Muon Collider Lectures

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Outline

PART I – Introduction and the R&D Effort

- Introduction
  - Why a Muon Collider?
  - Muon Colliders vs Electron-positron Colliders
  - Physics Reach
  - U.S Muon Accelerator Program
- Elements of a Muon Accelerator Facility
- Key R&D Challenges
- Recent Technology Development Highlights

PART II – A Closer Look at Muon Cooling

- Units and Definitions
- Cooling Channel Design
  - Solenoid Focusing
  - Transverse Cooling
  - Longitudinal Cooling
  - Emittance Exchange
- Conclusion

PART I

INTRODUCTION AND THE R&D EFFORT

INTRODUCTION

Why a Muon Collider?

- First – why a lepton collider?
  - In proton (or proton-antiproton) collisions, composite particles (hadrons), made up of quarks and gluons, collide
    - The fundamental interactions that take place are between individual components in the hadrons
    - These components carry only a fraction of the total energy of the particles
    - For p-p collisions, the effective interaction energies are O(10%) of the total center-of-mass (CoM) energy of the colliding protons
    - Thus a 14 TeV CoM energy at the LHC probes an energy scale E < 2 TeV
  - Electrons (and positrons) as well as muons are fundamental particles (leptons)
    - Leptons are point-like particles
    - Their energy and quantum state are well understood during the collision
    - When the leptons and anti-leptons collide, the reaction products probe the full CoM energy
    - Thus a few TeV lepton collider can provide a precision probe of the full energy range of fundamental processes that are discovered at the LHC

Muon ($\mu^+\mu^-$) Colliders vs Electron-Positron Colliders (I)

- Now – why a muon collider?
- s-Channel Production
  - When 2 particles annihilate with the correct quantum numbers to produce a single final state. Examples:

    $e^+e^- \rightarrow Higgs$ **OR** $\mu^+\mu^- \rightarrow Higgs$
  - The cross section for this process scales as m^2 of the colliding particles, so:

$$\sigma\left(\mu^+\mu^- \rightarrow H\right) = \left(\frac{m_\mu}{m_e}\right)^2 \times \sigma\left(e^+e^- \rightarrow H\right) = \left(\frac{105.7\,MeV}{0.511\,MeV}\right)^2 \times \sigma\left(e^+e^- \rightarrow H\right)$$

$$\sigma\left(\mu^+\mu^- \rightarrow H\right) = 4.28 \times 10^4 \sigma\left(e^+e^- \rightarrow H\right)$$

  - Thus a muon collider offers the potential to probe the Higgs resonance directly
    - The luminosity required is not so large
    - A precision scan capability is particularly interesting in the case of a richer Higgs structure (eg, a Higgs doublet)

Muon ($\mu^+\mu^-$) Colliders vs Electron-Positron Colliders (II)

- Synchrotron Radiation
  - In a circular machine, the energy loss per turn due to synchrotron radiation can be written as:

$$\Delta E_{turn} = \left(\frac{4\pi mc^2}{3}\right)\left(\frac{r_0}{\rho}\right)\beta^3\gamma^4$$

where ρ is the bending radius

$$\rho \propto \frac{\beta\gamma}{B} \Rightarrow \boxed{\Delta E_{turn} \propto B\gamma^3}$$

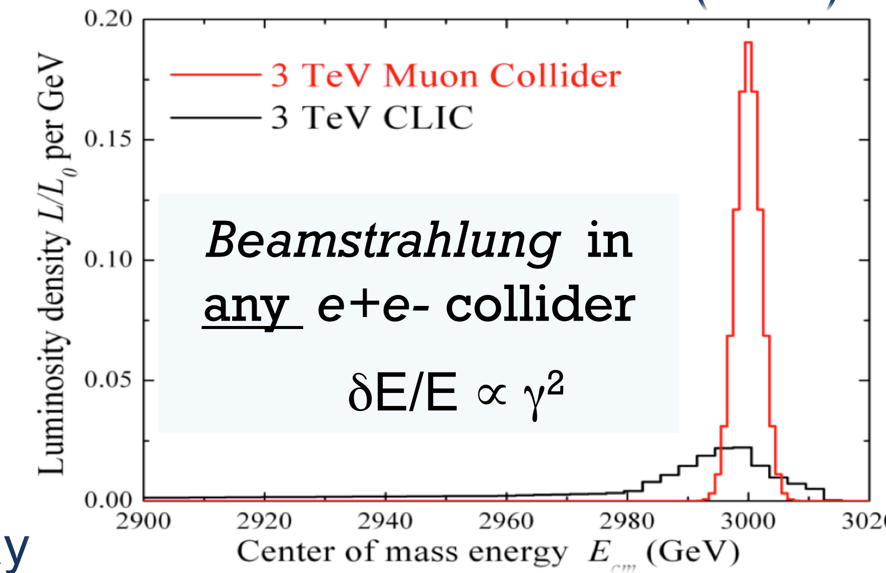
  - If we are interested in reaching the TeV scale, an e^+e^- circular machine is not feasible due to the large energy losses

    Solution 1: e^+e^- linear collider

    Solution 2: Use a heavier lepton – eg, the muon

Muon ($\mu^+\mu^-$) Colliders vs Electron-Positron Colliders (III)

- Beamstrahlung
  - When electrons and positrons collide, the interaction of the particles in one beam with the electromagnetic fields of the other beam results in the radiation of photons (synchrotron radiation) ⇨ *beamstrahlung*
  - This broadens the energy distribution of colliding particles and lowers the fraction of collisions that are near the nominal center-of-mass (CoM) energy
  - The beamstrahlung effect is negligible for a muon collider ⇨ most luminosity is produced near the nominal CoM energy

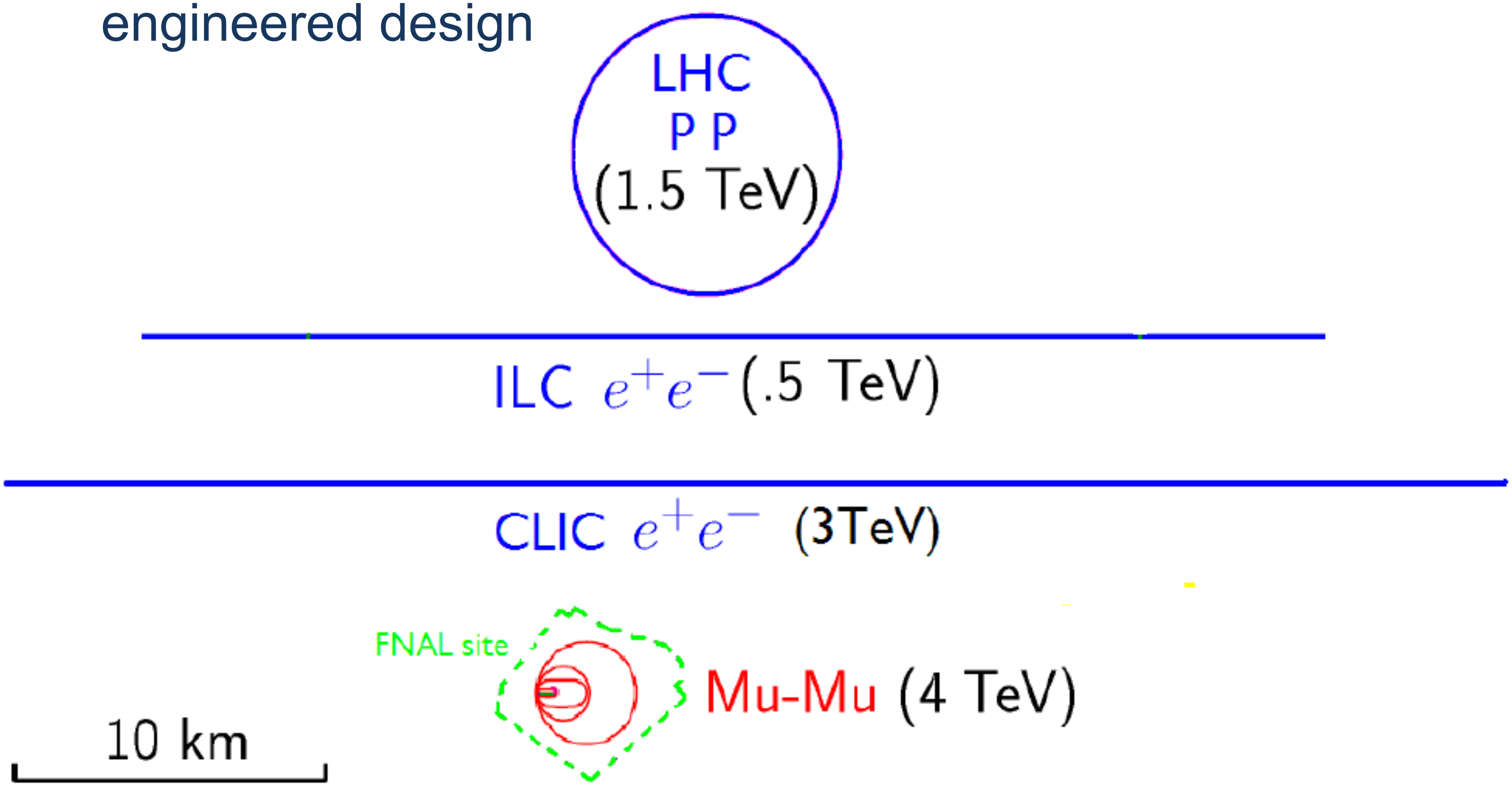


- Implications for a Higgs Factory
  - With negligible beamstrahlung, it may be possible to directly probe the width of the Higgs
  - Expected width of a standard Higgs is ~4.5 MeV
  - 125 GeV muon collider lattices with $\Delta E/E \sim 3\times10^{-5}$ (3.8 MeV) have been designed

Circular Colliders vs Linear Colliders

- Circular machines offer a number of advantages
  - Many crossings at an interaction point
    - Luminosity multiplier
    - For a TeV-scale muon collider, expect to have O(1000) crossings for each bunch
  - Multiple detectors can be used
    - Luminosity multiplier
    - Improved systematics understanding of the detectors
  - The additional integrated luminosity from multiple crossings allows larger transverse emittances than are needed for a linear collider. Machine tolerances become much easier
  - Acceleration can utilize multiple passes through the RF system
  - Overall, the beam and wall power for a circular machine can be significantly less than that for a linear collider

- The footprint of a muon collider can be much smaller than other facilities
  - Provides for a more flexible sight choice
  - Has the potential to provide cost savings in a fully engineered design



Luminosity

- The principle parameter driver is the production of luminosity at a single collision point

$$\mathcal{L} = \frac{N^2 f_{coll}}{4\pi\sigma_x\sigma_y}\mathcal{H}_D$$

Linear Collider Form

- where

  N is the number of particles per bunch (*assumed equal for all bunches*)

  f_{coll} is the overall collision rate at the interaction point (IP)

  σ_x and σ_y are the horizontal and vertical beam sizes (*assumed equal for all bunches*)

  $\mathcal{H}_D$ is the luminosity enhancement factor

- Ideally we want:
  - High intensity bunches
  - High repetition rate
  - Small transverse beam sizes

ILC Parameters at the Interaction Point

- The parameters at the interaction point have been chosen to provide a nominal luminosity of $2\times10^{34}\ \mathrm{cm^{-2}s^{-1}}$. With

  $N = 2\times10^{10}$ particles/bunch

  $\sigma_x \sim 640$ nm $\Leftrightarrow$ $\beta_x^* = 20$ mm, $\varepsilon_x = 20$ pm-rad

  $\sigma_y \sim 5.7$ nm $\Leftrightarrow$ $\beta_y^* = 0.4$ mm, $\varepsilon_y = 0.08$ pm-rad

  $\mathcal{H}_D \sim 1.7$

$$\mathcal{L} = \frac{N^2 f_{coll}}{4\pi\sigma_x\sigma_y}\mathcal{H}_D = \left(1.4\times10^{30}\,cm^{-2}\right)\times f_{coll}$$

- In order to achieve the desired luminosity, an average collision rate of ~14kHz is required (we will return to this parameter shortly). The beam sizes at the IP are determined by the strength of the final focus magnets and the emittance, phase space volume, of the incoming bunches.

- A number of issues impact the choice of the final focus parameters. For example, the beam-beam interaction as two bunches pass through each other can enhance the luminosity, however, it also disrupts the bunches. If the beams are too badly disrupted, safely transporting them out of the detector to the beam dumps becomes quite difficult. Another effect is that of beamstrahlung which leads to significant energy losses by the particles in the bunches and can lead to unacceptable detector backgrounds. Thus the above parameter choices represent a complicated optimization.

Muon Collider Luminosity

- For a muon collider, we can write the luminosity as:

$$\mathcal{L} = \frac{N^2 f_{coll}}{4\pi\sigma_x\sigma_y} = \frac{N^2 n_{turns} f_{bunch}}{4\pi\sigma_\perp^2}$$

- For the 1.5 TeV muon collider design, we hav
  - N = 2×10^{12} particles/bunch
  - $\sigma_{x,y}$ ~ 4.18 μm ⇔ β^* = 10 mm, $\varepsilon_{x,y}(norm)$ = 25 μm-rad
  - n_{turns}~1000
  - f_{bunch}=15 Hz

$$\mathcal{L} \approx \frac{N^2 n_{turns} f_{bunch}}{4\pi\sigma_\perp^2} \approx 2.7\times10^{34}\,cm^{-2}s^{-1}$$

- But this is optimistic since we've assumed N is constant for ~1000 turns when it's actually decreasing. The anticipated luminosity for this case is $\sim1\times10^{34}\,cm^{-2}s^{-1}$.

Challenges for a $\mu^+\mu^-$ Collider

- Make muons from the decay of pions
- With pions made from protons on a target
- To avoid excessive proton power, we must capture a large fraction of pions made

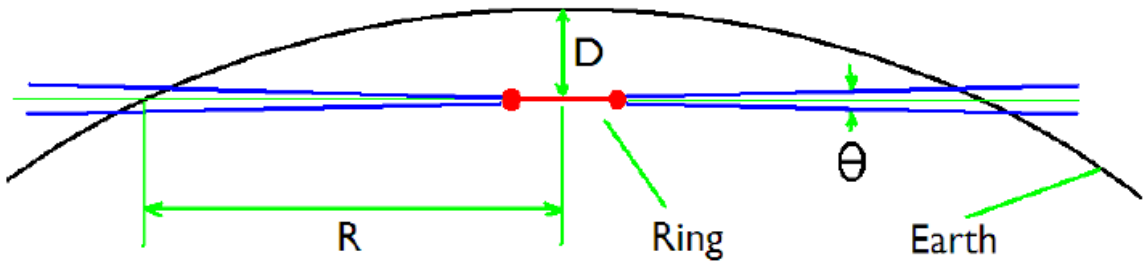
- Capture both forward and backward decays and loses polarization
- The phase space of the pions is now very large:
  - a transverse emittance of 20 pi mm and
  - a longitudinal emittance of 2 pi m
- Emittances must be somehow be cooled by a factor $\approx 10^7$!
  - $\approx$ 1000 in each transverse direction and
  - $\approx$ 40 in longitudinal direction

Cooling Options

- Electrons are typically cooled (damped) by synchrotron radiation but muons radiate too little ($\Delta E \propto 1/m^3$)
- Protons are typically cooled by:
  - a co-moving cold electron beam too slow
  - Or by stochastic methods too slow

- Ionization cooling is probably the only hope
- Although optical stochastic cooling has been studied does not look good

Neutrino Radiation



$$\text{Radiation} \;\propto\; \frac{E_\mu \; I_\mu \; \sigma_\nu}{\theta \; R^2} \;\propto\; \frac{I_\mu \; \gamma^3}{D}$$

$$\text{Radiation} \;\propto\; \frac{\mathcal{L} \; \beta_\perp}{\Delta\nu \;\; < \mathrm{B} >} \; \frac{\gamma^2}{D} \tag{6}$$

For fixed $\Delta\nu$, $\beta_\perp$ and $< B >$; and $\mathcal{L} \propto \gamma^2$:

$$\text{Radiation} \;\propto\; \frac{\beta_\perp}{\Delta\nu \;\; < B > \;\; D} \, \gamma^4 \tag{7}$$

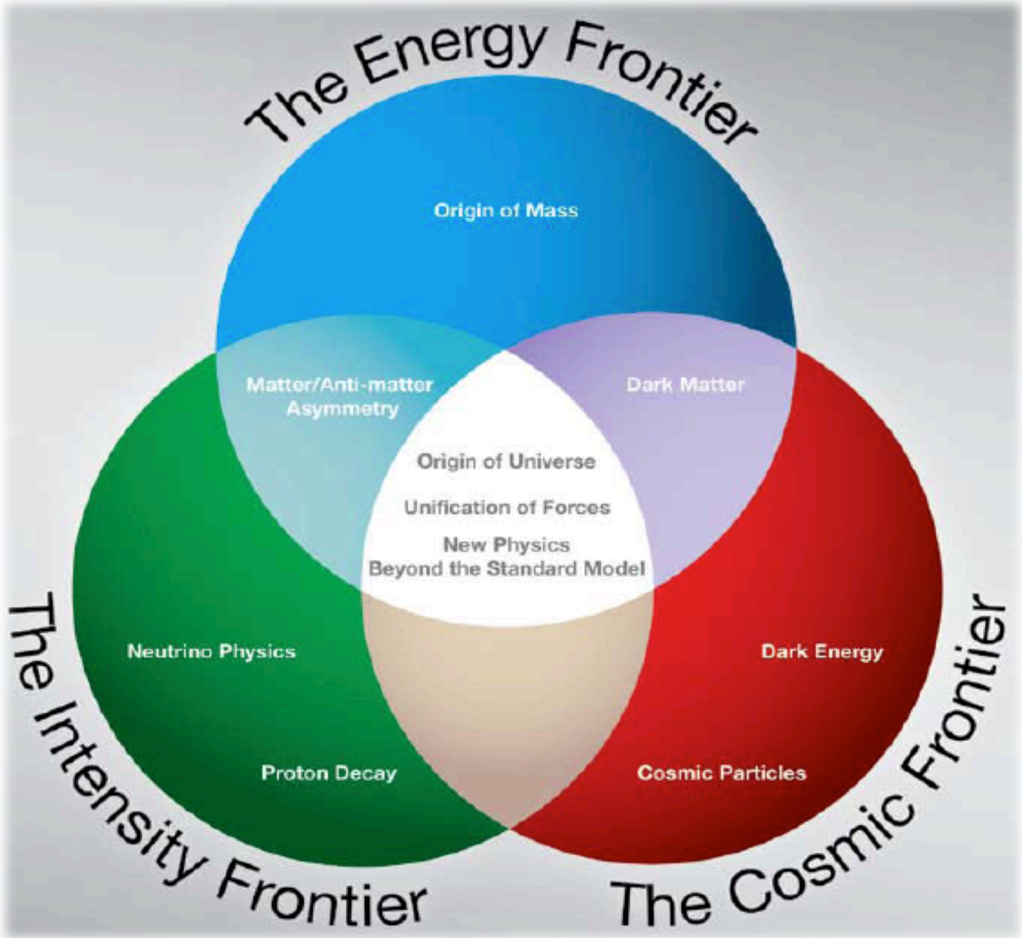
For 3 TeV: D=135 m R=40 Km $\beta_\perp$=5 mm

The Physics Reach of a Muon Accelerators

High intensity μ beams have the potential to enable key measurements on 2 frontiers:

The Intensity Frontier:
with well-characterized ν beams for precise, high sensitivity studies

The Energy Frontier:
with colliders capable of reaching the multi-TeV scale



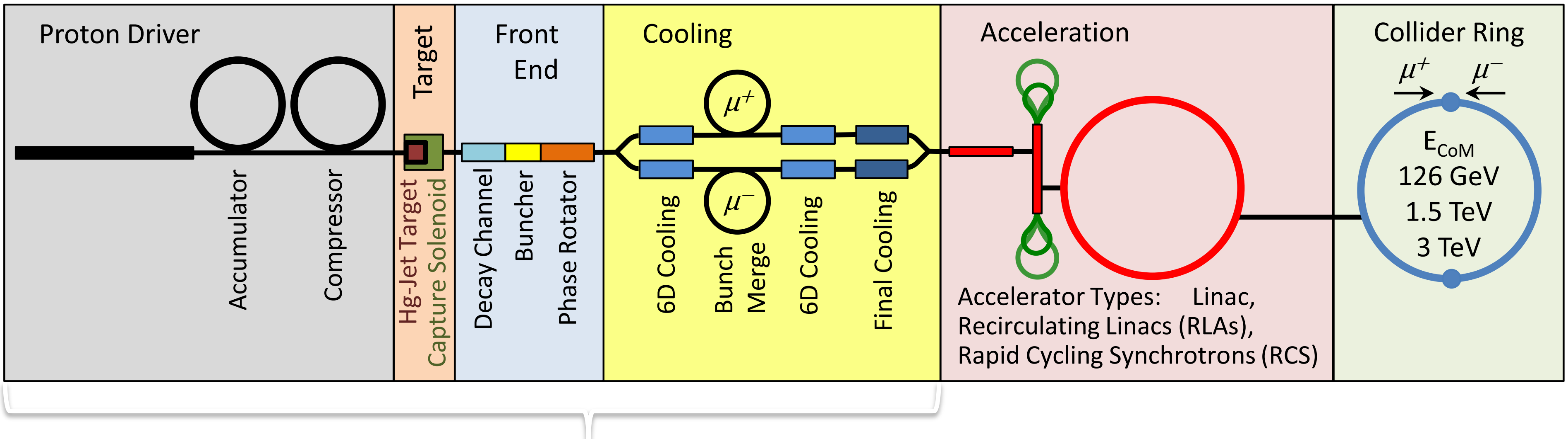
Priorities of the U.S. Muon Accelerator Program

