

Parameter Optimisation

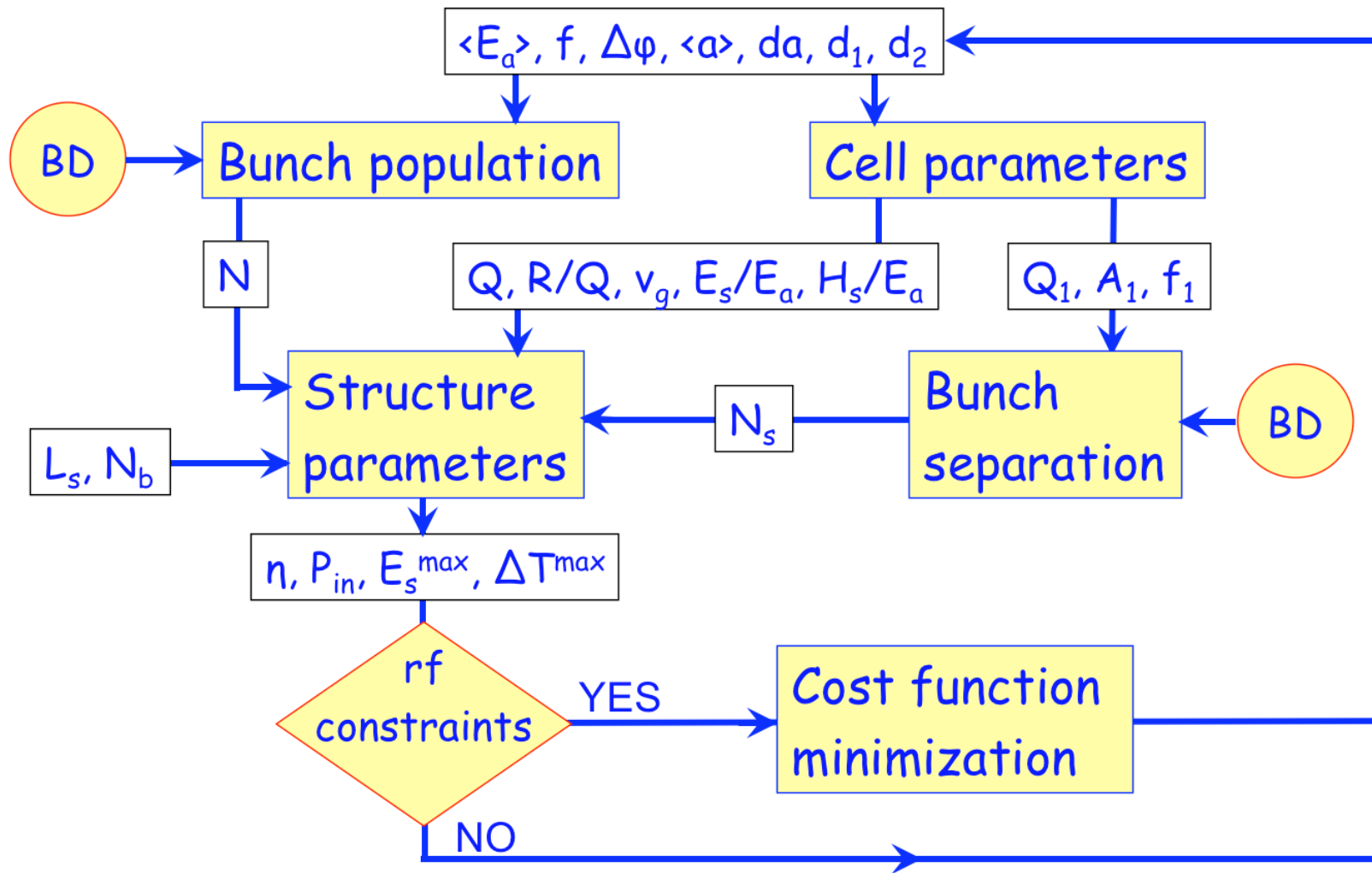
D. Schulte

Linear Collider School, October/November 2010

Overview

- Parameter optimisation requires to remember the previous lectures
- We will go through the relevant steps again

Work Flow as seen by RF Expert (Alexej Grudiev)



Luminosity

Simplified treatment and approximations used throughout

$$\mathcal{L} = H_D \frac{N^2 f_{rep} n_b}{4\pi\sigma_x\sigma_y}$$

$$\mathcal{L} \propto H_D \frac{N}{\sqrt{\beta_x \epsilon_x} \sqrt{\beta_y \epsilon_y}} \eta P$$

$$\epsilon_x = \epsilon_{x,DR} + \epsilon_{x,BC} + \epsilon_{x,BDS} + \dots$$

$$\epsilon_y = \epsilon_{y,DR} + \epsilon_{y,BC} + \epsilon_{y,linac} + \epsilon_{y,BDS} \\ + \epsilon_{y,growth} + \epsilon_{y,offset} \dots$$

$$\sigma_{x,y} \propto \sqrt{\beta_{x,y} \epsilon_{x,y} / \gamma}$$

$$N f_{rep} n_b \propto \eta P$$

typically $\epsilon_x \gg \epsilon_y$,
 $\beta_x \gg \beta_y$

Fundamental limitations from

- beam-beam: $N / \sqrt{\beta_x \epsilon_x}$, $N / \sqrt{\beta_x \epsilon_x \beta_y \epsilon_y}$
- emittance generation and preservation:
 $\sqrt{\beta_x \epsilon_x}$, $\sqrt{\beta_y \epsilon_y}$
- main linac RF: η

Potential Limitations

- Efficiency η :
depends on beam current that can be transported
Decrease bunch distance \Rightarrow long-range transverse wakefields in main linac
Increase bunch charge \Rightarrow short-range transverse and longitudinal wakefields in main linac, other effects
- Horizontal beam size σ_x
beam-beam effects, final focus system, damping ring, bunch compressors
- vertical beam size σ_y
damping ring, main linac, beam delivery system, bunch compressor, need to collide beams, beam-beam effects
- Will try to show how to derive $L_{bx}(f, a, \sigma_a, G)$

Beam Size Limit at IP

- The vertical beam size had been $\sigma_y = 1 \text{ nm}$ (BDS)
 \Rightarrow challenging enough, so keep it $\Rightarrow \epsilon_y = 10 \text{ nm}$
- Fundamental limit on horizontal beam size arises from beamstrahlung
 Two regimes exist depending on beamstrahlung parameter

$$\Upsilon = \frac{2\hbar\omega_c}{3E_0} \propto \frac{N\gamma}{(\sigma_x + \sigma_y)\sigma_z}$$

$\Upsilon \ll 1$: classical regime, $\Upsilon \gg 1$: quantum regime

At high energy and high luminosity $\Upsilon \gg 1$

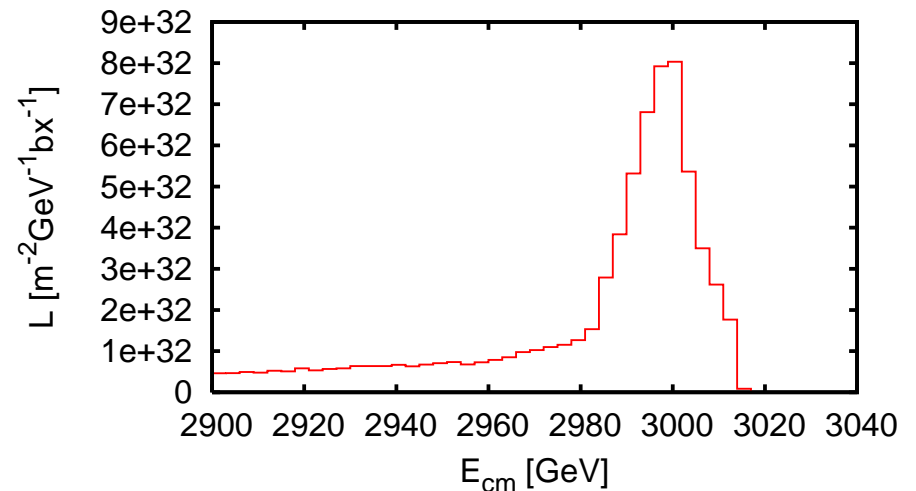
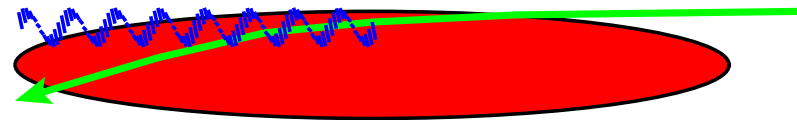
$$\mathcal{L} \propto \Upsilon\sigma_z/\gamma P\eta$$

