

Structure of dimension-six derivative interactions in pseudo Nambu-Goldstone N Higgs doublet models

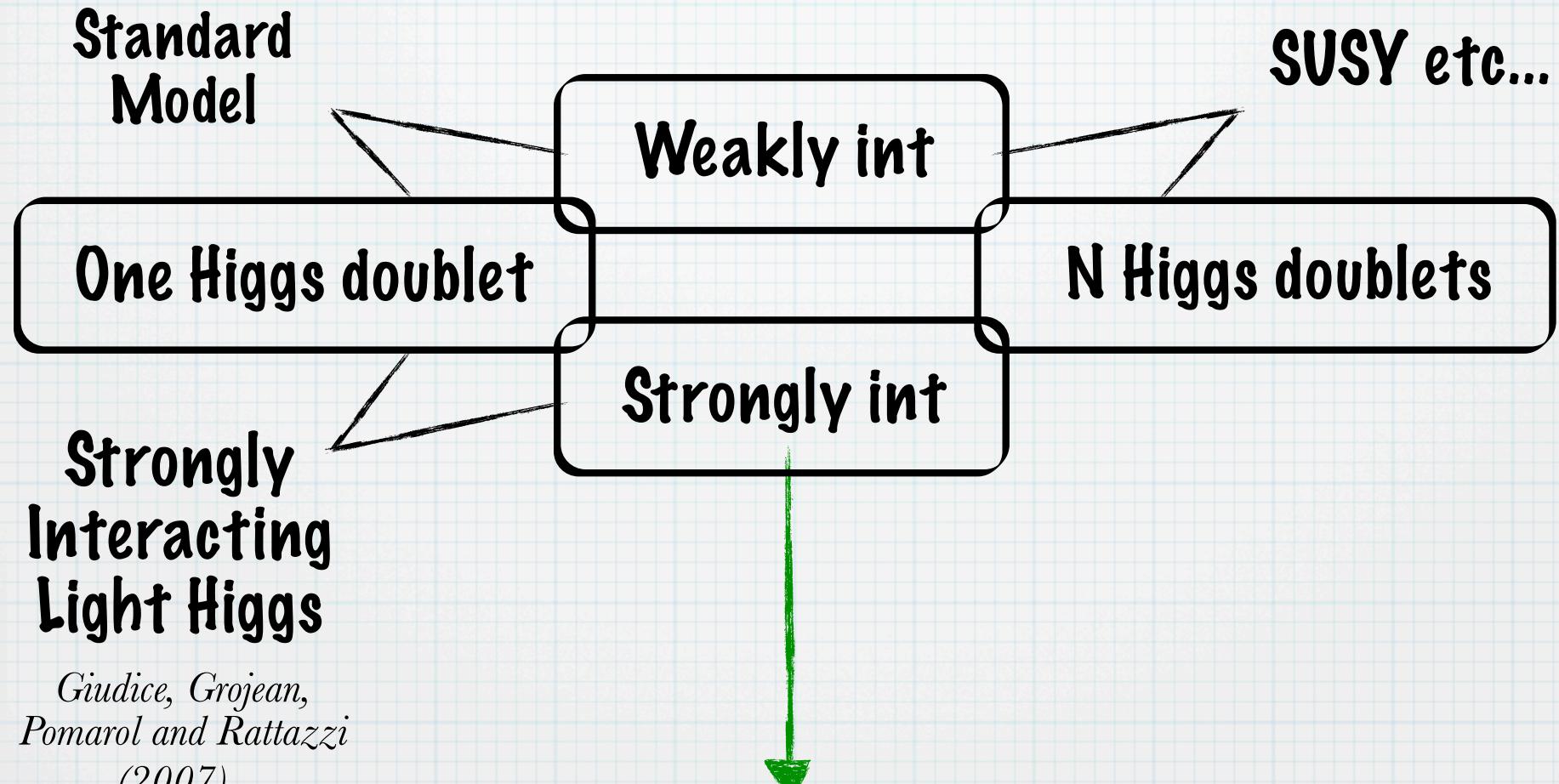
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KILC 2012
2012/04/25

Collaborators

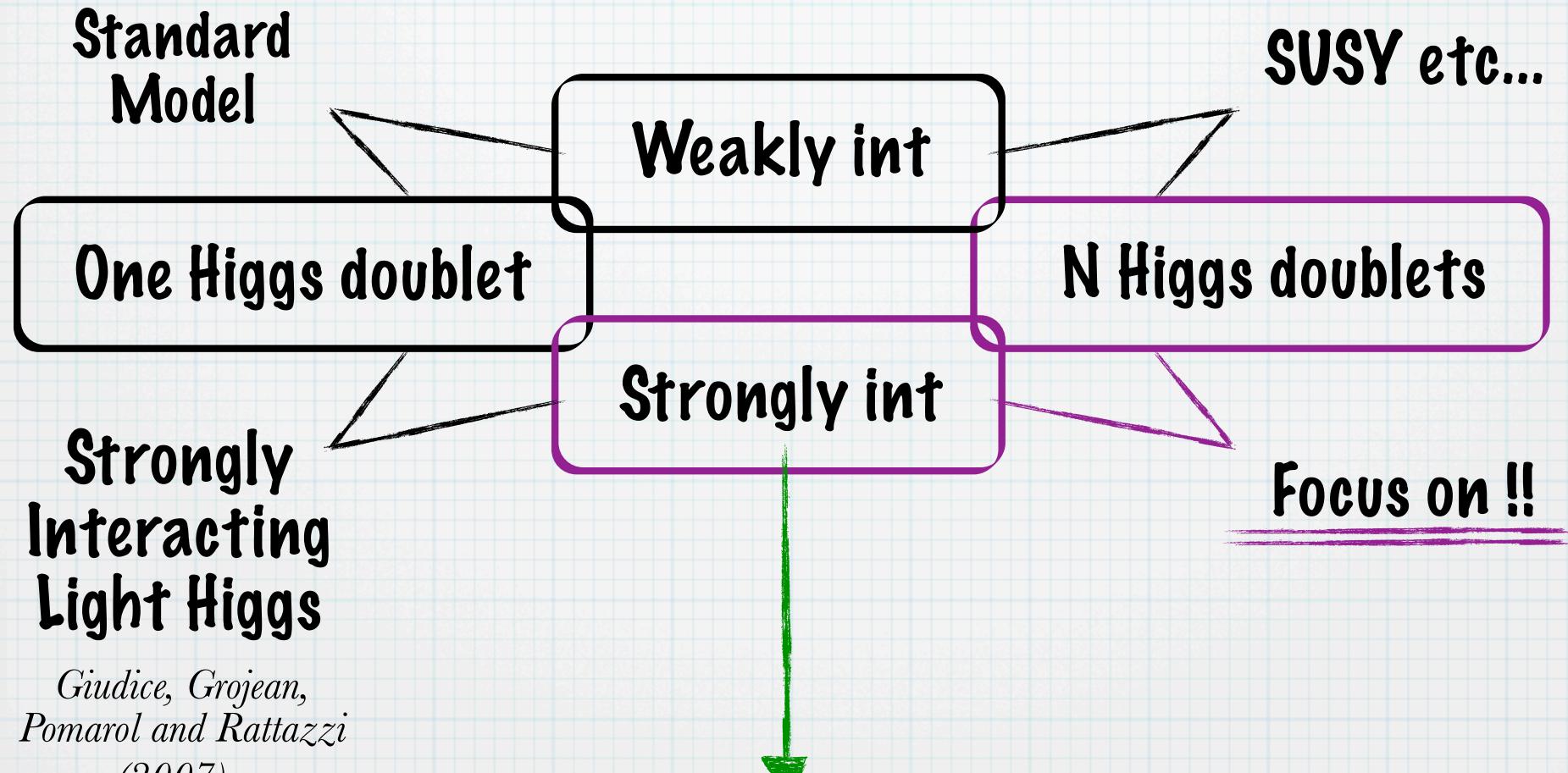
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Possibilities of the Higgs sector



The Higgs doublet arises as pseudo NG fields

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Strongly Interacting Light Higgs (SILH)

Giudice, Grojean, Pomarol and Rattazzi (2007)

Higgs doublet is associated with a global symmetry breaking

$\begin{pmatrix} \text{Little Higgs} \\ \text{Holographic Higgs} \\ \vdots \end{pmatrix} \rightarrow \text{Nonlinear sigma model (NL\!}\Sigma\!\text{M)}$

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Pseudo NG Higgs doublet Lagrangian

$$\mathcal{L}_{\text{SILH}} = \underbrace{\mathcal{L}_{\text{dim4}} + \mathcal{L}_{\text{dim6}} \left(\frac{H}{f}, \frac{D_\mu}{\Lambda} \right) + \dots}_{\text{SM-like Deviation from the SM}} \quad \begin{aligned} f &: \text{decay constant} \\ \Lambda &: \text{cutoff scale} \end{aligned}$$

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Dimension-six derivative int of the Higgs doublet

$$\frac{c_H}{2f^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H)$$

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† Correction to the kinetic term

$$\frac{1}{2} \left(1 + c_H \frac{v^2}{f^2} \right) \partial_\mu h \partial^\mu h$$

$$h \rightarrow \frac{1}{\sqrt{1 + c_H(v^2/f^2)}} h$$



Modification of
the Higgs couplings

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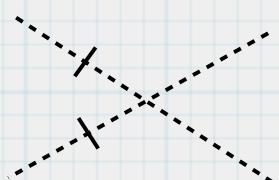
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Modification of
the Higgs couplings

† Scalar four point derivative int



$$\propto c_H \frac{E^2}{f^2}$$



Large contribution
in the high E region

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Modification of
couplings

† Scalar derivative int

$$V_L \quad V_L \\ \text{---} \quad \text{---} \\ V_L \quad V_L$$
$$\propto c_H \frac{E^2}{f^2}$$

Large contribution
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Contents of my talk

Focus on the dim 6 derivative interactions

→ Generalize to N Higgs doublets case

† Introduction (Importance of the dim 6 int)