1. Demonstrate the feasibility of key concepts that would allow us to build a multi-TeV collider
2. Continue to develop the critical elements of the NF and MC designs
3. Support the ongoing accelerator R&D and concept demonstration program
4. Establish close coordination with the detector and HEP experimental community
5. As able, continue to support fundamental technical development in the field that has the potential to contribute significantly to the machine design

Overarching goal during this phase of the program is to ***Establish Conceptual Feasibility***

ELEMENTS OF A MUON ACCELERATOR FACILITY

Muon Collider Block Diagram



Proton source:
For example Fermilab's PROJECT X at 4 MW, with 2±1 ns long bunches

Goal:
O(10^{21}) muons/year within the acceptance of an accelerator

Collider: $\sqrt{s}$ = 3 TeV
Circumference = 4.5km
$L = 3\times10^{34}$ cm^{-2}s^{-1}
μ/bunch = 2×10^{12}
$\sigma(p)/p$ = 0.1%
$\varepsilon_{\perp N}$ = 25 μm, $\varepsilon_{//N}$=72 mm
β^* = 5mm
Rep. Rate = 12 Hz

R.B,Palmer

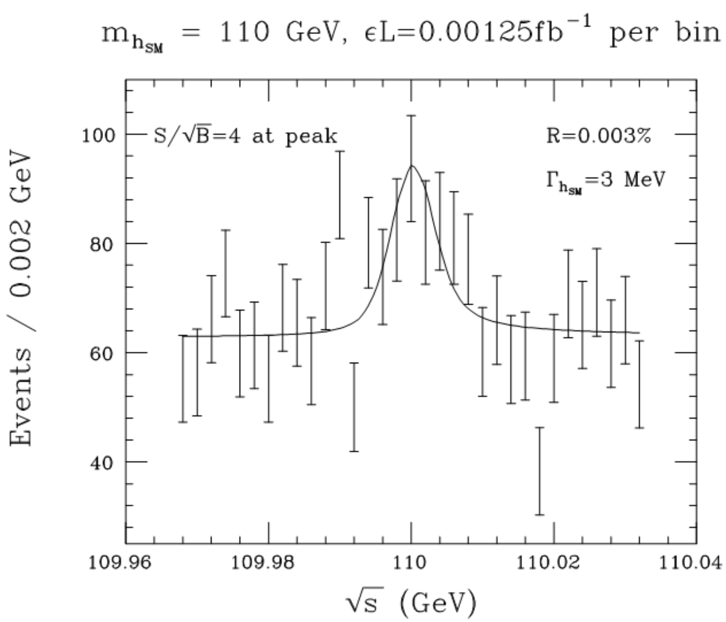
Muon Collider parameters

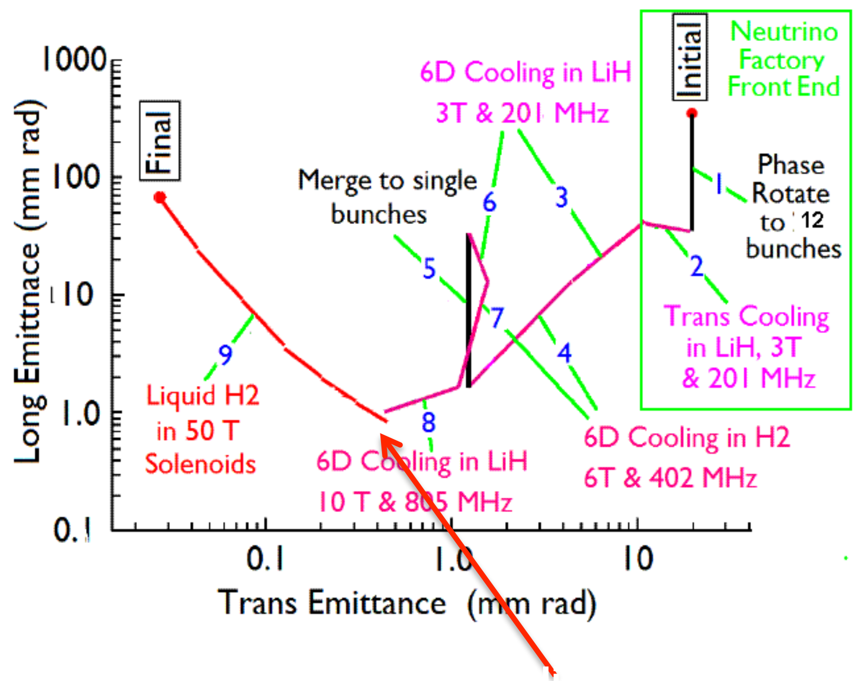
Depends on cooling scenario: $10^{31} \Rightarrow \sim 10^{32}$ cm^{-2}sec^{-1}

	Higgs[1]	Design	Design	Extrap[2]	
C of m Energy	0.126	1.5	3	6	TeV
Luminosity	0.002	1	4	12	10^{34} cm^{-2}sec^{-1}
Muons/bunch	2	2	2	2	10^{12}
Total muon Power	1.2	7.2	11.5	11.5	MW
Ring circumference	0.3	2.6	4.5	6	km
β^* at IP = σ_z	80	10	5	2.5	mm
rms momentum spread	0.004	0.1	0.1	0.1	%
Repetition Rate	30	15	12	6	Hz
Proton Driver power	4	4	3.2	1.6	MW
Muon Trans Emittance	300	25	25	25	μm
Muon Long Emittance	2	72	72	72	mm

126 GeV Higgs Factory (D.Neuffer)

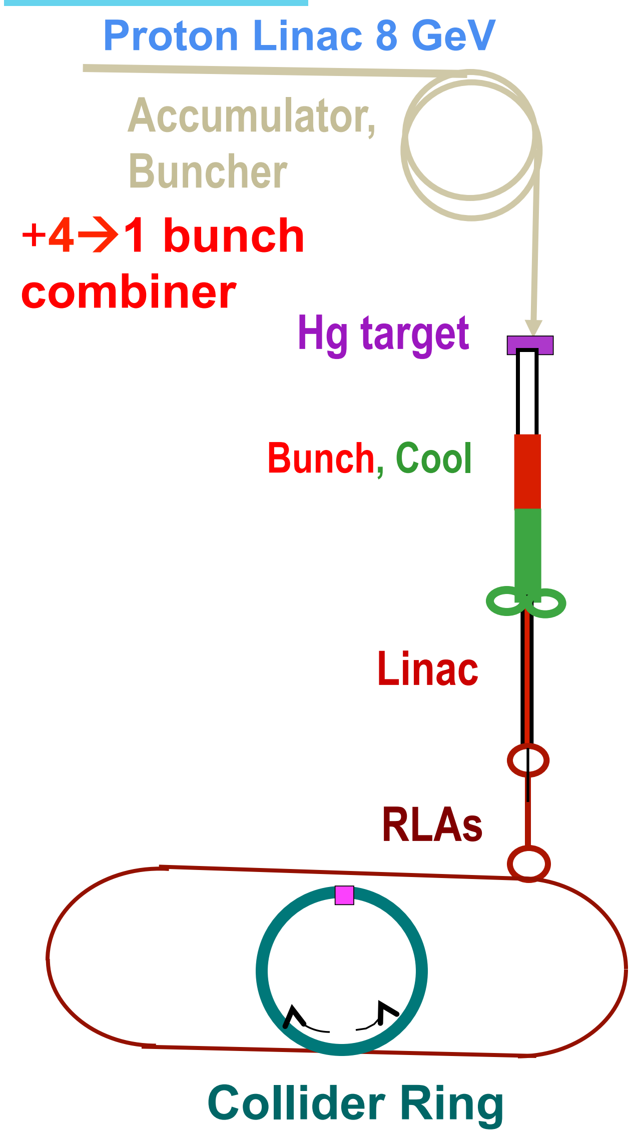
s-channel coupling of Muons to HIGGS with high cross sections:
Muon Collider of with L = 10^{32} $cm^{-2}s^{-1}$ @ 63 GeV/beam (50000 Higgs/year)
Competitive with e+/e- Linear Collider with L = 2. 10^{34} $cm^{-2}s^{-1}$ @ 126 GeV/beam





D.Neuffer
AAC12
NuFACT12

Higgs MC Parameters

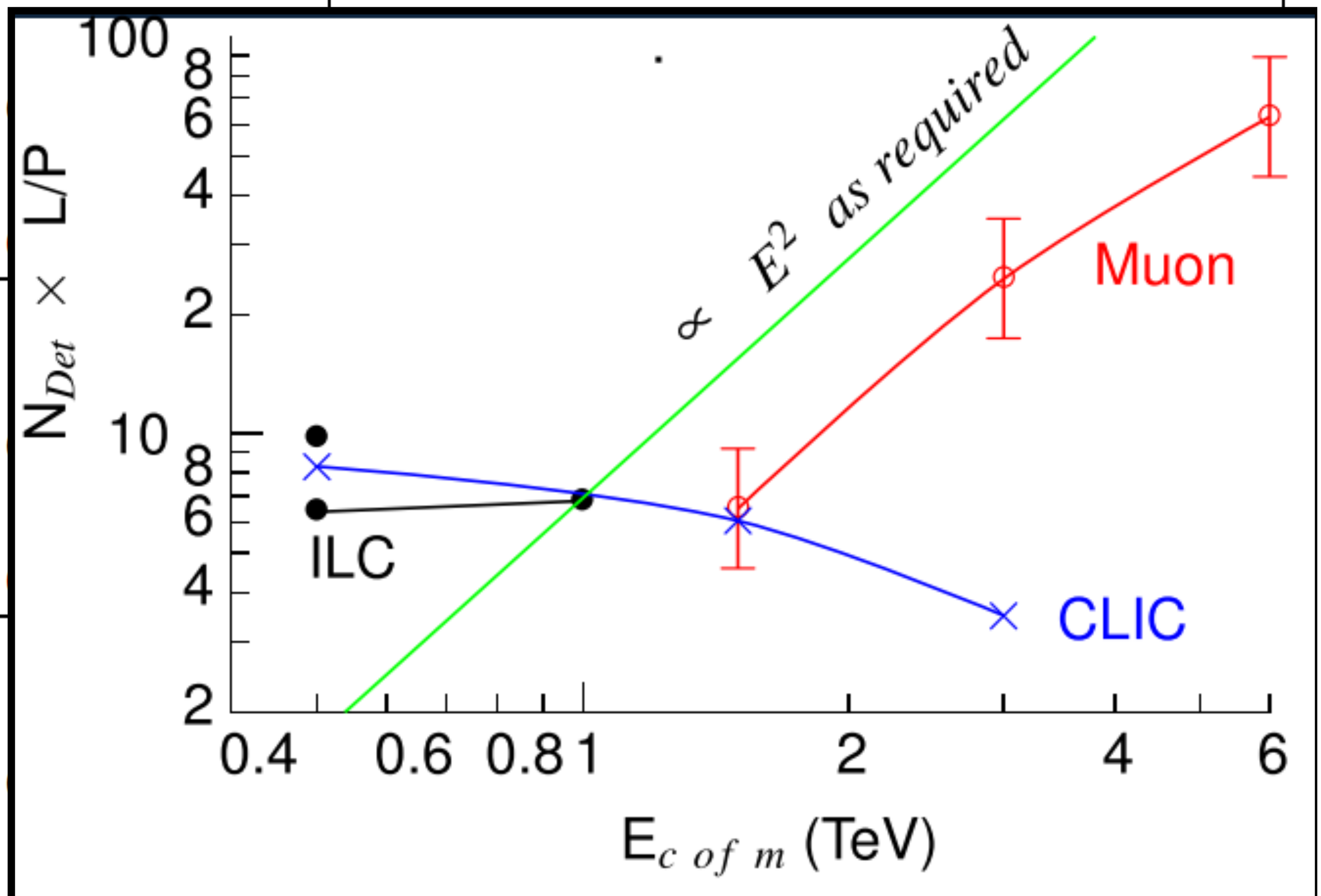


Parameter	Symbol	Value
Proton Beam Power	P_p	4 MW
Bunch frequency	F_p	15 Hz
Protons per bunch	N_p	$4\times5\times10^{13}$
Proton beam energy	E_p	8 GeV
Number of muon bunches	n_B	1
$\mu^{+/-}$/ bunch	N_μ	5×10^{12}
Transverse emittance	$\varepsilon_{t,N}$	0.0002m
Collision β^*	β^*	0.05m
Collision β_{max}	β^*	1000m
Beam size at collision	$\sigma_{x,y}$	0.02cm
Beam size (arcs)	$\sigma_{x,y}$	0.3cm
Beam size IR quad	σ_{max}	4cm
Collision Beam Energy	$E_{\mu+},E_{\mu\_}$	62.5(125geV total)
Storage turns	N_t	1000
Luminosity	L_0	10^{32}

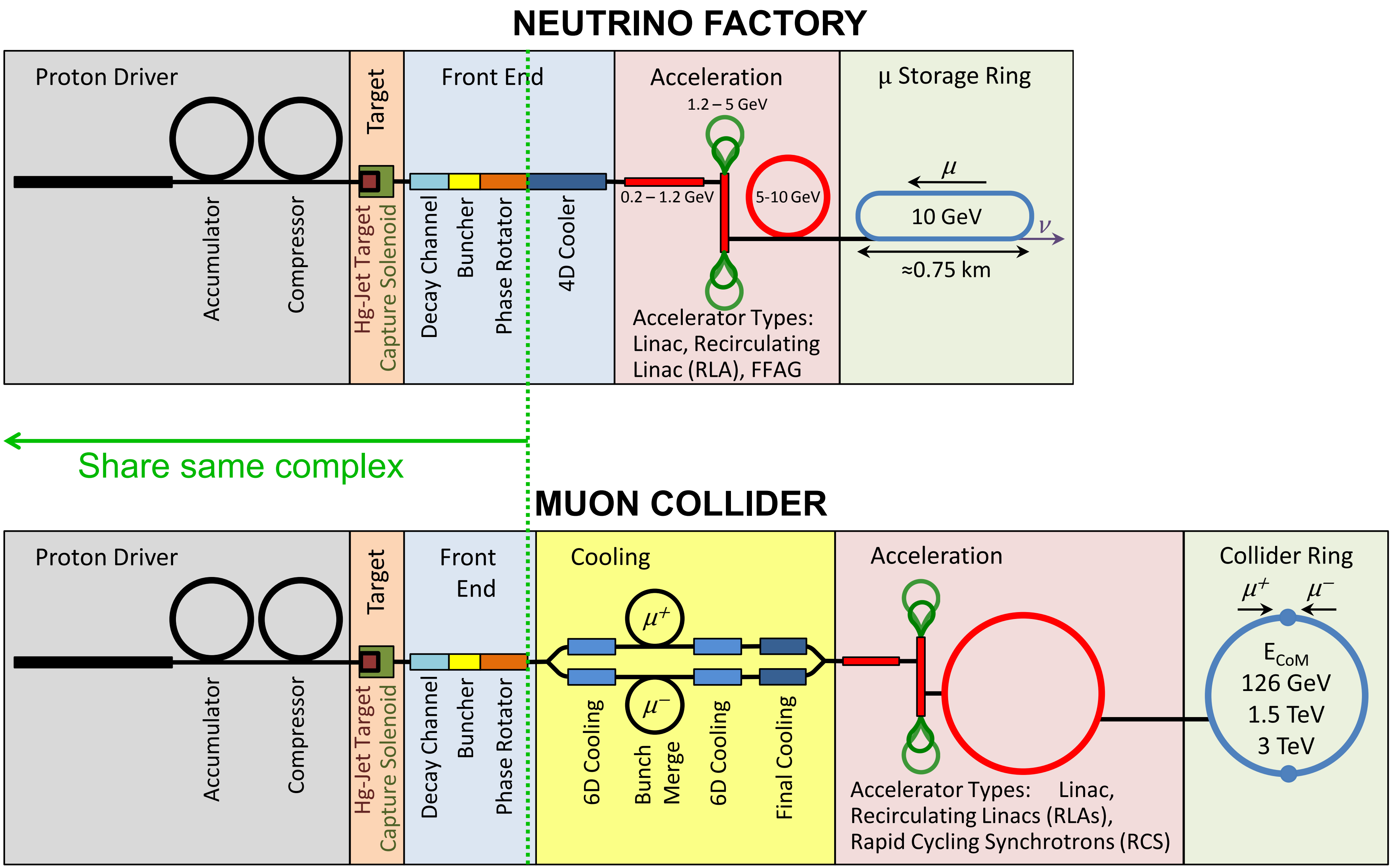
Luminosity Production Metric vs E_{CM}

R. Palmer
MAP DOE Review 2012

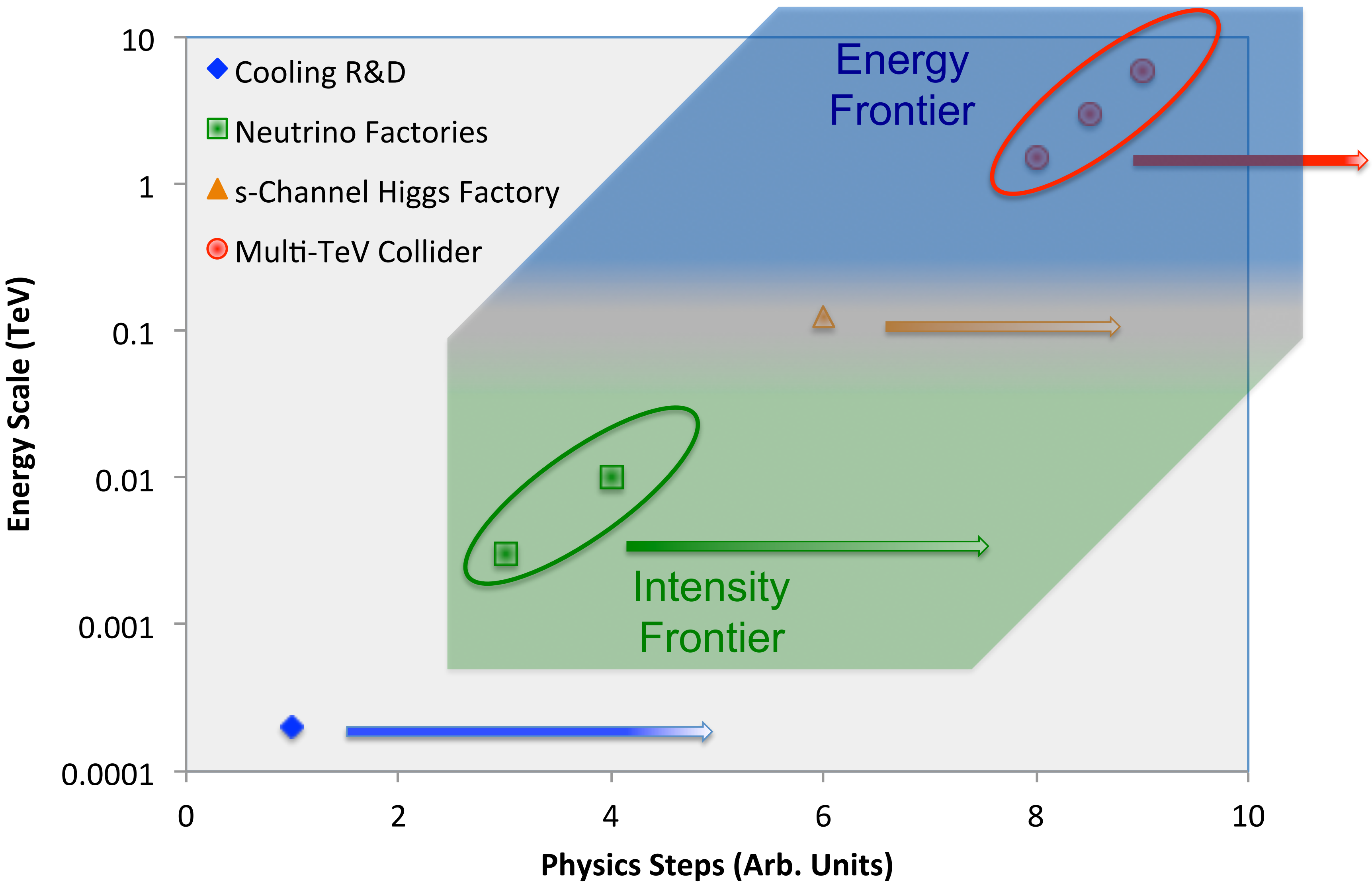
$E_{c\ of\ m}$	TeV	0.5	1.0	1.5	3.0	6.0
ILC		BL HL	HE			
Detectors N_{Det}						
Luminosity	10^{34}					
Wall Power	MW					
$N_{Det} \times L/P$	10^{34}					
CLIC						
Detectors N_{Det}						
Luminosity	10^{34}					
Wall Power	MW					
$N_{Det} \times L/P$	10^{34}					
Muon						
Detectors N_{Det}						
Luminosity	10^{34}					
Wall Power	MW					
$N_{Det} \times L/P$	10^{34} cm^{-2}s^{-1}GW^{-1}			9.2	35	89



