Associated production of light gravitinos at future linear colliders

Kentarou Mawatari



arXiv:1106.5592 KM, B.Oexl(VUB), Y. Takaesu(KEK)

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Outlines

- Gravitino phenomenology at colliders
- Gravitino in a Monte Carlo event generator
- Associated production of light gravitinos in future linear colliders:

$$- e^+e^- \to \tilde{\chi}_1^0 \tilde{G} \to \gamma \tilde{G} \tilde{G}$$

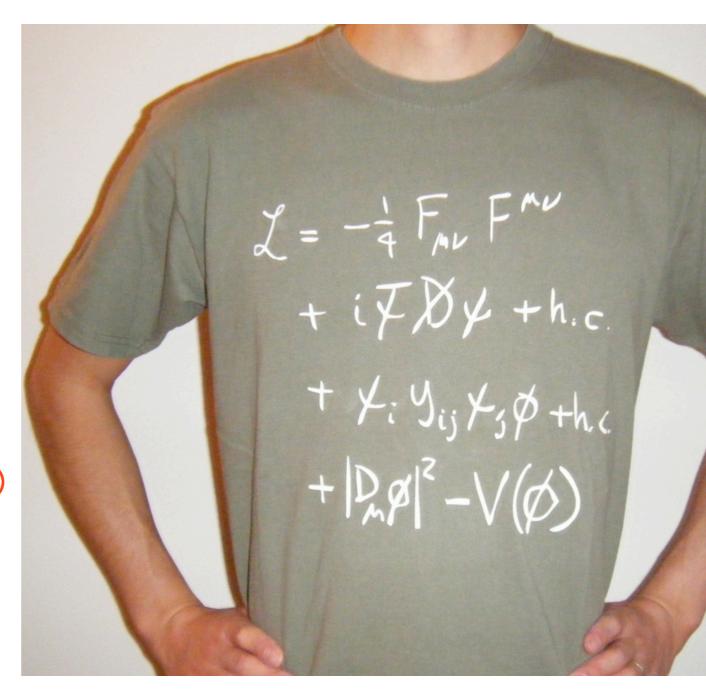
$$-e^-\gamma \to \tilde{e}^-\tilde{G} \to e^-\tilde{G}\tilde{G}$$

Gravitinos

- spin-3/2 superpartners of gravitons in local supersymmetric extensions to the Standard Model (Supergravity).
- If SUSY breaks spontaneously, gravitinos absorb massless spin-1/2 goldstinos and become massive by the super-Higgs mechanism.

- SU(2)xU(1) gauge symmetry
 - spontaneously broken

- discovered in 1983
- established the EW theory



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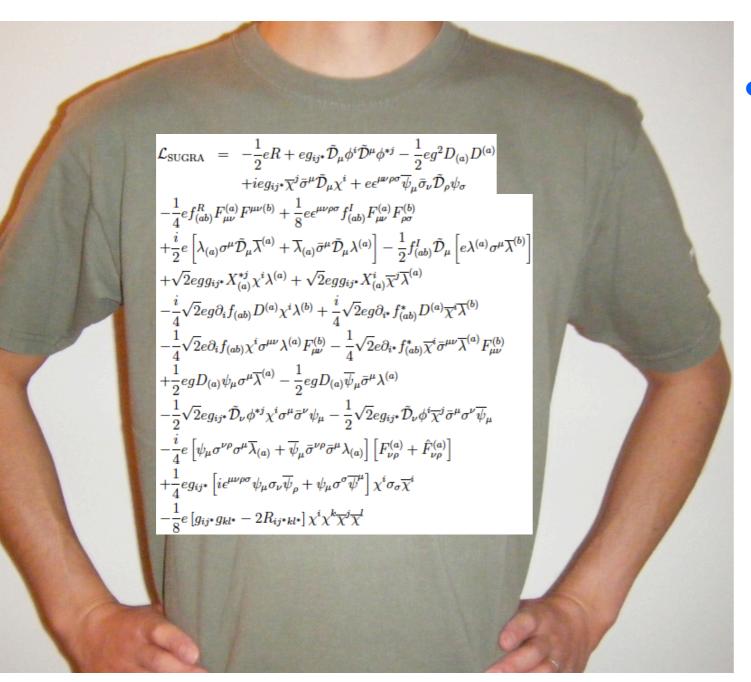
```
(spin)
----- +3/2
----- +1/2
(Goldstino)
----- -1/2
super-Higgs mechanism
---- -3/2 Massive gravitinos
```

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- **→** discover in 201? (??)
- establish supergravity!!



- Local supersymmetry
 - spontaneously broken

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(Goldstino)
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super-Higgs mechanism
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- **→** discover in 201? (??)
- establish supergravity!!



* copied from hep-ph/9503210 by T.Moroi.

Mass of the gravitino

 related to the SUSY breaking scale as well as the Planck scale

$$m_{3/2} \sim (M_{\rm SUSY})^2 / M_{\rm Pl}$$

 This implies that the gravitino can take a wide range of mass, depending on the SUSY breaking scale, from eV up to scales beyond TeV, and provide rich phenomenology in particle physics as well as in cosmology.

Collider phenomenology for a gravitino LSP

- The low-scale SUSY breaking can naturally happen in gaugemediated SUSY breaking scenarios, where the gravitino is often the LSP and can play an important role even for collider signatures.
- The phenomenology depends so much on what the NLSP is.
 - In the minimal model of gauge mediation, the lightest neutralino and the lighter stau are often the NLSP.
 - A chargino, sneutrino, gluino, and squark can also be NLSP in, e.g., general gauge mediation models, split SUSY models, ...

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- A simulation tool for gravitinos is needed for systematic analyses.

