Electron source for Linear Colliders

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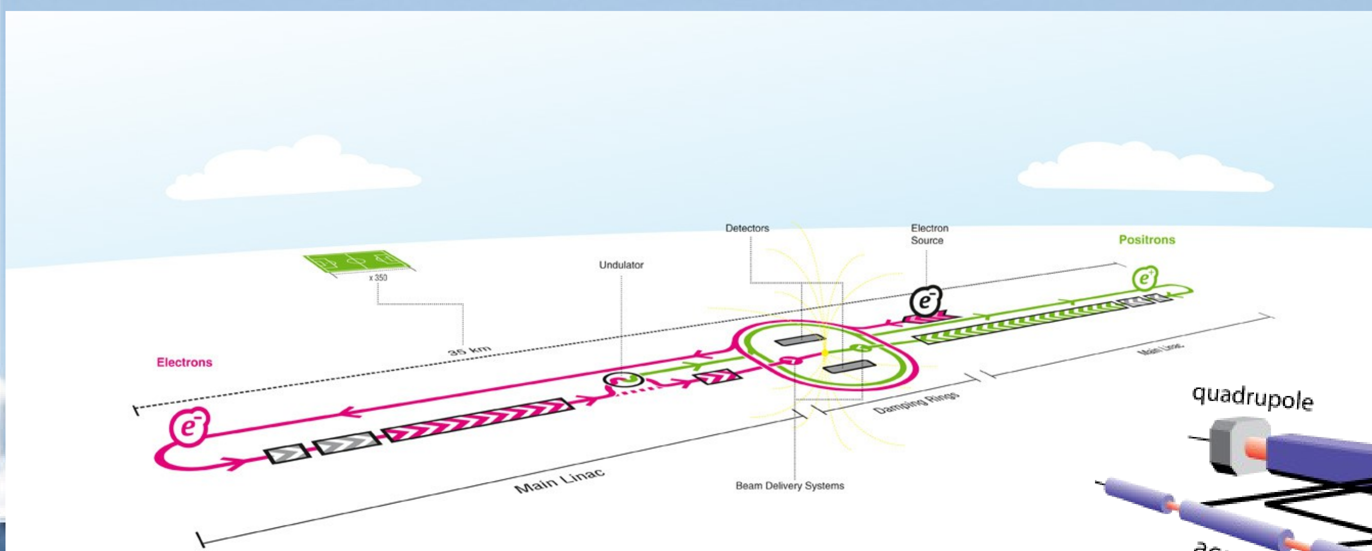

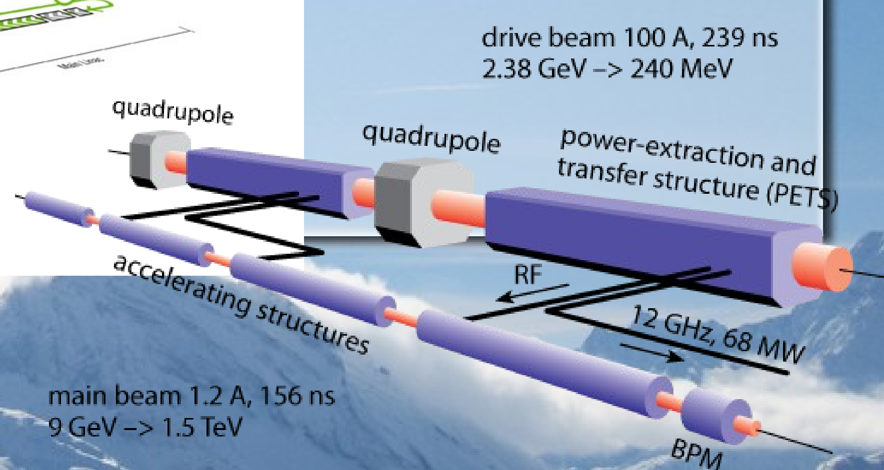

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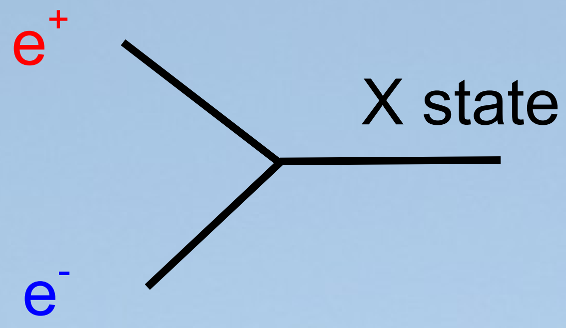
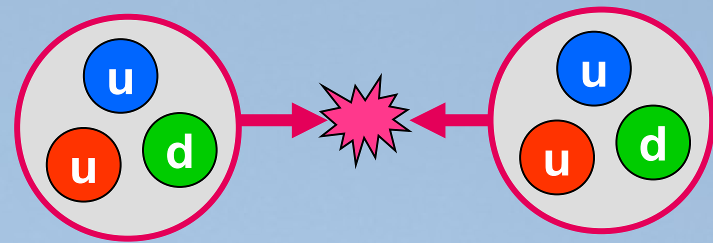
Lepton Collider

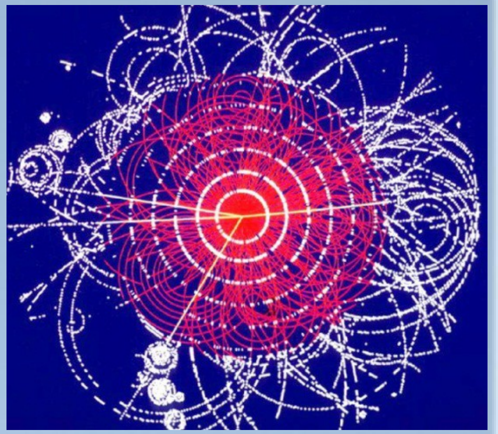


- e+e- collision is a simple interaction.
- The initial states is well defined.
- Easy to reconstrcut the final states.
- This full reconstruction is powerful and essential for e^+e^- colliders.