Muon Collider - Neutrino Factory Comparison



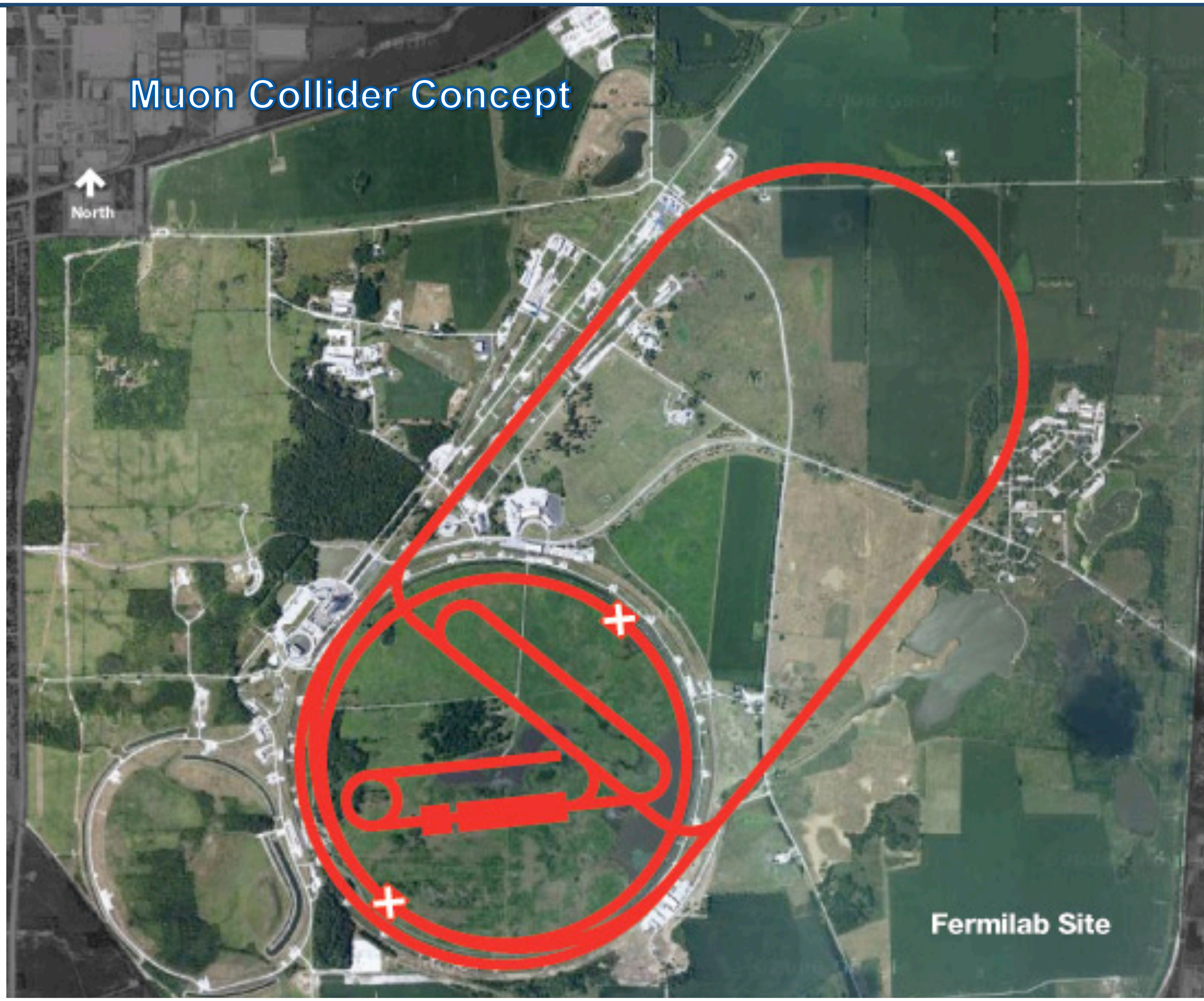
Muon Accelerator Physics Scope



Accelerator	Energy Scale
Cooling Channel	**~200 MeV**
MICE	*160-240 MeV* ⇦
Muon Storage Ring	**3-4 GeV**
νSTORM	*3.8 GeV*
Intensity Frontier ν Factory	**10-25 GeV**
Low Energy NF	*10 GeV*
IDS-NF 2.0	*25 GeV*
Current IDS-NF	*10 GeV* ⇦
s-Channel Higgs Factory	**~126 GeV CoM** ⇦
Energy Frontier μ Collider	**> 1 TeV CoM**
Opt. 1	*1.5 TeV CoM* ⇦
Opt. 2	*3 TeV CoM* ⇦
Opt. 3	*6 TeV CoM*

Program Baselines

A Muon Collider on the Fermilab Site



A challenging, but promising, R&D program is required!

THE R&D CHALLENGES

Design Studies

- Proton Driver
- Front End
- Cooling
- Acceleration and Storage
- Collider
- Machine-Detector Interface
- Work closely with physics and detector efforts

Technical R&D

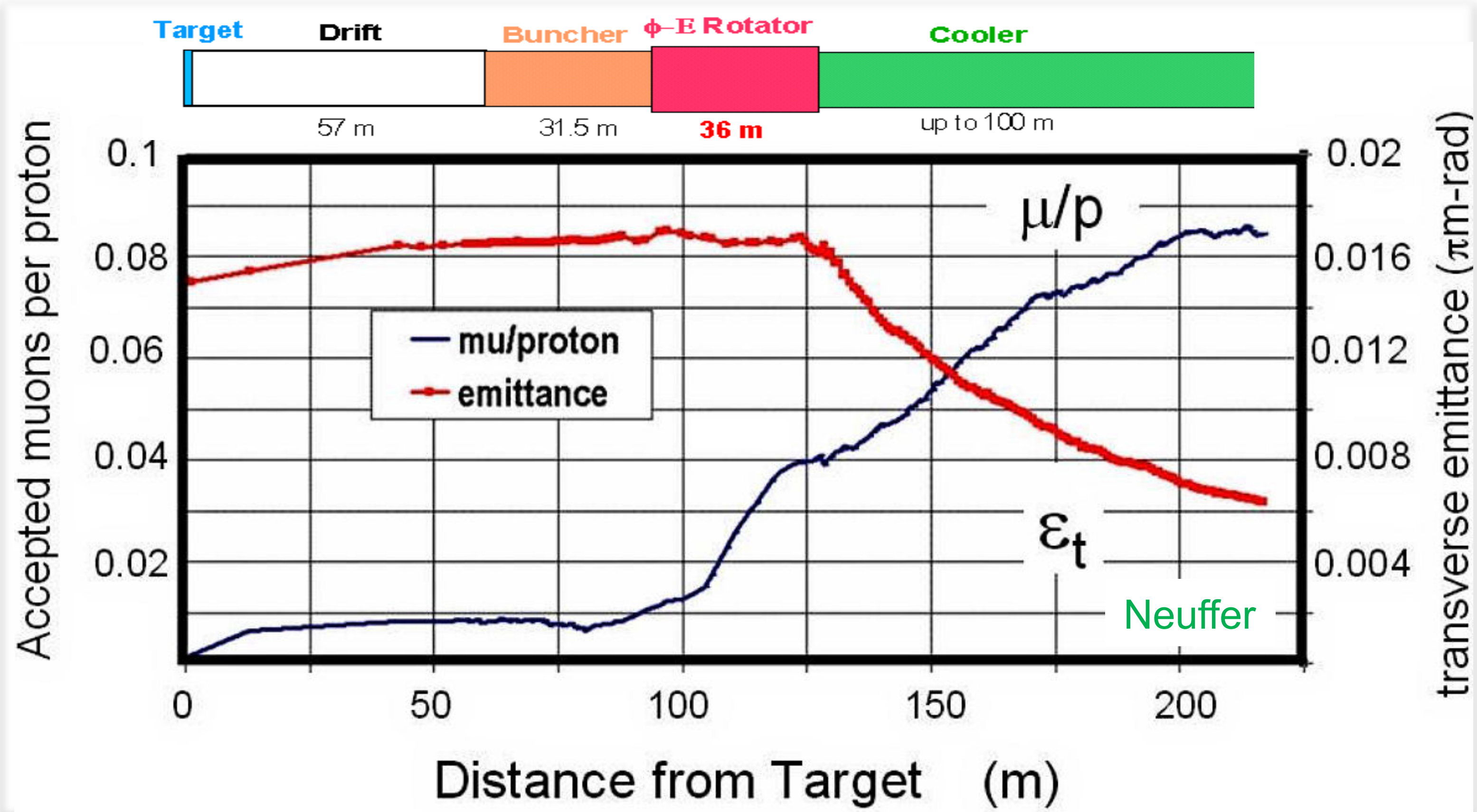
- RF in magnetic fields
- SCRF for acceleration chain (eg, 200 MHz cavities)
- High field magnets
  - Utilizing HTS technologies
- Targets & Absorbers
- MuCool Test Area (MTA)

Major System Demonstration

- The Muon Ionization Cooling Experiment – MICE
  - Major U.S. effort to provide key hardware: RF Cavities and couplers, Spectrometer Solenoids, Coupling Coils
  - Experimental and Operations Support

Technical Challenges – Tertiary Production

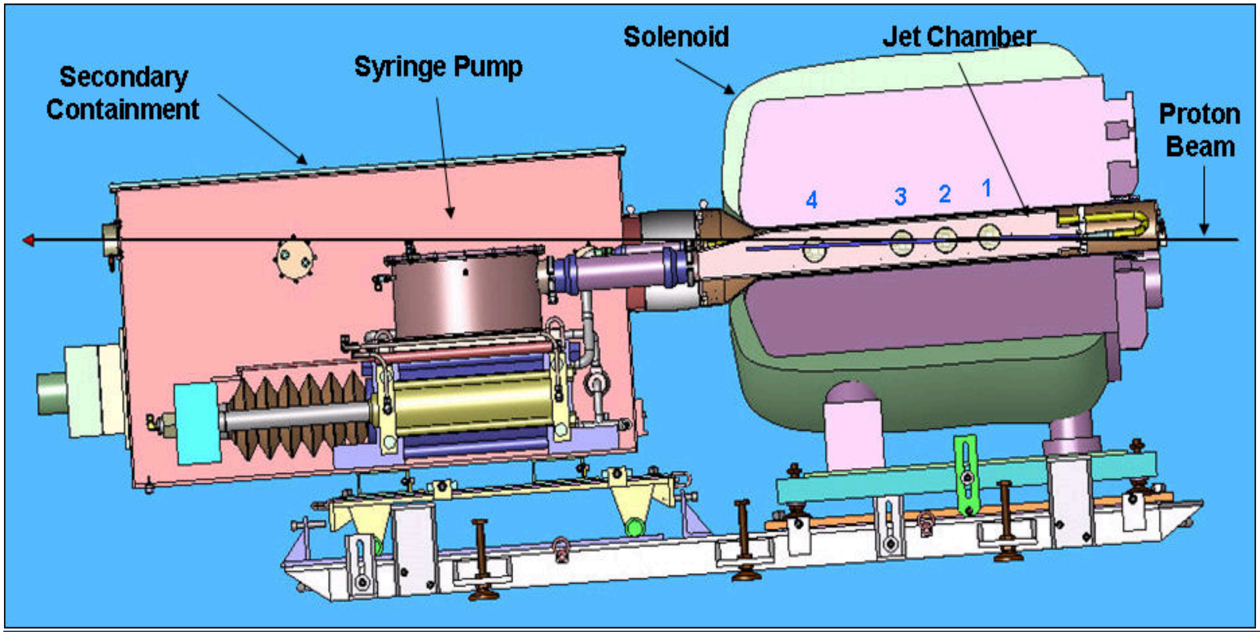
- A MW-scale proton source and target facility are required



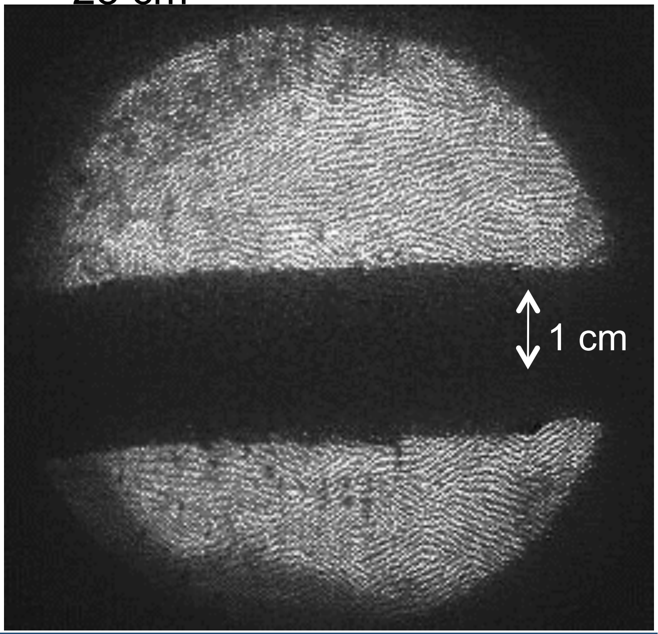
- A multi-MW proton source, *e.g.*, Project X, will enable $O(10^{21})$ muons/year to be produced, bunched and cooled to fit within the acceptance of an accelerator.

Technical Challenges - Target

- The MERIT Experiment at the CERN PS
  - Proof-of-principle demonstration of a liquid Hg jet target in high-field solenoid in Fall `07
  - Demonstrated a 20m/s liquid Hg jet injected into a 15 T solenoid and hit with a 115 KJ/ pulse beam!
    - ➪ Technology OK for beam powers up to 8 MW with a repetition rate of 70 Hz!



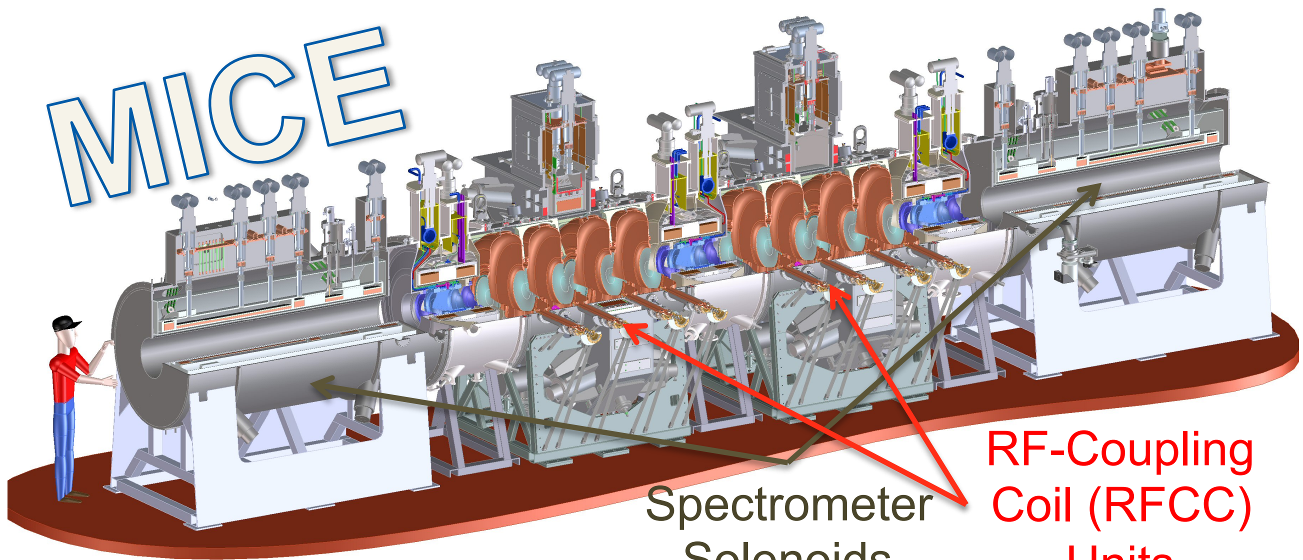
Hg jet in a 15 T solenoid with measured disruption length: ~ 28 cm



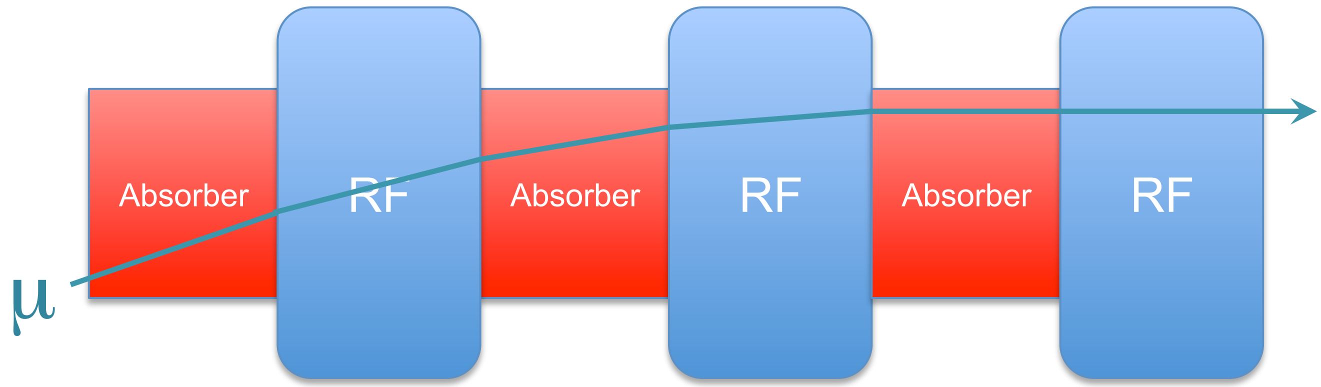
Technical Challenges - Cooling

- Tertiary production of muon beams ⇨
  - Initial beam emittance intrinsically large
  - Cooling mechanism required, but no radiation damping
- Muon Cooling ⇨ Ionization Cooling
  - dE/dx energy loss in materials
  - RF to replace p_{long}

The Muon Ionization Cooling Experiment: Demonstrate the method and validate our simulations



Technical Challenges - Cooling

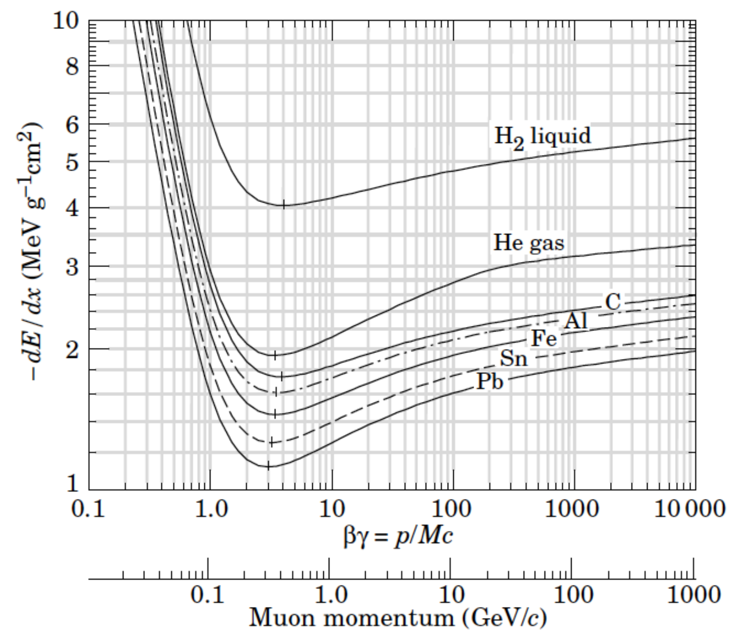


Ionization Energy Loss: $$\Delta E = \left\langle \frac{dE}{dx} \right\rangle \Delta s$$

