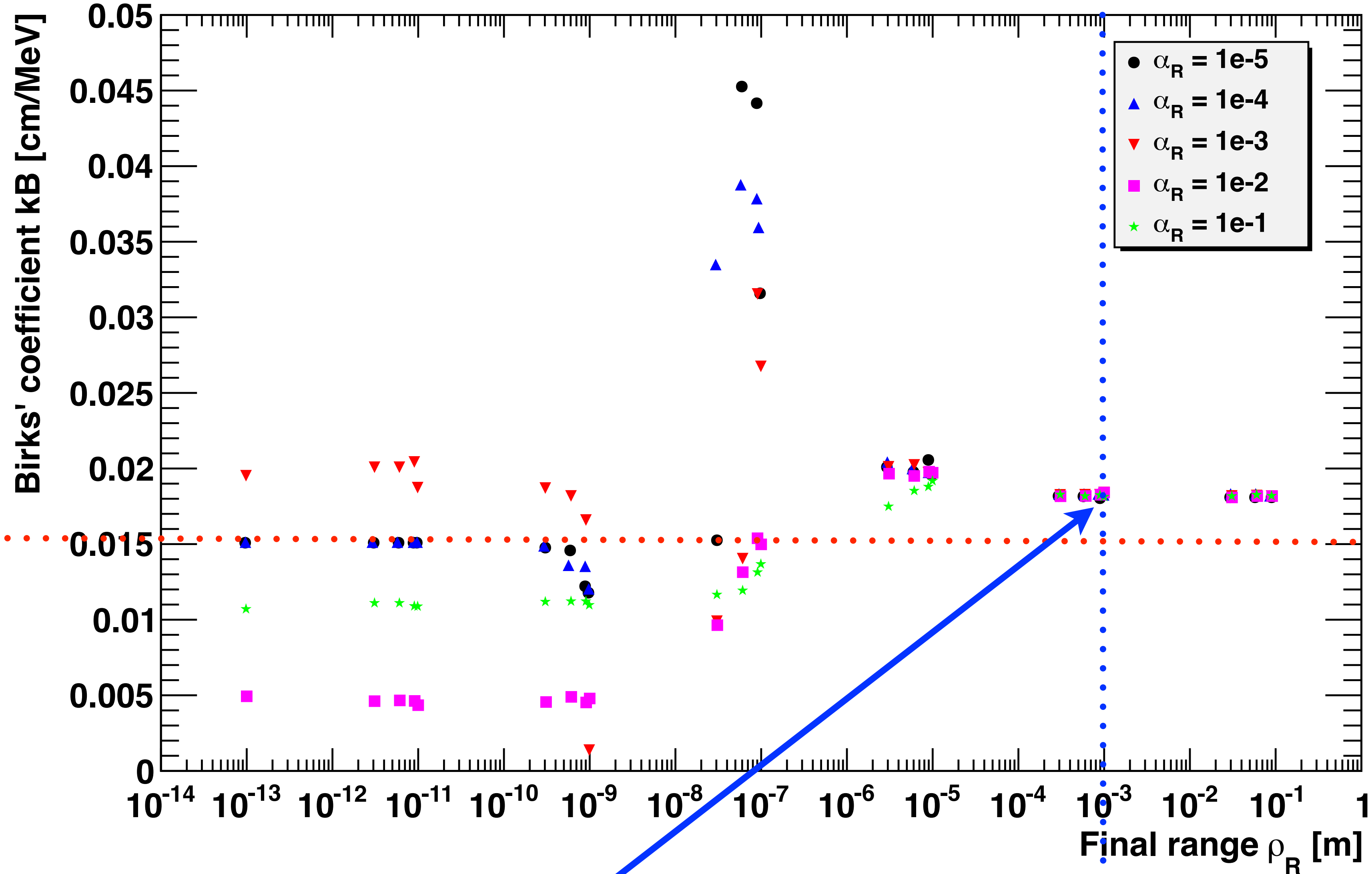