Gravitino in MadGraph/MadEvent

- "HELAS and MadGraph with spin-3/2 particles (gravitinos)"
 K. Hagiwara (KEK), KM, Y. Takaesu (KEK); EPJC71(2011) [1010.4255]
- "HELAS and MadGraph with goldstinos" KM,Y.Takaesu (KEK); EPJC71(2011) [1101.1289]
 - ▶ We added new HELAS fortran subroutines for massive spin-3/2 gravitinos and goldstinos and their interactions, and implemented them into MadGraph/MadEvent (MG/ME) so that arbitrary amplitudes with external gravitinos/goldstinos can be generated automatically.
 - MG/ME v4.5 supports spin-3/2 as well as spin-0, I/2, I, and 2. [HELAS and MG/ME w/ spin-2 particles by Hagiwara, Kanzaki, Q.Li, KM, EPJC(2008)]

Gravitino mass limits from colliders

K. Nakamura et al. (Particle Data Group), JP G 37, 075021 (2010) and 2011 partial update for the 2012 edition (URL: http://pdg.lbl.gov)

LIGHT \tilde{G} (Gravitino) MASS LIMITS FROM COLLIDER EXPERIMENTS

The following are bounds on light (\ll 1 eV) gravitino indirectly inferred from its coupling to matter suppressed by the gravitino decay constant.

Unless otherwise stated, all limits assume that other supersymmetric particles besides the gravitino are too heavy to be produced. The gravitino is assumed to be undetected and to give rise to a missing energy (\cancel{E}) signature.

VALUE (eV)	CL%	DOCUMENT ID	TECN	COMMENT
 ◆ ◆ We do not 	use the fo	ollowing data for a	verages, fits,	limits, etc. • • •
$> 1.09 \times 10^{-5}$	95	¹ ABDALLAH	05B DLPH	$e^+e^- ightarrow \ \widetilde{G} \widetilde{G} \gamma$
$> 1.35 \times 10^{-5}$	95	² ACHARD	04E L3	$e^+e^- ightarrow \ \widetilde{G} \widetilde{G} \gamma$
$> 1.3 \times 10^{-5}$		³ HEISTER	03C ALEP	$e^+e^- ightarrow \ \widetilde{G} \widetilde{G} \gamma$
$>11.7 \times 10^{-6}$	95	⁴ ACOSTA	02H CDF	$p\overline{p} \rightarrow \widetilde{G}\widetilde{G}\gamma$
$> 8.7 \times 10^{-6}$	95	⁵ ABBIENDI,G	00D OPAL	$e^+e^- ightarrow \ \widetilde{G} \widetilde{G} \gamma$
$>10.0 \times 10^{-6}$	95	⁶ ABREU	00z DLPH	$e^+e^- ightarrow \ \widetilde{G} \widetilde{G} \gamma$
$>11 \times 10^{-6}$	95	⁷ AFFOLDER	00J CDF	$p\overline{p} \rightarrow \widetilde{G}\widetilde{G} + \text{jet}$
$> 8.9 \times 10^{-6}$	95	⁸ ACCIARRI	99R L3	$e^+e^- ightarrow \ \widetilde{G}\widetilde{G}\gamma$
$> 7.9 \times 10^{-6}$	95	⁹ ACCIARRI	98V L3	$e^+e^- ightarrow \ \widetilde{G}\widetilde{G}\gamma$
$> 8.3 \times 10^{-6}$	95	⁹ BARATE	98J ALEP	$e^+e^- ightarrow \ \widetilde{G} \widetilde{G} \gamma$

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$m_{3/2} > 10^{-5} \mathrm{eV} = 10^{-14} \mathrm{GeV}$							