\Rightarrow partial suppression of beamstrahlung

\Rightarrow coherent pair production

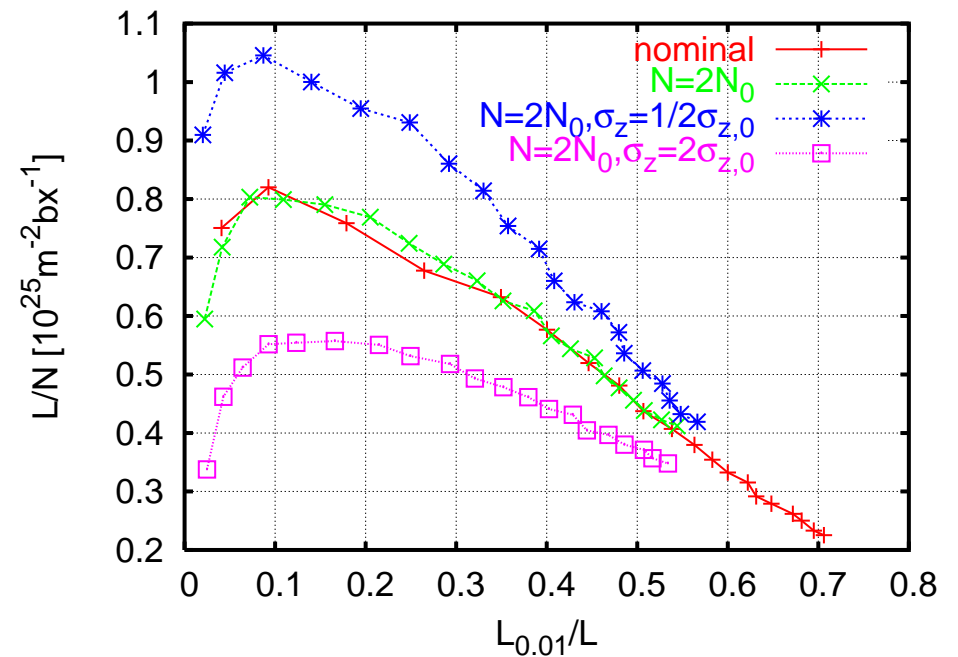
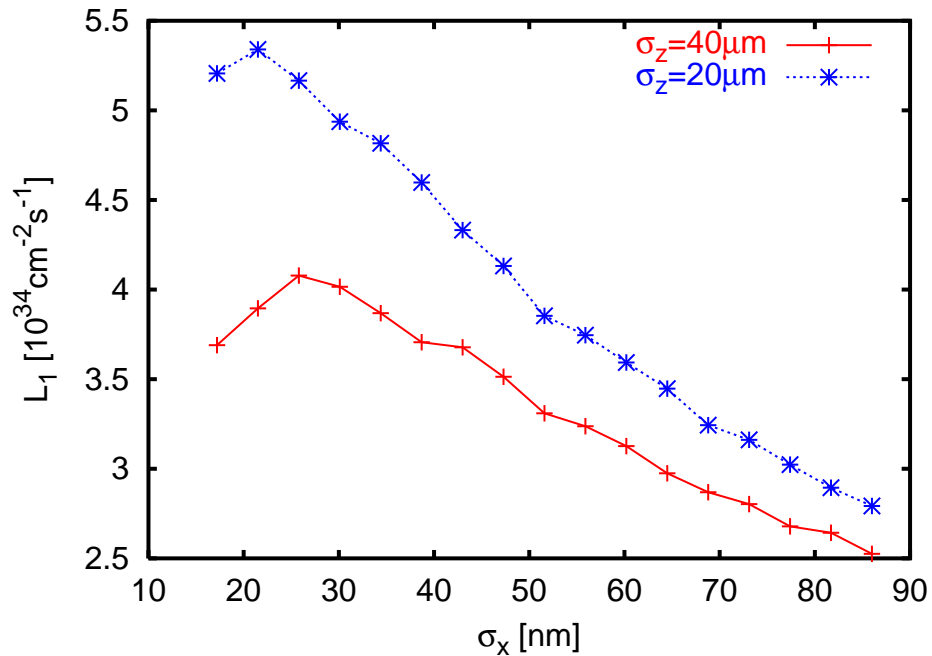
In CLIC $\langle \Upsilon \rangle \approx 6$, $N_{coh} \approx 0.1N$

\Rightarrow somewhat in quantum regime



\Rightarrow Use luminosity in peak as figure of merit

Luminosity Optimisation at IP



Total luminosity for $\Upsilon \gg 1$

$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y} \eta \propto \frac{n_\gamma^{3/2}}{\sqrt{\sigma_z}} \frac{\eta}{\sigma_y}$$

large $n_\gamma \Rightarrow$ higher $\mathcal{L} \Rightarrow$ degraded spectrum

chose n_γ , e.g. maximum $L_{0.01}$ or $L_{0.01}/L = 0.4$ or ...

$$\mathcal{L}_{0.01} \propto \frac{\eta}{\sqrt{\sigma_z} \sigma_y}$$

Other Beam Size Limitations

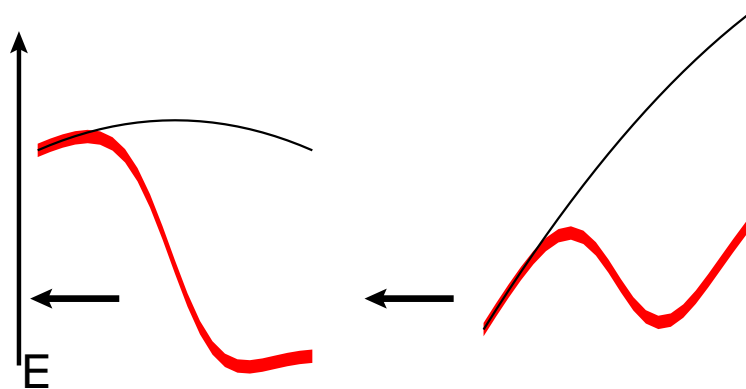
- Final focus system squeezes beams to small sizes with main problems:
 - beam has energy spread (RMS of $\approx 0.35\%$) \Rightarrow avoid chromaticity
 - synchrotron radiation in bends \Rightarrow use weak bends \Rightarrow long system
 - radiation in final doublet (Oide Effect)
 - Large $\beta_{x,y} \Rightarrow$ large nominal beam size
 - Small $\beta_{x,y} \Rightarrow$ large distortions
 - Beam-beam simulation of nominal case: effective $\sigma_x \approx 40$ nm, $\sigma_y \approx 1$ nm
- \Rightarrow lower limit of $\sigma_x \Rightarrow$ for small N optimum n_γ cannot be reached
- new FFS reaches $\sigma_x \approx 40$ nm, $\sigma_y \approx 1$ nm
- Assume that the transverse emittances remain the same
 - not strictly true
 - emittance depends on charge in damping ring (e.g. $\epsilon_x(N = 2 \times 10^9) = 450$ nm, $\epsilon_x(N = 4 \times 10^9) = 550$ nm)

Beam Dynamics Work Flow

- The parameter optimisation has been performed keeping the main linac beam dynamics tolerances at the same level as for the original 30 GHz design
 - The minimum spot size at the IP is dominated by BDS and damping ring
 - adjusted N/σ_x for large bunch charges to respect beam-beam limit
 - For each of the different frequencies and values of a/λ a scan in bunch charge N has been performed
 - the bunch length has been determined by requiring the final RMS energy spread to be $\sigma_E/E = 0.35\%$ and running 12° off-crest
 - the transverse wake-kick at $2\sigma_z$ has been determined
 - the bunch charge which gave the same kick as the old parameters has been chosen
 - The wakefields have been calculated using some formulae from K. Bane
 - used them partly outside range of validity
 - \Rightarrow but still a good approximation, confirmed by RF experts
- $\Rightarrow N$ and $L_{bx}(f, a, \sigma_a, G)$ given to RF experts

Beam Loading and Bunch Length

- Aim for shortest possible bunch (wakefields)
- Energy spread into the beam delivery system should be limited to about 1% full width or 0.35% RMS
- Multi-bunch beam loading compensated by RF
- Single bunch longitudinal wakefield needs to be compensated
⇒ accelerate off-crest



- Limit around average $\Delta\Phi \leq 12^\circ$
⇒ $\sigma_z = 44 \mu\text{m}$ for $N = 3.72 \times 10$

Specific Wakefields

- Longitudinal wakefields contain more than the fundamental mode
- We will use wakefields based on fits derived by Karl Bane

l length of the cell

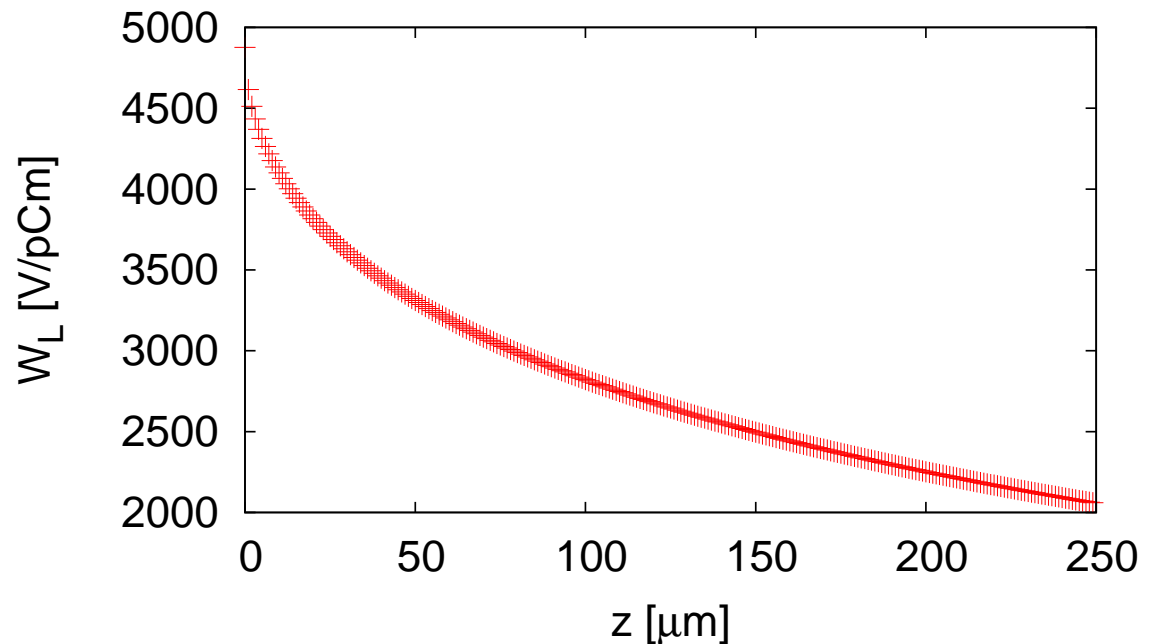
a radius of the iris aperture

g length between irises

$$s_0 = 0.41a^{1.8}g^{1.6} \left(\frac{1}{l}\right)^{2.4}$$

$$W_L = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{\frac{s}{s_0}}\right)$$

