Phenomenology of the Higgs sector of Type II Seesaw Model

Abdesslam Arhrib

Faculté des Sciences et Techniques Tangier, Morocco

Based on: arXiv:1105.1925 (PRD'11) and work in progress

Benbrik, Chabab, Moultaka, Peyranère, Rahili, Ramadan

Plan

- Motivations
- Higgs Triplet Model (HTM) and its Electroweak Symmetry Breaking
- Dynamical constraints:
  Potential Bounded From Bellow (BFB)
  Perturbative unitarity constraints
- $H^{\pm}$ and $H^{\pm\pm}$ effects in $h \to \gamma\gamma$
- Extracting triplet vev at ILC: $e^+e^- \to W^{\pm}H^{\mp}$,
  $e^+e^- \to W^*Z^* \to e^+\nu_e H^-$, $e^-e^- \to W^{-*}W^{-*} \to \nu_e\nu_e H^{--}$
- Conclusions

Motivations

Neutrino masses require the presence of the higher dimensional operator ($\Delta L = 2$):

$$\mathcal{L}_{eff} = Y_{eff} \frac{LLHH}{\Lambda} \ ,$$

After EWSB by the Higgs vev (v):

$$m_\nu = Y_{eff} \frac{v^2}{\Lambda} \ .$$

The smallness of neutrino masses tells us that either Λ is very large $\Lambda \gg v$, or Y_{eff} must be very small.

Motivations...

$$M_\Delta^2 Tr(\Delta^\dagger \Delta) + \mu(H^T i\tau_2 \Delta^\dagger H) \ ,$$

$$\boxed{m_\nu = Y_\nu \mu v_d^2 / M_\Delta^2}$$

- If $m_\nu \approx 1$ eV with $Y_\nu \approx 1$, then $M_\Delta \approx \mu \approx 10^{14-15}$ GeV not testable at the ILC/LHC
- If $m_\nu \approx 1$ eV and $M_\Delta \approx 1$ TeV , $Y_\nu \mu \approx 10^{-8}$ GeV
- small μ can be viewed as soft breaking term of lepton number

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- small μ can be viewed as soft breaking term of lepton number
- Real Triplet Higgs could be a viable cold dark matter (CDM) candidate if it has no vev
  M. Cirelli, N. Fornengo and A. Strumia, NPB 753(2006);
  M. Cirelli, A. Strumia and M. Tamburini, NPB 787(2007)

Higgs Triplet Model HTM

It consists of standard Higgs weak doublet H and a scalar field Δ transforming as a triplet under $SU(2)_L$ with $Y_\Delta = 2$
$H \sim (1,2,1)$ and $\Delta \sim (1,3,2)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \qquad \text{and} \qquad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H,\Delta) + \mathcal{L}_{\text{Yukawa}}$$

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The most general renormalizable potential is:

$$V = M_\Delta^2 Tr\Delta^\dagger\Delta - m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta)$$
$$+\lambda_2 (Tr\Delta^\dagger\Delta)^2 + \lambda_3 Tr(\Delta^\dagger\Delta)^2 + \lambda_4 H^\dagger \Delta\Delta^\dagger H + \mu H^T i\tau_2 \Delta^\dagger H + h$$

- The inclusion of the μ term eliminates the Majoron.

Electroweak symmetry breaking

$$\Delta = \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} 0 \\ v_d \end{pmatrix}$$

one finds after minimization of the potential:

$$M_\Delta^2 = \frac{2\mu v_d^2 - \sqrt{2}(\lambda_1+\lambda_4)v_d^2 v_\Delta - 2\sqrt{2}(\lambda_2+\lambda_3)v_\Delta^3}{2\sqrt{2}v_\Delta}$$

$$m_H^2 = \frac{\lambda v_d^2}{4} - \sqrt{2}\mu v_\Delta + \frac{(\lambda_1+\lambda_4)}{2}v_\Delta^2$$

- After EWSB: 2 CP-even, h, H, one CP-odd A, a pair of $H^\pm$ and a pair of doubly charged Higgs $H^{\pm\pm}$
- 10-3 independents parameters: 5 masses, μ and v_Δ

Doublet-Triplet mixing

- $H^{\pm\pm}$ is pure triplet
- $H^{\pm}$ and A are dominated by the triplet δ fields,
  the mixing is small: $v_\Delta/v \leq 0.03$
- h and H are mixtures of doublet ϕ and triplet δ fields,

$$\tan 2\alpha = \frac{2\mathcal{M}_{12}^2}{\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2} \approx v_\Delta/v$$

maximal mixing is possible for $\mathcal{M}_{11}^2 = \mathcal{M}_{22}^2$
(when h and H are close to degenerate)
[A. Akeroyd and C.W.Chiang PRD'10]
[P. Dey, A.Kundu and B.Mukhopadhyaya, J.Phys'09]

Constraint on triplet vev

$$M_Z^2 = \frac{(g^2 + g'^2)(v_d^2 + 4v_\Delta^2)}{4} = \frac{g^2(v_d^2 + 4v_\Delta^2)}{4\cos^2\theta_W}$$

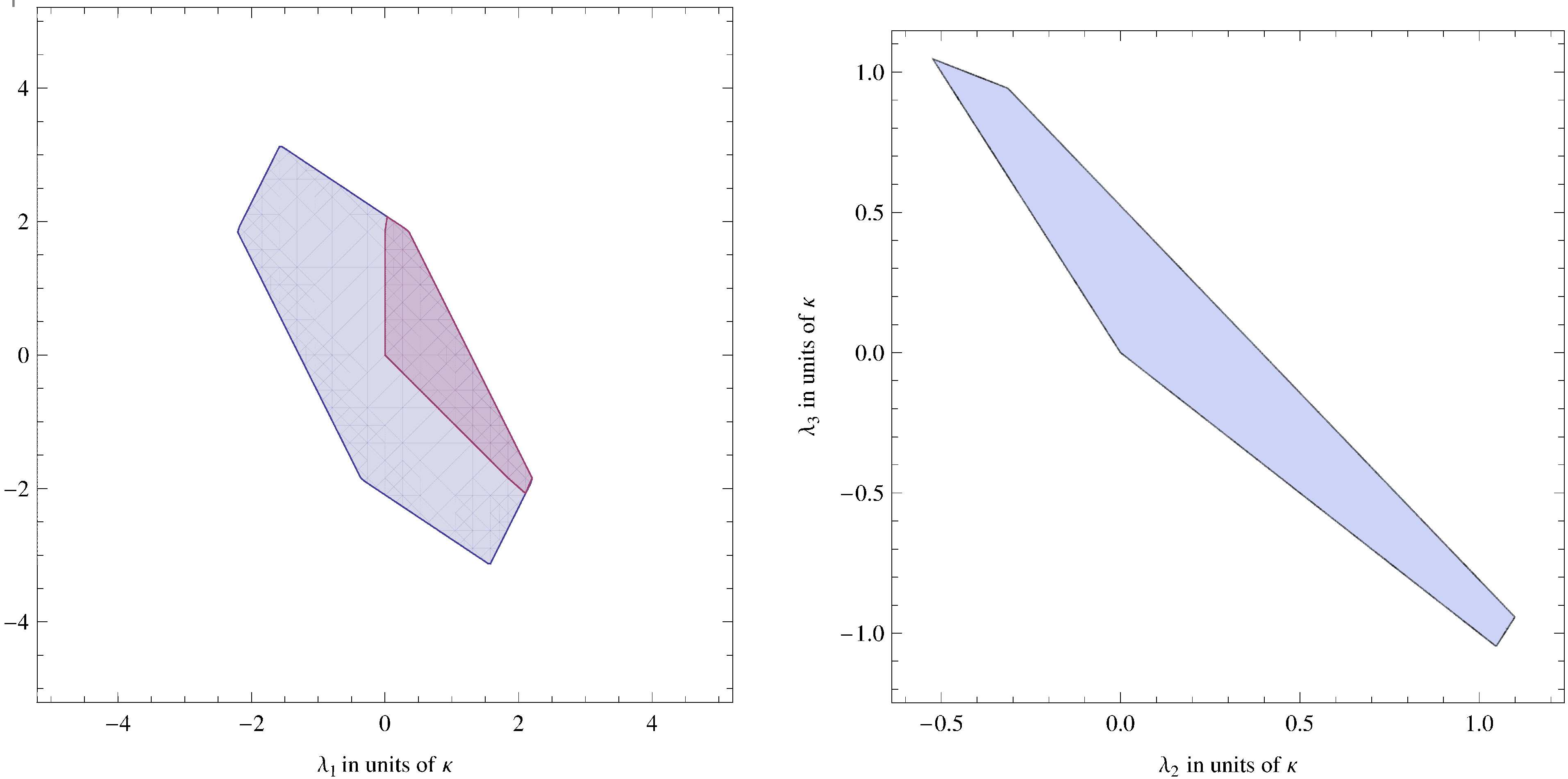
$$M_W^2 = \frac{g^2(v_d^2 + 2v_\Delta^2)}{4}$$