Hadron collider and Lepton Collider

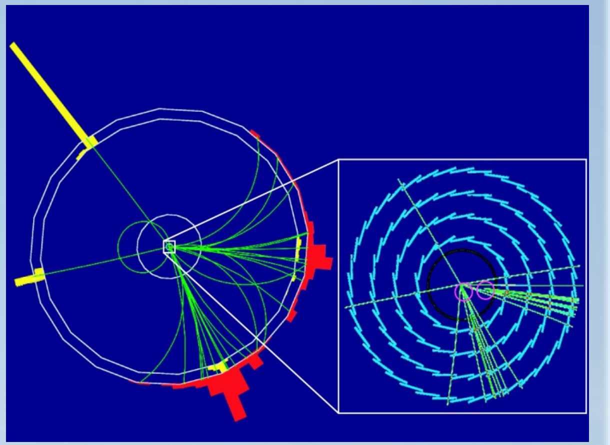
Hadron Collider





- Collision between composite particles (protons).
- Initial state is not defined.
- Extremely high energy, high event rate, large noise.

Lepton Colliders



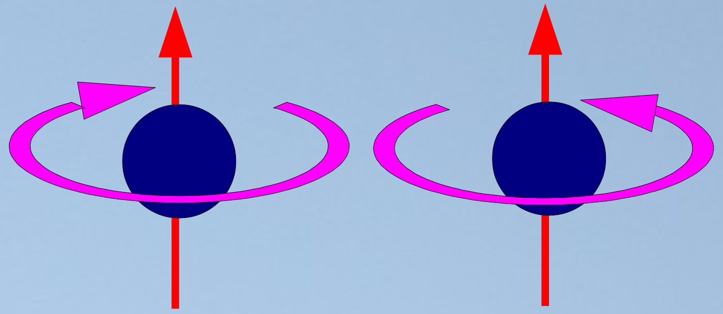
- Collision between elementary particles (leptons).
- The initial state is well defined.
- Full reconstruction of events.

Well defined initial states

- Electron and position are spin ½ fermions. Two eigen spin states.
- In SU(2)xU(1) gauge theory, these two spin eigen states are different particles which has different weak Iso-spin and hyper charge.

$$l_L \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad I_W = \frac{1}{2}, \quad Y_W = -1$$

$$e_R \quad I_W = 0, \quad Y_W = -2$$

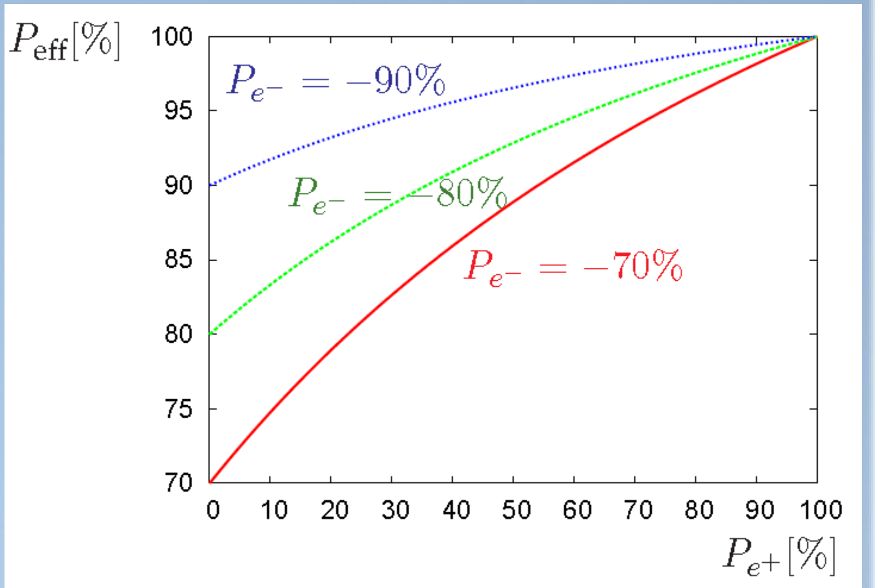


- Ideal well defined initial states means that the beam contains only one spin state.
- Practicall, the beam should be polarized.

