SUSY-Yukawa Sum Rule at the LHC and the ILC

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Blanke, Curtin, MP, 1004.5350 [hep-ph], PRD
Saelim, MP, work in progress

Anatomy of Standard Model Extensions at the Electroweak Scale

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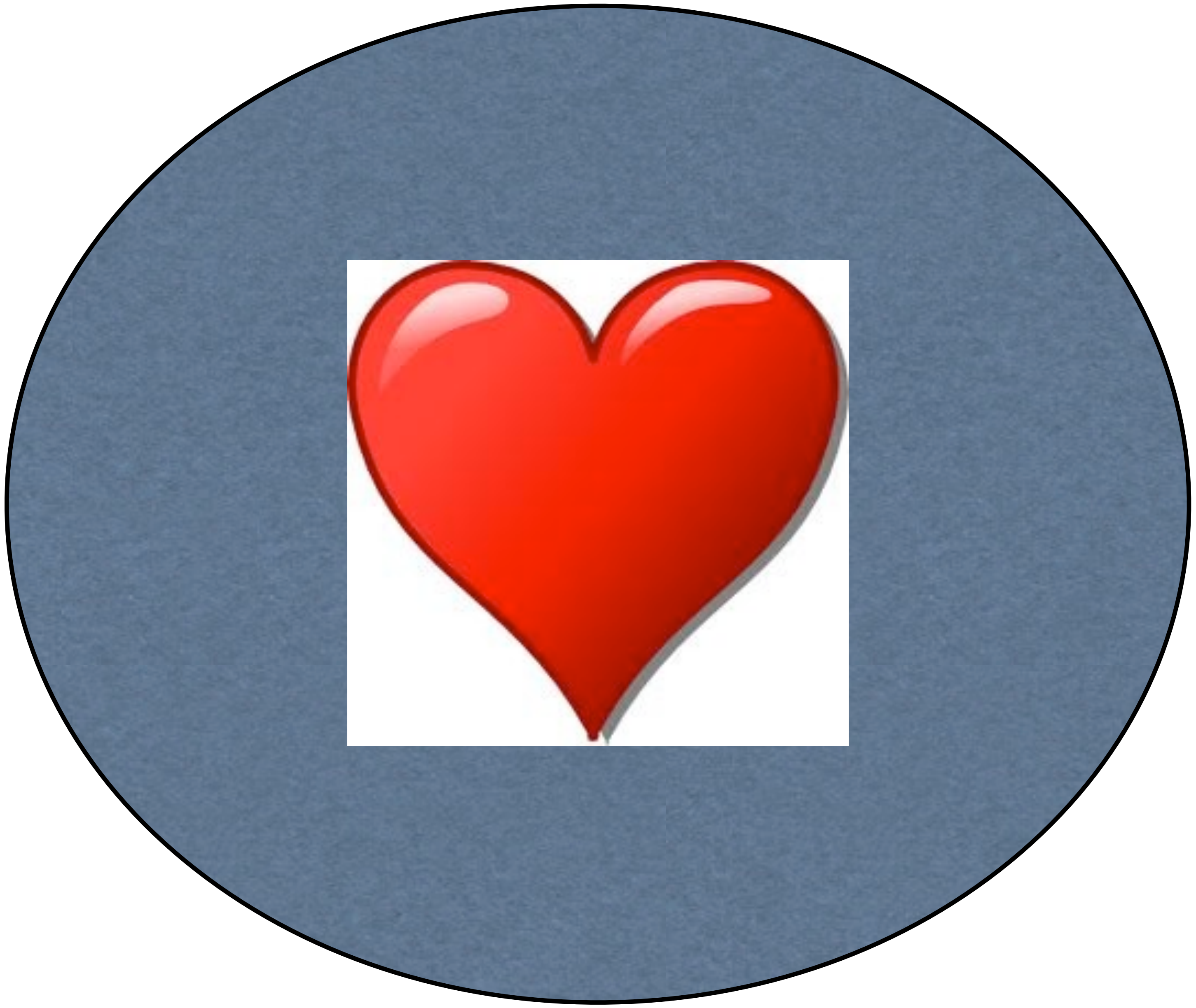
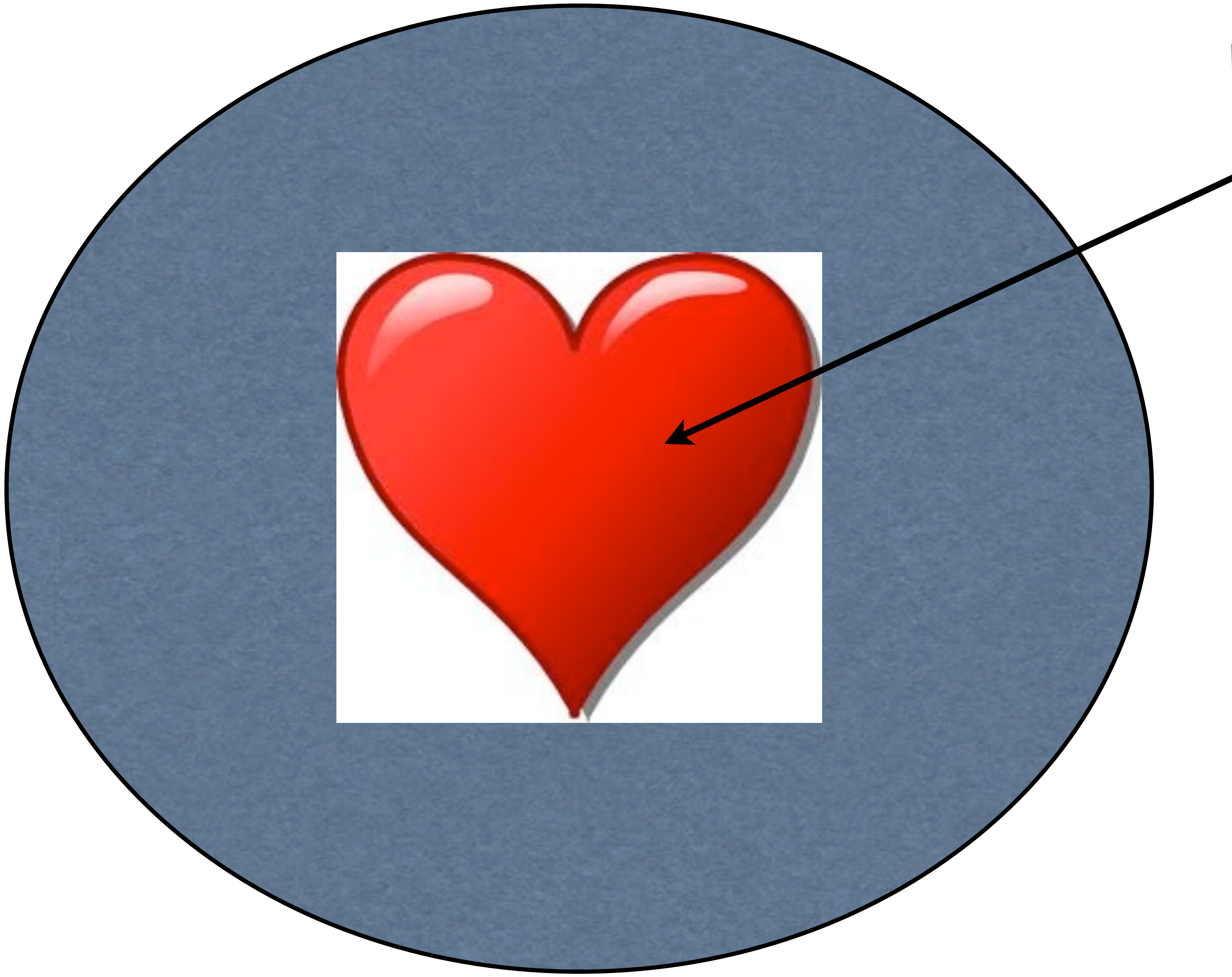


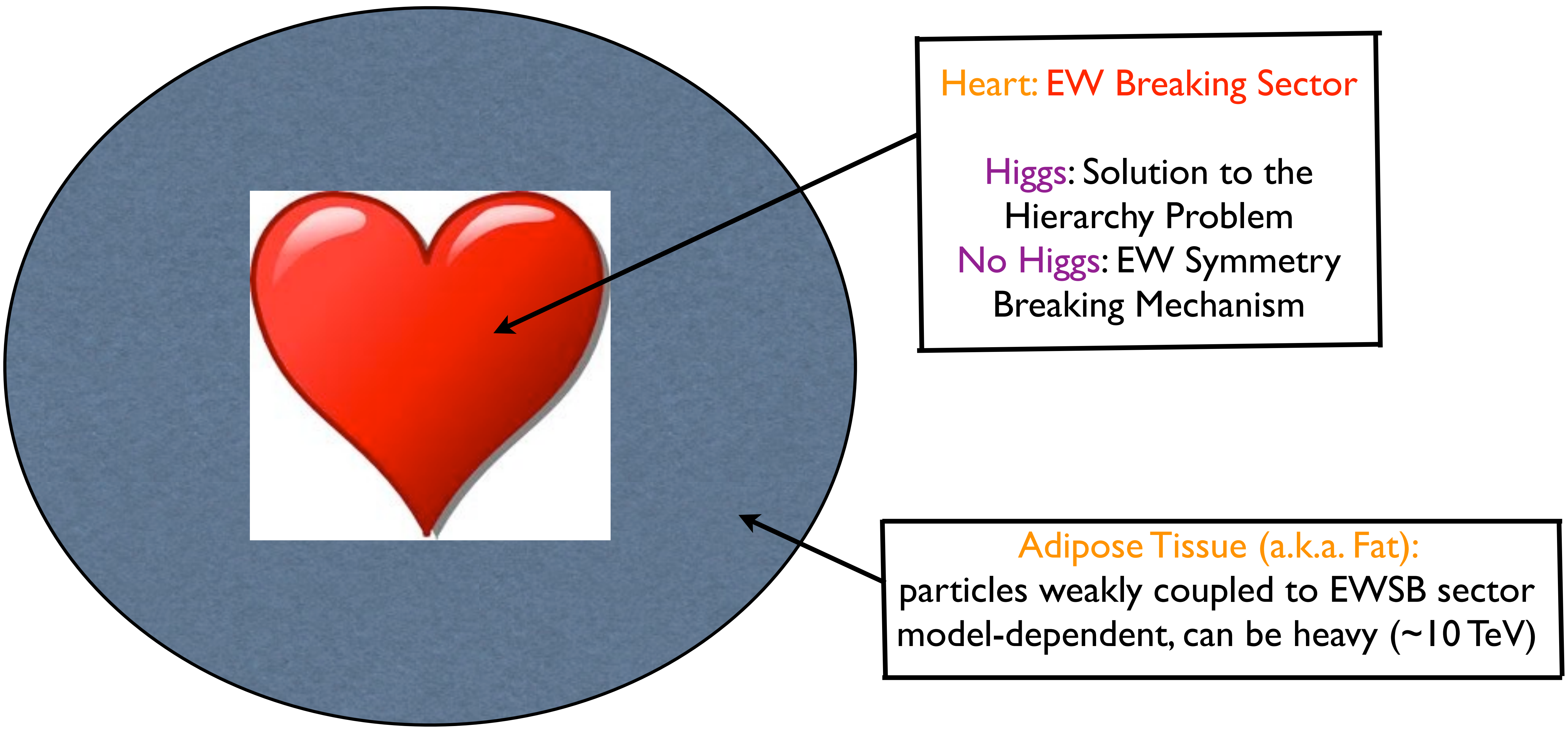
Anatomy of Standard Model Extensions at the Electroweak Scale