whence the modified form of the ρ parameter:

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2} = \frac{v_d^2 + 2v_\Delta^2}{v_d^2 + 4v_\Delta^2} \simeq 1 - 2\frac{v_\Delta^2}{v_d^2} \approx 1 + \delta\rho$$

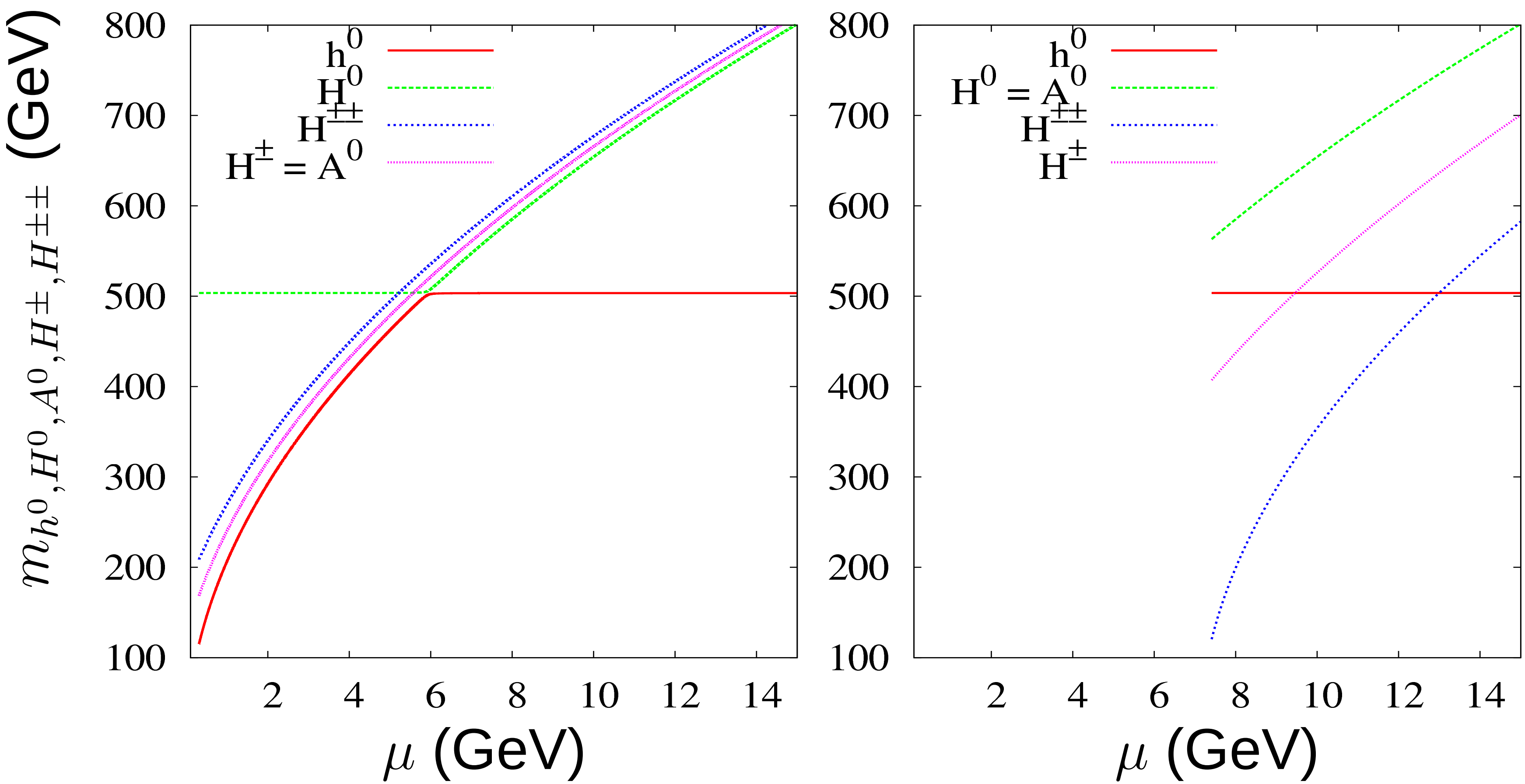
At the 2 σ level, $\rho_0 = 1.0004^{+0.0029}_{-0.0011}$ (or $\rho_0 = 1.0008^{+0.0017}_{-0.0010}$), one gets an upper bound on $v_\Delta \leq 2.5$–4.6 GeV.