Single photon + missing energy at LEP

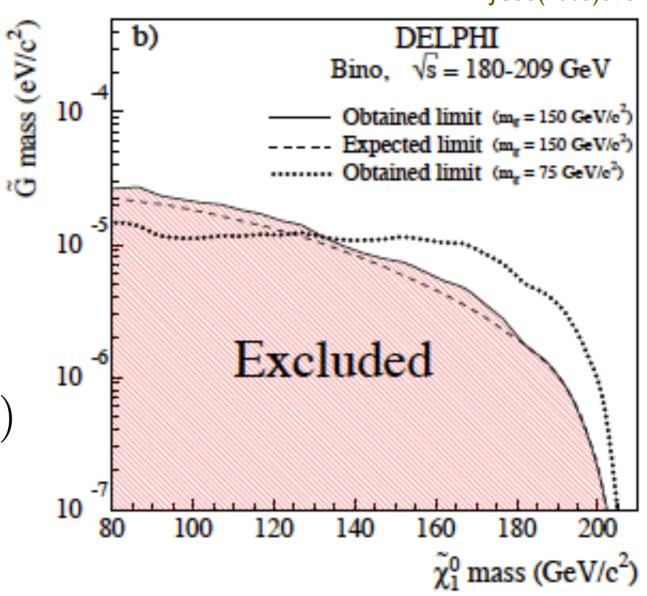
EPJC38(2005)395

 a neutralino NLSP with a gravitino LSP

•
$$e^+e^- \to \tilde{\chi}_1^0 \tilde{G} \to \gamma \tilde{G} \tilde{G}$$

-
$$\sigma \propto 1/m_{3/2}^2$$

-
$$\sigma=\sigma(m_{3/2},m_{\tilde{\chi}_1^0},m_{\tilde{e}})$$

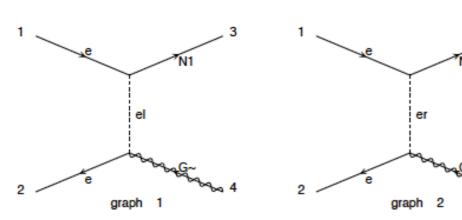


Neutralino-gravitino productions at e⁺e⁻ colliders

$$e^+e^- \to \tilde{\chi}_1^0 \tilde{G} \to \gamma \tilde{G} \tilde{G}$$

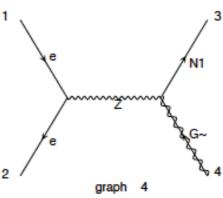
Diagrams by MadGraph

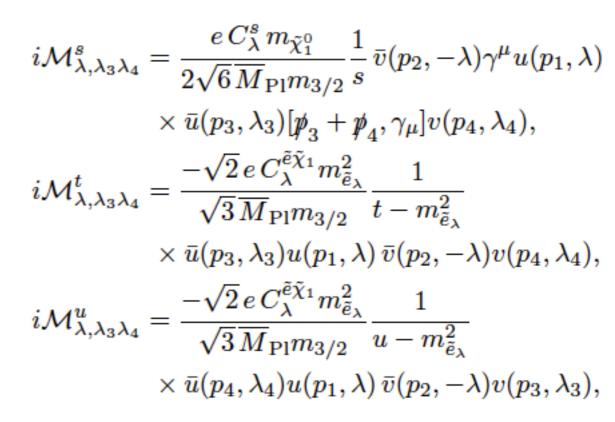
- e+ -> n1 gro

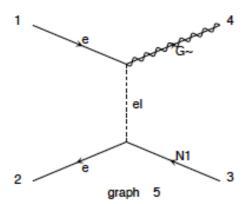


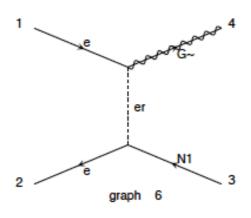
$$e^-\left(p_1, \frac{\lambda_1}{2}\right) + e^+\left(p_2, \frac{\lambda_2}{2}\right) \to \tilde{\chi}_1^0\left(p_3, \frac{\lambda_3}{2}\right) + \tilde{G}\left(p_4, \frac{\lambda_4}{2}\right)$$

$$\begin{split} \mathcal{M}_{\lambda,\lambda_3\lambda_4} &= \mathcal{M}^s_{\lambda,\lambda_3\lambda_4} + \mathcal{M}^t_{\lambda,\lambda_3\lambda_4} + \mathcal{M}^u_{\lambda,\lambda_3\lambda_4} \\ & \text{with } \lambda = \lambda_1 = -\lambda_2 \end{split}$$









Helicity amplitudes

$$i\mathcal{M}_{\lambda,\lambda_3\lambda_4} = \frac{-e}{\sqrt{6}\,\overline{M}_{\rm Pl} m_{3/2}} \sqrt{\beta}\,s\,\hat{\mathcal{M}}_{\lambda,\lambda_3\lambda_4}$$

KM, Oexl, Takaesu [1106.5592]

λ	$\lambda_3\lambda_4$			$\hat{\mathcal{M}}^s$	$\hat{\mathcal{M}}^t$	$\hat{\mathcal{M}}^u$	
±	土干	$(1+\cos\theta)$	[$\frac{m_{\tilde{\chi}}^2}{s}C_{\pm}^s$		$-\frac{m_{\tilde{e}_{\pm}}^2}{u-m_{\tilde{e}_{\pm}}^2}C_{\pm}^{\tilde{e}\tilde{\chi}_1}$]
\pm	 士	$-(1-\cos\theta)$	[$\frac{m_{\tilde{\chi}}^2}{s}C_{\pm}^s$	$-\frac{m_{\tilde{e}_{\pm}}^2}{t-m_{\tilde{e}_{\pm}}^2}C_{\pm}^{\tilde{e}\tilde{\chi}_1}$]
\pm	土土	$\pm \frac{m_{\tilde{\chi}}}{\sqrt{s}} \sin \theta$	[C^s_\pm	$-\frac{m_{\tilde{e}_{\pm}}^2}{t-m_{\tilde{e}_{\pm}}^2}C_{\pm}^{\tilde{e}\tilde{\chi}_1}$]
\pm	干干	$\mp \frac{m_{\tilde{\chi}}}{\sqrt{s}} \sin \theta$	[C^s_\pm		$-\frac{m_{\tilde{e}_{\pm}}^2}{u-m_{\tilde{e}_{\pm}}^2}C_{\pm}^{\tilde{e}\tilde{\chi}_1}$]

Table 1. The reduced helicity amplitudes $\hat{\mathcal{M}}_{\lambda,\lambda_3\lambda_4}$ for $e_{\lambda}^-e_{-\lambda}^+ \to \tilde{\chi}_{1\lambda_3}^0 \tilde{G}_{\lambda_4}$.

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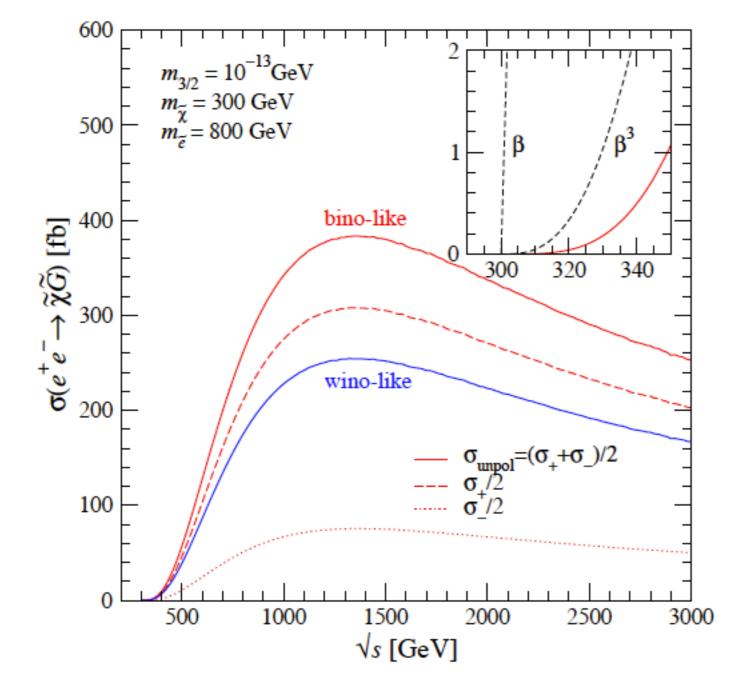
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Table 1. The reduced helicity amplitudes $\hat{\mathcal{M}}_{\lambda,\lambda_3\lambda_4}$ for $e_{\lambda}^-e_{-\lambda}^+ \to \tilde{\chi}_{1\lambda_3}^0 \tilde{G}_{\lambda_4}$.

