

# Shower Fractal Dimensional Analysis at PFA Oriented Calorimeter

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# Outline

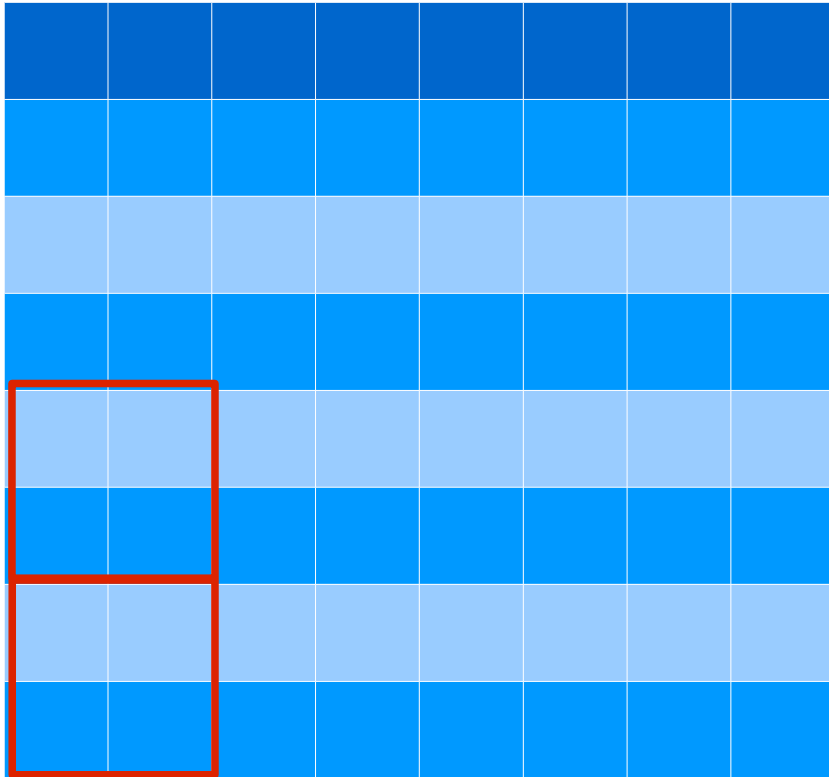


- Introduction:
  - Fractal Dimension of particle shower
  - Analysis with Full Simulated data
- Fractal dimensional analysis at CALICE DHCAL data
- One step further: Fractal dimension at SDHCAL...
- Summary & to do

# Shower particle: to interact or not



shower ~ self similar

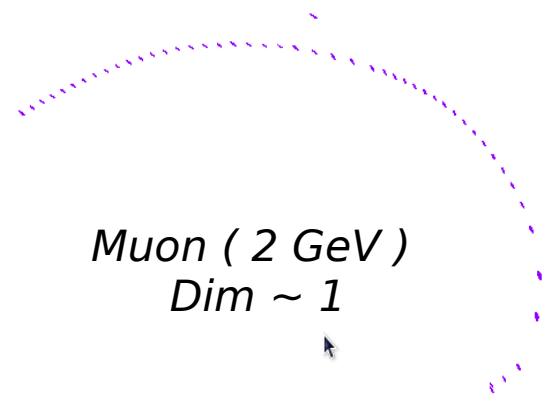


Measure shower **Fractal Dimension (FD)** at high granularity calorimeter

- Varying scale by grouping neighbouring cells
- Count Number of hits at different scale (*define*  $RN_x = N_{1\text{mm}}/N_{x\text{mm}}$ )

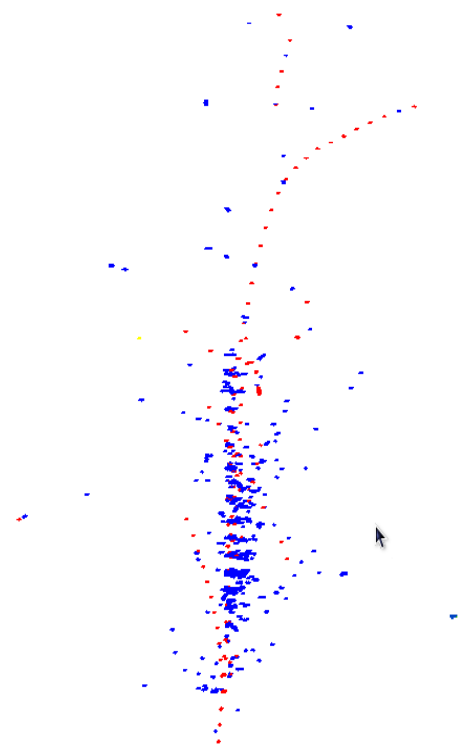
Test sample:  
2-40 GeV particles ( $e^+$ ,  $K_{\text{long}}$ ,  $\pi$ ,  $\mu^+$ ,  $p$ )  
normally injected into GRPC DHCAL with 1mm cell size

# Fractals in Nature

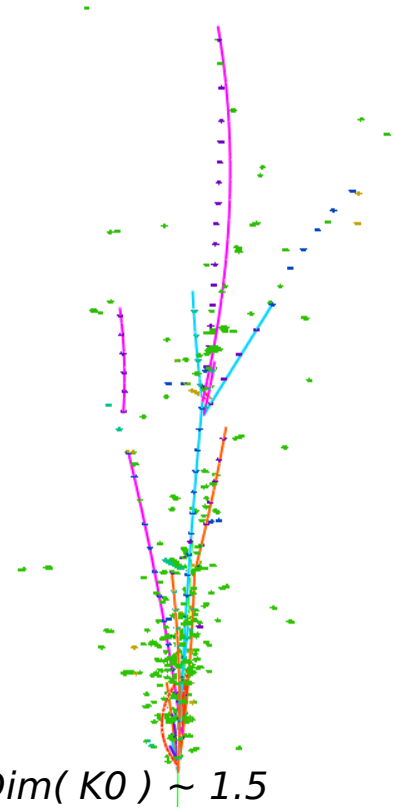


Muon ( 2 GeV )  
Dim ~ 1

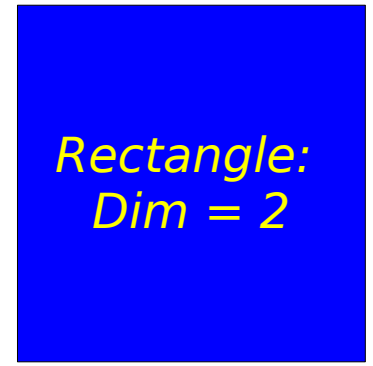
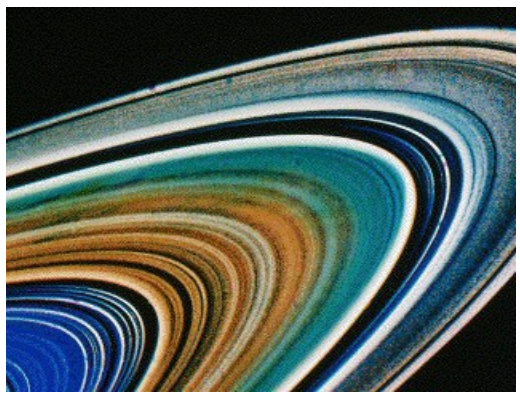
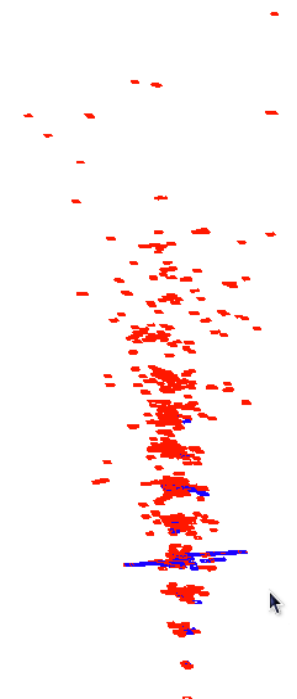
Straight line:  
Dim = 1



Hadrons:  $Dim(\pi) < Dim(K0) \sim 1.5$



Positron ( 40GeV )  
Dim ~ 1.75

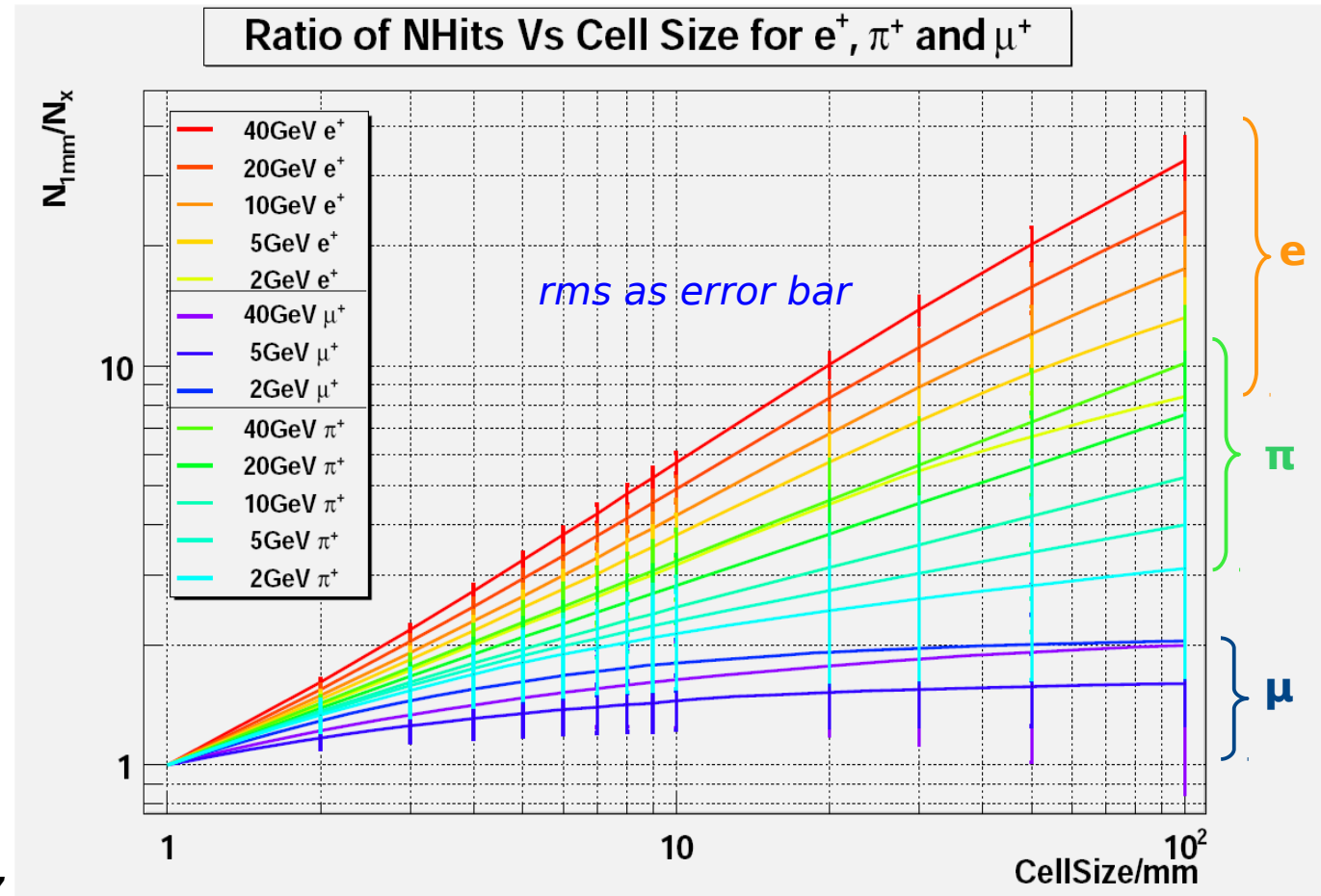


Rectangle:  
Dim = 2

- Characteristic constant based on energy/PID:

$$FD = 1 + \left\langle \frac{\ln RN_a}{\ln a} \right\rangle$$

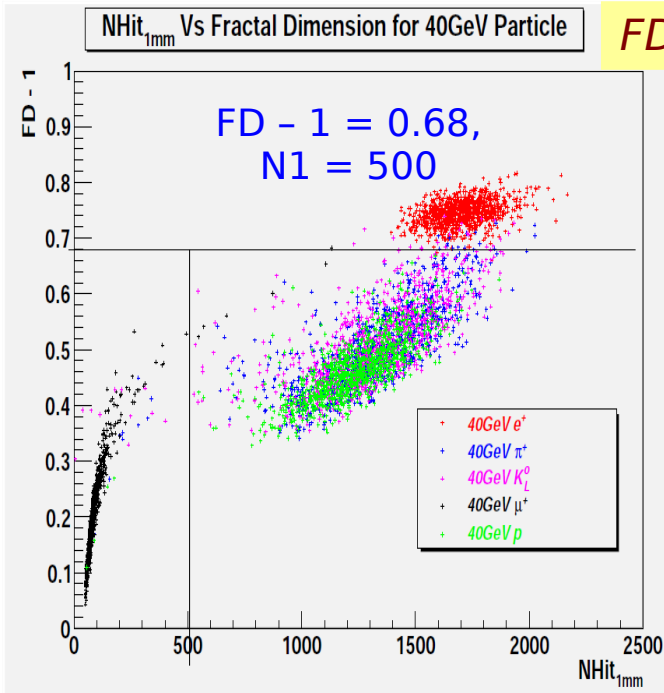
- $a$ : ratio between ganged cell size and initial cell size
- Initial cell size: 1mm in simulation
- Changed Cell Sizes: 2 - 10, 20, 30, 50, 60, 90, 120, 150mm.



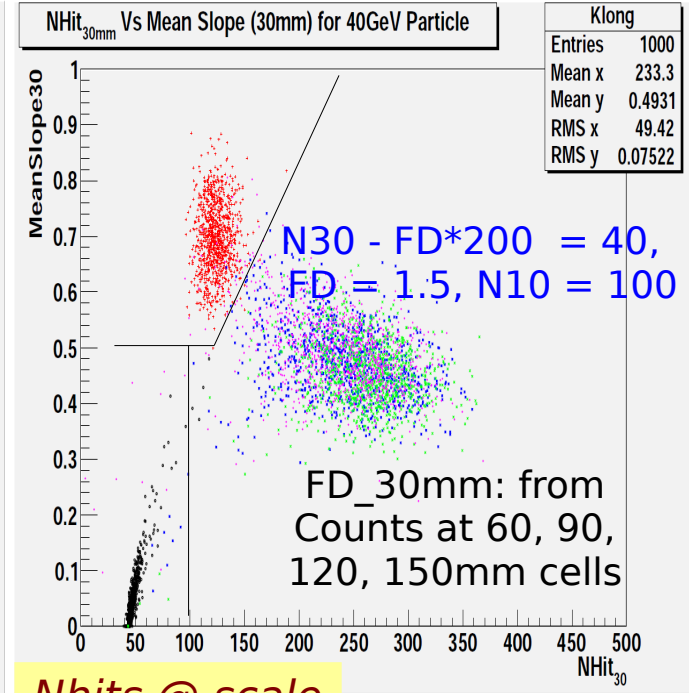
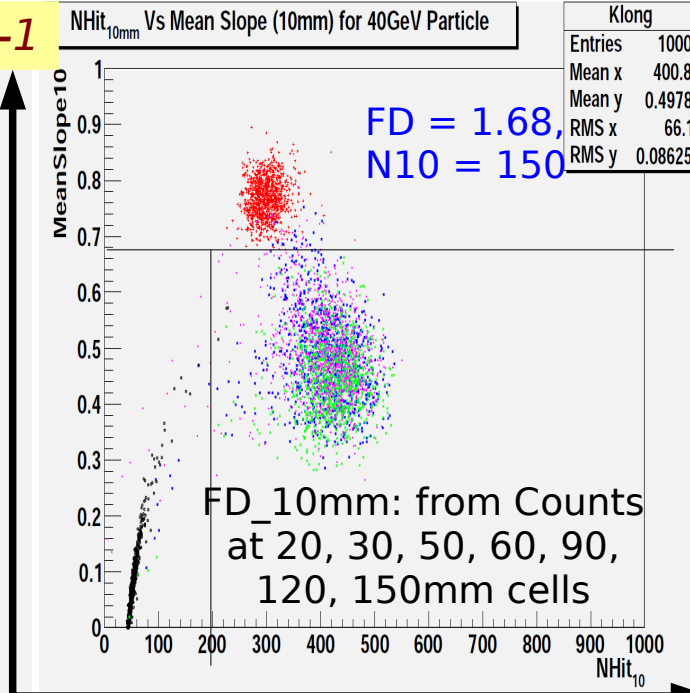
## 1 mm

## 10 mm

## 30 mm



**FD-1**



**Nhits @ scale**

FD together with other info ( Nhits ): Clear separation at different scales

*Remark: Energy dependent Cuts, easier for charged particles*

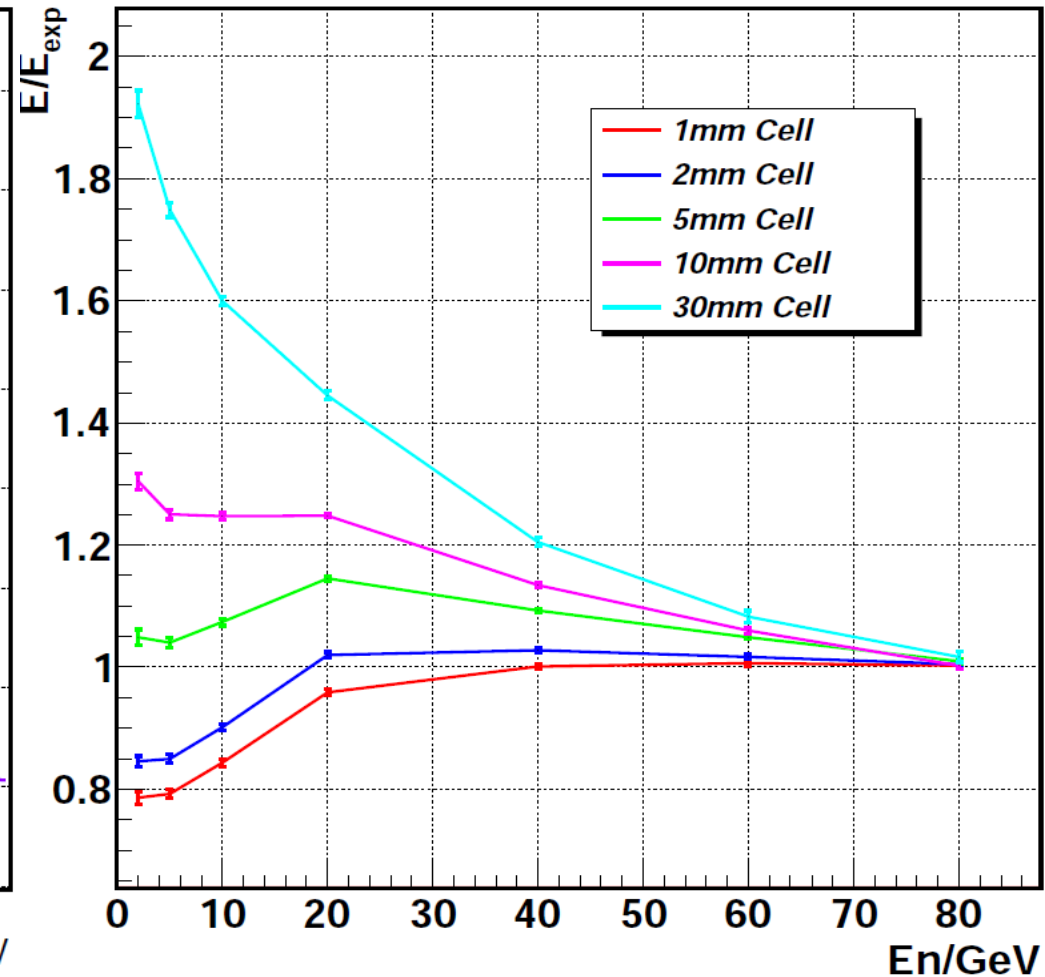
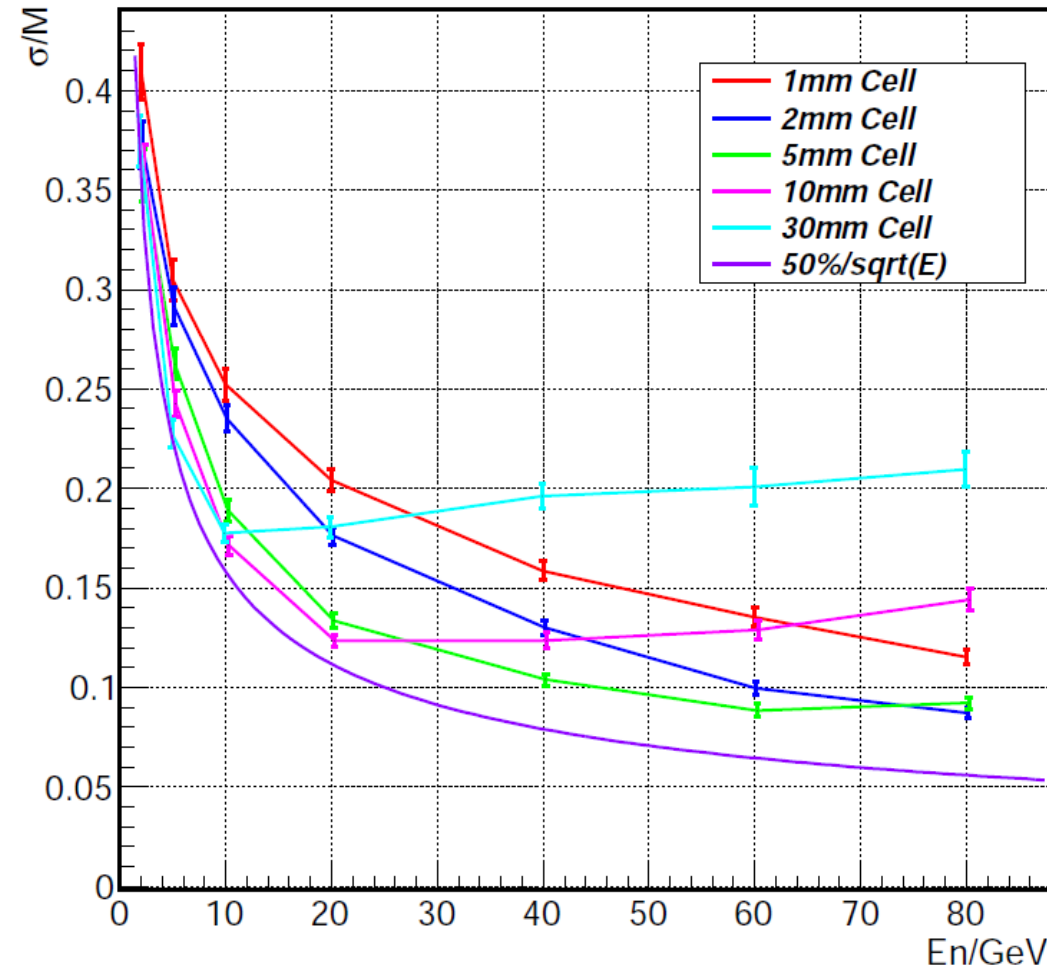
1mm	e+	$\mu$	h
e+	998	0	2
$\mu$	1	994	5
h	15	14	971