- Use CLIC structure parameters



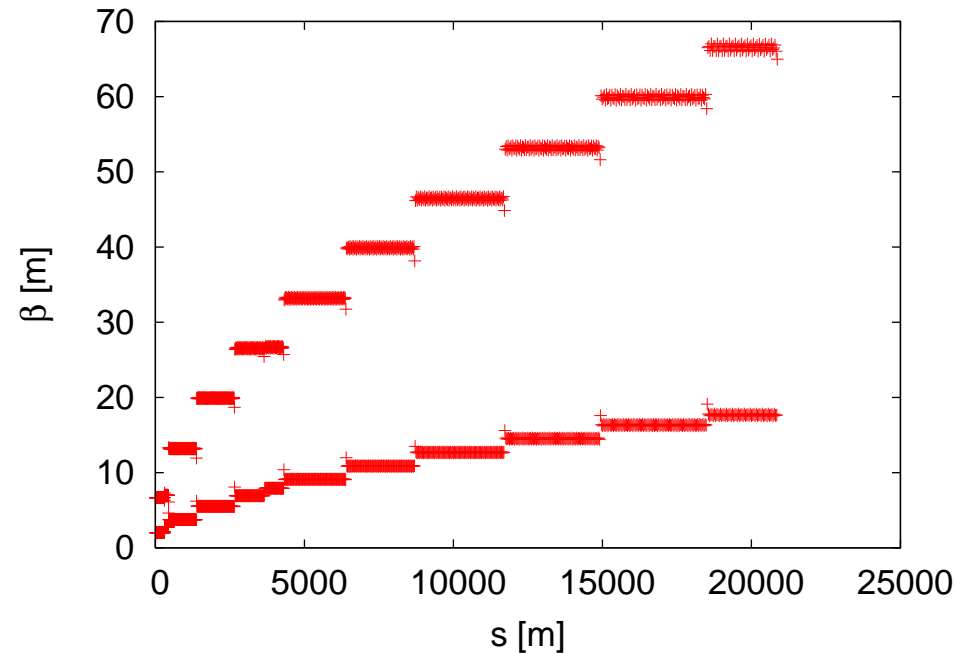
- Summation of an infinite number of cosine-like modes
 - calculation in time domain or approximations for high frequency modes

Recipe for Choosing the Bunch Parameters

- Decide on the average RF phase
 - OK, we fix 12°
- Decide on an acceptable energy spread at the end of the linac
 - OK, we chose 0.35%
- Determine $\sigma_z(N)$
 - chose a bunch charge
 - vary the bunch length until the final energy spread is acceptable
 - chose next charge
- Determine which bunch charge (and corresponding bunch length) can be transported stably

CLIC Lattice Design

- Used $\beta \propto \sqrt{E}$, $\Delta\Phi = \text{const}$
 - balances wakes and dispersion
 - roughly constant fill factor
 - phase advance is chosen to balance between wakefield and ground motion effects
- Preliminary lattice
 - made for $N = 3.7 \times 10^9$
 - quadrupole dimensions need to be confirmed
 - some optimisations remain to be done
- Total length 20867.6m
 - fill factor 78.6%



- 12 different sectors used
- Matching between sectors using 7 quadrupoles to allow for some energy bandwidth

CLIC Fill Factor

- Want to achieve a constant fill factor
 - to use all drive beams efficiently
- Scaling $f = f_0 \sqrt{E/E_0}$ yields

$$L_q \propto \frac{E}{\sqrt{\frac{E}{E_0}}} \propto \sqrt{E}$$

using a quadrupole spacing of $L = L_0 \sqrt{E/E_0}$ leads to

$$\frac{L_q}{L} \propto \frac{\sqrt{E}}{\sqrt{E}} \propto \text{const}$$

- ⇒ The choice allows to maintain a roughly constant fill factor
- ⇒ It maximises the focal strength along the machine

Magnet Considerations

- The maximum strength of a focusing magnet is limited
 - for a normal conducting design rule of thumb is 1 T at the poletip

⇒ Required integrated magnet strength is

$$\frac{\text{T}}{\text{m}} \frac{E}{0.3 \text{ GeV}} \frac{\text{m}}{f}$$

- For CLIC poletip radius is given by practical considerations of magnet design $a \approx 5 \text{ mm}$ yielding a gradient of 200 T/m
- We chose about 10% of the machine to be quadrupoles
 - ⇒ fill factor is $\approx 80\%$
 - 10% are lost for flanges (mainly on structures)
- Use $L_0 = 1.5 \text{ m}$ and $f_0 = 1.3 \text{ m}$ yields

$$\eta_q = \frac{E_0}{0.3 \text{ GeV}} \frac{\text{T/m}}{200 \text{ T/m}^2} \frac{\text{m}}{f_0} \frac{1}{L_0}$$
$$\Rightarrow \eta_q \approx 7.7\%$$

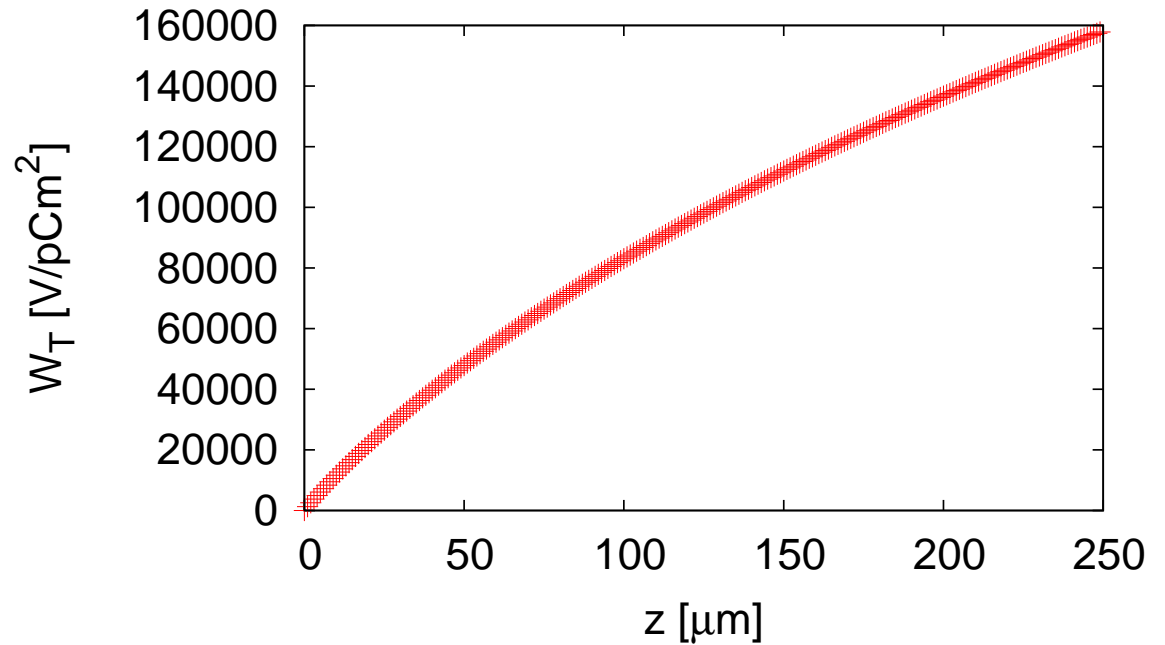
- We use discrete lengths hence we loose a bit more

Example of a Transverse Wakefield (CLIC)

Fit obtained by K. Bane

For short distances the wake-field rises linear

Summation of an infinite number of sine-like modes with different frequencies



$$s_0 = 0.169a^{1.79}g^{0.38} \left(\frac{1}{l}\right)^{1.17}$$

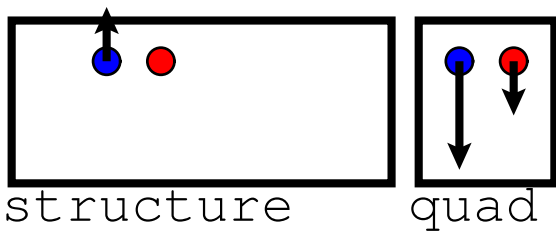
$$w_{\perp}(z) = 4\frac{Z_0cs_0}{\pi a^4} \left[1 - \left(1 + \sqrt{\frac{z}{s_0}} \right) \exp\left(-\sqrt{\frac{z}{s_0}}\right) \right]$$

$$w_{\perp}(z) \approx 4\frac{Z_0cs_0}{\pi a^4} \left[1 - \left(1 + \sqrt{\frac{z}{s_0}} \right) \left(1 - \sqrt{\frac{z}{s_0}} \right) \right] = 4\frac{Z_0cs_0}{\pi a^4} \left[1 - \left(1 - \frac{z}{s_0} \right) \right] = 4\frac{Z_0cz}{\pi a^4}$$