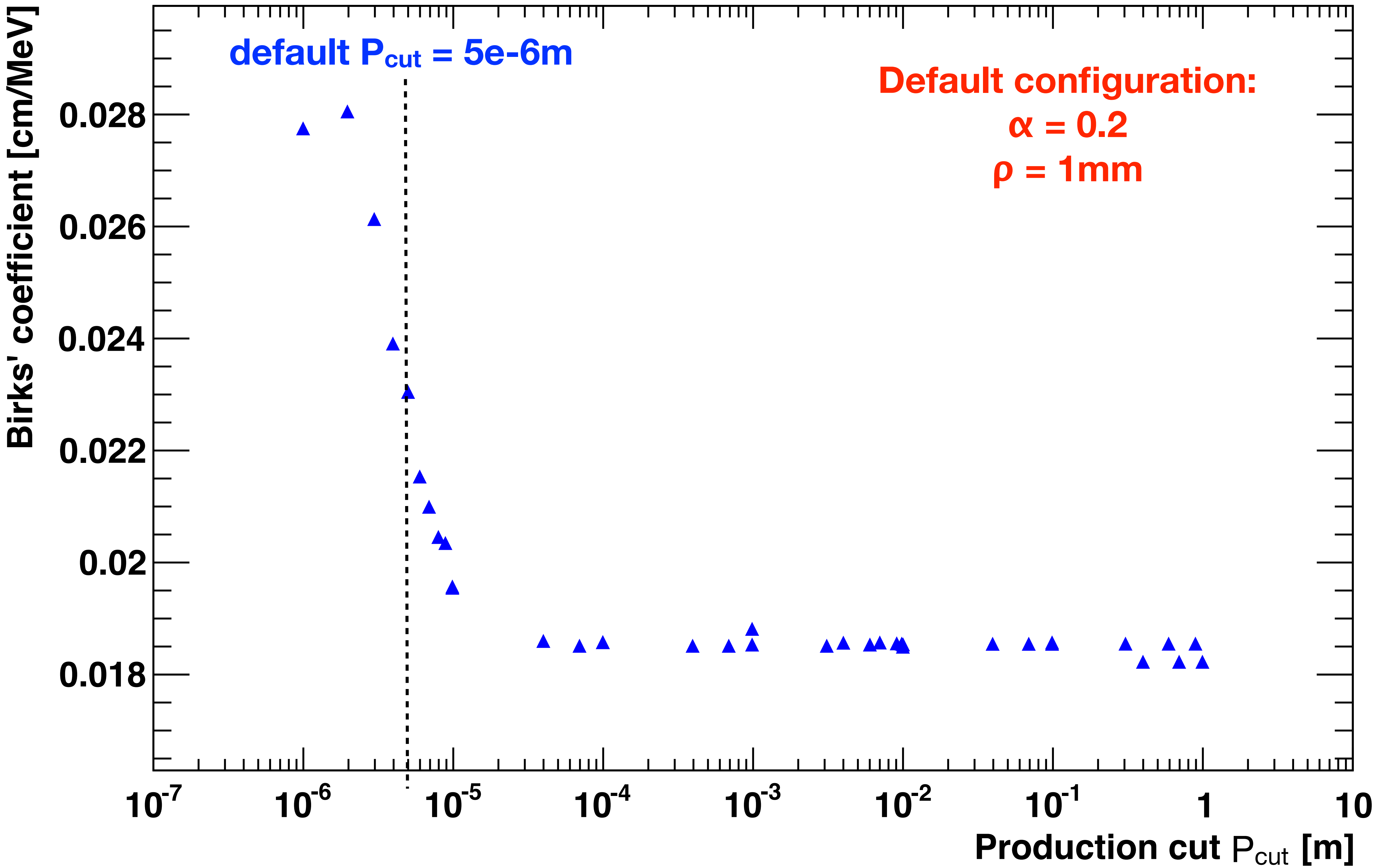