Heart: EW Breaking Sector

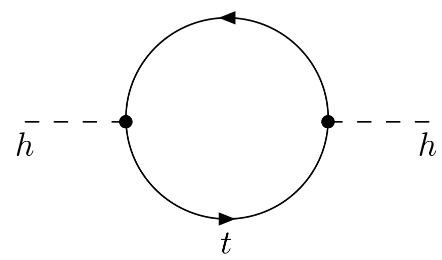
Higgs: Solution to the Hierarchy Problem
No Higgs: EW Symmetry Breaking Mechanism

Anatomy of Standard Model Extensions at the Electroweak Scale

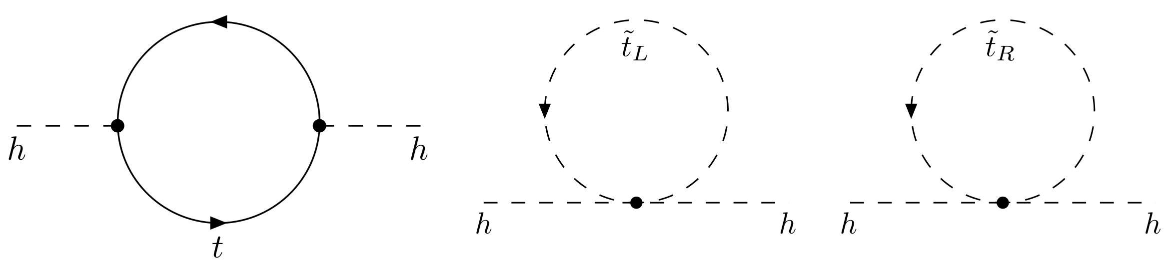


- To prove SUSY, test its heart: solution to hierarchy problem

- Focus on the top sector - largest SM Higgs coupling, must be at the weak scale (unless very finely tuned)



$$-\frac{3y_t^2}{8\pi^2}\Lambda^2 + A\log\Lambda + \ldots$$

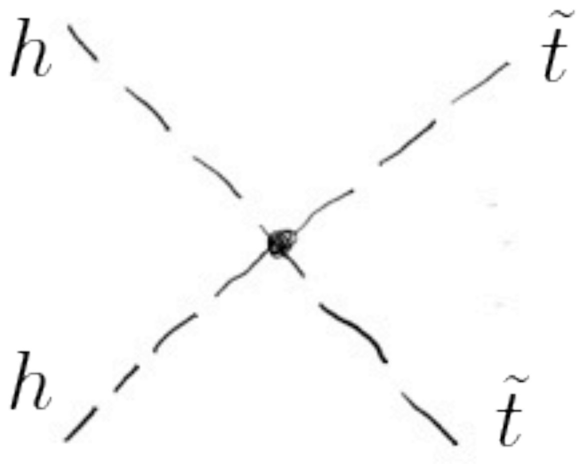


$$0 \times \Lambda^2 + \frac{3m_{\tilde{t}}^2}{8\pi^2}\log\Lambda + \ldots$$

- Why does it work:

$$\mathcal{L}_{\rm MSSM} = y_t h\bar{t}t + y_t^2 h^2 \left(|\tilde{t}_L|^2 + |\tilde{t}_R|^2\right) + \ldots$$

The same constant - sharp prediction! Test it?

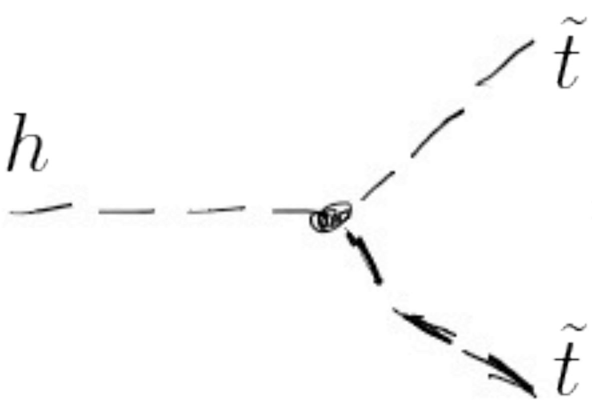


$= y_t^2$

Impossible to measure the quartic at the LHC!

[Challenge: prove me wrong!]

But: $h = v + h^0 + \ldots$ → cubic: $y_t^2 v h^0 |\tilde{t}|^2$



$= v y_t^2$

Still, (probably) impossible to measure at the LHC!

[Maybe Higgsstrahlung in stop production? **ILC?**]

But also: $V_{\rm SUSY} = y_t^2 v^2 \left(|\tilde{t}_L|^2 + |\tilde{t}_R|^2 \right)$ stop mass terms!

Problem: many other contributions to stop masses (both SUSY and SUSY-breaking)

$$V = (\tilde{t}_L^*, \tilde{t}_R^*)\ M^2 \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

$$M^2 = \begin{pmatrix} \underline{m_t^2} + M_{3L}^2 + \Delta_u & \sqrt{2} m_t \sin\beta (A_t - \mu \cot\beta) \\ \sqrt{2} m_t \sin\beta (A_t - \mu \cot\beta) & \underline{m_t^2} + M_{\tilde{t}_R}^2 + \Delta_{\bar{u}} \end{pmatrix}$$

Physical observables: mass eigenstates

$$\tilde{t}_1 = \cos\theta_t\, \tilde{t}_L + \sin\theta_t\, \tilde{t}_R$$
$$\tilde{t}_2 = -\sin\theta_t\, \tilde{t}_L + \cos\theta_t\, \tilde{t}_R$$

Observables: m_{t1}, m_{t2}, θ_t [Convention: $m_{t1} < m_{t2}$]

Express (11) matrix element in terms of eigenvalues + mixing angle:

$$m_t^2 + M_{3L}^2 + \Delta_u \; = \; m_{t1}^2 \cos^2\theta_t + m_{t2}^2 \sin^2\theta_t$$

big and unknown!

BUT, Sbottom masses have the same structure with the same M_{3L}^2 (enforced by $SU(2)_L$)

$$m_b^2 + M_{3L}^2 + \Delta_d \; = \; m_{b1}^2 \cos^2\theta_b + m_{b2}^2 \sin^2\theta_b$$

$$\boxed{\begin{aligned} m_t^2 - m_b^2 \; = \; & m_{t1}^2 \cos^2\theta_t + m_{t2}^2 \sin^2\theta_t \\ & - m_{b1}^2 \cos^2\theta_b - m_{b2}^2 \sin^2\theta_b - m_W^2 \cos 2\beta \end{aligned}}$$

“SUSY-Yukawa sum rule”

Dimensionless version:

$$\Upsilon = \frac{m_{\tilde{t}1}^2 \cos^2\theta_t + m_{\tilde{t}2}^2 \sin^2\theta_t - m_{\tilde{b}1}^2 \cos^2\theta_b - m_{\tilde{b}2}^2 \sin^2\theta_b}{v^2}$$

SUSY Prediction (at tree level):

$$\begin{aligned} \Upsilon_{\rm SUSY}^{\rm tree} &= \frac{1}{v^2}\left(\hat{m}_t^2 - \hat{m}_b^2 + m_Z^2 \cos^2\theta_W \cos 2\beta\right) \\ &= \begin{cases} 0.39 & \text{for } \tan\beta = 1 \\ \underline{0.28} & \text{for } \tan\beta \to \infty \end{cases} \end{aligned}$$

[Note: β dependence is $\tan^{-2}\beta$ in the large-$\tan\beta$ limit]

Allowed range outside SUSY? Consider arbitrary perturbative quartic:

$$\lambda|\tilde{t}|^2 h^2, \quad \lambda \leq 16\pi^2 \quad \Rightarrow \quad \underline{\Upsilon < 8\pi^2}$$

Loop Corrections:

Physical (pole) masses

Observable: $$\Upsilon = \frac{m_{t1}^2 \cos^2\theta_t + m_{t2}^2 \sin^2\theta_t - m_{b1}^2 \cos^2\theta_b - m_{b2}^2 \sin^2\theta_b}{v^2}$$

-We can define $\Upsilon(\mu)$ in terms of running masses/mixings evaluated at scale μ