Results



Left: $\lambda_2 = \lambda_3 = 0$, Right: $\lambda_1 = \lambda_4 = 0$

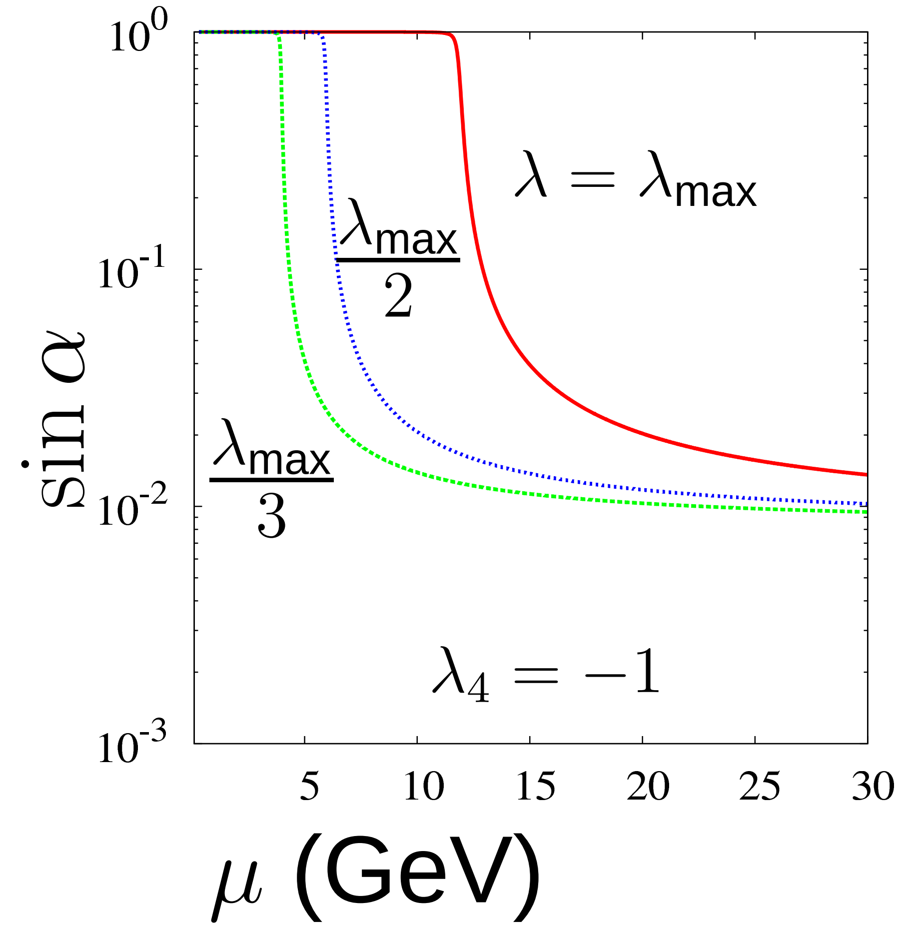
$M_{H^{\pm\pm}} > m_{H^{\pm}} > m_A$, $m_A > m_{H^{\pm}} > m_{H^{\pm\pm}}$



Higgs boson masses as a function of μ with $v_{\Delta} = 1$ GeV, $\lambda = 8\pi/3$, $\lambda_1 = 0.5$, $\lambda_2 = \lambda_3 = 0.1$, $\lambda_4 = -1$ (left) and $\lambda_4 = 10$ (right)

$$m^2_{H^{\pm}} - m^2_{H^{\pm\pm}} \approx m^2_{H^0} - m^2_{H^{\pm}} \approx \lambda_4\, v_d^2/4$$

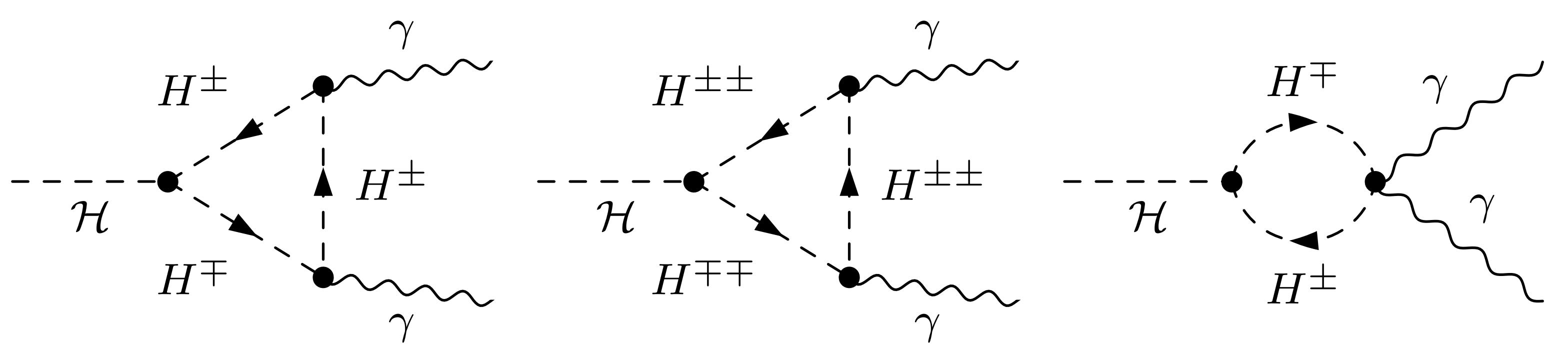
$$h^0 = \sin\alpha \Re e(\phi^0) + \cos\alpha \Re e(\delta^0)$$



$\lambda_{max} = 16\pi/3$, $\lambda_2 = \lambda_3 = 0.1$, $\lambda_1 = 0.5, v_\Delta = 1$ GeV

$h \to \gamma\gamma$

- In the low mass region $m_h \in [110, 140]$ GeV: Due to the high QCD back-grounds, the LHC is betting on the clean diphoton channel: $gg \to h \to \gamma\gamma$.
- In the SM, $h \to \gamma\gamma$ is dominated by W loops
- $H^{\pm\pm}$ and $H^{\pm}$ loops can interfere constructively or destructively with W



$h \to \gamma\gamma$ amplitude

$$\Gamma(h^0 \to \gamma\gamma) = \frac{G_F \alpha^2 M_h^3}{128\sqrt{2}\pi^3} \Big| \sum_f N_c Q_f^2 g_{hff} A_{1/2}(\tau_f) + g_{hWW} A_1(\tau_W)$$

$$- \frac{M_W}{g} \Big(\frac{g_{hH^\pm H^\mp}}{m_{H^\pm}^2} A_0(\tau_{H^\pm}) + 4 \frac{g_{hH^{\pm\pm} H^{\mp\mp}}}{m_{H^{\pm\pm}}^2} A_0(\tau_{H^{\pm\pm}}) \Big)$$

