Beam Dynamics with Account of Environment Effects in Future e+e- Colliders : New Results

B.B. Levchenko

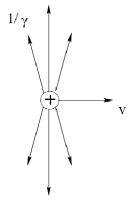
Institute of Nuclear Physics, Moscow State University

levtchen@mail.desy.de

I. Real Fields in Free space

Field of a Circular Cylinder

The field produced rapidly moving single charge



$$\vec{E} = \kappa \frac{q\,\gamma}{r^2}\Big[\frac{1-\beta^2}{1-\beta^2\,sin^2\,\theta}\Big]^{3/2}\frac{\vec{\mathbf{r}}}{r}$$

The radial electric field of a bunch shaped as a circular cylinder of length L with a uniform charge density ρ

$$E_\perp(r,\xi,z) = \kappa\rho\gamma\{zI_1 + (L-z)I_2\}$$

with

$$I_1 = \int\int \frac{(r-\sigma\cos(\xi-\phi))\,\sigma d\sigma d\phi}{(r^2+\sigma^2-2r\sigma\cos(\xi-\phi))\sqrt{\gamma^2 z^2+r^2+\sigma^2-2r\sigma\cos(\xi-\phi)}}$$

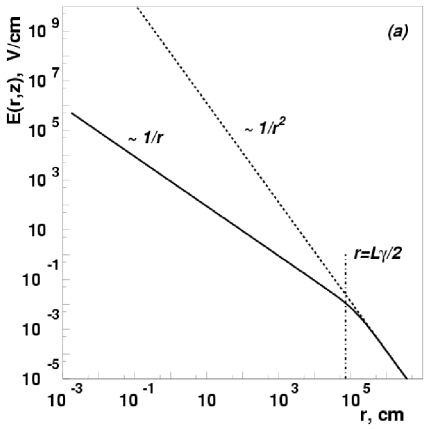
$$I_2 = \int\int \frac{(r-\sigma\cos(\xi-\phi))\,\sigma d\sigma d\phi}{(r^2+\sigma^2-2r\sigma\cos(\xi-\phi))\sqrt{\gamma^2 (L-z)^2+r^2+\sigma^2-2r\sigma\cos(\xi-\phi)}}$$

▸▸In ultra-relativistic limit, $\gamma >> 1$, simplified to (Fig. A)

$$E_\perp(r,z) = \kappa\frac{qN\gamma}{Lr}\Big\{\frac{z}{\sqrt{r^2+\gamma^2z^2}}\Big(1+\frac{3}{8}\frac{b^2}{r^2}C_1^2\Big) + \frac{L-z}{\sqrt{r^2+\gamma^2(L-z)^2}}\Big(1+\frac{3}{8}\frac{b^2}{r^2}C_2^2\Big)\Big\} \ast$$

with

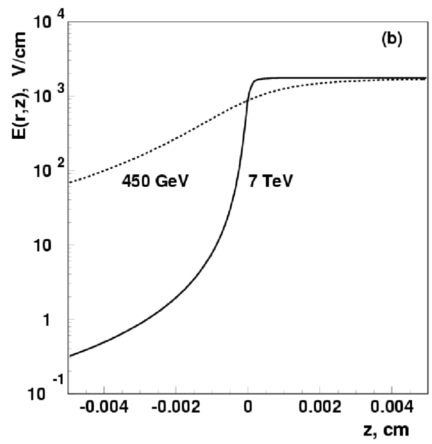
$$C_1 = \Big[1+\frac{\gamma^2z^2}{r^2}\Big]^{-1} \qquad C_2 = \Big[1+\gamma^2(L-z)^2/r^2\Big]^{-1}$$



A. The transverse profile of E field generated by a bunch. The $1/r^2$ behavior is restored only at the distance of several kilometers. Parameters of the bunch corresponds to the LHC proton beam.

In a very narrow transition region beyond the bunch tail, the field strength decreases rapidly (Fig. B)

$$E_\perp(r,z) \approx \kappa\frac{\lambda}{r}\Big(1-\frac{r^2}{2\gamma^2(L-z)^2}\Big)$$



B. The radial field variations with z near the bunch tail at fixed r=1 cm. Parameters corresponds to the LHC proton beam.