-The tree-level sum rule applies to $\Upsilon(\mu)$ as long as $\mu \gg M_{susy}, v$

- Corrections are power-suppressed: $\mathcal{O}(M_{susy}^2/\mu^2)$

$$\Upsilon = \Upsilon(M_{\rm susy}) + \frac{A}{16\pi^2}\log\frac{M_{\rm susy}}{m_{\tilde{t},\tilde{b}}} + B_{\rm thresh}$$

0.28

depend on all SUSY masses

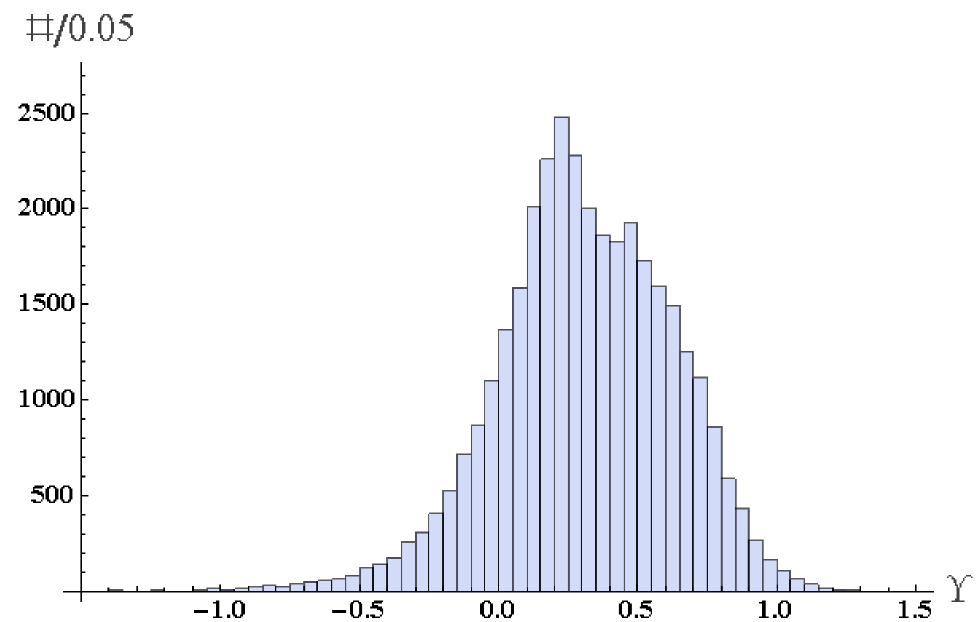


FIG. 2: Distribution of Υ for a `SuSpect` random scan of pMSSM parameter space. Scanning range was $\tan\beta \in (5, 40)$; $M_A, M_1 \in (100, 500)$ GeV; $M_2, M_3, |\mu|, M_{QL}, M_{tR}, M_{bR} \in (M_1 + 50$ GeV$, 2$ TeV$)$; $|A_t|, |A_b| < 1.5$ TeV; random sign(μ). EWSB, neutralino LSP, and experimental constraints ($m_H, \Delta\rho$, $b \to s\gamma$, a_μ, $m_{\tilde{\chi}_1^\pm}$ bounds) were enforced.

- "Order-one" corrections, due to the few-% level cancellation in the tree-level sum rule

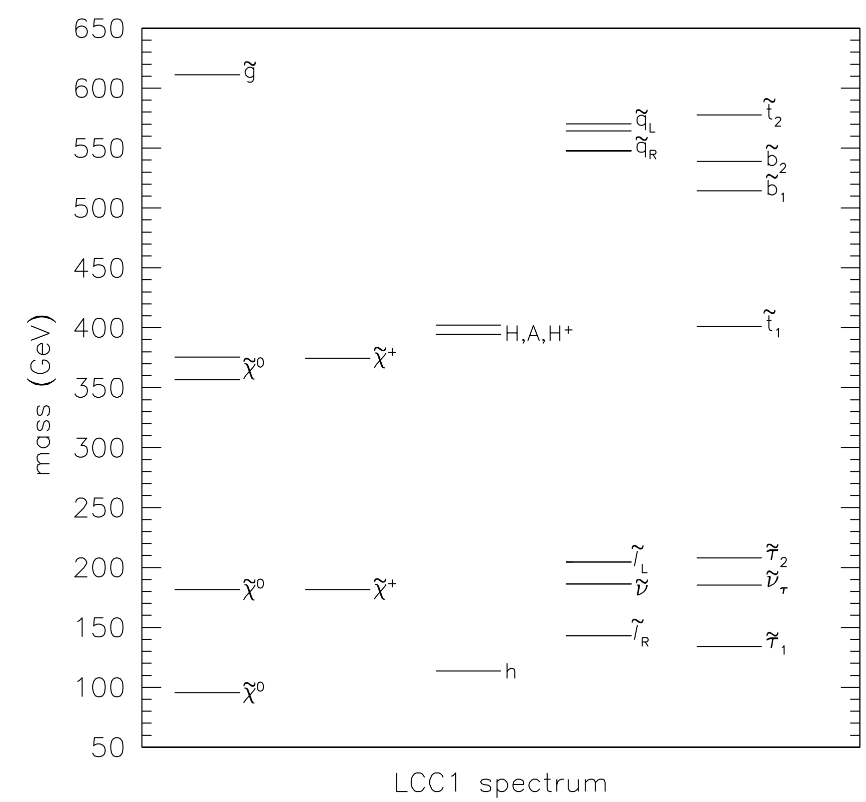
- Still, predicted range << range allowed outside SUSY

- The prediction gets sharper as more superpartner masses are measured!

Improving Theoretical Prediction of the Sum Rule with Data

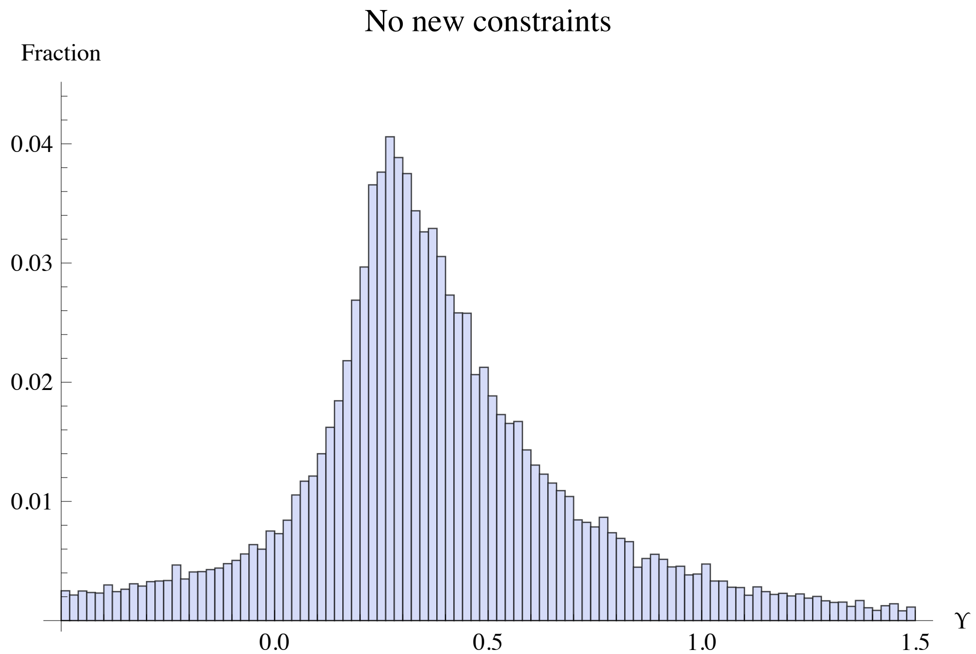
[MP, Saelim, in progress]

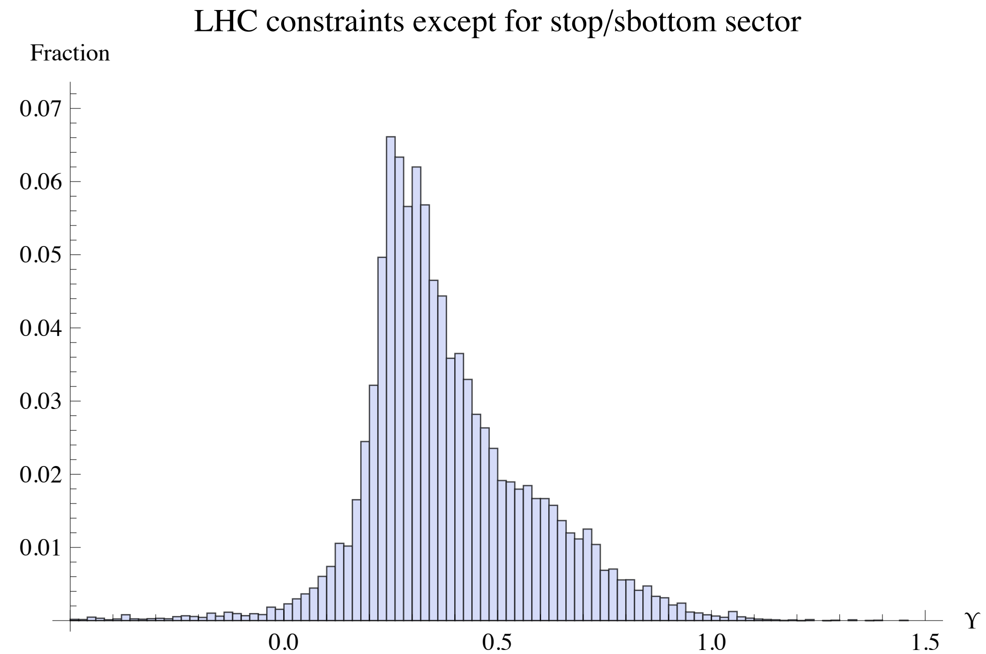
- Measuring MSSM parameters reduces the range of possible loop corrections, leads to sharper prediction of the sum rule
- Example: assume LCC1 point*, use projected LHC and ILC measurement errors from Baltz, Battaglia, Peskin, Wizansky, hep-ph/0602187
- Scan pMSSM parameter space using Markov Chain Monte Carlo approach
- Compute Υ for each point in the scan