 The t- and u-amplitudes become dominant as the selectron mass increases.

Total cross sections (the collision energy)

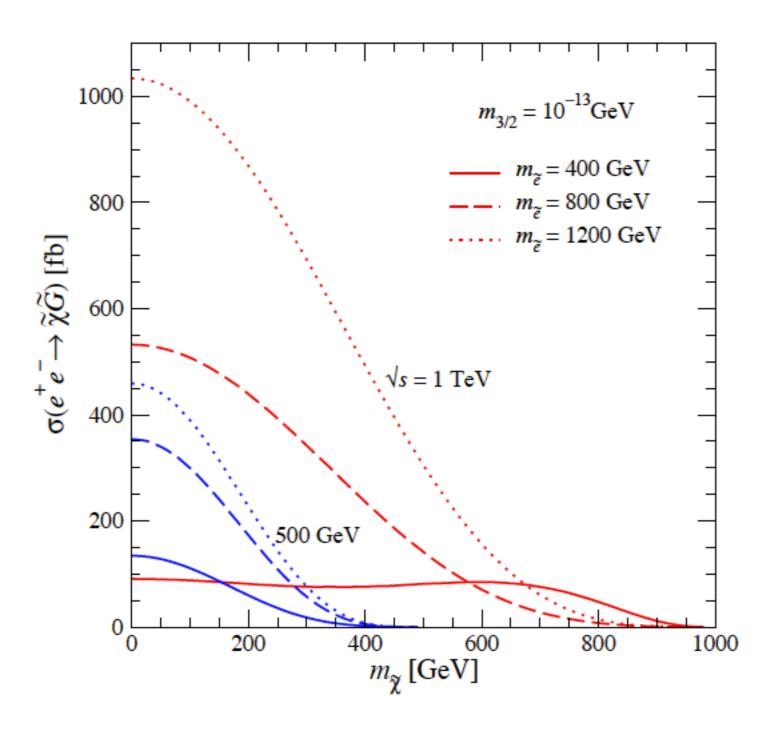


- The cross section scales with $(m_{3/2})^{-2}$.
- The threshold excitation is $\sigma \sim \beta^4$.
- The ratio of the polarized and unpolarized cross sections is roughly given by

$$\frac{\sigma_{\pm}}{2\sigma_{\rm unpol}} \sim \frac{|C_{\pm}^{\tilde{e}\tilde{\chi}_1}|^2}{|C_{+}^{\tilde{e}\tilde{\chi}_1}|^2 + |C_{-}^{\tilde{e}\tilde{\chi}_1}|^2}$$

→ neutralino mixing

Total cross sections (the neutralino mass)



- The cross sections are strongly suppressed as the neutralino mass is approaching the collider energy.
- The cross sections are quite sensitive to the selectron masses, even if the collider energy cannot reach them.

Angular distributions

14

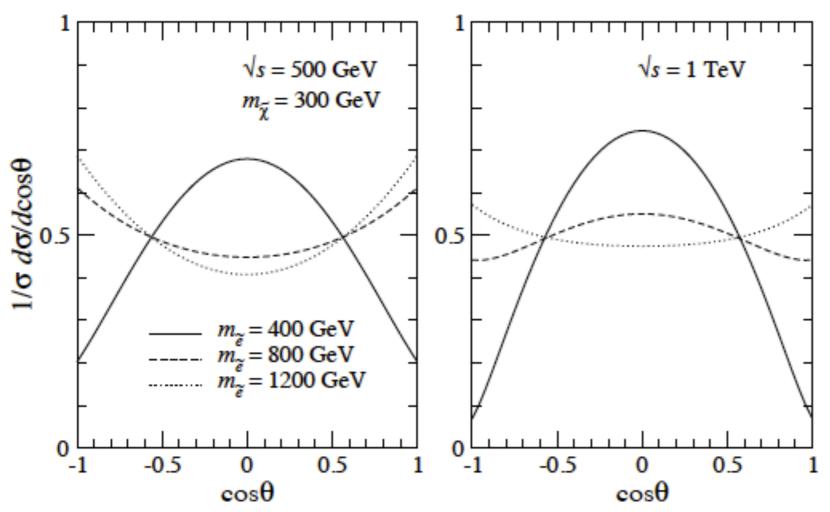


Fig. 4. Normalized angular distributions of neutralinos in $e^+e^- \to \tilde{\chi}_1^0 \tilde{G}$ at $\sqrt{s} = 500$ GeV (left) and 1 TeV (right) for $m_{\tilde{\chi}_1^0} = 300 \text{ GeV}$, where the selectron masses are taken to be 400, 800 and 1200 GeV.

Not only the cross section but also the angular distribution is sensitive to the selectron masses.

Kinematical cuts and beam polarizations

σ [fb]		$(P_{e^-}, P_{e^+}) =$	(0, 0)	(0.9, 0)	(0.9, -0.6)
$\sqrt{s} = 500 \text{ GeV}$	$m_{\tilde{e}} = 400 \text{ GeV}$		15	23	37
	800 GeV		48	75	119
	$1200~{ m GeV}$		64	100	159
	SM background		1592	178	94
$\sqrt{s} = 1 \text{ TeV}$	$m_{\tilde{e}} = 400 \text{ GeV}$		72	112	177
	800 GeV		320	494	785
	$1200~{ m GeV}$		642	1002	1582
	SM background		1443	149	65

Table 2. Cross sections in fb for the signal $e^+e^- \to \tilde{\chi}_1^0 \tilde{G} \to \gamma \tilde{G} \tilde{G}$, assuming $B(\tilde{\chi}_1^0 \to \gamma \tilde{G}) = 1$, with $m_{\tilde{G}} = 10^{-13}$ GeV and $m_{\tilde{\chi}_1^0} = 300$ GeV and for the SM background $e^+e^- \to \gamma \nu \bar{\nu}$ at $\sqrt{s} = 500$ GeV and 1 TeV, with different beam polarizations. The minimal cuts in (19) and the Z-peak cut in (20) are taken into account.