10mm	e+	$\mu$	h
e+	1000	0	0
$\mu$	0	995	5
h	17	14	969

30mm	e+	$\mu$	h
e+	1000	0	0
$\mu$	0	996	4
h	18	11	971

**OUT**

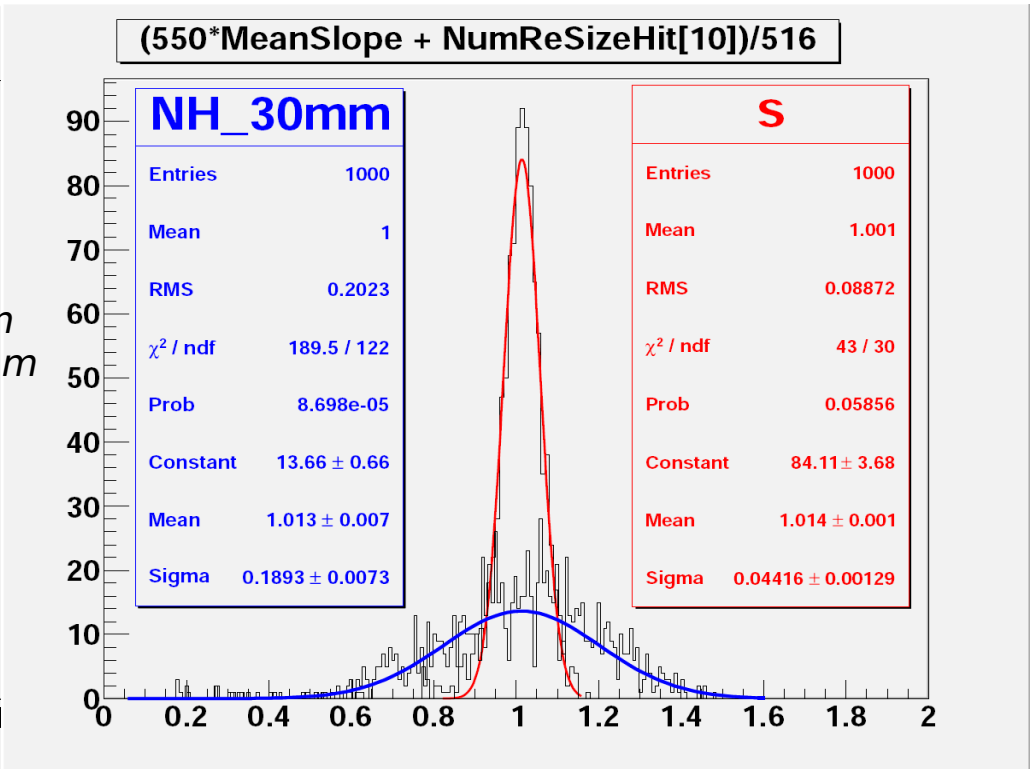
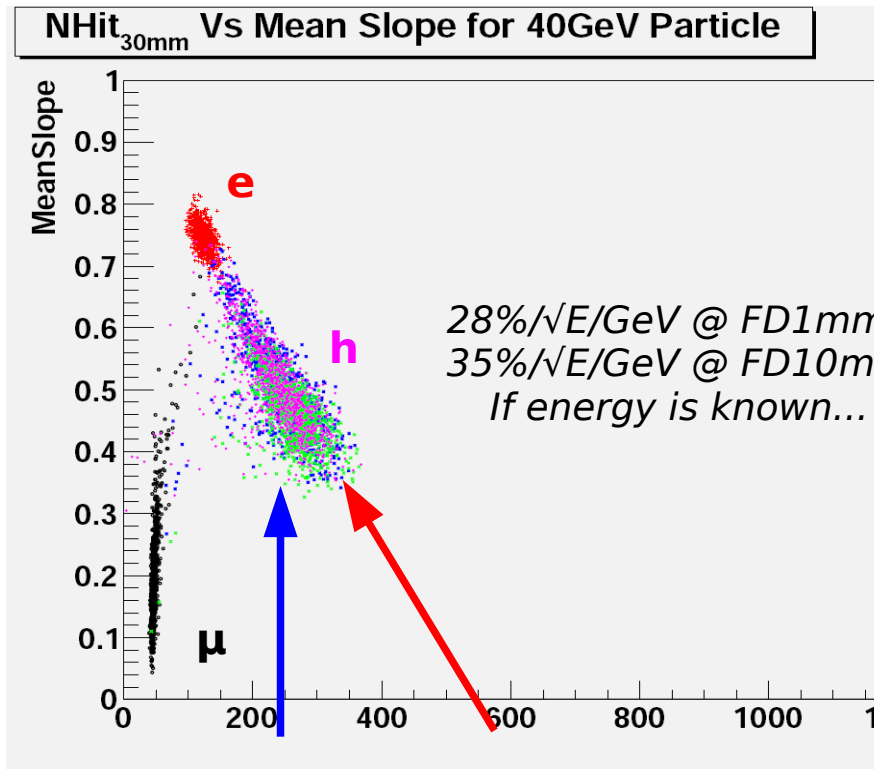
# Energy Estimation: with Naive Counting



$\sigma/M$ : Large cell better at low energy & Smaller cell at high energy.

Linearity: Better at 2 - 5 mm cell, strong saturation effects at larger cell...

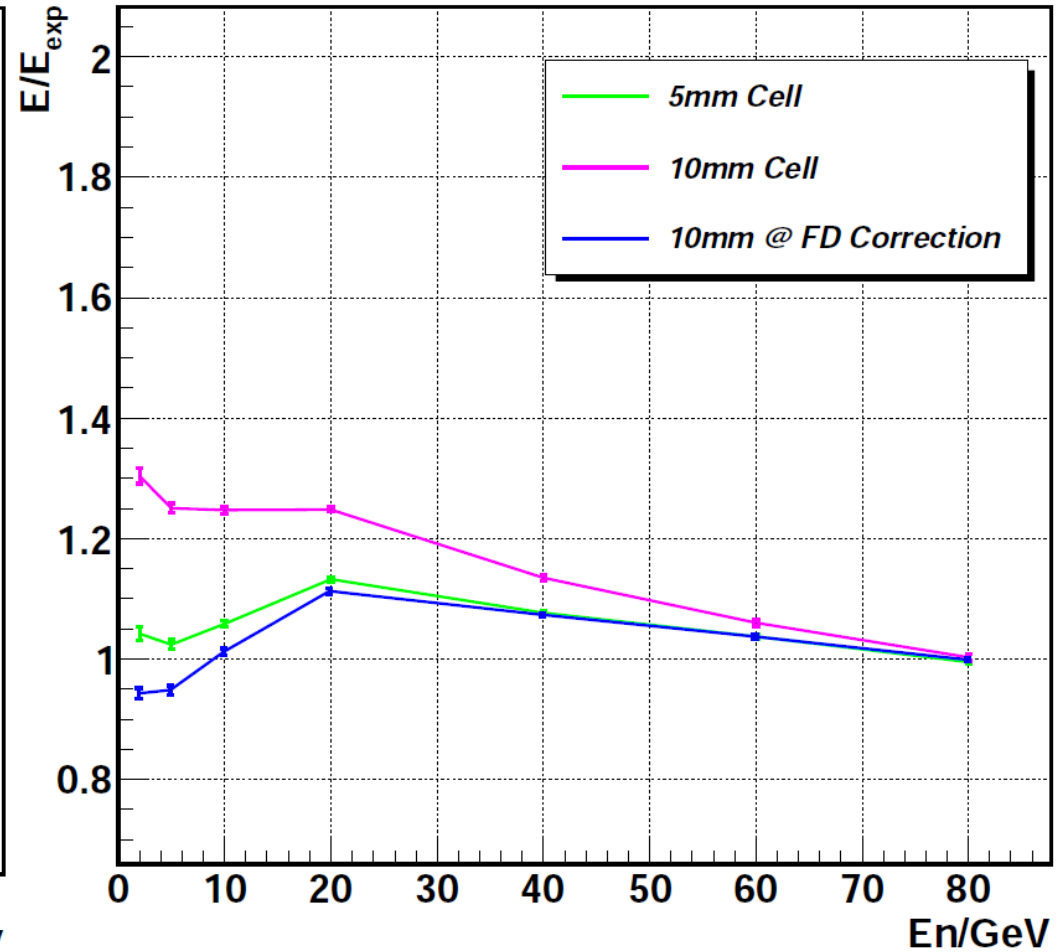
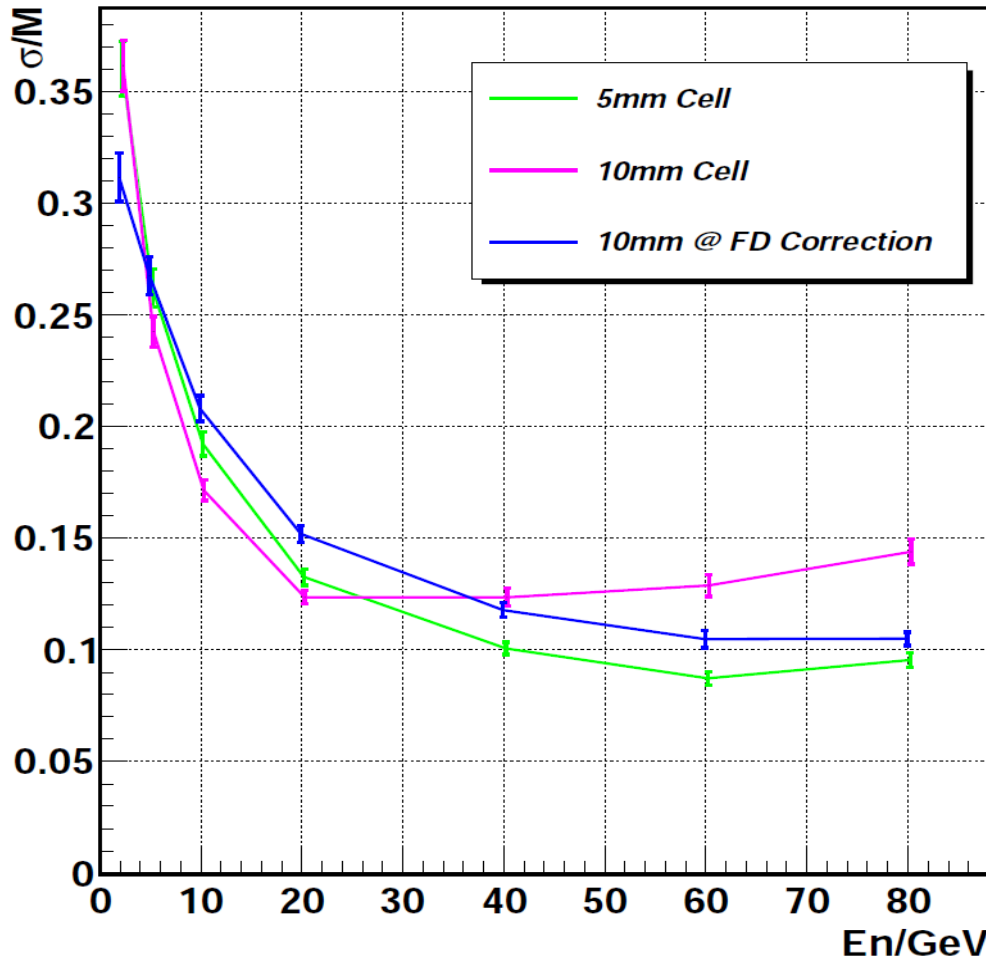
Naively: 5mm seems a nice choice (as EM & hadronic hits are compensated)...



- **Strong correlation** at FD vs Nhit (large scale): only loose shower makes lots of hits!
- For example: compensation based on NH\_30mm & FD1mm:  

$$E = a * NH_{30} + b * FD \sim 30\%/\sqrt{E}!$$
 But...
- $b = b(E) \sim kE$ . To improve track-cluster **matching** ?
- A set of energy independent ( LO ) estimator:  $E = a' * NH_x / (1 - FD * b')$



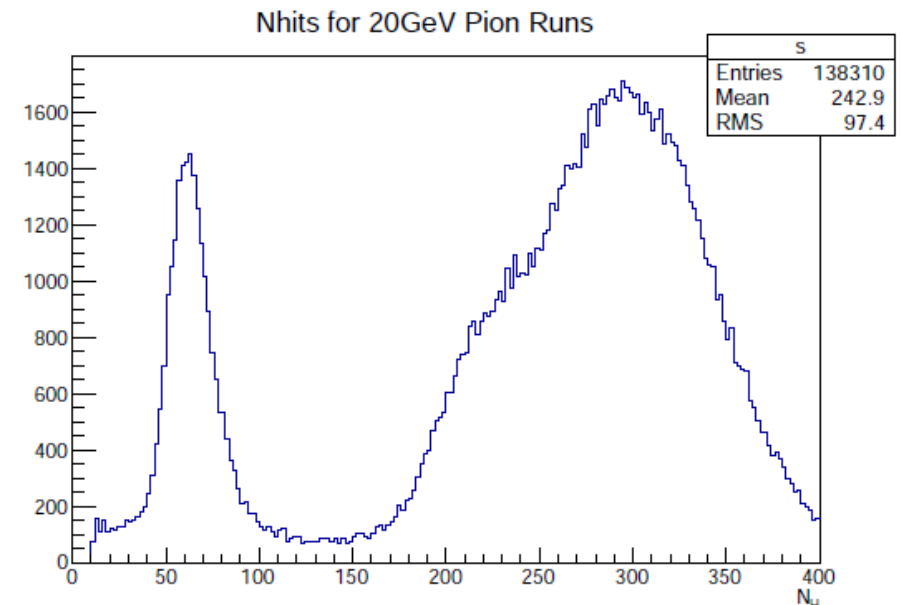


Hand put Energy Estimator with FD:  $NH10/(1-0.65*FD10)$   
 Energy resolution improved at high energy:  $\sim$  saturation effect correction  
 Linearity improved: close to 5mm Cell

Pion/ Combine	2GeV	4GeV	8GeV	10GeV	12GeV	16GeV	20GeV	25GeV	32GeV
Run Number	600094 600095 600096	600086 600087 600089 600091 600092	600082 600083 600084	600097 600098	600073 600075 600076 600079 600080	600063 600067 600069	600054 600055 600058 600059 600062	600052 600049 600050 600053	600032 600034 600037 600038 600040 600043 600048
Statistic	48.2k	116.5k	87.7k	33.3k	103.3k	22.2k	138.3k	144.1k	112.2k

Using 2010 DHCAL test beam data:  
 Event pre-selection: Nhits > 10  
 Statistic: 210k Muon (610036, 38,  
 39, 47, 64) + 807k Mixed

Fractal Dimension calculation: with  
 Nhits at 20mm - 80mm ( 7 points )



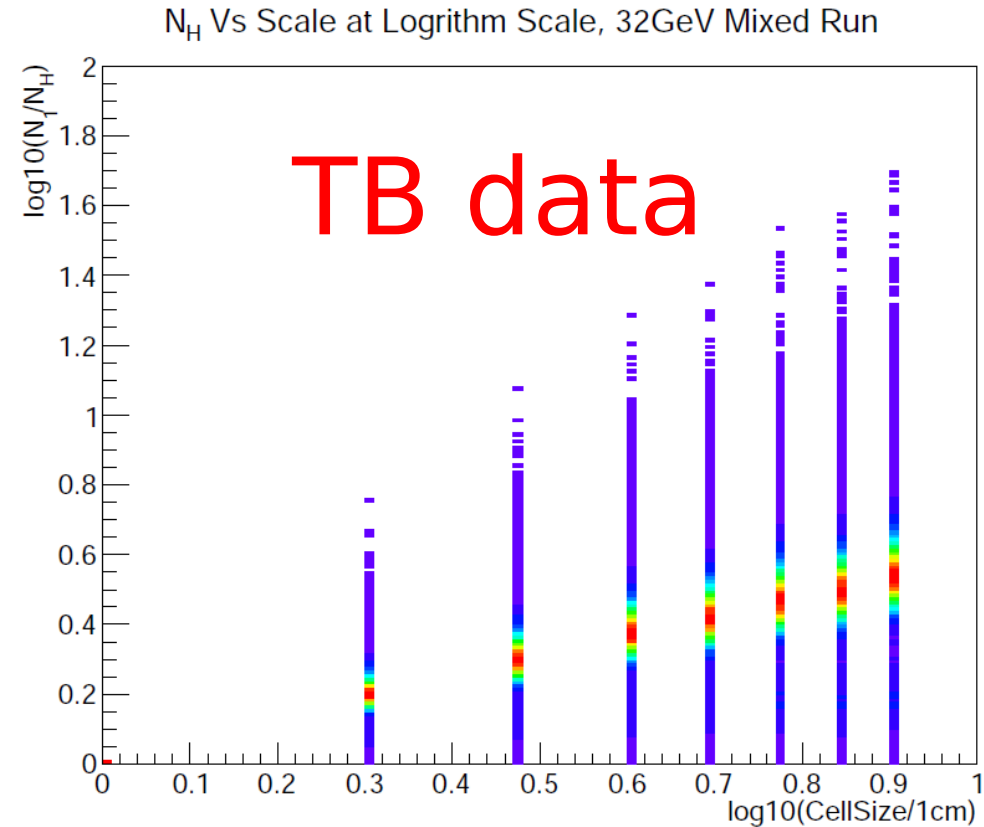
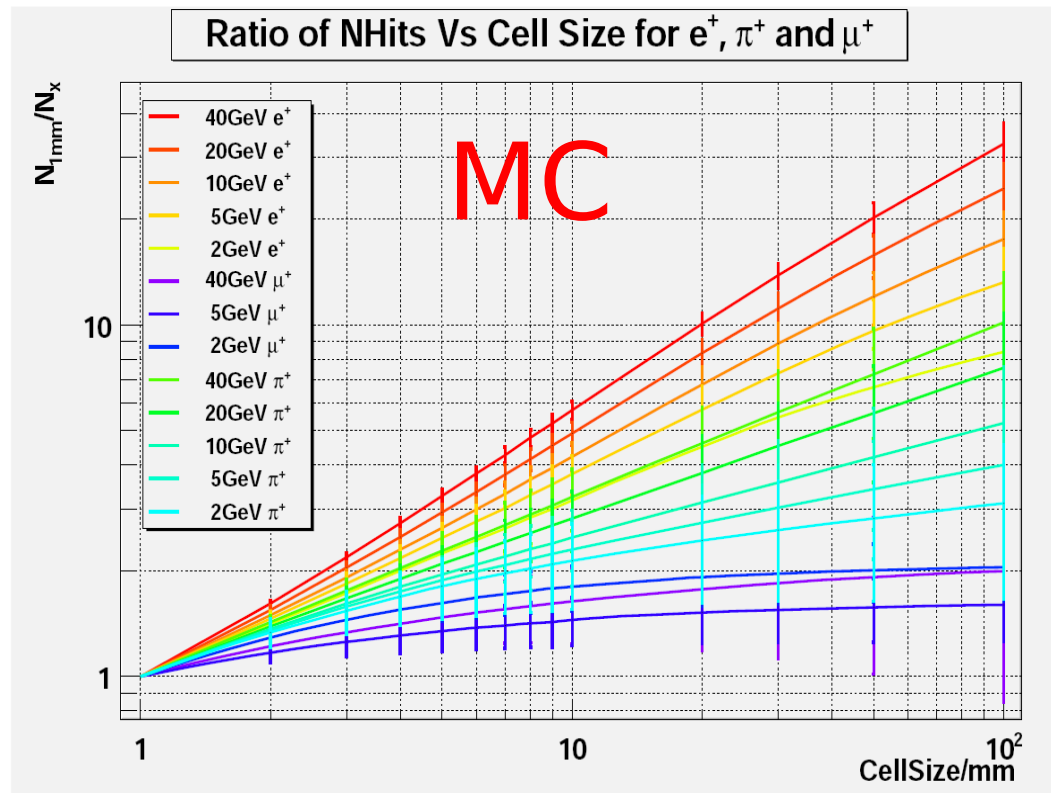
FD method, from MC:

**PID**: promising, with capability to tag detailed interaction information

Energy Estimation:

Charge particle: Resolution largely improved...