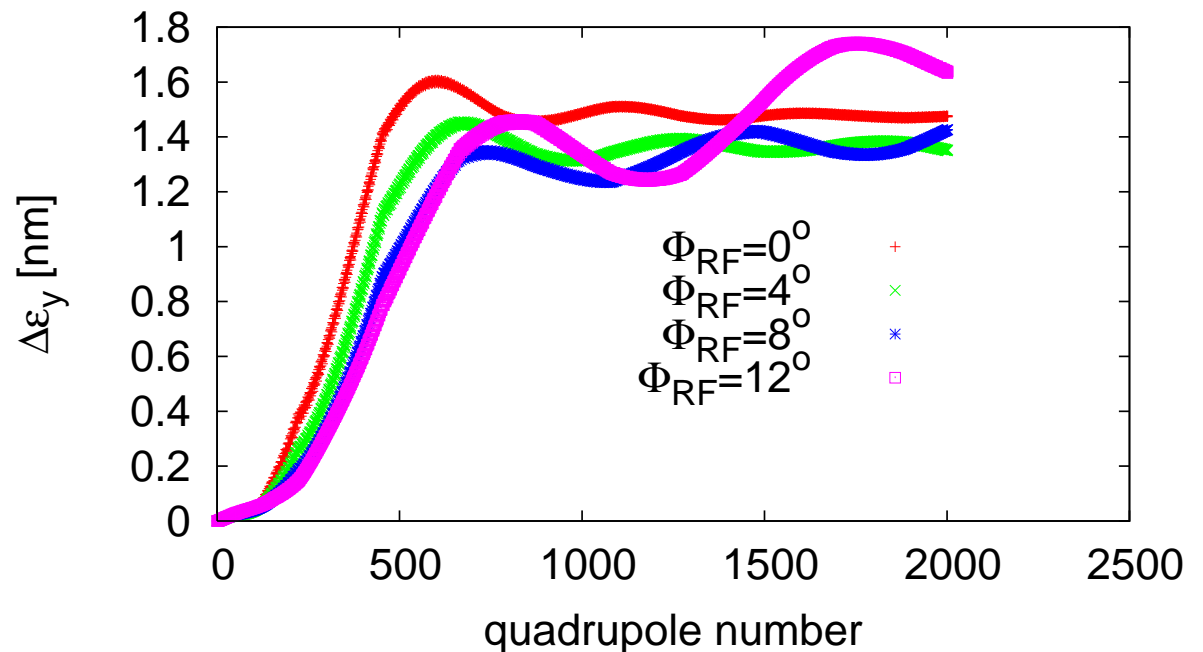
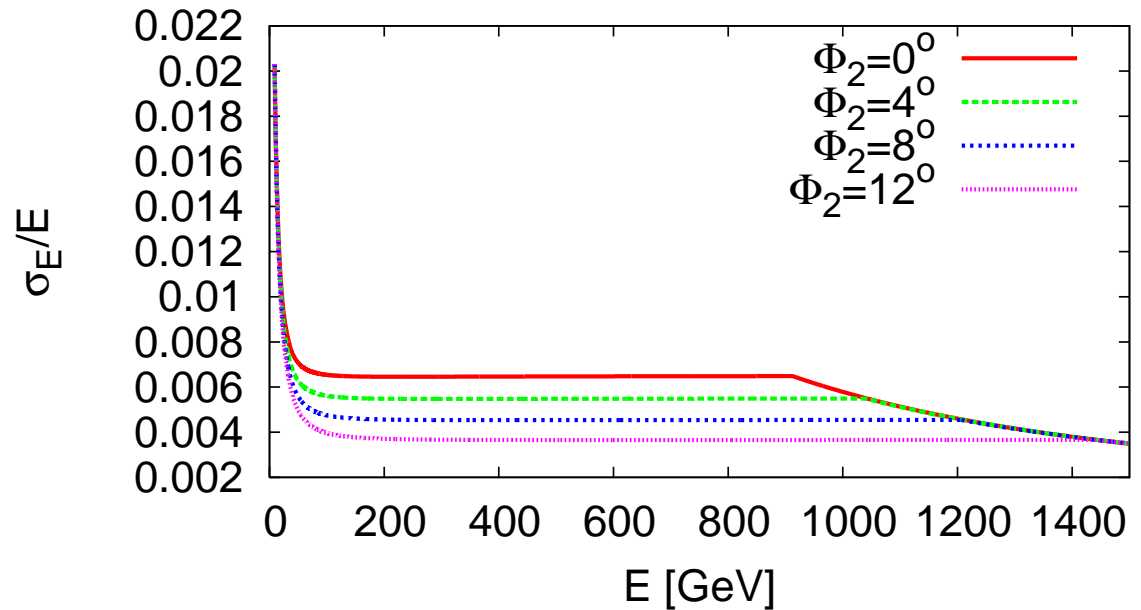
Energy Spread and Beam Stability

- Trade-off in fixed lattice
 - large energy spread is more stable
 - small energy spread is better for alignment

⇒ Beam with $N = 3.7 \times 10^9$ can be stable



⇒ Tolerances are not a unique number

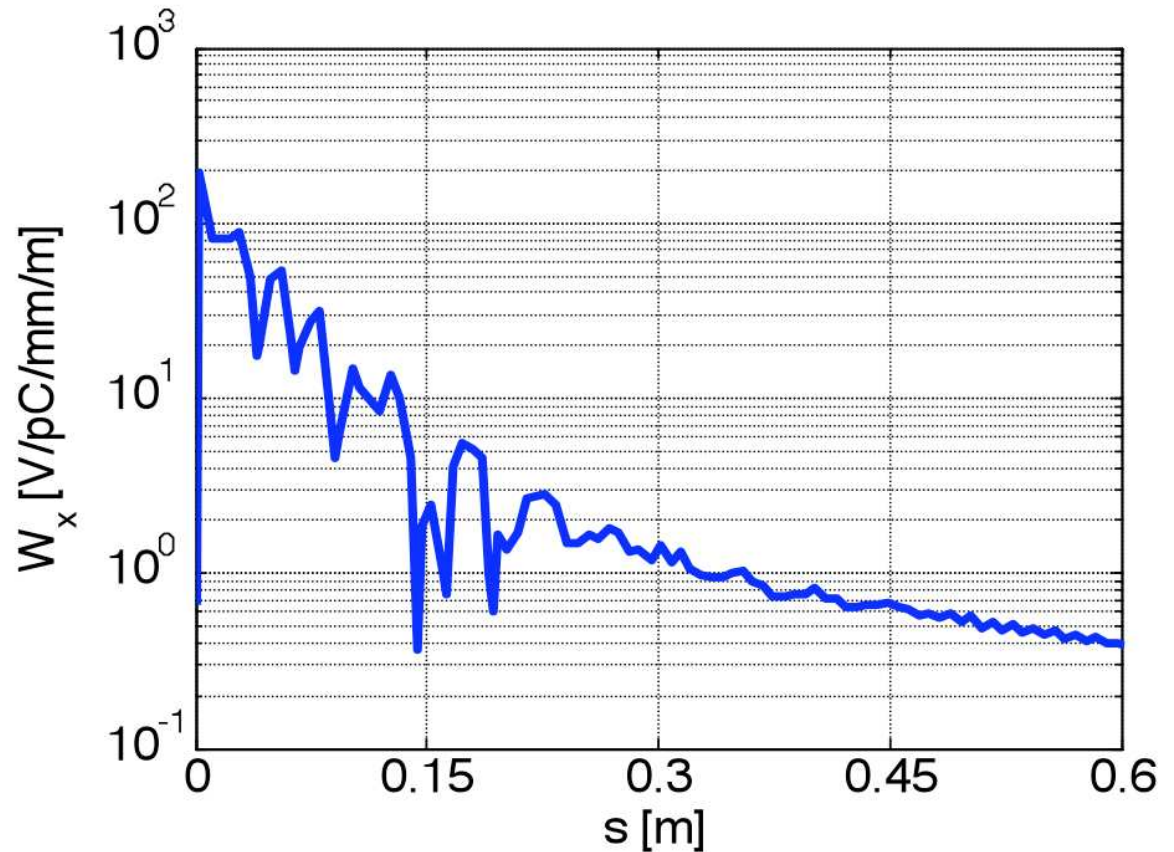


Remember: Multi-Bunch Wakefields

- Long-range transverse wakefields have the form

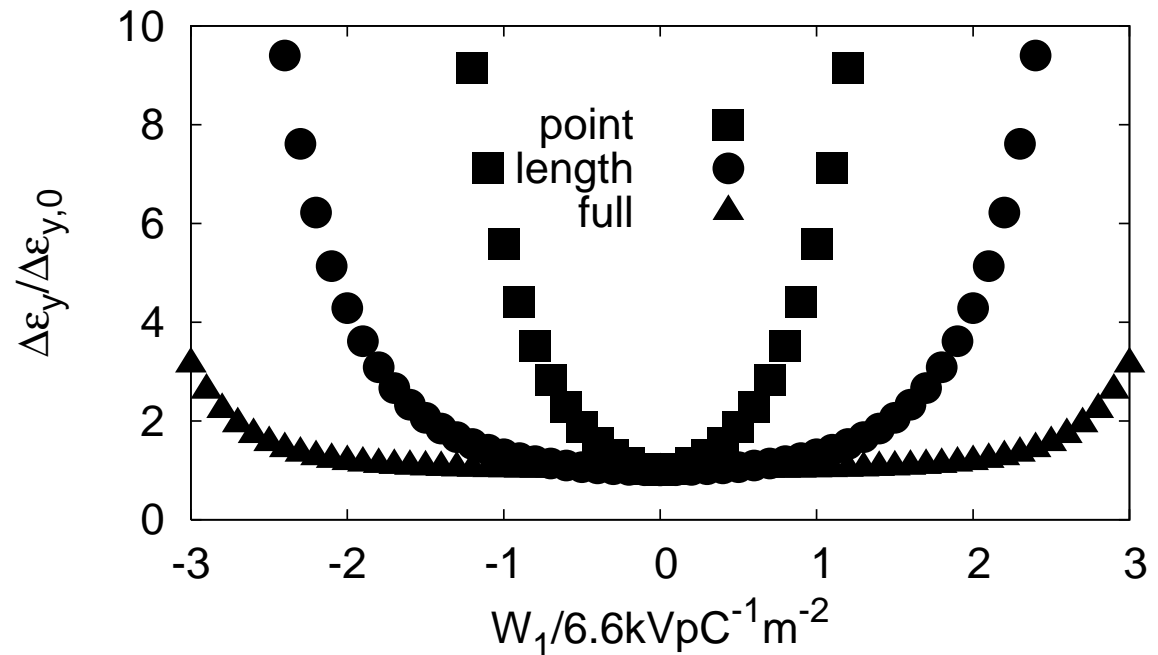
$$W_{\perp}(z) = \sum_i^{\infty} 2k_i \sin\left(2\pi \frac{z}{\lambda_i}\right) \exp\left(-\frac{\pi z}{\lambda_i Q_i}\right)$$

