Lecture 3: Electron Damping Rings

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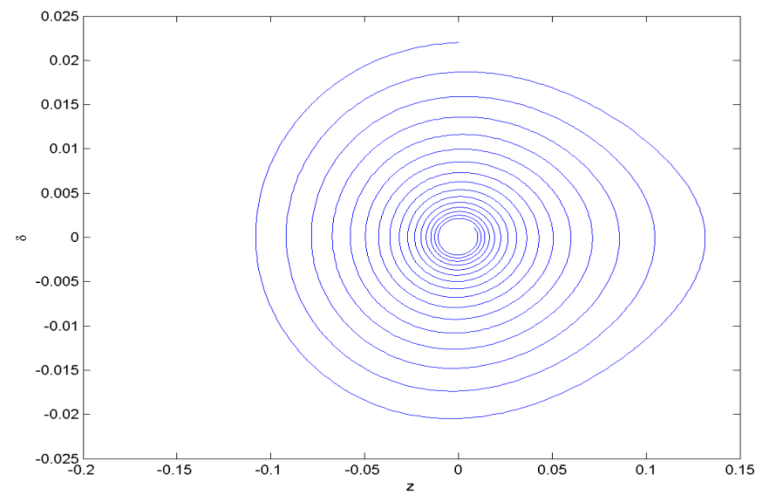
Longitudinal Radiation Damping

For a single RF in a ring, every turn we have

$$\begin{cases} \delta_{n+1} = \delta_n + \dfrac{eV_{RF}}{E_0}\sin(k_{RF} z_n + \phi_s) - \dfrac{U_0}{E_0} - D_s \delta_n \\ z_{n+1} = z_n - \alpha C \delta_{n+1} \end{cases}$$

D_s is due to the fact that the energy loss depends in the deviation of the energy from the synchronous particle.

RF Bucket



⇒

$$\begin{cases} \dot{\delta} = \dfrac{eV_{RF} k_{RF}}{T_0 E_0} \cos\phi_s z - D_s \delta \\ \dot{z} = -\dfrac{\alpha C}{T_0} \delta \end{cases}$$

Synchrotron tune is given by

$$\nu_s = \sqrt{\frac{h\alpha}{2\pi} \frac{eV_{RF}}{E_0} \cos\phi_s},$$

where $\omega_s = \nu_s \omega_0$.

Synchrotron Radiation

Instantaneous radiated power is given by

$$P_\gamma = \frac{2}{3} r_e mc^2 \frac{c\beta^4\gamma^4}{\rho^2},$$

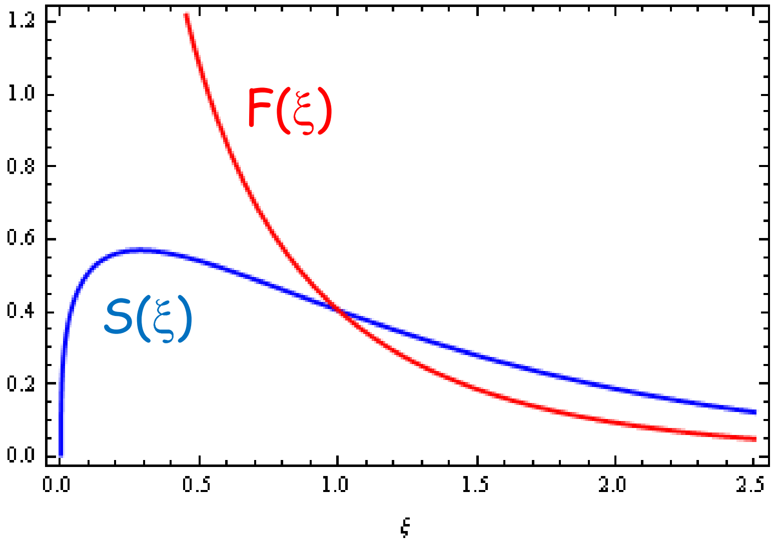
and spectrum,

$$\frac{dP_\gamma}{d\omega} = \frac{P_\gamma}{\omega_c} S(\frac{\omega}{\omega_c}),$$

where $\omega_c = 3c\gamma^3/2\rho$ and S is defined as,

$$S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_\xi^\infty K_{5/3}(\bar{\xi})d\bar{\xi}.$$