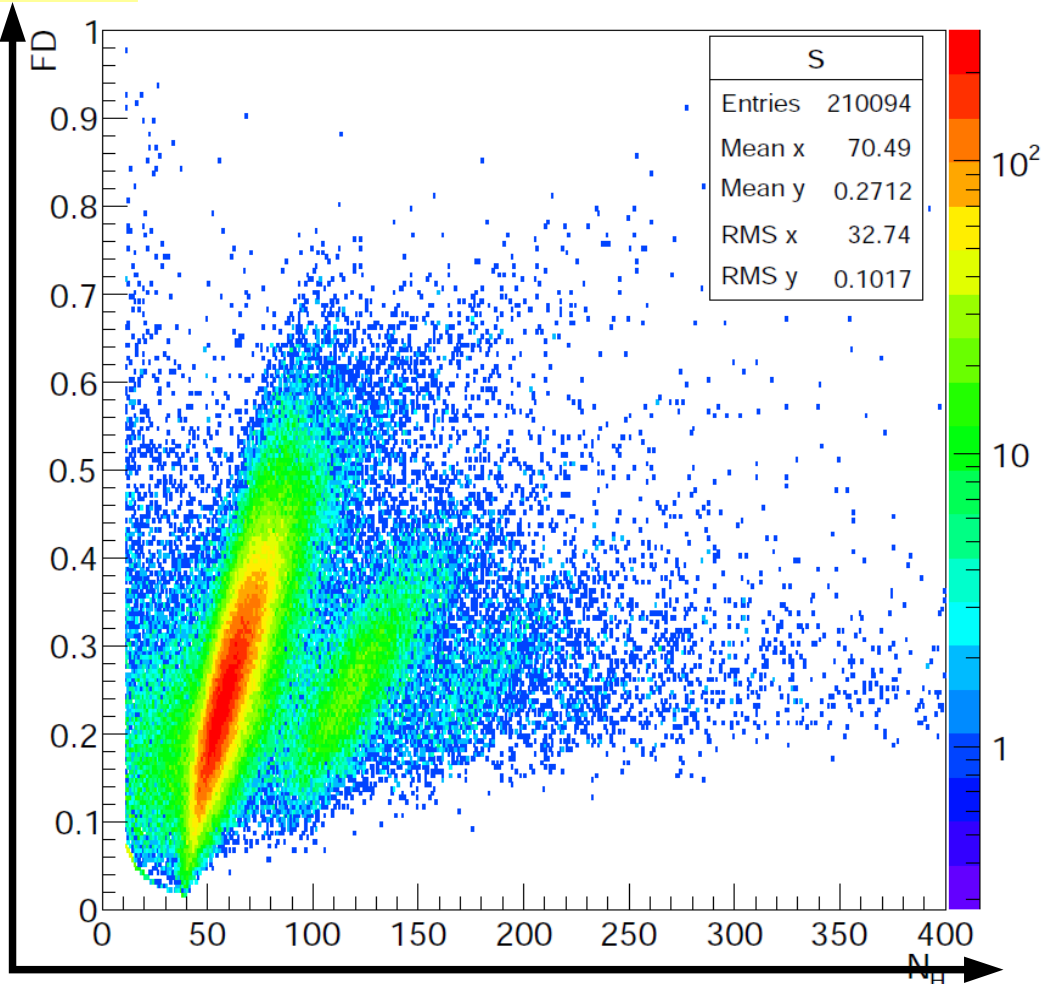
Neutral hadron: Slightly improve resolution/linearity



Slightly Saturated as starting from 1cm cell size

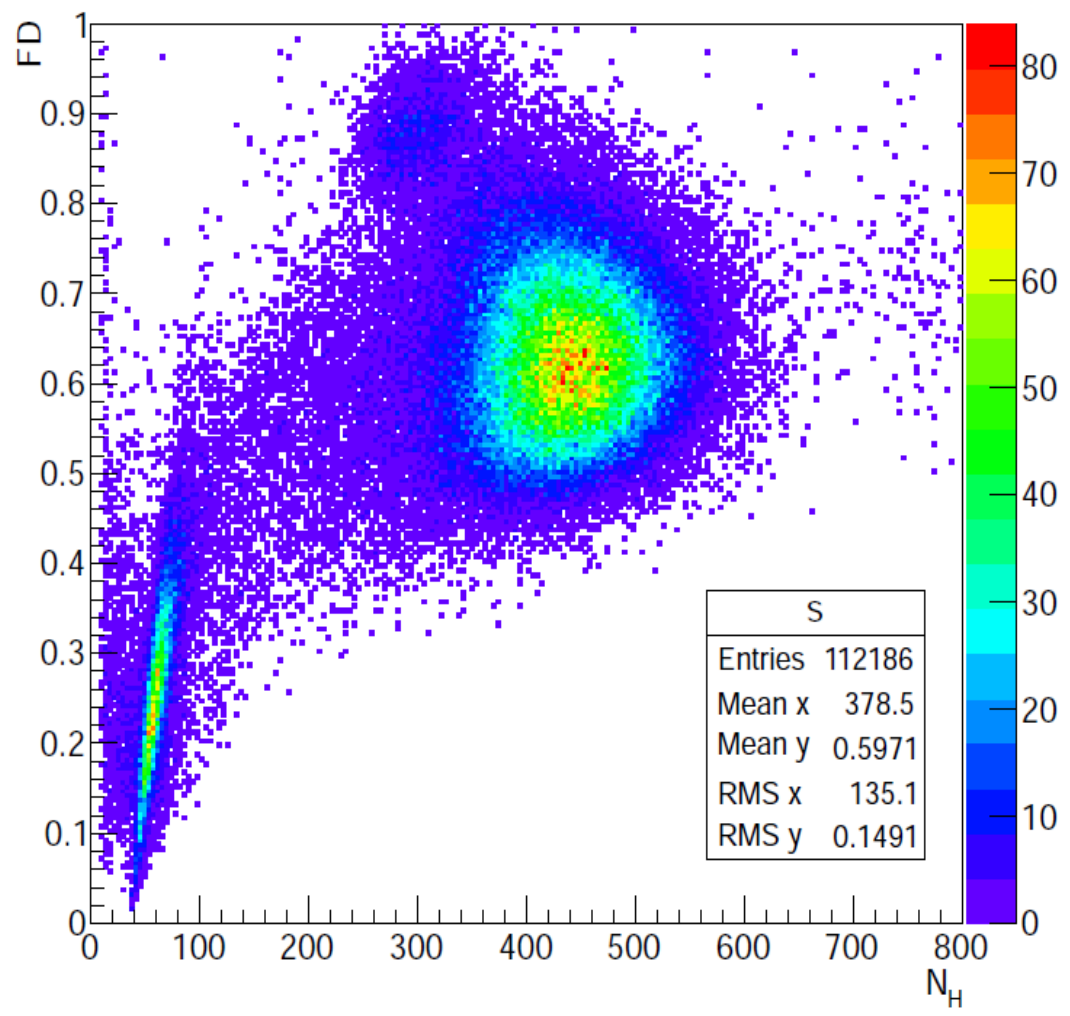
**FD-1** Nhits Vs FD @ Muon (610036, 38, 39, 47, 64)

Nhits Vs FD @ 32GeV (600032, 34, 37, 38, 40, 43, 48)

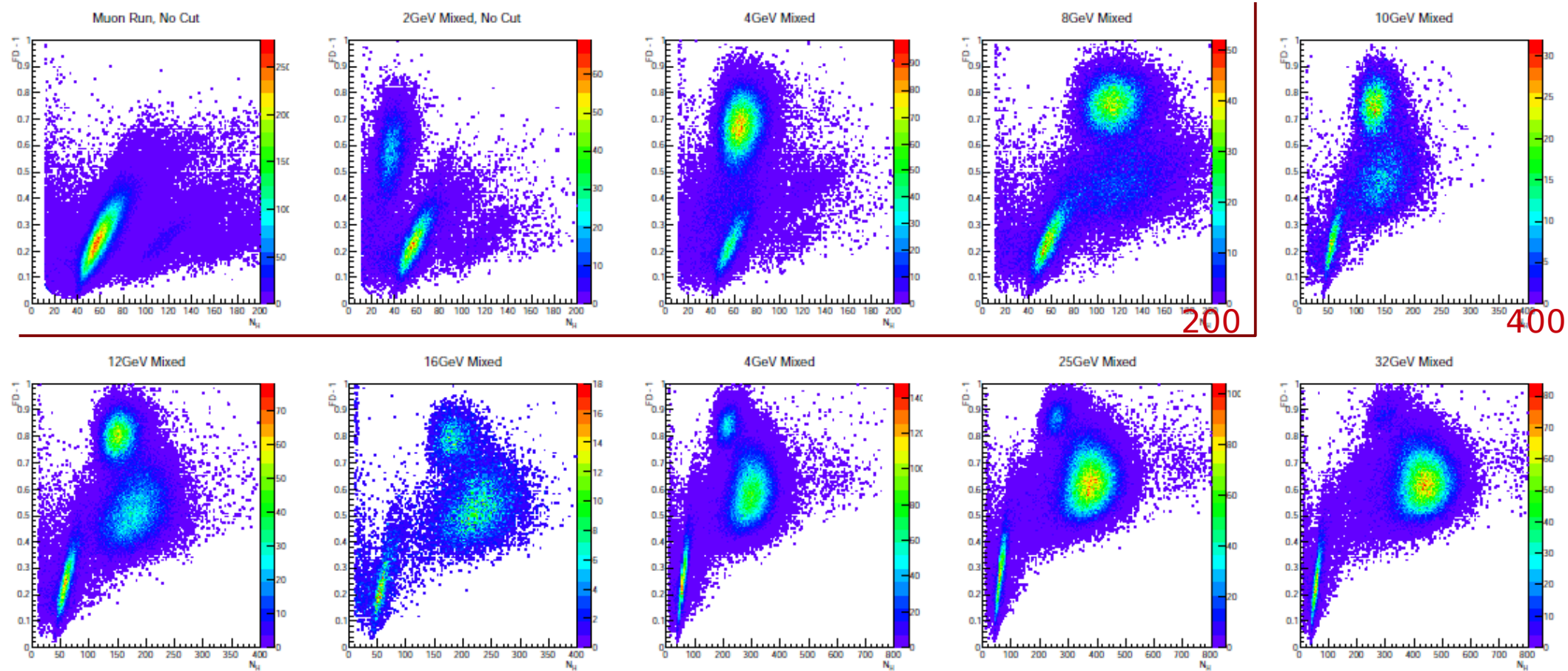


*Nhits @ scale*

Muon Run:  
double/multiple particle events & strong interaction in a few events  
( large FD + large Nhits )



Energetic Pion Run:  
Clear separation between Mip, Positron and Pion

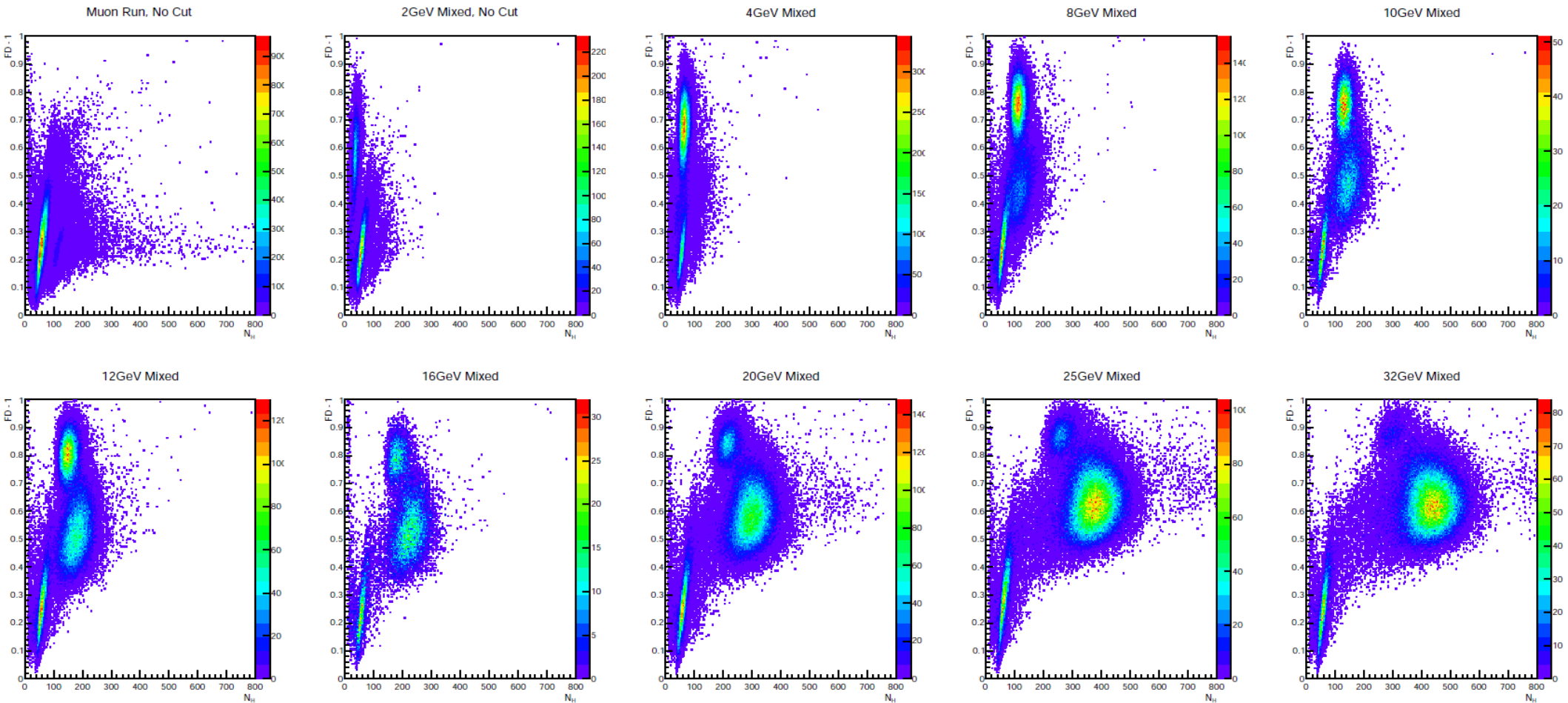


For all events with  $N_{hits} > 10$

**Muon Run:** significant double event component

**Mixed Run @ 2, 4GeV:** Clear separation between positron and MIP component, with significant double events

**Mixed Run @  $E_n > 4\text{GeV}$ :** Clear separation between EM, MIP & Hadron component



For all events with  $N_{hits} > 10$

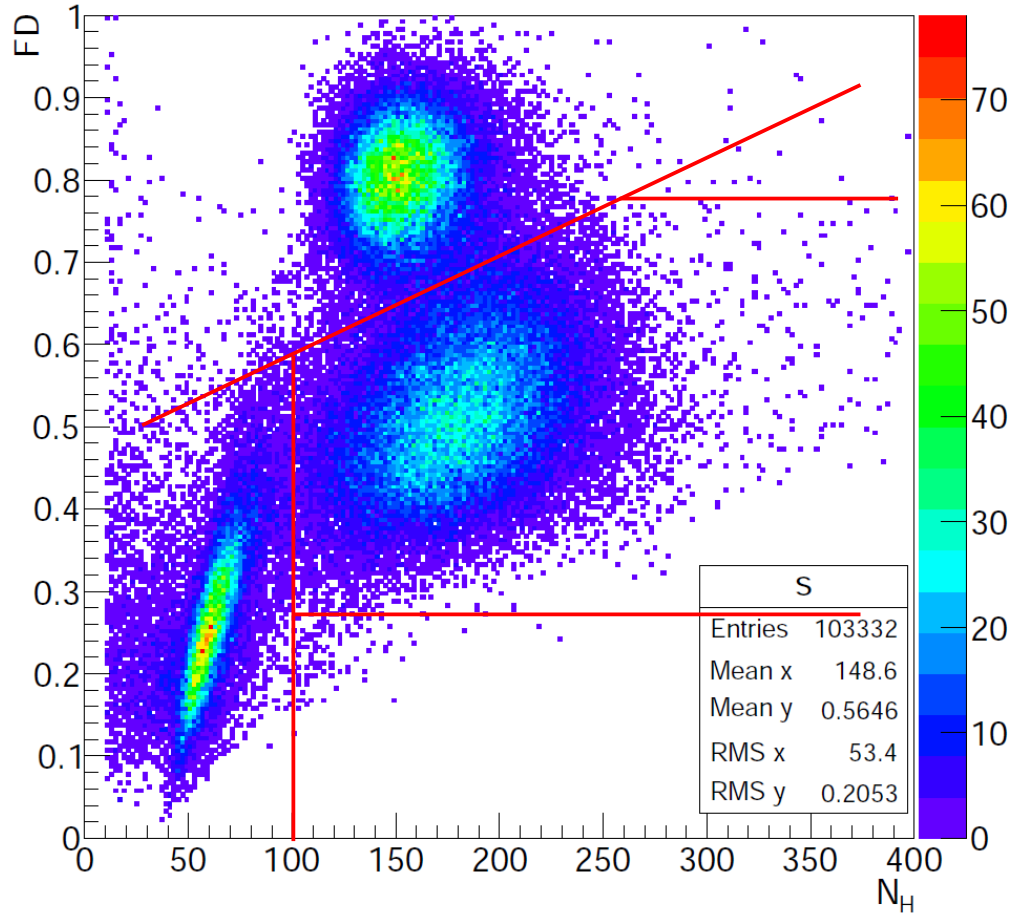
**Muon Run:** significant double event component

**Mixed Run @ 2, 4GeV:** Clear separation between positron and MIP component, with significant double events

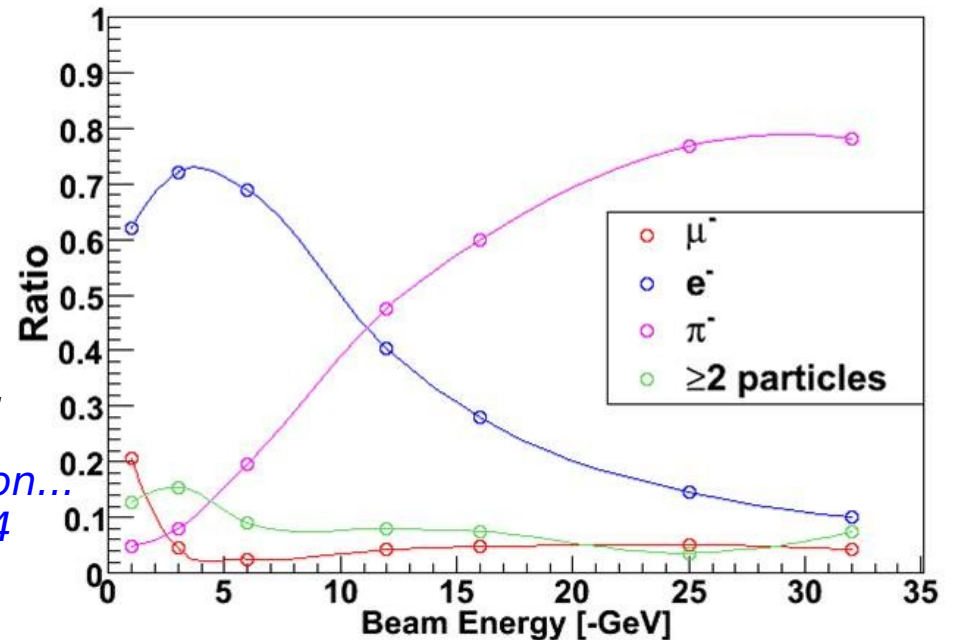
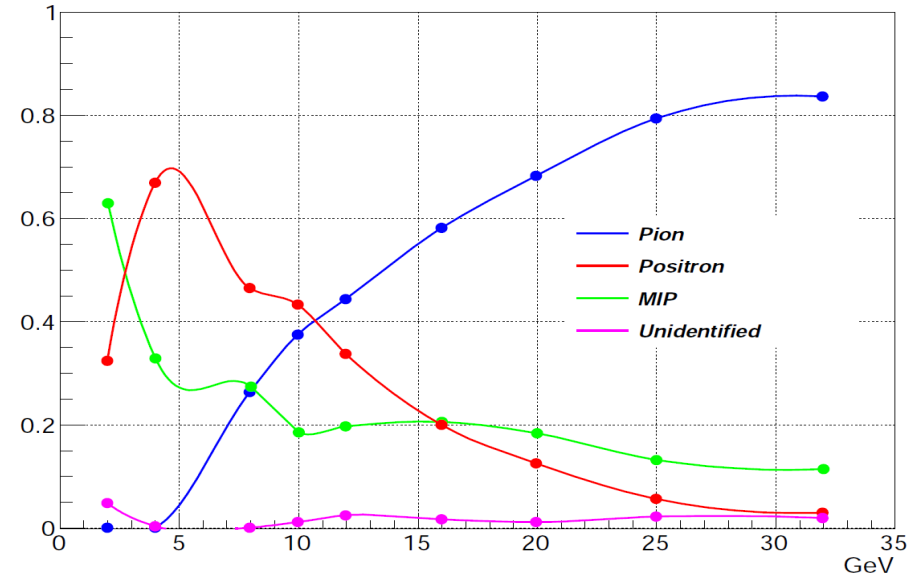
**Mixed Run @  $E_n > 4\text{GeV}$ :** Clear separation between EM, MIP & Hadron component



Nhits Vs FD @ 12GeV (600073, 75, 76, 79, 80)



Beam Contamination



No cleaning, No identification on double events...

Low energy (2, 4GeV) pions regard as MIP

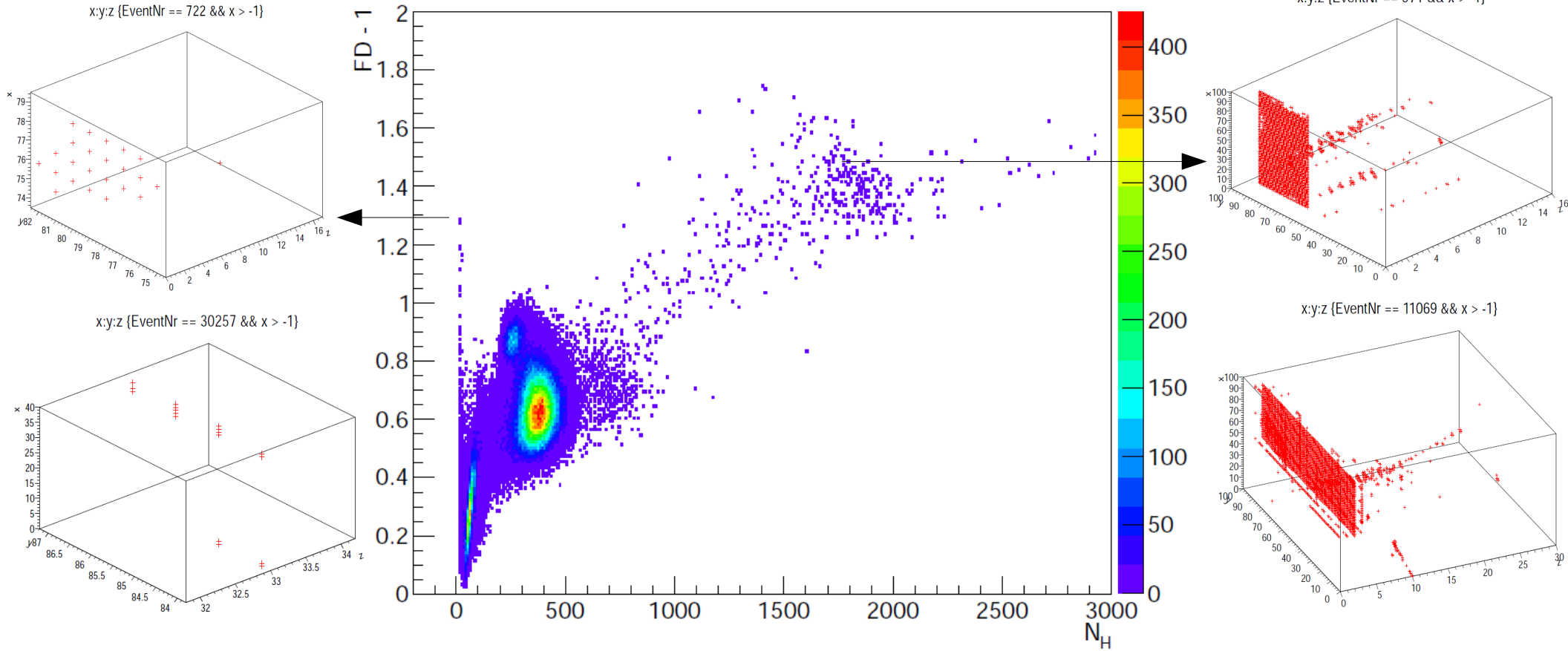
Remark: contamination can depends on rate selection...