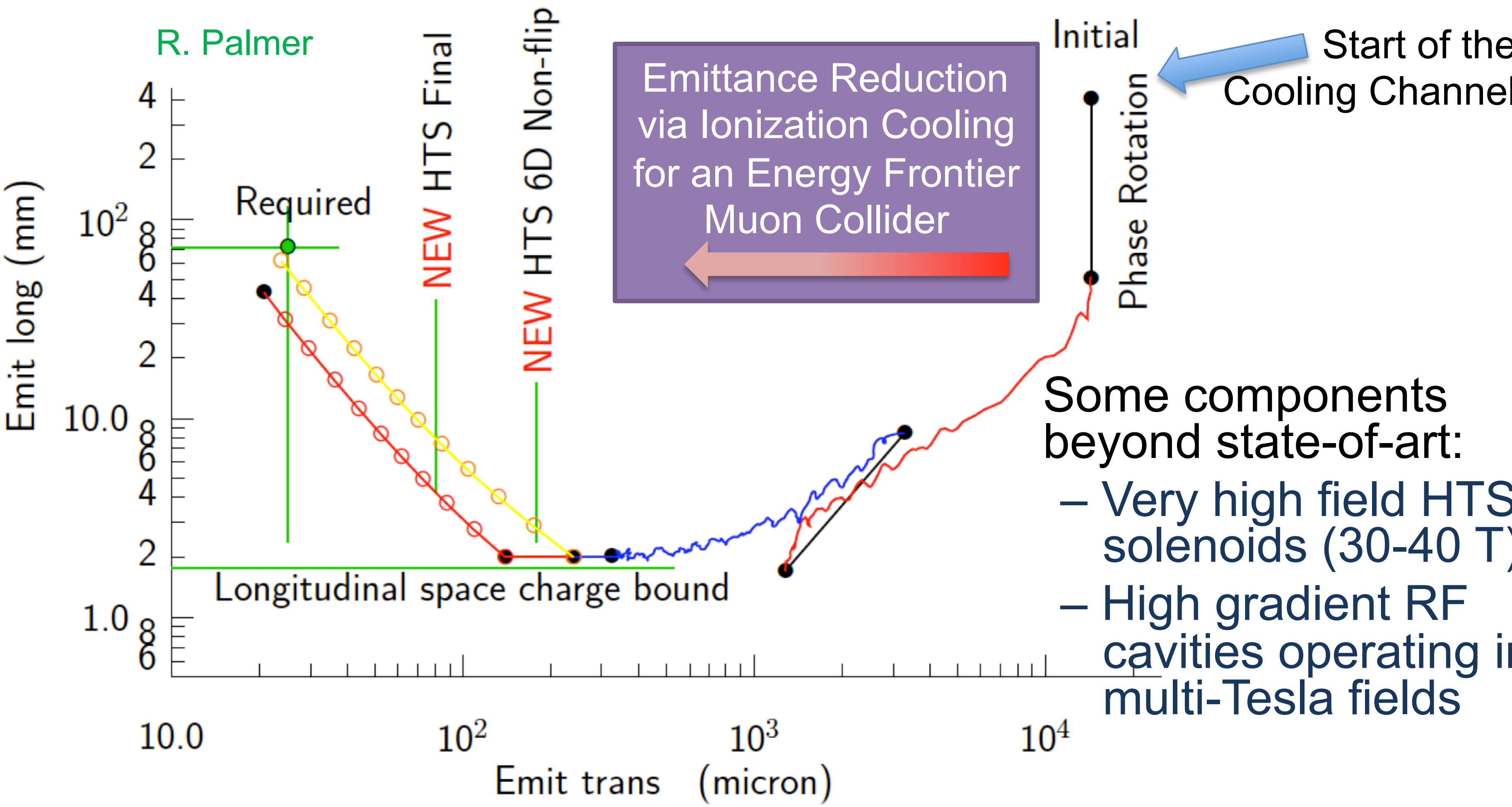
Competes with multiple scattering.

Equilibrium emittance:

$$\frac{d\varepsilon_N}{ds} \approx -\frac{1}{\beta^2}\left\langle \frac{dE_\mu}{ds} \right\rangle \frac{\varepsilon_N}{E_\mu} + \frac{\beta_\perp (0.014GeV)^2}{2\beta^3 E_\mu m_\mu X_0}$$



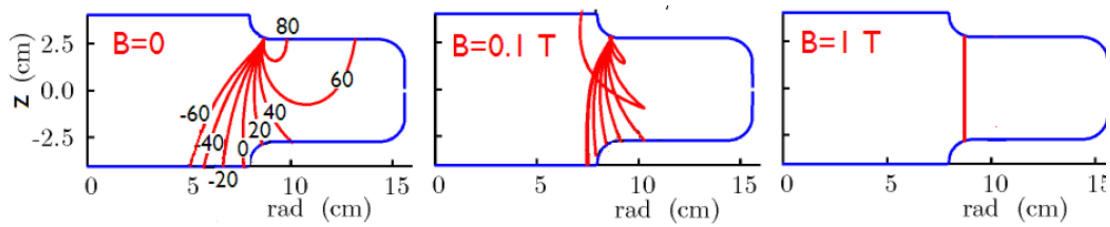
- Development of a cooling channel design to reduce the 6D phase space by a factor of $O(10^6)$ $\longrightarrow$ luminosity of $O(10^{34})$ cm^{-2} s^{-1}



Some components beyond state-of-art:
- Very high field HTS solenoids (30-40 T)
- High gradient RF cavities operating in multi-Tesla fields

Technical Challenges – RF

- A Viable Cooling Channel requires
  - Strong focusing and a large accelerating gradient to compensate for the energy loss in absorbers
    - ⇨ Large B- and E-fields superimposed



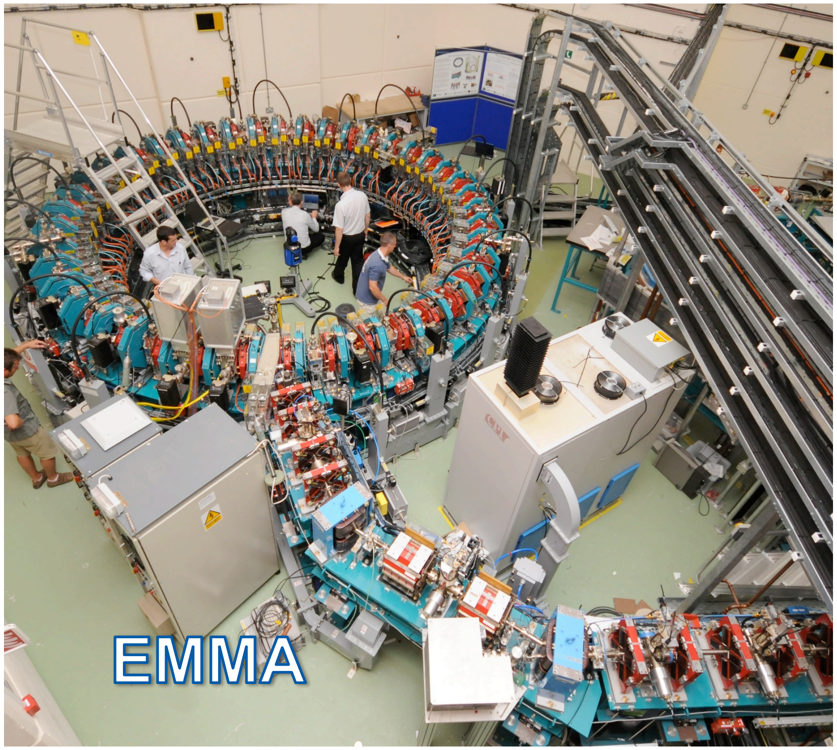
1. "Dark Current" electrons accelerated and focused by magnetic field
2. Damage spots by thermal fatigue causing breakdown

- Operation of RF cavities in high magnetic fields is a necessary element for muon cooling

- Control RF breakdown in the presence of high magnetic fields
- The MuCool Test Area (MTA) at Fermilab is actively investigating operation of RF cavities in the relevant regimes
- Development of concepts to mitigate this problem are being actively pursued

Technical Challenges - Acceleration

- Muons require an ultrafast accelerator chain
  ➪ *Beyond the capability of most machines*
- Several solutions for a muon acceleration scheme have been proposed:



Superconducting Linacs

Recirculating Linear Accelerators (RLAs)
- JLAB proposal for scaled electron demonstration machine

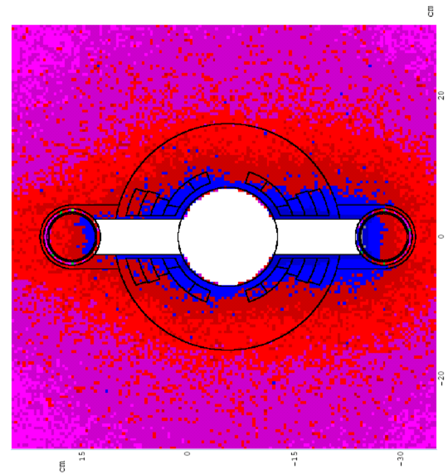
Fixed-Field Alternating-Gradient (FFAG) Machines
- ➢ EMMA at Daresbury Lab is a test of the promising non-scaling type

Rapid Cycling Synchrotrons (RCS/VRCS)

Hybrid Machines

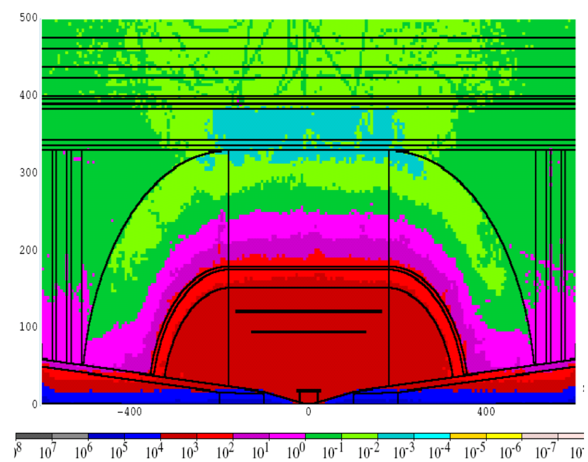
Technical Challenges – Ring, Magnets, Detector

- Emittances are relatively large, but muons circulate for ~1000 turns before decaying
  - Lattice studies for 1.5 TeV and 3 TeV CoM



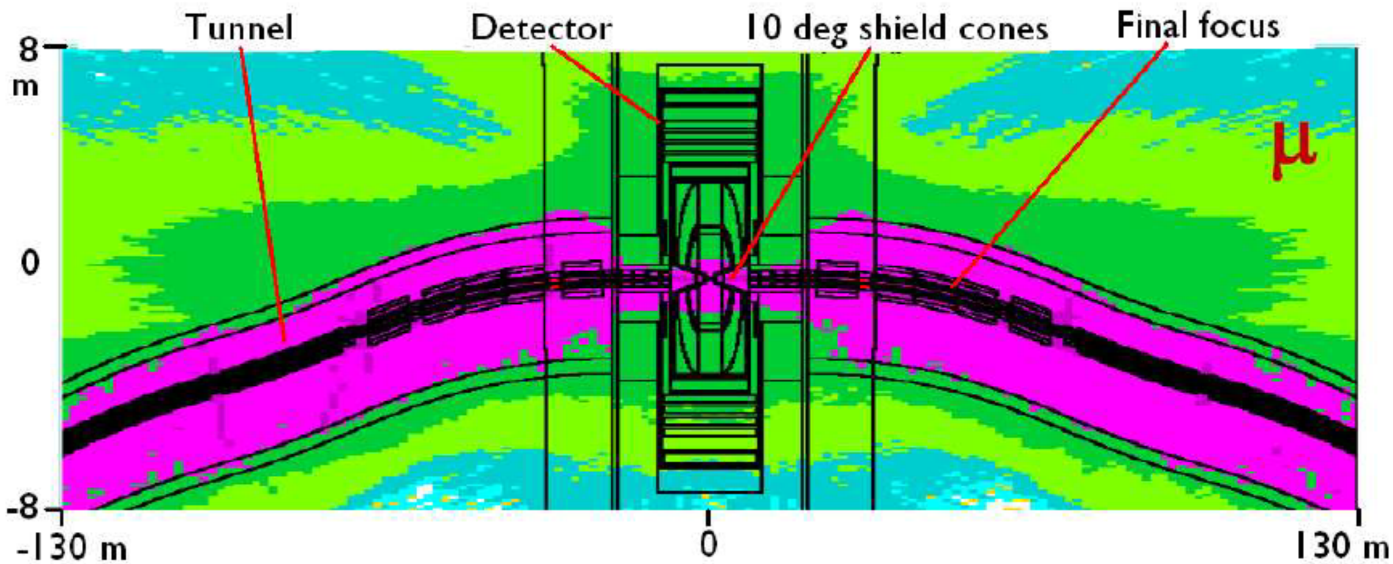
MARS energy deposition map for 1.5 TeV collider dipole

- High field dipoles and quadrupoles must operate in high-rate muon decay backgrounds
  - Magnet designs under study



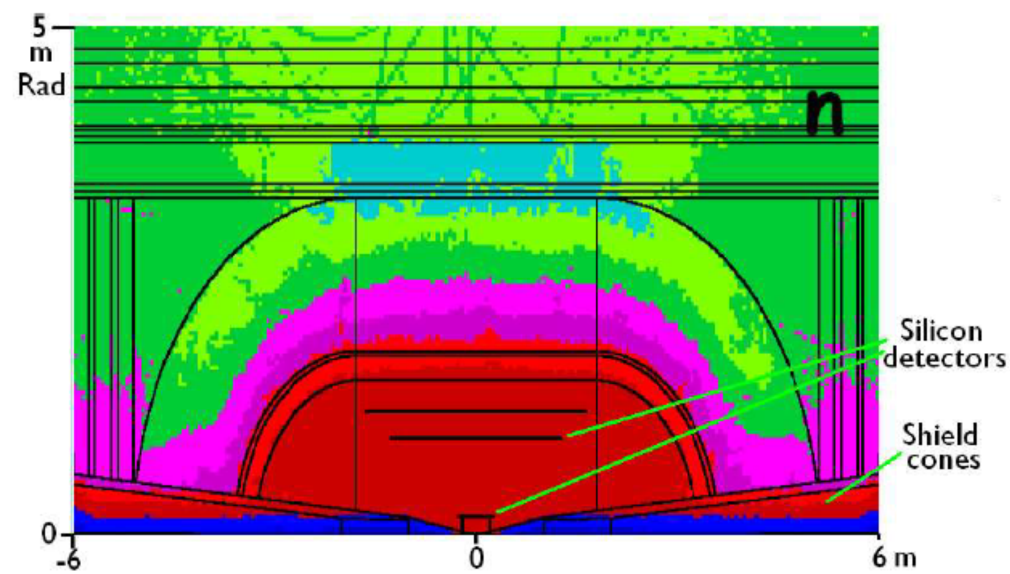
- Detector shielding & performance
  - Initial studies for 1.5 TeV, then 3 TeV
  - Shielding configuration
  - MARS background simulations
- Detector studies

Detector Shielding



Fluence at first silicon tracker
10% of LHC
(at $10^{34}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}$)

Worse than e^+e^-
but appears acceptable



RECENT R&D PROGRESS – SOME TECHNOLOGY HIGHLIGHTS

Recent Progress I – MICE Magnets

Spectrometer Solenoids

1st SS

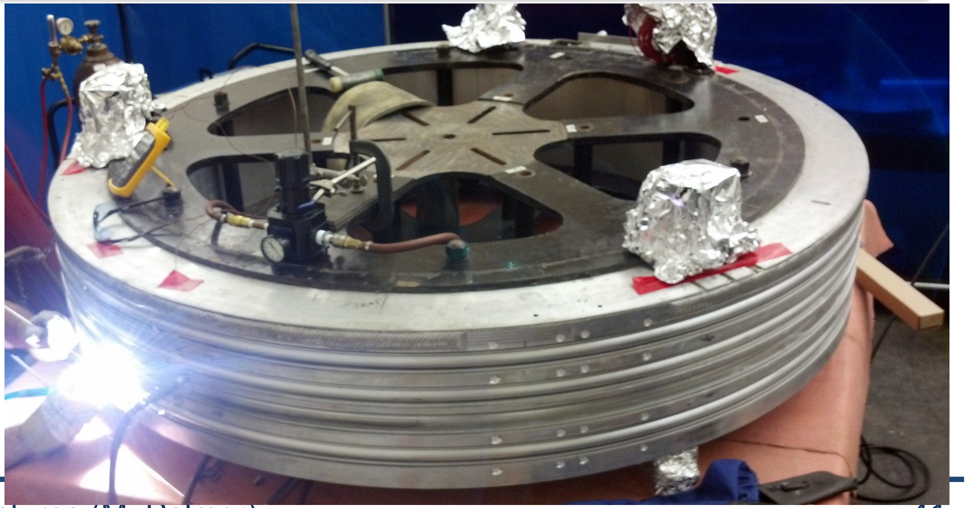
- Cooldown and preliminary operational tests complete
- Ready for full current training and field mapping

2nd SS

- Cold mass and shield being assembled into vacuum vessel

⇨ Support MICE Step IV (2013-)

Coupling Coils



- First Coupling Coil cold mass being prepared (at LBNL) for training (at FNAL) in new Solenoid Test Facility

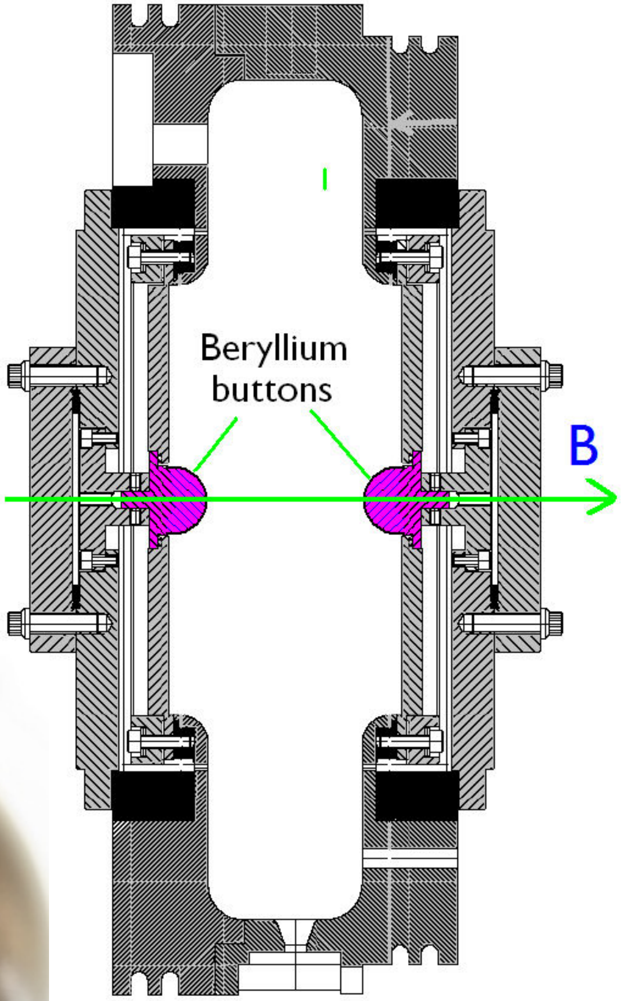
⇨ Support MICE Step V/VI (2016-)

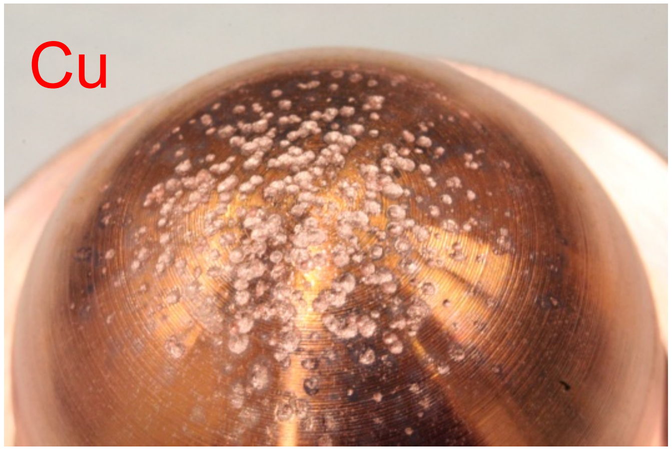
Recent Progress II – Cavity Materials

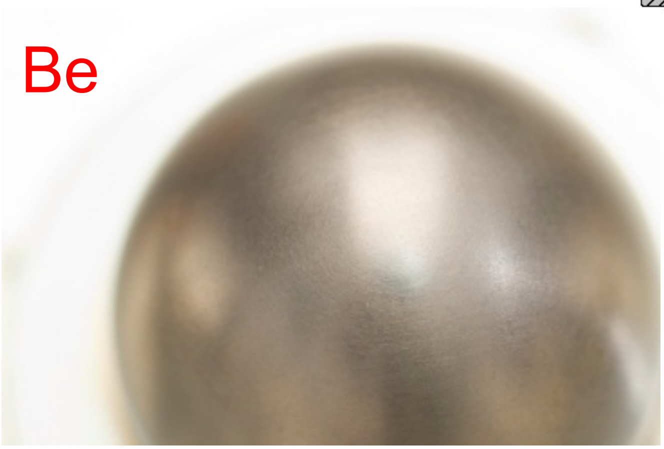
Breakdown tests with Be and Cu Buttons

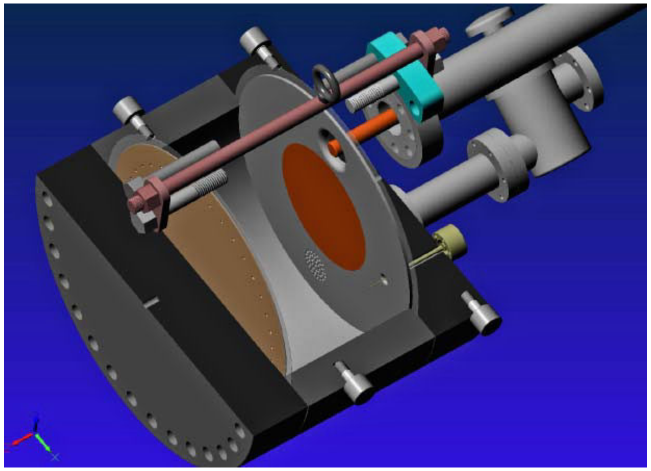
- Both reached ~31 MV/m
- Cu button shows significant pitting
- Be button shows minimal damage