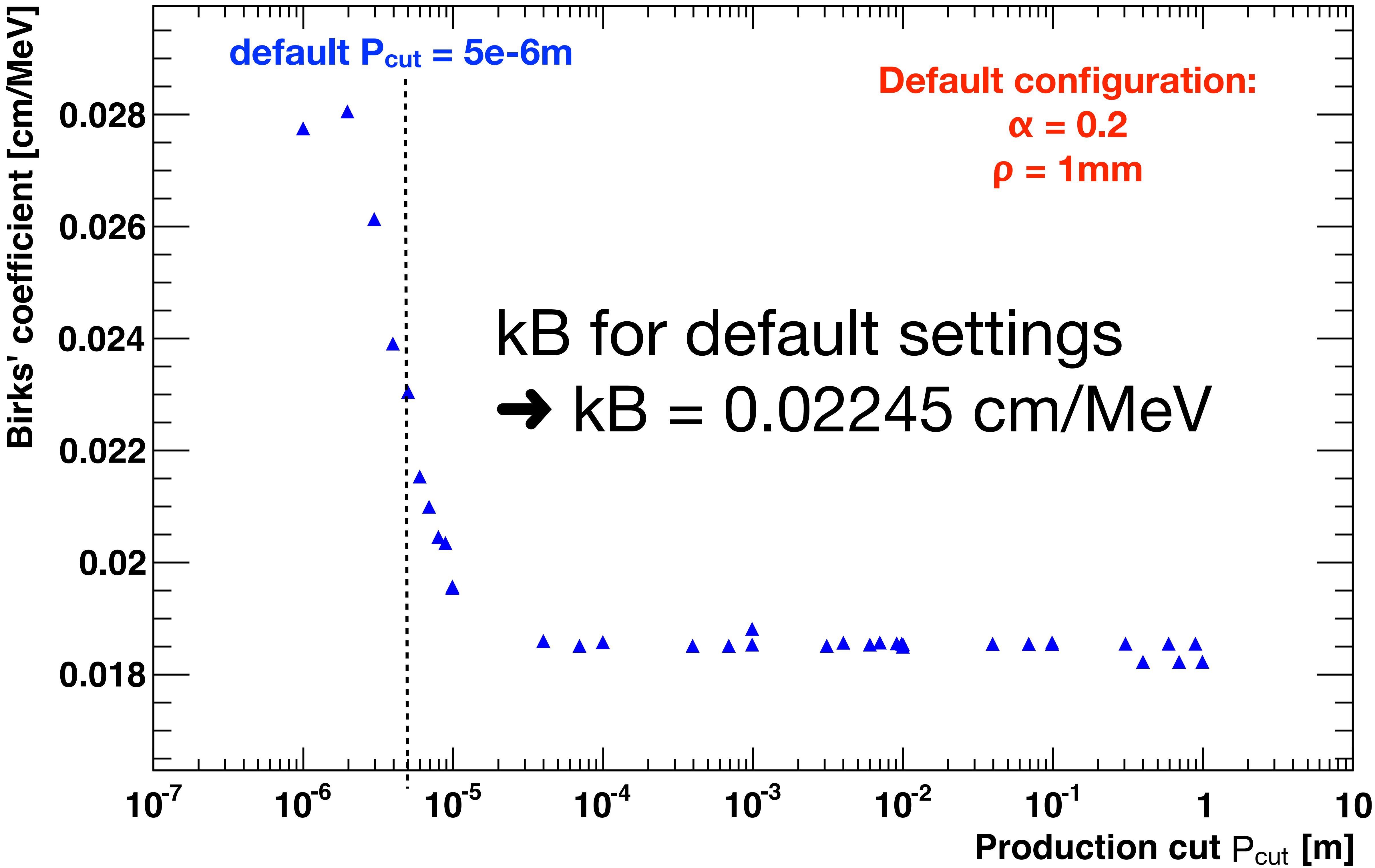
20 % higher kB value at default ρ:
kB = 0.018 cm/MeV