Detail cuts can be traced on spare slides P31 - 34

# FD & Typical Patterns



Nhits Vs FD @ 25GeV Mixed

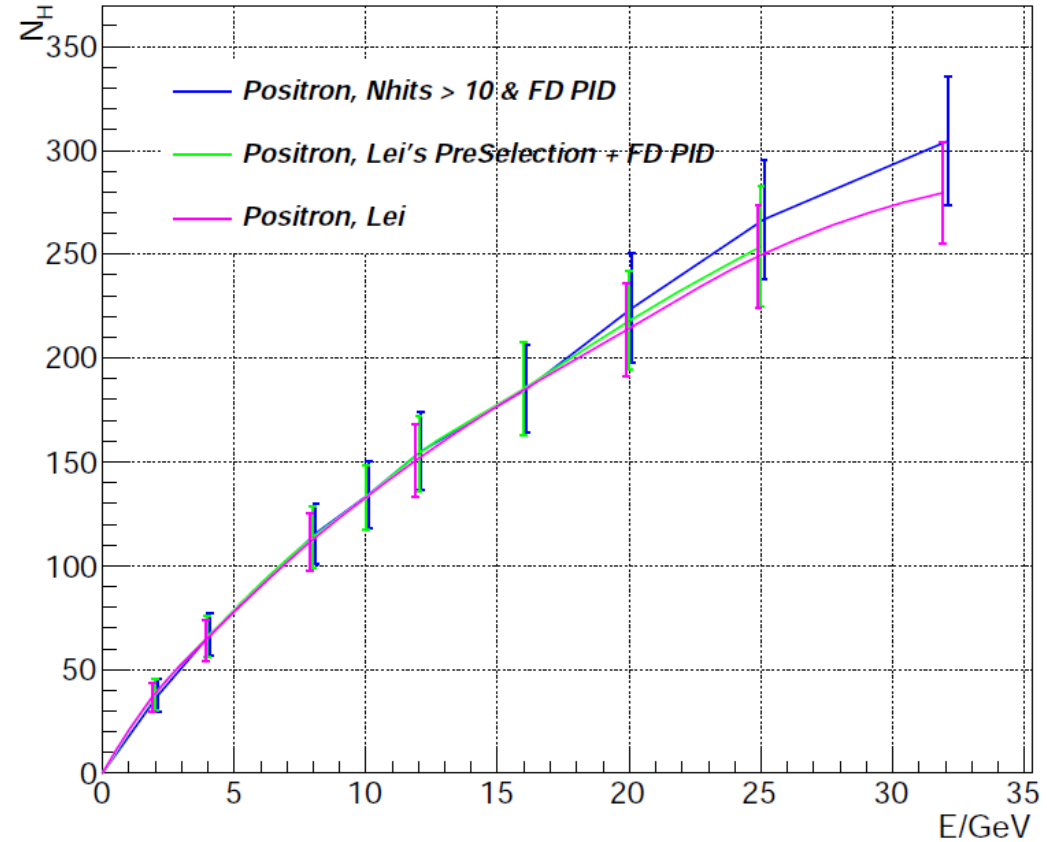
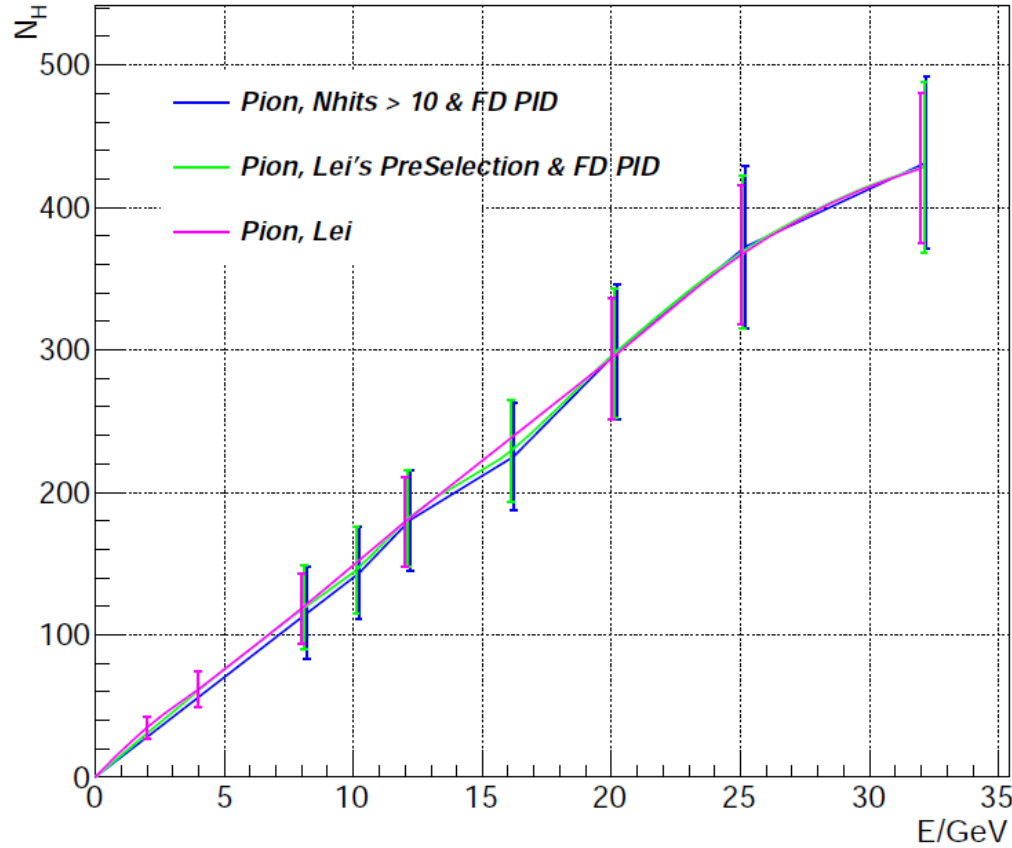


DHCAL Runs: pretty clean... but not completely free of noises...



Energy Response of Pion

Energy Response of Positron



Energy Response:

*Basically agrees, especially with same pre selection  
 Non-linear behaviour of positron and high energy pion  
 Pion: lower response at 16GeV...*

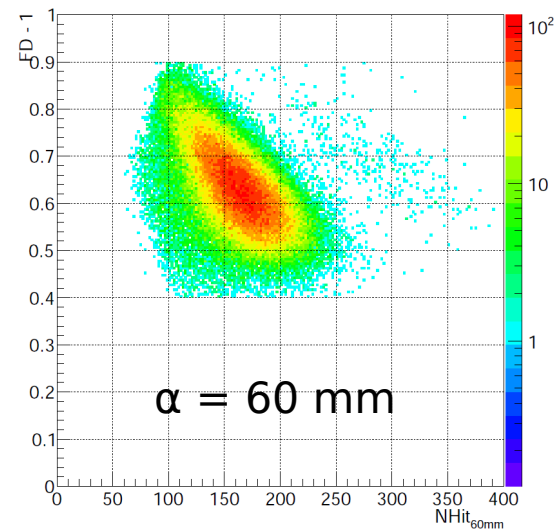
Lei's pre selection

- \* Exactly 1 cluster in layer 1
- \* Not more than 4 hits in layer 1
- \* At least 3 layers with hits
- \* No hits within 2 cm to layer edges

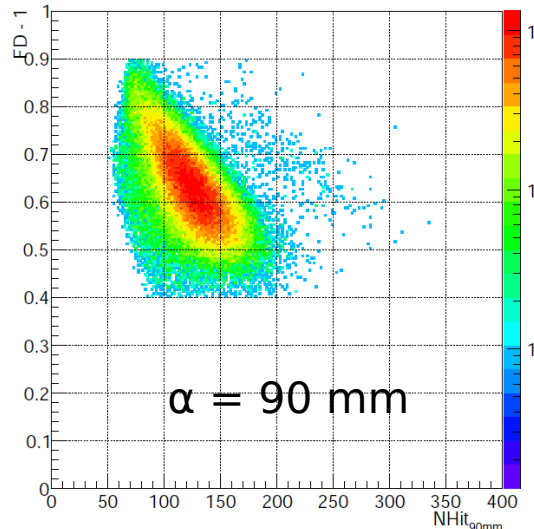
# Track-Cluster matching (with known track E)



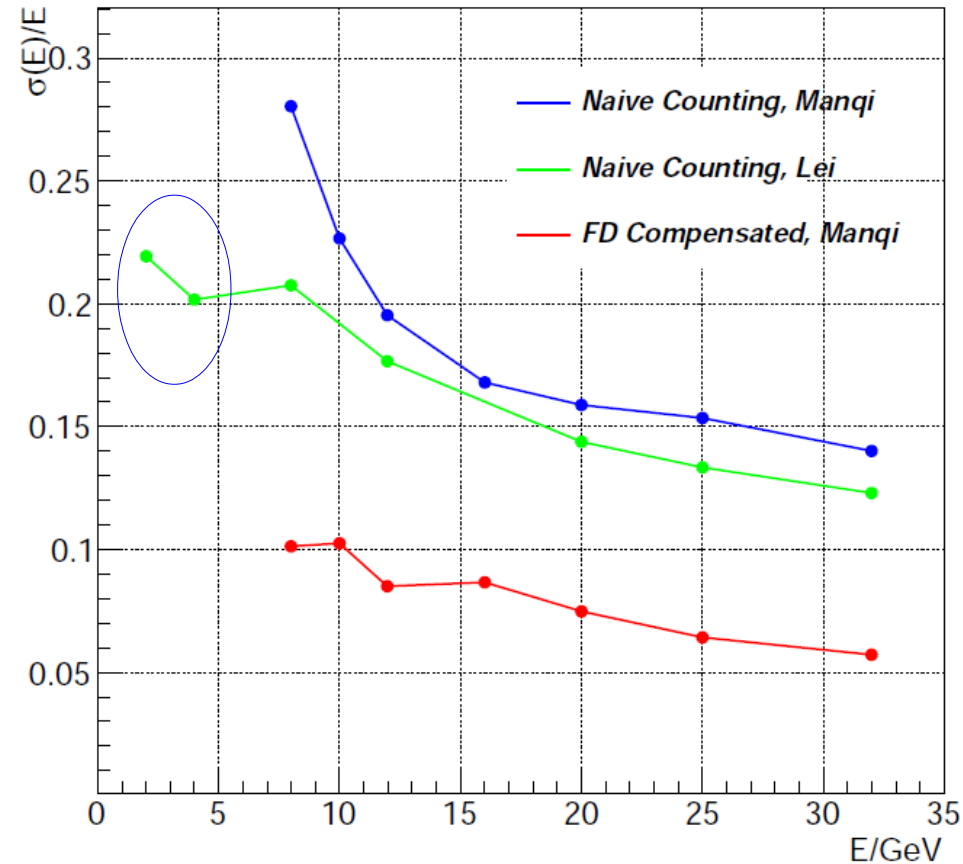
NHit<sub>60mm</sub> Vs FD @ 32GeV Pion



NHit<sub>90mm</sub> Vs FD @ 32GeV Pion



Pion Energy Resolution



- FD & Nhits(large\_scale) (extreme case: Num of fired layer ): strong correlation holds for test beam data

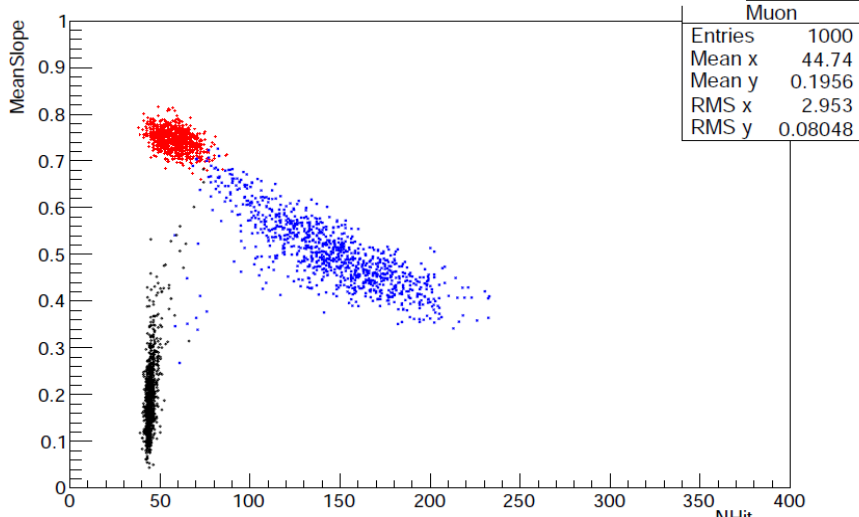
- Hand put energy estimator

$$E = N * ( NH_{90mm} + 10 * E * FD )$$

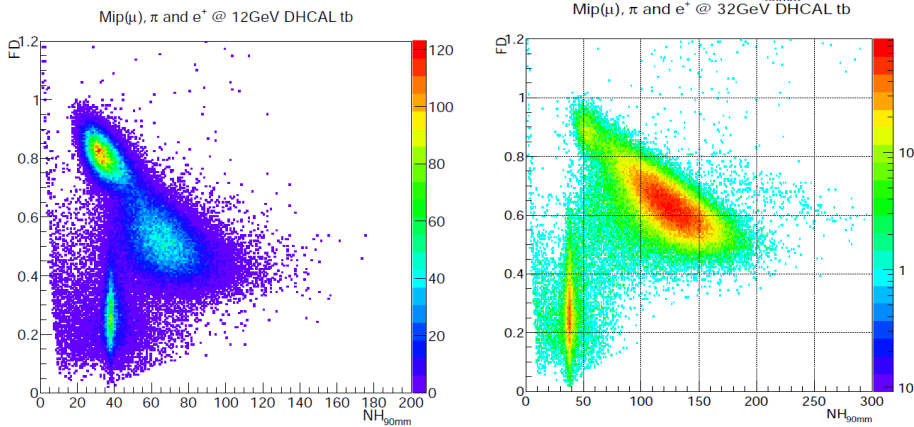
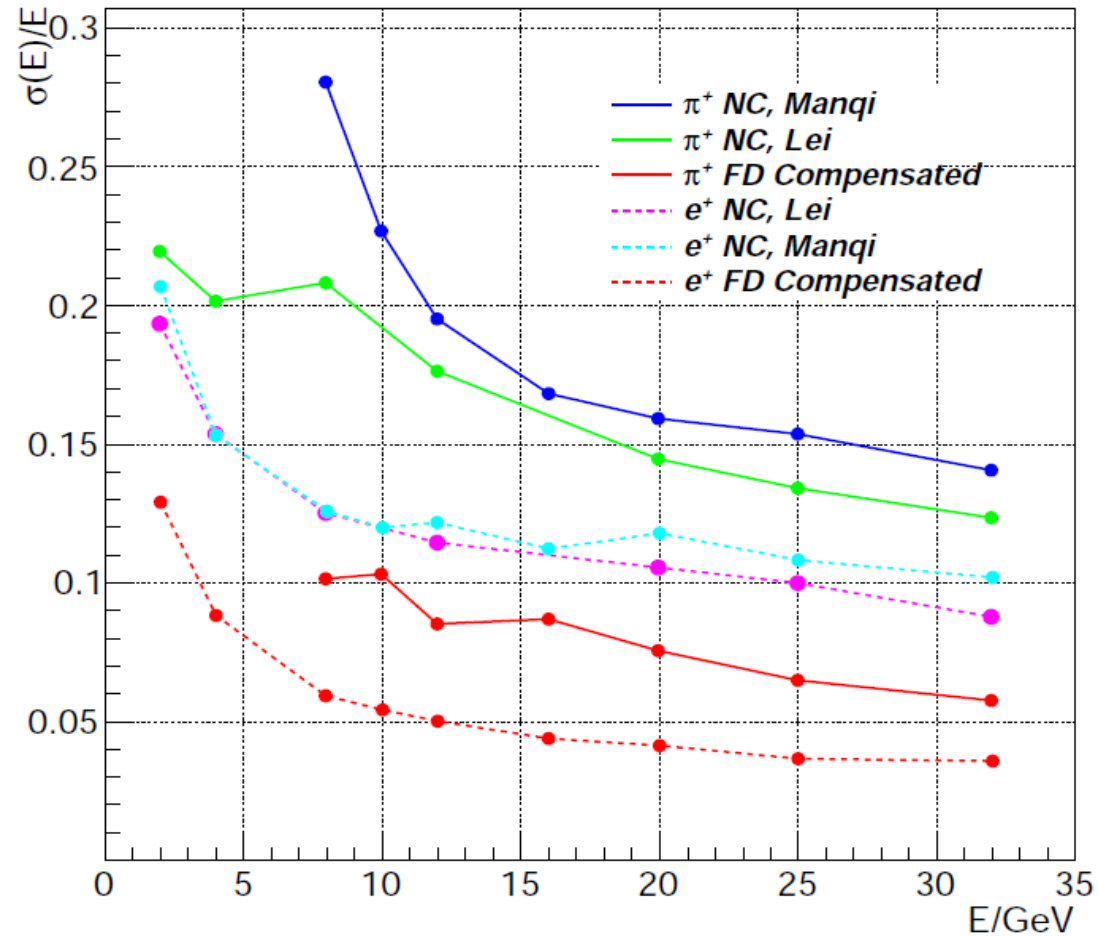
Energy Resolution easily improved by a factor of 2...

*The difference between naive counts is due to different cut chain*

NHit<sub>90mm</sub> Vs Mean Slope for 40GeV Particle

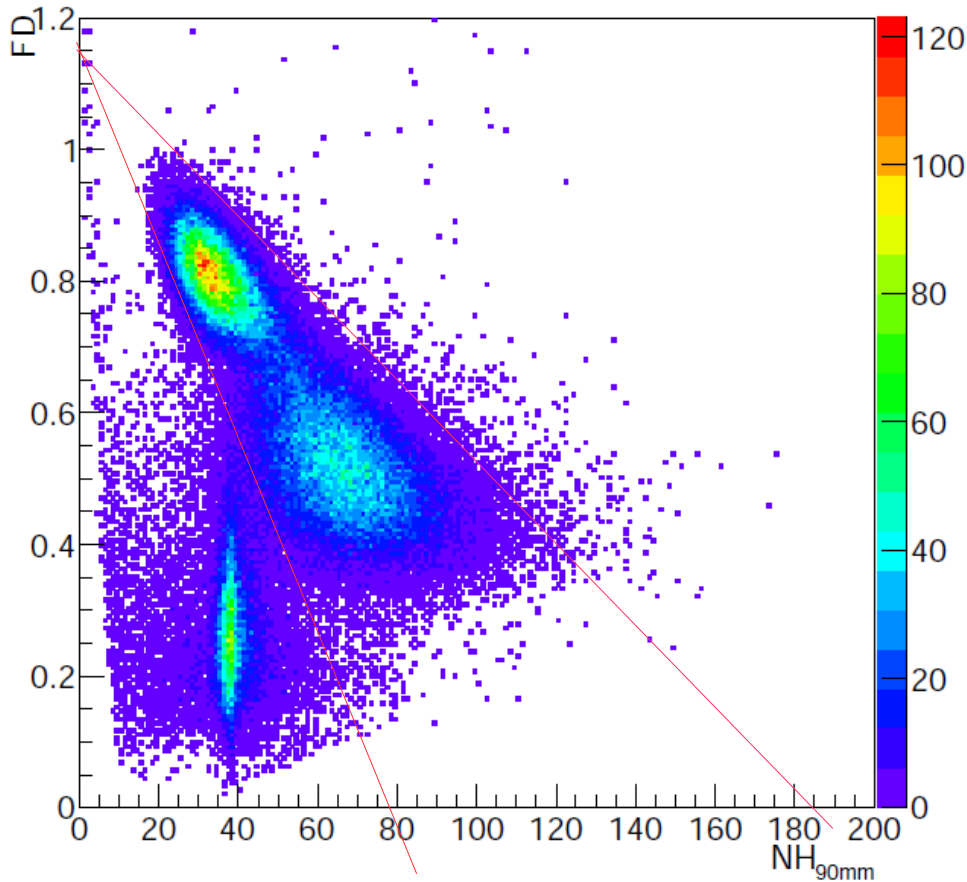


$\pi^+/e^+$  Energy Resolution

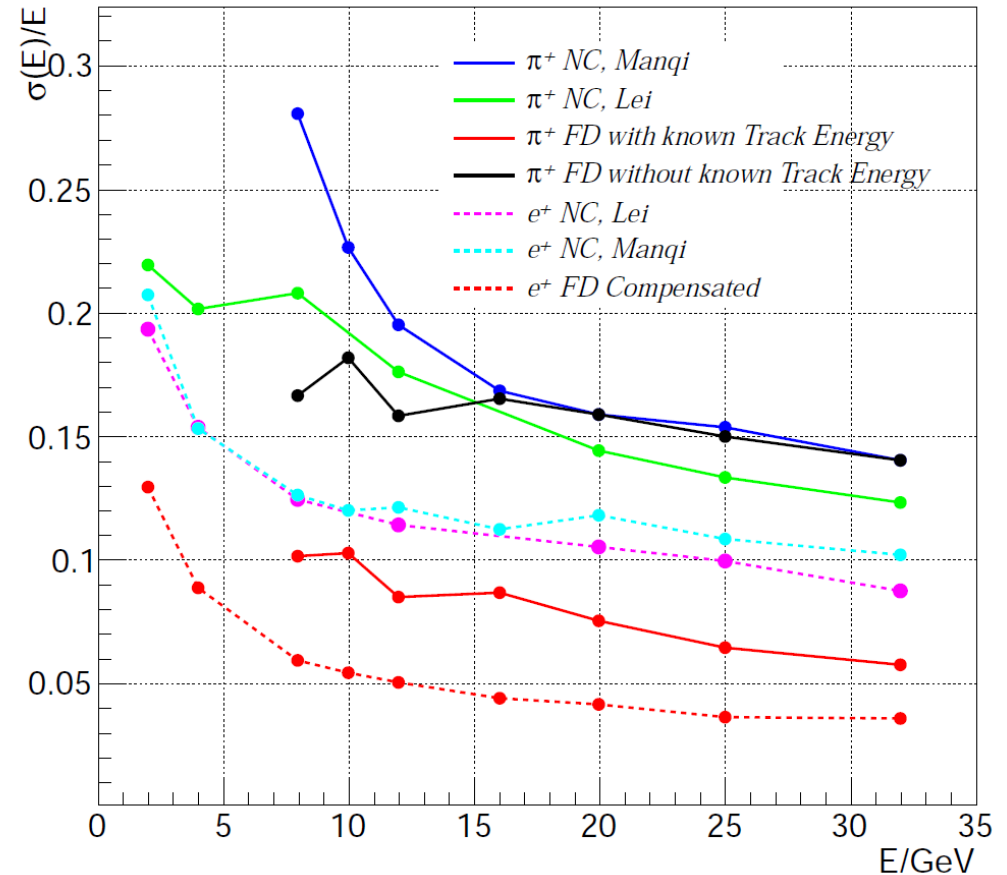


- $e^+$  and  $\pi^+$ : continuous & aligned distribution on the Nhits(large scale) - FD plane ( extreme case is  $\pi^+ + n \rightarrow \pi^0 + p$  ): Estimate positron energy with same estimator as pion.
- Track-cluster matching can be improved by the use of FD, but complete performances still have to be studied

