Problems to be solved:

Tune, Phase advance measurements

Neglecting betatron oscillations after the injection process or beam instabilities we can assume that the particle beam will circulate on a stable closed orbit in the accelerator.
In order to measure the betatron tune a transverse oscillation of the beam has to be excited and then observed with one or several position sensitive instruments. Depending on the choice of the excitation method and the observation instrument many different ways of measuring the tune with different qualities can be imagined. The following table gives a brief summary.

Beam Excitation Signal	Treatment of measured oscillation	Comments
Consecutive random noise kicks	Fourier transform	Work horse of operational tune measurement, precision 10-3…10-4
Single kick	Fourier transform, sliding Fourier transform, wavelet analysis	Observation of damped oscillation, large initial amplitude (emittance dilution in hadron machines after filamenation), access to non-linearities Precision depends on damping time
Sin wave excitation with slowly varying frequency (CHIRP)	Synchronous detection of beam motion	Measurement of full beam transfer function (network analyser) Precision only limited by beam stability and measurement time, precision typical typical 10-4
Continuous sin wave excitation on beam resonance	Phase locked loop (PLL) circuit keeps beam exciter on resonance	Best tool to trace tune changes during machine transitions (acceleration…) Information of full beam spectrum not available, precision depends on bandwidth of PLL, 10-6 can be
“AC-dipole”excitation Sin wave excitation OUTside frequencies of beam resonance	Fourier transform of beam motion	acts like a fast orbit changing dipole magnet, can create large amplitudes for diagnostics, no emittance blowup

Question 1)

Discuss the table with amongst you and draw the time envelope of the beam excitation signal and the beam response for the first 3 methods:

a) broad band noise excitation and frequency analysis of the resulting oscillation.
b) single kick stimulus and frequency analysis of resulting oscillation
c) swept sine wave excitation and vector analysis of resulting oscillation

For a) What happens in case one does not use broad band (white) noise? Imagine the extreme case that the bandwidth of the "noise" spectrum is reduced to a single sinusoidial line.

Another way of looking at tune measurements is in frequency domain:

One considers the beam as a linear resonating oscillator, which responds to a stimulus like a filter. One can hence draw the spectral intensity of the beam exciter and the spectral density of the resulting beam oscillation. The ratio of both functions corresponds to the transfer function of the beam (as resonator) and the betatron tune can be measured as the maximum of this transfer function.

Question 2)

Repeat the discussion as in 1, but this time produce qualitative drawings in frequency domain for the input spectra and the output spectra. Indicate in the ratio of these spectra (i.e. in the "measured beam transfer functions") the betatron tune.

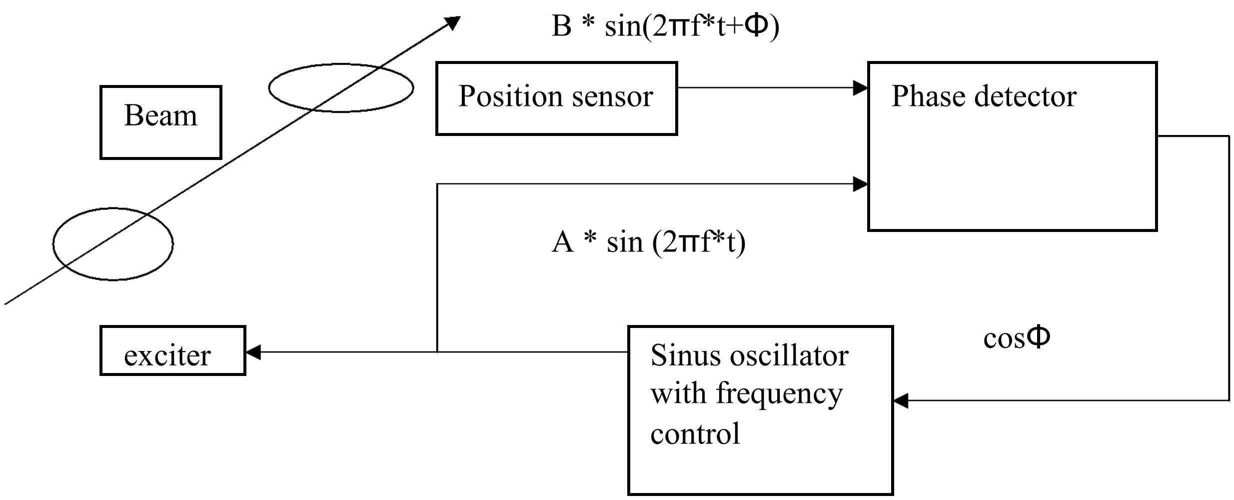
Question 3)

Draw the transfer function of the beam as resonator as amplitude and phase as function of frequency. What physics parameters determine the resonance frequency and the width of these functions?

The very simple electronic circuit (below) (called **PLL** for **P**hase **L**ocked **L**oop) excites the beam (via an electromagnetic deflection element) permanently with a sin-wave. In the return path the beam oscillation is measured with a position sensitive pickup. The phase detector measures the phase between beam exciter and beam response. The phase information is feed back to the frequency of the driving exciter, such that the circuit maintains an oscillation on the center of the resonance, even if this resonance changes as a function of time.