$$P \equiv \frac{N_R - N_L}{N_R + N_L}$$

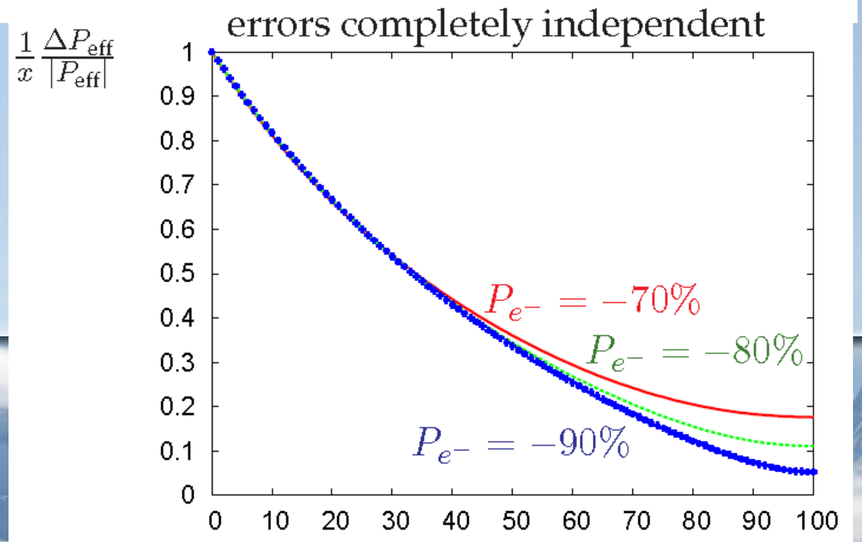
Effective Polarization

- One of the beam has to be polarized.
- If both beams were polarized, polarization effectively could be close to 100% with less ambiguity.



$$P_{eff} \equiv \frac{P_e - P_p}{1 - P_e P_p}$$

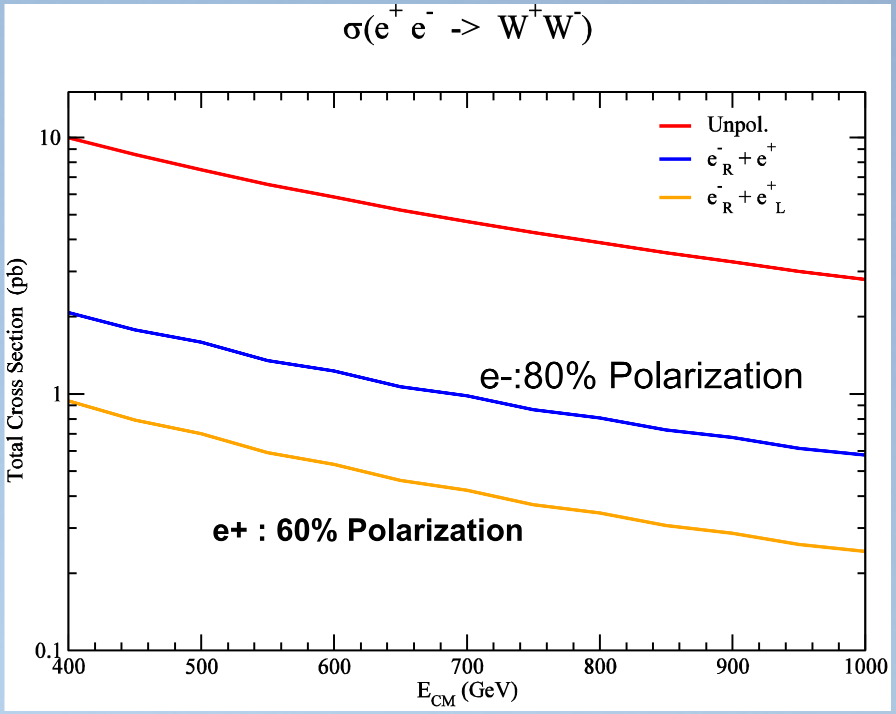
$$\frac{\Delta P_{eff}}{P_{eff}} = \frac{1 - P_e P_p}{1 + P_e P_p} \frac{\Delta P_e}{P_e}$$



G. Moortgat-pick

Polarization

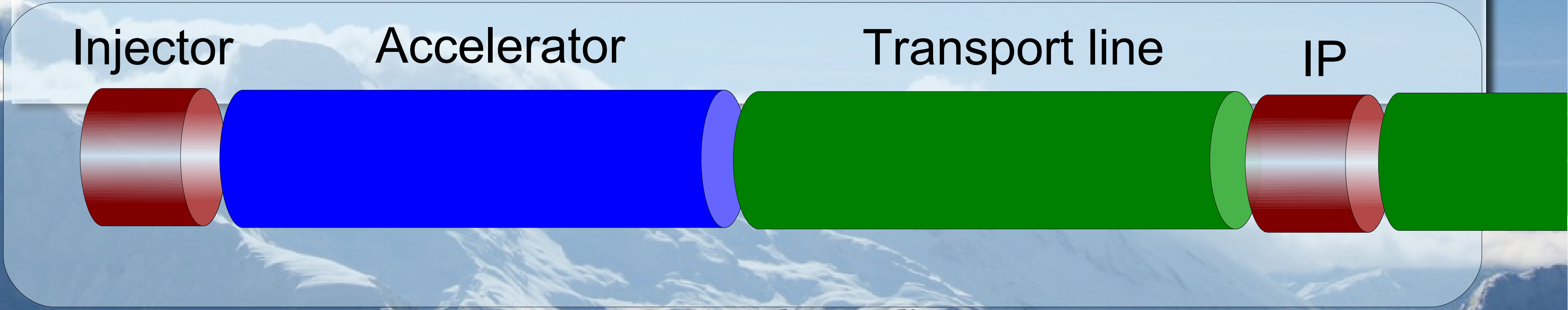
- In e+e- collider, WW-scattering is the biggest background.
- Polarized electron (and also positron) can compensate this background.
- Polarization is important not only to define the initial state, but also to improve the sensitivity for new discovery.