Normalized power spectrum S and photon number spectrum F



$K_{5/3}$ is the modified Bessel function. Then the quantum distribution function is

$$n(u) = \frac{P_\gamma}{u_c^2} F(\frac{u}{u_c}), F(\xi) = S(\xi)/\xi, u_c = \omega_c \hbar.$$

key →

$$\dot{N}_{ph} < u^2 > = \frac{55}{24\sqrt{3}} u_c P_\gamma$$

Radiation Damping Time

As we have illustrated that energy loss as a function of the energy deviation results in the radiation damping, it easily see that the damping increments are given by

$$\alpha_x = -\frac{1}{2}\frac{<P_\gamma>}{E_0}(1-\vartheta) = -\frac{1}{2}\frac{<P_\gamma>}{E_0}J_x,$$

$$\alpha_y = -\frac{1}{2}\frac{<P_\gamma>}{E_0} = -\frac{1}{2}\frac{<P_\gamma>}{E_0}J_y,$$

$$\alpha_s = -\frac{1}{2}\frac{<P_\gamma>}{E_0}(2+\vartheta) = -\frac{1}{2}\frac{<P_\gamma>}{E_0}J_s,$$

where

$$\vartheta = \frac{<\frac{\eta_x}{\rho^3}(1+2\rho^2 K_1)>_s}{<\frac{1}{\rho^2}>_s}$$

← Only important one combined function magnets are used. Note that $K_1 < 0$ reduces the horizontal emittance.

J_x, J_y, and J_s are called the damping partitions and $J_x+J_y+J_s=4$. The damping time is given by $\tau=|1/\alpha|$.

Energy Spread and Emittance

Balance between the quantum excitation and radiation damping results in an equilibrium Gaussian distribution with relative energy spread σ_δ and horizontal emittance ε_x:

$$\sigma_\delta^2 = \frac{\tau_s}{2E_0^2} < \dot{N}_{ph} < u^2 >>_s = C_q \frac{\gamma^2}{J_s} \frac{\langle 1/\rho^3 \rangle_s}{\langle 1/\rho^2 \rangle_s},$$

$$\varepsilon_x = \frac{\tau_x}{4E_0^2} < \dot{N}_{ph} < u^2 > \mathcal{H}_x >_s = C_q \frac{\gamma^2}{J_x} \frac{\langle \mathcal{H}_x/\rho^3 \rangle_s}{\langle 1/\rho^2 \rangle_s},$$

where

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc},$$

and

$$\mathcal{H}_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$$

- The quantum constant $C_q = 3.8319 \times 10^{-13}$ m for electron
- γ is the Lorentz factor (energy)

Minimization of Emittance

For an electron ring without damping wigglers, the horizontal emittnace is given by

$$\varepsilon_0 = F \frac{C_q \gamma^2}{I_x} \theta^3$$

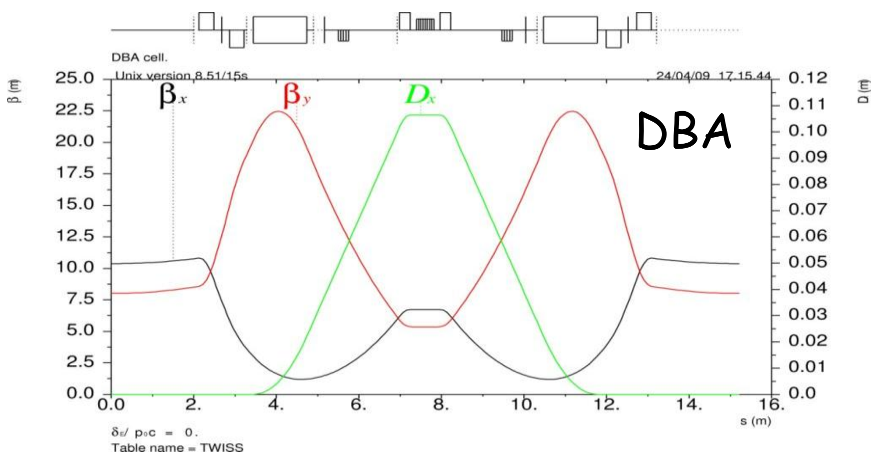
where F is a form factor determined by choice of cell and θ is bending angle of dipole magnet in cell. In general, stronger focusing makes F smaller. Often there is a minimum achievable value of F for any a given type of cell. For example, we have