† Dim 6 derivative int from the NLΣM

Derive $SU(2)_L \times U(1)_Y$ inv form of the int

Count degrees of freedom (DOF) of the int

† Scatterings of V_L and Higgs in $N=2$

Nonlinear representation

Think global symmetry G breaks into H

X^a : broken generators

T^i : unbroken generators

Maurer-Cartan 1-form

$$U^\dagger \partial_\mu U \equiv i\alpha_{\perp\mu}^a X^a + i\alpha_{\parallel\mu}^i T^i \quad \text{where } U = e^{i\Pi^a X^a/f}$$

NG field

Nonlinear sigma model Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{NL}\Sigma\text{M}} &= \frac{f^2}{2} \text{Tr} [\alpha_{\perp\mu} \alpha_{\perp}^\mu] \\ &= \frac{1}{2} (\partial_\mu \Pi^a) (\partial^\mu \Pi^a) + \frac{1}{f^2} \mathcal{T}^{abcd} \underbrace{\Pi^a \Pi^b (\partial_\mu \Pi^c) (\partial^\mu \Pi^d)}_{\dim 6} + \dots \end{aligned}$$

$$\mathcal{T}^{abcd} \equiv -\frac{1}{6} f^{aci} f^{bdi} - \frac{1}{24} f^{ace} f^{bde}$$

dim 6

Higgs NLΣM Lagrangian

N=1

$G \rightarrow H \supset \text{SM}$

Pick up only Higgs fields as NGBs ($\Pi^a \rightarrow h^a$)

$$\mathcal{L}_{\text{NLΣM}}^{Higgs} = \frac{1}{2}(\partial_\mu h^a)(\partial^\mu h^a) + \frac{1}{f^2} \mathcal{T}^{abcd} h^a h^b (\underline{\partial_\mu h^c})(\underline{\partial^\mu h^d}) + \dots$$

$$\begin{aligned} \mathcal{T}^{abcd} &= -\frac{1}{6} f_{\underline{aci}}^{\underline{f}} f_{\underline{bdi}}^{\underline{f}} - \frac{1}{24} f_{\underline{ace}}^{\underline{f}} f_{\underline{bde}}^{\underline{f}} \\ &\sim \sum_{SO(4)} T_{\underline{ac}}^{SO(4)} T_{\underline{bd}}^{SO(4)} \end{aligned}$$

dim 6

$a, b, c, d = 1 \sim 4$

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$$\begin{aligned} \mathcal{T}^{abcd} &= -\frac{1}{6} f_{aci}^{faci} f_{bdi}^{bdi} - \frac{1}{24} f_{ace}^{face} f_{bde}^{bde} \\ &\sim \sum_{SO(4)} T_{ac}^{SO(4)} T_{bd}^{SO(4)} \end{aligned}$$

$a, b, c, d = 1 \sim 4$

dim 6

$$\mathcal{T}^{abcd} h^a h^b (\partial_\mu h^c)(\partial^\mu h^d) \sim \sum_{SO(4)} \left(\overrightarrow{h}^T (T^{SO(4)}) \partial_\mu \overrightarrow{h} \right) \left(\overrightarrow{h}^T (T^{SO(4)}) \partial_\mu \overrightarrow{h} \right)$$

$$\overrightarrow{h} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 + ih_2 \\ h_3 + ih_4 \end{pmatrix}$$

Higgs NLΣM Lagrangian

General N

$G \rightarrow H \supset \text{SM}$

Pick up only Higgs fields as NGBs ($\Pi^a \rightarrow h^a$)

$$\mathcal{L}_{\text{NLΣM}}^{Higgs} = \frac{1}{2}(\partial_\mu h^a)(\partial^\mu h^a) + \frac{1}{f^2} \mathcal{T}^{abcd} h^a h^b (\partial_\mu h^c)(\partial^\mu h^d) + \dots$$

$$\begin{aligned} \mathcal{T}^{abcd} &= -\frac{1}{6} f_{\underline{aci}}^{\underline{bdi}} - \frac{1}{24} f_{\underline{ace}}^{\underline{bde}} \\ &\sim \sum_{SO(4N)} T_{\underline{ac}}^{SO(4N)} T_{\underline{bd}}^{SO(4N)} \end{aligned}$$

$a, b, c, d = 1 \sim 4N$

dim 6

$$\mathcal{T}^{abcd} h^a h^b (\partial_\mu h^c)(\partial^\mu h^d) \sim \sum_{SO(4N)} \left(\overrightarrow{h}^T (T^{SO(4N)}) \partial_\mu \overrightarrow{h} \right) \left(\overrightarrow{h}^T (T^{SO(4N)}) \partial_\mu \overrightarrow{h} \right)$$

$$\overrightarrow{h} = \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_{4N} \end{pmatrix} \quad H_i = \frac{1}{\sqrt{2}} \begin{pmatrix} h_{1+4(i-1)} + i h_{2+4(i-1)} \\ h_{3+4(i-1)} + i h_{4+4(i-1)} \end{pmatrix}$$

Classification of SO(4) generator

N=1

$$\mathcal{T}^{abcd} h^a h^b (\partial_\mu h^c) (\partial^\mu h^d) \sim \sum_{SO(4)} \left(h^a (T^{SO(4)})_{ac} \partial_\mu h^c \right) \left(h^b (T^{SO(4)})_{bd} \partial^\mu h^d \right)$$

→ Want to identify $SU(2)_L \times U(1)_Y$ invariant form

Classify SO(4) generators $(\alpha, \beta = 1, 2, 3)$

$$\begin{array}{ccc} [3,1] & [1,3] & SU(2)_L \times SU(2)_R \\ \left\{ T_{(i,j)}^{SO(4)} \right\} = \left\{ T^{L\alpha}, T^{R\beta} \right\} \end{array}$$

Classification of SO(4N) generator

General N

$$\mathcal{T}^{abcd} h^a h^b (\partial_\mu h^c) (\partial^\mu h^d) \sim \sum_{SO(4N)} \left(h^a (T^{SO(4N)})_{ac} \partial_\mu h^c \right) \left(h^b (T^{SO(4N)})_{bd} \partial^\mu h^d \right)$$

→ Want to identify $SU(2)_L \times U(1)_Y$ invariant form

Classify SO(4N) generators ($\alpha, \beta = 1, 2, 3$)

$$\left\{ T_{(i,j)}^{SO(4N)} \right\} = \left\{ T_{(i,j)}^{L\alpha}, T_{(i,j)}^{R\beta}, U_{(i,j)}, S_{(i,j)}^{\alpha\beta} \right\}$$

$[3,1]$ $[1,3]$ $[1,1]$ $[3,3]$ $SU(2)_L \times SU(2)_R$

$$4 \begin{pmatrix} T_{(1,1)} & T_{(1,2)} & \cdots & T_{(1,N)} \\ T_{(2,1)} & T_{(2,2)} & & \\ \vdots & & \ddots & \\ T_{(N,1)} & & & T_{(N,N)} \end{pmatrix}$$

$SU(2)_L \times U(1)_Y$ invariant form

How to construct $SU(2)_L \times U(1)_Y$ invariant terms

† Sum over left & right indices

e.g. $\left(T_{(i,j)}^{L\alpha}\right)_{ac} \left(T_{(k,l)}^{L\alpha}\right)_{bd}, \quad \left(S_{(i,j)}^{\alpha\beta}\right)_{ac} \left(S_{(k,l)}^{\alpha\beta}\right)_{bd}, \dots$

† Leave only σ^3 of $SU(2)_R$

e.g. $\left(T_{(i,j)}^{R3}\right)_{ac} \left(T_{(k,l)}^{R3}\right)_{bd}, \quad \left(S_{(i,j)}^{\alpha 3}\right)_{ac} \left(S_{(k,l)}^{\alpha 3}\right)_{bd}, \dots$

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N=1

Low, Rattazzi and Vichi (2009)

$$T^{abcd} = \alpha^L (T^{L\alpha})_{ac} (T^{L\alpha})_{bd} + \alpha^R (T^{R\alpha})_{ac} (T^{R\alpha})_{bd} + \beta (T^{R3})_{ac} (T^{R3})_{bd}$$



Determined by symmetry breaking pattern G/H

$SU(2)_L \times U(1)_Y$ invariant form

How to construct $SU(2)_L \times U(1)_Y$ invariant terms

† Sum over left & right indices

e.g. $\left(T_{(i,j)}^{L\alpha}\right)_{ac} \left(T_{(k,l)}^{L\alpha}\right)_{bd}, \quad \left(S_{(i,j)}^{\alpha\beta}\right)_{ac} \left(S_{(k,l)}^{\alpha\beta}\right)_{bd}, \dots$

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General N

$$T_{abcd}^I = a_{iiii}^L \left(T_{(i,i)}^{L\alpha}\right)_{ac} \left(T_{(i,i)}^{L\alpha}\right)_{bd} + a_{iiii}^R \left(T_{(i,i)}^{R\beta}\right)_{ac} \left(T_{(i,i)}^{R\beta}\right)_{bd} + a_{iiii}^Y \left(T_{(i,i)}^{R3}\right)_{ac} \left(T_{(i,i)}^{R3}\right)_{bd}$$

$$\begin{aligned} T_{abcd}^{II} = & a_{iiij}^L \left(T_{(i,i)}^{L\alpha}\right)_{ac} \left(T_{(i,j)}^{L\alpha}\right)_{bd} + a_{iiij}^R \left(T_{(i,i)}^{R\beta}\right)_{ac} \left(T_{(i,j)}^{R\beta}\right)_{bd} + a_{iiij}^{LS} \left(T_{(i,i)}^{L\alpha}\right)_{ac} \left(S_{(i,j)}^{\alpha 3}\right)_{bd} \\ & + a_{iiij}^Y \left(T_{(i,i)}^{R3}\right)_{ac} \left(T_{(i,j)}^{R3}\right)_{bd} + a_{iiij}^{YU} \left(T_{(i,i)}^{R3}\right)_{ac} \left(U_{(i,j)}\right)_{bd} \end{aligned}$$