Mip( $\mu$ ),  $\pi$  and  $e^+$  @ 12GeV DHCAL tb

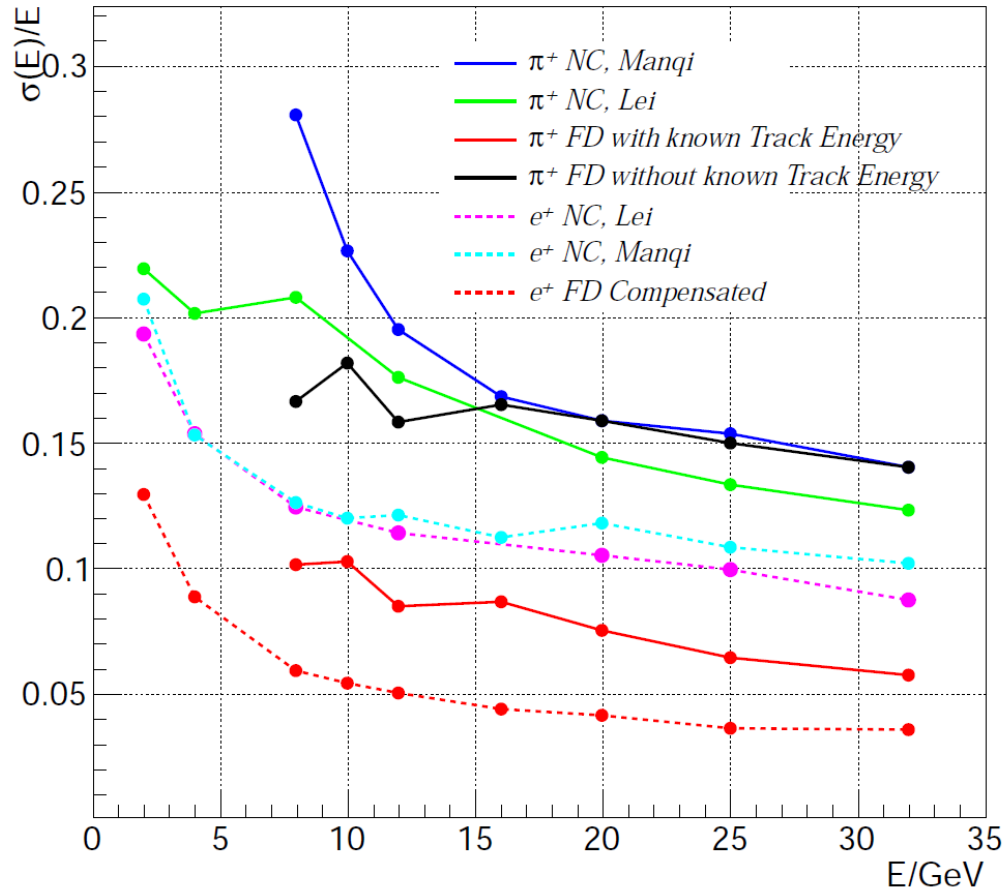


$\pi^+/e^+$  Energy Resolution

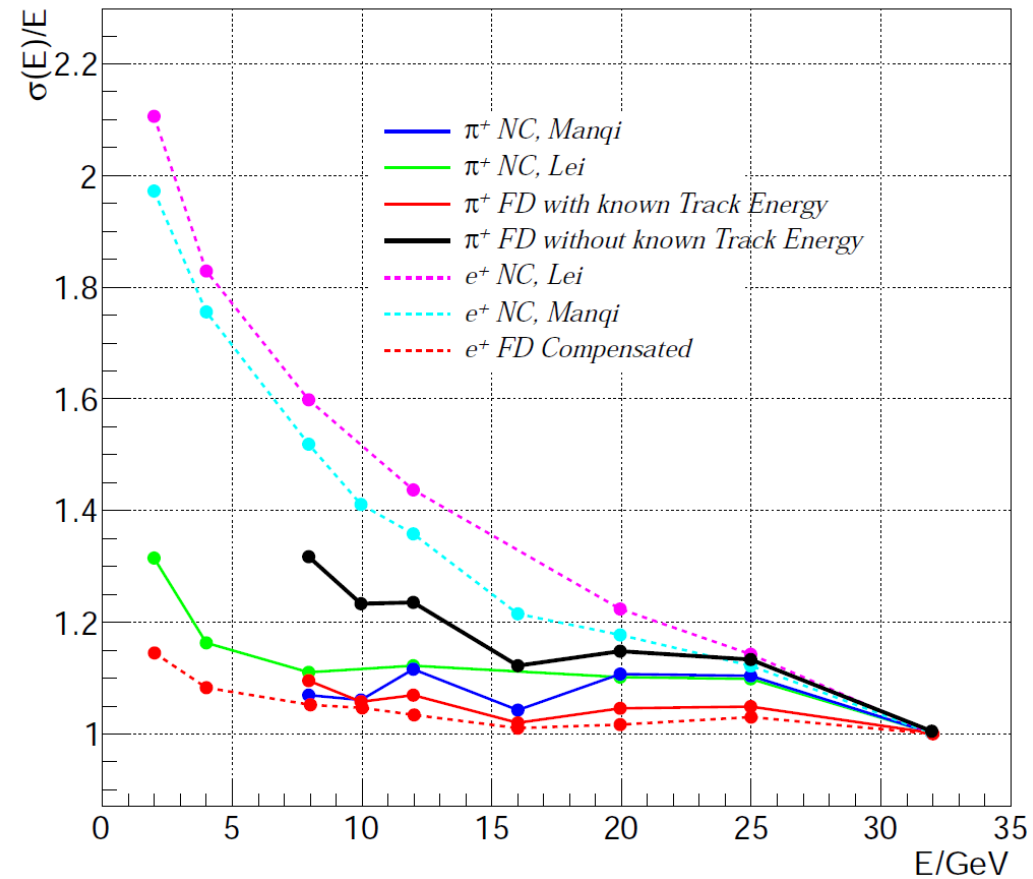


- Without knowledge on initial energy  $E \sim NH/(a - FD)$  : Projection from the invariance point on FD axis to  $NH_{hit}$  axis.
- EM & Hadronic distribution has the same boundary (  $FD \sim$  contamination of EM interaction inside hadronic shower )  $\sim$  Hard limit: **Measure Hadronic as precisely as EM component?**

$\pi^+/e^+$  Energy Resolution



$\pi^+/e^+$  Energy Response Linearity



**Charged particle:** Significantly improve energy resolution & linearity ( i.e, positron saturation effects corrected )

**“Neutral” particle:** Energy Resolution could be improved at low energy, but lose a bit the linearity... contrary to MC prediction.

Objective: to find optimised threshold and energy estimator, with best linearity and resolution of particle energy

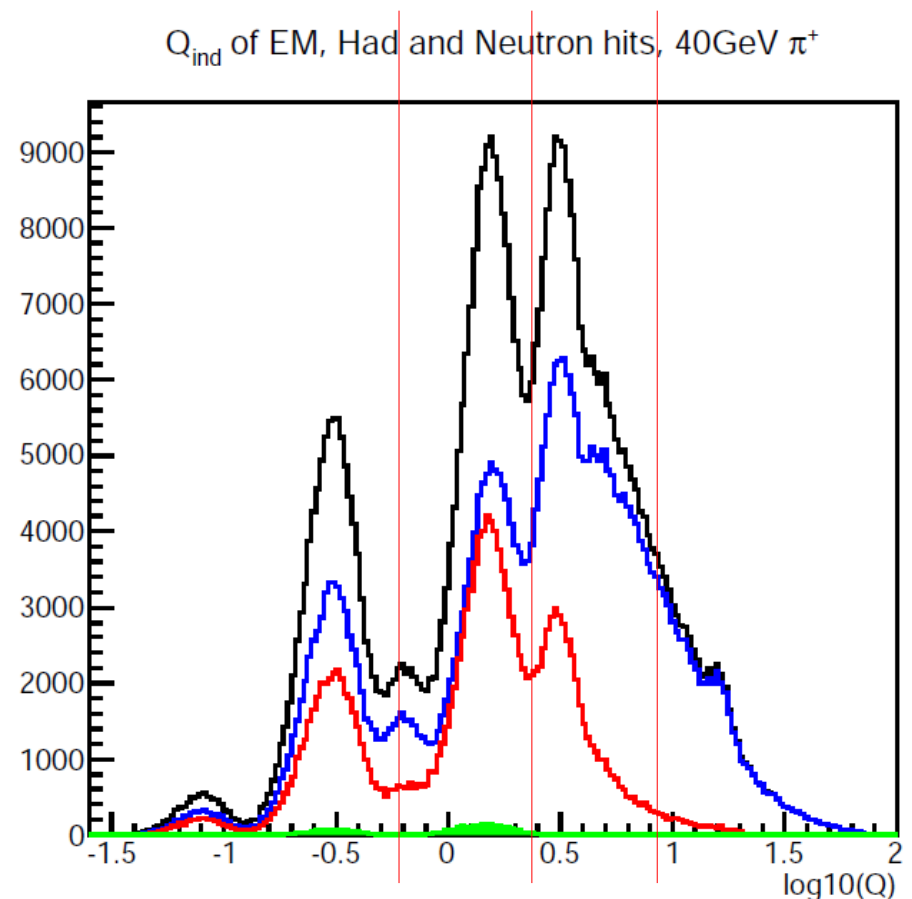
$$E_n = E_n ( \text{Resize\_NH}(i), \text{FD}(i), \dots ); i = (1), 2, 3, 4$$

To study correlations @ different thresholds  
( Code done )

Open questions:

Besides e/h ratio, any information else  
Presented by FD?

...

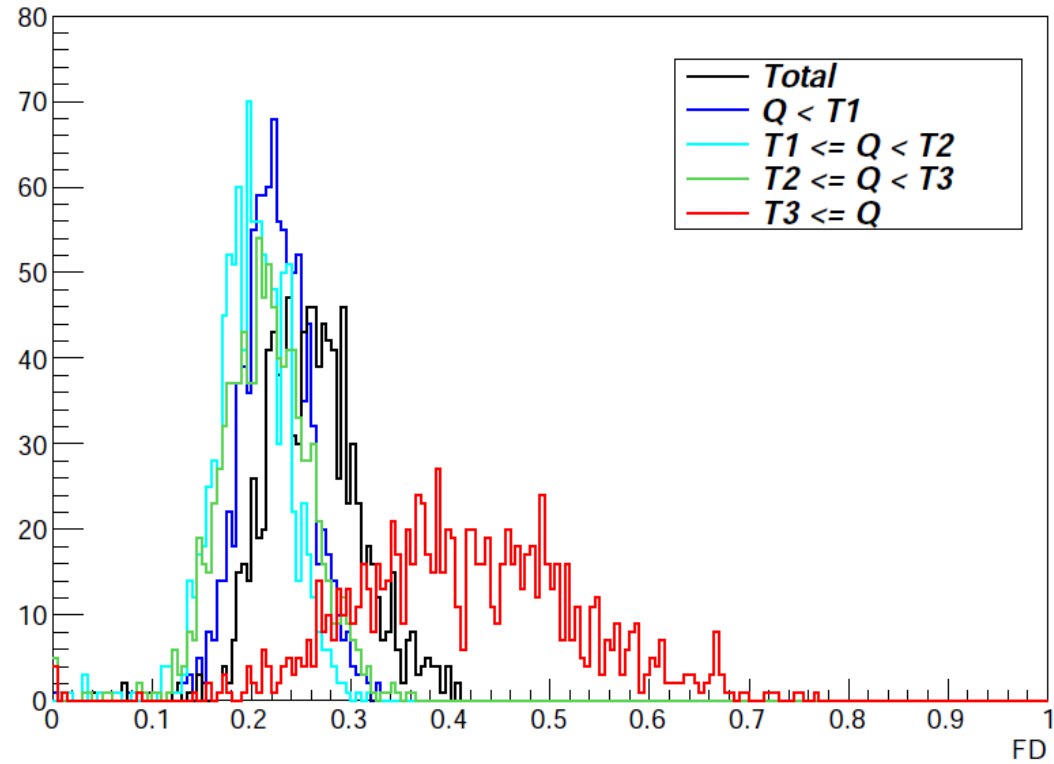
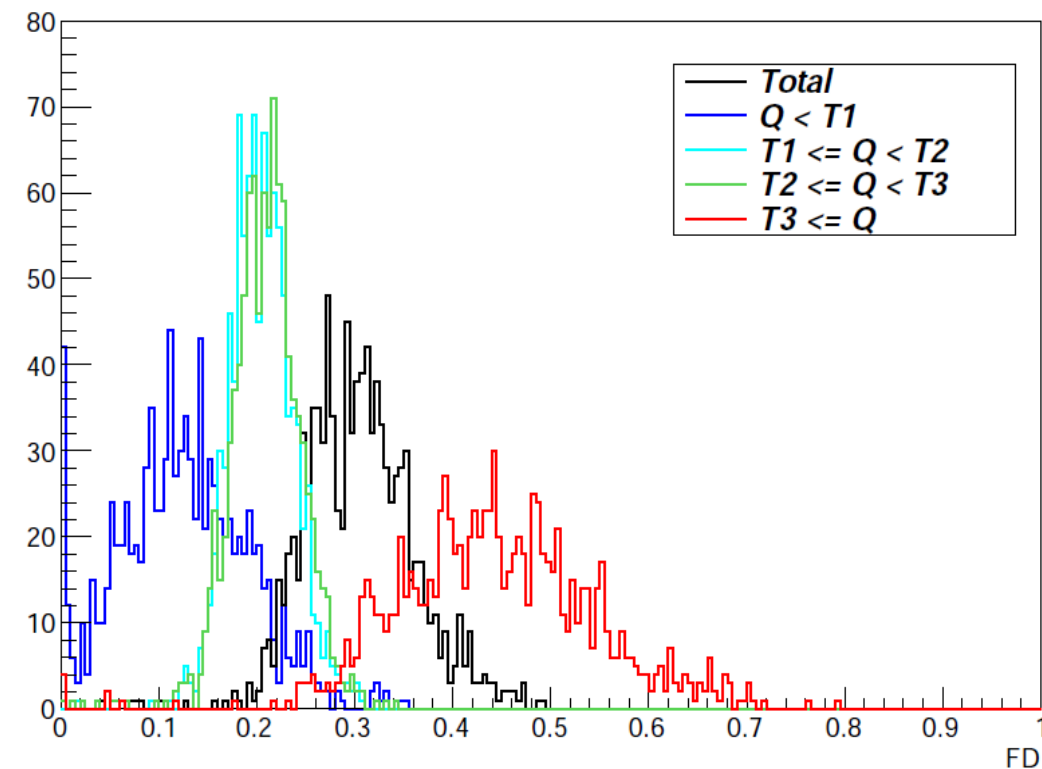


Thresholds: 0.2, 1.0, 2.5 pC

FD of different SDHCAL hits, 40GeV  $\pi^+$

Thresholds: 0.8, 2.2, 4.5 pC

FD of different SDHCAL hits, 40GeV  $\pi^+$



Significantly different behaviour...

Most interesting part:  $Q > T3$  hits - Core of EM interaction?

# Summary & To do

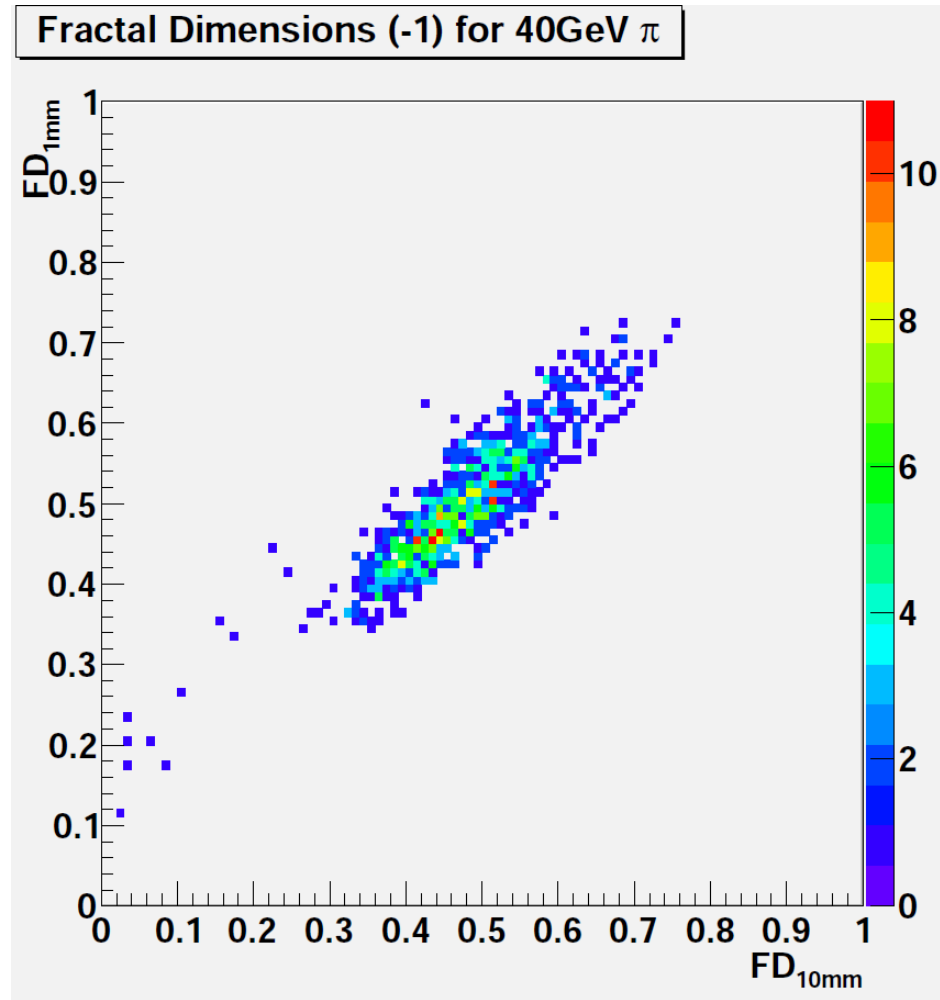
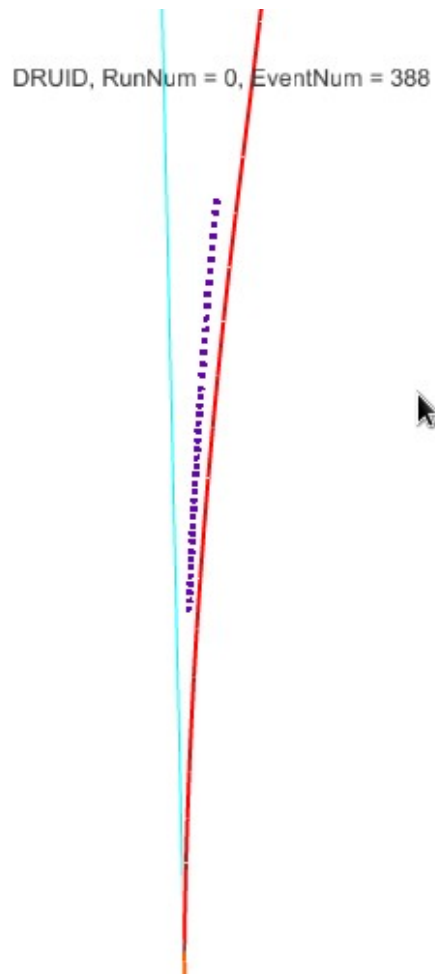


- Fractal Dimensional: **Validated @ simulation and real data**
  - Roughly repeat:
    - Fermi Lab Beam contamination measurement in previous CALICE TB
    - e+ and pi energy resolution (compare to Lei's Granada slides)
  - PID: *Čerenkov seems no longer needed for the prototype...*
  - Energy Estimation:
    - With known track energy: resolution easily improved by a factor of 2
    - Possibility to measure hadronic shower energy as precisely as EM shower?
- To do:
  - Better understanding to FD
    - FD @ ECAL, SDHCAL, AHCAL...
    - FD Vs Geometry...
  - Note & Paper

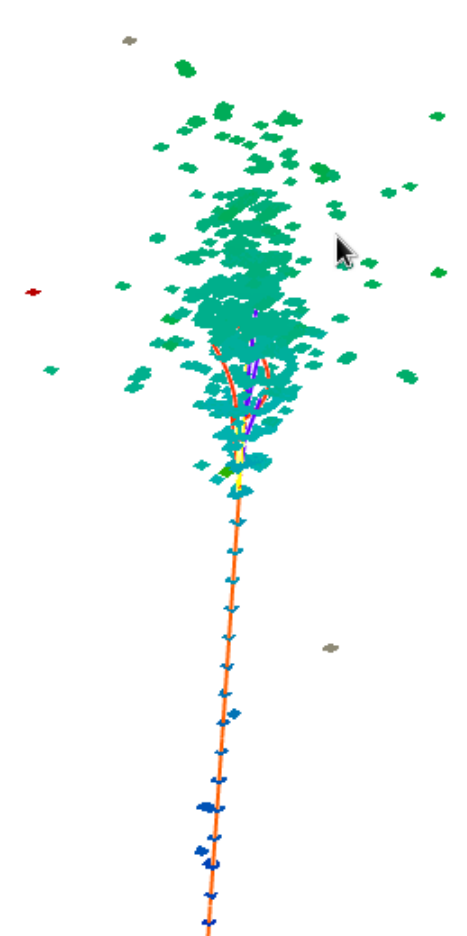


# Spare slides

# Extreme Cases: Pion



DRUID, RunNum = 0, EventNum = 112



- Pion: MIP, Pion decay;
- EM interaction ( $\pi + n = p + \pi^0$ ); partially identified by interaction point tagging