default ρ
(1mm)

Production Cut Dependence



Production Cut Dependence



kB for default settings

➜ kB = 0.02245 cm/MeV

Electron summary

“standalone”
calculation:
step size: 1nm

kB = 0.0151 cm/MeV

→

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“standalone” calculation:

step size: 1nm

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GEANT4/MOKKA:

step-size: ρ = *1mm*, α = *0.2*

\+

production cut: R_{cut} = 5µm

kB = 0.02245 cm/MeV

Electron summary

“standalone” calculation:

step size: 1nm

kB = 0.0151 cm/MeV

GEANT4/MOKKA:

step-size: ρ = *1mm*, α = *0.2*

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production cut: R_{cut} = 5μm

kB = 0.02245 cm/MeV

180% larger than current value!

(Currently in MOKKA: 0.007943 cm/MeV)

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“standalone” calculation:

step size: 1nm

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step-size: ρ = *1mm*, α = *0.2*

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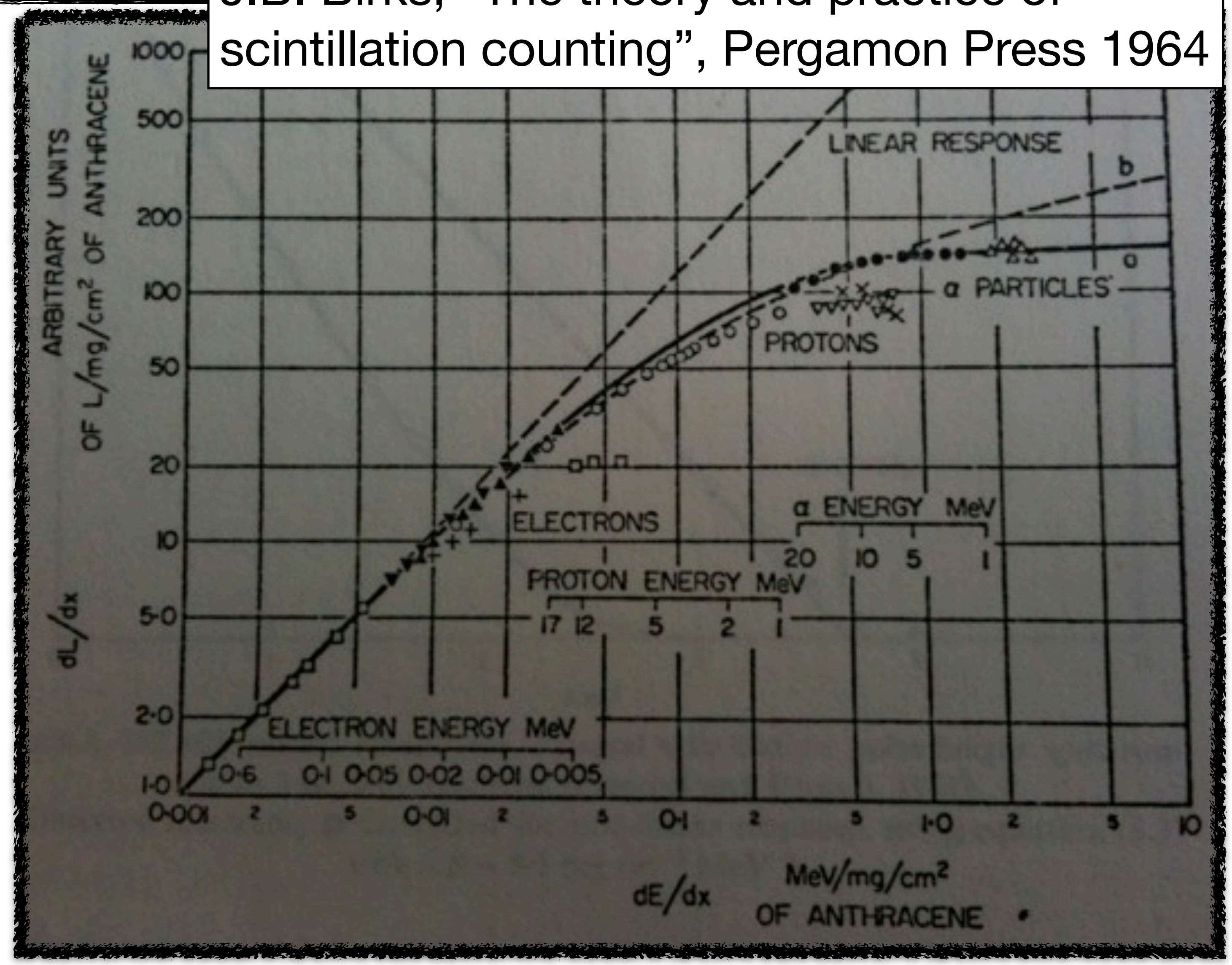
What about other particles?

Birks' coefficient for other particles

J.B. Birks, "The theory and practice of scintillation counting", Pergamon Press 1964



- Currently, only measurement for electrons
- Reasonable assumption:
  The value of Birks' coefficient is identical for all particles
  if it is calculated "correctly"
- Correctly means:
  - Small step-size δ_x
  - Large range cut P_{cut}

➡ *Assumption for all particles:*
kB = 0.0151 cm/MeV

➡ Transform kB values for usage in GEANT4/MOKKA

Most of experimental data fits to solid line **a** (Birks' Formula).

$$\frac{dL}{dx} = \frac{S\frac{dE}{dx}}{1 + kB\frac{dE}{dx}}$$

Data generation

Particle type *(e.g. proton)*

kB = 0.0151 cm/MeV

S = 29807 a.u.

GEANT4

small step-size (α, ρ)

large production cut P_{cut}

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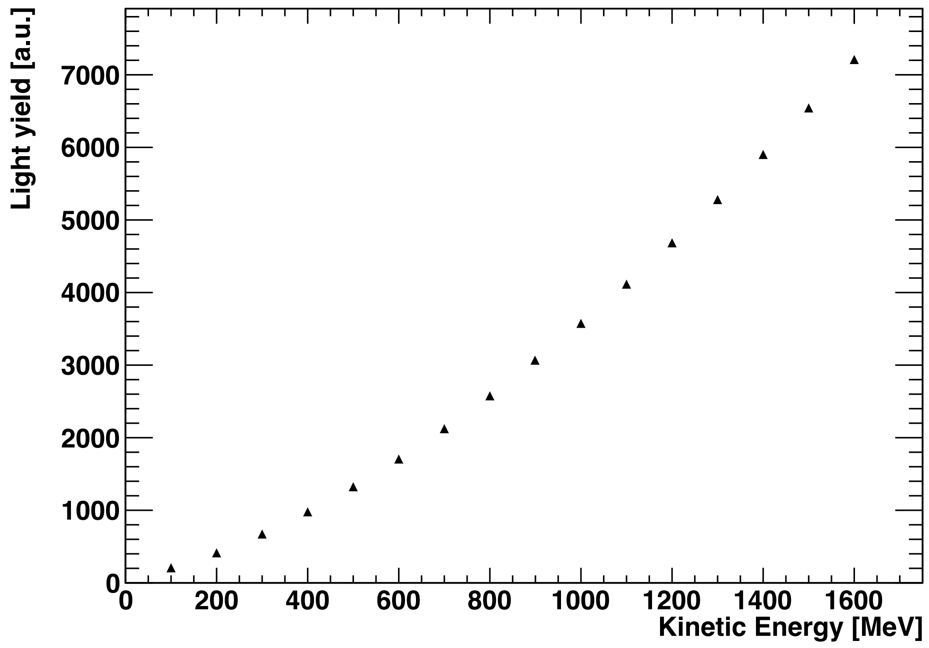
"artificial" data set