$$\begin{aligned} T_{abcd}^{III} = & a_{ijij}^L \left(T_{(i,i)}^{L\alpha}\right)_{ac} \left(T_{(j,j)}^{L\alpha}\right)_{bd} + a_{ijij}^L \left(T_{(i,j)}^{L\alpha}\right)_{ac} \left(T_{(i,j)}^{L\alpha}\right)_{bd} + a_{ijij}^R \left(T_{(i,i)}^{R\beta}\right)_{ac} \left(T_{(j,j)}^{R\beta}\right)_{bd} \\ & + a_{ijij}^R \left(T_{(i,j)}^{R\beta}\right)_{ac} \left(T_{(i,j)}^{R\beta}\right)_{bd} + a_{ijij}^S \left(S_{(i,j)}^{\alpha 3}\right)_{ac} \left(S_{(i,j)}^{\alpha 3}\right)_{bd} + a_{ijij}^{SS} \left(S_{(i,j)}^{\alpha\beta}\right)_{ac} \left(S_{(i,j)}^{\alpha\beta}\right)_{bd} \\ & + a_{ijij}^{LS} \left(T_{(i,j)}^{L\alpha}\right)_{ac} \left(S_{(i,j)}^{\alpha 3}\right)_{bd} + a_{ijij}^Y \left(T_{(i,i)}^{R3}\right)_{ac} \left(T_{(j,j)}^{R3}\right)_{bd} + a_{ijij}^Y \left(T_{(i,j)}^{R3}\right)_{ac} \left(T_{(i,j)}^{R3}\right)_{bd} \\ & + a_{ijij}^U \left(U_{(i,j)}\right)_{ac} \left(U_{(i,j)}\right)_{bd} + a_{ijij}^{YU} \left(T_{(i,j)}^{R3}\right)_{ac} \left(U_{(i,j)}\right)_{bd} \end{aligned}$$

⋮

Degrees of freedom

N=1

Low, Rattazzi and Vichi (2009)

$$E^{ac} \equiv (\delta^{a1}\delta^{c2} + \delta^{a3}\delta^{c4}) - (a \leftrightarrow c)$$

$$\mathcal{T}^{abcd} = -\frac{\alpha^L + \alpha^R}{4} (\delta^{ab}\delta^{cd} - \delta^{ad}\delta^{bc}) + \frac{\alpha^L - \alpha^R}{4} \epsilon^{abcd} - \frac{\beta}{4} E^{ac} E^{bd}$$

$$\mathcal{L}_{\text{NL}\Sigma\text{M}}^{Higgs} \supset \frac{1}{f^2} \mathcal{T}^{abcd} h^a h^b (\partial_\mu h^c) (\partial^\mu h^d)$$

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∴ Bose sym

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		Re	Im
fundamental	→ Only $SU(2)_L \times U(1)_Y$ inv	3	1
composite	→ Nonlinear rep	2	0

$$\begin{aligned}
 & \frac{c^H}{f^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H) \\
 & \frac{c^T}{f^2} (H^\dagger \overleftrightarrow{D}_\mu H) (H^\dagger \overleftrightarrow{D}^\mu H) \\
 & \frac{c^r}{f^2} H^\dagger H (D_\mu H)^\dagger (D^\mu H) \\
 & \frac{c^{HT}}{f^2} \partial_\mu (H^\dagger H) (H^\dagger \overleftrightarrow{D}^\mu H)
 \end{aligned}$$

Degrees of freedom

General N

$$\mathcal{L}_{\text{NL}\Sigma\text{M}}^{Higgs} \supset \frac{1}{f^2} \mathcal{T}^{abcd} h^a h^b (\partial_\mu h^c) (\partial^\mu h^d)$$

	Re	Im
Only $SU(2)_L \times U(1)_Y$	$\frac{3}{2}N^2(N^2 + 1)$	$\frac{1}{2}N^2(3N^2 - 1)$
Nonlinear rep	$\frac{1}{2}N^2(N^2 + 3)$	$\frac{1}{2}N^2(N^2 - 1)$

→ DOF reduction in the Nonlinear rep case !

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Derive $SU(2)_L \times U(1)_Y$ inv form of the int

Count degrees of freedom (DOF) of the int

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Dim 6 derivative int in 2HDM

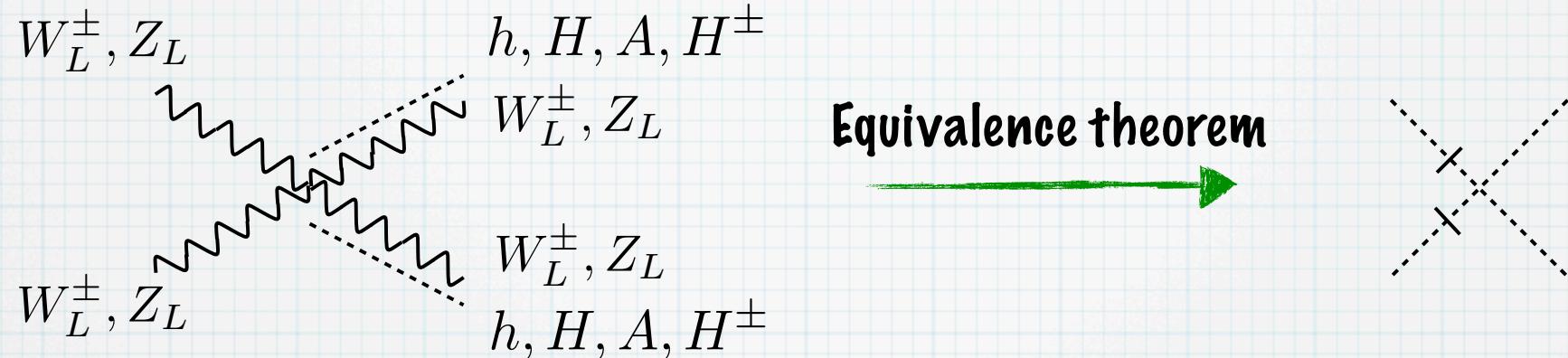
Consider the custodial invariant theory

$$\begin{aligned}\mathcal{L}_{\text{2HDM}}^{\text{dim 6}} = & \frac{c_{1111}^H}{2f^2} O_{1111}^H + \frac{c_{1112}^H}{f^2} (O_{1112}^H + O_{1121}^H) + \frac{c_{1122}^H}{f^2} O_{1122}^H + \frac{c_{1221}^H}{f^2} O_{1221}^H \\ & + \frac{c_{1212}^H}{2f^2} (O_{1212}^H + O_{2121}^H) + \frac{c_{2221}^H}{f^2} (O_{2212}^H + O_{2221}^H) + \frac{c_{2222}^H}{2f^2} O_{2222}^H \\ & - \frac{c_{1221}^H - c_{1212}^H}{3f^2} O_{1122}^T + \frac{c_{1221}^T}{f^2} O_{1221}^T - \frac{3c_{1221}^T - c_{1221}^H + c_{1212}^H}{6f^2} (O_{1212}^T + O_{2121}^T)\end{aligned}$$

$$\begin{aligned}\mathcal{O}_{ijkl}^H &= \partial_\mu (H_i^\dagger H_j) \partial^\mu (H_k^\dagger H_l) \\ \mathcal{O}_{ijkl}^T &= (H_i^\dagger \overleftrightarrow{D}_\mu H_j) (H_k^\dagger \overleftrightarrow{D}^\mu H_l)\end{aligned}$$

Coefficients $\frac{c^{H,T}}{f^2}$: 8 real DOF

Scatterings of V_L and Higgs



High E region ($s, t \gg m_V, m_{Higgs}$)

- Massless approximation
- Dim 6 derivative int dominates

Parameters $\frac{c^{H,T}}{f^2}, \alpha, \beta$: 10 real DOF

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_0^3 \\ h_0^7 \end{pmatrix} \quad \begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} h^4 \\ h^8 \end{pmatrix}$$

Scatterings of V_L and Higgs

Cross sections

$$V_L V'_L \rightarrow LL'$$

$$\begin{aligned}\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}} &= \frac{s+t}{f^2} C_1(\beta) \\ \mathcal{M}(W_L^+ W_L^- \rightarrow hh)_{\text{cust}} &= \frac{s}{f^2} C_2(\alpha, \beta) \\ \mathcal{M}(W_L^+ W_L^- \rightarrow Z_L Z_L)_{\text{cust}} &= \frac{s}{f^2} C_1(\beta), \\ \mathcal{M}(Z_L Z_L \rightarrow W_L^+ W_L^-)_{\text{cust}} &= \frac{s}{f^2} C_1(\beta) \\ \mathcal{M}(Z_L Z_L \rightarrow hh)_{\text{cust}} &= \frac{s}{f^2} C_2(\alpha, \beta) \\ \mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ Z_L)_{\text{cust}} &= \frac{t}{f^2} C_1(\beta) \\ \mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ W_L^+)_{\text{cust}} &= -\frac{s}{f^2} C_1(\beta)\end{aligned}$$

$$V_L V'_L \rightarrow LH$$

$$\begin{aligned}\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}} &= \frac{s+t}{f^2} C_3(\beta) \\ \mathcal{M}(W_L^+ W_L^- \rightarrow hH)_{\text{cust}} &= \frac{s}{f^2} C_4(\alpha, \beta) \\ \mathcal{M}(W_L^+ W_L^- \rightarrow hA)_{\text{cust}} &= -i \frac{s+2t}{3f^2} s_{\alpha-\beta} (c_{1221}^H - c_{1212}^H) \\ \mathcal{M}(W_L^+ W_L^- \rightarrow Z_L A)_{\text{cust}} &= \frac{s}{f^2} C_3(\beta) \\ \mathcal{M}(Z_L Z_L \rightarrow W_L^+ H^-)_{\text{cust}} &= \frac{s}{f^2} C_3(\beta) \\ \mathcal{M}(Z_L Z_L \rightarrow hH)_{\text{cust}} &= \frac{s}{f^2} C_4(\alpha, \beta) \\ \mathcal{M}(W_L^+ Z_L \rightarrow H^+ h)_{\text{cust}} &= -i \frac{s+2t}{3f^2} s_{\alpha-\beta} (c_{1221}^H - c_{1212}^H) \\ \mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ A)_{\text{cust}} &= \frac{t}{f^2} C_3(\beta) \\ \mathcal{M}(W_L^+ Z_L \rightarrow H^+ Z_L)_{\text{cust}} &= \frac{t}{f^2} C_3(\beta) \\ \mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ H^+)_{\text{cust}} &= -\frac{s}{f^2} C_3(\beta)\end{aligned}$$