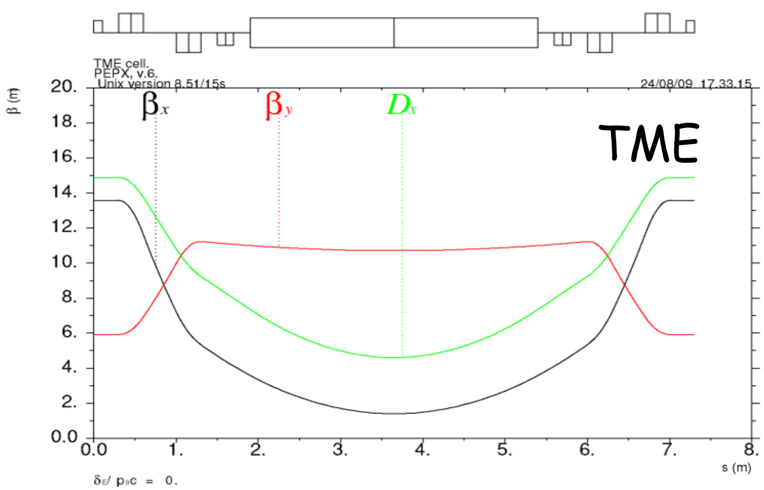
$$F_{min}^{DBA} = \frac{1}{4\sqrt{15}}$$

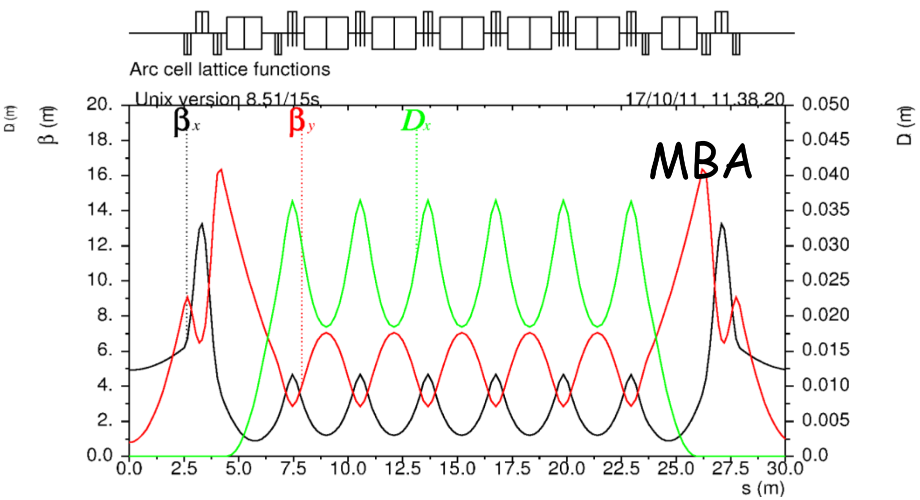
$$F_{min}^{TME} = \frac{1}{12\sqrt{15}}$$

There is a factor of three between the minimum values of DBA and TME cells. That's the price paid for an achromat, namely fixing the dispersion and its slop at one end of dipole.

Types of Periodic Cell







Emittance Reduction Using Damping Wiggler

$$\frac{\varepsilon}{\varepsilon_0} = \frac{1 + \dfrac{4C_q}{15\pi J_x} N_p \dfrac{\beta_x}{\varepsilon_0 \rho_w} \gamma^2 \dfrac{\rho_0}{\rho_w} \theta_w^3}{1 + \dfrac{1}{2} N_p \dfrac{\rho_0}{\rho_w} \theta_w}$$

N_p, is the total number of wiggler poles

β_x, is the average horizontal beta function in the wiggler

ρ_w, is wiggler bending radius at the peak field

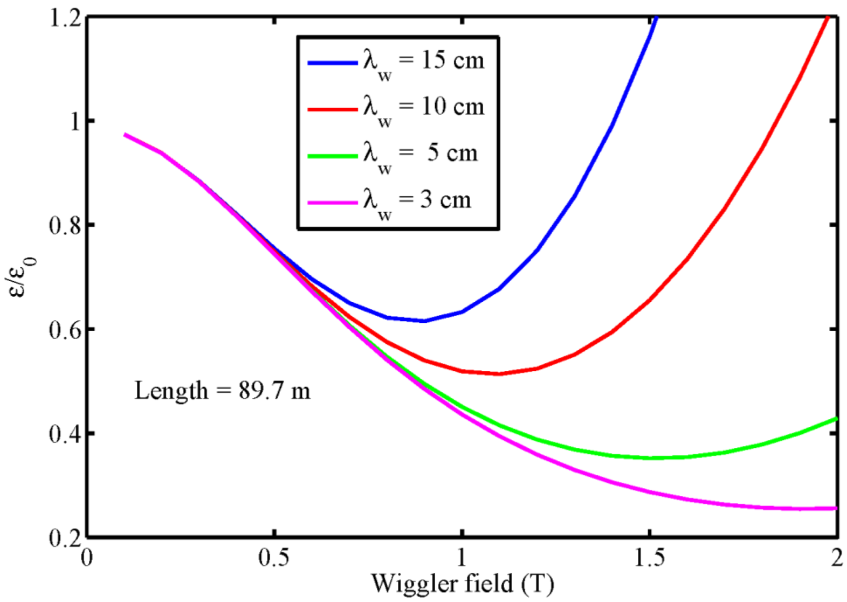
$\theta_w = \dfrac{\lambda_w}{2\pi\rho_w}$, is the peak trajectory angle in the wiggler

