Homework 1:

1. A) Make a non-relativistic approximation of the Hamiltonian

$$H = e\phi + c[m^2c^2 + (\vec{p} - e\vec{A}/c)^2]^{1/2},$$

and show the equation of motion is indeed the Lorentz equation.

B) Repeat the derivation without the approximation.

2. In a rectangular coordinate system, given $A_x = A_y = 0$ and

$$A_s = -Re[\sum_{n=1} \frac{1}{n}(b_n + ia_n)(x+iy)^n].$$

Derive a formula for corresponding magnetic field. How a_n and b_n are related in terms of placing the physical magnets for n=1, 2, …?

3. Derive the six-dimensional transfer map for a defocusing (in x) quadruple. Extract its linear matrix and show it is indeed symplectic.

4. With the thin lens approximation, show that the maximum and minimum values of the β function for the simple FODO cell are given by

$$\beta_{max} = 2F(\frac{1+\sin(\mu/2)}{1-\sin(\mu/2)})^{1/2},$$

$$\beta_{min} = 2F(\frac{1-\sin(\mu/2)}{1+\sin(\mu/2)})^{1/2}.$$

Evaluate these for a quadrupole spacing of 100 m and phase advance per cell of 80^0.

5. Given α, β at position s and the betatron tune ν in a ring, calculate the tune shift and β change due to a thin quadrupole at that position.