with GRACE System Developed by Computational Physics Group in KEK

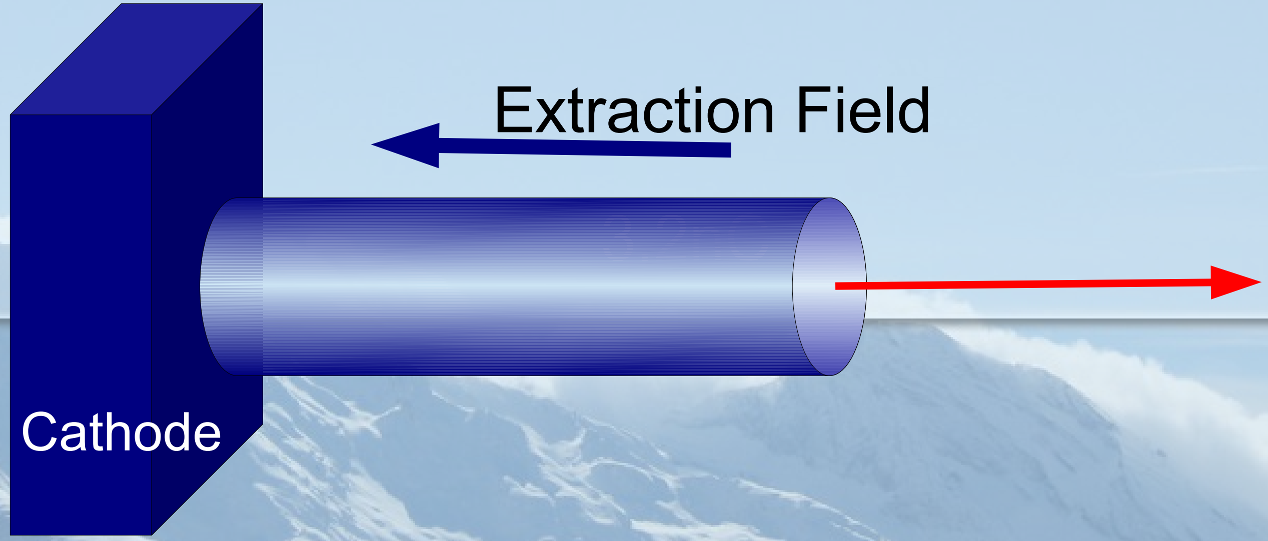
Injector

- What is the injector?
  - Generate accelerate-able particle beams;
- What is the accelerate-able beams?
  - Right amount : Charge
  - Right shape : Beam size, emittance, bunch length
  - Right direction: along beam line
  - Right time : timing, phase



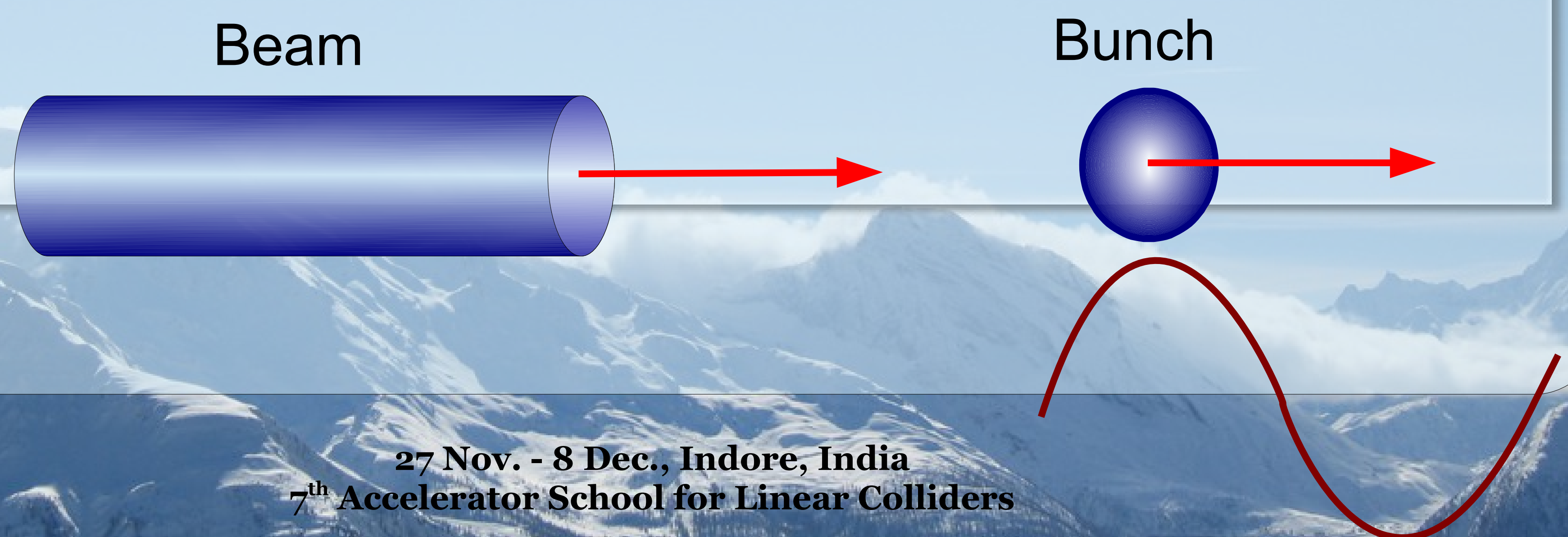
Electron Gun

- What is electron gun?
  - Generate electron beam
    - Right amount : Charge
    - Right shape : Beam size, emittance, bunch length
    - Right direction: beam line
    - Right time : timing, phase