⇨ Materials choices offer the possibility of more robust operation in magnetic fields



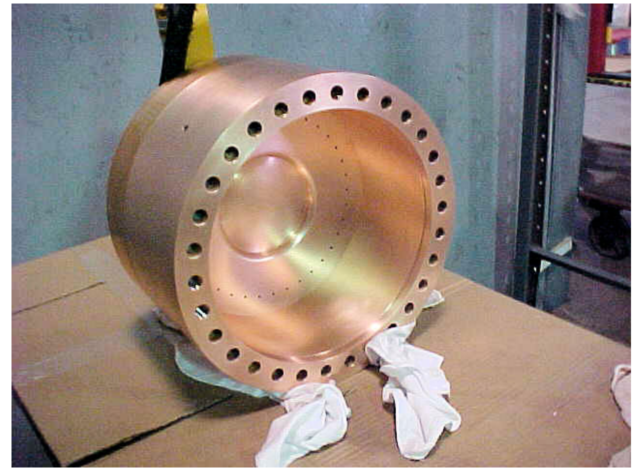


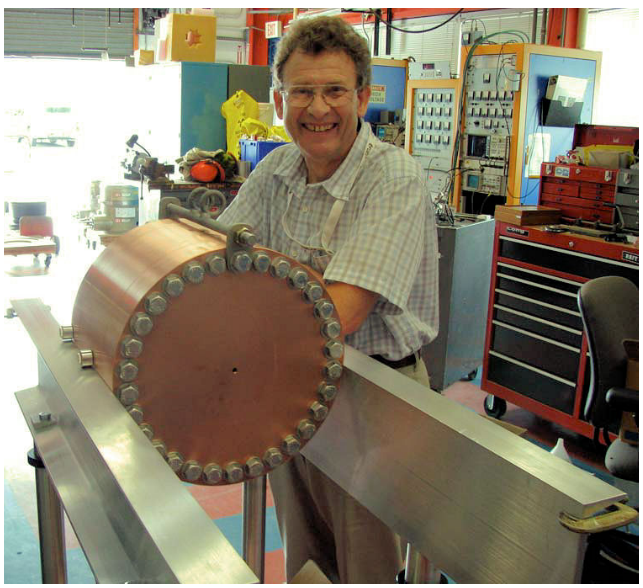




All-Seasons Cavity

(designed for both vacuum and high pressure operation)

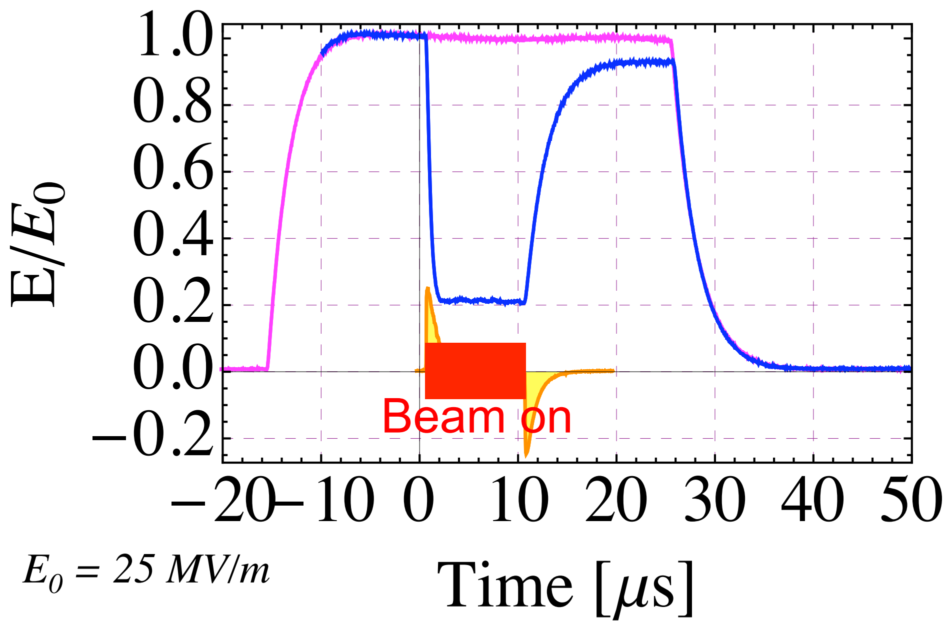




- Vacuum Tests at B = 0 T & B = 3 T
  - Two cycles: $B_0 \Rightarrow B_3 \Rightarrow B_0 \Rightarrow B_3$
- No difference in maximum stable operating gradient
  - Gradient $\approx$ 25 MV/m
- Demonstrates possibility of successful operation of vacuum cavities in magnetic fields with careful design

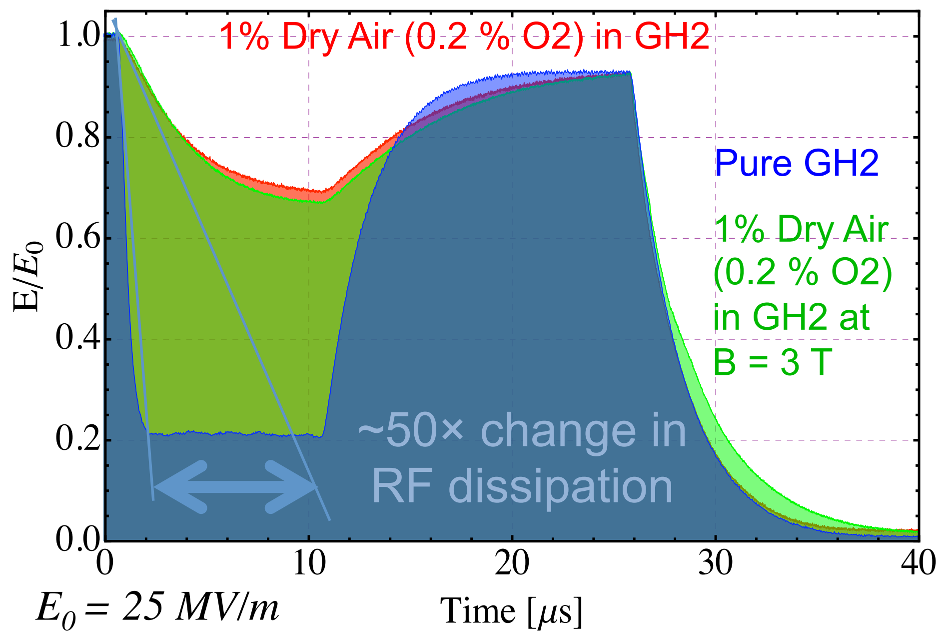
Recent Progress IV: High Pressure RF

- Gas-filled cavity
  - Can moderate dark current and breakdown currents in magnetic fields
  - Can contribute to cooling
  - Is loaded, however, by beam-induced plasma



$E_0 = 25\ MV/m$

- Electronegative Species
  - Dope primary gas
  - Can moderate the loading effects of beam-induced plasma by scavenging the relatively mobile electrons

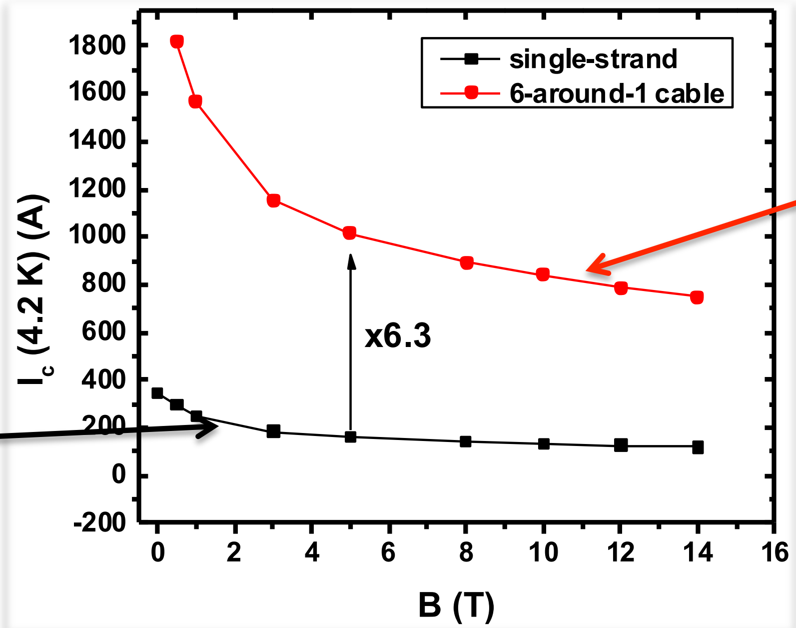


$E_0 = 25\ MV/m$

Recent Progress V: High Field Magnets

Progress towards a demonstration of a final stage cooling solenoid:

- Demonstrated 15+ T (16+ T on coil)
  - ~25 mm insert HTS solenoid
  - BNL/PBL YBCO Design
  - Highest field ever in HTS-only solenoid (by a factor of ~1.5)
- Will soon begin preparations for a test with HTS insert + mid-sert in NC solenoid at NHFML ⇨ >30 T



PART II

A CLOSER LOOK AT MUON COOLING

with acknowledgments to Bob Palmer

UNITS AND DEFINITIONS

Units and Useful Conversions

- Generally will set up any equations in MKS units
- However, we typically will still want to talk about:
  - Energies and masses in MeV, GeV, and TeV
  - Momenta as MeV/c
- This just requires some careful grouping of our variables and constants. So remember that:

  (pc/e): has units of volts

  (E/e): has units of volts

  (mc^2/e): has units of volts
- We can write the bending radius in a magnetic field as:

$$\rho = \frac{p}{eB} = \frac{(pc/e)}{Bc}$$

So, for a 3 GeV/c particle in a 5 Tesla field, the radius of curvature is simply:

$$\rho = \frac{3\times 10^9 V}{5T \times 3\times 10^8 m/s} = 2m$$

- We will use normalized emittances:

$$\varepsilon_n = \frac{Phase\ Space\ Area \times c/e}{\pi \times mc^2/e}$$

- The phase space can be transverse (x,p_x), (y,p_y) or longitudinal $(z,\Delta p_z)$, where the z coordinates are with respect to the bunch center:

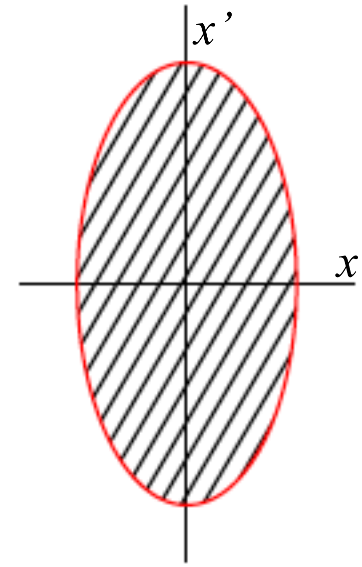
$$\varepsilon_\perp = \frac{\pi\sigma_{p_x(c/e)}\sigma_x}{\pi\left(mc^2/e\right)} = \beta_v\gamma\sigma_\theta\sigma_x \qquad [m \times rad]$$

$$\varepsilon_{\|} = \frac{\pi\sigma_{p_z(c/e)}\sigma_z}{\pi\left(mc^2/e\right)} = \beta_v\gamma\frac{\sigma_p}{p}\sigma_z \qquad [m \times rad]$$

where $\beta_v = v/c$

- We will not use geometric emittances $(\varepsilon_0 = \sigma_\theta\sigma_x)$ in this lecture

β Function Concepts (I)



- The transverse beta of a beam is just given by the width/height of the phase ellipse

$$\beta_\perp = \frac{\sigma_x}{\sigma_\theta}$$

- By combining this with the expressions for emittance, we can write:

$$\sigma_x = \sqrt{\varepsilon_\perp \beta_\perp \left(\frac{1}{\beta_v \gamma}\right)}$$

$$\sigma_\theta = \sqrt{\frac{\varepsilon_\perp}{\beta_\perp}\left(\frac{1}{\beta_v \gamma}\right)}$$

- The beta of a lattice can also be defined for a repeating lattice where the beta of the beam is matched to the lattice.

β Function Concepts (II)

- Generally we define beta functions for a lattice which has continuous focusing. In such a case, periodic solutions of the form:

$$u = A\sin\left(\frac{z}{\beta}\right) \qquad u' = \frac{A}{\beta}\cos\left(\frac{z}{\beta}\right)$$

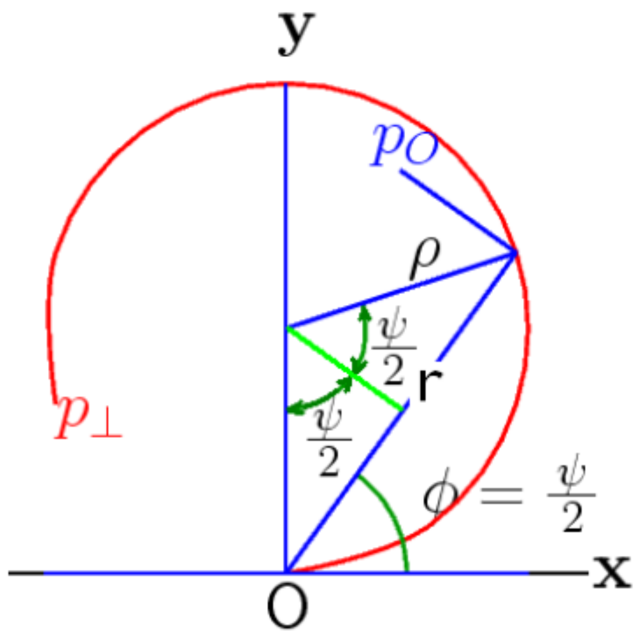
can be written. These equations of motion describe a particle moving in an ellipse in phase space.

- If the particles in an ensemble have $\beta_{beam} = \beta_{lattice}$ then the particles move around the ellipse and the shape of the beam remains constant ⇨ the beam is *matched* to the lattice
- If $\beta_{beam} \neq \beta_{lattice}$, then β_{beam} will oscillate around $\beta_{lattice}$ ⇨ "beta beat"

SOLENOID FOCUSING

Transverse Motion in a Long Solenoid (I)

- Field: Assume B_z=*constant*
- Consider a particle starting on at the origin O, with no longitudinal momentum, but finite transverse momentum. Since the particle starts at the origin, it has no initial angular momentum in this frame. The particle's motion can be described as:



$$\rho = \frac{p_\perp c/e}{cB_z}$$

$$x = \rho \sin(\psi)$$

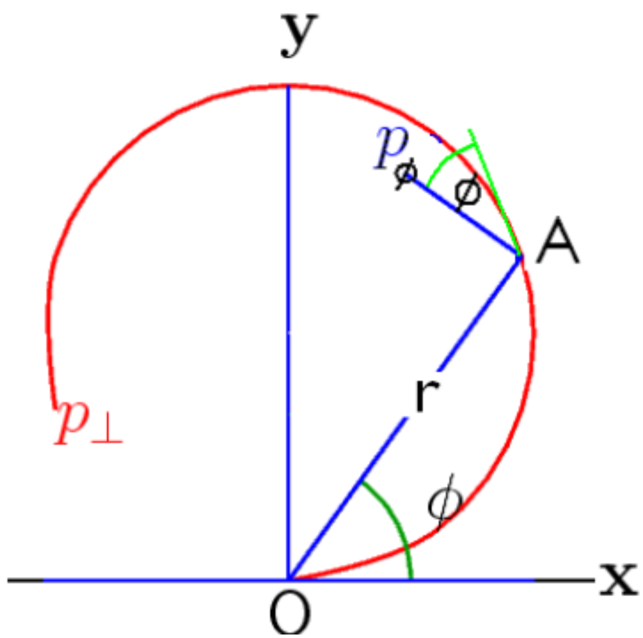
$$y = \rho\left[1 - \cos(\psi)\right]$$

$$r = 2\rho \sin\left(\frac{\psi}{2}\right) = 2\rho \sin(\phi)$$

- Note that r is sinuosoidal, as is x, but oscillates with half the frequency of x.

Transverse Motion in a Long Solenoid (II)

- If we now solve for the angular momentum of the particle, we obtain:



$$p_\phi = -p_\perp \sin(\phi)$$

$$r = 2\rho \sin(\phi)$$

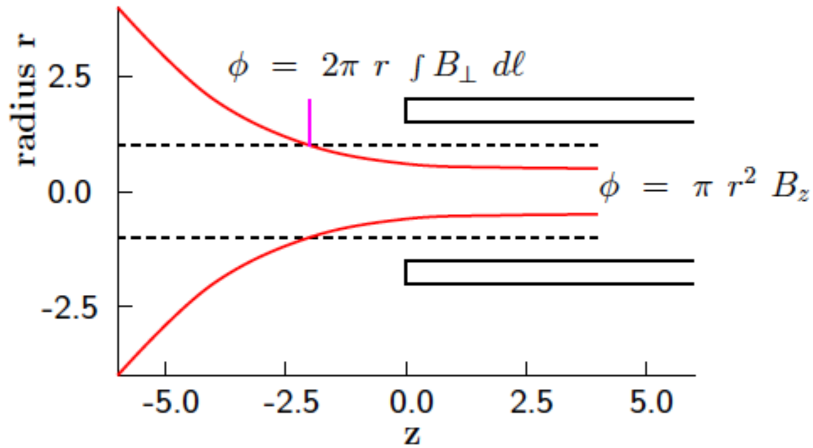
$$\Rightarrow p_\phi = -p_\perp \frac{r}{2\rho}$$

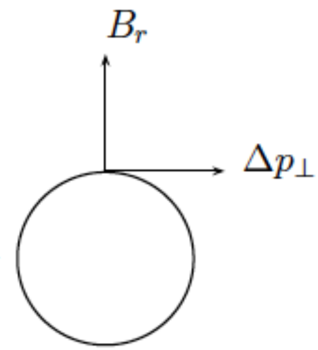
- Using the expression for ρ from the previous slide, we then obtain:

$$p_\varphi \, c/e = -\frac{rc}{2} B_z$$

- Consider a particle entering a solenoid

$$\Delta p_\phi \, c/e = \int B_r dz$$
$$= -\frac{rc}{2}\Delta B_z$$





- If the particle has no initial angular momentum, this implies:

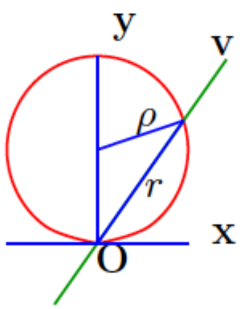
$$p_\phi \, c/e = -\frac{rc}{2}\Delta B_z$$

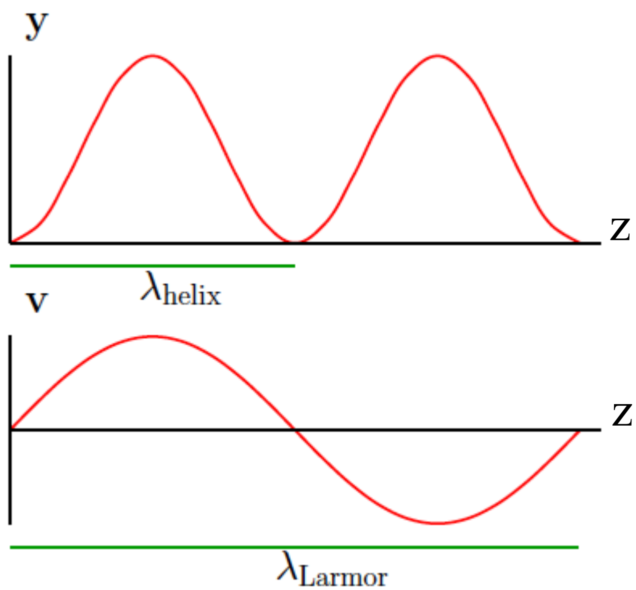
which is exactly the condition required for a helix that passes through the axis of the solenoid.