$$V_L V'_L \rightarrow HH'$$

$$\begin{aligned}\sigma(W_L^+ W_L^- \rightarrow H^+ H^-)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{2}{3} (C_5(\beta)^2 - C_5(\beta)(c_{1221}^H - c_{1122}^H) + (c_{1221}^H - c_{1122}^H)^2) \\ \sigma(W_L^+ W_L^- \rightarrow HH)_{\text{cust}} &= \frac{s}{32\pi f^4} C_6(\alpha, \beta)^2 \\ \sigma(W_L^+ W_L^- \rightarrow AA)_{\text{cust}} &= \frac{s}{32\pi f^4} C_5(\beta)^2 \\ \sigma(W_L^+ W_L^- \rightarrow HA)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{2}{27} \cos^2(\alpha - \beta) (c_{1221}^H - c_{1212}^H)^2 \\ \sigma(Z_L Z_L \rightarrow H^+ H^-)_{\text{cust}} &= 2\sigma(W_L^+ W_L^- \rightarrow AA)_{\text{cust}} \\ \sigma(Z_L Z_L \rightarrow HH)_{\text{cust}} &= \sigma(W_L^+ W_L^- \rightarrow HH)_{\text{cust}} \\ \sigma(Z_L Z_L \rightarrow AA)_{\text{cust}} &= \frac{s}{32\pi f^4} (c_{1122}^H - 3c_{1221}^T)^2 \\ \sigma(W_L^+ Z_L \rightarrow H^+ H)_{\text{cust}} &= \sigma(W_L^+ W_L^- \rightarrow HA)_{\text{cust}} \\ \sigma(W_L^+ Z_L \rightarrow H^+ A)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{2} \left((C_5(\beta) - c_{1122}^H + c_{1221}^H - 3c_{1221}^T)^2 \right. \\ &\quad \left. + \frac{1}{3} \left(C_5(\beta) + \frac{c_{1221}^H + 2c_{1212}^H}{3} - c_{1122}^H + c_{1221}^T \right)^2 \right)\end{aligned}$$

Scatterings of V_L and Higgs

Cross sections

$$V_L V'_L \rightarrow LL'$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}} = \frac{s+t}{f^2} C_1(\beta)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow hh)_{\text{cust}} = \frac{s}{f^2} C_2(\alpha, \beta)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L Z_L)_{\text{cust}} = \frac{s}{f^2} C_1(\beta).$$

$$\mathcal{M}(Z_L Z_L)$$

$$\mathcal{M}(Z_L)$$

$$\mathcal{M}(W_L^+ Z_L)$$

$$\mathcal{M}(W_L^+ W_L^+ -)$$

$$V_L$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ H)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow h)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow hA)_{\text{cust}} = -i \frac{s}{3f^2} s_{\alpha-\beta} (c_{1221} - c_{1212})$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L A)_{\text{cust}} = \frac{s}{f^2} C_3(\beta)$$

$$\mathcal{M}(Z_L Z_L \rightarrow W_L^+ H^-)_{\text{cust}} = \frac{s}{f^2} C_3(\beta)$$

$$\mathcal{M}(Z_L Z_L \rightarrow hH)_{\text{cust}} = \frac{s}{f^2} C_4(\alpha, \beta)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow H^+ h)_{\text{cust}} = -i \frac{s+2t}{3f^2} s_{\alpha-\beta} (c_{1221}^H - c_{1212}^H)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ A)_{\text{cust}} = \frac{t}{f^2} C_3(\beta)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow H^+ Z_L)_{\text{cust}} = \frac{t}{f^2} C_3(\beta)$$

$$\mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ H^+)_{\text{cust}} = -\frac{s}{f^2} C_3(\beta)$$

$$V_L V'_L \rightarrow HH'$$

$$(c_{1222}^H + (c_{1221}^H - c_{1122}^H)^2)$$

10 independent processes !

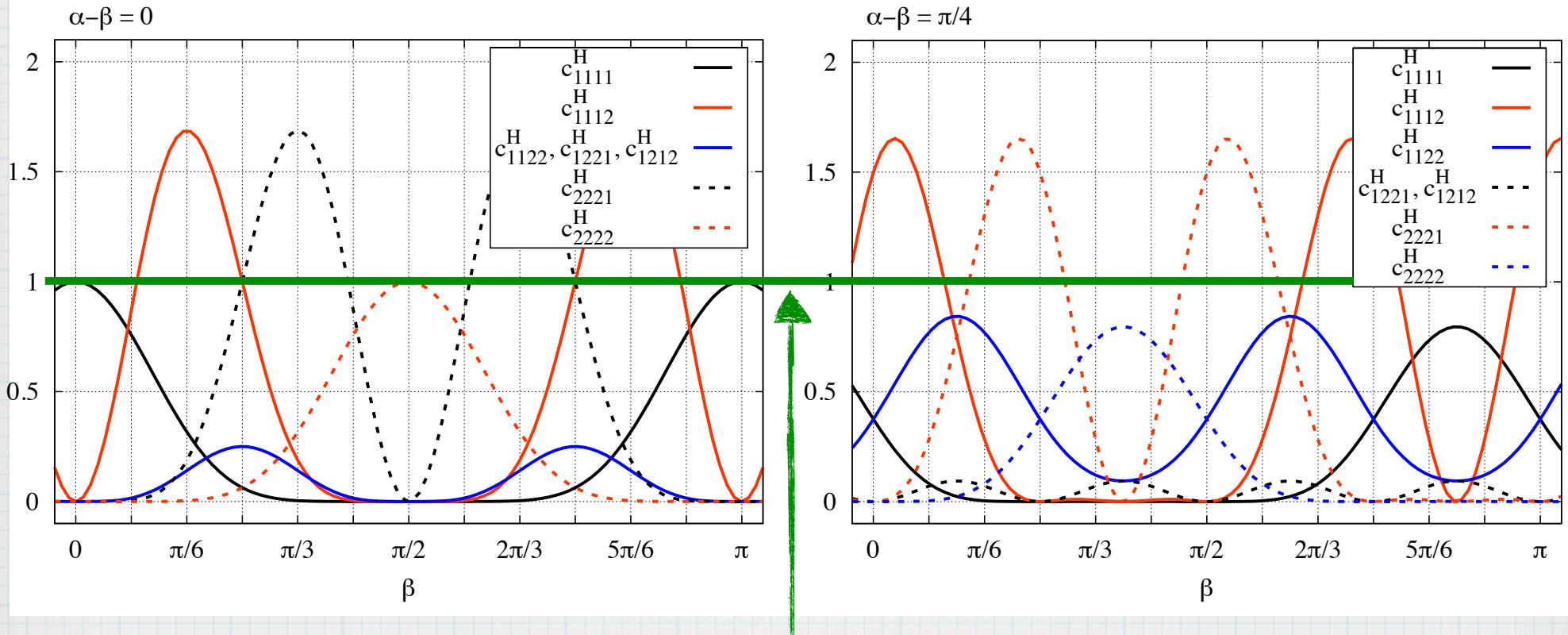


10 real parameters !

$$\begin{aligned} \sigma(W_L^+ Z_L \rightarrow H^+ A)_{\text{cust}} &= \frac{1}{32\pi f^4} \frac{1}{2} \left((C_5(\beta) - c_{1122}^* + c_{1221}^* - 3c_{1221}^T)^2 \right. \\ &\quad \left. + \frac{1}{3} \left(C_5(\beta) + \frac{c_{1221}^H + 2c_{1212}^H}{3} - c_{1122}^H + c_{1221}^T \right)^2 \right) \end{aligned}$$

Scatterings of V_L and Higgs

$$\frac{\sigma(W_L^+ W_L^- \rightarrow h h)_{\text{cust}}}{\sigma(W_L^+ W_L^- \rightarrow h h)_{\text{SILH}}}$$