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GEANT4
small step-size (α, ρ)
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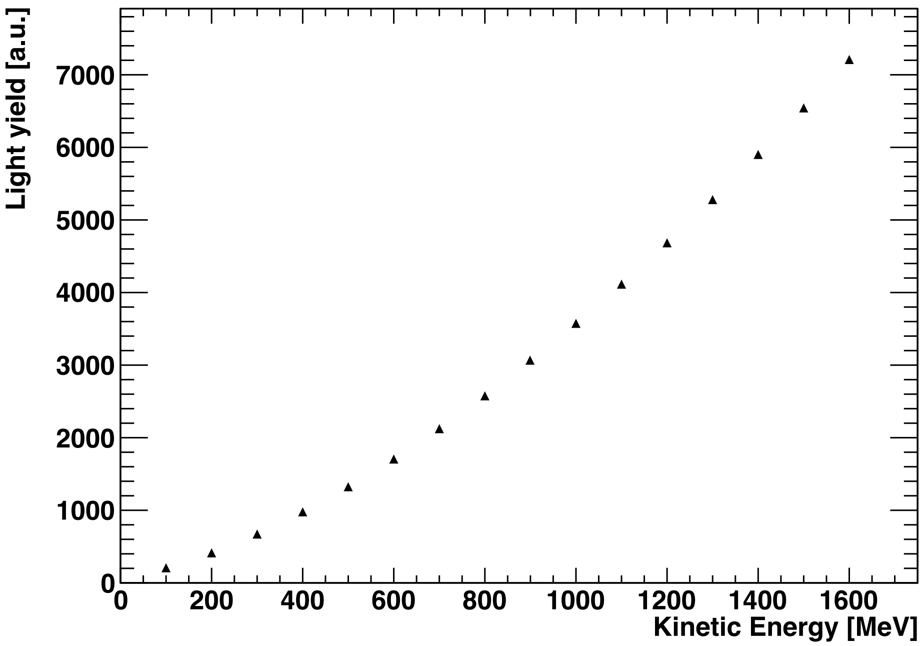
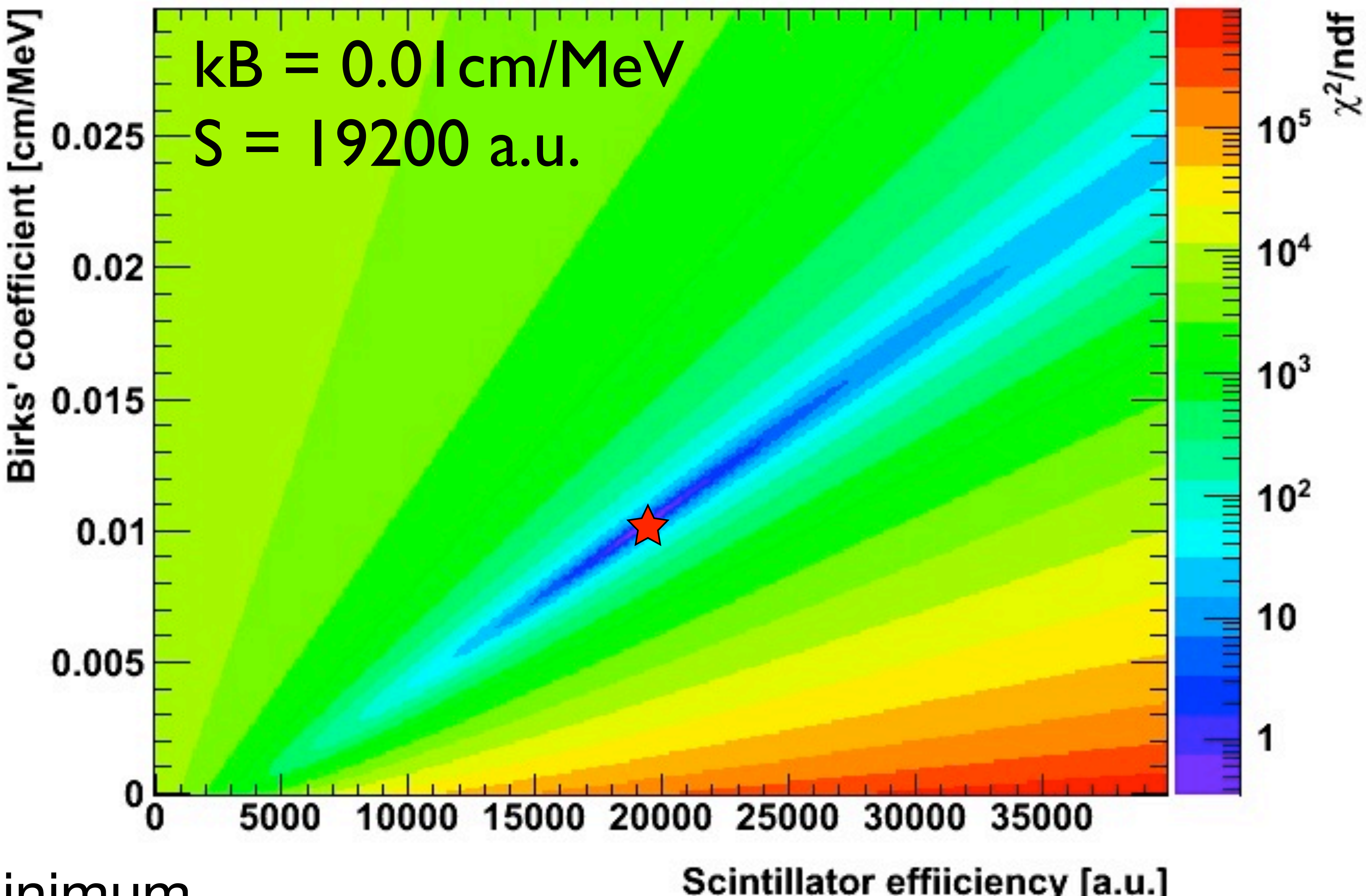