λ_w, is the wiggler period length

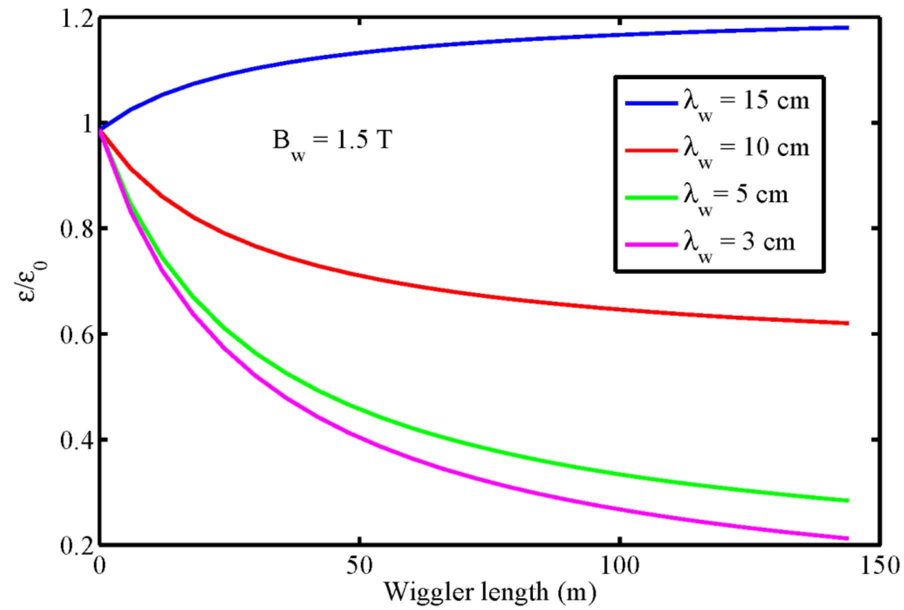
Optimization of Wigglers Parameters

Emittance = 11 pm-rad at 4.5 GeV with
parameters λ_w=5 cm, B_w=1.5 T

Wiggler Field Optimization



Wiggler Length Optimization



Average beta function at the wiggler section is 12.4 meter.

Intra-Beam Scattering

The growth rate in the relative energy spread σ_δ is given by

$$\frac{1}{T_p} = \frac{r_e^2 c N_b (log)}{16\gamma^3 \varepsilon_x^{3/4} \varepsilon_y^{3/4} \sigma_z \sigma_\delta^3} < \sigma_H g(\alpha)(\beta_x \beta_y)^{-1/4} >,$$

where N_b is the bunch population and (log) the Coulomb log factor and the other factors are defined by

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{\mathcal{H}_x}{\varepsilon_x}, \alpha = \sqrt{\frac{\varepsilon_y \beta_x}{\varepsilon_x \beta_y}},$$