▸▸The space-time distribution of E field is well approximated by a step-like function

$$E_{cir}(r,z,t) = \kappa\frac{2\lambda}{r}\Big[\theta(z-\beta ct)-\theta(z-\beta ct-L)\Big]$$

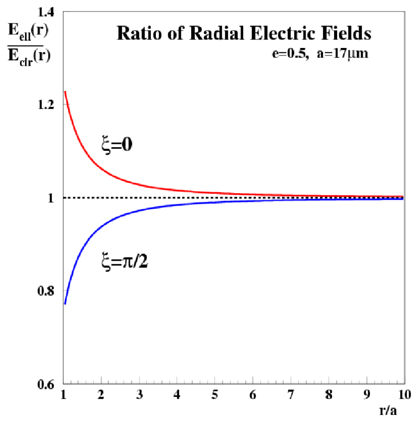
Field of an Elliptical Cylinder

With the same reasoning as above, the radial electric field produced by a rapidly moving elliptic bunch of length L, the eccentricity e and the semi-axis a

$$E_{ell}(r,z,t) = \kappa\frac{2\lambda}{r}\left(1+\frac{a^2e^2}{r^2}\cos 2\xi\right)\left[\theta(z-\beta ct)-\theta(z-\beta ct-L)\right]$$

The azimutal field variation is essential only at r ~ a. For lager r, the angular dependence vanish rapidly (Fig. C)

$$\frac{E_{ell}(r,z,t)}{E_{cir}(r,z,t)} = 1+\frac{a^2e^2}{r^2}\cos 2\xi$$



C. The ratio of radial electric fields produced by an elliptic bunch and a circular bunch of length L.

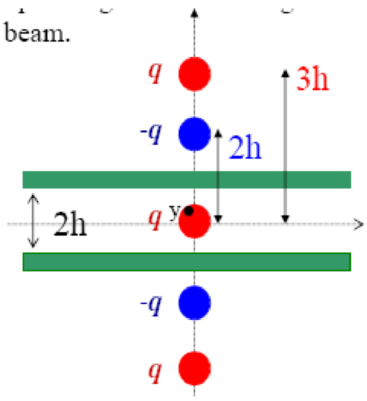
Theorem

- In the ultra-relativistic limit, γ→∞, the external fields of a bunch with a linear charge density λ(z) governed by the law

$$E(r,z,t) = \frac{2\kappa}{r}\lambda(z-\beta ct) \qquad B_\phi = -\frac{\beta}{c}E(r,z,t)$$

II. Image Fields Generated by a Bunch Between Perfectly Conducting Plates

Fields from Image Charges: Exact Solution



A relativistic bunch moves between infinitely wide parallel perfectly conducting plates.

☛Unsolved classical problem: Precisely summarize fields of an infinite series of mirrored image charges, displaced from the symmetry plane

$$E_{\perp,image}(x,\bar{x}) = 2\kappa\lambda\cdot\sum_k^{\infty}\left(\frac{1}{2kh-x_1}-\frac{1}{2kh+x_1}\right)-\sum_m^{\infty}\left(\frac{1}{2mh-x_2}-\frac{1}{2mh+x_2}\right)$$

- The exact solution is provided by the electric field structure function Λ depending only on normalized variables δ=x/h, δ=x/h

$$\Lambda(\delta,\bar{\delta}) = \frac{1}{2}\left[\frac{\pi}{2}\cdot\frac{\cos(\frac{\pi}{2}\bar{\delta})}{\sin(\frac{\pi}{2}\delta)-\sin(\frac{\pi}{2}\bar{\delta})}-\frac{1}{\delta-\bar{\delta}}\right]$$

The image field must be added to the direct field of the bunch to meet the boundary conditions

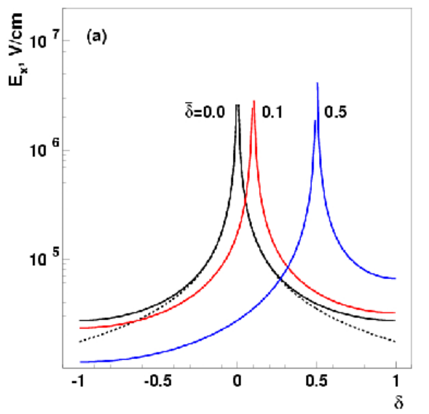
$$E_{\perp,tot}(x,\bar{x}) \;=\; E_{\perp,dir} + E_{\perp,image} = \frac{\pi\kappa\lambda}{h}\cdot\frac{\cos(\frac{\pi}{2}\bar{\delta})}{\sin(\frac{\pi}{2}\delta)-\sin(\frac{\pi}{2}\bar{\delta})}$$