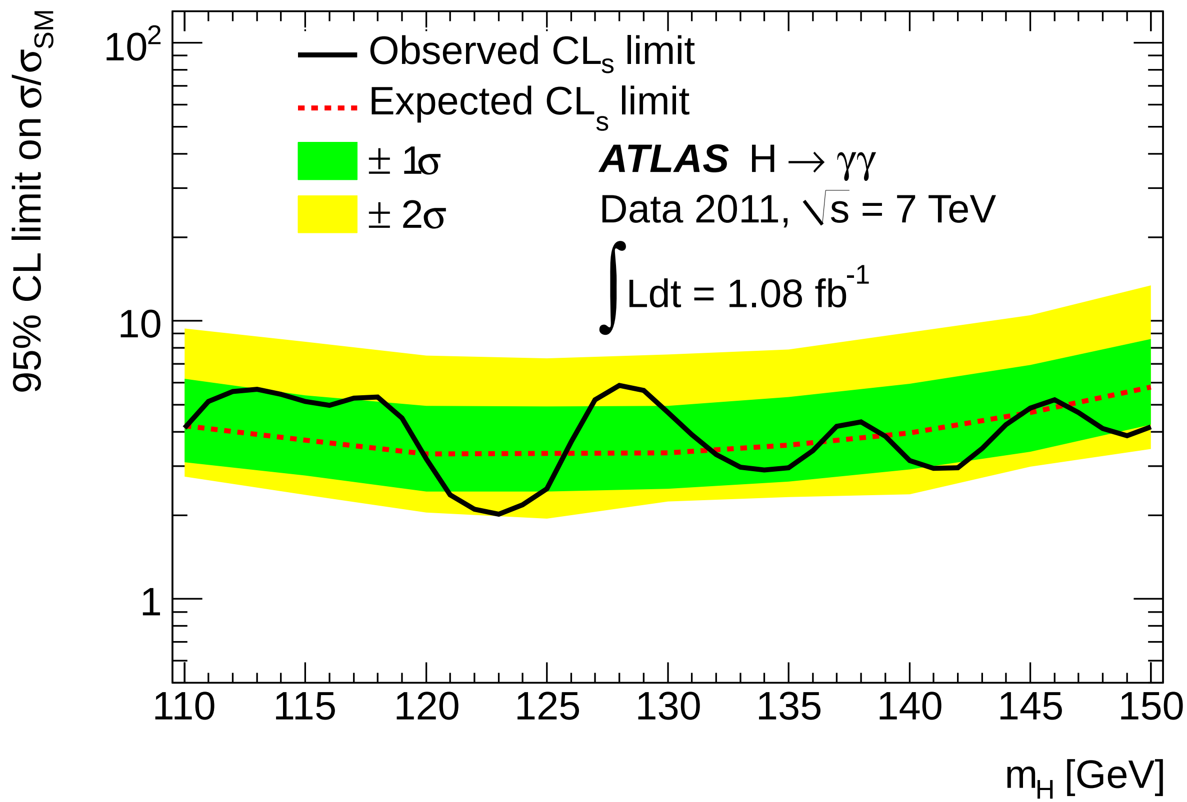
with

$$g_{h^0 H^{++} H^{--}} = -2(\lambda_2 v_\Delta s_\alpha + \lambda_1 v_d c_\alpha) \approx -\lambda_1 v_d + \ldots$$

$$g_{h^0 H^+ H^-} = -\frac{1}{2}(2\lambda_1 + \lambda_4) v_d + \ldots$$

ATLAS and CMS limits on diphoton

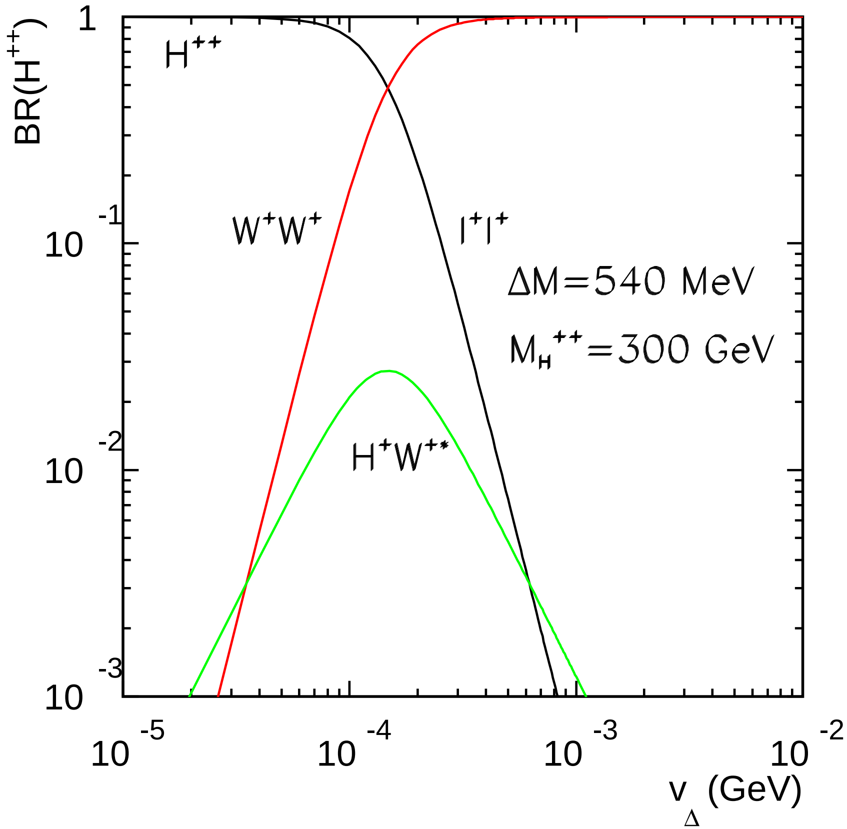
$$R_{\gamma\gamma} = \frac{[\sigma(gg\to h)\times Br(h^0\to\gamma\gamma)]_{HTM}}{[\sigma(gg\to h)\times Br(h^0\to\gamma\gamma)]_{SM}} < 2 \to 6$$



95% CL upper limits on a SM-like Higgs boson production cross-section, relative to the SM cross-section

CMS limits on $H^{\pm\pm}$

With 0.98 fb^{-1}, $m_{H^{\pm\pm}} > 313$ GeV
if $Br(H^{\pm\pm} \to l^{\pm}l'^{\pm}) = 100\%$, $l = \mu, e$.
This limit is weakened to 254 GeV in case of $e^{\pm}\tau^{\pm}$ final state.



(Tao Han'08)

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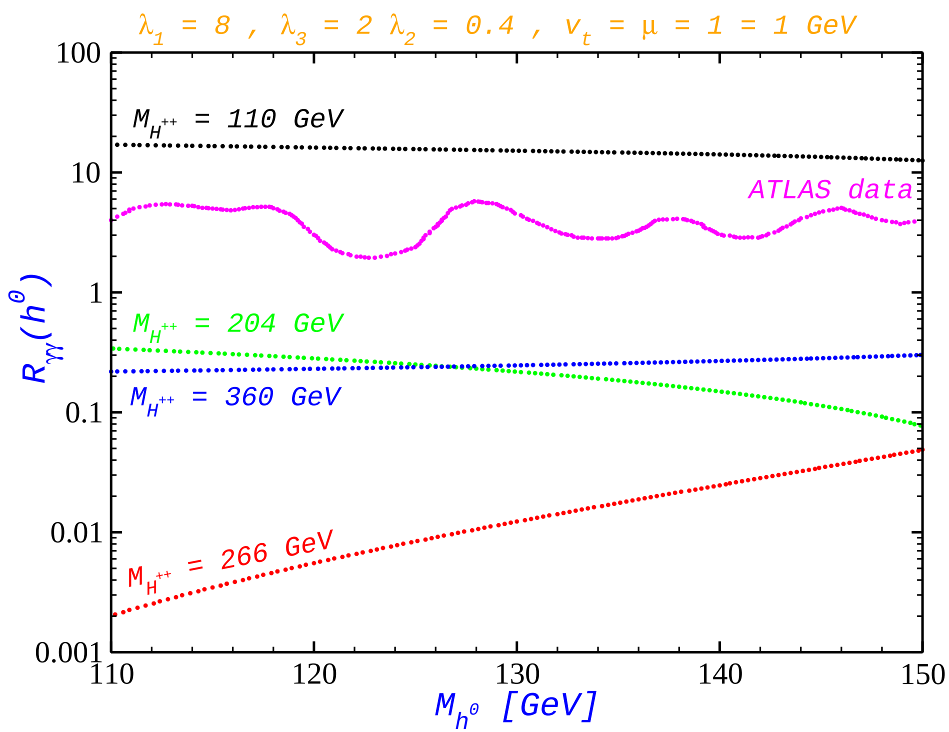

