

AS Sources Homework  
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 Rev. 0: November 12, 2011

1. Space Charge Derivation: Derive the space charge equations on slides 30 and 31. Show that the SC forces go to zero as  $\gamma$  gets large.
2. Derive the Spin Rotation equations on slide 111:

$$\theta_{Bend} = \frac{B(kG)L(m)}{33.359E(GeV)} \text{ rad} \quad (14')$$

$$\theta_{prec} = \frac{E(GeV)}{0.44065} \theta_{Bend} \quad (15')$$

$$\phi_{rot} = \frac{B(kG)L(m)}{33.359E(GeV)} \quad (16')$$

$$\theta_{prec_{\mathcal{E}}} = \left[ \left( \frac{\gamma^2 - 1}{\gamma} \right) \left( \frac{g}{2} \right) - \gamma \right] \theta_{Bend_{\mathcal{E}}} \quad (22)$$

The Lorentz force is given by Jackson (11.168) in cgs units as

$$\frac{d\vec{\beta}}{dt} = \frac{e}{\gamma mc} [\vec{\mathcal{E}} + \vec{\beta} \times \vec{B} - \vec{\beta}(\vec{\beta} \cdot \vec{B})] \quad \{\text{cgs}\} \quad (1)$$

wherein vectors are indicated by an arrow,  $\vec{\mathcal{E}}$  is the electric field,  $\vec{B}$  the magnetic field, and  $\vec{\beta} = \frac{\vec{v}}{c}$ . Convert (1) to MKS by replacing  $\vec{B}$  by  $c\vec{B}$ ; also multiply both sides by  $c$  to convert  $\vec{\beta}$  to  $\vec{v}$  and change  $e$  to  $q$  for aesthetics

$$\frac{d\vec{v}}{dt} = \frac{q}{\gamma mc} [c\vec{\mathcal{E}} + \vec{v} \times \vec{B} - \vec{\beta}(\vec{v} \cdot \vec{B})]. \quad \{\text{MKS}\} \quad (2)$$

The BMT equation is given by Jackson (11.170) in cgs units as

$$\frac{d\vec{s}}{dt} = \frac{e}{mc} \vec{s} \times \left[ \left( \frac{g}{2} - 1 + \frac{1}{\gamma} \right) \vec{B} - \left( \frac{g}{2} - 1 \right) \frac{\gamma}{\gamma + 1} \vec{\beta}(\vec{\beta} \cdot \vec{B}) - \left( \frac{g}{2} - \frac{\gamma}{\gamma + 1} \right) \vec{\beta} \times \vec{\mathcal{E}} \right]. \quad \{\text{cgs}\} \quad (3)$$

Convert (3) to MKS by replacing  $\vec{B}$  by  $c\vec{B}$  and change  $e$  to  $q$

$$\frac{d\vec{s}}{dt} = \frac{q}{m} \vec{s} \times \left[ \left( \frac{g}{2} - 1 + \frac{1}{\gamma} \right) \vec{B} - \left( \frac{g}{2} - 1 \right) \frac{\gamma}{\gamma + 1} \vec{\beta}(\vec{\beta} \cdot \vec{B}) - \left( \frac{g}{2} - \frac{\gamma}{\gamma + 1} \right) \frac{\vec{\beta} \times \vec{\mathcal{E}}}{c} \right]. \quad \{\text{MKS}\} \quad (4)$$

For  $c=2.998 \times 10^8$  m/s,  $mc^2=0.511$  MeV,  $\left(\frac{g}{2} - 1\right) = 1.16 \times 10^{-3}$ , BL in units of kG-m, and E in GeV (14)-(16) become

3. Laser Power: Assuming  $QE = 2e-5$  for a copper cathode, how much peak and average laser power is required to generate 250pC of charge at 120 Hz in the LCLS? How much laser energy is absorbed within a depth of one wavelength of the surface?
4. What is the underlying physics for Moeller scattering? How does Moeller scattering work?
5. Extra Credit: Thermal Diffusion, slides 119-121: What is the energy distribution as a function of time in a copper cathode at  $t=2$ ps for an incident 2 ps long laser pulse with an energy of DU (joules)?