With an increase of δ, the field gradient across the bunch significantly increases

$$\partial E/\partial x \sim 1/\cos^2(\pi\delta/2)$$

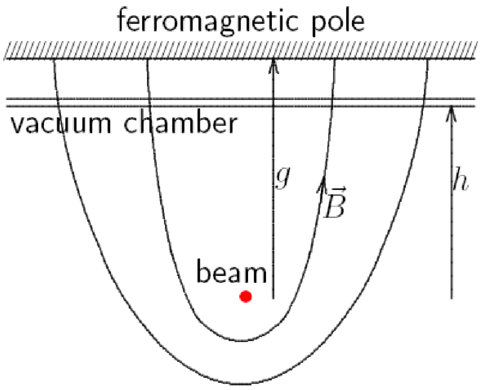
At opposite ends of the bunch diameter the difference $\Delta E(\delta)$

$$\Delta E(0.1) = 4360 \text{ V/cm}, \quad \Delta E(0.5) = 27300 \text{ V/cm}$$



Electric field distribution between parallel conducting plates for a number of bunch offset values.

Magnetic Images



The ferromagnetic boundaries are represented by a pair of infinitely wide parallel surfaces at x=±g. The DC field penetrates the vacuum camber, the AC fields do not penetrate.

Magnetic images can be treated in much the same way.

The magnetic field structure function is

$$H(\eta,\bar{\eta}) \;=\; \frac{1}{2}\Big[\frac{1}{\eta-\bar{\eta}} - \frac{\pi}{2}\cdot\frac{\cos(\frac{\pi}{2}\eta)}{\sin(\frac{\pi}{2}\eta)-\sin(\frac{\pi}{2}\bar{\eta})}\Big]$$

We distinguish between the DC and AC image field

$$B_{y,image,DC}(x,\bar{x}) \;=\; \frac{4\kappa\lambda\beta}{cg}\cdot\mathcal{B}\cdot H(\eta,\bar{\eta})$$

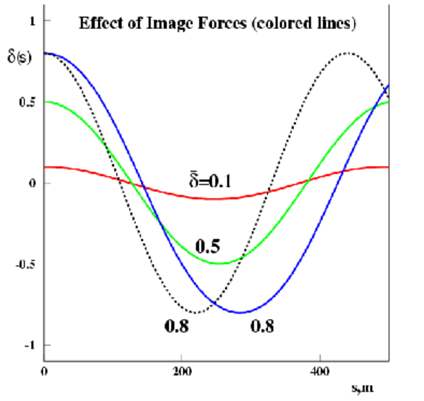
$$B_{y,image,AC}(x,\bar{x}) \;=\; -\frac{4\kappa\lambda\beta}{ch}\cdot(1-\mathcal{B})\cdot\Lambda(\delta,\bar{\delta})$$

To satisfy the boundary conditions, the image fields must be added to the direct field

$$\begin{aligned} B_{y,tot}(x,\bar{x}) \;&=\; B_y + B_{y,image,DC} + B_{y,image,AC} \\ &=\; -\frac{\pi\kappa\lambda\beta}{ch}\Big\{\frac{(1-\mathcal{B})\cos(\frac{\pi}{2}\bar{\delta})}{\sin(\frac{\pi}{2}\delta)-\sin(\frac{\pi}{2}\bar{\delta})} + \frac{h}{g}\cdot\frac{\mathcal{B}\cos(\frac{\pi}{2}\eta)}{\sin(\frac{\pi}{2}\eta)-\sin(\frac{\pi}{2}\bar{\eta})}\Big\}\end{aligned}$$

III. Applications

Coherent Motion and the Tune Shift



The coherent oscillation of the bunch under influence of the linear focusing and image forces (colored solid lines)

Direct space-charge fields, as well as fields due to image charges and currents shift the betatron frequencies.