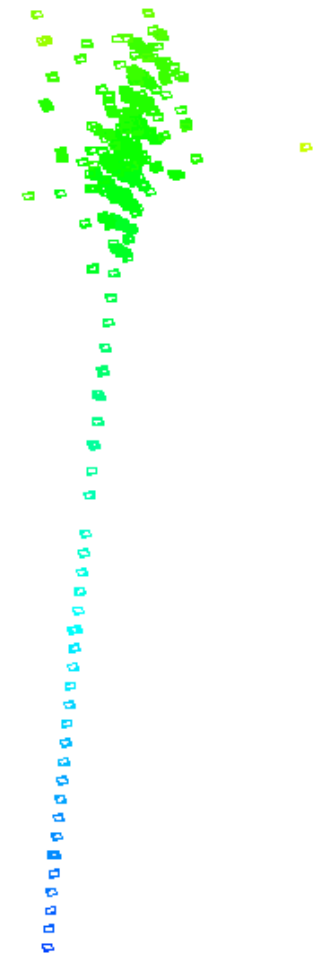
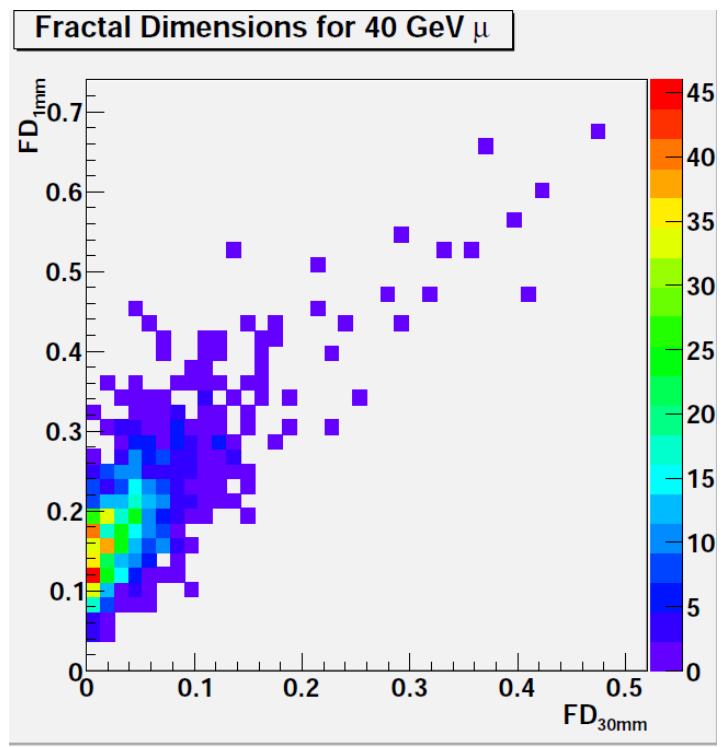
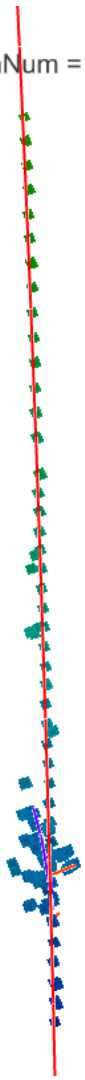
# Extreme Cases: Muon



DRUID, RunNum = 0, EventNum = 535

DRUID, RunNum = 0, EventNum = 547

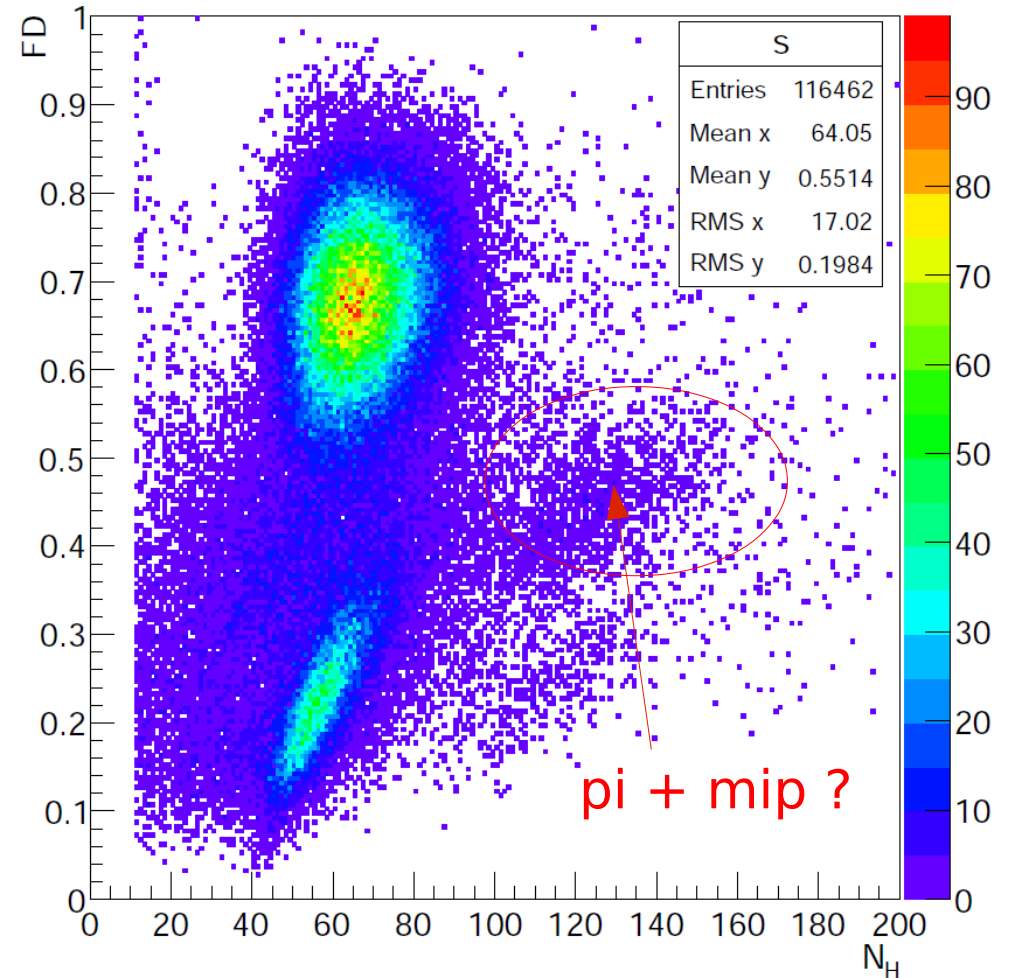
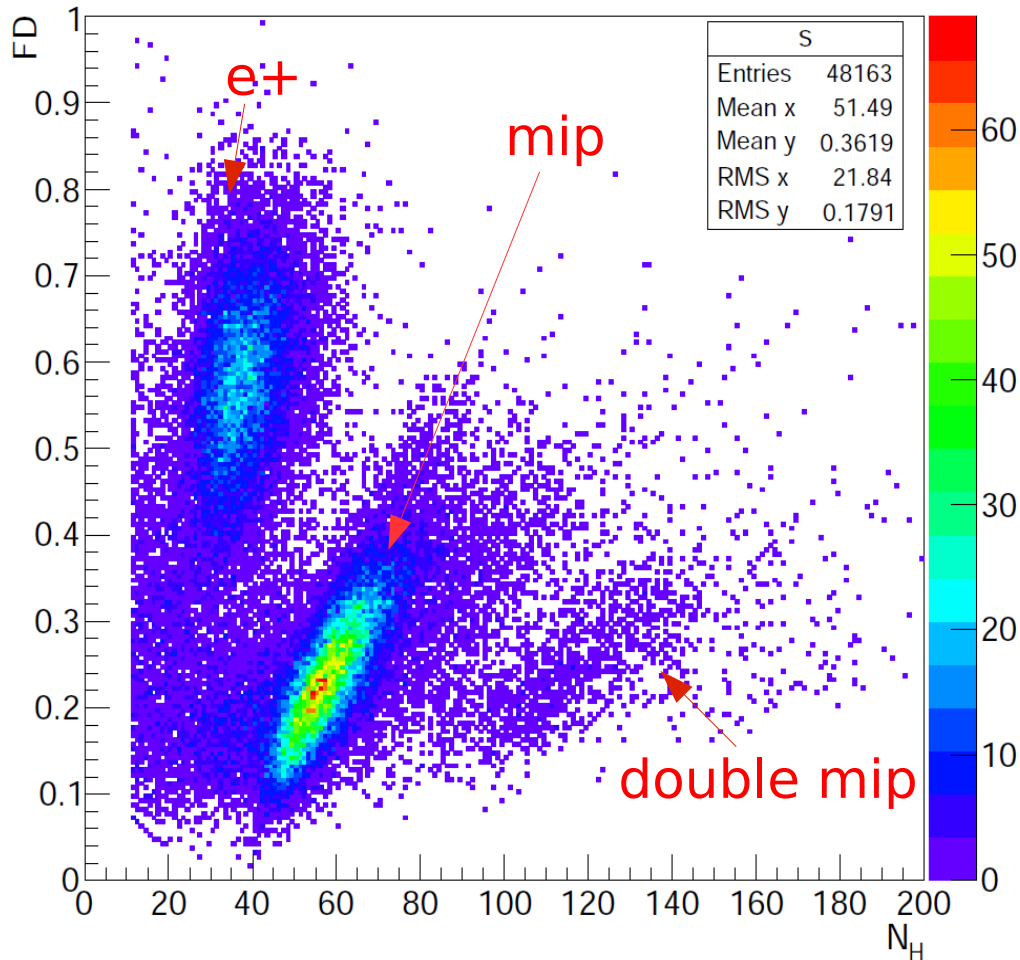
DRUID, RunNum = 0, EventNum = 367



Together with  $N_{hit}$  information: to identify Muon radiation & String noise...

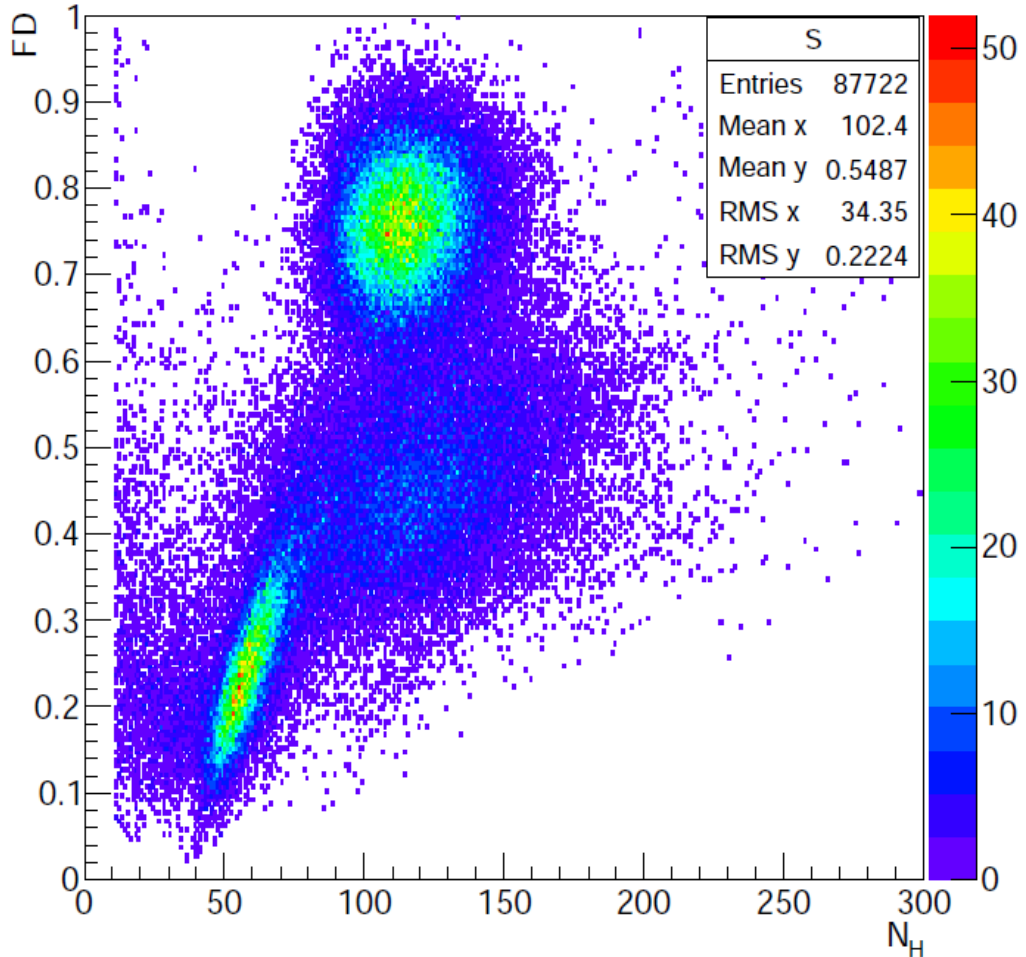
Nhits Vs FD for 2GeV Mixed beam (600094, 95, 96)

Nhits Vs FD @ 4GeV (600086, 87, 89, 91, 92)

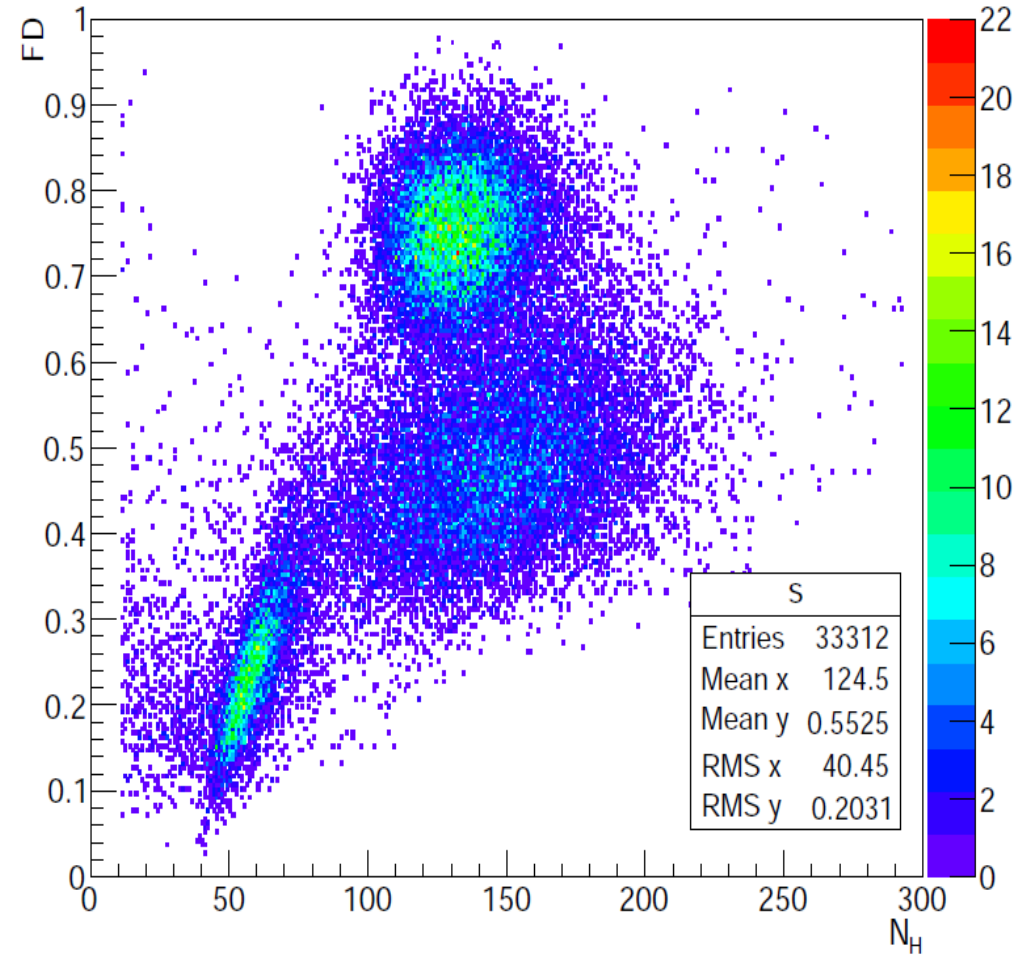


Low Energy Mixed Run:  
 Significant Positron component  
 Low contamination of pion & difficult to identify... reliable MC input should help

Nhits Vs FD @ 8GeV (600082, 83, 84)



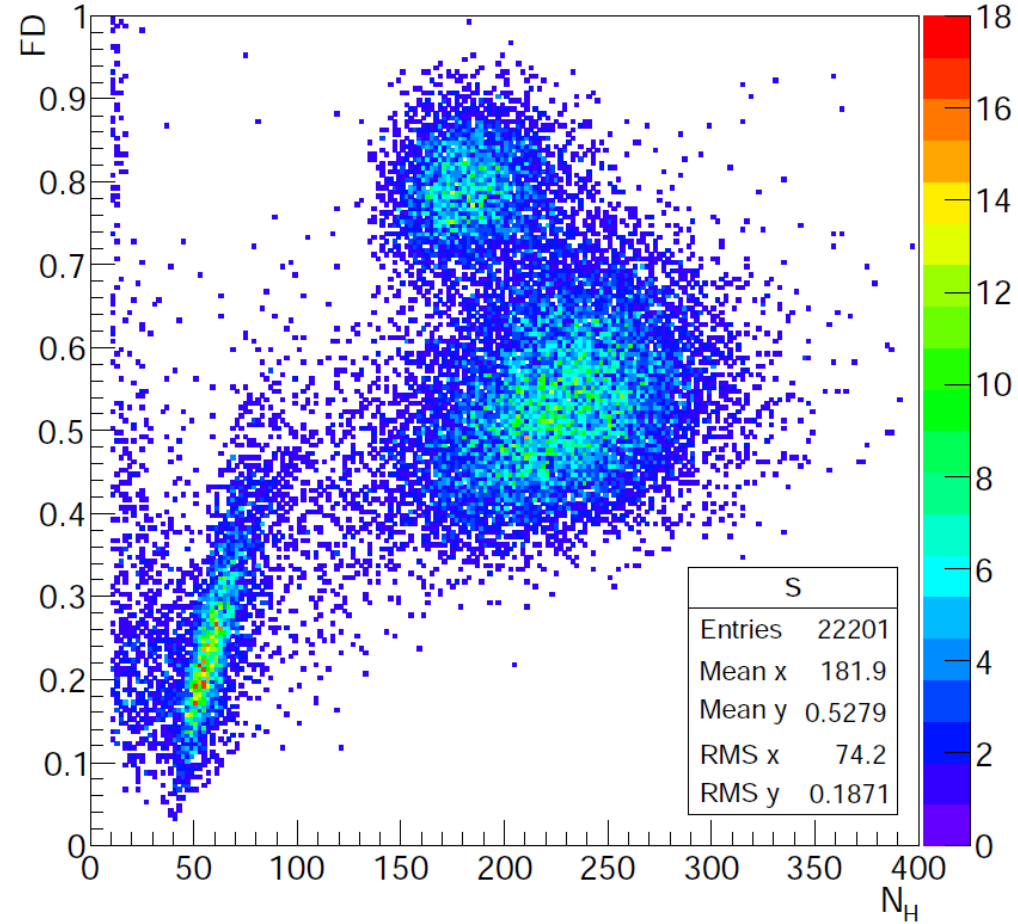
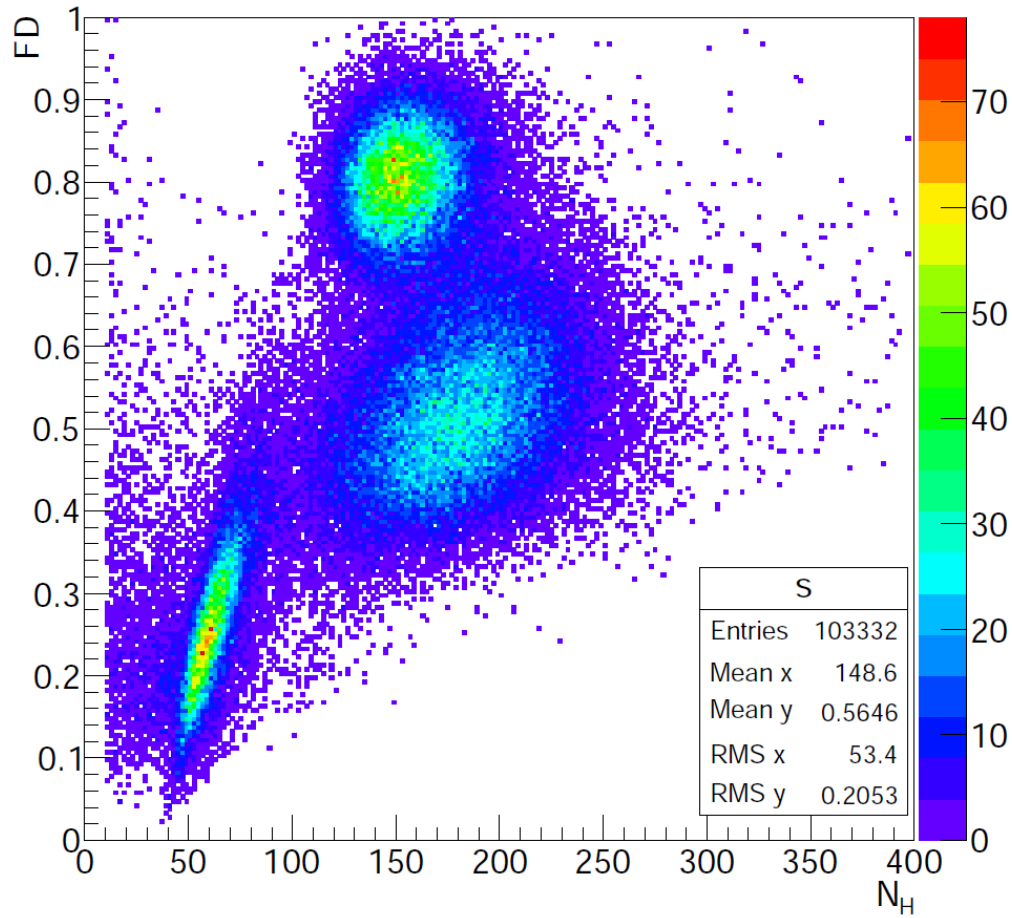
Nhits Vs FD @ 10GeV (600097, 98)



From 8GeV: Start to have good  $\mu$ - $\pi$  separation.  
 Could be improved with more dedicated FD definition