- In practice need to consider only a limited number of modes
- Their impact can be reduced by detuning and damping



Multi-Bunch Jitter

- If bunches are not point-like the results change
 - an energy spread leads to a more stable case
- Simulations show
 - point-like bunches
 - bunches with energy spread due to bunch length
 - including also initial energy spread

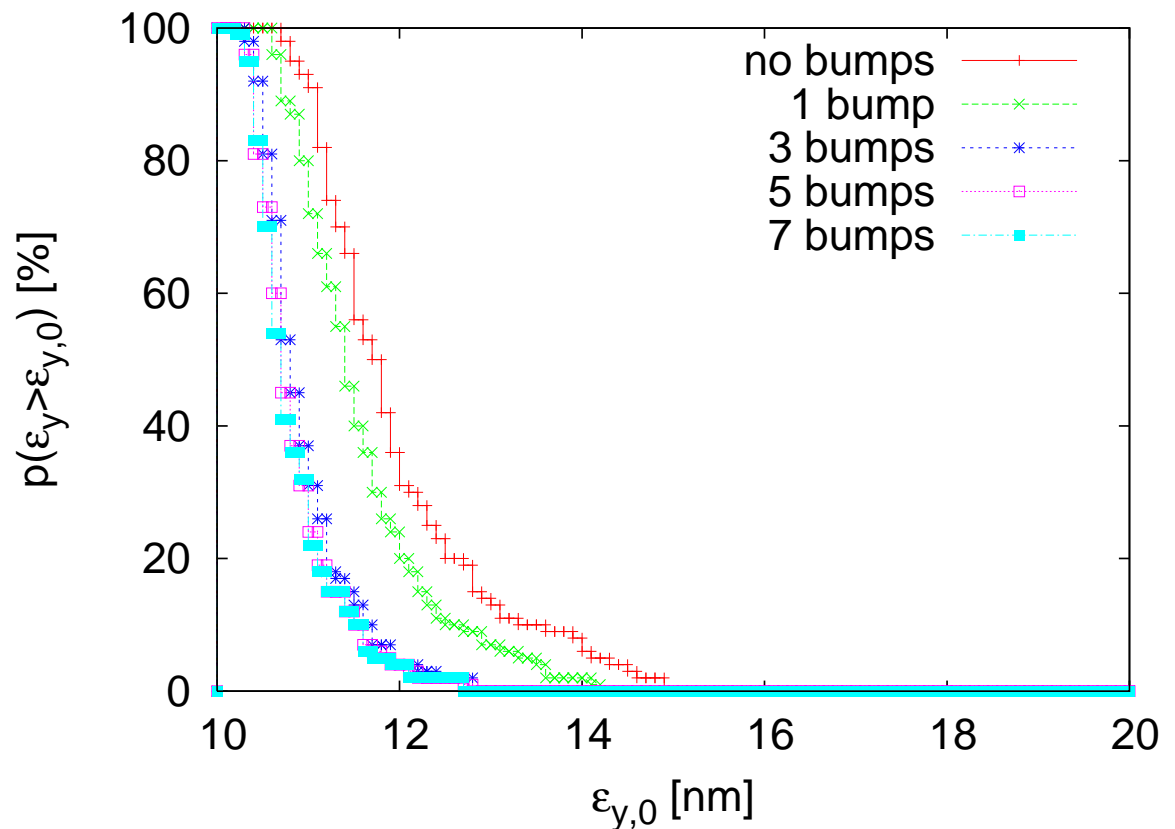


⇒ Point-like bunches is a pessimistic assumption for the dynamic effects

Final Emittance Growth (CLIC)

imperfection	with respect to	symbol	value	emitt. growth
BPM offset	wire reference	σ_{BPM}	14 μm	0.367 nm
BPM resolution	wire reference	σ_{res}	0.1 μm	0.04 nm
accelerating structure offset	girder axis	σ_4	10 μm	0.03 nm
accelerating structure tilt	girder axis	σ_t	200 μradian	0.38 nm
articulation point offset	wire reference	σ_5	12 μm	0.1 nm
girder end point	articulation point	σ_6	5 μm	0.02 nm
wake monitor	structure centre	σ_7	5 μm	0.54 nm
quadrupole roll	longitudinal axis	σ_r	100 μradian	≈ 0.12 nm

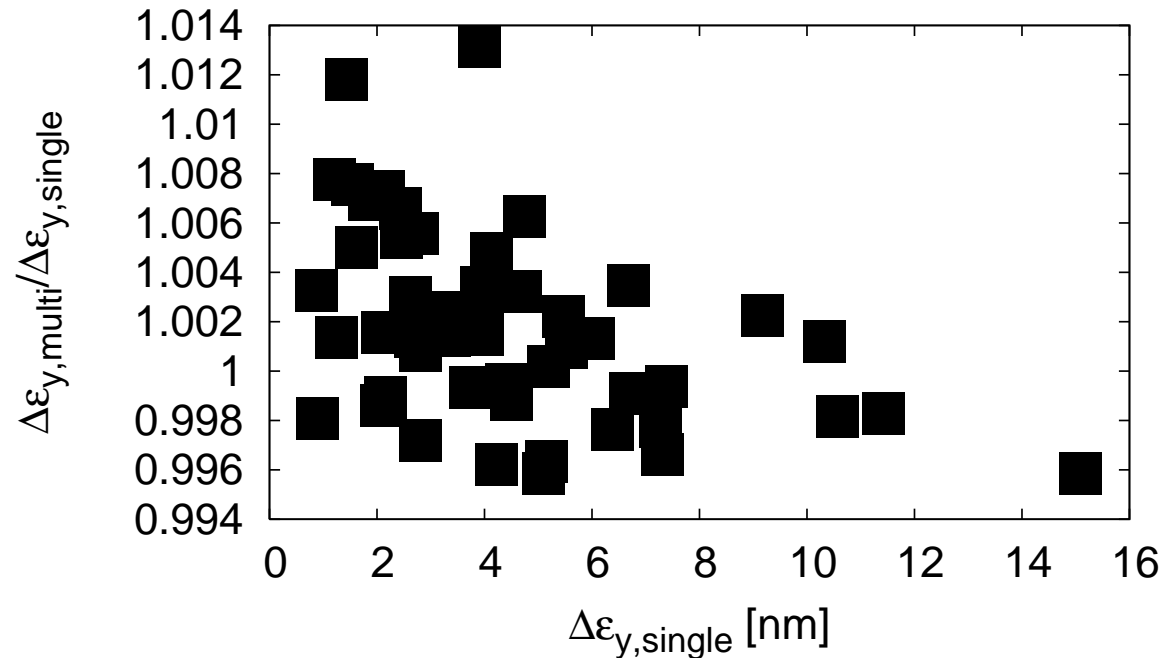
- Selected a good DFS implementation
 - trade-offs are possible
- Multi-bunch wakefield misalignments of 10 μm lead to $\Delta\epsilon_y \approx 0.13$ nm
- Performance of local pre-alignment is acceptable



Multi-Bunch Static Imperfections

- In CLIC

- we misalign all structures
- perform one-to-one steering with a multi-bunch beam
- perform one-to-one steering with a single bunch
- compare the emittance growth



CLIC Example of Fast Imperfection Tolerances

- Many sources exist

Source	Luminosity budget	Tolerance
Damping ring extraction jitter	1%	
Magnetic field variations	?%	
Bunch compressor jitter	1%	
Quadrupole jitter in main linac	1%	$\Delta\epsilon_y = 0.4 \text{ nm}$ $\sigma_{jitter} \approx 1.8 \text{ nm}$
Structure pos. jitter in main linac	0.1%	$\Delta\epsilon_y = 0.04 \text{ nm}$ $\sigma_{jitter} \approx 800 \text{ nm}$
Structure angle jitter in main linac	0.1%	$\Delta\epsilon_y = 0.04 \text{ nm}$ $\sigma_{jitter} \approx 400 \text{ nradian}$
RF jitter in main linac	1%	
Crab cavity phase jitter	1%	$\sigma_\phi \approx 0.01^\circ$
Final doublet quadrupole jitter	1%	$\sigma_{jitter} \approx 0.1 \text{ nm}$
Other quadrupole jitter in BDS	1%	
...	?%	

RF Constraints

- To limit the breakdown rate and the severeness of the breakdowns
- The maximum surface field has to be limited

$$\hat{E} < 260 \text{ MV/m}$$

- The temperature rise at the surface needs to be limited

$$\Delta T < 56 \text{ K}$$

- The power flow needs to be limited
 - related to the badness of a breakdown

empirical parameter is

$$P/(2\pi a)\tau^{\frac{1}{3}} < 18 \frac{\text{MW}}{\text{mm}} \text{ns}^{\frac{1}{3}}$$