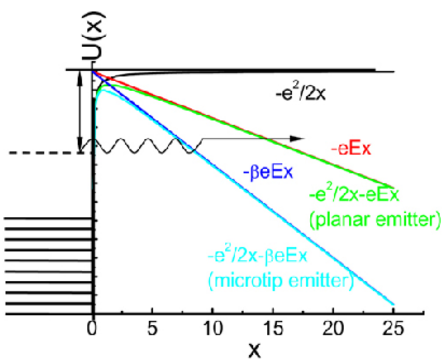
The coherent tune shift

$$\Delta\nu_x^{(coh)} = -\frac{2r_p R J\langle\beta\rangle}{q\beta c\gamma}\left[\left(\frac{1}{B\beta^2\gamma^2}+1\right)\frac{\xi_1(\bar{\delta}_0)}{h^2}+\frac{\xi_2(\bar{\eta}_0)}{g^2}\right] \quad *$$

The incoherent tune shift for the x-motion

$$\Delta\nu_x^{(inc)} = -\frac{2r_p R J\langle\beta\rangle}{q\beta c\gamma}\left[\frac{1}{B\beta^2\gamma^2}\left(\frac{1}{2a^2}+\frac{\epsilon_1(\bar{\delta})}{h^2}\right)+\left(\frac{\epsilon_1(\bar{\delta})}{h^2}+\frac{\epsilon_2(\bar{\delta})}{g^2}\right)\right] \quad *$$

Field Emission in CLIC Initiated by a Bunch

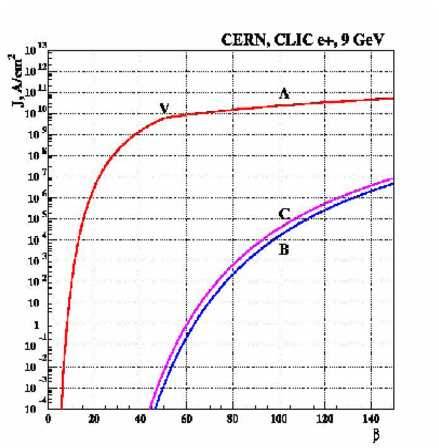


Application of a high electric field to a conductor produces a triangular shaped potential energy barrier through which electrons may tunnel.

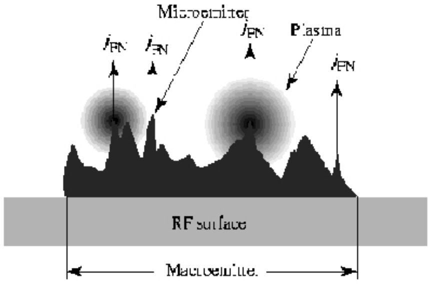
In addition to known electron sources, electron field emission intensified by multipacting can make a dominat contribution to the build-up of the electron cloud in a beam transport system.

In the Fowler-Nordheim theory with an accounting of the image force, the current density of field emission of electrons (FEC) is

$$J_{FN}(F) = A\frac{F^2}{\varphi\cdot t^2(y)}\exp\left\{-B\frac{\varphi^{3/2}}{F}\vartheta(y)\right\}$$



Densities of the electron current as a function of the field enhancement factor. Electrons are extracted by the bunch field from the copper irises in the CLIC RF structures at T=300 K. A) RF field +beam field; B) Only beam field; C) Main linac, inside a quadrupole beam pipe with the inner radius 2.35 mm. The kink V shows the value of surface field at which electrons escape the surface freely (the potential barrier vanishes).



Origin of the anomalous high FEC. The surface quality is characterized by the field enhancement factor βFN.
The local field strength $F=\beta FNE$

Only at β_{FN} below 7, the electron emission is negligible. However, the emission current at $\beta_{FN}=20$ increases by 10 order of magnitude ! The tunneling time Tt from the iris surface is of 2.5E-14 s and should be compared with time Tb the bunch travels the distance of own length at the velocity of light, Tb = 3.7E-13 s. There is enough time to extract electrons and accelerate them in the field of a single bunch.

IV. Summary

- We derive new expressions for electric and magnetic self-fileds produced by a relativistic bunch shaped as circular or elliptical cylinder with uniform charge density. In the ultra-relativistic limit, the radial electric field to a good accuracy is independent of the bunch shape at r>10a.

- We solved the classical problem of summing image fields generated by a relativistic bunch of charged particles moving with offset between infinitely wide parallel perfectly conducting plates.

- The exact solutions are represented by the structure function $\Lambda(\delta, \delta)$ of electric images and the structure function $H(\eta, \acute{\eta})$ of magnetic images, depending only on the normalized variables.