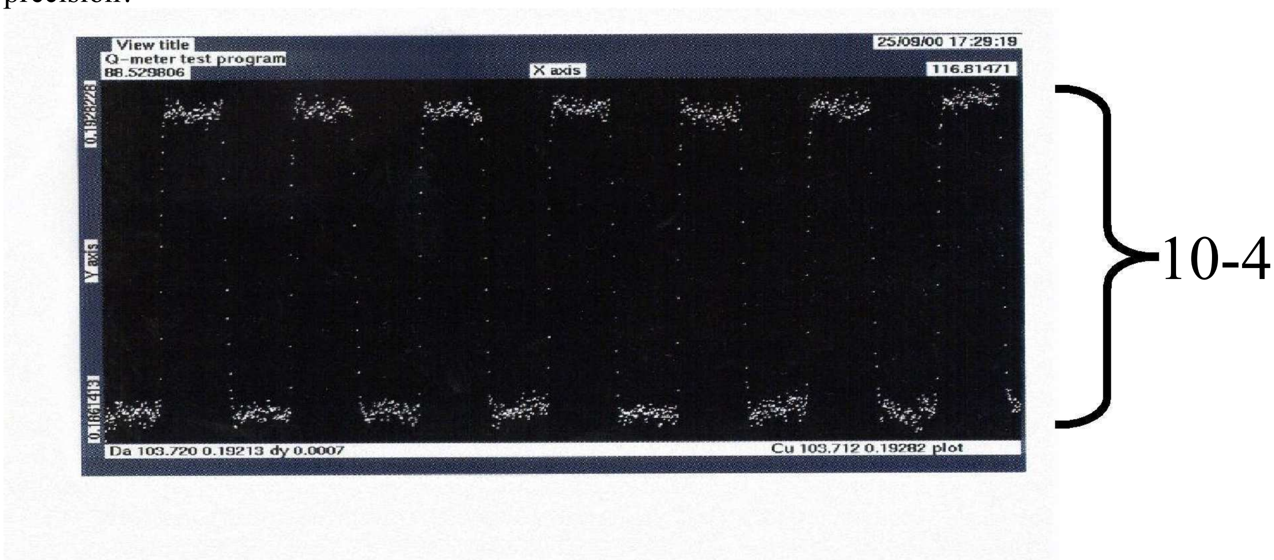
Question 4)
If the phase detector of the above circuit is made, such that its output is proportional to cosΦ, then there will be no action on the frequency o the oscillator for Φ=90. Use the answer of question 3 to indicate to what frequency the PLL circuit will lock in this case. If you had to build such a phase detector, how would you do that?
By taking a second output of the controlled oscillator, which is 90 degrees shifted in phase (called a "quadrature oscillator" in the literature), one can construct another observable, which is proportional to sinΦ. This variable is at a maximum, in case the PLL has locked to the resonance.

Question 5)
Draw a sketch, how the signal proportional to sinΦ could be used in order to regulate the amplitude of the beam exciting signal. Hint: Insert a 4-quadrant analog multiplier in to the output of the oscillator.



As closure of this exercise a measurement of the tunes in LEP during a rectangular current modulation in a LEP quadrupole.

Question7) Knowing the amount of current modulation in the quadrupole and the resulting tune change, what beam quantity is measured with this experiment? What precision does one need in the tune measurement, to measure the observable to 1% precision?



Question 8:
What is the equivalent measurement of tune in a linac?
How is it done?

Optional, if you feel like it.

Chromaticity measurements

The "classical" method to measure chromaticity is the individual measurement of the betatron tune over a given range of beam momenta. The chromaticity is obtained from a linear fit to the data.

Question 9: In practice people very often do not go through the problem of collecting the data for a linear fit; they just measure the difference of two tune readings for two beam momenta. Calculate the requirements on precision for the tune measurements for such a simple measurement, in case the beam momentum is varied by +/- 10^{-3}. The requested precision in chromaticity is half a unit.

By how much can in practice the beam momentum be varied?

Is the requested precision in tune measurements attainable with a single kick stimulus and a FFT analysis of the resulting beam oscillation?

The following list gives the keywords for other methods, which are used (have been used) for chromaticty diagnostics: Method a) has already been treated with the problem 1.

a) Tune Difference for different beam momenta
b) Width of tune peak or damping time
c) amplitude ratio of synchrotron sidebands
d) creation of periodic energy variations and PLL tune tracking

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e) bunchspectrum variations during betatron oscillations
f) Head-tail phase advance (same as e), but in time domain)

Question 10: Discuss the above list amongst yourself and try to understand the basic measurement principles.

Question 11: The following figure shows a nice example of the chromaticities in LEP (top: horizontal, bottom: vertical) by a triangular modulation of the Rf-frequency. The measurement shows the variation of the chromaticity during the beta squeeze of LEP.

a) What is a beta squeeze? Recall what you have heard about low beta insertions.
b) Calculate the minimum and maximum chromaticity during the shown beta-squeeze? Assume highly relativistic beams.

(The Rf-Frequency was 354 MHz, α = 3 10^(-4), modulation depth 70 Hz, take as full scale of the vertical axis 0.004)