$$g(\alpha) = \alpha^{(0.021 - .0044 \ln \alpha)}.$$

And the horizontal growth rate is given by

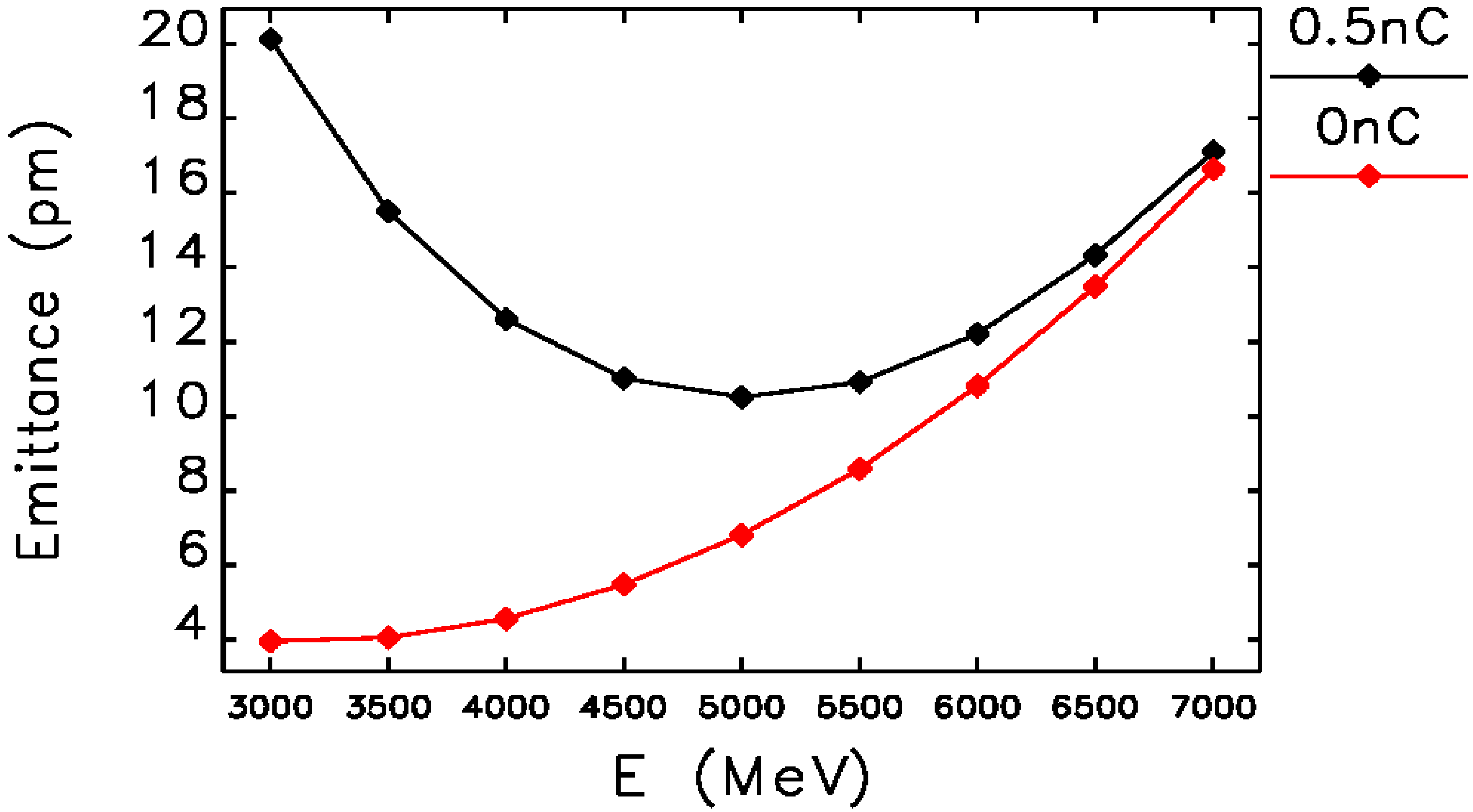
$$\frac{1}{T_x} = \frac{\sigma_\delta^2}{\varepsilon_x} < \mathcal{H}_x \Delta(\frac{1}{T_p}) > .$$

Combined with synchrotron radiation

$$\varepsilon_x = \frac{\varepsilon_{x0}}{1 - \tau_x / T_x}, \sigma_\delta^2 = \frac{\sigma_{\delta 0}^2}{1 - \tau_s / T_p},$$

$$\varepsilon_y = \kappa \varepsilon_x$$

Optimization of Energy



Touschek Lifetime

When a pair of electrons go through a hard scattering, their momentum changes are so large that they are outside the RF bucket or the momentum aperture. This process results in a finite lifetime of a bunched beam. The lifetime is given by

$$\frac{1}{\mathcal{T}} = \frac{r_e^2 c N_b}{8\sqrt{\pi}\gamma^4 \varepsilon_x \varepsilon_y \sigma_z \sigma_\delta} < \sigma_H F(\delta_m) > ,$$