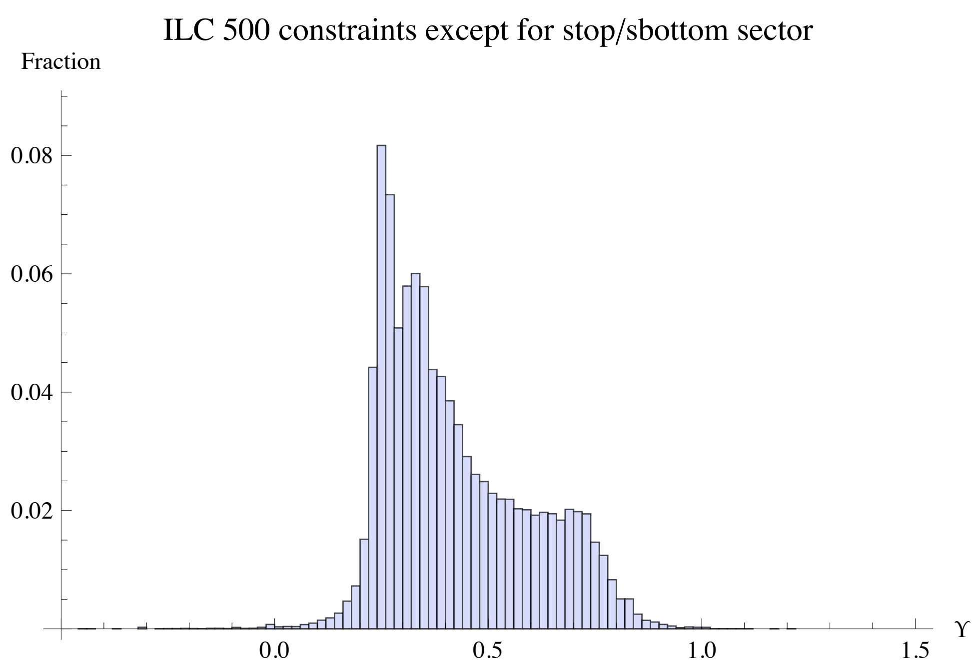


* - Yes, I know, it is now ruled out... It's just an example.

Results (PRELIMINARY!!!)







Pre-LHC: $\Upsilon_{th} = 0.18 \pm 0.85$

Post-LHC: $\Upsilon_{th} = 0.37 \pm 0.39$

Post-ILC: $\Upsilon_{th} = 0.42 \pm 0.19$

Measuring Stop and Sbottom Masses at the LHC

[Blanke, Curtin, MP, 1004.5350]

- We study two reactions: $pp \to \tilde{g}\tilde{g}, \quad \tilde{g} \to \bar{b}\tilde{b}, \quad \tilde{b} \to b\tilde{\chi}_1^0$
  $$pp \to \tilde{t}\tilde{t}^*, \quad \tilde{t} \to t\tilde{\chi}_1^0$$
- Both reactions are "generic": they occur in large parts of parameter space (though not guaranteed, of course)
- To simplify things, we choose the MSSM parameter point* such that both reactions (a) have branching ratios of 1, and (b) have no significant SUSY backgrounds

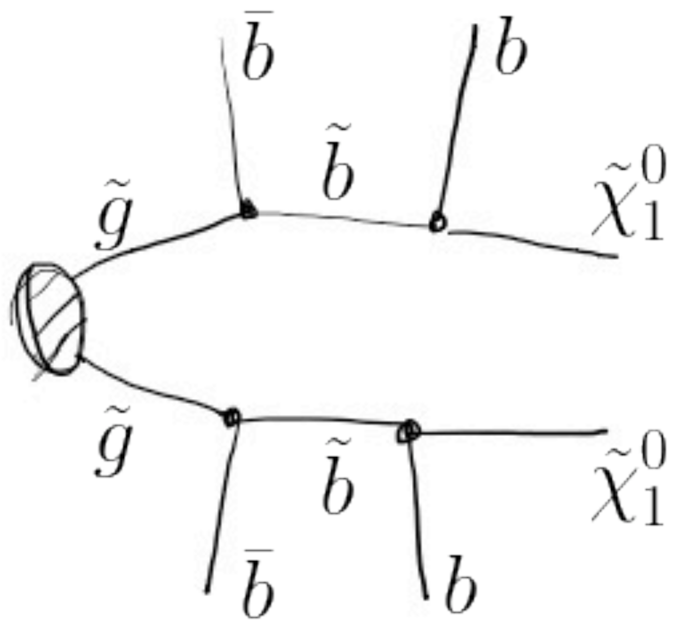
$\tan\beta$	M_1	M_2	M_3	μ	M_A	M_{Q3L}	M_{tR}	A_t
10	100	450	450	400	600	310.6	778.1	392.6

⇨

m_{t1}	m_{t2}	s_t	m_{b1}	m_{b2}	s_b	$m_{\tilde{g}}$	$m_{\tilde{\chi}_1^0}$
371	800	-0.095	341	1000	-0.011	525	98

* - Yes, I know, it is now ruled out... It's just an example.

Process 1: $pp \to \tilde{g}\tilde{g}, \quad \tilde{g} \to \bar{b}\tilde{b}, \quad \tilde{b} \to b\tilde{\chi}_1^0$