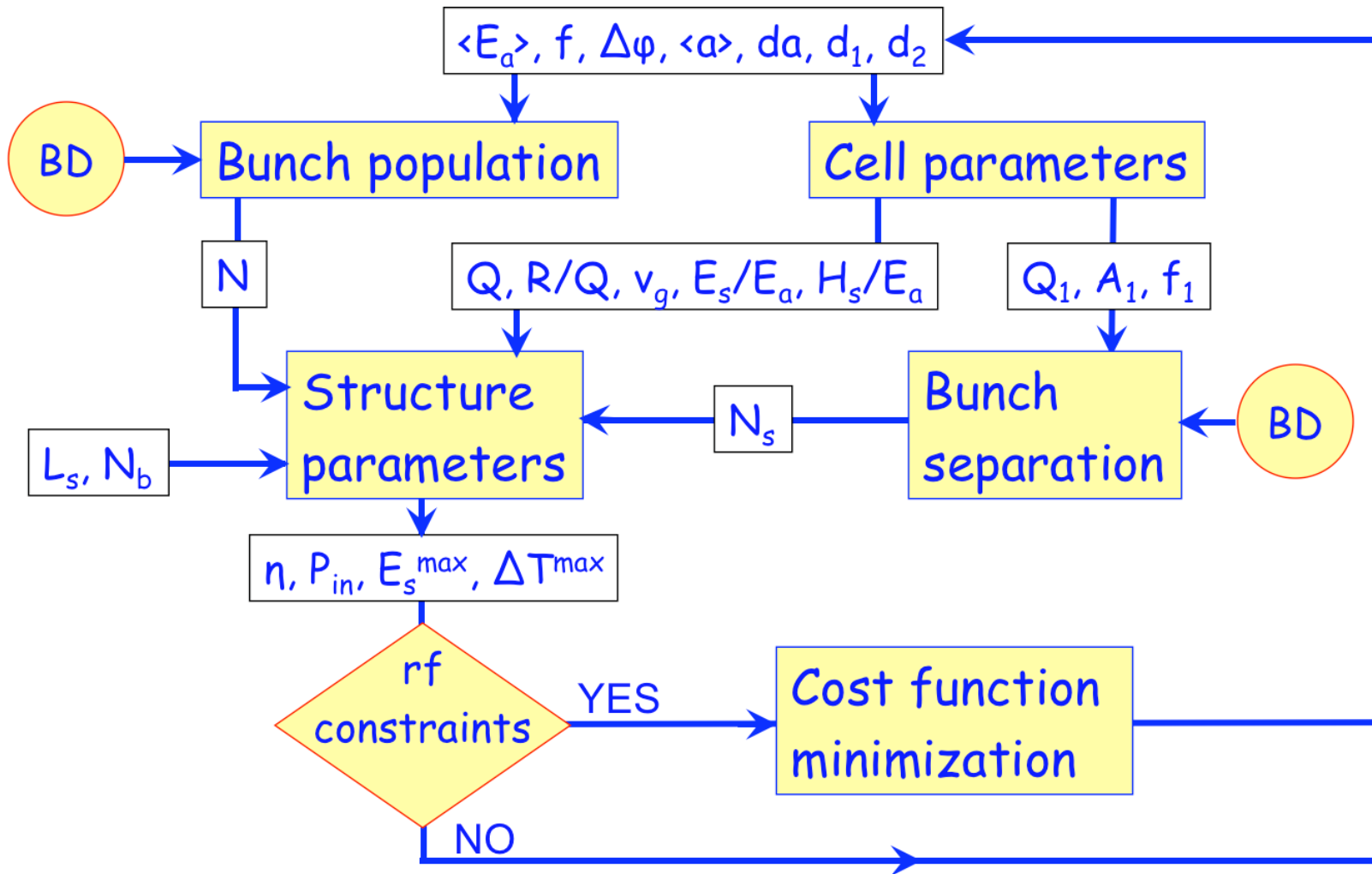
RF Work Flow

- Calculate RF properties of cells with different a/λ
 - structures can be constructed by interpolating between these values
- Remove all structures with a too high surface field
- Determine the pulse length supported by the structure
- Estimate long-range wake and chose bunch distance
 - bunch charge is given by beam dynamics
- Calculate RF to beam efficiency for the structure

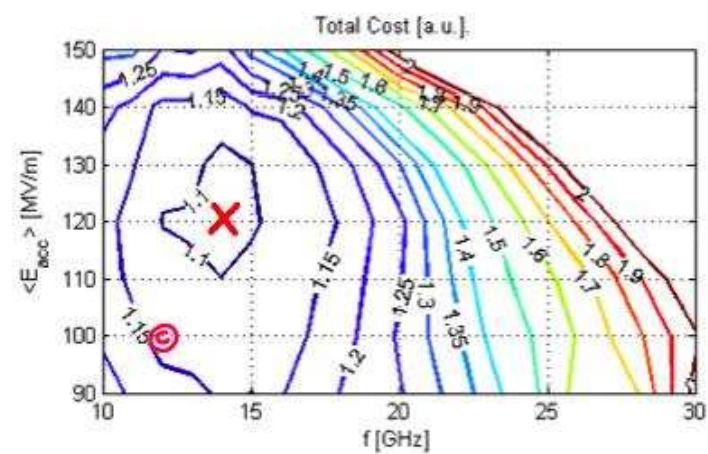
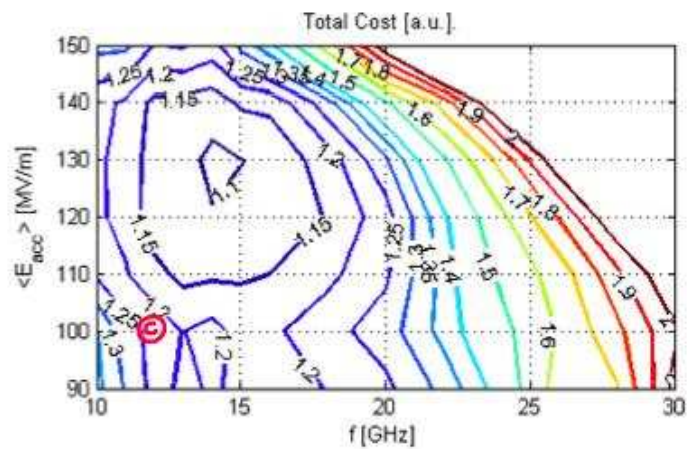
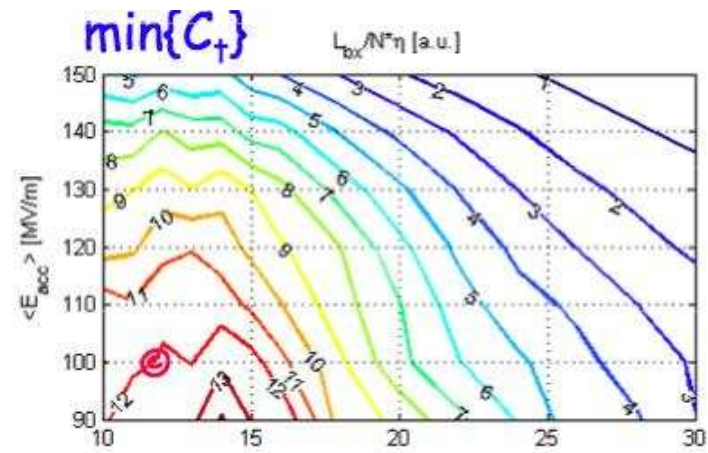
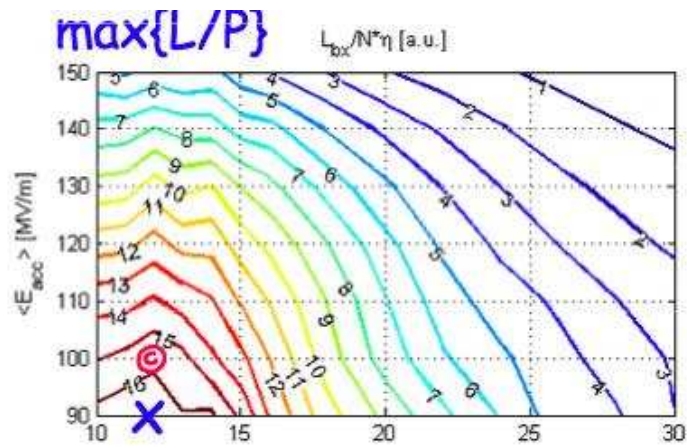
Cost Model

- The machine should be optimised for lowest cost
 - power consumption will also limit the choice
- A simplified cost model can be developed
 - e.g. cost per unit length of linac
 - energy to be stored in drive beam accelerator modulators per pulse
 - ...
- With this model one can identify the cheapest machine