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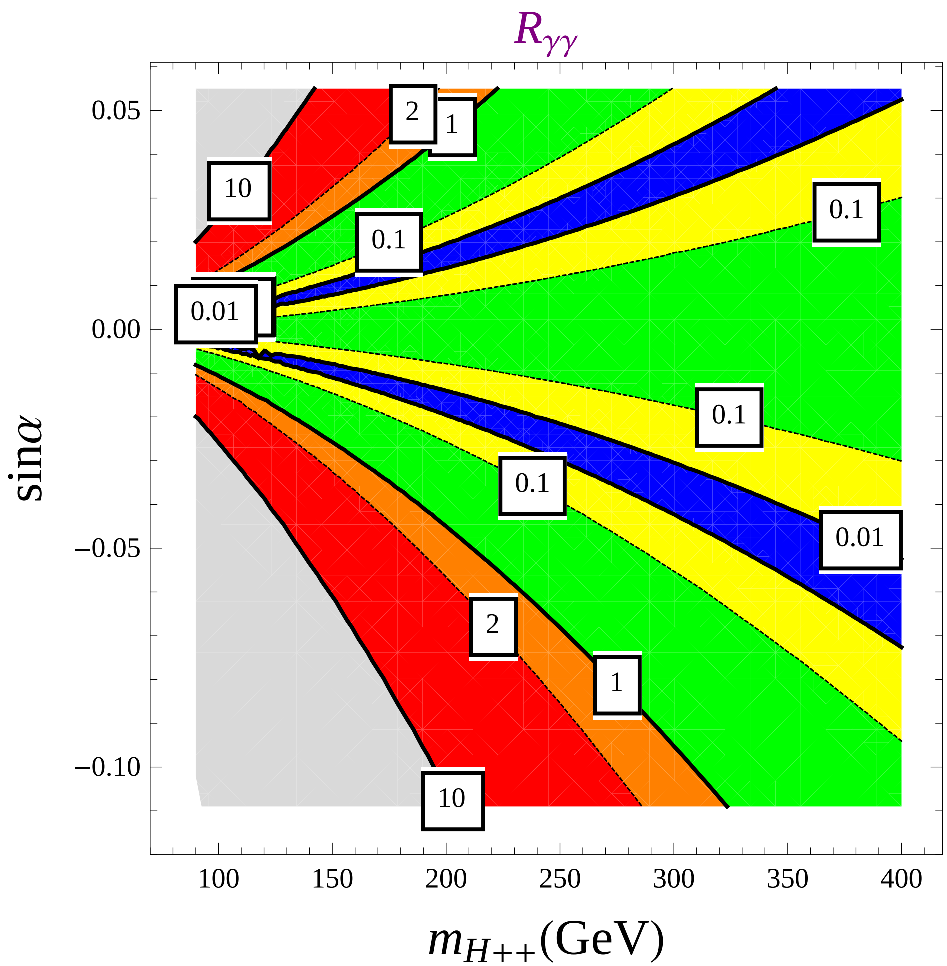
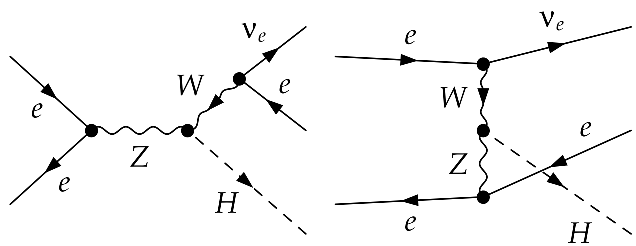