$\sigma(\tilde{g}\tilde{g}) = 11.6 \text{ pb}$ ➡ high rate √

Final state: 4 b-jets + MET

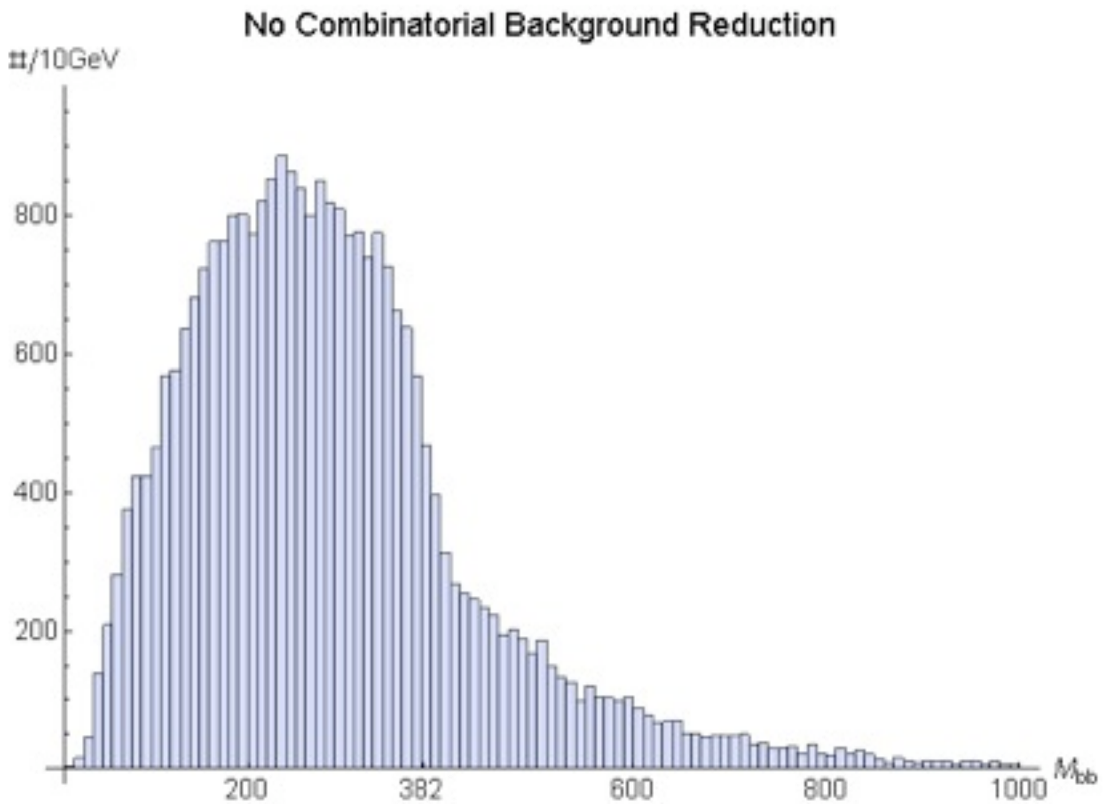
SM Backgrounds: $Z/W + 4j, \; t\bar{t}$

Cuts (standard): 4 b-tags, plus

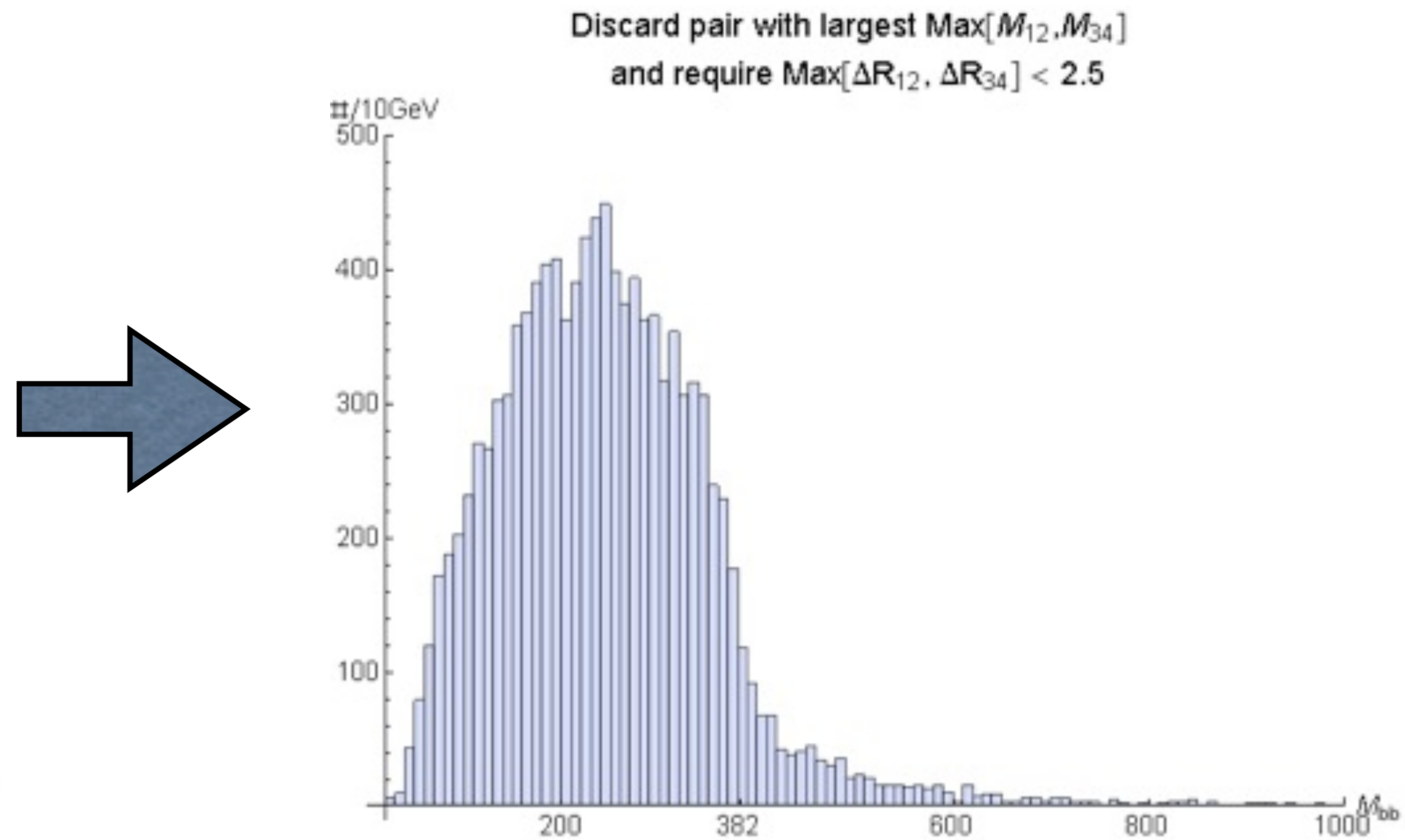
$$\not{E}_T > 200 \text{ GeV},$$
$$p_T^b > 40 \text{ GeV}$$
$$p_T^{\max} > 100 \text{ GeV}$$
$$|\eta^b| \leq 2.5$$

After cuts: $\sigma_{\rm sig} = 480 \text{ fb}, \quad \sigma_{\rm bg} \approx 35 \text{ fb}$ ➡ Ignore backgrounds

Kinematic Edge



[6 values in each event, 4 are from wrong pairings]



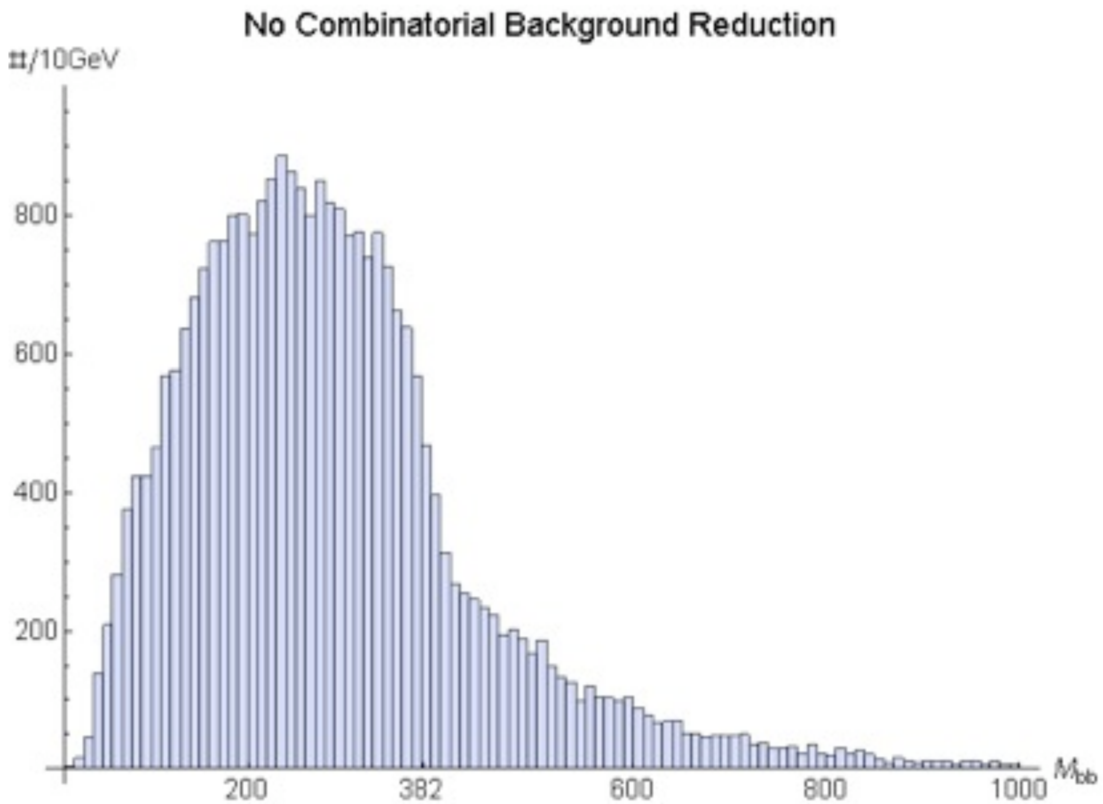
[cleaned up with cuts]

Theory:

$$M_{bb}^{\max} = \sqrt{\frac{(m_{\tilde{g}}^2 - m_{b1}^2)(m_{b1}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{b1}^2}} = 382.3 \text{ GeV}.$$

(

Kinematic Edge



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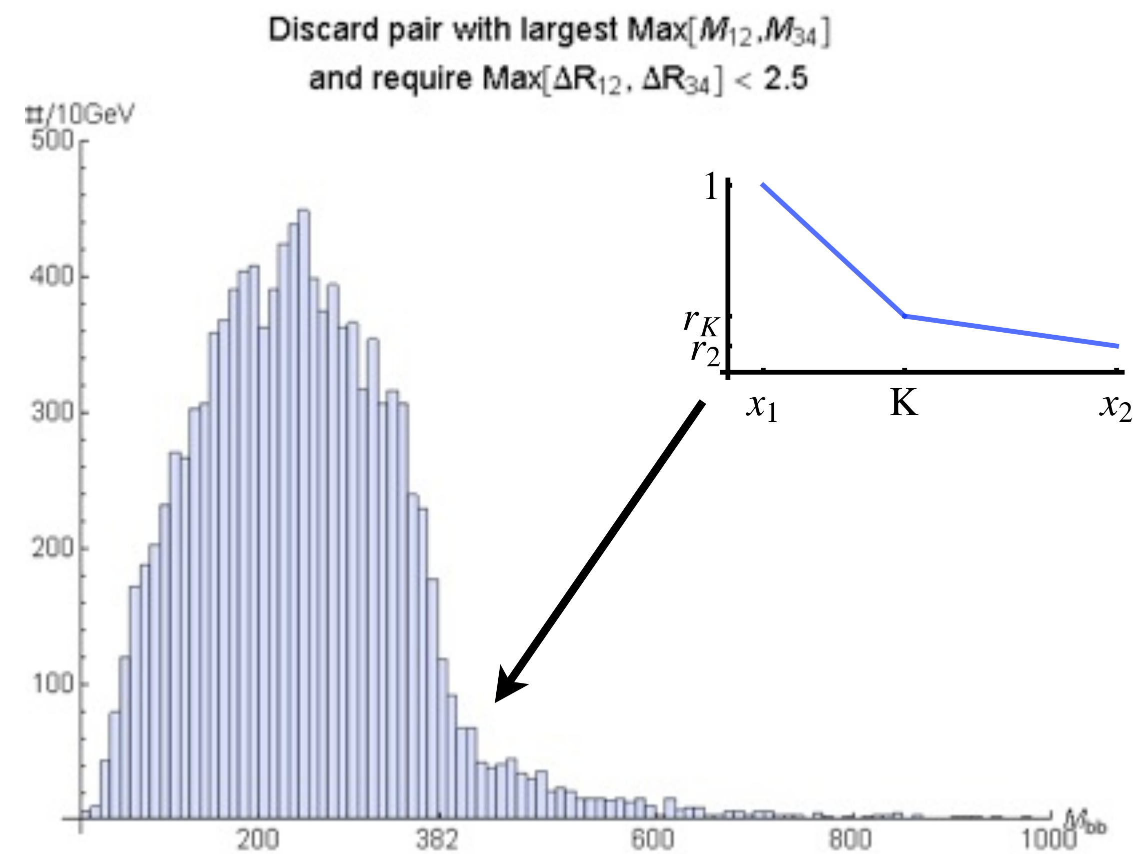


[cleaned up with cuts]

Theory:

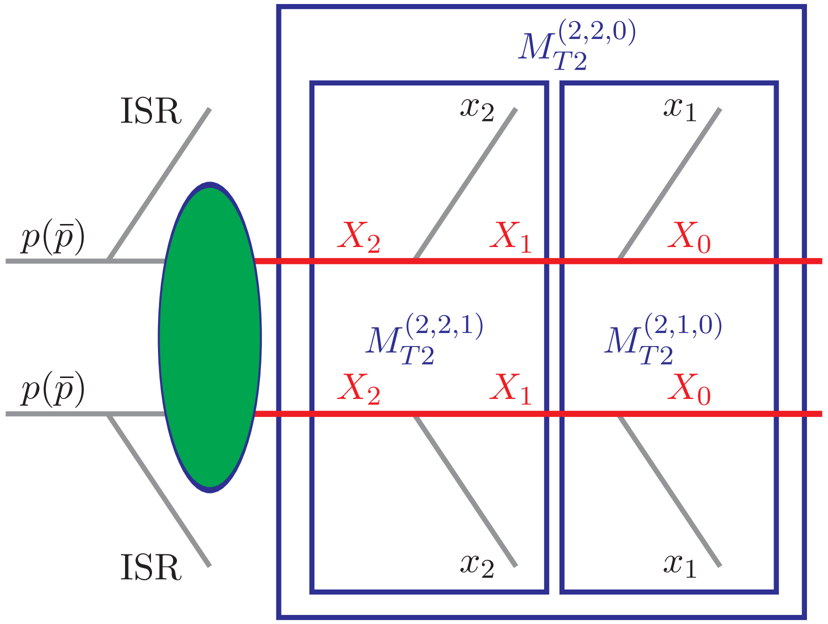
$$M_{bb}^{\max} = \sqrt{\frac{(m_{\tilde{g}}^2 - m_{\tilde{b}1}^2)(m_{\tilde{b}1}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{b}1}^2}} = 382.3 \text{ GeV}.$$

Measurement (10 fb-1, 14 TeV):

$$M_{bb}^{\max} = (395 \pm 5) \text{ GeV} \checkmark$$

x3 - systematics

MT2 and Subsystem MT2's



(b)

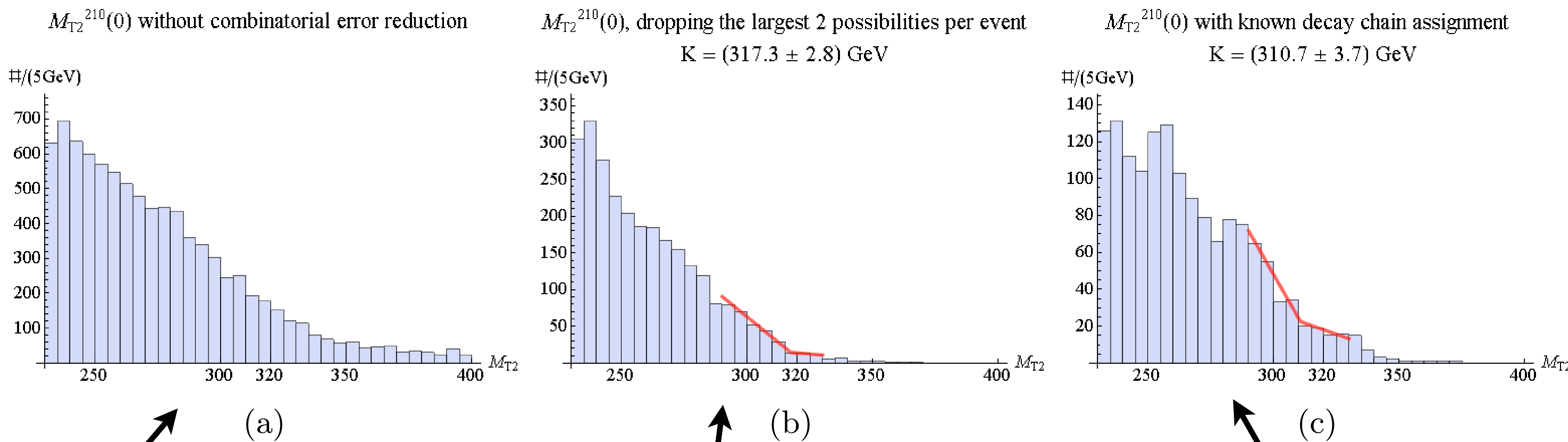
Theory predictions:

$$M_{T2}^{210}(0)^{\max} = \frac{[(m_{\tilde{b}1}^2 - m_{\tilde{\chi}_1^0}^2)(m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2)]^{1/2}}{m_{\tilde{g}}} = 320.9 \text{ GeV}$$

$$M_{T2}^{220}(0)^{\max} = m_{\tilde{g}} - m_{\tilde{\chi}_1^0}^2/m_{\tilde{g}} = 506.7 \text{ GeV}.$$

[Note: we did not find large- $\tilde{M}$ endpoints very useful, but did not try to optimize $\tilde{M}$]

Example: Subsystem MT2



6 values per event, 5 incorrect

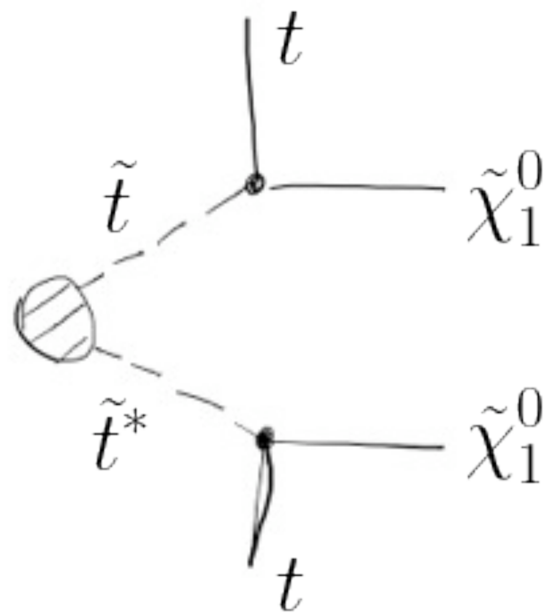
4 values per event, by hand

4 values per event, 3 incorrect

Theory: 320.9 GeV

Measured: $(314 \pm 14) \text{ GeV}$

Process 2: $pp \to \tilde{t}\tilde{t}^*, \quad \tilde{t} \to t\tilde{\chi}_1^0$



$\sigma = 2$ pb

Final state: 2 tops (both had.) + MET

SM Background: $Zt\bar{t}$ $\sigma = 135$ fb

No kinematic edges, single MT2 endpoint:

$$M_{T2}^{\max}(0) = \frac{M_{\tilde{t}}^2 - M_{\tilde{\chi}_1^0}^2}{M_{\tilde{\chi}_1^0}} = 336.7 \text{ GeV}$$

Measurement (100 fb-1, 14 TeV):

$$(340 \pm 4) \text{ GeV}$$

Put Everything Together:

Process 1:

$$
\begin{aligned}
M_{bb}{}^{\max}_{\text{meas}} &= (395 \pm 15)\ \text{GeV}\,, \\
M_{T2}^{210}(0)^{\max}_{\text{meas}} &= (314 \pm 14)\ \text{GeV}\,, \\
M_{T2}^{220}(0)^{\max}_{\text{meas}} &= (492 \pm 14)\ \text{GeV}\,.
\end{aligned}
$$

Process 2:

$$M_{T2}(0)^{\max}_{\text{meas}} = (340 \pm 4)\ \text{GeV}.$$

mass	theory	median	mean	68% c.l.	95% c.l.	process
m_{b_1}	341	324	332	(316, 356)	(308, 432)	I
$m_{\tilde{g}}$	525	514	525	(508, 552)	(500, 634)	I
$m_{\tilde{\chi}_1^0}$	98	–	–	(45, 115)	(45, 179)	I + LEP
m_{t_1}	371	354	375	(356, 414)	(352, 516)	I + II

TABLE I: Mass measurements (all in GeV), assuming Gaussian edge measurement uncertainties. We imposed the lower bound $m_{\tilde{\chi}_1^0} > 45$ GeV, which generically follows from the LEP invisible Z decay width measurement [17].

If we assume that t1 and b1 are exactly left-handed:

$$\Upsilon'_{\text{meas}} = \frac{1}{v^2}\left(m_{t1}^2 - m_{b1}^2\right) = 0.525^{+0.20}_{-0.15}$$

[theory prediction, with rad. cor., is 0.42]

Error Bar Inflation:

mass	theory	median	mean	68% c.l.	95% c.l.	process
m_{b_1}	341	324	332	(316, 356)	(308, 432)	I
$m_{\tilde{g}}$	525	514	525	(508, 552)	(500, 634)	I
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TABLE I: Mass measurements (all in GeV), assuming Gaussian edge measurement uncertainties. We imposed the lower bound $m_{\tilde{\chi}_1^0} > 45$ GeV, which generically follows from the LEP invisible Z decay width measurement [17].

$$\Upsilon'_{\rm meas} = \frac{1}{v^2}\left(m_{t1}^2 - m_{b1}^2\right) = 0.525^{+0.20}_{-0.15}$$

5-10% errors on masses

40% error on the sum rule

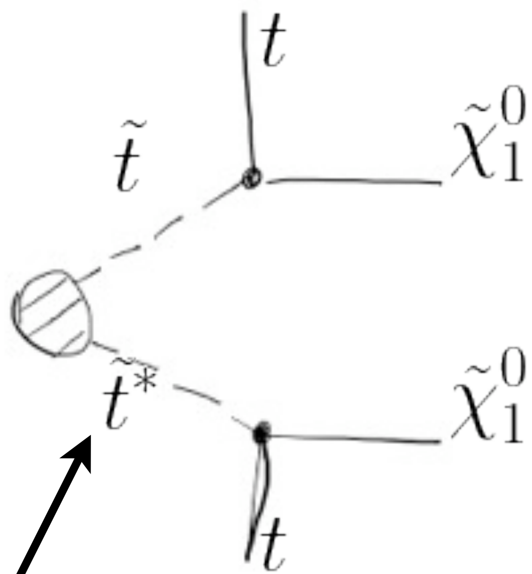
Due to the SU(2) cancellation in the sum rule:

$$(371)^2 - (341)^2 \sim (170)^2$$

Precise mass measurements are key, ILC can do it!

LHC Stop Mixing Angle Measurement?