Work Flow



Results



Results 2

$$FoM = L_{bx}/N \cdot n$$

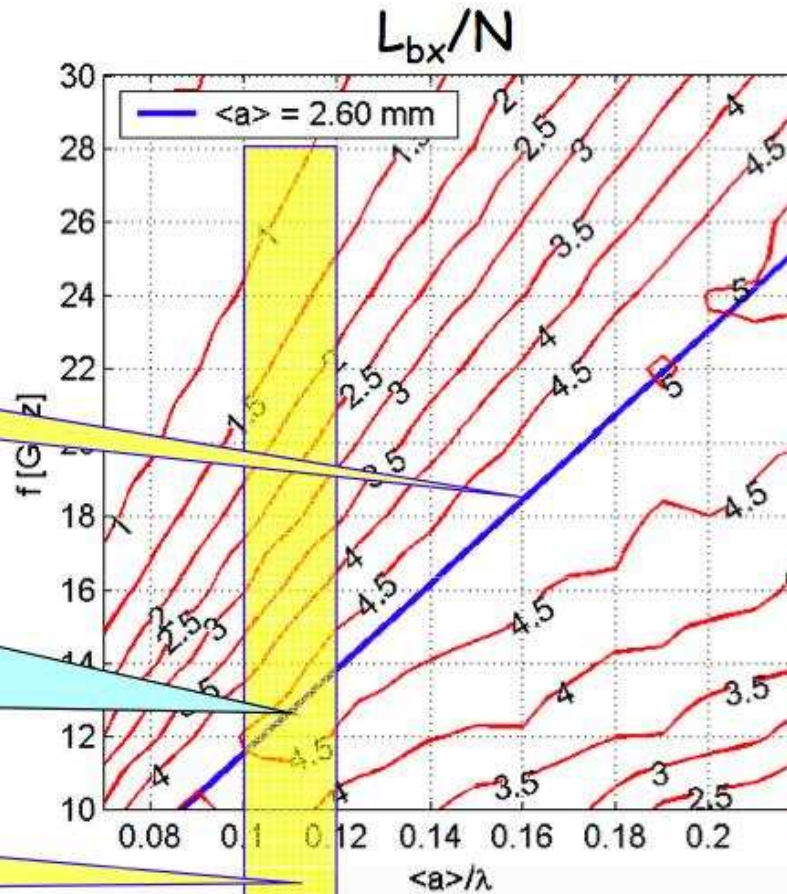
BD

RF

BD optimum aperture:
 $\langle a \rangle = 2.6 \text{ mm}$

Why X-band?
 Crossing gives
 optimum frequency

RF optimum aperture:
 $\langle a \rangle / \lambda = 0.1 \div 0.12$

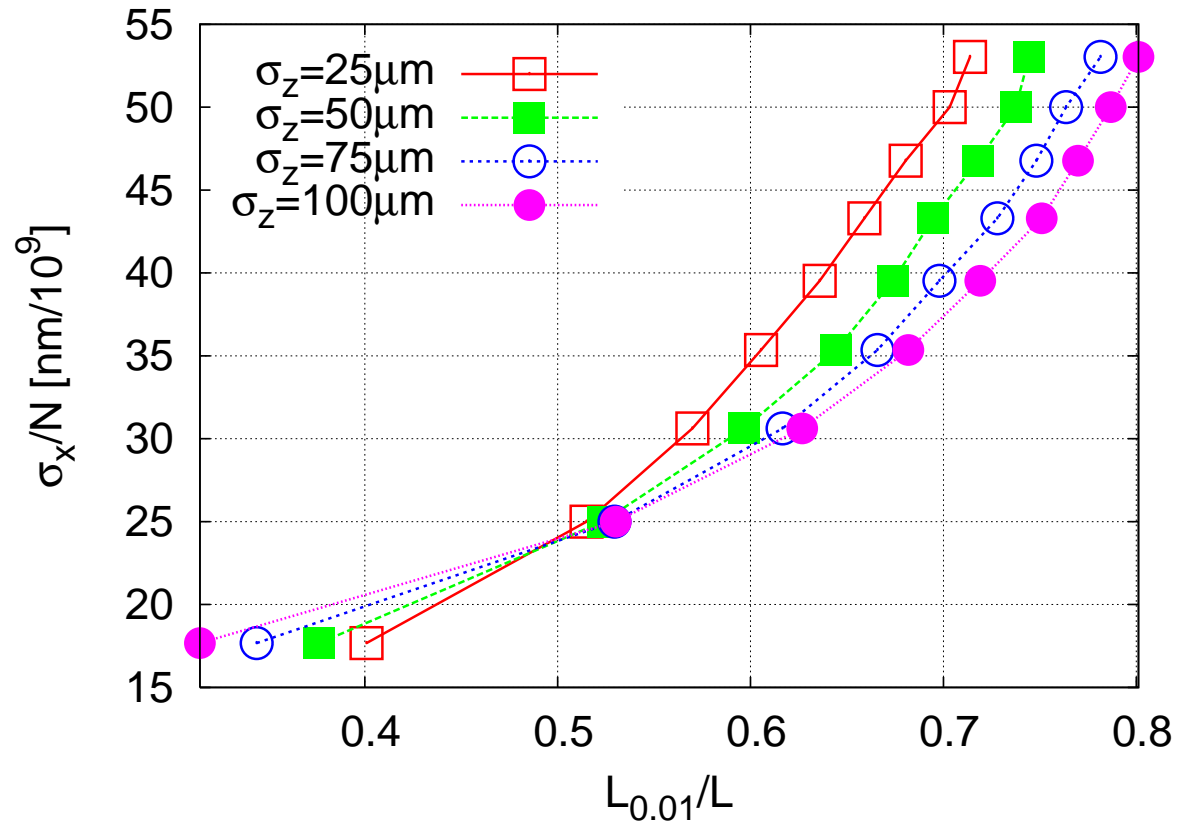


Lattice at Lower Energy

Required Beam Size (CLIC 500GeV)

- Roughly constant luminosity spectrum quality for constant N/σ_x
- Required beam size is between 25 and 40 nm for beam with $N = 10^9$ particles
 - scales with the square of the charge
- For $\beta_x = 10$ mm and $N = 4 \times 10^9$ requires $\epsilon_x \approx 1 \mu\text{m}$

$$\epsilon_{x,opt} \approx \left(\frac{N}{4 \times 10^9} \right)^2 \frac{10 \text{ mm}}{\beta_x} \mu\text{m}$$



Relative Luminosity

- Relevant parameter is

$$D = \frac{\beta_x}{\text{mm}} \frac{\epsilon_x}{\mu\text{m}} \left(\frac{10^9}{N} \right)^2$$

$$\frac{L_{bx}}{N} \propto \frac{1}{\sqrt{D}}$$

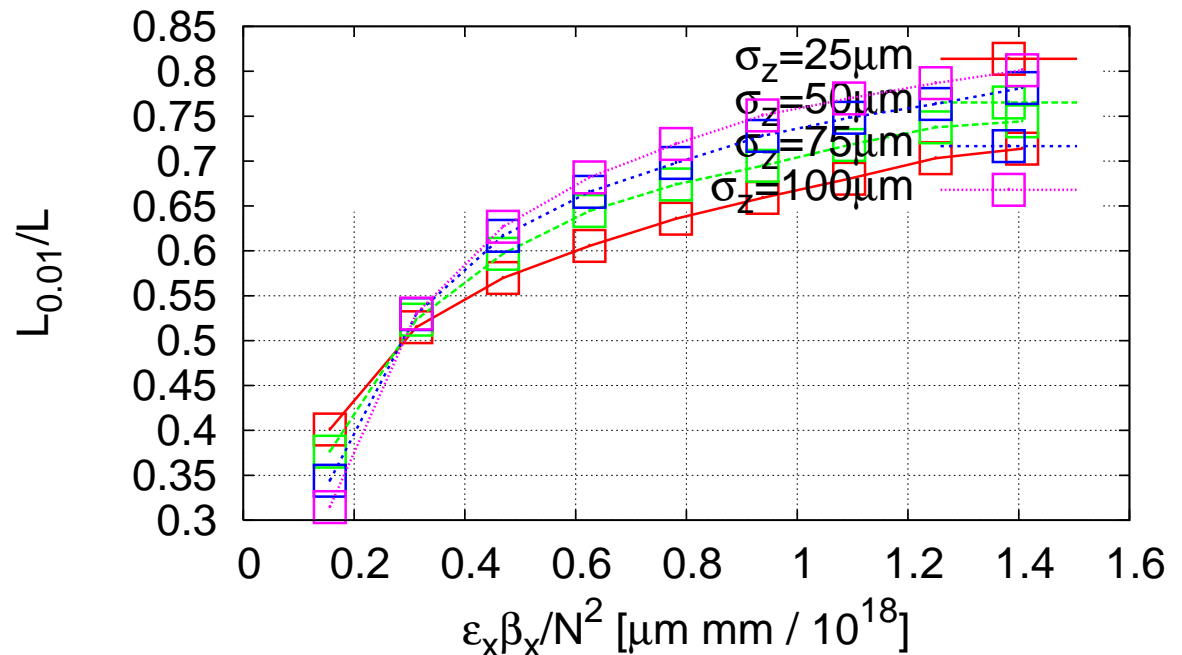
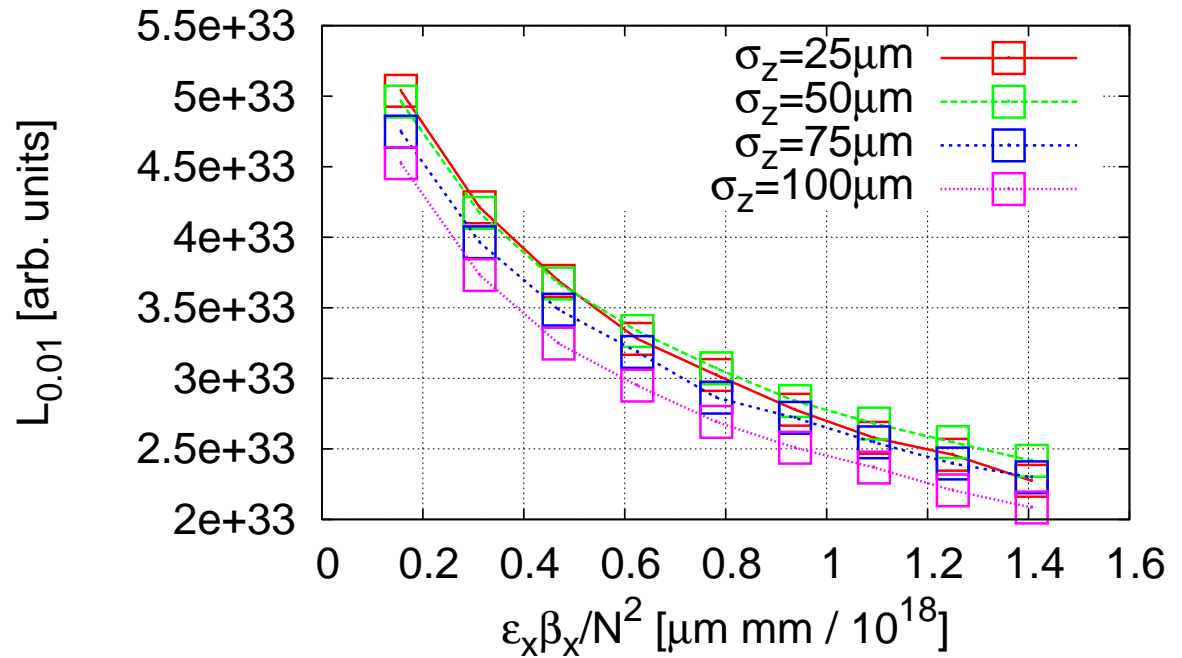
- Require this value to be in the range 0.3–0.7

- $\approx 30\%$ more luminosity for lower value

- NLC had $N = 7.5 \times 10^9 \beta_x = 10 \text{ mm}$ and $\epsilon_x = 4 \mu\text{m}$