- Define a coordinate system u, v which rotates about the axis by the angle ϕ. For a particle in that frame which initially has no angular momentum, it will remain in the u=0 plane – this is the Larmor Frame

The Larmor Plane

- If we consider a plane containing the particle and the axis of the solenoid (again for a particle which initially has no angular momentum) this plane is known as the Larmor Plane





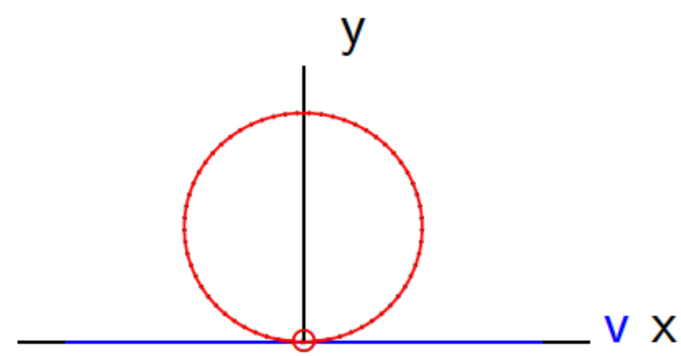
$$y = \rho\left[1 - \cos(\psi)\right]$$

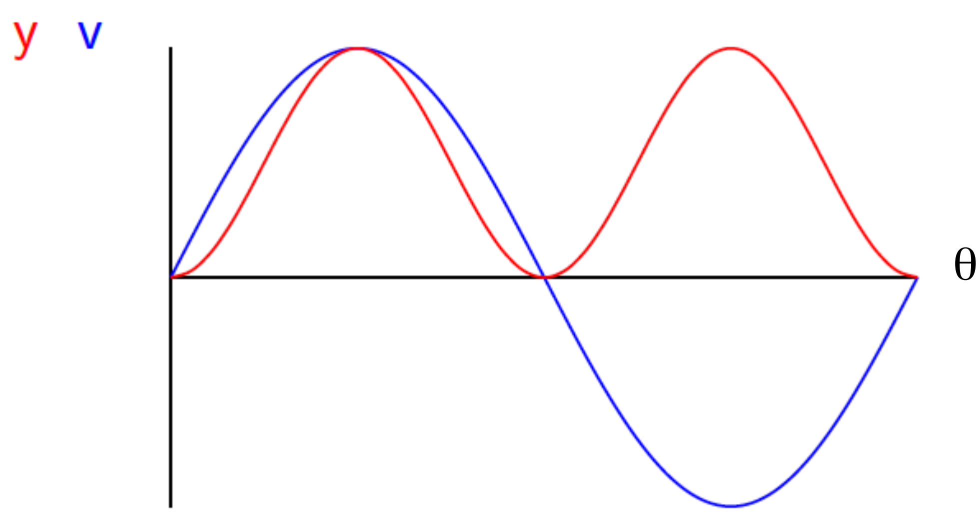
$$v = 2\rho \sin(\phi)$$

$$\frac{\lambda_{Helix}}{2\pi} = \frac{dz}{d\psi} = \rho\frac{p_z}{p_\perp} = \frac{p_z\, c/e}{cB_z}$$

$$\frac{\lambda_{Larmor}}{2\pi} = \beta_{Lattice} = \frac{dz}{d\phi} = 2\rho\frac{p_z}{p_\perp} = 2\frac{p_z\, c/e}{cB_z}$$

Larmor Motion





- For a particle moving in an axially symmetric solenoidal field, $B_z(z)$
  - Define a transverse frame with axes u and v rotating rotating about the axis which moves longitudinally with the particle

$$\frac{d\phi}{dz} = -\frac{cB_z(z)}{2\left(\frac{p_z c}{e}\right)}$$

  - In this frame the focusing force is given by:

$$\frac{1}{\eta} = \frac{d^2 r}{dz^2} = -\left[\frac{cB_z(z)}{2(p_z c/e)}\right]^2 r \qquad \Rightarrow \text{Solenoid Focusing} \propto \frac{B^2}{p^2}$$

where r is the distance from the axis

  - The equivalent expression for a quadrupole is

$$\frac{1}{\eta} = \frac{d^2 r}{dz^2} = -\left[\frac{Gc}{(p_z c/e)}\right] r$$

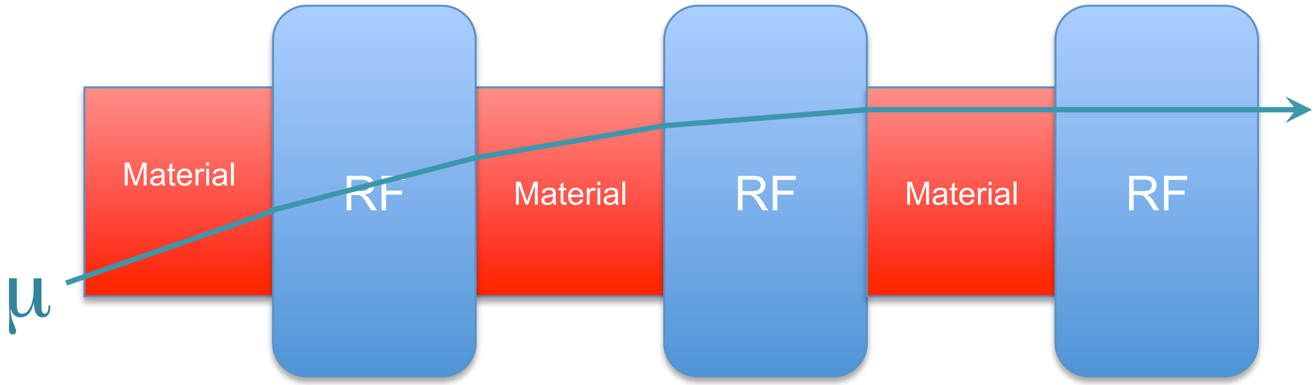
⇨ NOTE: solenoid focusing (unlike a quad) is independent of sign and is stronger for lower momenta

Solenoid Focusing Summary

- In a long solenoid: a particle moves along a helix of wavelength λ_{helix}
- In the Larmor Plane: a particle oscillates with wavelength $\lambda_{Larmor} = 2\lambda_{helix}$
- For motion in the Larmor plane
  - It focuses towards the axis
  - It has a focusing force proportional to B^2/ρ^2
  - A particle that starts in the Larmor plane stays in the Larmor plane
- At sufficiently low momentum, solenoid focusing is always stronger than quadrupole focusing
- Solenoids focus in *both* planes, unlike quadrupoles which focus in one plane and defocus in the other
- A solenoid can focus very large transverse emittances, with large angles (a radian or more), and thus are very well-suited for focusing in an ionization cooling channel

IONIZATION COOLING

Transverse Ionization Cooling



$$p_{\perp,f}(material) < p_{\perp,i} \qquad p_{\perp,f}(RF) \approx p_{\perp,i}$$

$$p_{\parallel,f}(material) < p_{\parallel,i} \qquad p_{\parallel,f}(RF) > p_{\parallel,i}$$

- Emittance Cooling $\varepsilon_{x,y} = \beta_v \gamma \sigma_{\theta_x,\theta_y} \sigma_{x,y}$
  - In the absence of Coulomb scattering (and any other emittance growth mechanisms), σ_θ and $\sigma_{x,y}$ are not affected by energy loss. However, p and $\beta\sigma$ are reduced. Thus we have:

$$\frac{d\varepsilon}{\varepsilon} = \frac{dp}{p} = \frac{dE}{\beta_v^2 E}$$

Thus, the cooling rate improves at lower energies

Contributions to Emittance Heating

$$\varepsilon_{x,y} = \beta_v \gamma \sigma_{\theta_x,\theta_y} \sigma_{x,y}$$

- We will consider scattering processes
- Note: between scatters, a drift will conserve emittance due to Liouville's Theorem
- A scatter will:
  - Leave $\sigma_{x,y}$ unchanged
  - Cause an increase in σ_θ
  - The corresponding change in emittance is:

$$\Delta(\epsilon_{x,y})^2 = \gamma^2 \beta_v^2 \, \sigma_{x,y}^2 \Delta(\sigma_\theta^2)$$

$$2\epsilon \, \Delta\epsilon = \gamma^2 \beta_v^2 \left(\frac{\epsilon \beta_\perp}{\gamma \beta_v} \right) \Delta(\sigma_\theta^2)$$

$$\Delta\epsilon = \frac{\beta_\perp \gamma \beta_v}{2} \Delta(\sigma_\theta^2)$$

Coulomb Scattering

e.g. from Particle data booklet

$$\Delta(\sigma_\theta^2) \approx \left(\frac{14.1\ 10^6}{[pc/e]\beta_v}\right)^2 \frac{\Delta s}{L_R}$$

$$\Delta\epsilon = \frac{\beta_\perp}{\gamma\beta_v^3}\,\Delta E\ \left(\left(\frac{14.1\ 10^6}{2[mc^2/e]_\mu}\right)^2 \frac{1}{L_R dE/ds}\right)$$

Defining $$C(mat,E) = \frac{1}{2}\left(\frac{14.1\ 10^6}{[mc^2/e]_\mu)}\right)^2 \frac{1}{L_R\, d\gamma/ds}$$

then $$\frac{\Delta\epsilon}{\epsilon} = dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat,E)$$

- Equating the expressions for the emittance growth rate due to scattering and the emittance damping rate due to cooling gives:

$$\frac{dE}{\beta_v^2 E} = C(mat, E)\frac{\beta_\perp dE}{\varepsilon \gamma \beta_v^3}$$

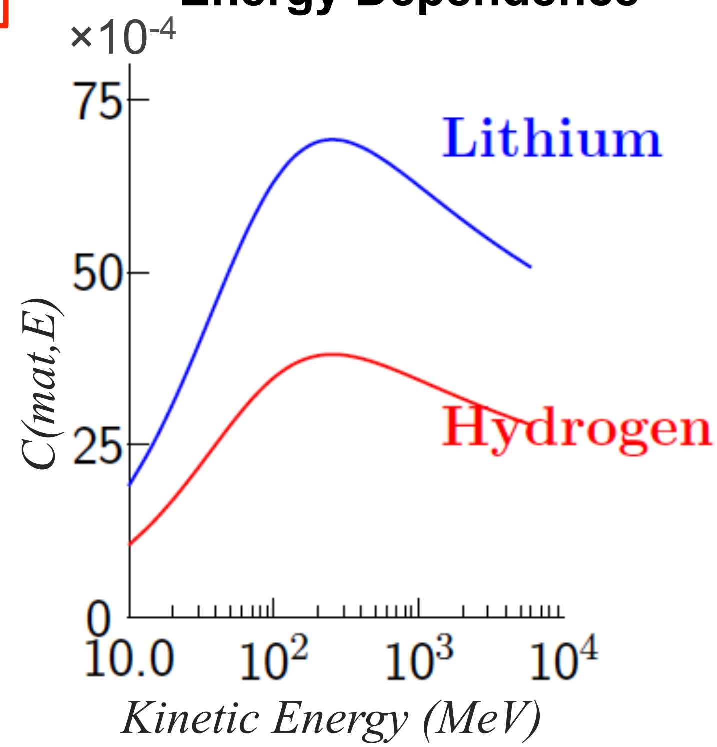
$$\Rightarrow \boxed{\varepsilon_0 = C(mat, E)\frac{\beta_\perp}{\beta_v}}$$

where ε_0 is the *equilibrium emittance*

Values at Ionization Minimum

Material	T (°K)	ρ (kg/m³)	dE/dx (MeV/m)	L_R (m)	C_0 ($\times 10^{-4}$)
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

Energy Dependence



Cooling Channel Optimization (I)

- Material:
  - Liquid H_2 has, by far, the best performance, but comes with challenges
    - Cryogenic liquid
    - Safety issues
    - Requires windows
  - LiH is the second best material
    - Doesn't need windows nor cryogenics
- Cooling Channel Energy:
  - At lower energies, C is smaller, but longitudinal heating occurs
  - The initial cooling for a collider or neutrino factory uses an energy near the minimum ionizing point
  - In the final cooling section, an energy of ~10 MeV is employed (see next section)

Cooling Channel Optimization: Rate of Cooling

$$\frac{d\varepsilon}{\varepsilon} = \left(1 - \frac{\varepsilon_{\min}}{\varepsilon}\right)\frac{dp}{p}$$

- Choice of β:
  - Naively, cooling rate appears best with $\varepsilon_{\min} \ll \varepsilon$, but this can cause problems due to non-linearities when large values of σ_θ result
- Beam Divergence Angles and Required Aperture
  - Recall

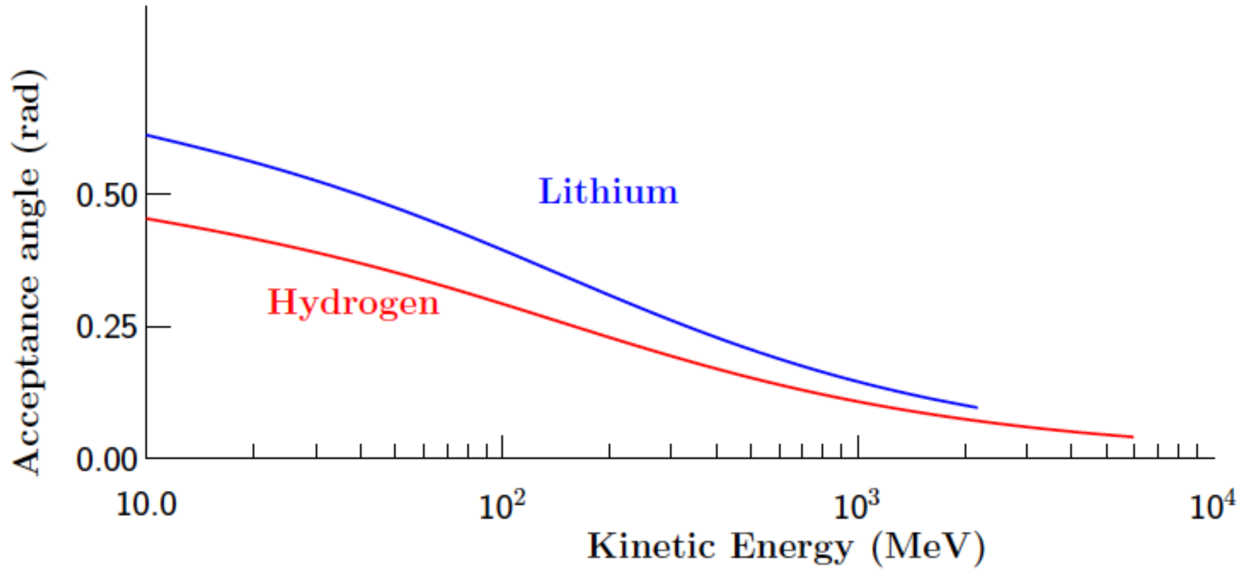
$$\sigma_\theta = \sqrt{\frac{\varepsilon_\perp}{\beta_\perp}\left(\frac{1}{\beta_v \gamma}\right)} = \sqrt{\frac{C(mat,E)}{\beta_v^2 \gamma}}$$

  - If we assume that we obtain only half of the maximum cooling rate and also require that the angular aperture provide at least a 3σ spread, we obtain:

$$A_\theta = 3\sqrt{\frac{2C(mat,E)}{\beta_v^2 \gamma}}$$

Cooling Channel Optimization: Aperture

- Plot of A_θ for Li and H_2 as a function of energy



- NOTE: For low energies, the required angular acceptance is VERY large!
- In realistic lattice configurations it is doubtful whether $A_\theta > 0.3$ is feasible

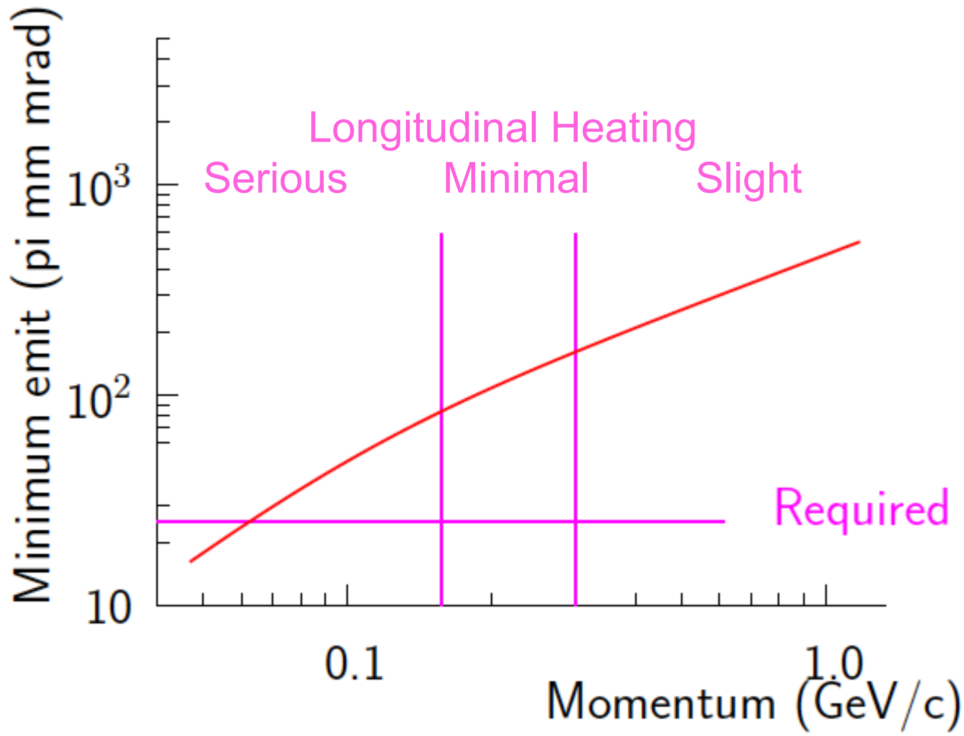
Cooling Channel Optimization: Focusing and Final Cooling

- Recall $$\beta_{\perp} = \frac{2 p_z c / e}{c B_z}$$

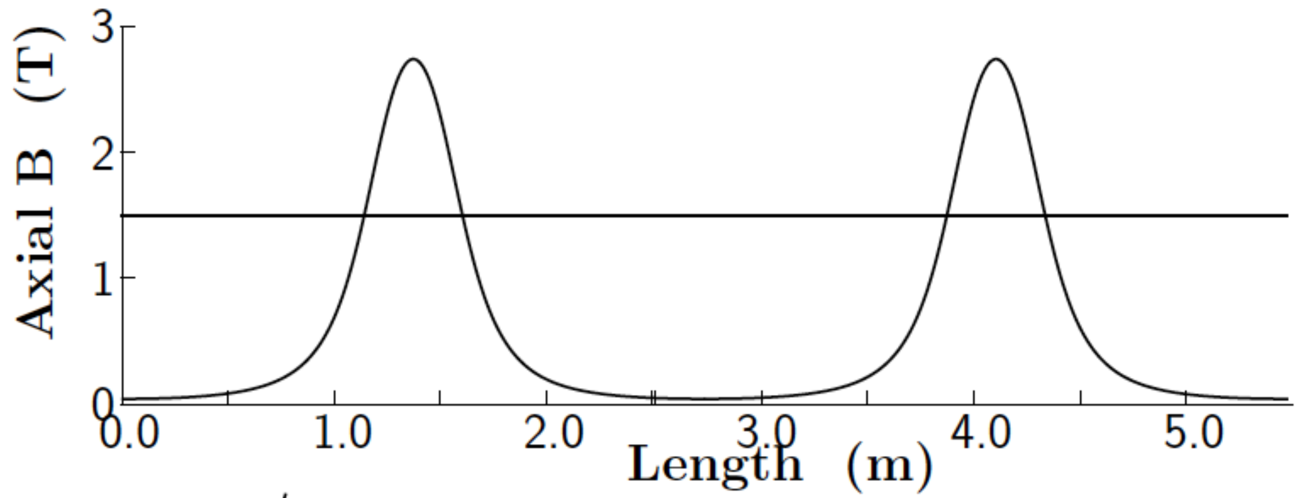
- Thus we can write $$\varepsilon_{x,y}(\min) = \frac{2C(mat,E)\gamma m_{\mu} c^2 / e}{c B_z}$$