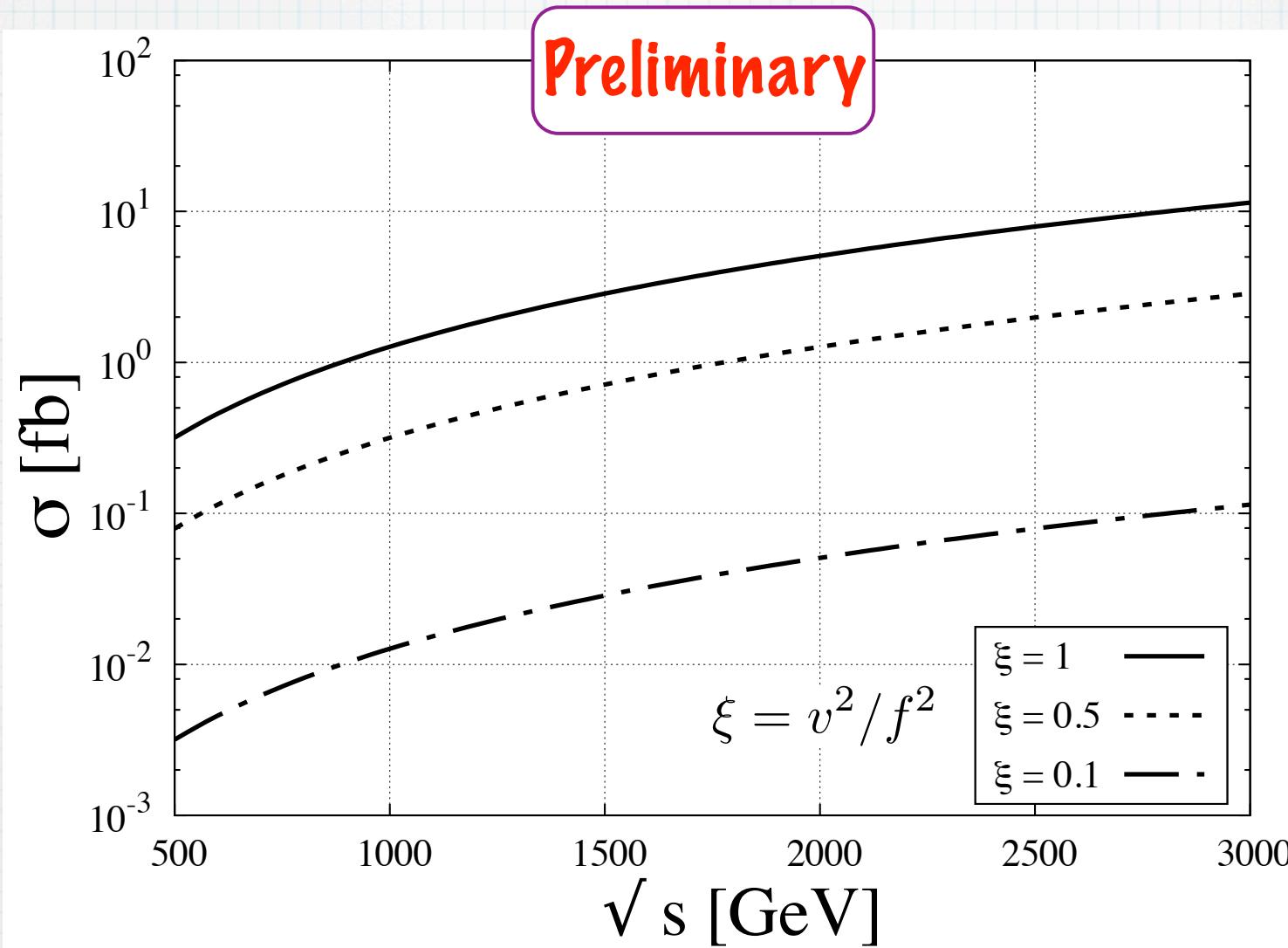


Only one Higgs doublet

$\alpha - \beta$ dependence can discriminate 2HDM from SILH !!

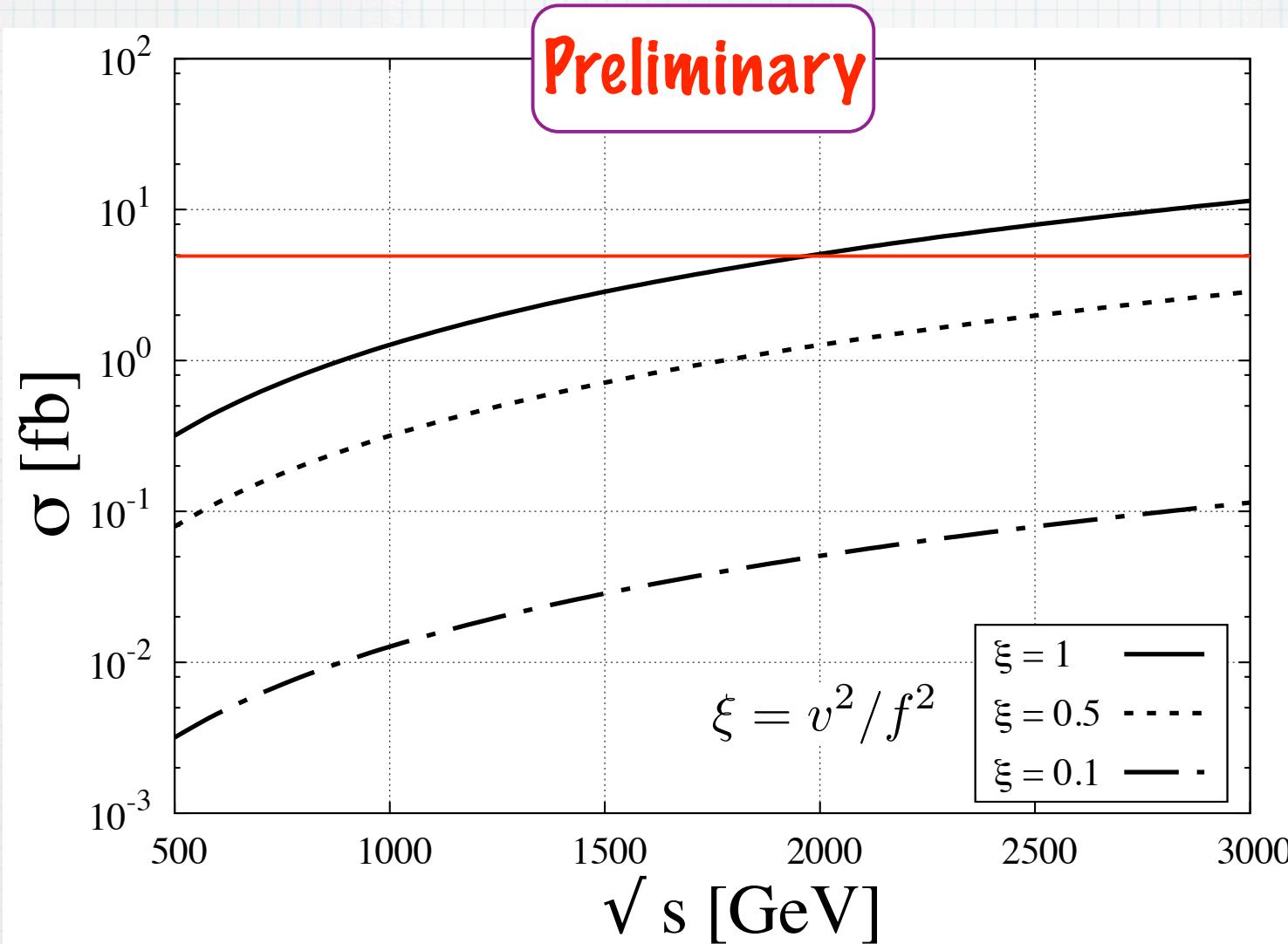
Toward collider experiments

$e^+ e^- \rightarrow W_L W_L v\bar{v} \rightarrow hhv\bar{v}$ process in SO(5)/SO(4)



Toward collider experiments

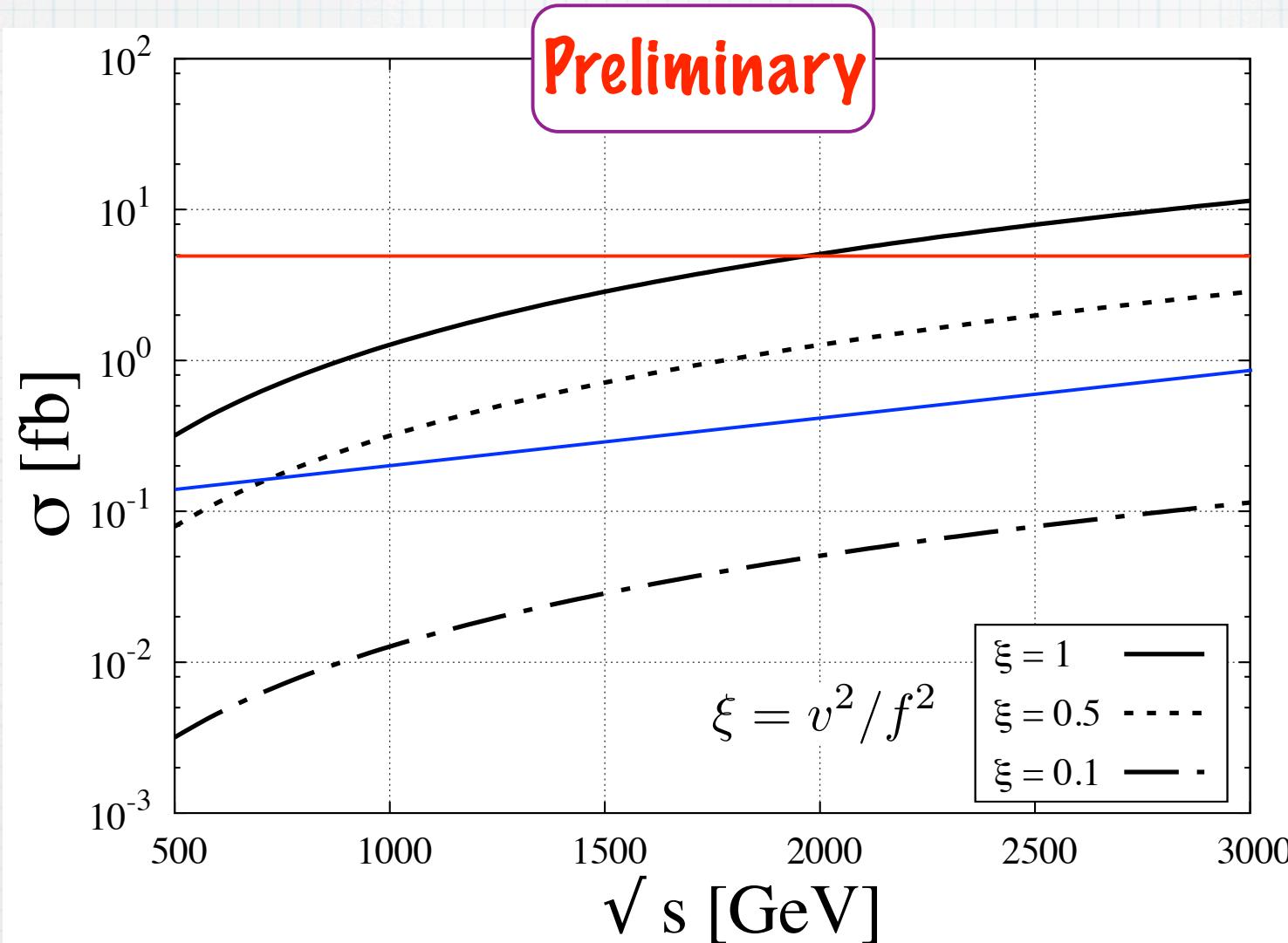
$e^+ e^- \rightarrow W_L W_L v\bar{v} \rightarrow hhv\bar{v}$ process in SO(5)/SO(4)



$p p \rightarrow W_L W_L jj \rightarrow hhjj$
@LHC 14[TeV]
 $\xi = 1$

Toward collider experiments

$e^+ e^- \rightarrow W_L W_L v\bar{v} \rightarrow hhv\bar{v}$ process in SO(5)/SO(4)