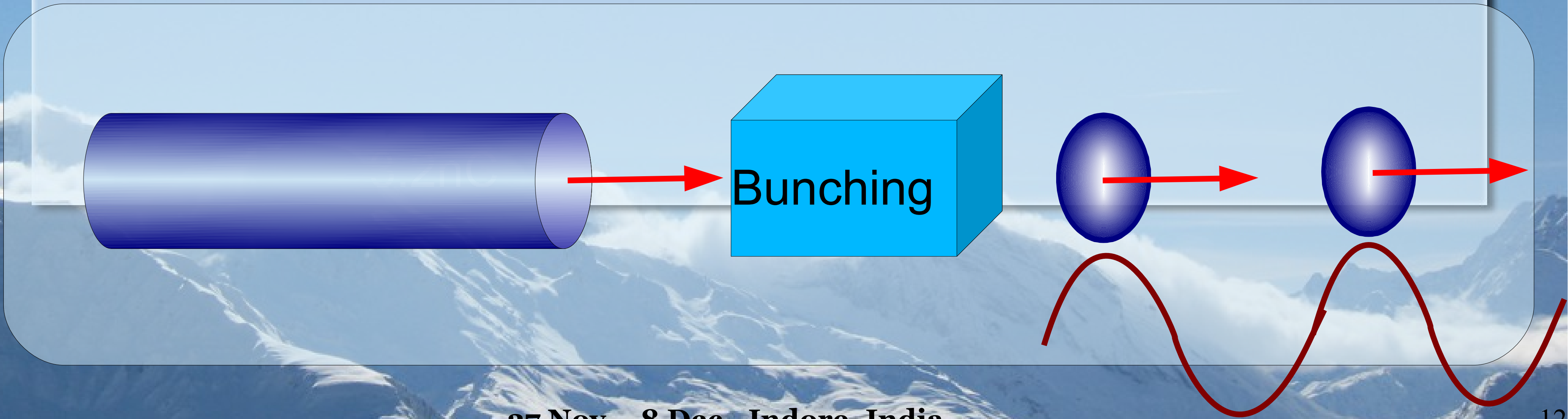
Bunching (1)

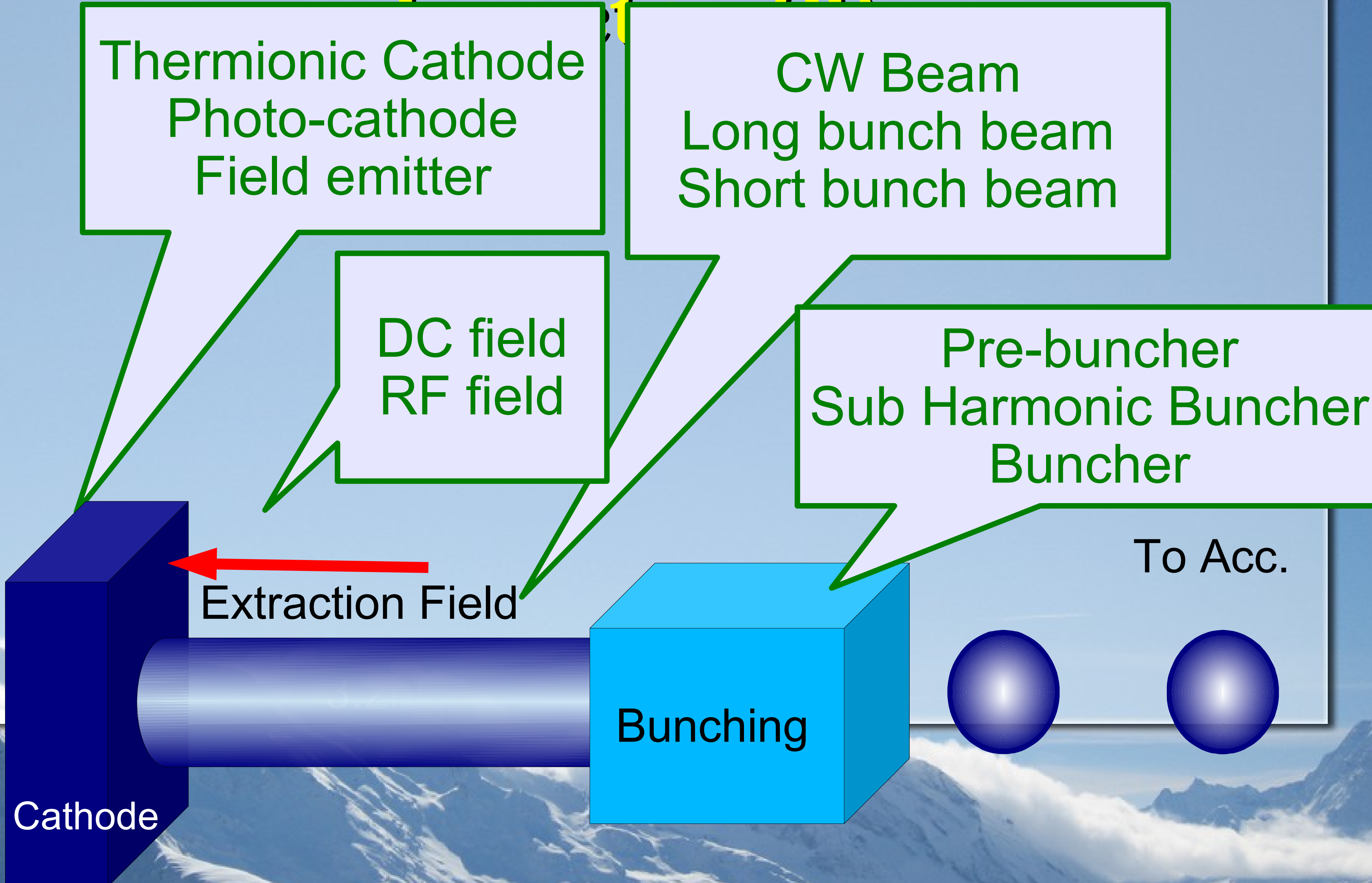
- Bunching : Bunch the beam. What is the bunch?
  - Beam : collimated particle flow.
  - Bunch : collimated and clustered particle flow. The length should be short enough comparing to the RF period for uniform acceleration.