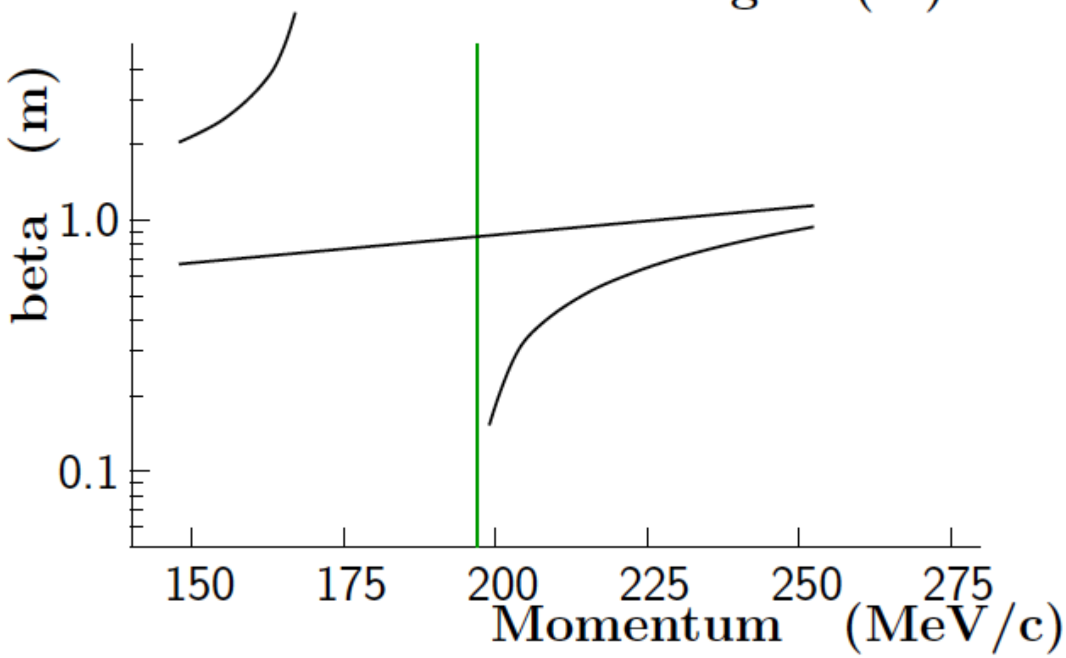
- Note: Transverse emittance target for collider is ~25μm

- The plot at the right shows that the target transverse emittance cannot be obtained without going to low energies and allowing some heating of the longitudinal emittance
  ⇨ a careful balance for the final cooling stage. Higher B-fields (~30-40 T) help here



Effects of Periodicity in Lattice

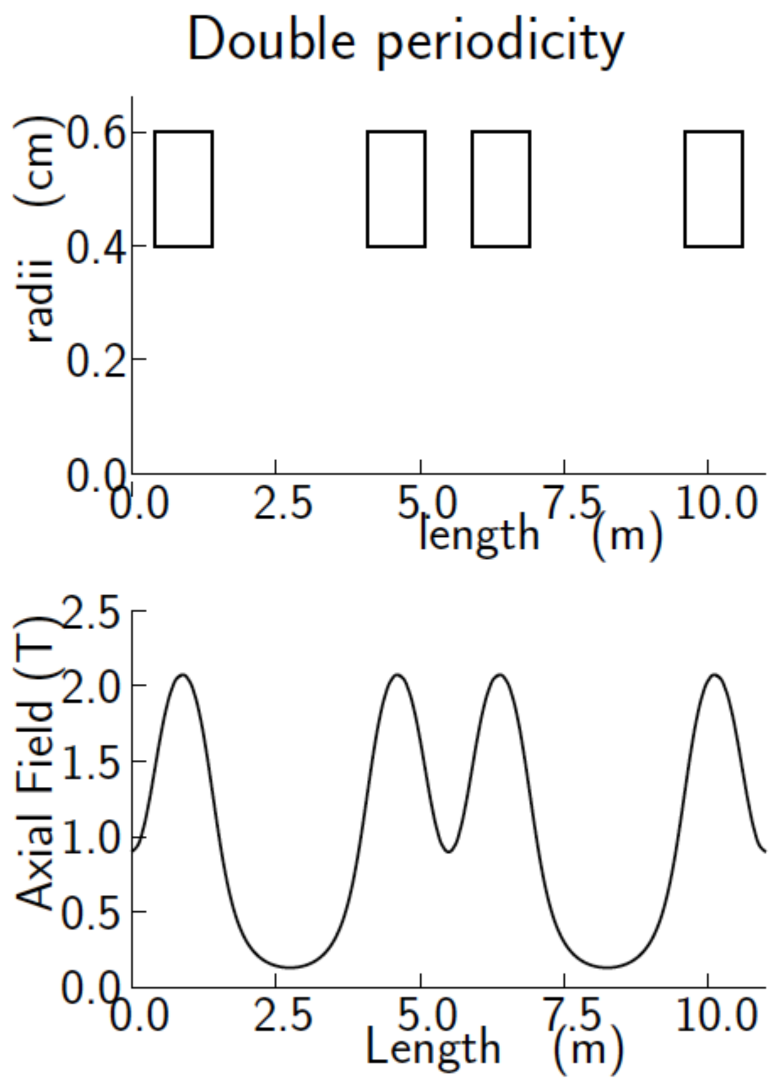


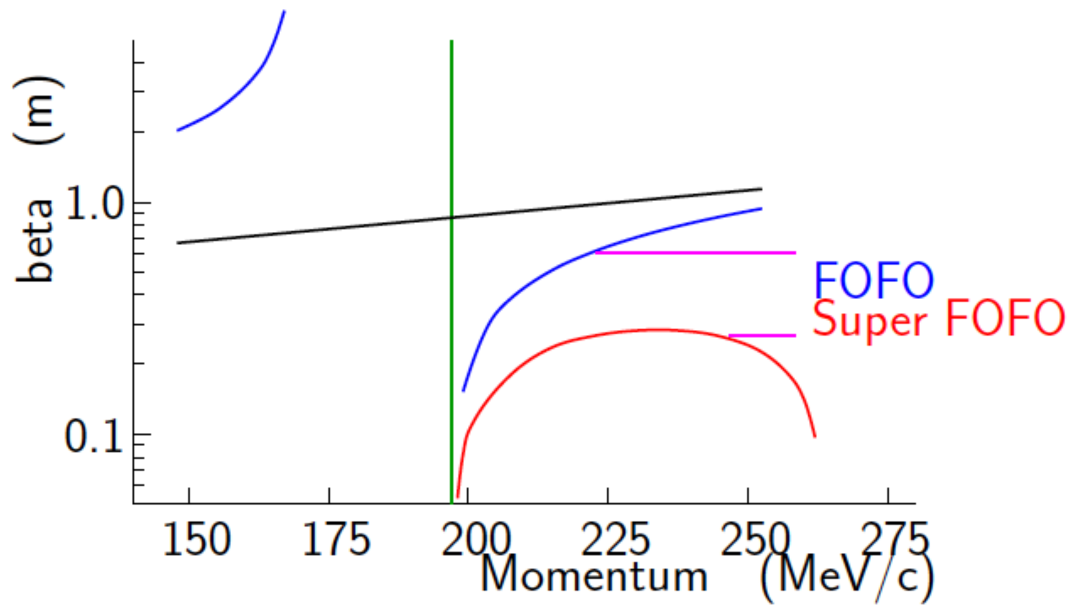


- Resonances introduced
- Betas reduced locally
- But only over small momentum range

Solenoid fields are alternated to avoid a buildup of angular momentum

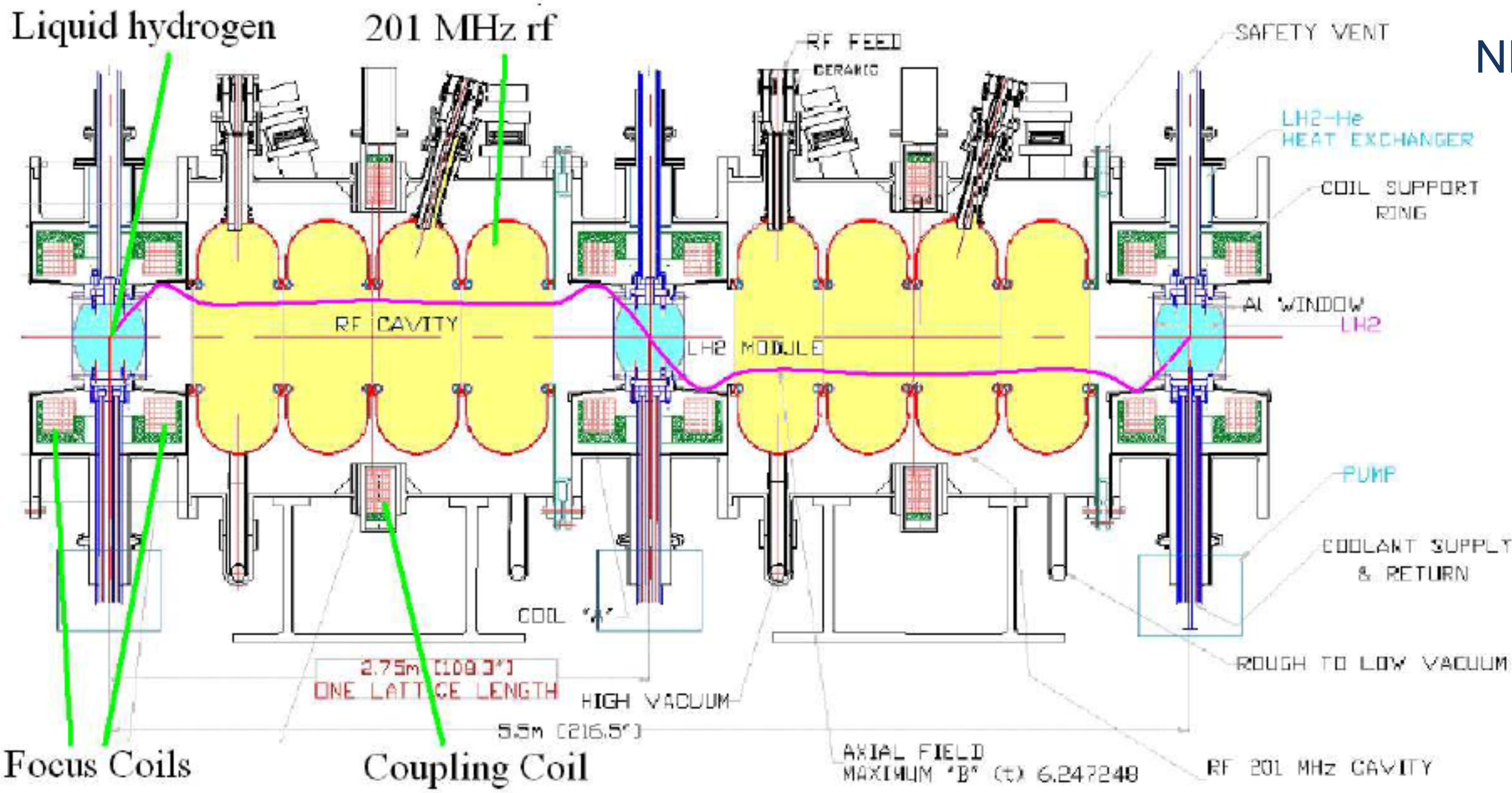
Super FOFO





- Beta lower over finite momentum range
- Beta lower by about 1/2 solenoid

SFOFO for Neutrino Factory and MICE Demo



NF Study 2

- This is the lattice to be tested in Muon Ionization Cooling Experiment (MICE) at RAL
- Study 2 the lattice is modified vs. length to lower $\beta_\perp$ as ϵ falls, keeping σ_θ and ϵ/ϵ_o more constant, thus maintains cooling rate

Summary of Transverse Cooling

- The optimum "absorber" material for a cooling channel is hydrogen (gas or liquid)
  - This offers operational and safety challenges
- Cooling requires very large angular acceptances
  - Solenoid focusing is well-suited for this requirement
  - Betas can be lowered by adding periodicity [but at the expense of reduced momentum acceptance]
- Final cooling with a target transverse emittance of 25μm is possible if have high magnetic fields and operate at low energies ⇨ must accept an increase in longitudinal emittance in ths situation

LONGITUDINAL COOLING

Partition Functions

- When dealing with synchrotron radiation, typically work with the "radiation integrals" and "partition functions". You will learn about this in detail during the DR lectures.
- For now, we would like to introduce the partition functions to look at the features of 6D cooling

$$J_{x,y,z} = \frac{\left(\Delta\varepsilon_{x,y,z} / \varepsilon_{x,y,z}\right)}{\left(\Delta p / p\right)}$$

$$J_6 = J_x + J_y + J_z$$

where $\Delta\varepsilon$ and Δp are changes due to the energy loss mechanism

- For discrete function electron synchrotrons, you will learn in the DR lectures that $J_x \approx J_y = 1$ *and* $J_z = 2$
- For muon ioniztion cooling, $J_x = J_y = 1$ but J_z is small or negative

Generalized Expression for the Transverse Emittance

- We saw previously that

$$\frac{\Delta\varepsilon_{x,y}}{\varepsilon_{x,y}} = \frac{\Delta p}{p}$$

  which corresponds to $J_x = J_y = 1$

- More generally, with $J_{x,y} \neq 1$ we can write

$$\frac{\Delta\varepsilon_{x,y}}{\varepsilon_{x,y}} = \frac{1}{J_{x,y}} \frac{\Delta p}{p}$$

- In this case, the expression for the minimum emittance then becomes:

$$\varepsilon_{x,y}(\min) = \frac{\beta_{\perp}}{J_{x,y}\beta_v} C(mat, E)$$

Longitudinal Cooling/Heating from the shape of the dE/dx Curve

- The longitudinal emittance can be written as:

$$\varepsilon_z = \gamma\beta_v \frac{\sigma_p}{p}\sigma_z = \frac{1}{m}\sigma_p\sigma_z = \frac{1}{m}\sigma_E\sigma_t = c\sigma_\gamma\sigma_t$$

  where σ_t is the rms bunch length in time.

- Note that a particle interaction with matter will not change σ_t, so that the change in emittance will only come from the energy change in the interaction:

$$\frac{\Delta\varepsilon_z}{\varepsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma} = \frac{\sigma_\gamma \Delta s \dfrac{d\left(d\gamma/ds\right)}{d\gamma}}{\sigma_\gamma} = \Delta s \frac{d\left(d\gamma/ds\right)}{d\gamma}$$

- We also can write:

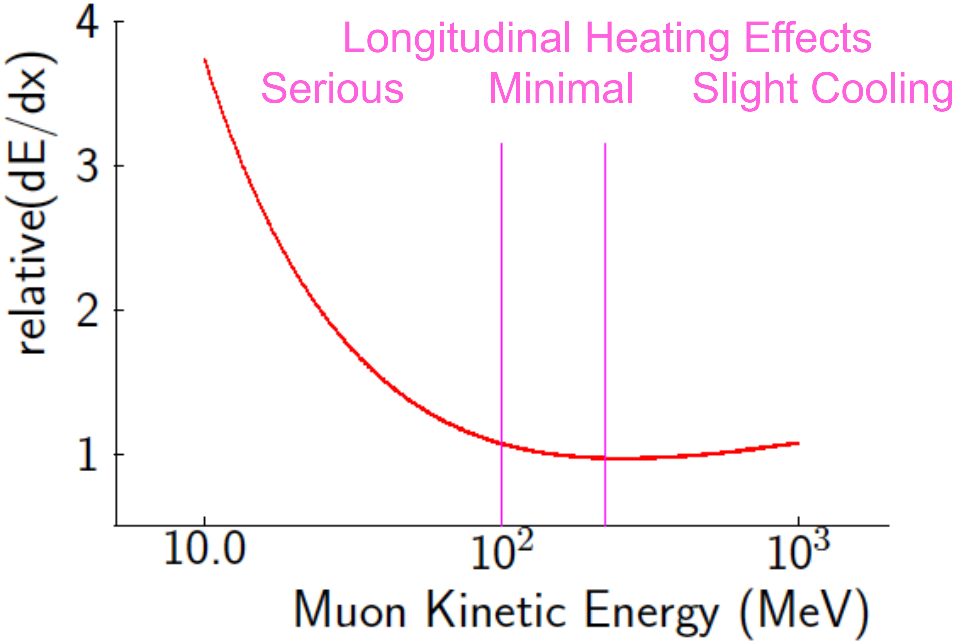
$$\frac{\Delta p}{p} = \frac{\Delta\gamma}{\beta_v^2\gamma} = \frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)$$

Longitudinal Heating and Cooling

- The partition function, J_z, can then be written as:

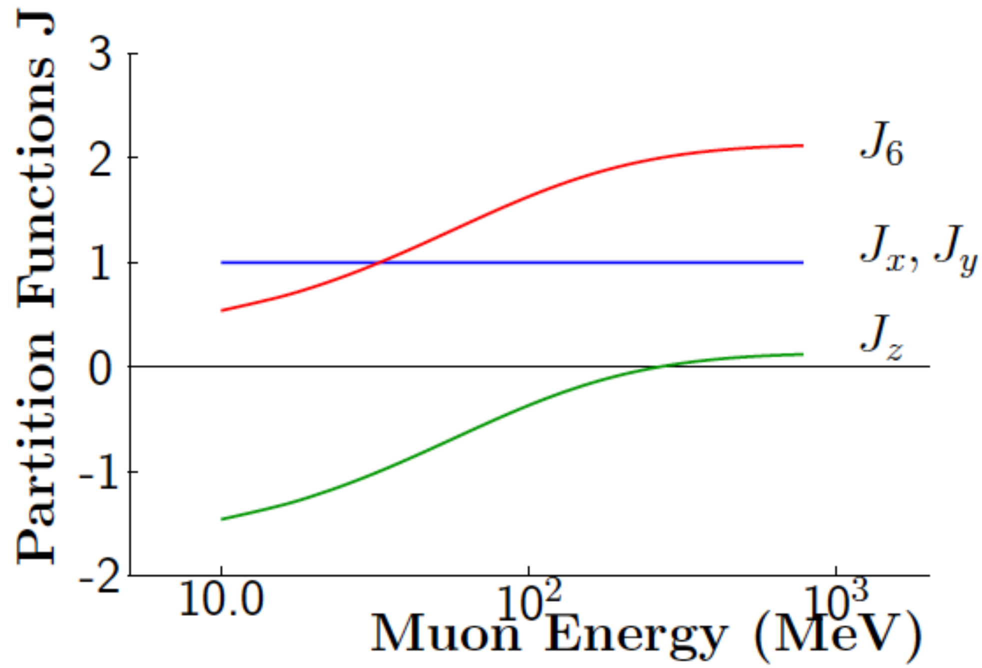
$$J_z = \frac{\Delta\varepsilon_z/\varepsilon_z}{\Delta p/p} = \frac{\left(\Delta s \dfrac{d(d\gamma/ds)}{d\gamma}\right)}{\dfrac{\Delta s}{\beta_v^2\gamma}\left(\dfrac{d\gamma}{ds}\right)} = \frac{\left(\beta_v^2\gamma \dfrac{d(d\gamma/ds)}{d\gamma}\right)}{\left(\dfrac{d\gamma}{ds}\right)}$$

- The relative energy loss as a function of energy is shown for the example of Li:



Energy Dependence of the Ionization Cooling Partition Functions

- J_z is seen to be strongly negative a low energies (longitudinal heating) and becomes slightly positive above 300 MeV/c



- More acceleration per cooling decrement at higher energies makes cooling at energies of ~200 MeV preferable

- Final cooling still requires moving to very low beam energies. However, the overall 6D cooling remains significant at these low energies.