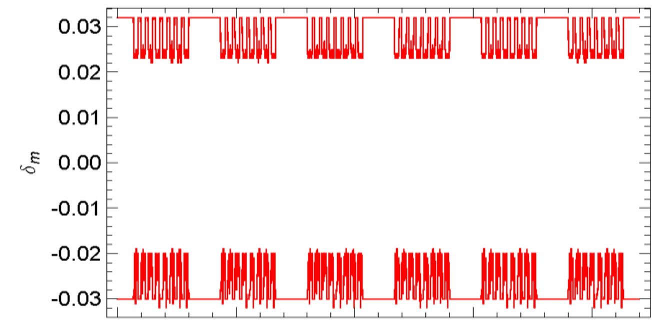
with

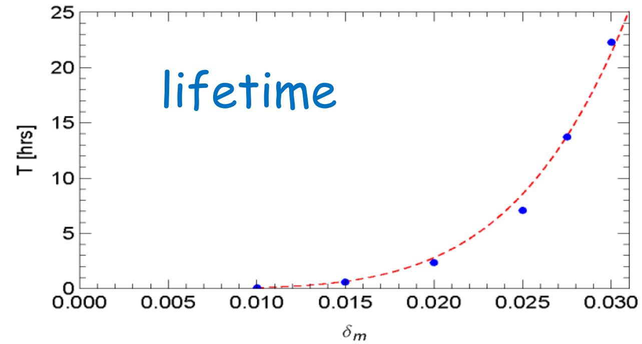
$$F(\delta_m) = \int_{\delta_m^2}^{\infty} \frac{d\tau}{\tau^{3/2}} e^{-\tau B_+} I_0(\tau B_-)\left[\frac{\tau}{\delta_m^2} - 1 - \frac{1}{2}\ln\left(\frac{\tau}{\delta_m^2}\right)\right],$$

$$B_\pm = \frac{1}{2\gamma^2}\left|\frac{\beta_x(\beta_x\varepsilon_x + \eta_x^2\sigma_\delta^2)}{\varepsilon_x(\beta_x\varepsilon_x + \beta_x\mathcal{H}_x\sigma_\delta^2)} \pm \frac{\beta_y}{\varepsilon_y}\right|,$$

where δ_m is the momentum acceptance.

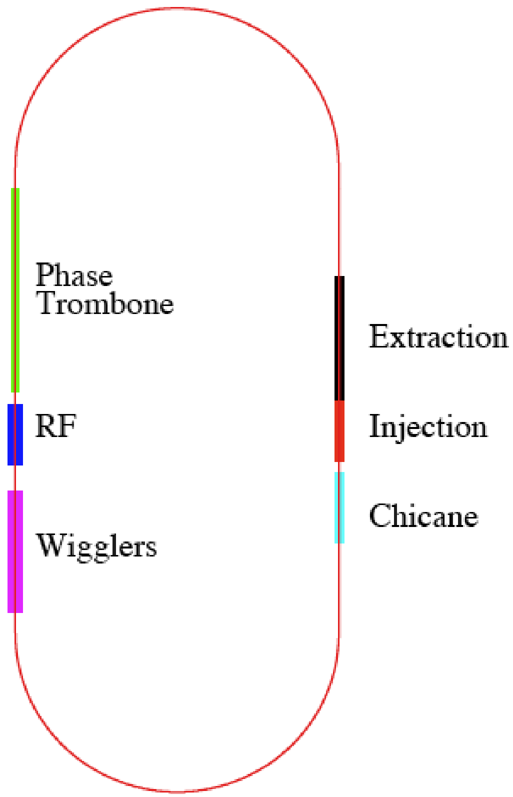
momentum aperture





New ILC Damping Ring Baseline Lattice

DTC01 layout



Usually damping rings lattices have a racetrack layout with long straight sections including RF cavities, injection, extraction and long wiggler sections

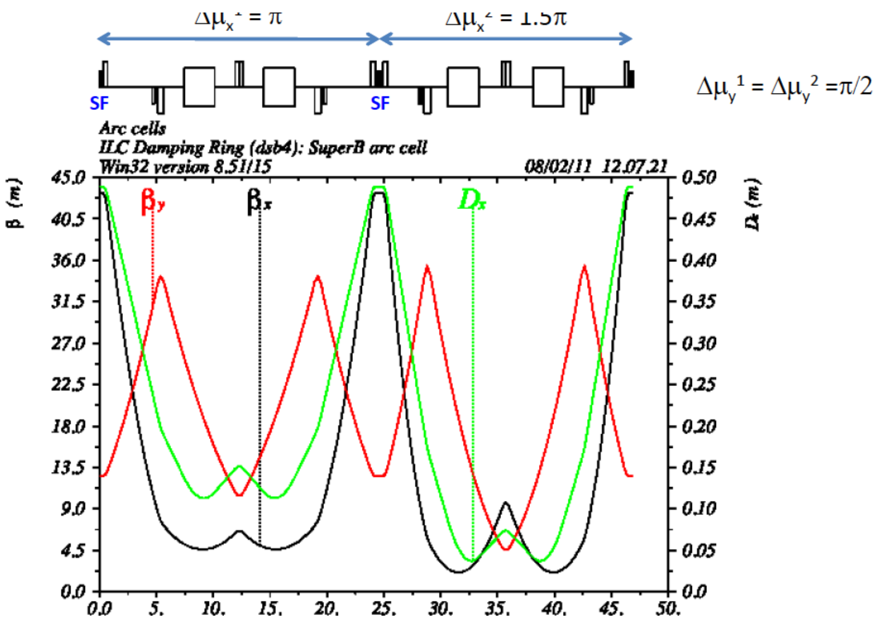
1. Circumference = 3242.9m, 712m straights
2. ~ 6 phase trombone cells
3. 54 – 1.92m long wigglers
   wiggler period = 32cm
   12-poles
   B_{max} = 2.1T
4. Space for 16 RF cavities
   Cryostats for upper and lower positron rings are interleaved