[MP, Weiler, 0811.1024; Shelton, 0811.0569]



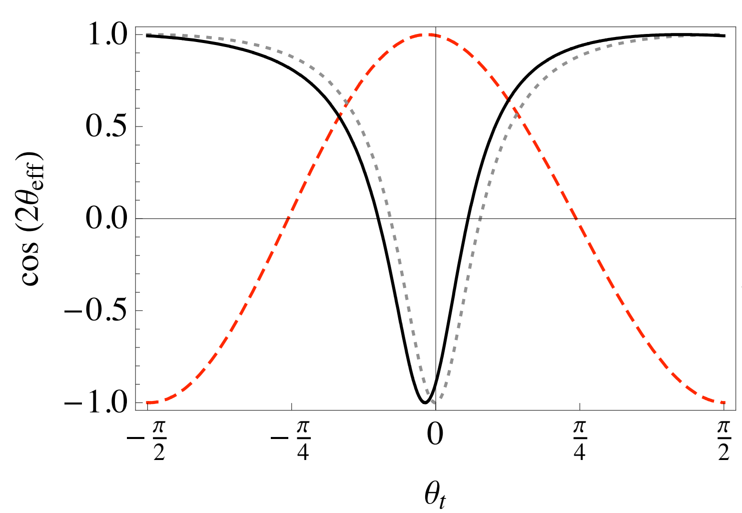
$\tilde{t}_1$ only, if $m_{t1} - m_{t2}$ large!

- Top decays before hadronization
  ⇨ polarization is observable!

- Top polarization is same as stop handedness if $\chi_1^0 = \tilde{B}, \tilde{W}^3$, or opposite if $\chi_1^0 = \tilde{H}_u^0, \tilde{H}_d^0$

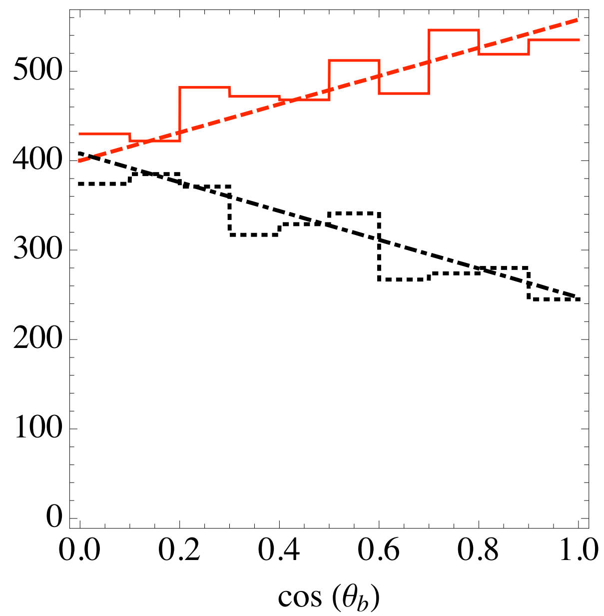
- Top polarization determined by the "effective mixing angle"

$$\tan\theta_{\rm eff}^{1j} = \frac{y_t N_{j4}\cos\theta_t - \frac{2\sqrt{2}}{3} g' N_{j1}\sin\theta_t}{\sqrt{2}\left(\frac{g}{2}N_{j2} + \frac{g'}{6}N_{j1}\right)\cos\theta_t + y_t N_{j4}\sin\theta_t}$$



Knowledge of neutralino mixing angles is required to get θ_t

Before cuts:



After cuts:

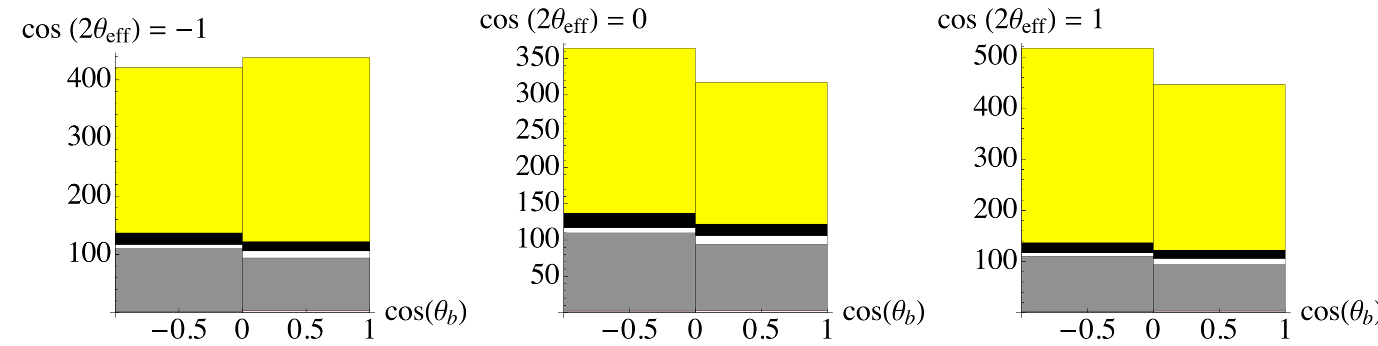


Figure 5: Angular distributions of events in the angle θ_b. The different contributions correspond to (from top to bottom): signal (yellow), $4j + W^-$ (black), $2j + 2b + W^-$ (white), $t\bar{t}(\mu^-)$ (gray), $t\bar{t}(\tau^- \to \mu^-)$ (light red). The event numbers correspond to 10 fb^{-1} integrated luminosity at the LHC.

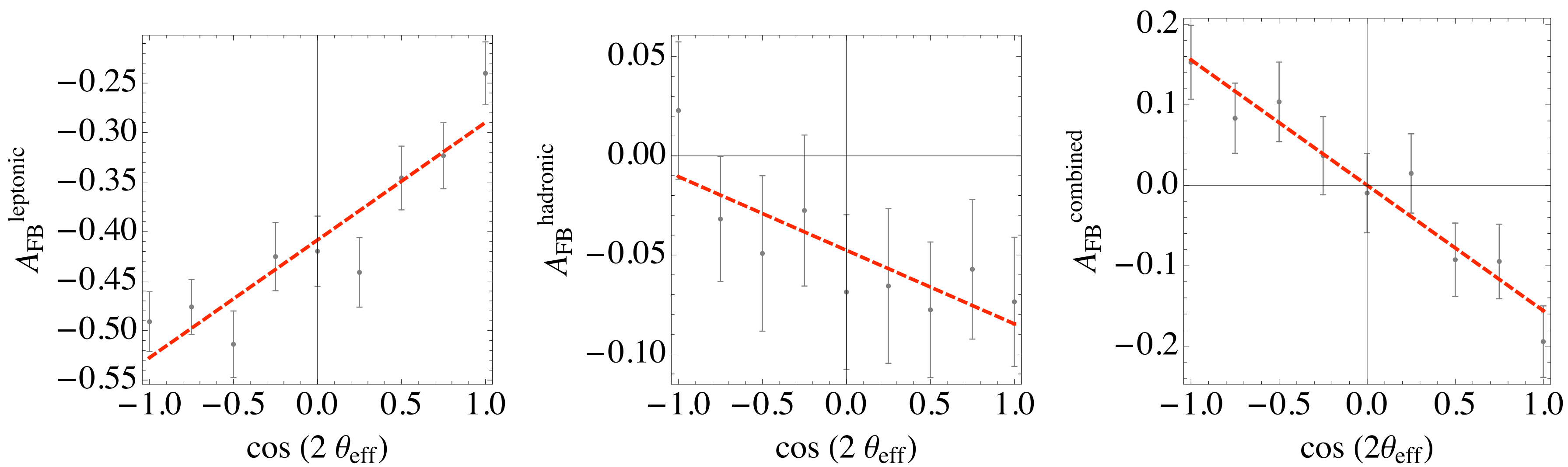


Figure 7: Leptonic, hadronic, and combined forward-backward asymmetries, as a function of the angle θ_{eff}. The error bars indicate statistical errors for 10 fb^{-1} integrated luminosity.

[Parton-level analysis; ISR complicates things further - Plehn et al, 1006.2833]

Stop Mixing from Gluino Decays?

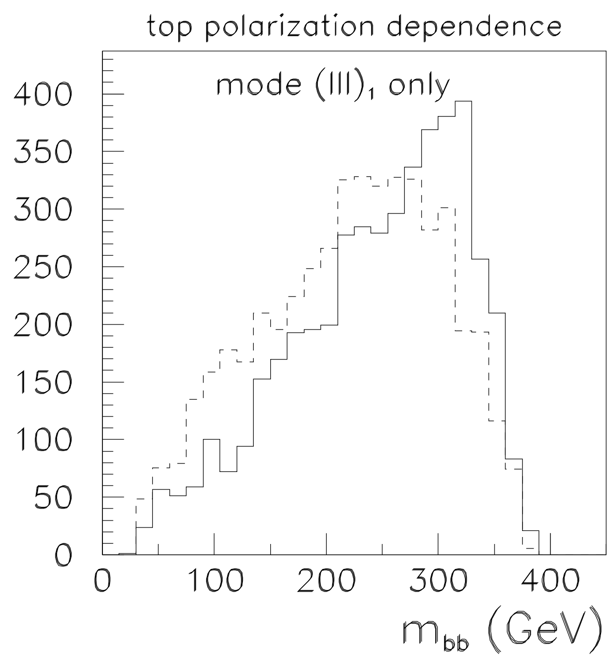


FIG. 22: Distribution of m_{bb} in the decay chain $(\mathrm{III})_1$. The (dashed) line is for $\tilde{t}_1 = \tilde{t}_L(\tilde{t}_R)$, and 400 GeV$< m_{tb} <$ 470 GeV. We use the mass spectrum in the sample point A1 in Table I, and the normalization is arbitrary.