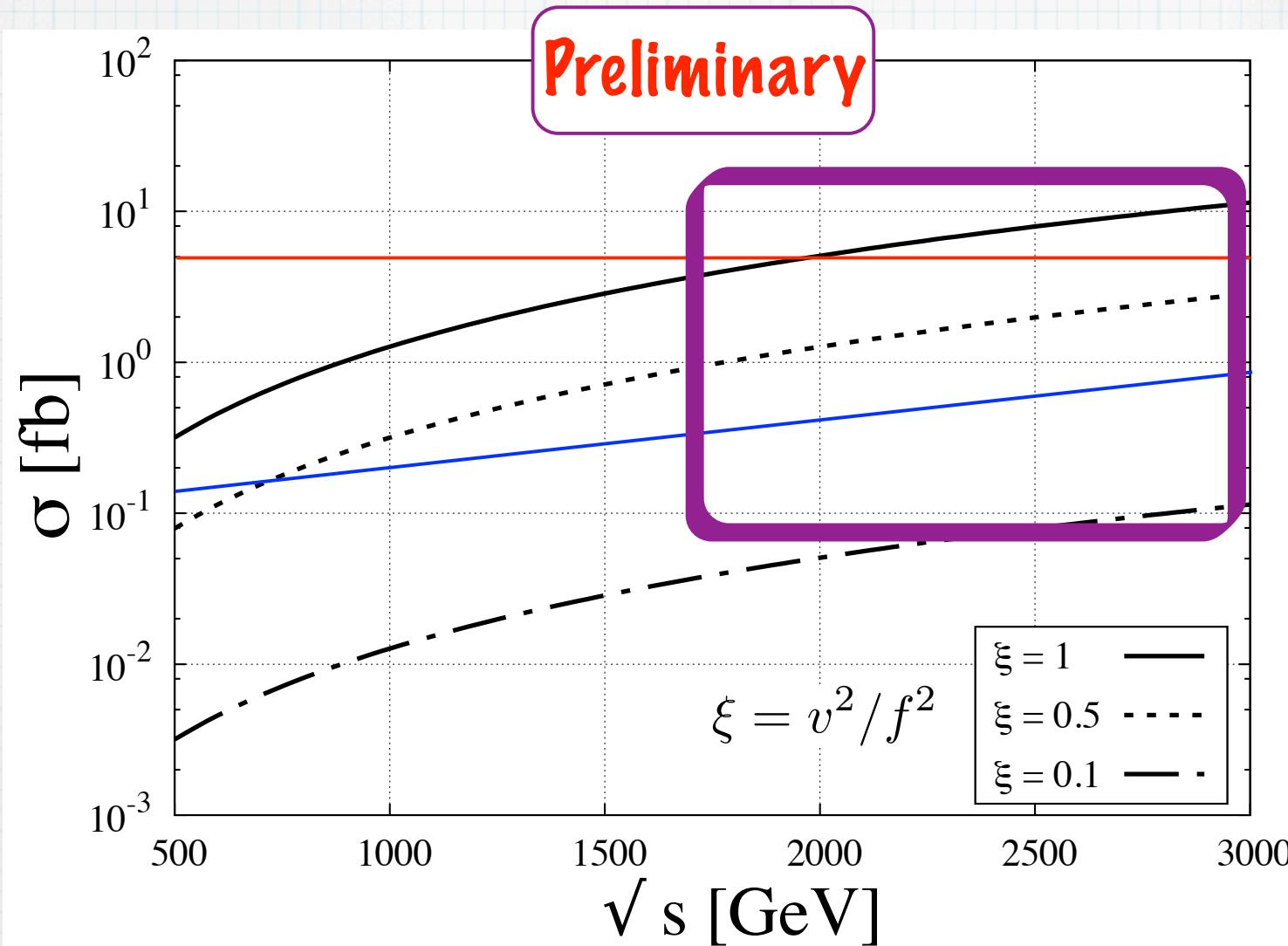


$p p \rightarrow W_L W_L jj \rightarrow hhjj$
@LHC 14[TeV]
 $\xi = 1$

$ee \rightarrow hhv\bar{v}$
SM

Toward collider experiments

$e^+ e^- \rightarrow W_L W_L v\bar{v} \rightarrow hhv\bar{v}$ process in SO(5)/SO(4)

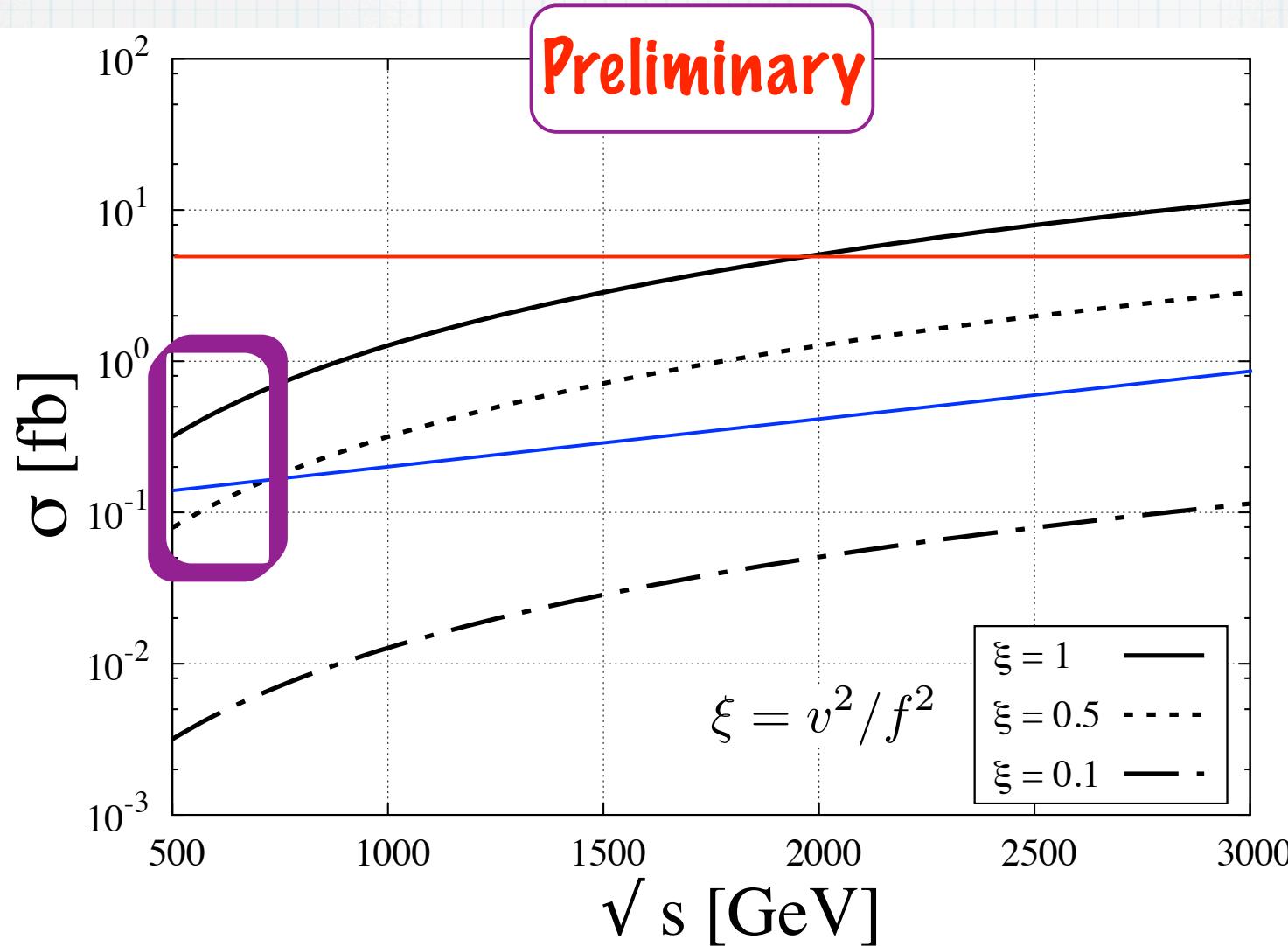


$pp \rightarrow W_L W_L jj \rightarrow hhjj$
@LHC 14[TeV]
 $\xi = 1$

$ee \rightarrow hhvv$
SM

Toward collider experiments

$e^+ e^- \rightarrow W_L W_L v\bar{v} \rightarrow hhv\bar{v}$ process in SO(5)/SO(4)



$p p \rightarrow W_L W_L jj \rightarrow hhjj$
@LHC 14 [TeV]
 $\xi = 1$

$ee \rightarrow hhvv$
SM

Summary

Pseudo NG N Higgs doublet models

† Analyzed dim 6 derivative interactions

→ $SU(2)_L \times U(1)_Y$ invariant form

→ DOF

	Re	Im
$SU(2)_L \times U(1)_Y$	$\frac{3}{2}N^2(N^2 + 1)$	$\frac{1}{2}N^2(3N^2 - 1)$
nonlinear rep	$\frac{1}{2}N^2(N^2 + 3)$	$\frac{1}{2}N^2(N^2 - 1)$

† Calculated scatterings σ of V_L and Higgs ($N = 2$)

→ Growth with energy $\sigma \propto \frac{s}{f^4}$

→ Discrimination 2HDM from SILH with $\alpha - \beta$

† Let's study the collider signatures !!