GEANT4
default α, ρ, P_{cut}

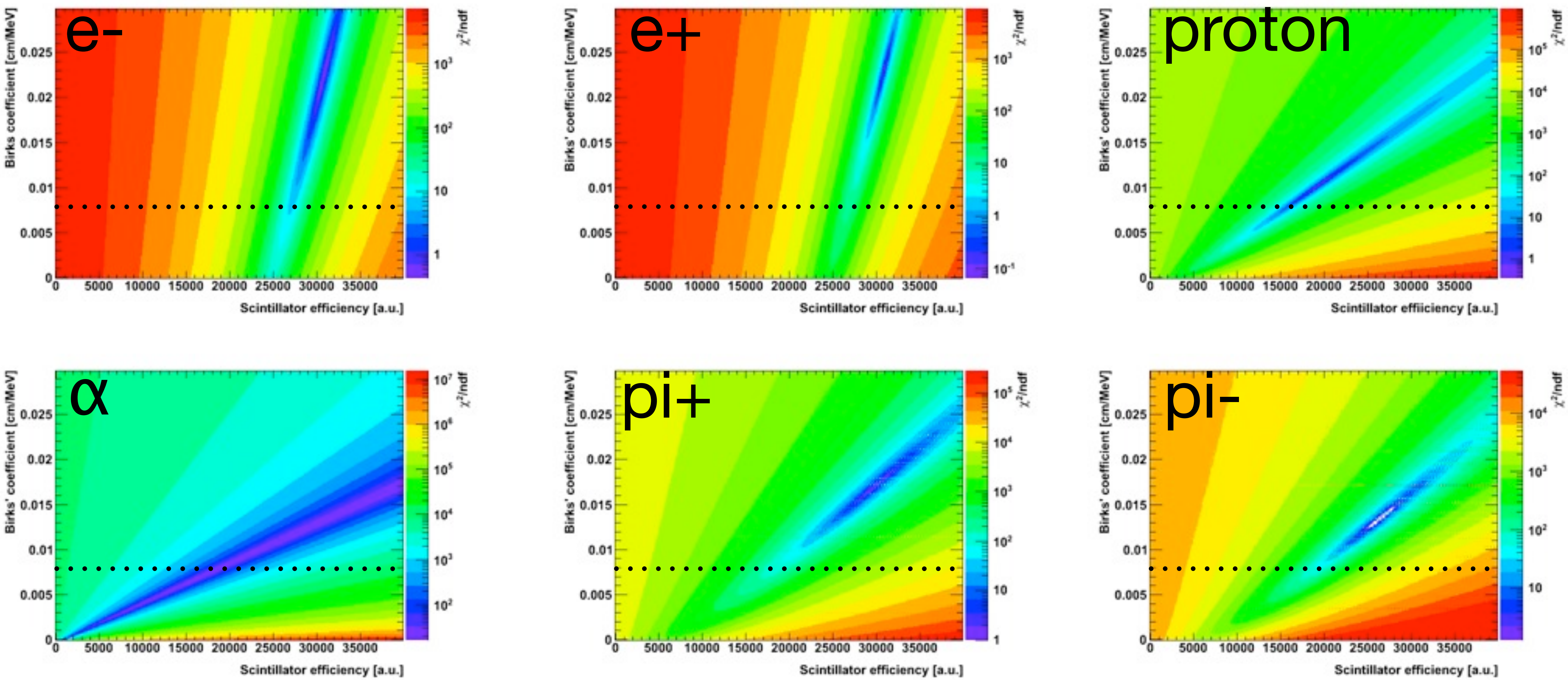
chi squared distribution

Proton chi-squared distr.