The minimal cuts:

$$E_{\gamma} > 0.03 \sqrt{s}, \quad |\eta_{\gamma}| < 2,$$

The Z-peak cut:

$$E_{\gamma} > 0.03 \sqrt{s}, \quad |\eta_{\gamma}| < 2, \qquad E_{\gamma} < \frac{s - m_Z^2}{2\sqrt{s}} - 5\Gamma_Z,$$

• With beam polarizations, the signal is enhanced, while the background coming from the t-channel W-exchange can be reduced.

Mono-photon distributions (energy)

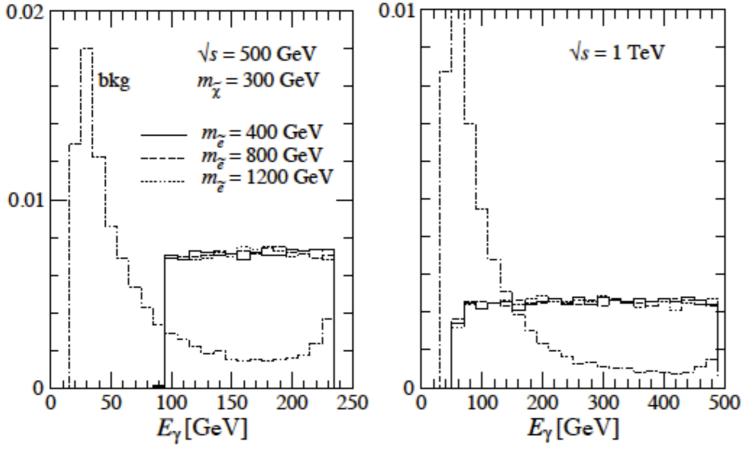


Fig. 5. Normalized energy distributions of the photon for $e^+e^- \to \tilde{\chi}_1^0 \tilde{G} \to \gamma \tilde{G} \tilde{G}$ at $\sqrt{s} = 500$ GeV (left) and 1 TeV (right), where $m_{\tilde{e}_{\pm}} = 400$ (solid), 800 (dashed) and 1200 (dotted) GeV with $m_{\tilde{\chi}_1^0} = 300$ GeV are considered. The kinematical cuts in (20) and (21) and the beam polarizations $(P_{e^-}, P_{e^+}) = (0.9, -0.6)$ are taken into account. Those of the SM background are also shown by dot-dashed lines.

 The neutralino decays into a photon and a gravitino is isotropic in the neutralino rest frame.

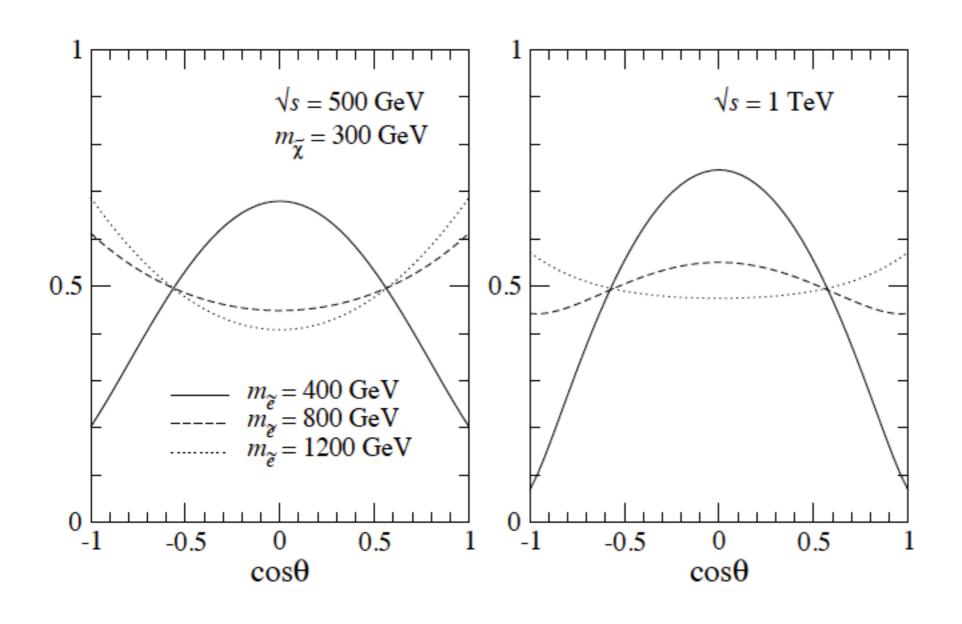
flat energy distribution

The range of the energy is

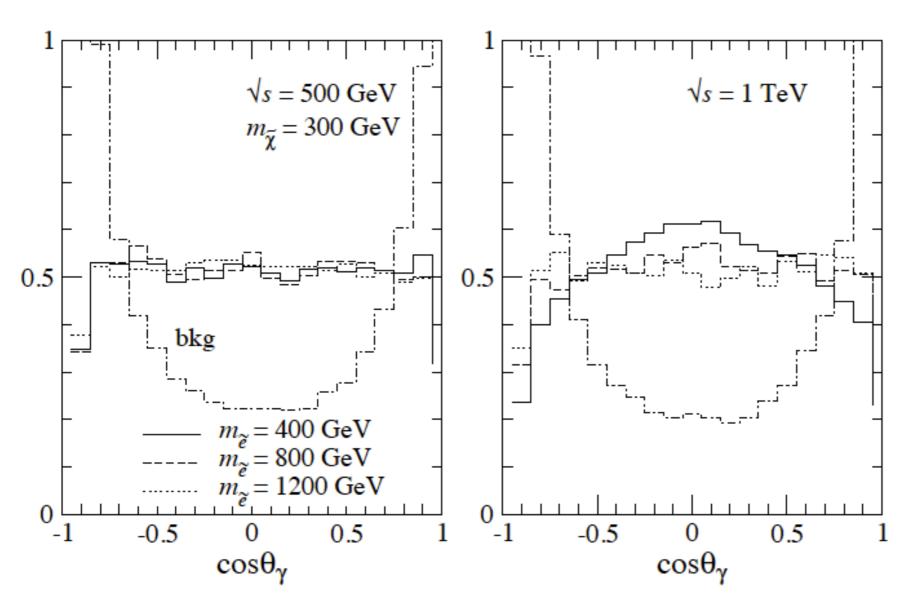
$$\frac{m_{\tilde{\chi}}^2}{2\sqrt{s}} < E_{\gamma} < \frac{\sqrt{s}}{2}$$

neutralino mass

Mono-photon distributions (angle)