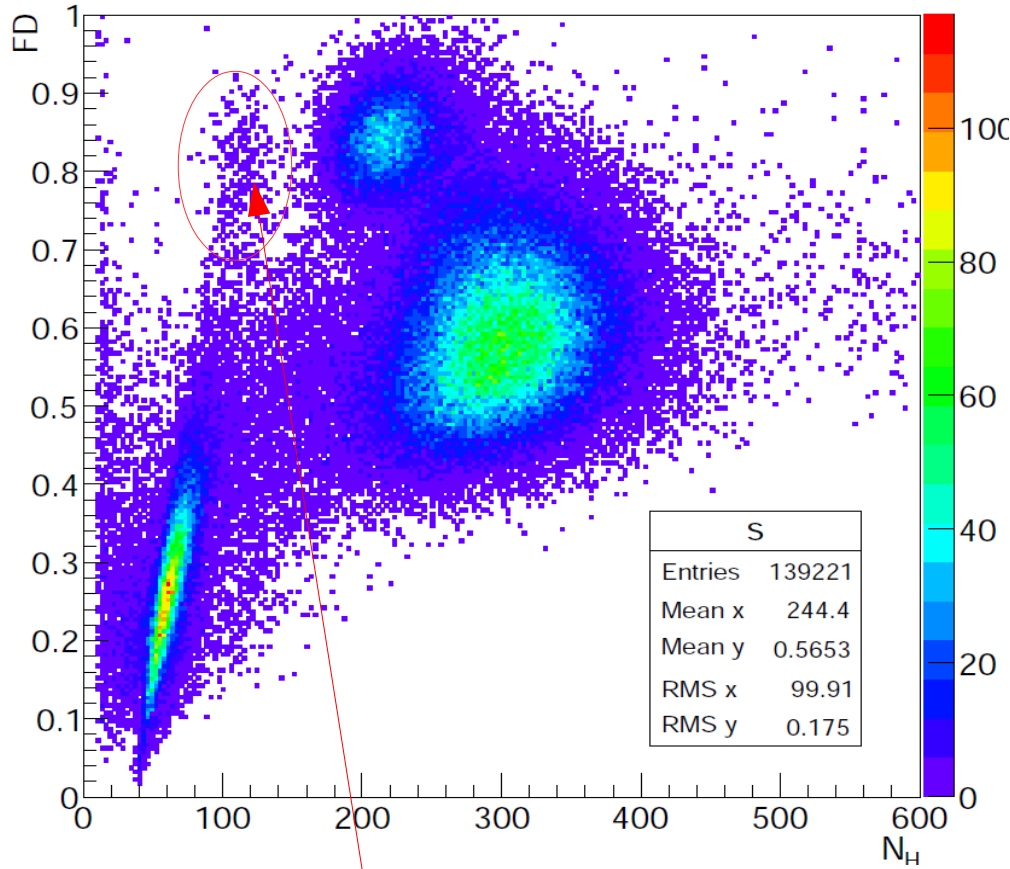
Nhits Vs FD @ 12GeV (600073, 75, 76, 79, 80)

Nhits Vs FD @ 16GeV (600063, 67, 69)

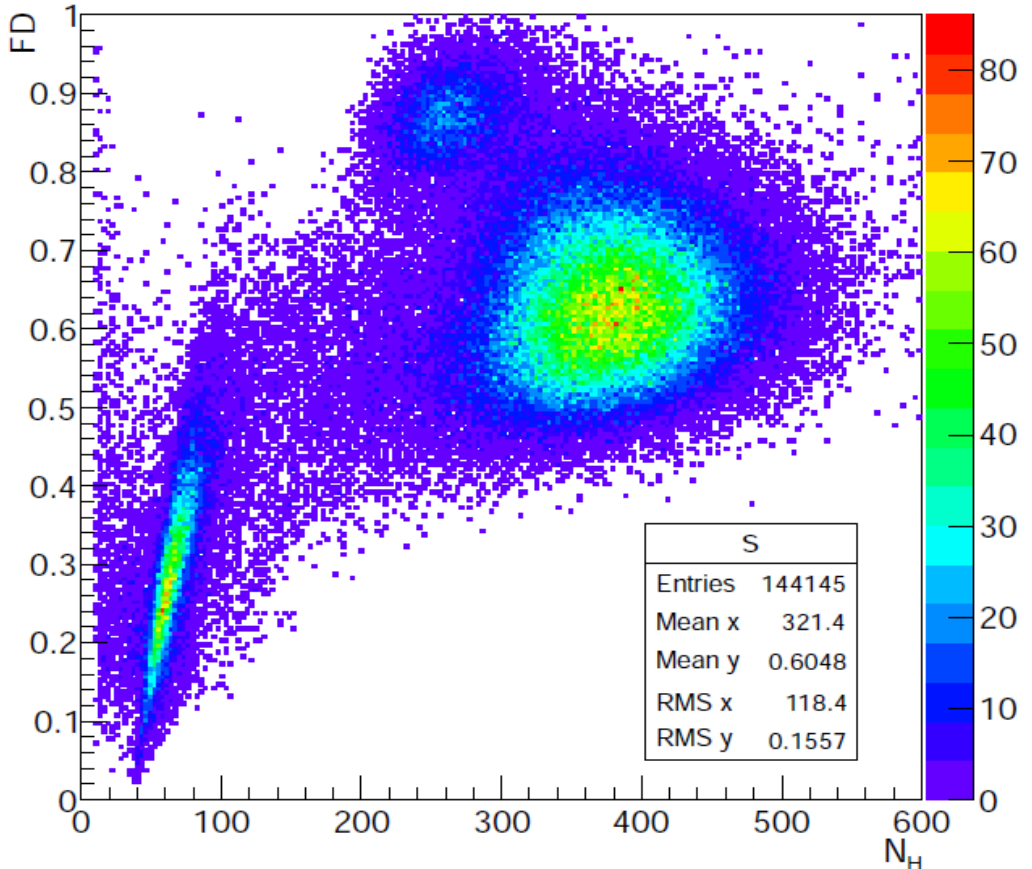


Nhits Vs FD @ 20GeV (600054, 55, 57, 58, 59, 62)

Nhits Vs FD @ 25GeV (600049, 50, 52, 53)

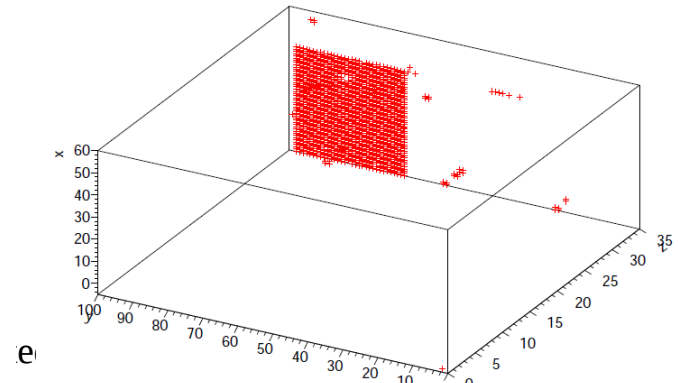
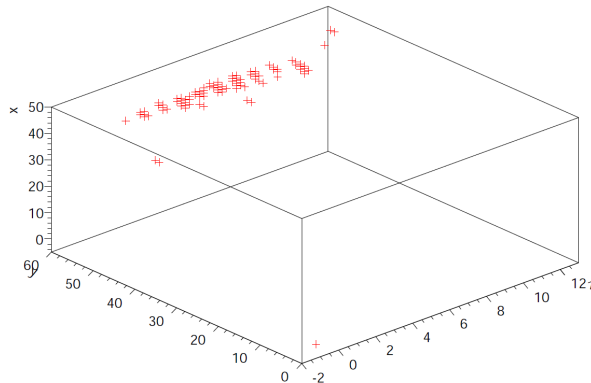


x:y:z {EventNr == 950}

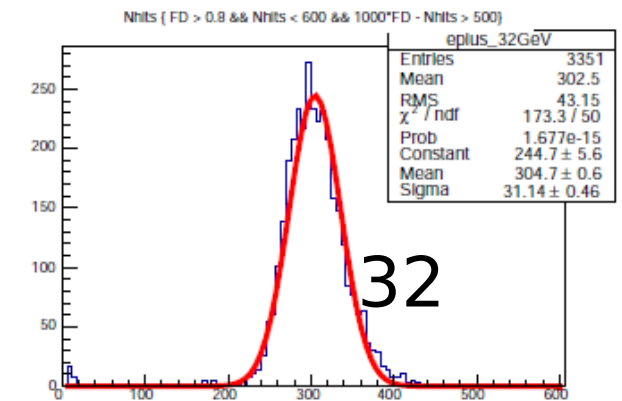
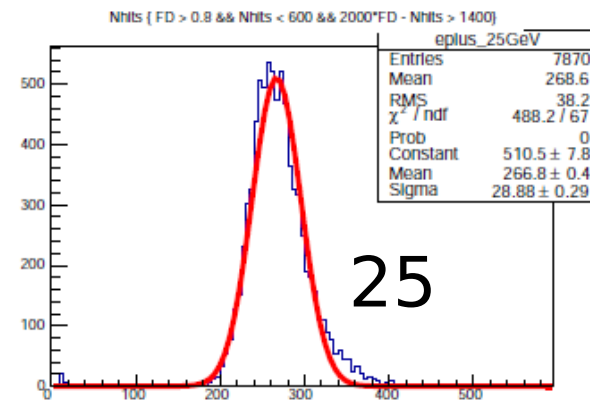
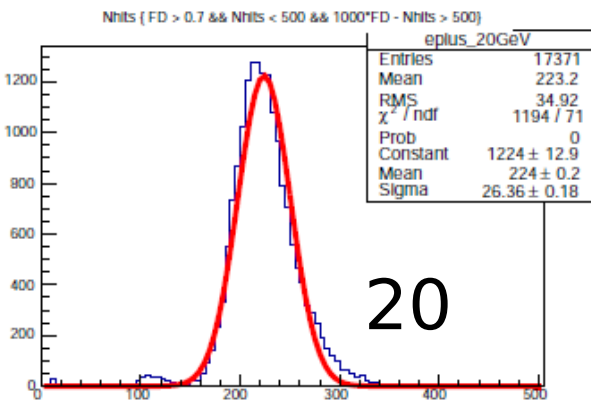
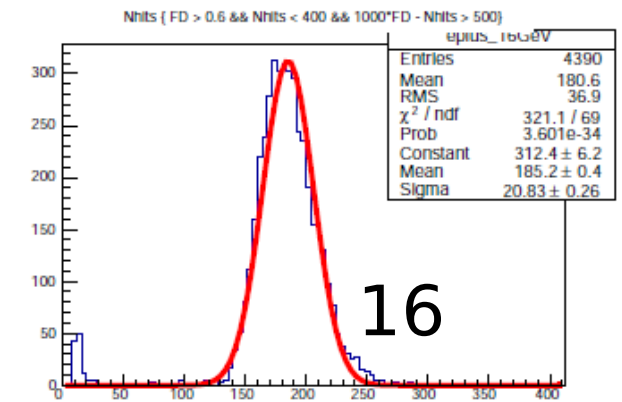
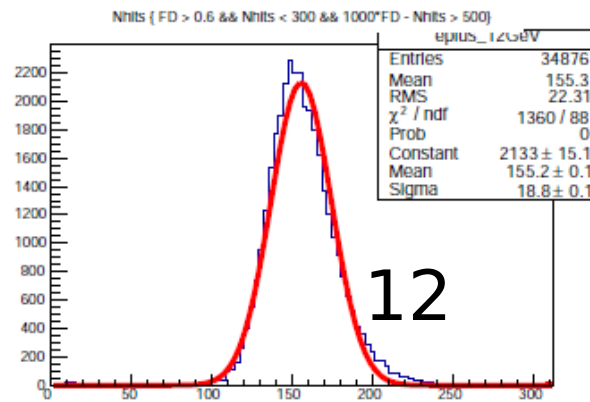
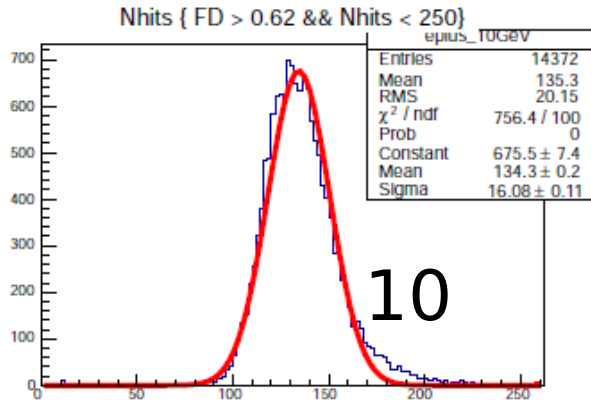
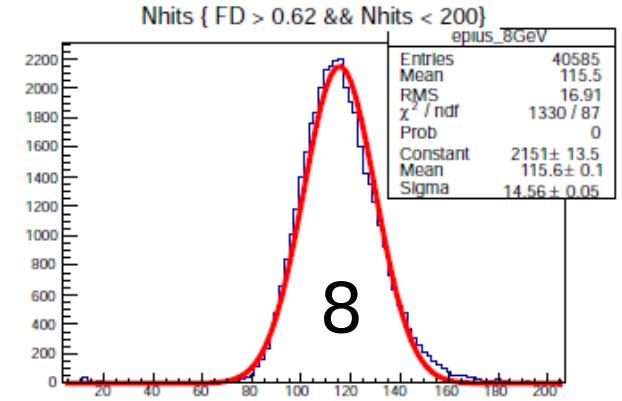
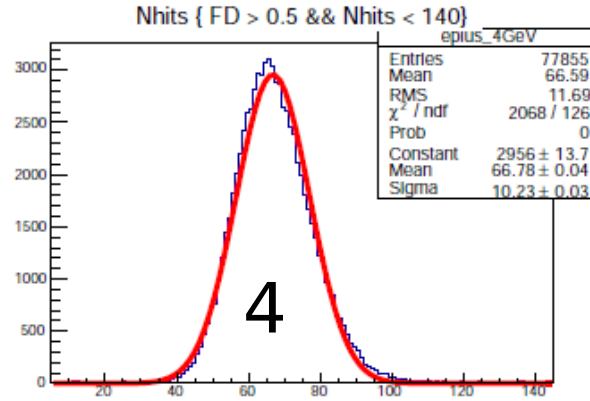
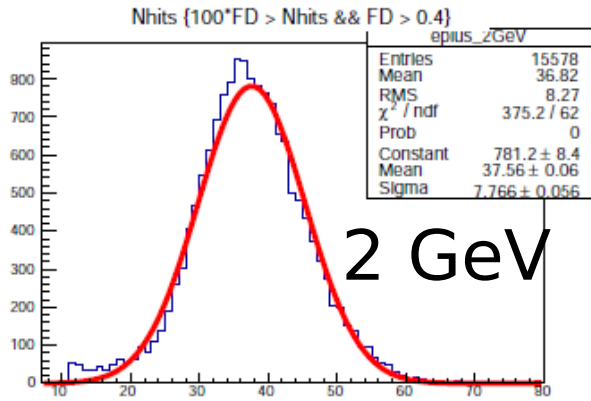


x:y:z {EventNr == 901}

From Run 600057:  
Low energy positron?  
Mip multiplicity?



# Nhits Response: Positron



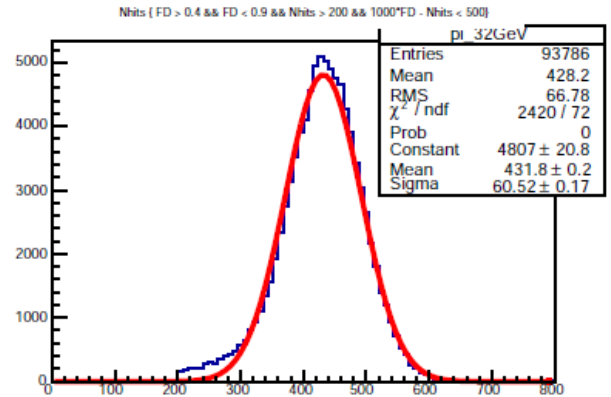
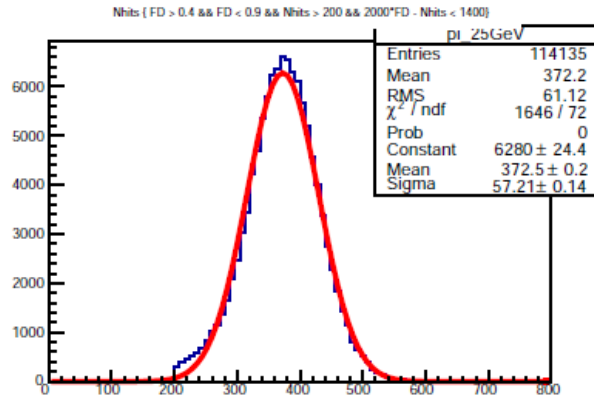
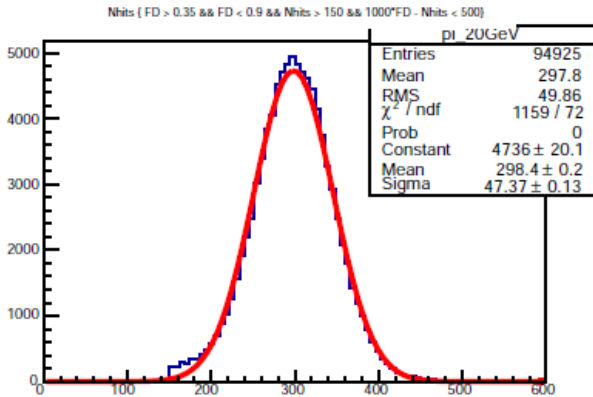
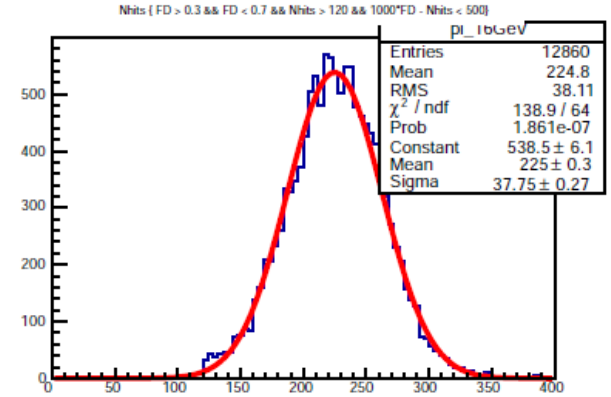
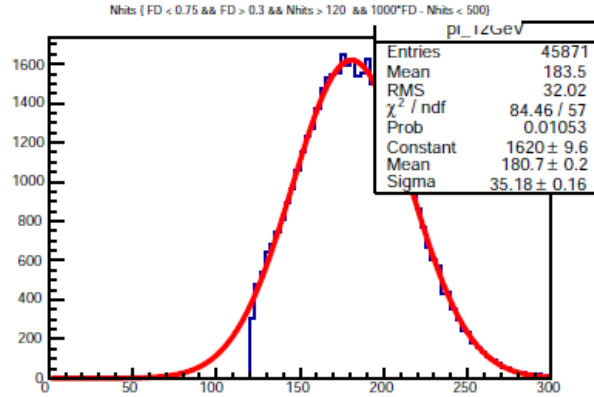
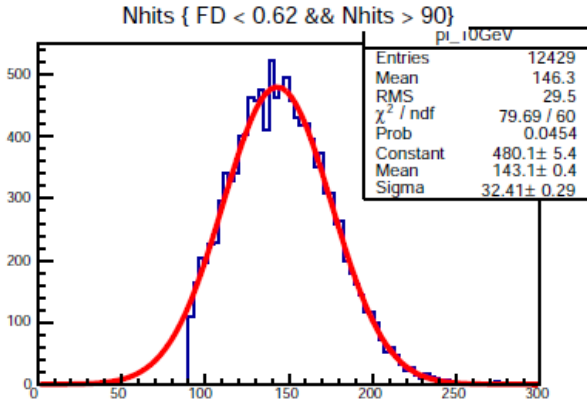
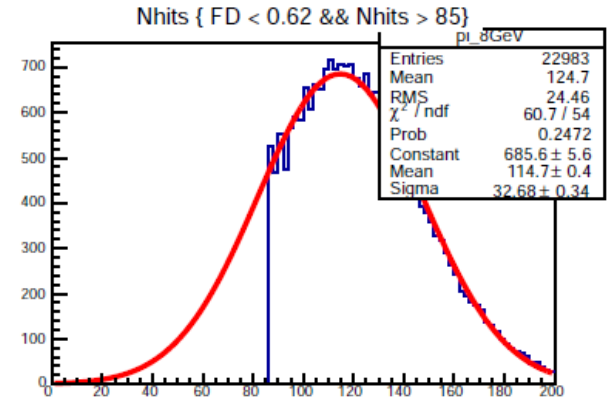