- $D = 0.7$

\Rightarrow close to optimum



Beam Jitter at Lower Energy

- Two main limitations
 - local beam stability
 - integrated residual effect along the machine
- To keep the local beam stability constant yields the limitation
 - $Nw_{\perp}(2\sigma_z) = \text{const}$
 - keeps the beam energy spread constant

- A second limitation arises from the integral effect

$$\frac{d}{ds} \frac{\Delta y' / \sigma'_y}{y / \sigma_y} \propto \frac{Nw_{\perp} \sigma_y}{E \sigma'_y}$$

- Integral using lattice scaling $\beta = \beta_0 \sqrt{E(s)/E_0}$ yields

$$\frac{\Delta y' / \sigma'_y}{y / \sigma_y} \propto \frac{Nw_{\perp} \beta_0}{G} \sqrt{\frac{E_f}{E_0}}$$

- $Nw_{\perp}(2\sigma_z) = \text{const}$ is stronger limitation as long as
 - $G \geq \sqrt{E_f/E_{f,0}} G_0$
 - For 500 GeV $G \geq 41 \text{ MV/m}$

Emittance Growth at Lower Energy

- Express structure induced emittance growth as function of energy and gradient

$$\frac{d}{ds} \frac{\Delta\epsilon(s)}{\epsilon} \propto \left(\frac{Nw_{\perp}(2\sigma_z)\Delta y L_{cav}}{E(s)} \frac{1}{\sigma'_y(s)} \right)^2 \frac{1}{L_{cav}}$$

using the lattice scaling $\beta = \beta_0 \sqrt{E(s)/E_0}$ one finds

$$\Delta\epsilon_{cav} \propto \frac{N^2 w_{\perp}^2 (2\sigma_z) \Delta y^2 \beta_0 L_{tot,cav}}{G} \sqrt{\frac{E_f}{E_0}}$$

⇒ Could increase $Nw_{\perp}(2\sigma_z)$ by factor 2.4 at 500 GeV

- for constant gradient

- For constant Nw_{\perp} and L_{cav} we find $G \geq 41$ MV/m
- For constant Nw_{\perp} and doubled L_{cav} we find $G \geq 82$ MV/m
 - but for $G = 50$ MV/m still only 1.6 times as high as at 3 TeV
- Dispersive emittance growth scales as

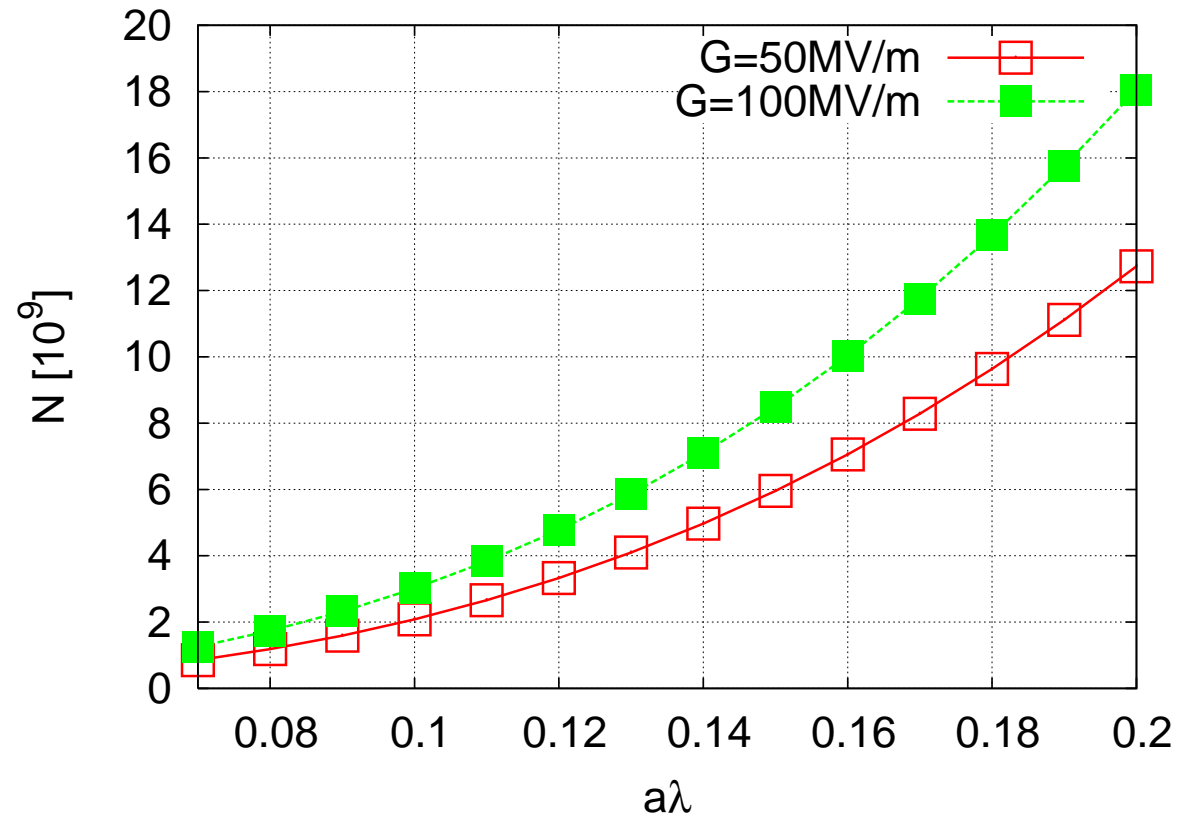
$$\Delta\epsilon_{tot,disp} \propto \frac{\Delta E^2 \Delta y^2}{G} \sqrt{\frac{E_f}{E_0}}$$

⇒ independent of structure length

- Total emittance growth should not increase much, first simulations confirm this

Aperture and Bunch Charge

- Procedure is similar to the one for 3 TeV
 - $\sigma_y(N)$ from single bunch longitudinal wake
 - N, σ_z from transverse single bunch wake
- Keep local beam stability constant
 - leads to less bunch charge than for 3 TeV
 - but longer bunches



Luminosity

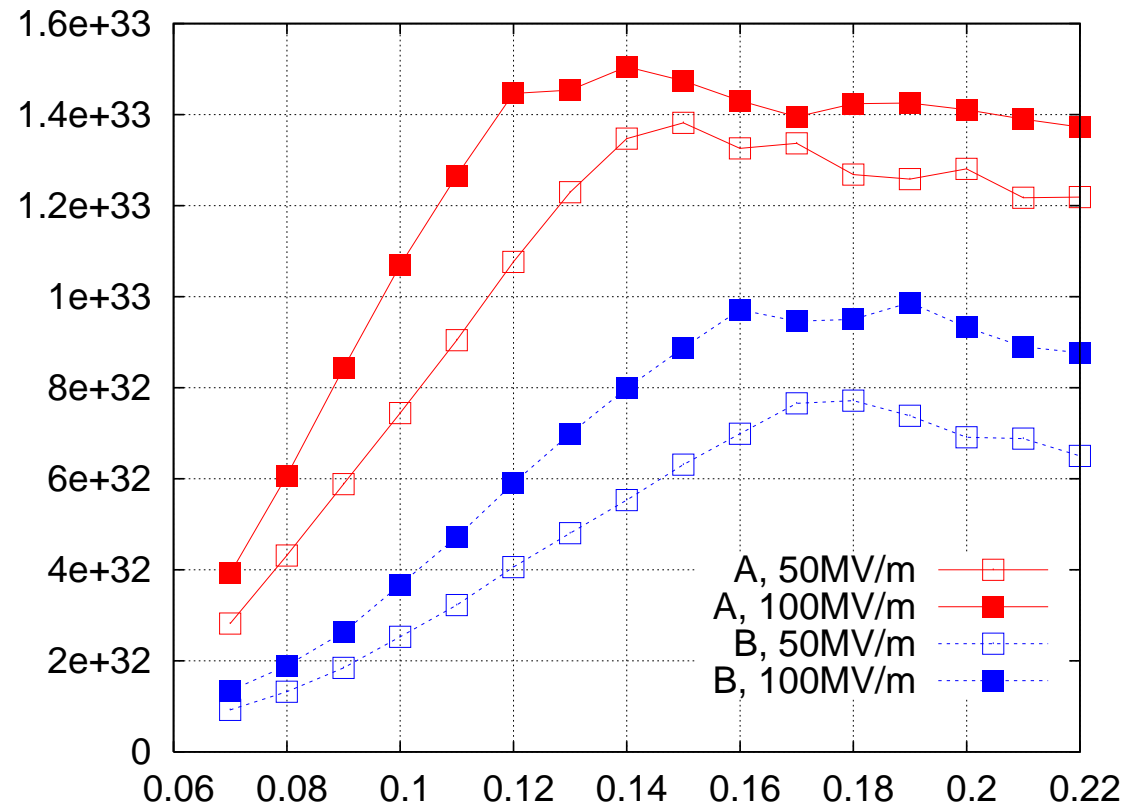
Assume the following

- case A

- emittance from 3 TeV
- beta-functions of $\beta_x = 10 \text{ mm}$ and $\beta_y = 0.1 \text{ mm}$ at the interaction point

- case B

- horizontal emittance from $\epsilon_x = 3 \mu\text{m}$ at the damping ring to $\epsilon_x = 4 \mu\text{m}$ at the interaction point
- vertical emittance from $\epsilon_y = 10 \text{ nm}$ at the damping ring to $\epsilon_y = 40 \text{ nm}$ at the interaction point
- beta-functions of $\beta_x = 8 \text{ mm}$ and $\beta_y = 0.1 \text{ mm}$ at the interaction point



Summary

- You had a glimpse on the most important main linac topics
- To really understand experiments are nice
 - a cheap way is to use a simulation code
 - and play with it

Thanks



Many thanks to you for listening (I hope) and to those who helped preparing lecture