Courtesy S. Guiducci and R. Bartolini

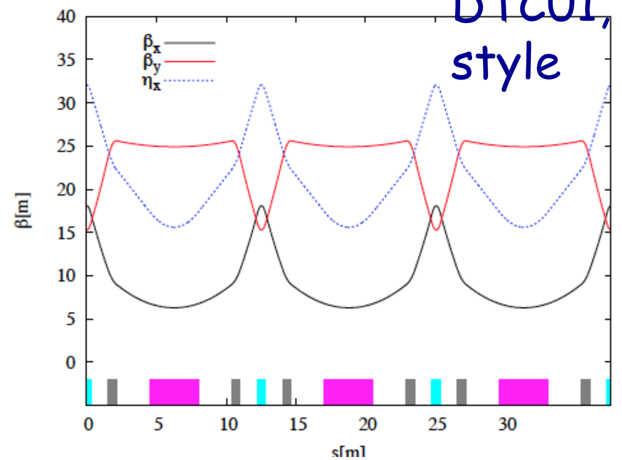
3.2 km Damping Ring - Lattice Comparison

Courtesy S. Guiducci and R. Bartolini

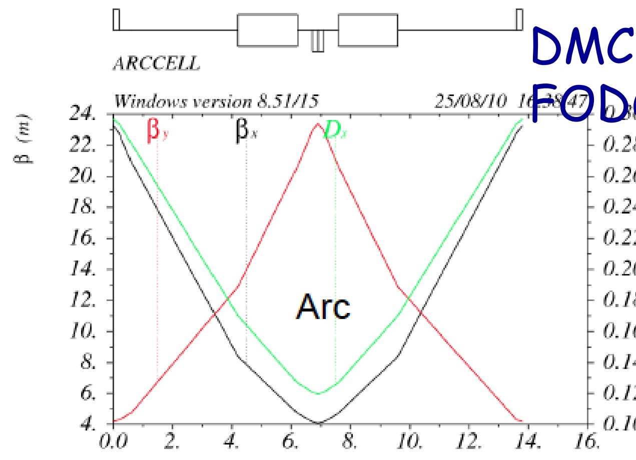
DSB3, SuperB-style



DTC01, TME-style



DMC3, FODO



All the arc cell styles satisfy emittance and damping time requirements.

S. Guiducci, M. E. Biagini, “A Low Emittance Lattice for The ILC 3 Km Damping Ring”, IPAC’10
D. Wang, J. Gao, Y. Wang, “A New Design for ILC 3.2 km Damping Ring Based on FODO Cell”, IPAC’10
D. Rubin, DR TBR, LNF July 2011, http://ilcagenda.linearcollider.org/conferenceDisplay.py?confId=5183
S. Guiducci et al., , “Updates to the International Linear Collider Damping Rings Baseline Design”, IPAC’11

3.2 km ILC damping ring main parameters comparison

	DSB3	DMC3	DTC01
Arc lattice	SuberB-style	FODO	TME-style
Energy (GeV)	5	5	5
Circumference (m)	3238	3220	3239
Horizontal Emittance (nm)	0.66	0.36	0.45
Damping time τ_x, τ_y (ms)	24	23	24
Energy spread %	0.12	0.13	0.11
Energy loss/turn U_0	4.5	4.7	4.5
$F_w = U_{0wiggler}/U_{0arc}$	3.5	10.8	4.6
Wiggler field (T)	1.6	1.6	1.5
Total wiggler length (m)	78	95	104