Bunching (2)

- Bunching: Shorten the longitudinal length of the beam.
  - Right amount : Charge
  - Right shape : Beam size, emittance,bunch length
  - Right direction: beam line
  - Right time : timing, phase




Thermionic Cathode
Photo-cathode
Field emitter
CW Beam
Long bunch beam
Short bunch beam
DC field
RF field
Pre-buncher
Sub Harmonic Buncher
Buncher
To Acc.
Extraction Field
Bunching
Cathode

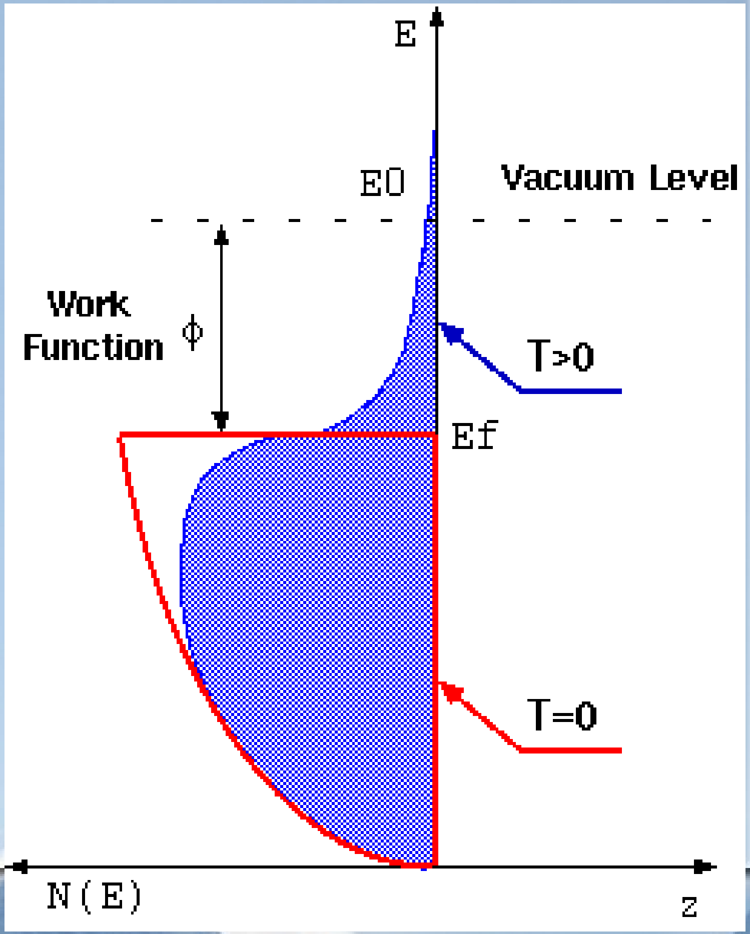
Electron Emission

Electron Emission (1)

- Thermal electron emission : Electron emission from the heated material (typically 1000 - 3000K).
- Field emission: Emission from the high field gradient surface.
- Photo-electron emission: Emission by photo-electron effect.
- Secondary electron emission: Emission induced by electron absorption.

Electronic States

- Electrons in a metal are confined in a well potential and distributed according to Fermi-Dirac Distribution.
- T=0: Electrons occupy the energy states up to Fermi-level (Fermi energy, E_f).
- T>0: Electron distribution extends to higher energy state due to the thermal energy.



Electronic States (2)

Electron density in a metal is product of state density D(ε) and distribution function f(ε),

$$n(\epsilon)=D(\epsilon)f(\epsilon) \qquad (1-1)$$

State density in phase space (x, v_x) – $(x+dx, v_x+dv_x)$.... is

$$D(\epsilon)=\frac{2\mathrm{m}^2}{h^3}dx\,dy\,dz\,dv_x\,dv_y\,dv_z \qquad (1-2)$$

Distribution function f(ε) is given by Fermi-Dirac function

$$f(\epsilon)=\frac{1}{\exp\left(\frac{\epsilon-\mu}{kT}\right)+1} \qquad (1-3)$$