★ Minimum

Chi-squared distributions



Different possibilities to combine results:

- *Common kB, S for all particles*
- *Particle specific kB, but common S*

······· current kB
0.007943cm/MeV

Summary of Birks' Coefficients

		e^-	e^+	proton	alpha	pi+	pi-
kB [cm/MeV]	**small step-size**	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151
S [1/MeV]		29807	29807	29807	29807	29807	29807
kB [cm/MeV]	**Mokka default** R_{cut} = 5e-6m	0.02245	0.023	0.0100	0.004*	0.017	0.014
S [1/MeV]		30852	30925	19200	9327*	30244	26586

* no clear minimum of chi^2 detectable (strong correlation between S and kB)

Conclusion & Outlook

- Birks' coefficient measured with electrons
- kB value needs to be adapted to step-size and production cut in GEANT4
- Assumption of common kB at small step-size and large cut allows to determine GEANT4 kB values of other particles
- To do
  - Possible solutions
    - common kB, S (put "average" kB into GEANT4)
    - common S, but particle specific kB (some "small" modifications neccessary)
  - Study impact of change in kB

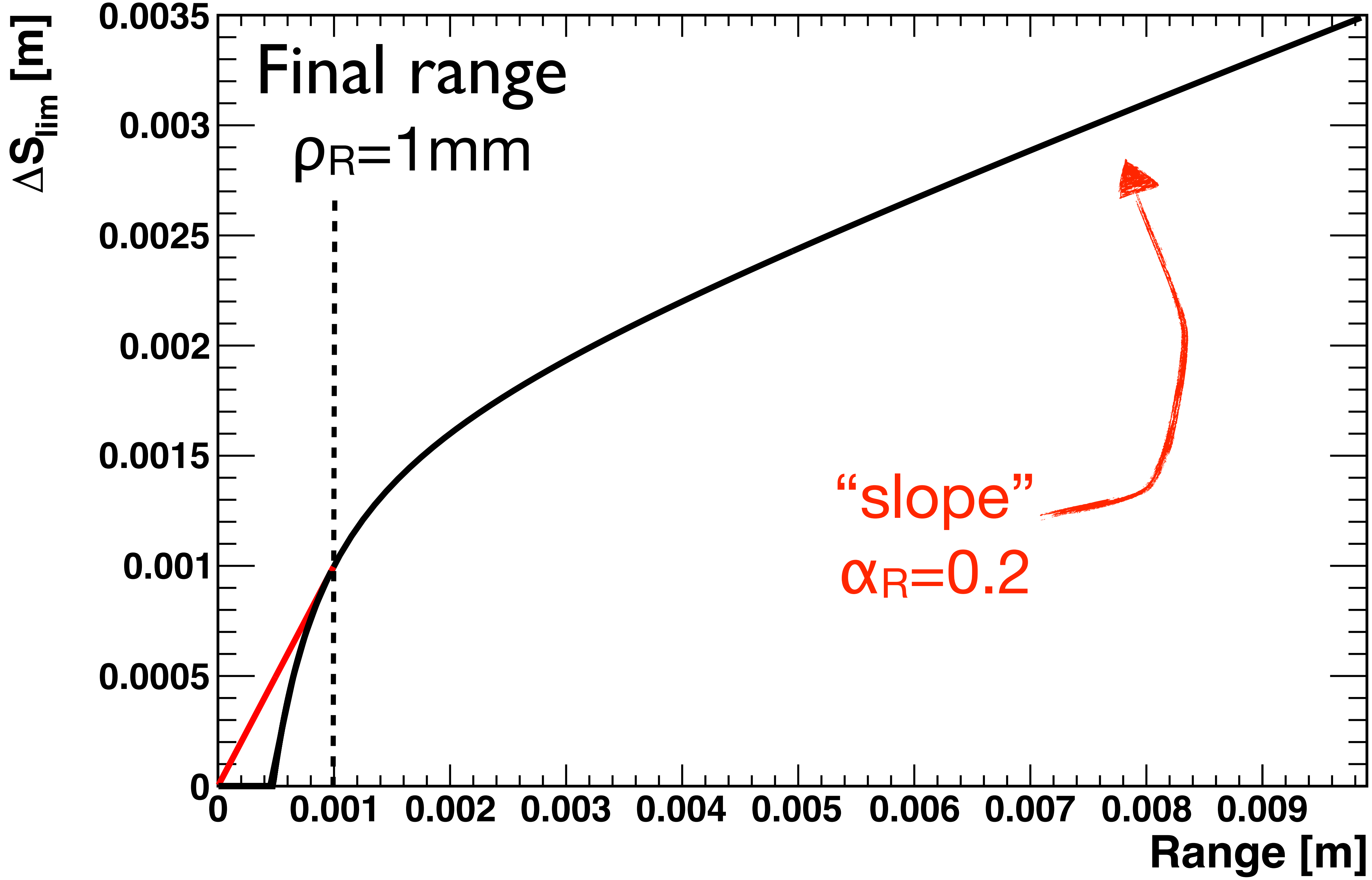
Thank you for your attention!