Courtesy S. Guiducci and R. Bartolini

ILC-CLIC Damping Ring comparisons

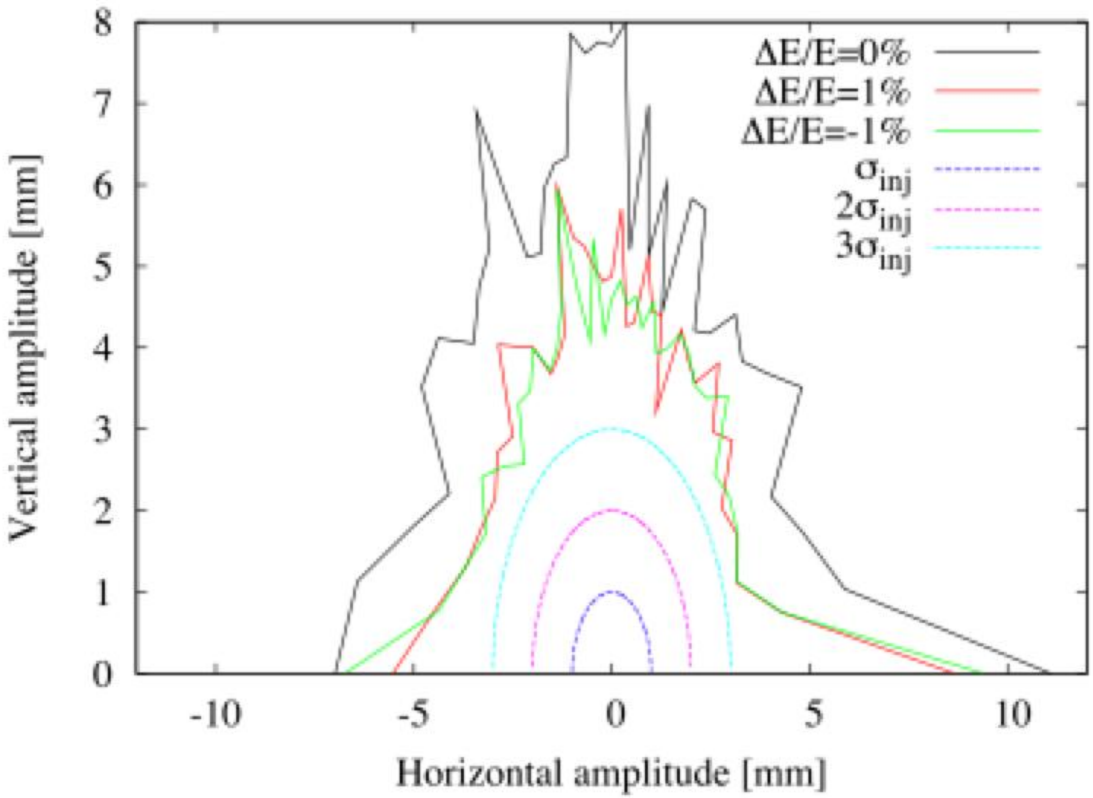
	ILC-DCO4	ILC-DTC01	CLIC
Arc lattice	Modified FODO	TME-style	Modified TME
Energy	5	5	2.86
Circumference (m)	6476	3239	493
Horizontal Emittance (nm)	0.45	0.45	0.079
Damping time tx (ms)	21	24	2.4
Energy spread	0.13	0.11	0.1
Energy loss/turn U_0 (MeV)	10.2	4.5	3.9
$F_w = U_{arc}/U_{wiggler}$	10.7	4.6	6.9
Wiggler field (T)	1.6	1.5	2.5
Total wiggler length (m)	216	104	152

M.Korostelev, A.Wolski, "DCO4 Lattice Design For 6.4 Km ILC Damping Rings", IPAC'10
Y. Papaphilippou et al., , "Lattice Options for the CLIC Damping Rings", IPAC'09

Courtesy S. Guiducci and R. Bartolini

ILC Damping Ring Dynamic Aperture

DTC01



For ILC damping ring the DA has to be **3s$_x$** of the “large” positron beam, which is **130s$_x$** of the stored beam

<u>Courtesy S. Guiducci and R. Bartolini</u>

Summary

- Radiation damping is the key of the damping rings
  - Damping time is mostly determined by wigglers
  - Emittance is mostly by energy, bending angles and cell. Wigglers are helpful.
- Intra-beam scattering becomes more important for smaller emittance
  - Require higher energy
- Touschek lifetime
  - Require large momentum aperture
- Positron damping ring
  - Require large dynamic aperture at injection

References

1. Matthew Sands, "The physics of electron storage rings," SLAC-121, November 1970.
2. Helmut Wiedemann, Particle Accelerator Physics I, "Basic Principles and Linear Beam Dynamics," Second Edition, Springer-Verlag Berlin, Heidelberg New York (1993).
3. Karl Bane, "A simplified model of intrabeam scattering," Proc. EPAC 2002, Pairs, France, pages 1443-1445, 2002.
4. A. Piwinski, "The Touschek effect in strong focusing storage rings," DESY 98-179, DESY, November 1998.