Back up

Maurer-Cartan 1-form

$$U^\dagger \partial_\mu U \equiv i\alpha_{\perp\mu}^a X^a + i\alpha_{\parallel\mu}^i T^i \quad \text{where } U = e^{i\Pi^a X^a / f}$$

Transformation law

$$\alpha_{\perp\mu}(\Pi) \xrightarrow{g} \alpha_{\perp\mu}(\Pi') = h(\Pi, g)\alpha_{\perp\mu}(\Pi)h^{-1}(\Pi, g) \quad : \text{homogeneous tr.}$$

$$\alpha_{\parallel\mu}(\Pi) \xrightarrow{g} \alpha_{\parallel\mu}(\Pi') = h(\Pi, g)\alpha_{\parallel\mu}(\Pi)h^{-1}(\Pi, g) + i^{-1}h(\Pi, g)\partial_\mu h^{-1}(\Pi, g) \quad : \text{inhomogeneous tr.}$$

Gauged

$$\bar{\alpha}_\mu = i^{-1} U^\dagger (\partial_\mu - iA_\mu) U$$

Invariant combinations

$$f^2 Tr\{\bar{\alpha}_{\perp\mu}(\Pi)\bar{\alpha}_{\perp}^\mu(\Pi)\}$$

$$Tr\{F_{\mu\nu}(\bar{\alpha}_{\parallel}) F^{\mu\nu}(\bar{\alpha}_{\parallel})\}, \quad Tr\{\bar{\alpha}_{\perp\mu}\bar{\alpha}_{\perp\nu} F^{\mu\nu}(\bar{\alpha}_{\parallel})\}$$

Bi-doublet notation

$$\Phi_i \equiv (i\sigma^2 H_i^* \quad H_i) \quad \text{where } \Phi_i \rightarrow L\Phi_i R^\dagger$$

SO(4N) fundamental rep \Leftrightarrow Bi-doublet rep

$$h^a \left(T_{(i,j)}^{L\alpha} \right)_{ac} \partial_\mu h^c = \frac{1}{8} \text{Tr} \left[\Phi_i^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi_j + \Phi_j^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi_i \right] \quad [\mathbf{3}, \mathbf{1}]$$

\uparrow \uparrow
 $\mathbf{SU(2)_L}$

$$h^a \left(T_{(i,j)}^{R\beta} \right)_{ac} \partial_\mu h^c = \frac{1}{8} \text{Tr} \left[\Phi_i \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi_j^\dagger + \Phi_j \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi_i^\dagger \right] \quad [\mathbf{1}, \mathbf{3}]$$

\uparrow \uparrow
 $\mathbf{SU(2)_R}$

$$h^a \left(U_{(i,j)} \right)_{ac} \partial_\mu h^c = \frac{i}{8} \text{Tr} \left[\Phi_j^\dagger \overleftrightarrow{\partial}_\mu \Phi_i + \Phi_j \overleftrightarrow{\partial}_\mu \Phi_i^\dagger \right] \quad [\mathbf{1}, \mathbf{1}]$$

$$h^a \left(S_{(i,j)}^{\alpha\beta} \right)_{ac} \partial_\mu h^c = \frac{i}{8} \text{Tr} \left[\Phi_i^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi_j \sigma^\beta + \Phi_i \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi_j^\dagger \sigma^\alpha \right] \quad [\mathbf{3}, \mathbf{3}]$$

\uparrow \uparrow \uparrow \uparrow
 $\mathbf{SU(2)_L}$ $\mathbf{SU(2)_R}$

Back up

$$\begin{aligned}
\mathcal{T}_{2\text{HDM}}^{abcd} = & a_{1111}^L \left(T_{(1,1)}^{L\alpha} \right)_{ac} \left(T_{(1,1)}^{L\alpha} \right)_{bd} + 2a_{1112}^L \left(T_{(1,1)}^{L\alpha} \right)_{ac} \left(T_{(1,2)}^{L\alpha} \right)_{bd} + 2a_{1122}^L \left(T_{(1,1)}^{L\alpha} \right)_{ac} \left(T_{(2,2)}^{L\alpha} \right)_{bd} \\
& + a_{1212}^L \left(T_{(1,2)}^{L\alpha} \right)_{ac} \left(T_{(1,2)}^{L\alpha} \right)_{bd} + 2a_{2221}^L \left(T_{(2,2)}^{L\alpha} \right)_{ac} \left(T_{(2,1)}^{L\alpha} \right)_{bd} + a_{2222}^L \left(T_{(2,2)}^{L\alpha} \right)_{ac} \left(T_{(2,2)}^{L\alpha} \right)_{bd} \\
& + a_{1111}^R \left(T_{(1,1)}^{R\alpha} \right)_{ac} \left(T_{(1,1)}^{R\alpha} \right)_{bd} + 2a_{1112}^R \left(T_{(1,1)}^{R\alpha} \right)_{ac} \left(T_{(1,2)}^{R\alpha} \right)_{bd} + 2a_{1122}^R \left(T_{(1,1)}^{R\alpha} \right)_{ac} \left(T_{(2,2)}^{R\alpha} \right)_{bd} \\
& + a_{1212}^R \left(T_{(1,2)}^{R\alpha} \right)_{ac} \left(T_{(1,2)}^{R\alpha} \right)_{bd} + 2a_{2221}^R \left(T_{(2,2)}^{R\alpha} \right)_{ac} \left(T_{(2,1)}^{R\alpha} \right)_{bd} + a_{2222}^R \left(T_{(2,2)}^{R\alpha} \right)_{ac} \left(T_{(2,2)}^{R\alpha} \right)_{bd} \\
& + a_{1212}^S \left(S_{(1,2)}^{\alpha 3} \right)_{ac} \left(S_{(1,2)}^{\alpha 3} \right)_{bd} + a_{1212}^{SS} \left(S_{(1,2)}^{\alpha \beta} \right)_{ac} \left(S_{(1,2)}^{\alpha \beta} \right)_{bd} + 2a_{1112}^{LS} \left(T_{(1,1)}^{L\alpha} \right)_{ac} \left(S_{(1,2)}^{\alpha 3} \right)_{bd} \\
& + a_{1212}^{LS} \left(T_{(1,2)}^{L\alpha} \right)_{ac} \left(S_{(1,2)}^{\alpha 3} \right)_{bd} + 2a_{2221}^{LS} \left(T_{(2,2)}^{L\alpha} \right)_{ac} \left(S_{(2,1)}^{\alpha 3} \right)_{bd} + 2a_{1111}^Y \left(T_{(1,1)}^{R3} \right)_{ac} \left(T_{(1,1)}^{R3} \right)_{bd} \\
& + 4a_{1112}^Y \left(T_{(1,1)}^{R3} \right)_{ac} \left(T_{(1,2)}^{R3} \right)_{bd} + 2a_{1122}^Y \left(T_{(1,1)}^{R3} \right)_{ac} \left(T_{(2,2)}^{R3} \right)_{bd} + a_{1212}^Y \left(T_{(1,2)}^{R3} \right)_{ac} \left(T_{(1,2)}^{R3} \right)_{bd} \\
& + 4a_{2212}^Y \left(T_{(2,2)}^{R3} \right)_{ac} \left(T_{(1,2)}^{R3} \right)_{bd} + 2a_{2222}^Y \left(T_{(2,2)}^{R3} \right)_{ac} \left(T_{(2,2)}^{R3} \right)_{bd} + a_{1212}^U \left(U_{(1,2)} \right)_{ac} \left(U_{(1,2)} \right)_{bd} \\
& + 4a_{1112}^{YU} \left(T_{(1,1)}^{R3} \right)_{ac} \left(U_{(1,2)} \right)_{bd} + a_{1212}^{YU} \left(T_{(1,2)}^{R3} \right)_{ac} \left(U_{(1,2)} \right)_{bd} + 4a_{2212}^{YU} \left(T_{(2,2)}^{R3} \right)_{ac} \left(U_{(1,2)} \right)_{bd}.
\end{aligned}$$

Custodial invariant condition

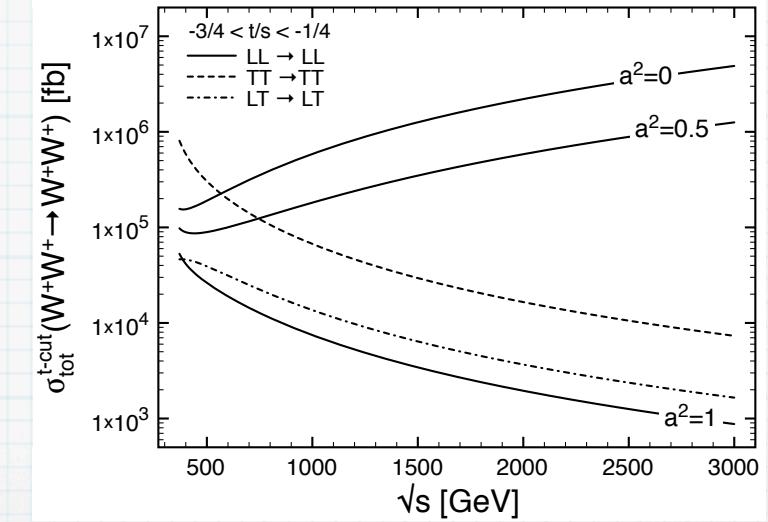
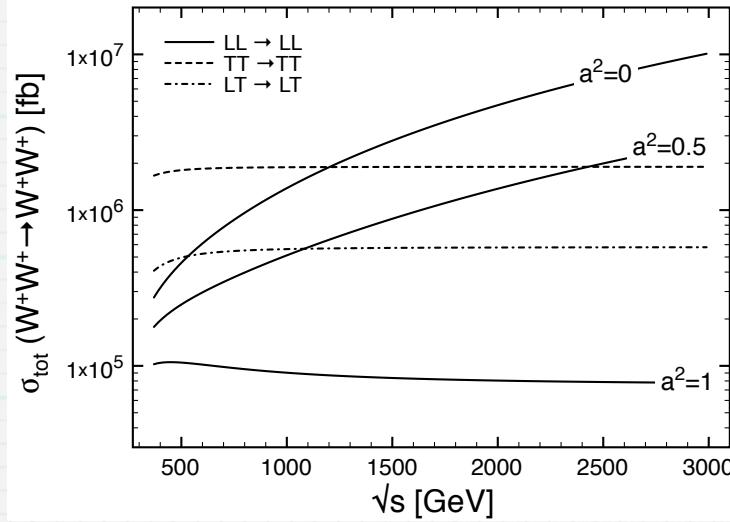
$$\begin{aligned}
a_{1112}^{YU} &= 0, & a_{1212}^{YU} &= 0, & a_{2212}^{YU} &= 0, \\
a_{1112}^{LS} &= 0, & a_{1212}^{LS} &= 0, & a_{2212}^{LS} &= 0, \\
a_{1111}^Y &= 0, & a_{1112}^Y &= 0, & a_{2212}^Y &= 0, & a_{2222}^Y &= 0, \\
a_{1212}^S &= a_{1212}^Y = -\frac{a_{1122}^Y}{2}.
\end{aligned}$$