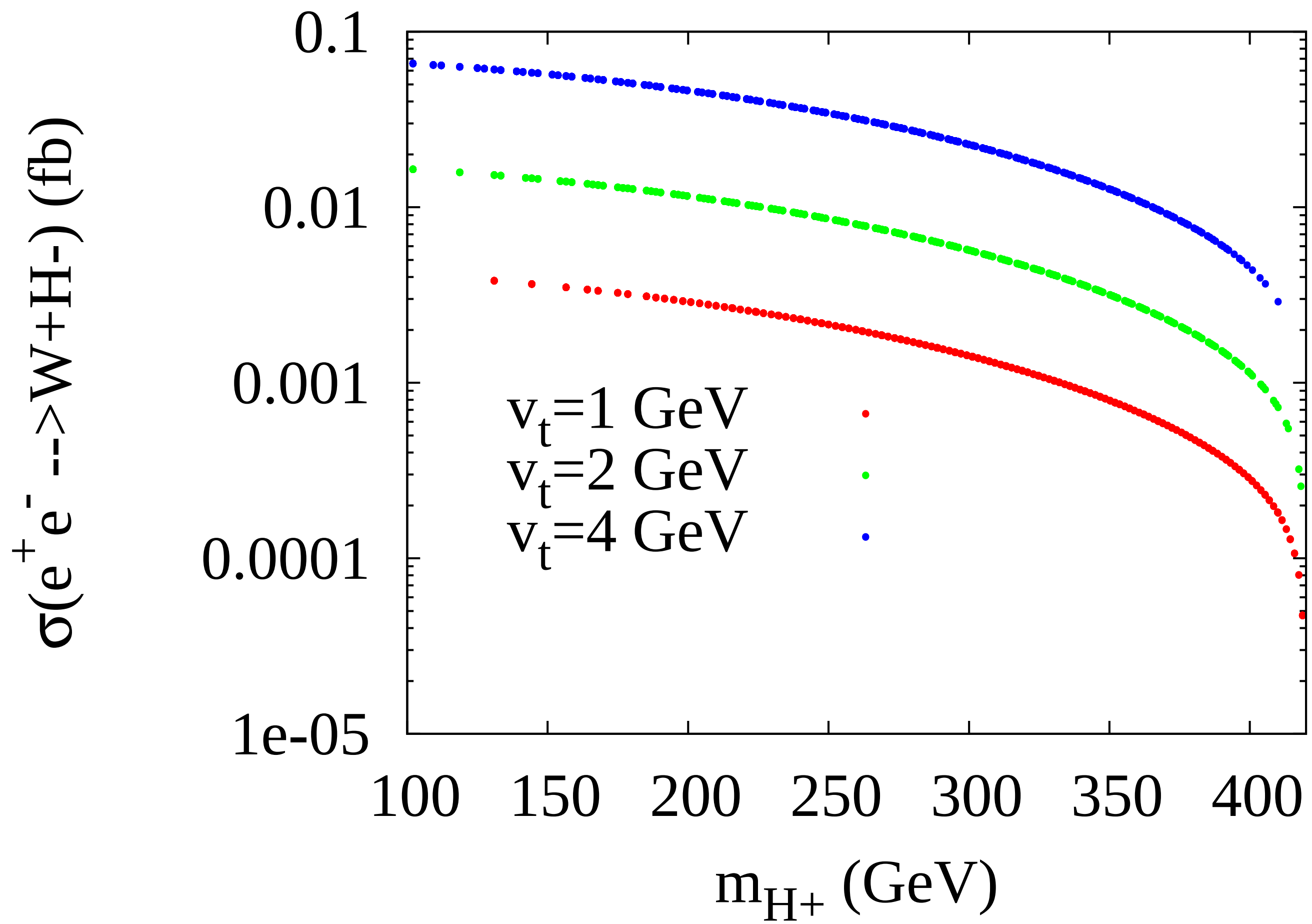
Extracting triplet vev v_Δ at ILC

$e \quad e \quad \rightarrow \quad \nu_e \quad e \quad H$

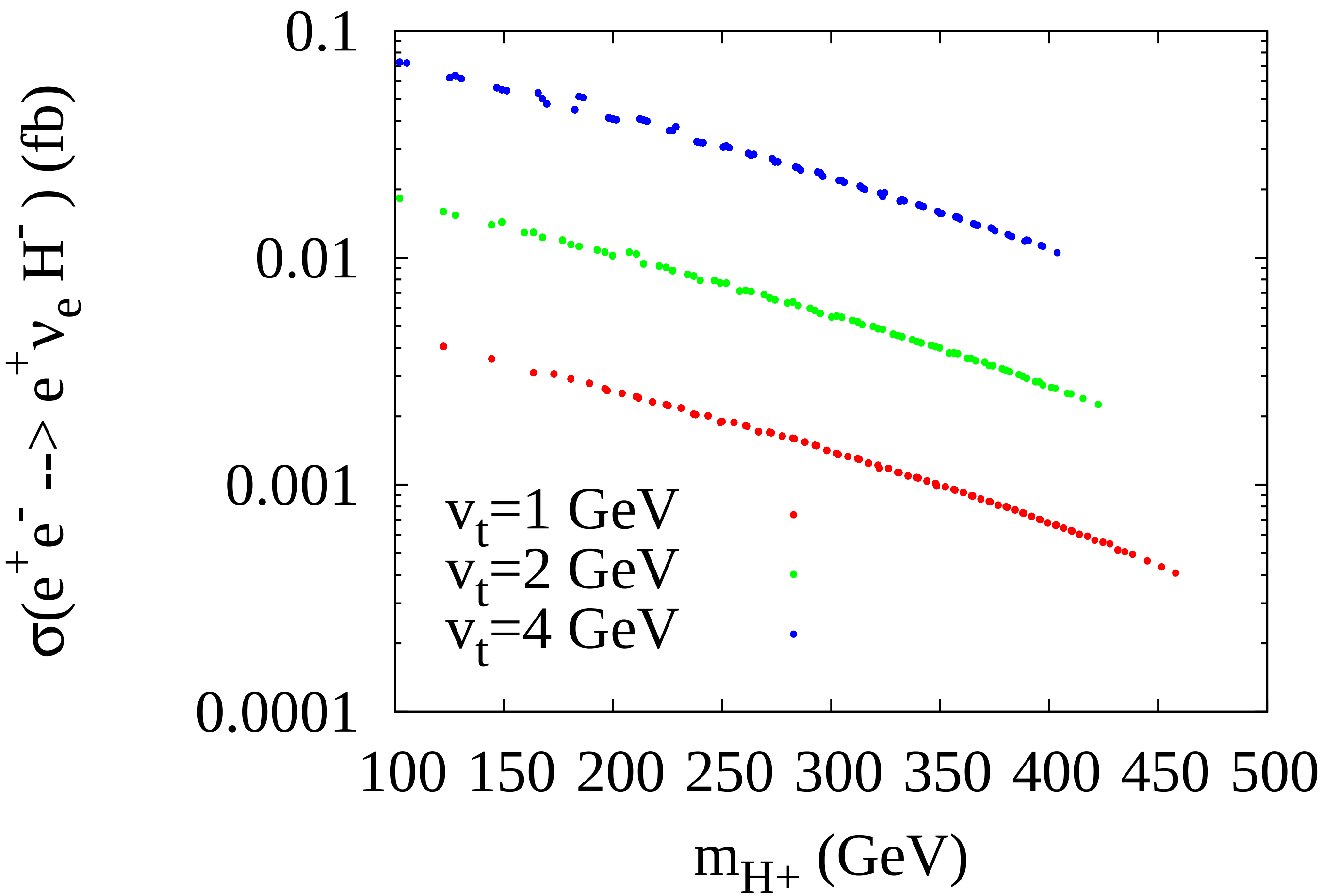


1. $e^+e^- \rightarrow Z^* \rightarrow W^\pm H^\mp$,
2. $e^+e^- \rightarrow W^* Z^* \rightarrow e^+ \nu_e H^-$,
3. $e^-e^- \rightarrow W^{-*} W^{-*} \rightarrow \nu_e \nu_e H^{--}$