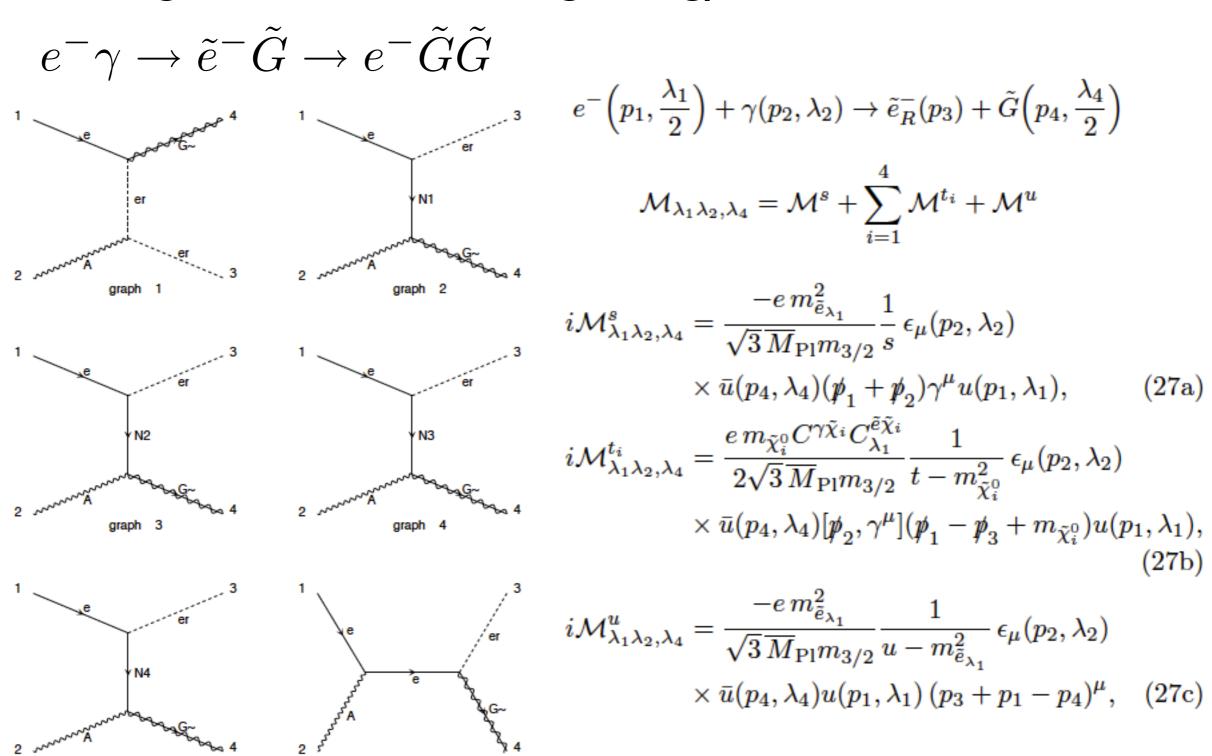
Mono-photon distributions (angle)



- The original neutralino distributions are flattened.
- When the decaying neutralino has a large momentum, the original neutralino directions can survive.

selectron mass

Single electron + missing energy in eγ collisions



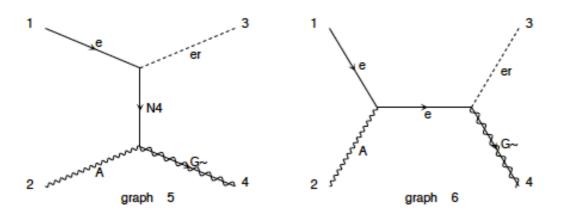
Single electron + missing energy in ey collisions

$$e^{-\gamma} \rightarrow \tilde{e}^{-\tilde{G}} \rightarrow e^{-\tilde{G}} \tilde{G}$$