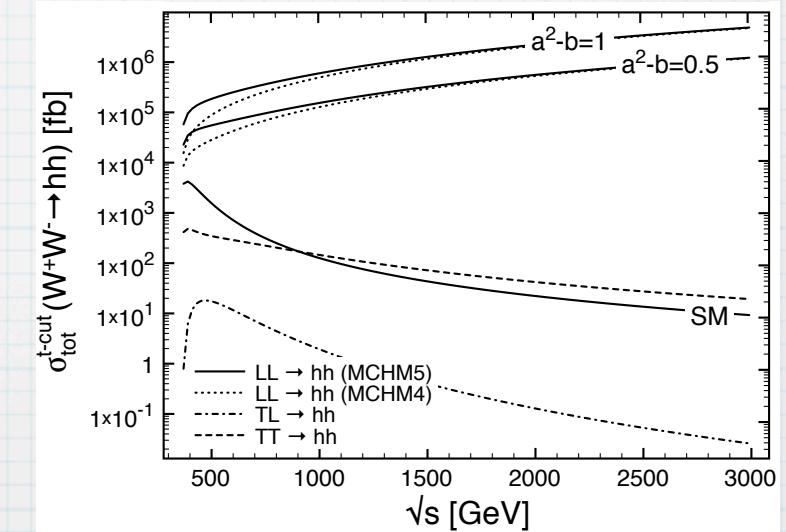
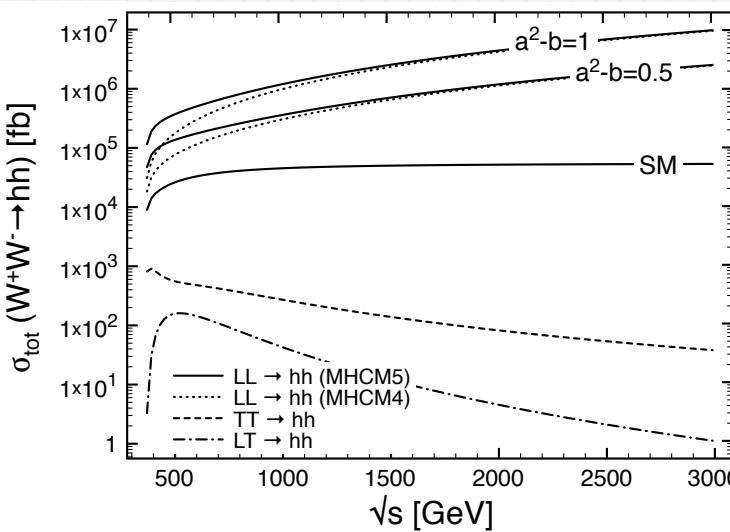
$$\begin{aligned}
c_{1111}^H &= \frac{3}{2}(a_{1111}^L + a_{1111}^R), \\
c_{1112}^H &= \frac{3}{2}(a_{1112}^L + a_{1112}^R), \\
c_{1122}^H &= \frac{3}{2}(a_{1212}^L + a_{1212}^R + a_{1212}^S + 2a_{1212}^{SS}), \\
c_{1221}^H &= \frac{3}{2}(a_{1122}^L + a_{1212}^R - a_{1212}^{SS}), \\
c_{1212}^H &= \frac{3}{2}(a_{1212}^L + a_{1122}^R - a_{1212}^S - a_{1212}^{SS}), \\
c_{2221}^H &= \frac{3}{2}(a_{2221}^L + a_{2221}^R), \\
c_{2222}^H &= \frac{3}{2}(a_{2222}^L + a_{2222}^R), \\
c_{1122}^T &= \frac{1}{2}(-a_{1122}^L + a_{1212}^L + a_{1122}^R - a_{1212}^R - a_{1212}^S), \\
c_{1221}^T &= \frac{1}{2}(-a_{1212}^L + a_{1122}^L + a_{1212}^U), \\
c_{1212}^T &= \frac{1}{2}(a_{1212}^R - a_{1122}^R + a_{1212}^S - a_{1212}^U).
\end{aligned}$$

Back up

$W^+W^+ \rightarrow W^+W^+$



$W^+W^- \rightarrow hh$

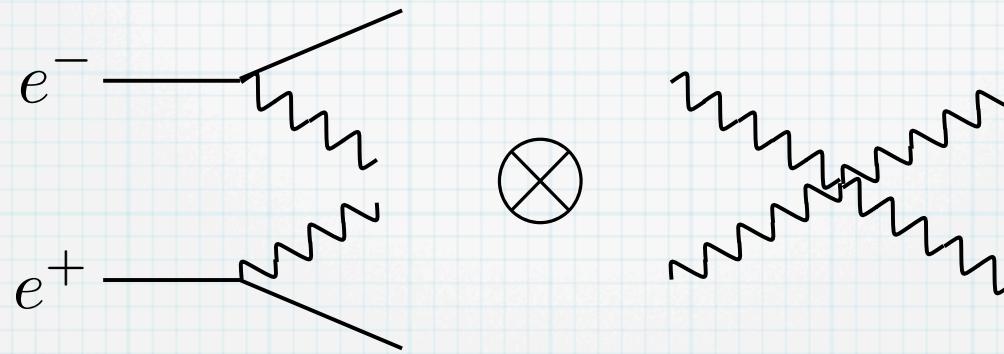


arXiv:1002.1011

Contino, Grojean, Moretti, Piccinini and Rattazzi (2010)

Back up

Effective boson approximation



Treat vector bosons as partons !

Cross section

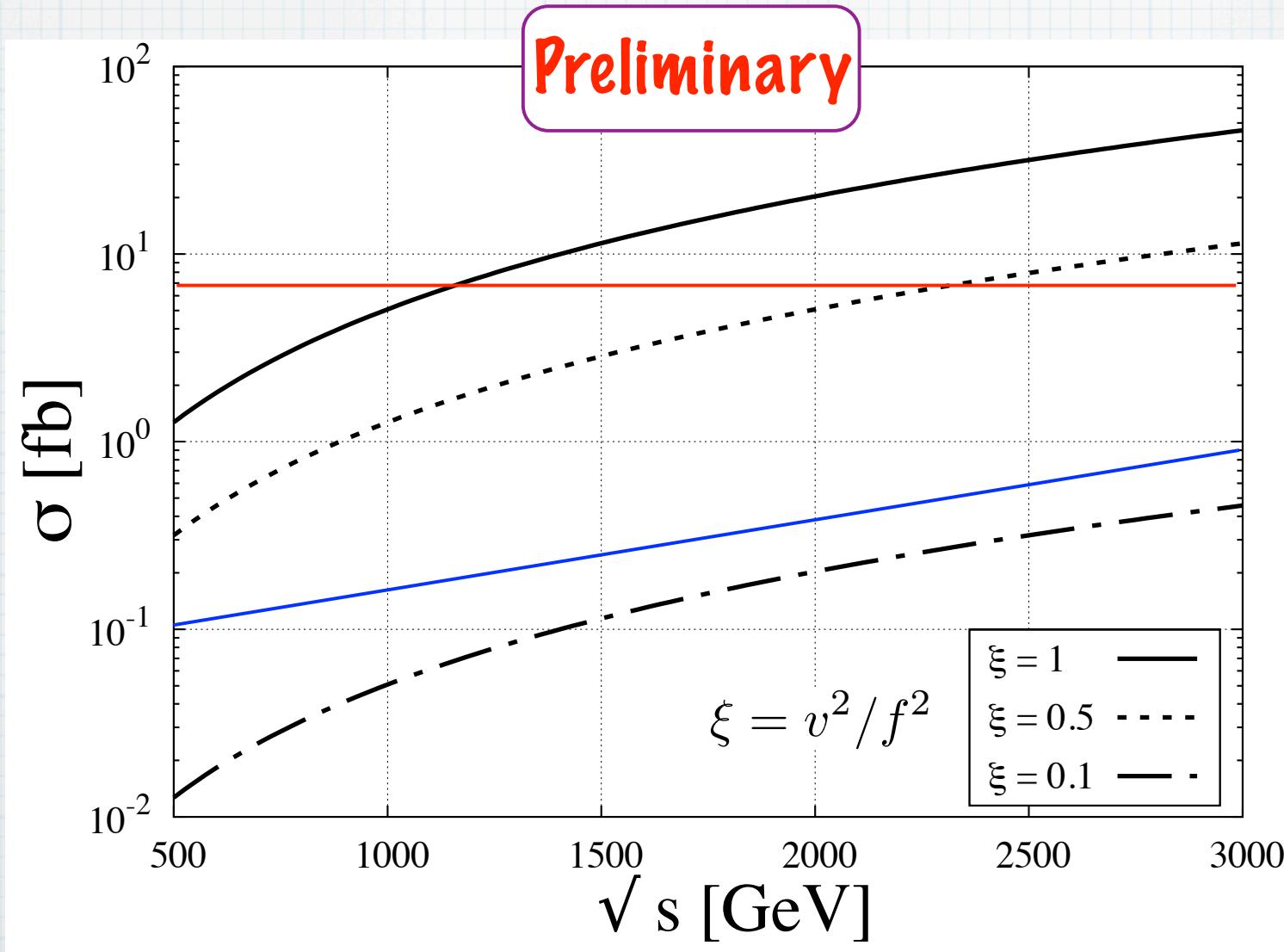
$$\sigma = \int dx_1 dx_2 f(x_1) f(x_2) \sigma_{\text{sub-process}}$$

PDF of longitudinal mode

$$f_L(x) \simeq \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x}$$

Back up

$e^+ e^- \rightarrow W_L W_L v\bar{v} \rightarrow hhv\bar{v}$ process in SU(6)/Sp(6) model



$pp \rightarrow W_L W_L jj \rightarrow hhjj$
@LHC 14[TeV]
 $\xi = 1$

$ee \rightarrow hhv\bar{v}$
via W fusion
SM