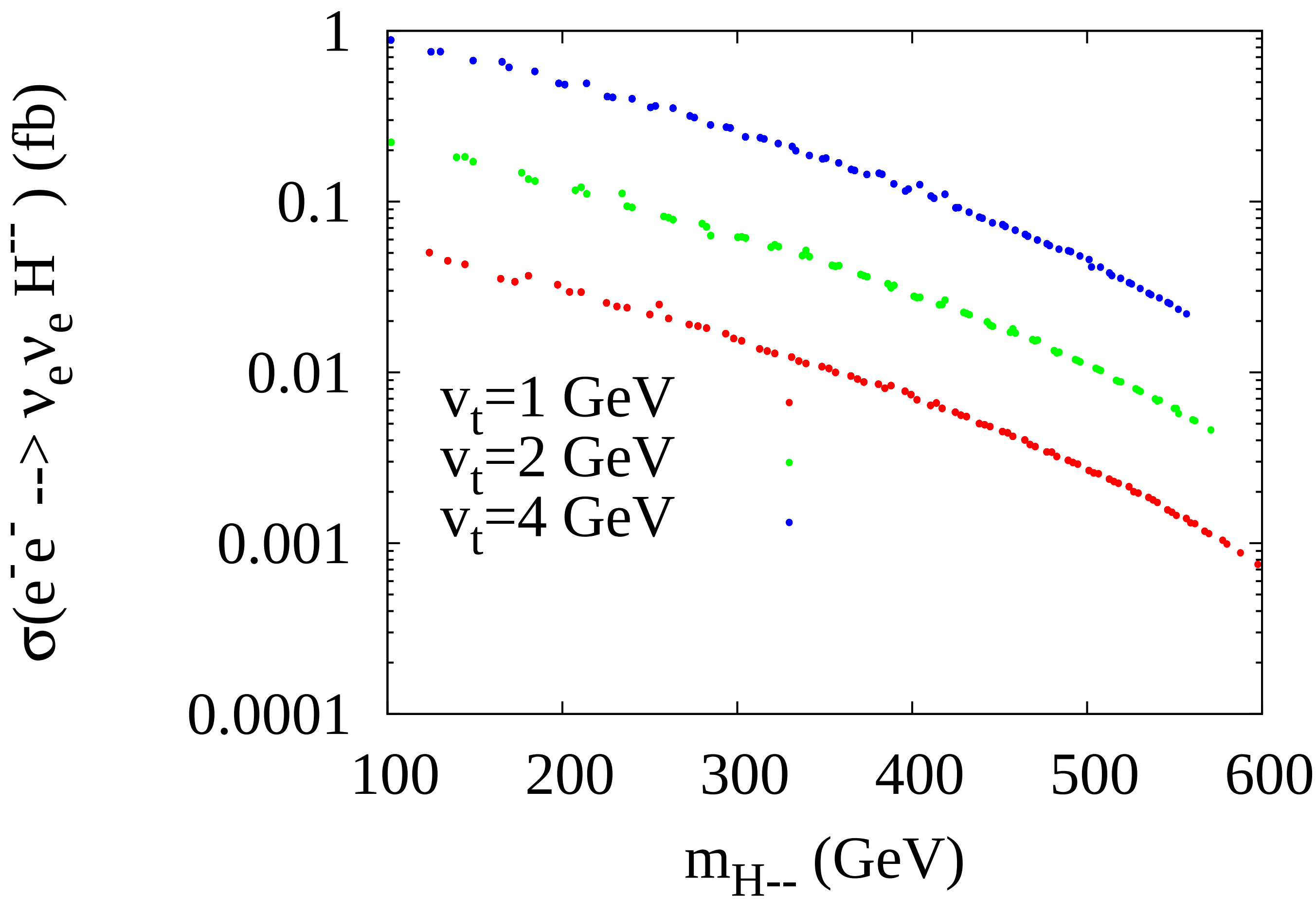
$e^+e^- \to Z^* \to W^\pm H^\mp$ @ 500 GeV



$e^+e^- \to W^*Z^* \to e^+\nu_e H^-$ @ 800 GeV



$e^-e^- \rightarrow W^{-*}W^{-*} \rightarrow \nu_e\nu_e H^{--}$ @800 GeV



Summary

- We identify two regimes:
  $\underline{\mu \geq \mu_c}$: h^0 SM-like could be accessible to the LHC, the other Higges are heavy.
  $\underline{\mu \leq \mu_c}$: The heavy CP-even H^0 becomes SM-like, the lighter states: $A^0, H^{\pm\pm}, H^{\pm}, h^0$, leading to a distinctive phenomenology at the colliders.
- Large splitting between Higgs exists and my allow Higgs to Higgs decays: $H^{\pm} \rightarrow W^{\pm *} A^0$, $H^{\pm\pm} \rightarrow W^{\pm *} H^{\pm}$...
- $h \rightarrow \gamma\gamma$ is very sensitive to $H^{\pm\pm}$ and can be used to set limits on the triplet parameters.
- extracting triplet vev from specific process at ILC

BFB: General proof

$$r \equiv \sqrt{H^\dagger H + Tr\Delta^\dagger\Delta} > 0$$

$$H^\dagger H \equiv r^2 \cos^2\gamma$$

$$Tr(\Delta^\dagger\Delta) \equiv r^2 \sin^2\gamma \quad ; \quad -\frac{\pi}{2} < \gamma < +\frac{\pi}{2}$$

$$Tr(\Delta^\dagger\Delta)^2/(Tr\Delta^\dagger\Delta)^2 \equiv \zeta \in [\frac{1}{2}, 1]$$

$$(H^\dagger\Delta\Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger\Delta) \equiv \xi \in [0, 1]$$

$$V_0^{(4)} = \frac{r^4\cos^4\gamma}{4}(\lambda + 4(\lambda_1 + \xi\lambda_4)\tan^2\gamma + 4(\lambda_2 + \zeta\lambda_3)\tan^4\gamma)$$

$$V(\chi) = a|\phi^0|^4 + b|\phi^0|^2|\delta^0|^2 + c|\delta^0|^4 \quad , \; \chi = |\phi^0|/|\delta^0|$$

$$= a + b\chi^2 + c\chi^4 = (\sqrt{a} - \sqrt{c}\chi^2)^2 + (b + 2\sqrt{ac})\chi^2 \Rightarrow a > 0 \;\&\; c > 0 \;\&\; b + 2\sqrt{ac} > 0$$

$$\lambda > 0 \;\&\; \lambda_2 + \zeta\lambda_3 > 0 \;\;\&\; \lambda_1 + \xi\lambda_4 + \sqrt{\lambda(\lambda_2 + \zeta\lambda_3)} > 0 \; \forall\zeta \in [\frac{1}{2}, 1], \xi \in$$

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$$\forall\zeta \in [\frac{1}{2}, 1], \forall\xi \in [0, 1]$$

$$\lambda > 0 \;\;\&\;\; \lambda_2 + \lambda_3 > 0 \;\;\&\;\; \lambda_2 + \frac{\lambda_3}{2} > 0$$

$$\&\;\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0 \;\;\&\;\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} > 0$$

$$\&\;\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0 \;\;\&\;\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} > 0$$

Spectrum and constraints on μ

Absence of tachyonic modes:

$$m_A^2 = \frac{\mu(v_d^2 + 4v_\Delta^2)}{\sqrt{2}v_\Delta} \Rightarrow \mu > 0$$

$$m_{H^{\pm\pm}}^2 = \frac{\sqrt{2}\mu v_d^2 - \lambda_4 v_d^2 v_\Delta - 2\lambda_3 v_\Delta^3}{2v_\Delta} \Rightarrow \mu > \frac{\lambda_4 v_\Delta}{\sqrt{2}} + \sqrt{2}\frac{\lambda_3 v_\Delta^3}{v_d^2}$$

$$m_{H^\pm}^2 = \frac{(v_d^2 + 2v_\Delta^2)[2\sqrt{2}\mu - \lambda_4 v_\Delta]}{4v_\Delta} \Rightarrow \mu > \frac{\lambda_4 v_\Delta}{2\sqrt{2}}$$

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From the CP-even sector, it is more involving:

$$-8\mu^2 v_\Delta + \sqrt{2}\mu(\lambda v_d^2 + 8\lambda_{14} v_\Delta^2) + 4(\lambda\lambda_{23} - \lambda_{14})v_\Delta^3 > 0$$

$$(\lambda_{14}^2 - \lambda\lambda_{23})\,\frac{2\sqrt{2}}{\lambda}\frac{v_\Delta^3}{v_d^2} + \mathcal{O}(v_\Delta^4) < \mu < \frac{\lambda}{4\sqrt{2}}\frac{v_d^2}{v_\Delta} + \sqrt{2}\lambda_{14}v_\Delta + \mathcal{O}(v_\Delta^2).$$

with $\lambda_{ij} = \lambda_i + \lambda_j$

Boundedness From Bellow (BFB)