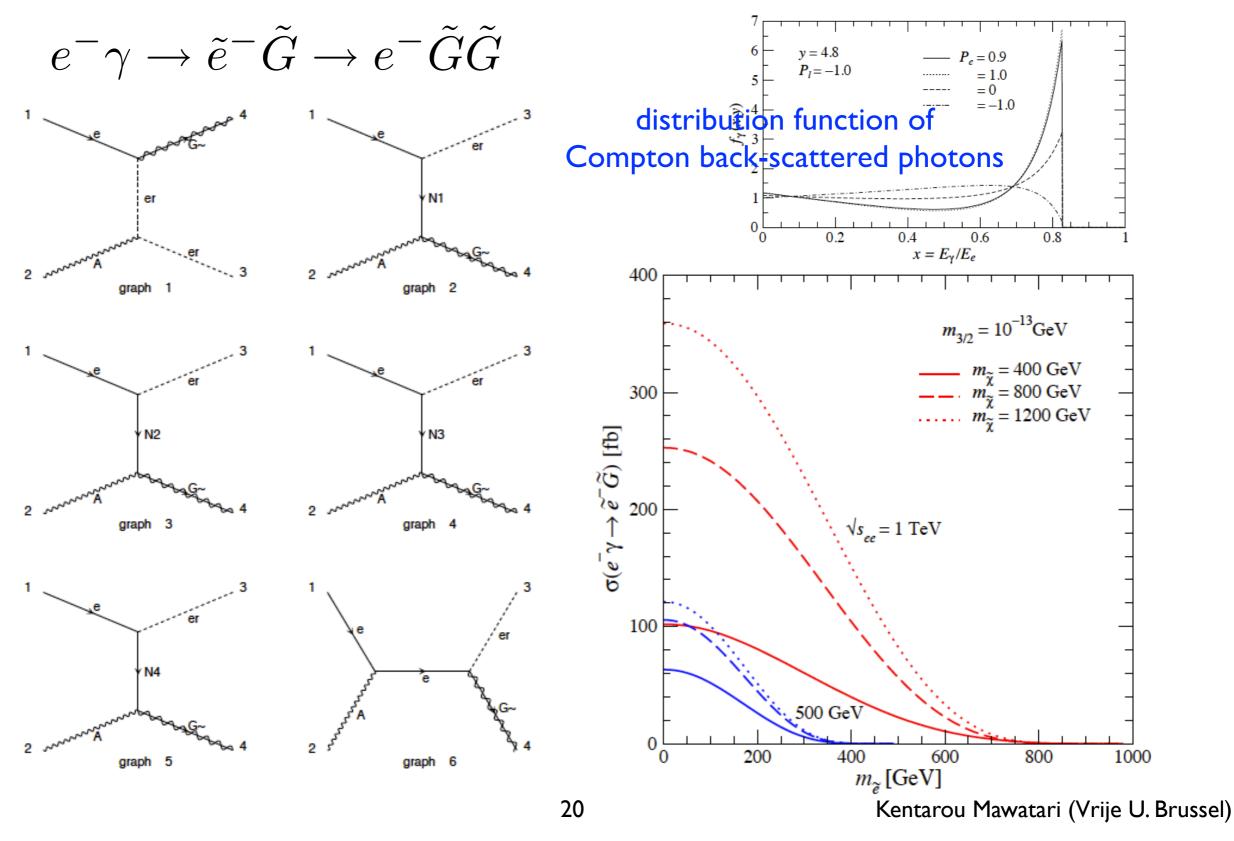
$$iM_{\lambda_1\lambda_2,\lambda_4} = \frac{-e}{\sqrt{6} \, \overline{M}_{\rm Pl} m_{3/2}} \sqrt{\beta} \, s \, \hat{M}_{\lambda_1\lambda_2,\lambda_4}$$

$$\frac{\lambda_1\lambda_2 \quad \lambda_4}{++-} \qquad \frac{\hat{M}^s}{2 \sin \frac{\theta}{2}} \quad \left[\begin{array}{ccc} \frac{m_{\tilde{e}}^s}{s} & -\sum_i C^{\gamma \tilde{\chi}_i} C^{\tilde{e}}_i \tilde{\chi}_i & \frac{m_{\tilde{\chi}_i}^s}{t-m_{\tilde{\chi}_i}^s} & +\frac{m_{\tilde{e}}^2}{u-m_{\tilde{e}}^2} \beta \frac{1+\cos \theta}{2} \end{array} \right]$$

Table 3. The reduced helicity amplitudes $\hat{\mathcal{M}}_{\lambda_1\lambda_2,\lambda_4}$ for $e_{\lambda_1}^-\gamma_{\lambda_2} \to \tilde{e}_R^-\tilde{G}_{\lambda_4}$.



Single electron + missing energy in eγ collisions



Single electron + missing energy in ey collisions

$$e^- \gamma \to \tilde{e}_R^- \tilde{G} \to e^- \tilde{G} \tilde{G} \Rightarrow e^- + E$$

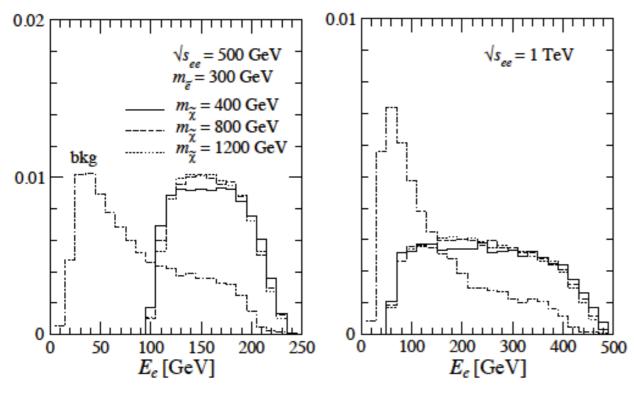
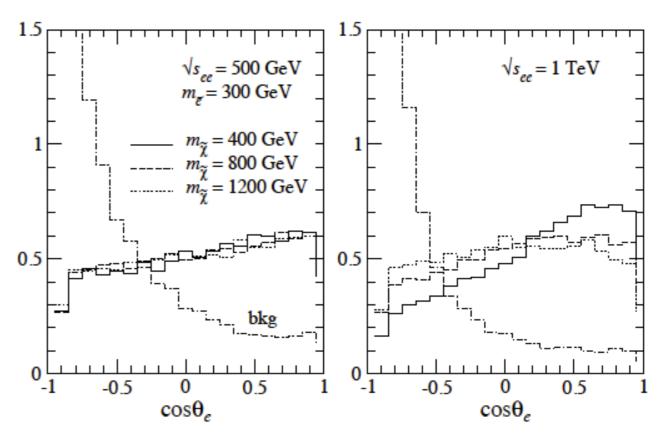


Fig. 11. Normalized energy distributions of the electron for $e^-\gamma \to \tilde{e}_R^-\tilde{G} \to e^-\tilde{G}\tilde{G}$ at $\sqrt{s_{ee}} = 500$ GeV (left) and 1 TeV Fig. 12. Normalized angular distributions of the electron in (right), where $m_{\tilde{\chi}} = 400$ (solid), 800 (dashed) and 1200 (dotted) GeV with $m_{\tilde{e}_R} = 300$ GeV are considered. The kinematical cuts in (35) and (36) and the electron beam polarization $P_{e^-} = 0.9$ are taken into account. Those of the SM background are also shown by dot-dashed lines.