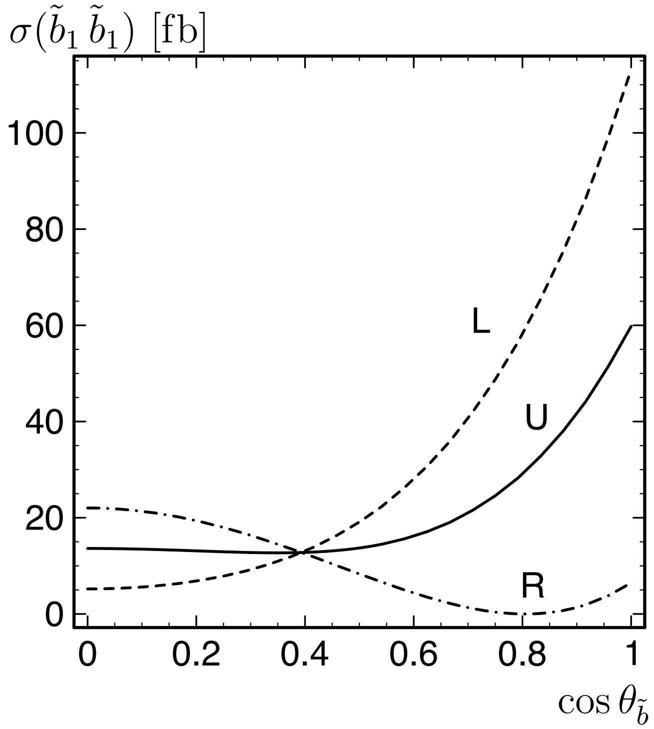
[Hisano, Kawagoe, Nojiri, hep-ph/0304214]

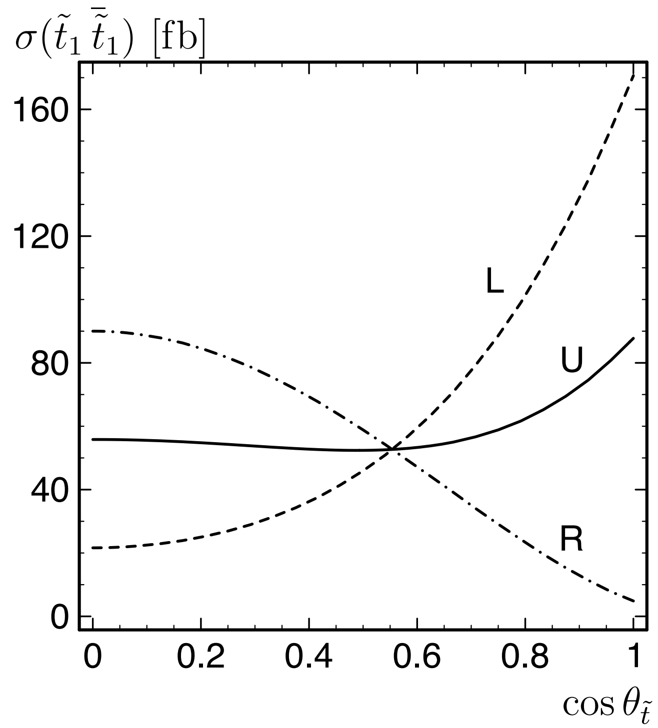
- Direct measurement of θ_t - gluino is a pure gaugino!
- Complicated final state, combinatoric issues
- More difficult if gluino is heavy
- More detailed, quantitative analysis is required to assess the LHC potential for this measurement

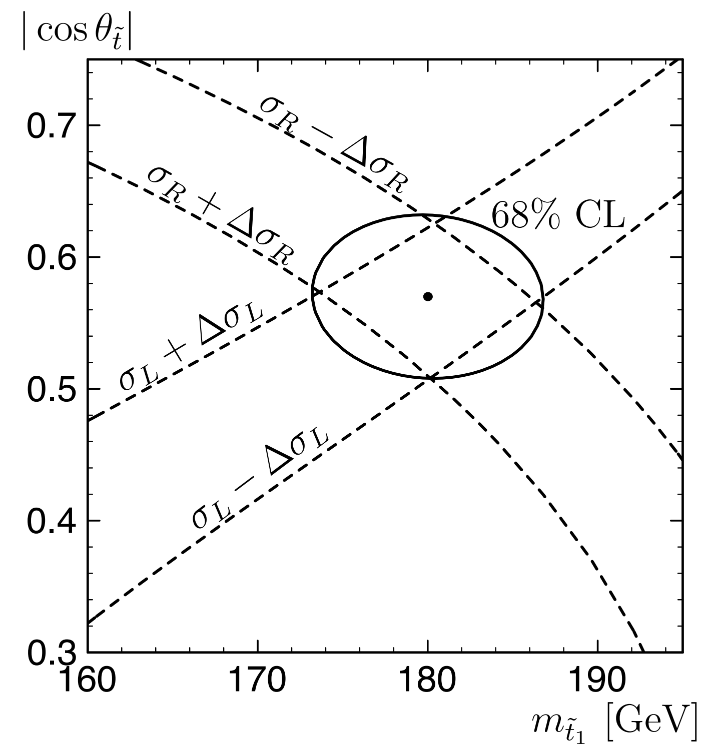
Sbottom Mixing Measurement at the LHC

Mixing Angle Measurements at the ILC

[Bartl, Eberl, Kraml, Majerotto, Porod, Sopczak, hep-ph/9701336]







$$
\begin{aligned}
m_{\tilde{t}_1} &= 180 \pm 7 \text{ GeV}, \\
\cos\theta_{\tilde{t}} &= 0.57 \pm 0.06.
\end{aligned}
$$

Conclusions

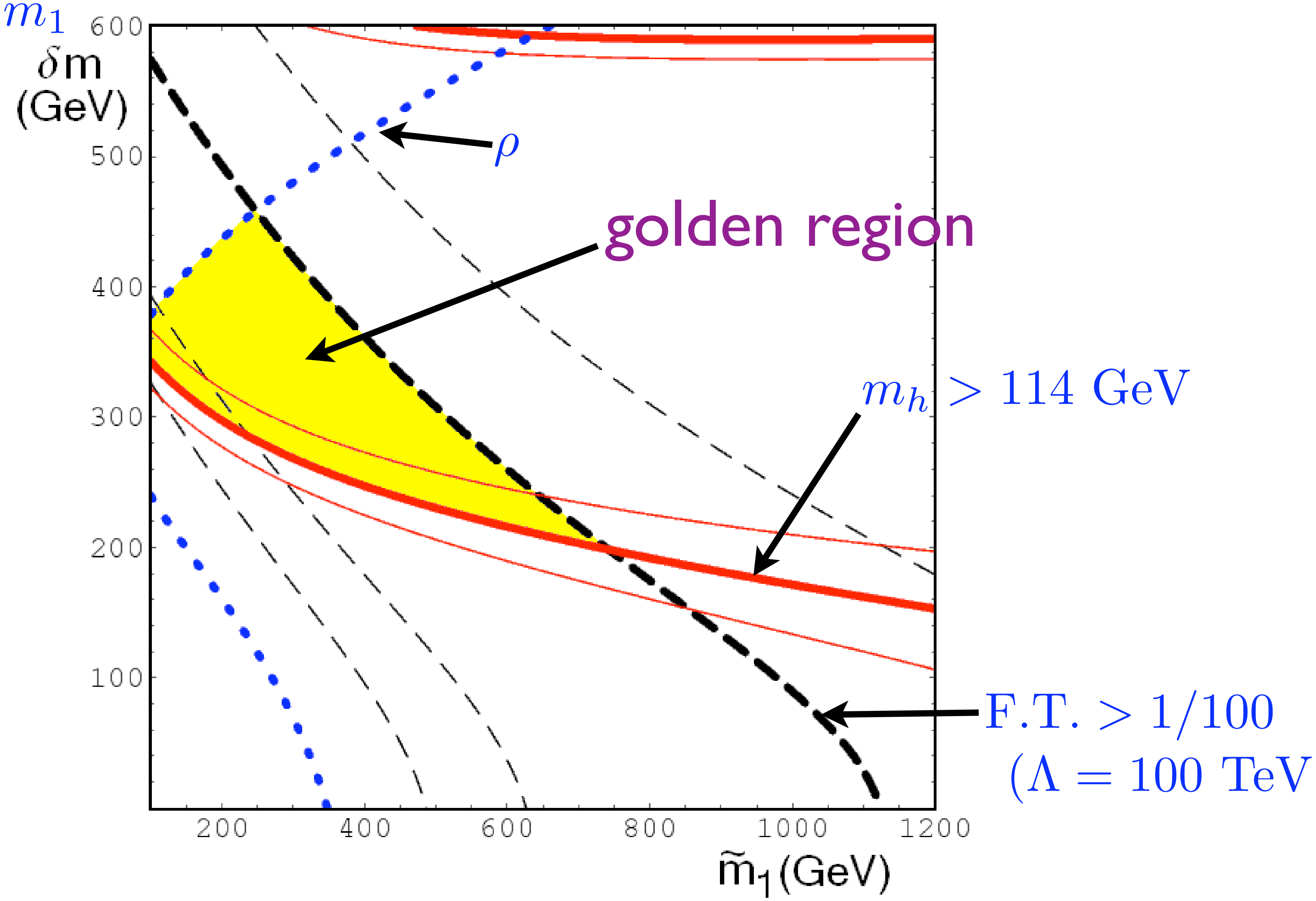
- Proving SUSY-Yukawa Sum Rule experimentally would provide a striking confirmation of SUSY and its role in electroweak symmetry breaking
- Unfortunately, this will be quite challenging at the LHC:
  - Error inflation requires precise mass measurements
  - Stop mixing angle measurement is hard, sbottom even harder
- ILC excels at this - a quantitative study would be very interesting!

Backup Slides

Stop Mass vs. Naturalness in the MSSM

[MP, Spethmann, hep-ph/0702038]

$\delta m = m_2 - m_1$



$\theta_t = \pi/4, \quad \tan\beta = 10$

Note: in the pMSSM ("without prejudice"), other squarks and gluinos can be >5 TeV without much fine-tuning