# Nhits Response: Pion

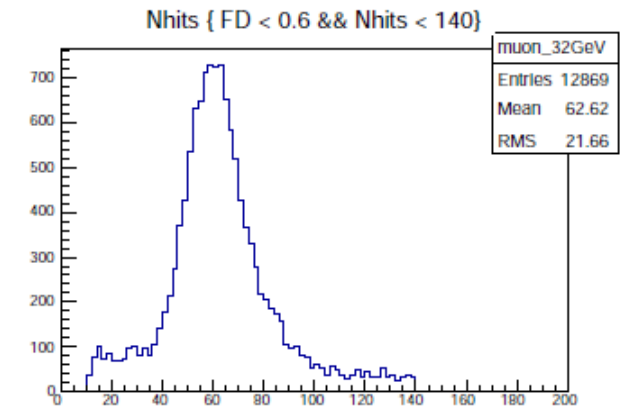
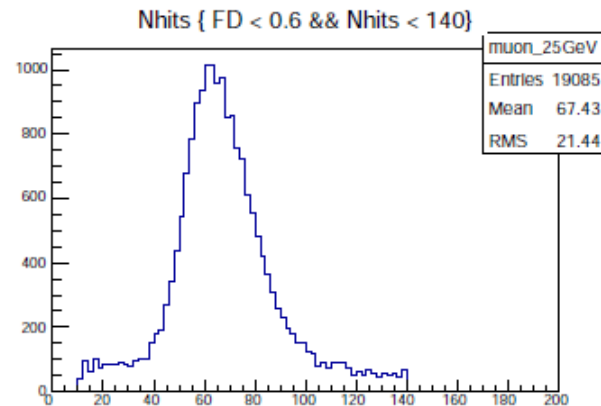
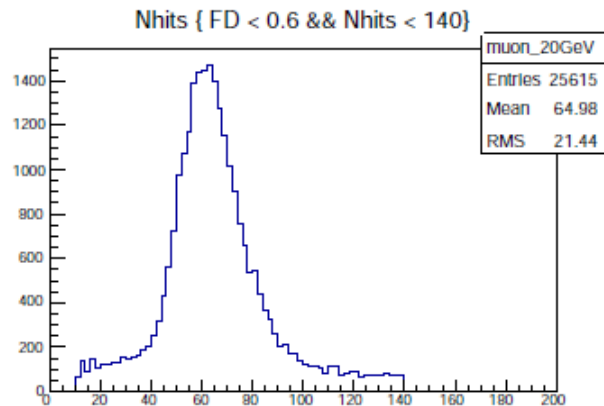
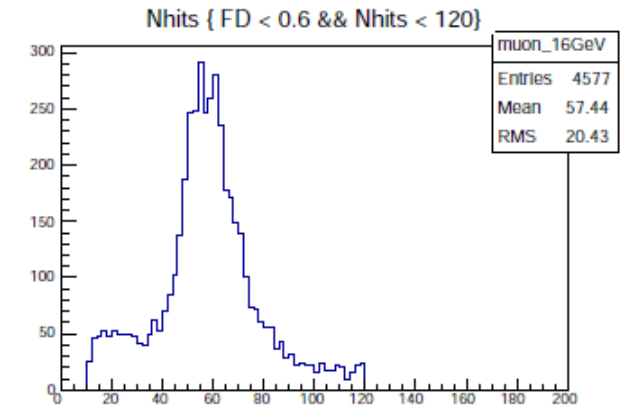
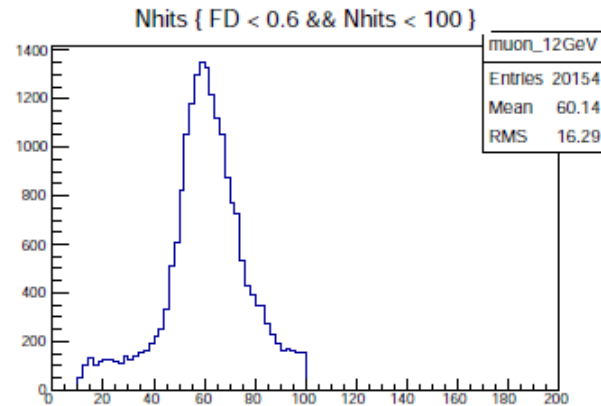
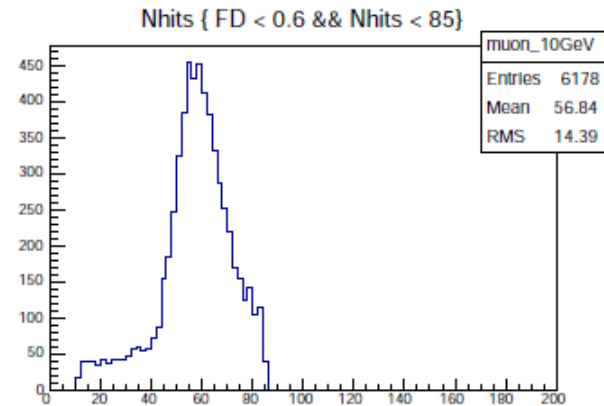
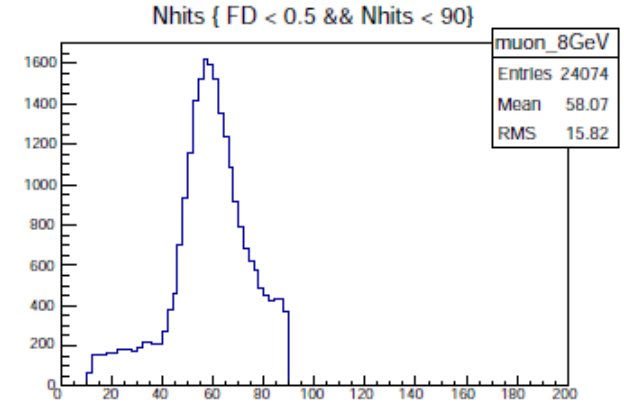
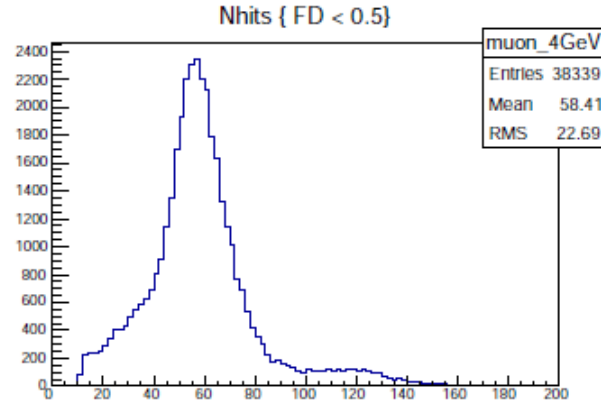
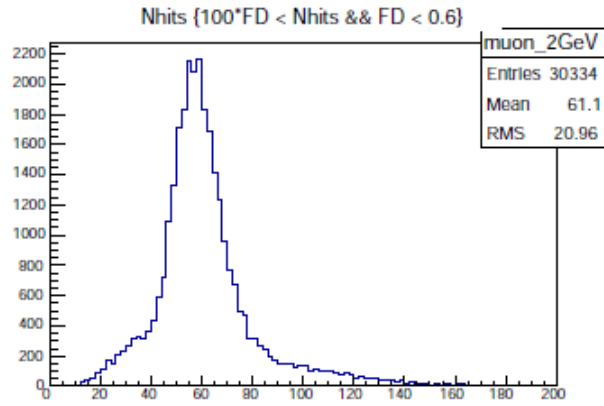


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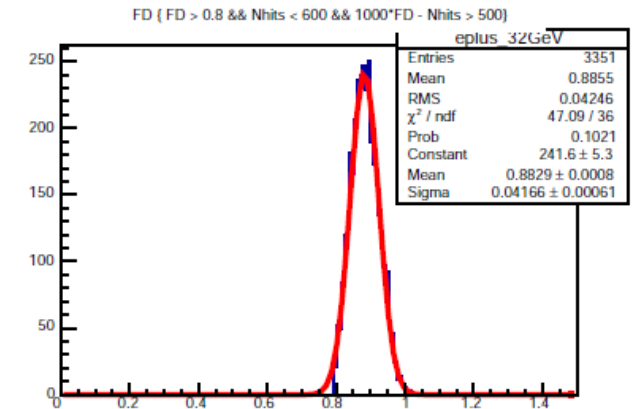
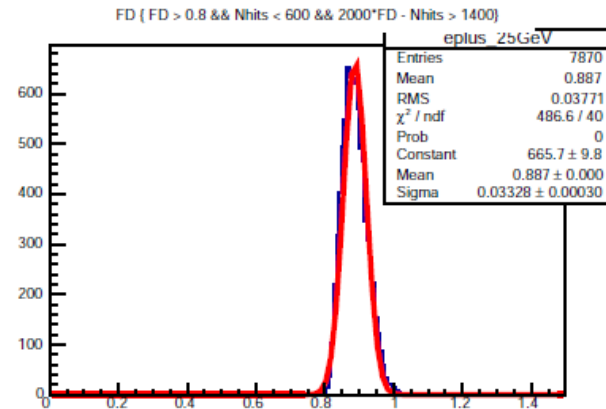
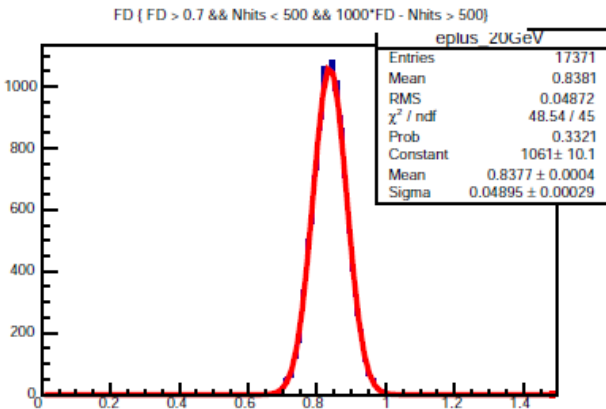
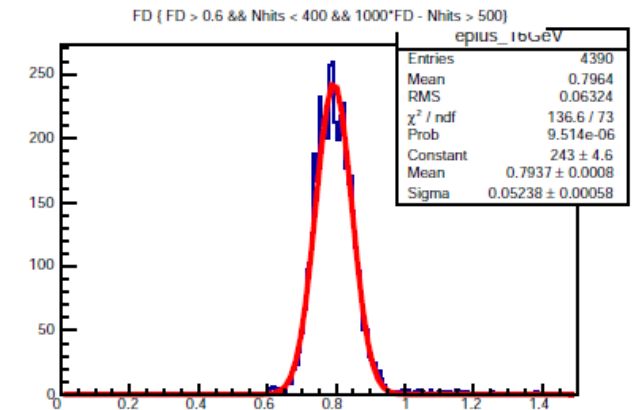
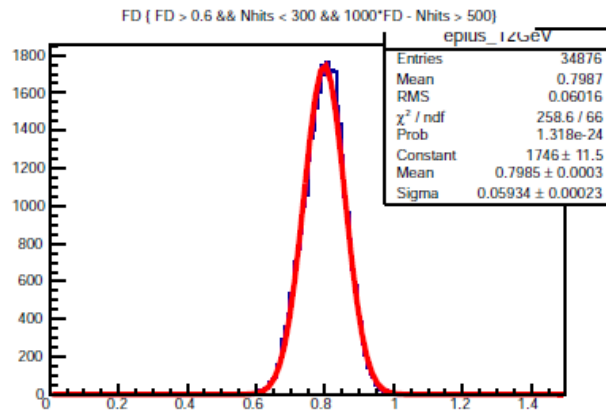
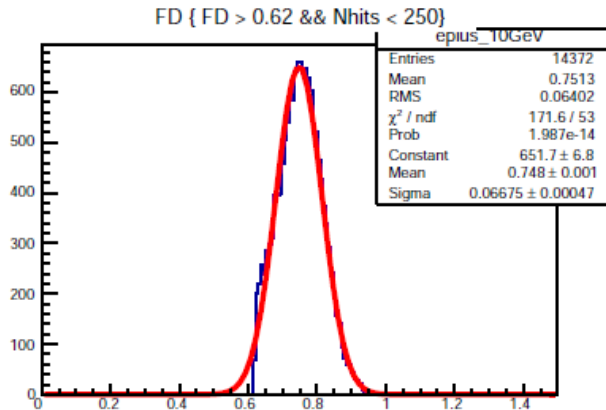
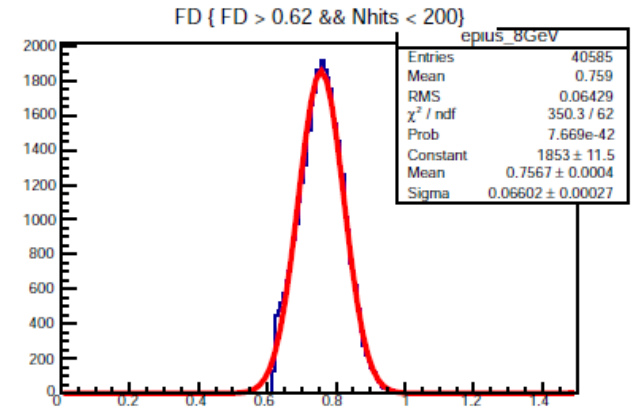
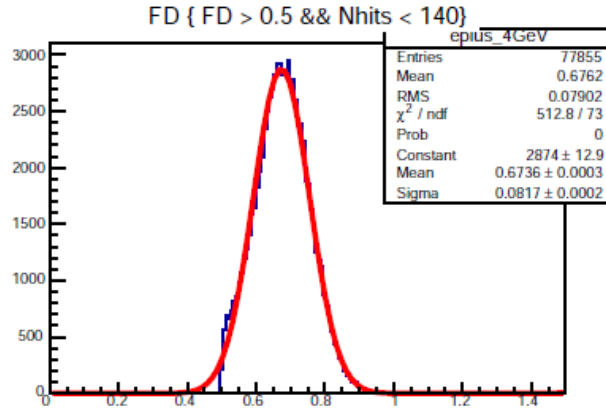
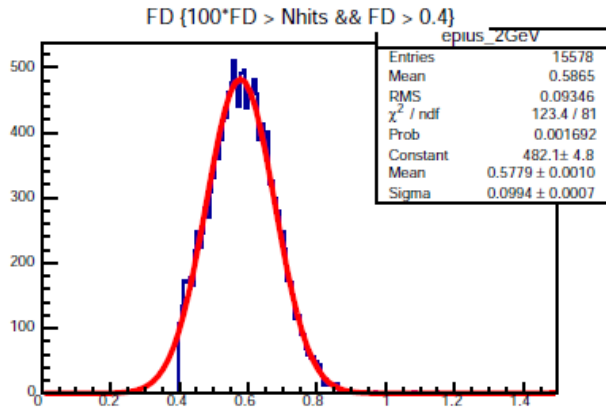
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# Nhits response: Muon



# FD Vs Energy: Positron

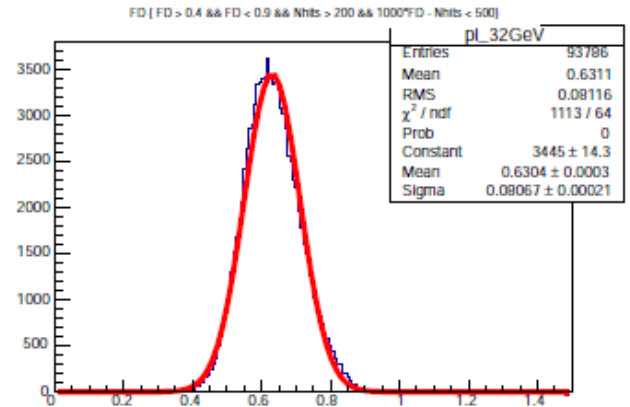
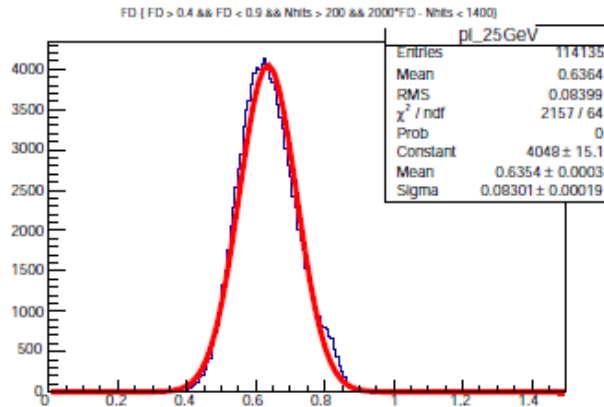
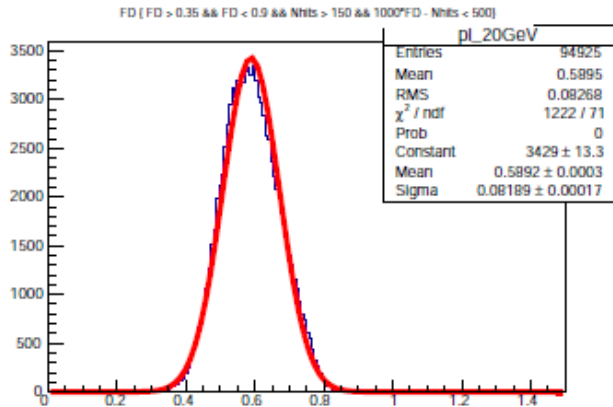
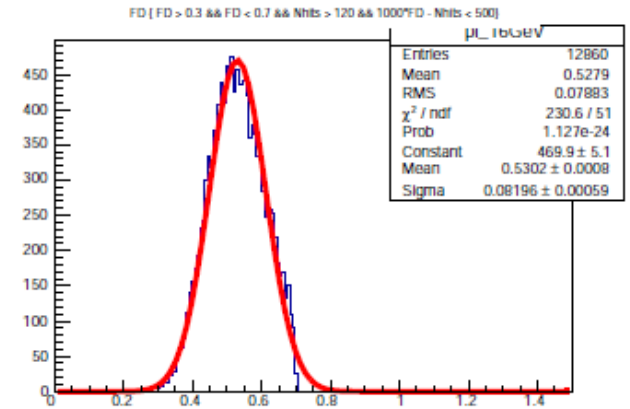
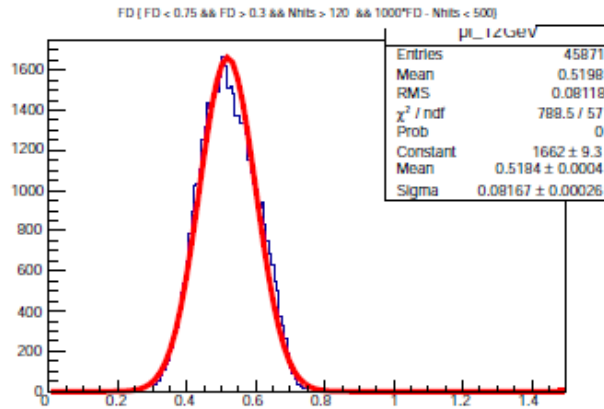
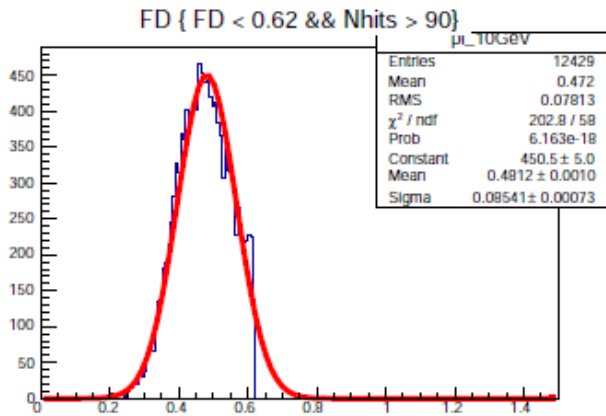
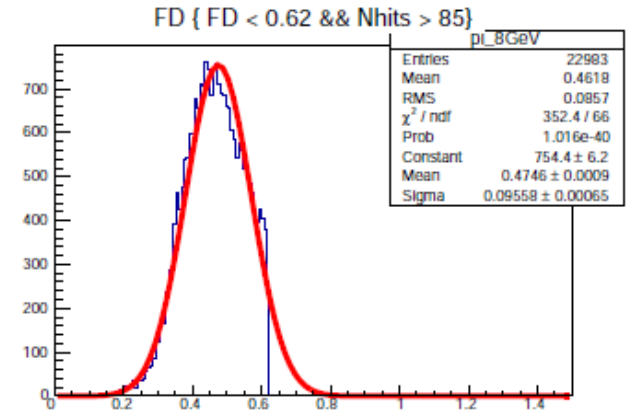


# FD Vs Energy: Pion

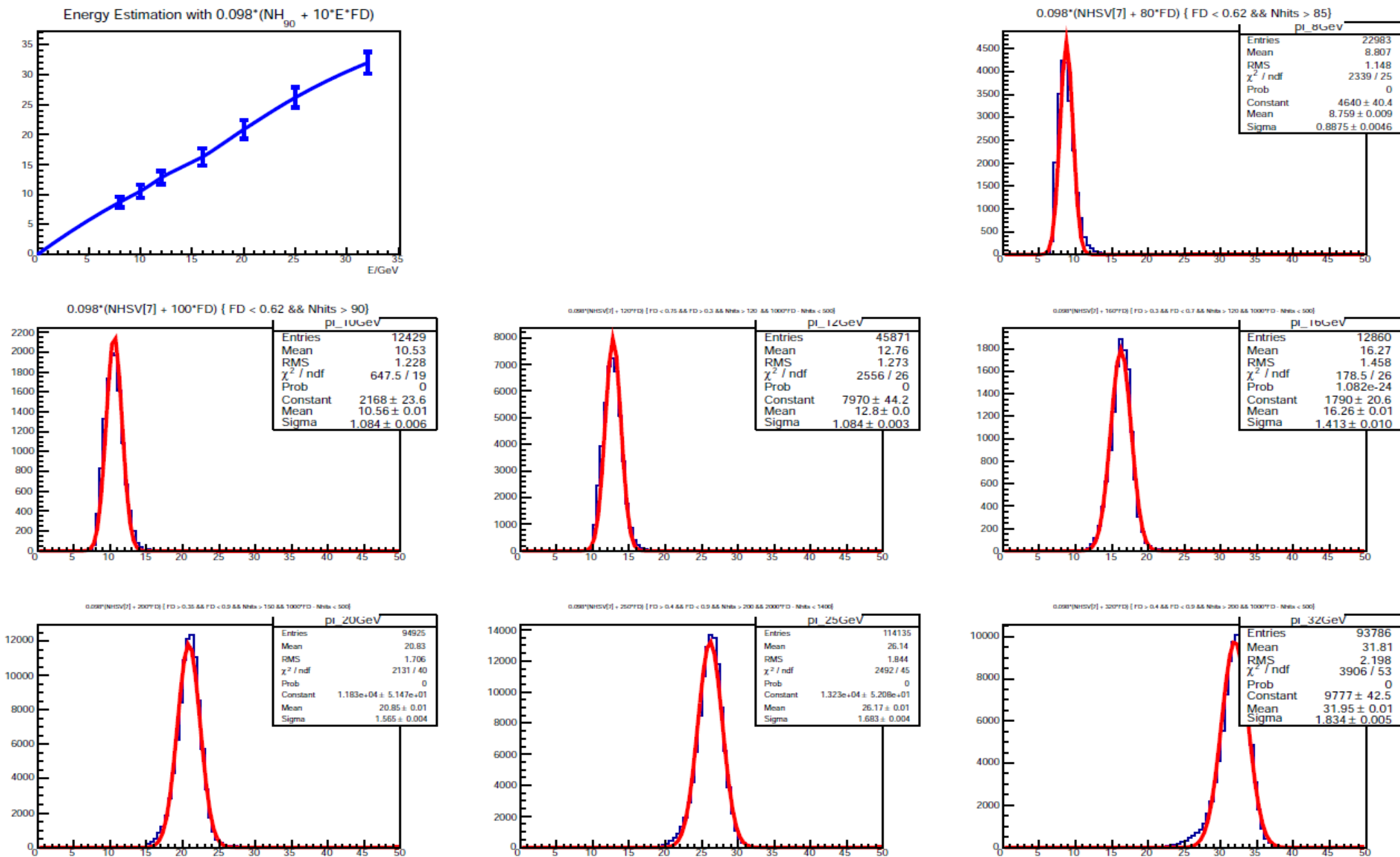


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# Pion Energy Measurement: with known track energy



# Positron Energy Measurement: with known track energy