the e^-e^- laboratory frame for $e^-\gamma \to \tilde{e}_R^-\tilde{G} \to e^-\tilde{G}\tilde{G}$. The detail is the same as Fig. 11.

t-channel neutralino mass

Summary

- Gravitinos can provide rich phenomenology in particle physics as well as in cosmology, and especially play an important role in collider signatures when it is the LSP. The phenomenology depends on what the NLSP is.
- We (Hagiwara, KM, Takaesu [1010.4255], KM, Takaesu [1101.1289])
 - added new HELAS fortran subroutines to calculate helicity amplitudes with massive gravitinos/goldstinos.
 - coded them in such a way that arbitrary amplitudes with external gravitinos/goldstinos can be generated automatically by MadGraph.
 (Our implementation was officially supported by MG/MEv4.5, and will be available in MG5 soon.)
- We (KM, Oexl, Takaesu [1106.5592])
 - restudied associated production of light gravitinos in future linear colliders.

$$e^+e^- \to \tilde{\chi}_1^0 \tilde{G} \to \gamma \tilde{G} \tilde{G}$$
 $e^- \gamma \to \tilde{e}^- \tilde{G} \to e^- \tilde{G} \tilde{G}$

showed that the energy and angular distributions of the photon/electron can explore the mass of the t-channel exchange particles as well as the mass of the decaying particle.

Back-up

Effective goldstino interaction Lagrangian

$$\mathcal{L}_{\tilde{G}} = \mp \frac{im_{\tilde{e}_{\pm}}^{2}}{\sqrt{3} \overline{M}_{Pl} m_{3/2}} \left[\bar{\psi}_{\tilde{G}} P_{\pm} \psi_{e} \, \phi_{\tilde{e}_{\pm}}^{*} - \bar{\psi}_{e} P_{\mp} \psi_{\tilde{G}} \, \phi_{\tilde{e}_{\pm}} \right] \\
- \frac{C^{V \tilde{\chi}_{i}} m_{\tilde{\chi}_{i}^{0}}}{4\sqrt{6} \overline{M}_{Pl} m_{3/2}} \bar{\psi}_{\tilde{G}} [\gamma^{\mu}, \gamma^{\nu}] \psi_{\tilde{\chi}_{i}^{0}} (\partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}), \quad (38)$$

$$\mathcal{L}_{eeV} = e \, \bar{\psi}_{e} \gamma^{\mu} [A_{\mu} - (g_{+} P_{+} + g_{-} P_{-}) Z_{\mu}] \psi_{e}, \quad (39)$$

$$\mathcal{L}_{\tilde{\chi}_{i}^{0}} e\tilde{e} = \pm \sqrt{2} e \, C_{\pm}^{\tilde{e} \tilde{\chi}_{i}} \left[\bar{\psi}_{\tilde{\chi}_{i}^{0}} P_{\pm} \psi_{e} \, \phi_{\tilde{e}_{\pm}}^{*} + \bar{\psi}_{e} P_{\mp} \psi_{\tilde{\chi}_{i}^{0}} \, \phi_{\tilde{e}_{\pm}} \right], \quad (40)$$

$$\mathcal{L}_{\tilde{e}\tilde{e}\gamma} = ie \, \phi_{\tilde{e}_{\pm}}^{*} \overleftarrow{\partial^{\mu}} \phi_{\tilde{e}_{\pm}} A_{\mu}, \quad (41)$$

$$C_{\lambda}^{s} = C^{\gamma \tilde{\chi}_{i}} - \frac{s}{s - m_{Z}^{2} + im_{Z} \Gamma_{Z}} g_{\lambda} C^{Z \tilde{\chi}_{i}}$$

$$C^{\gamma \tilde{\chi}_{i}} = U_{1i} \cos \theta_{W} + U_{2i} \sin \theta_{W}, \quad C^{Z \tilde{\chi}_{i}} = -U_{1i} \sin \theta_{W} + U_{2i} \cos \theta_{W}, \quad C^{\tilde{e} \tilde{\chi}_{i}} = T_{\pm}^{\tilde{e}} \frac{U_{2i}}{\sin \theta_{W}} + Y_{\pm}^{\tilde{e}} \frac{U_{1i}}{\cos \theta_{W}}, \quad C^{\tilde{e} \tilde{\chi}_{i}} = T_{\pm}^{\tilde{e}} \frac{U_{2i}}{\sin \theta_{W}} + Y_{\pm}^{\tilde{e}} \frac{U_{1i}}{\cos \theta_{W}}, \quad (41)$$

Neutralino decay

$$\tilde{\chi}_1^0\left(p_1, \frac{\lambda_1}{2}\right) \to \gamma(p_2, \lambda_2) + \tilde{G}\left(p_3, \frac{\lambda_3}{2}\right).$$
 (42)

The partial decay rate in the neutralino rest frame is given by

$$\Gamma = \frac{1}{2m_{\tilde{\chi}_1^0}} \frac{1}{2} \int \sum_{\lambda_{1,2,3}} |\mathcal{M}_{\lambda_1,\lambda_2\lambda_3}|^2 d\Phi_2, \tag{43}$$

and the helicity amplitudes are calculated as

$$\mathcal{M}_{+,++} = -\mathcal{M}_{-,--} = \frac{-C^{\gamma\tilde{\chi}_1} m_{\tilde{\chi}_1^0}^3}{\sqrt{3} \overline{M}_{\text{Pl}} m_{3/2}} \cos\frac{\theta^*}{2},$$

$$\mathcal{M}_{+,--} = \mathcal{M}_{-,++} = \frac{-C^{\gamma\tilde{\chi}_1} m_{\tilde{\chi}_1^0}^3}{\sqrt{3} \overline{M}_{\text{Pl}} m_{3/2}} \sin\frac{\theta^*}{2},$$
(44)

$$\Gamma(\tilde{\chi}_1^0 \to \gamma \tilde{G}) = \frac{|C^{\gamma \tilde{\chi}_1}|^2 m_{\tilde{\chi}_1^0}^5}{48\pi \overline{M}_{\rm Pl}^2 m_{3/2}^2}$$