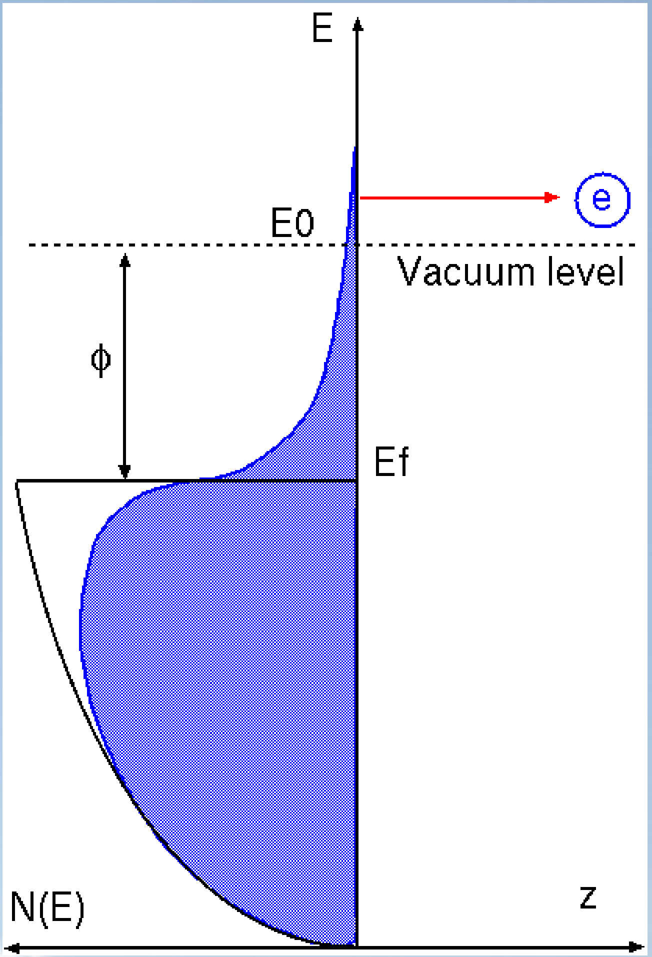
Number of electron with energy ε<E is

$$N(\epsilon)=\int_0^E f(\epsilon)D(\epsilon)d\epsilon \qquad (1-4)$$

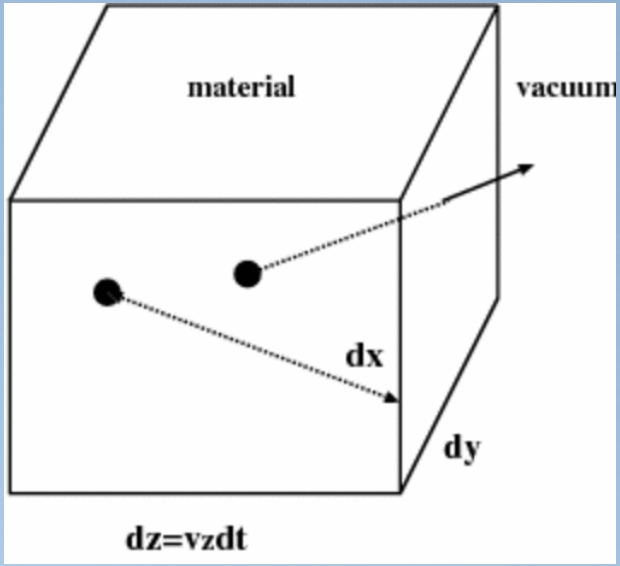
Thermal Electron Emission

- If the temperature is sufficiently high, so that electrons are distributed up to more than the vacuum level (E_0), the electrons escape out to the outside.
- The gap between the vacuum level and the Fermi energy is Work function, φ, which characterize the thermal emission.

$$E > E_0 = E_f + \phi$$



Emission Density (1)



Number of emitted electron:

In depth (z-direction)

$$z \le v_z \Delta t \qquad (1-5)$$

Kinetic energy for z-direction must be more than vacuum potential energy, μ+Φ

$$v_z \ge v_{vac} \equiv \sqrt{\frac{2(\mu+\phi)}{m}} \qquad (1-6)$$

Number of electron emitted from the cathode is give by

$$N = \int dx \int dy \int_0^{v_z \Delta t} dz \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z\, f(\epsilon) D(\epsilon) \quad (1-7)$$

Emission Density (2)

By integrating x, y, z and inserting distribution function,

$$N=\Delta x \Delta y \Delta t \frac{2m^3}{h^3}\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z \frac{v_z}{\exp\left(\frac{\epsilon-\mu}{kT}\right)+1} \qquad (1-8)$$

From this equation, emission density per unit time is obtained

$$\sigma \equiv \frac{N}{\Delta x \Delta y \Delta t} = \frac{2m^3}{h^3}\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z \frac{v_z}{\exp\left(\frac{\epsilon-\mu}{kT}\right)+1} \qquad (1-9)$$

Emission Density (4)

Because ε-μ≫kT, f(ε) is approximated as

$$\frac{1}{\exp\left(\frac{\epsilon-\mu}{kT}\right)+1}\sim\exp\left(\frac{\mu-\epsilon}{kT}\right) \qquad (1-10)$$

The density is simplified as

$$\sigma=\frac{2\mathrm{m}^3}{h^3}\int_{-\infty}^{+\infty}dv_x\int_{-\infty}^{+\infty}dv_y\int_{v_{vac}}^{+\infty}dv_z\,v_z\exp\left(\frac{\mu-\epsilon}{kT}\right) \qquad (1-11)$$

Replacing the energy with the velocity,

$$\epsilon=\frac{m}{2}\left(v_x^2+v_y^2+v_z^2\right)$$

$$\sigma=\frac{2\mathrm{m}^3}{h^3}\exp\left(\frac{\mu}{kT}\right)\int_{-\infty}^{+\infty}dv_x\int_{-\infty}^{+\infty}dv_y\int_{v_{vac}}^{+\infty}dv_z\,v_z\exp\left(\frac{-m\left(v_x^2+v_y^2+v_z^2\right)}{2\mathrm{kT}}\right) \qquad (1-12)$$

Emission Density (5)