Emittance Exchange

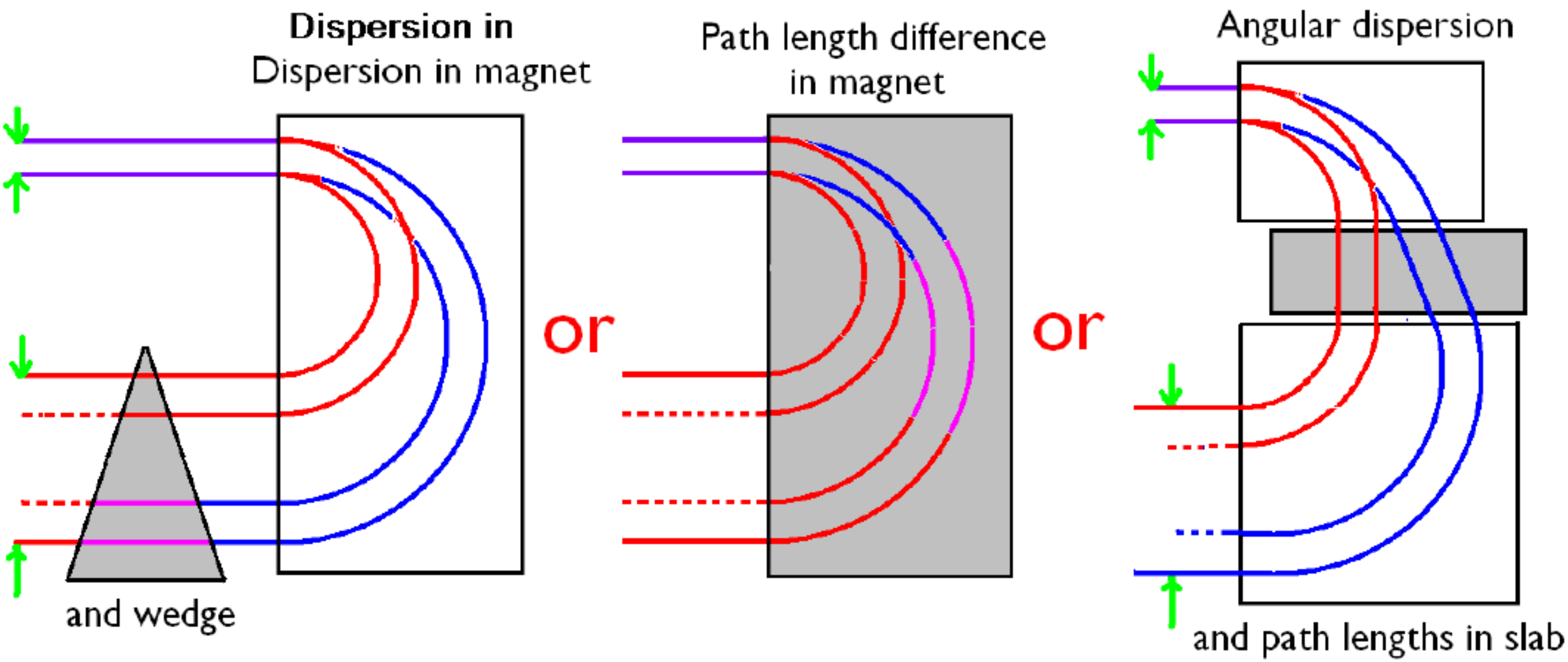
- In order to achieve the desired 6D beam emittance, a method to exchange emittance between the longitudinal and transverse dimensions is required.
  - This can be accomplished in electron/positron rings by combining focusing and bending.
  - Muons can use wedge absorbers and dispersion "tricks" to achieve the same result

- Note: These techniques must increase one partition number while decreasing another, because the overall sum is conserved.

- Typical values for muon cooling are:

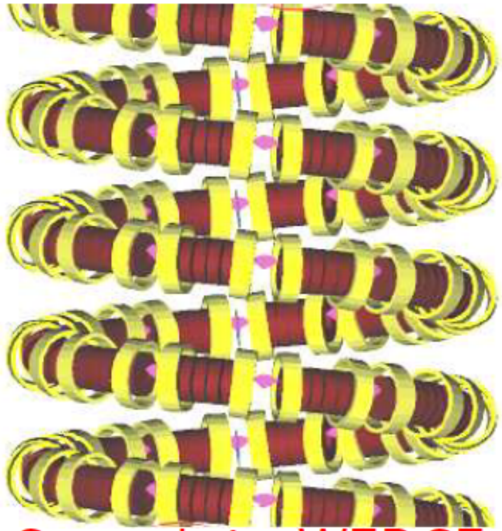
$$J_x + J_y + J_z = J_6 \approx 2$$

Emittance Exchange Methods

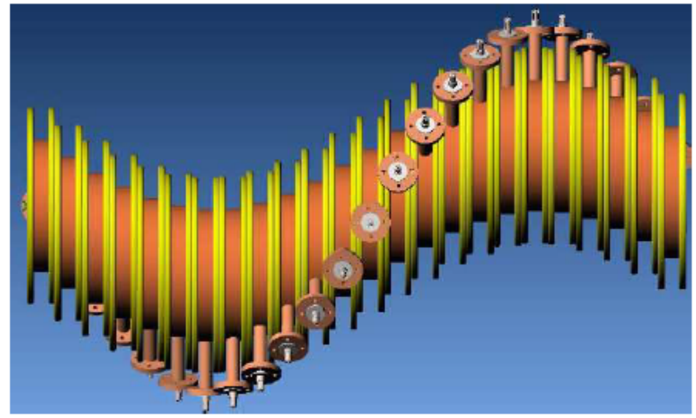


- In these examples, $\Delta p/p$ is reduced but σ_y is increased
  ⇨ the longitudinal emittance is reduced but the transverse emittance increases

6D Cooling Candidate Designs

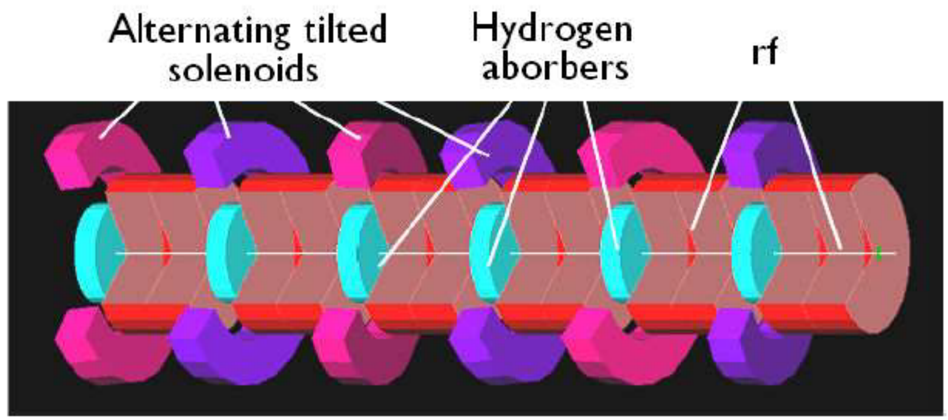


Guggenheim WEDGE



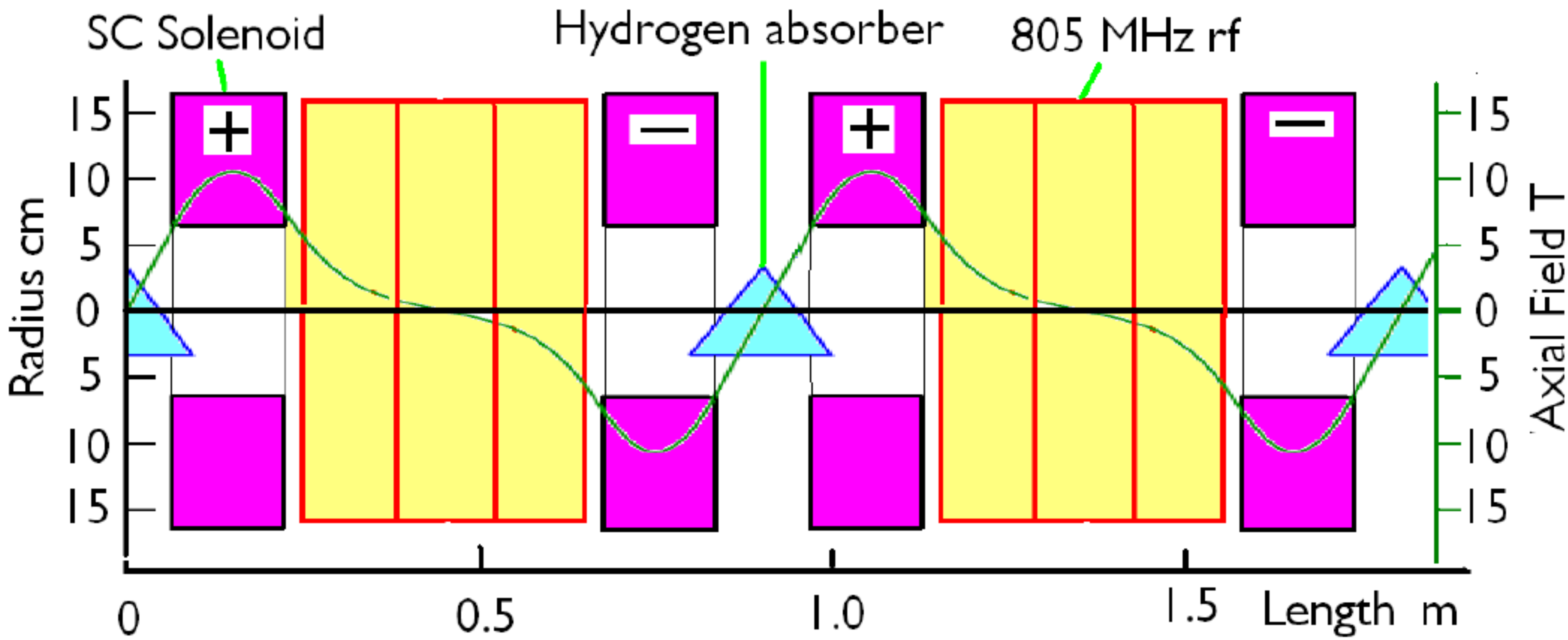
Helical Cooling Channel PATH

Snake SLAB

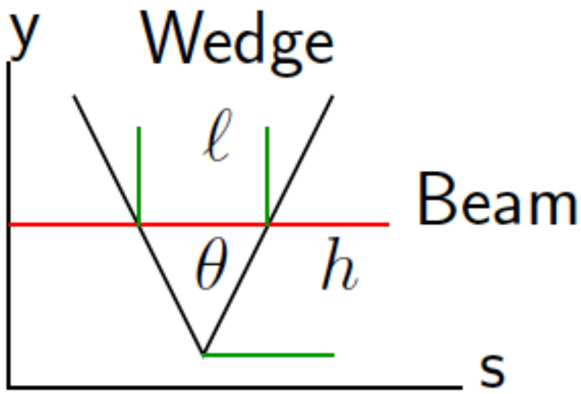


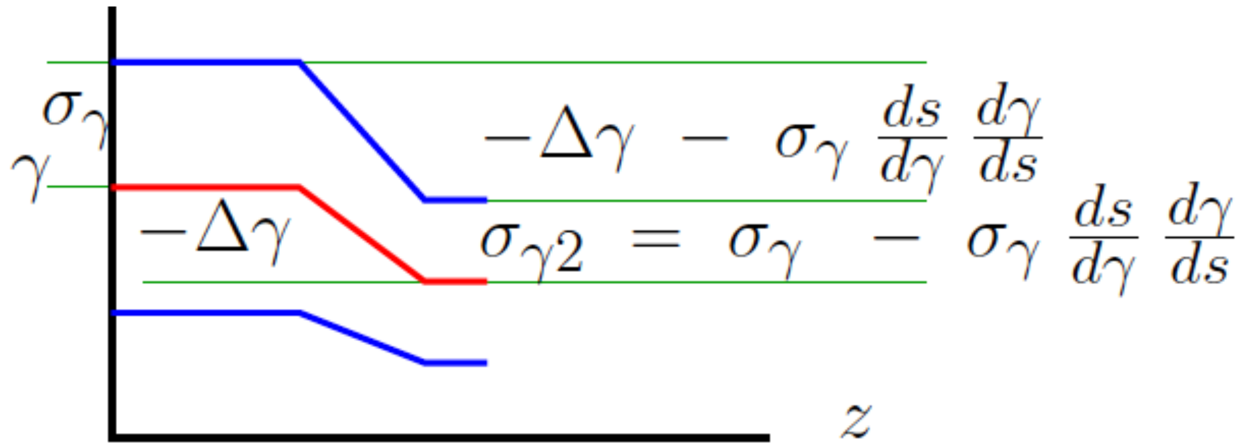
- Each of these examples has been simulated
- No design has had all of its outstanding issues resolved at the level that one can be selected as an official baseline design
- MAP is targeting an initial baseline selection within the next 18 months

Guggenheim Lattice Example (R. Palmer)



- Coils are tilted to generate a vertical bending field
  - Provide dispersion at the wedge aborbers
  - Generates the helical form





- Consider a wedge with center thickness ℓ and height h ($2h\tan(\theta/2) = \ell$) from the center, which is located in a region with dispersion D: $$D = \frac{dy}{dp/p} = \beta_v^2 \frac{dy}{d\gamma/p}\gamma$$
- We can write:

$$\frac{\Delta\varepsilon_z}{\varepsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma} = \frac{\sigma_\gamma \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right)}{\sigma_\gamma} = \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right) = \left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)$$

$$\frac{\Delta p}{p} = \frac{\Delta\gamma}{\beta_v^2\gamma} = \frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)$$

Longitudinal Cooling with Wedges and Dispersion (II)

- Thus:

$$\Delta J_z(\text{wedge}) = \frac{\Delta\varepsilon_z/\varepsilon_z}{\Delta p/p} = \frac{\left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)}{\frac{\ell}{\beta_v^2\gamma}\left(\frac{d\gamma}{ds}\right)} = \frac{D}{h}$$

$$J_z = J_z(\text{no wedge}) + \Delta J_z(\text{wedge})$$

- Note that J_x or J_y will have to change in the opposite direction of any change due to ΔJ_z since J_6 is an invariant.

Sources of Longitudinal Heating

Since $\epsilon_z = \sigma_\gamma \, \sigma_t \, c$, and t and thus σ_t is conserved in an interaction

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

Straggling $:\Delta(\sigma_\gamma) \approx \dfrac{\Delta\sigma_\gamma^2}{2\sigma_\gamma} \approx \dfrac{1}{2\sigma_\gamma} 0.06 \, \dfrac{Z}{A} \left(\dfrac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \dfrac{\beta_v^2}{2}\right) \rho \, \Delta s$

$\Delta E = E \, \beta_v^2 \, \dfrac{\Delta p}{p}$, so : $\Delta s = \dfrac{\Delta E}{dE/ds} = \dfrac{1}{dE/ds} E \, \beta_v^2 \, \dfrac{\Delta p}{p}$

giving:

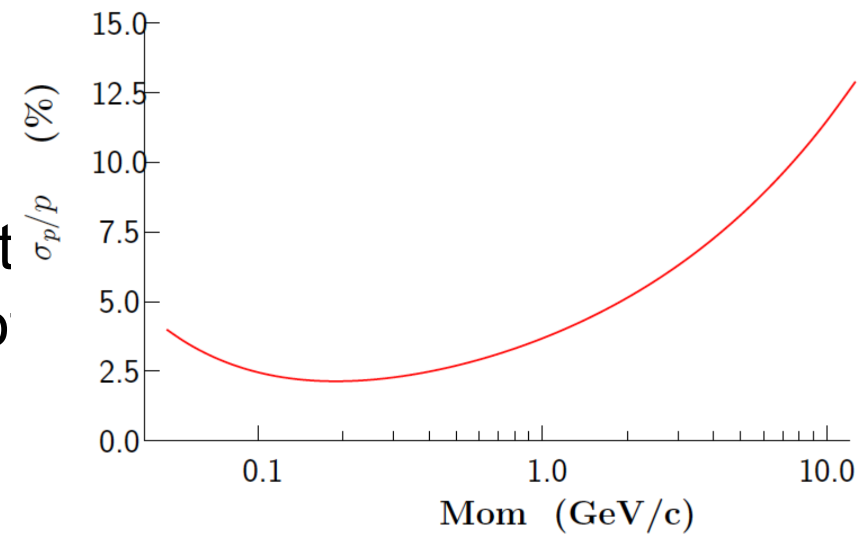
$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{0.06}{2\sigma_\gamma^2} \, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho \, \frac{\beta_v^2 \, E}{dE/ds} \, \frac{\Delta p}{p}$$

Equilibrium Longitudinal Emittance

- Longitudinal cooling term: $$\frac{\Delta\varepsilon_z}{\varepsilon_z} = -J_z \frac{\Delta p}{p}$$
- Equating this with the heating term gives an equilibrium fractional momentum spread of:

$$\frac{\sigma_p}{p} = \underbrace{\left(\frac{m_e}{m_\mu}\sqrt{\frac{0.06\,Z\rho}{2A\left(d\gamma/ds\right)}}\right)}_{\approx 1.36\%\text{ for } H_2}\sqrt{\frac{\gamma}{\beta_v^2}\left(1-\frac{\beta_v^2}{2}\right)\frac{1}{J_z}}$$

- Without emittance exchange, J_z would be small or negative and the equilibrium wouldn't exist. For $J_z \approx 2/3$ and hydrogen, the plot at the right is obtained, which shows a broad minimum around 200 MeV/c



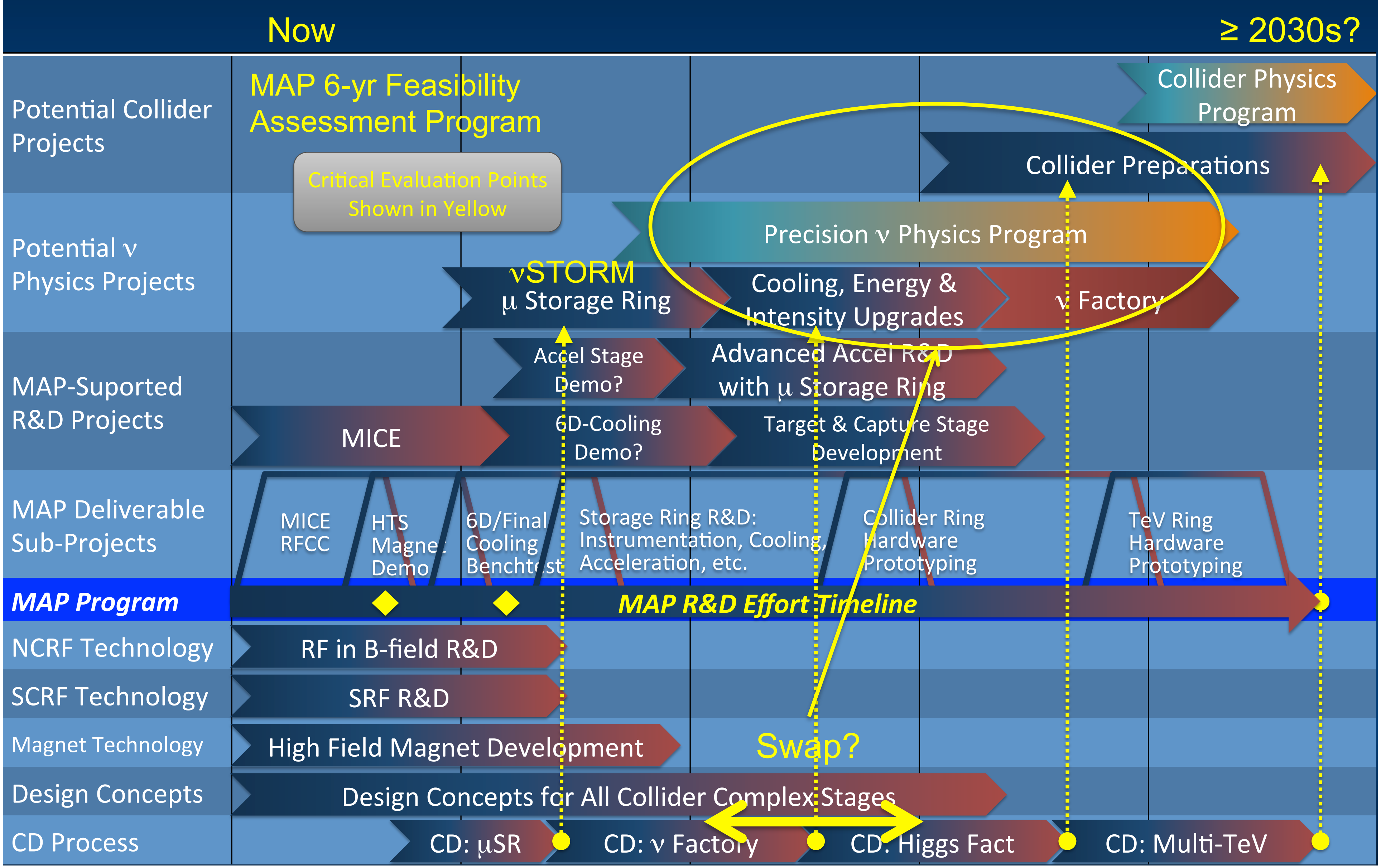
Longitudinal Cooling Summary

- Good 6D cooling can be obtained in a ring
  - Injection and extraction are challenging
  - A short bunch train is required
  - This led to the proposal of a helix design
- Guggenheim 6D Cooling Channel
  - Eliminates injection/extraction problem
  - Eliminates bunch train length problem
  - Allows tapering of channel to optimize performance
  - But uses considerably more hardware than a ring would require
- Helical Cooling Channel utilizing High Pressure RF Cavities
  - Engineering design to develop RF system and integrate it into the magnetic channel underway
  - Have now demonstrated the ability to operate cavities with an ionizing beam
- Snake
  - Unique in that this channel accepts both signs of muons
  - Do not have a design that reaches the targeted final emittances

Conclusion

- What is the timescale for considering a muon collider?
  - The U.S. Muon Accelerator Program is embarking on a 6 year effort to complete a detailed feasibility assessment for such a machine.
    - This includes design studies and technology R&D
  - If the feasibility assessment is positive, we will aim to get approval for a full conceptual design effort for a muon accelerator facility that can support both an intensity frontier neutrino program along with a collider.
  - The options being considered are outlined on the following slide

A “Technically Limited” Staging Scenario with Physics Output at Each Stage



Acknowledgments

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- Bob Palmer (BNL)

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