Stability of the vacuum ($V > V_{min}$) requires that the potential should be BFB. At large field values: $V \approx V^{(4)}(H,\Delta)$

$$
\begin{aligned}
V^{(4)}(H,\Delta) &= \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr\Delta^\dagger \Delta)^2 \\
&+ \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H
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$$

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- If we pick up neutral directions:

$$V_0^{(4)} = \frac{\lambda}{4}|\phi^0|^4 + (\lambda_2+\lambda_3)|\delta^0|^4 + (\lambda_1+\lambda_4)|\phi^0|^2|\delta^0|^2 =$$

$$[\frac{\sqrt{\lambda}}{2}|\phi^0|^2 - \sqrt{\lambda_2+\lambda_3}|\delta^0|^2]^2 + (\lambda_{14} + \sqrt{\lambda(\lambda_{23})})|\phi^0|^2|\delta^0|^2$$

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$$[\frac{\sqrt{\lambda}}{2}|\phi^0|^2 - \sqrt{\lambda_2 + \lambda_3}|\delta^0|^2]^2 + (\lambda_{14} + \sqrt{\lambda(\lambda_{23})})|\phi^0|^2|\delta^0|^2$$

- $\lambda > 0 \;\&\; \lambda_2 + \lambda_3 > 0 \;\&\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0$

BFB

What about the other 10 directions: (ϕ^0 , δ^{++}), (ϕ^0 ,δ^+), (ϕ^0 , ϕ^+), (δ^+ , ϕ^+) ...

- (ϕ^0 , δ^0) neutral direction:
  $\lambda > 0 \;\&\; \lambda_2 + \lambda_3 > 0 \;\&\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0$
- (ϕ^0 , δ^{++}) direction:
  $\lambda > 0 \;\&\; \lambda_2 + \lambda_3 > 0 \;\&\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} > 0$
- (δ^+ , ϕ^+) direction:
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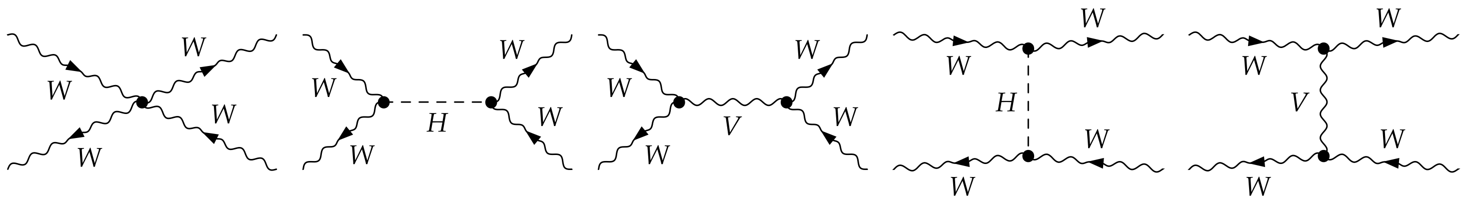
- (δ^+ , ϕ^+) direction:
  $\lambda > 0 \ \& \ \lambda_2 + \frac{\lambda_3}{2} > 0 \ \& \ \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} > 0$
- It is obvious that these sets are neither equivalent nor contained in the neutral direction. Neutral direction BFB constraint is neither necessary nor sufficient

Perturbative unitarity

In the SM: $W_L W_L$, $Z_L Z_L$, HH, $W_L H$, $Z_L H$



the scattering amplitude $\mathcal{M}$ can be written as:

$$\mathcal{M}(s,t,u) = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l(s)$$

Perturbative unitarity

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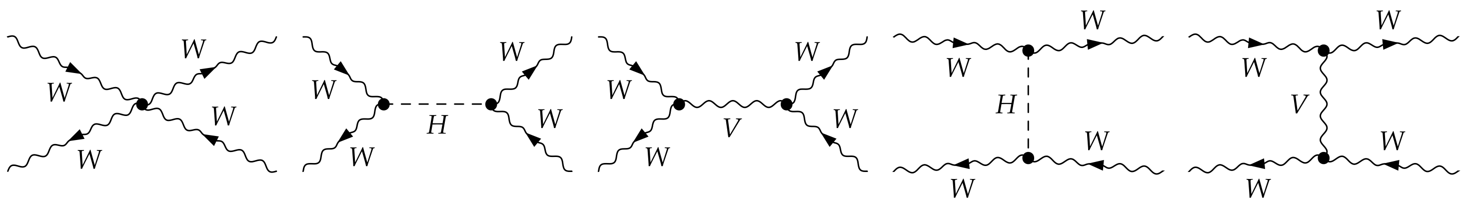

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$$\mathcal{M}(s,t,u) = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l(s)$$

If we limit ourselves to the ($J=0$) s–wave amplitude $a_0(s)$

$$a_0 = \frac{M_{\phi^0}^2}{v^2} \begin{pmatrix} 1 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \qquad |a_0| \leq 1$$

$M_{\phi^0} \leq 710$GeV

Perturbative unitarity

In the HTM: the scattering amplitude is 35×35 matrix which can be cast to 7 sub-matrix:

- $S_1(6 \times 6)$, $S_2(7 \times 7)$, $S_3(2 \times 2)$, (0-charge channels): $\delta^0\delta^0$, $\phi^+\phi^-$, $\delta^{++}\delta^{--}$
- $S_{(4)}(10 \times 10)$: (1-charge channels) : $\delta^0\phi^+$
- $S_{(5)}(7 \times 7)$: (2-charge channels) : $\phi^+\phi^+$
- $S_{(6)}(2 \times 2)$: (3-charge channels): $\delta^{++}\phi^+$
- $S_{(7)}(1 \times 1)$: (4-charge channels): $\delta^{++}\delta^{++}$

unitarity

$$|\lambda_1 + \lambda_4| \leq \kappa\pi \quad ; \quad |\lambda_1| \leq \kappa\pi \quad ; \quad |2\lambda_1 + 3\lambda_4| \leq 2\kappa\pi$$

$$|\lambda| \leq 2\kappa\pi \quad ; \quad |\lambda_2| \leq \frac{\kappa}{2}\pi \quad ; \quad |\lambda_2 + \lambda_3| \leq \frac{\kappa}{2}\pi$$

$$|2\lambda_1 - \lambda_4| \leq 2\kappa\pi \quad ; \quad |2\lambda_2 - \lambda_3| \leq \kappa\pi$$

$$|\lambda + 4\lambda_2 + 8\lambda_3 \pm \sqrt{(\lambda - 4\lambda_2 - 8\lambda_3)^2 + 16\lambda_4^2}\,| \leq 4\kappa\pi$$

$$|3\lambda + 16\lambda_2 + 12\lambda_3 \pm \sqrt{(3\lambda - 16\lambda_2 - 12\lambda_3)^2 + 24(2\lambda_1 + \lambda_4)^2}\,| \leq 4\kappa\pi$$

$\kappa = 16$ or 8, depending on : $|a_0| < 1$ or $|\Re a_0| < \frac{1}{2}$