References

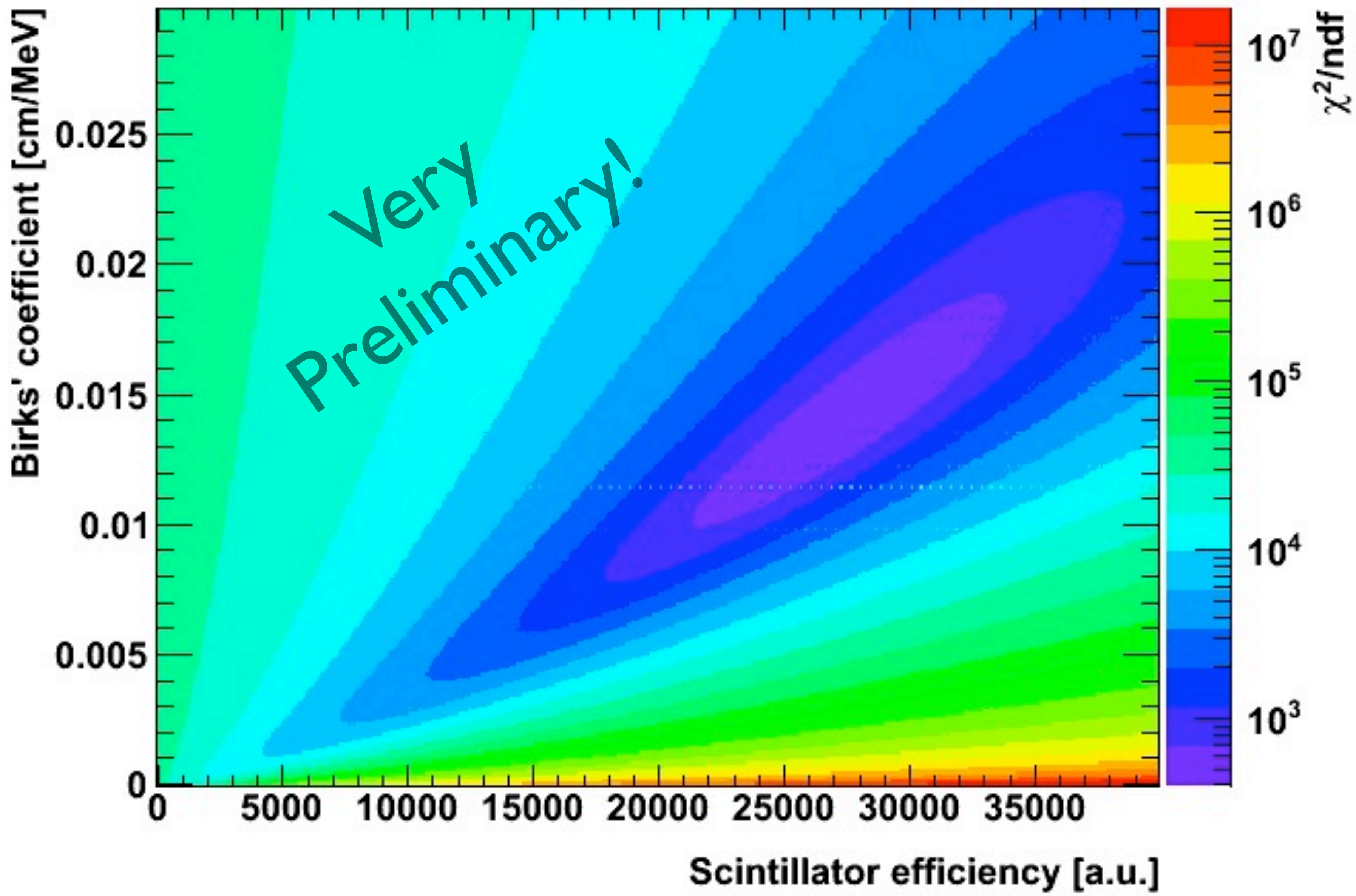
- [1] Stefan Wagner, "*Ionization Quenching by Low Energy Electrons in the Double Chooz Scintillators*", Diploma Thesis (2010)
- [2] M. Hirschberg et. al. *"Precise Measurement of Birks kB Parameter in Plastic Scintillators", IEEE Trans. Nucl. Sc., Vol. 39, No. 4, 1992*

Backup

Step Function



Total chi-squared



Naive approach: Add up chi square distributions

➜ Influence of weights needs to be studied