Integral for v_x and v_y can be performed as

$$\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \exp\left(\frac{-m\left(v_x^2+v_y^2\right)}{2kT}\right) = \frac{2\pi kT}{m} \qquad (1-13)$$

and for v_z as

$$\int_{v_{vac}}^{+\infty} dv_z v_z \exp\left(\frac{-mv_z^2}{2kT}\right) = \frac{kT}{m}\exp\left(\frac{-mv_{vac}^2}{2kT}\right) \qquad (1-14)$$

we obtain

$$\sigma = \frac{4\pi mk^2T^2}{h^3}\exp\left(-\frac{\phi}{kT}\right) \qquad (1-15)$$

Electric current density J is given by

$$J = \frac{4\pi emk^2T^2}{h^3}\exp\left(-\frac{\phi}{kT}\right) \qquad (1-16)$$

Richardson-Dushman Equation

$$J = AT^2 e^{-\frac{\phi}{kT}} \qquad (1-17)$$

$$A = \frac{4\pi emk^2}{h^3} = 1.20\times10^6 [A/m^2 K^2]$$

A : thermionic emission constant

T: Temperature (K)

k : Boltzmann constant ; 1.38E-23 (J/K)

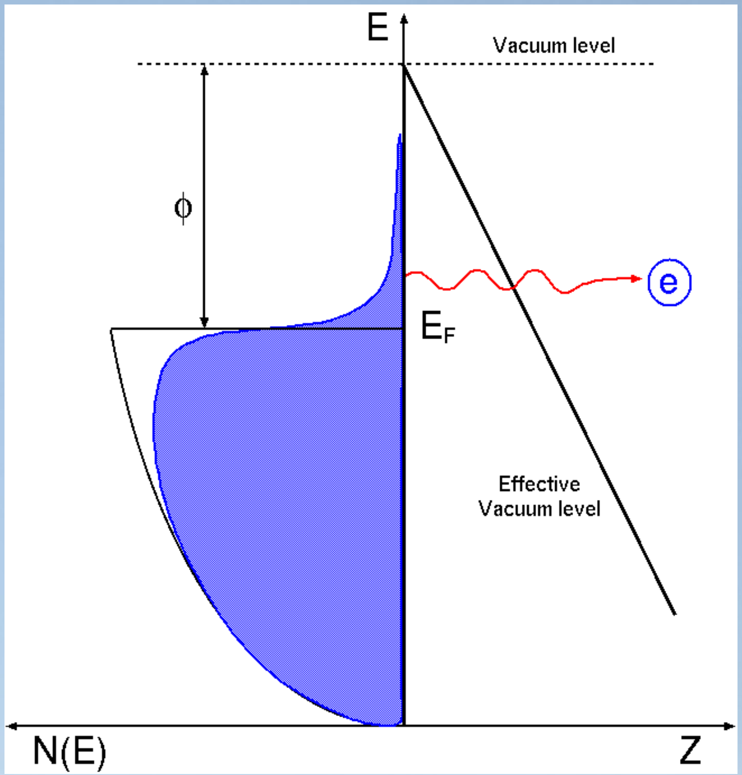
e : electronic charge

m : electron mass

h : Plank constant ; 6.63E-34 (Js)

Field Emission (1)

- FE is electron emission observed from cold (not hot) material when a high electric field is applied.
  - Large surface field makes the potential barrier very thin.
  - The tunnel current becomes significant with 1E+8 V/m.

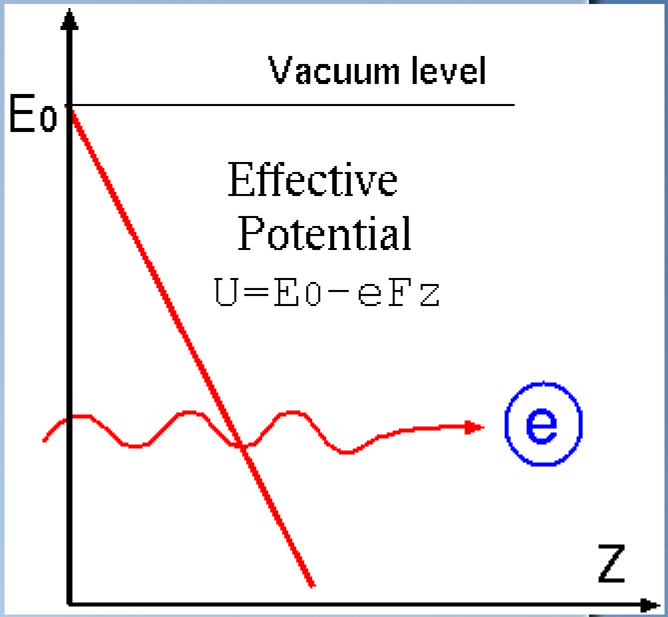


Field Emission (2)

$$J = e \int_0^\infty n(\epsilon_z) P(\epsilon_z) d\epsilon_z \qquad (1-18)$$

Electron density

Tunneling probability



Tunneling Probability by WKB method

$$P(\epsilon_z, F) = \exp\left[-\int_0^w \sqrt{\frac{8\mathrm{m}(2\pi)^2}{h^2}\{U(z)-\epsilon_z\}}\, dz \right]$$

$$= \exp\left[\frac{-8\pi\sqrt{2\mathrm{m}}}{3\mathrm{heF}} (E_0 - \epsilon_z)^{3/2} \right] \qquad (1-19)$$

$$U(z) = E_0 - eFz$$

$$w = \frac{E_0 - \varepsilon_z}{eF}$$

Field Emission (3)

By Taylor expansion,

$$(E_0-\epsilon_z)^{3/2}=\left[\phi+(\mu-\epsilon_z)\right]^{3/2}=\phi^{3/2}+\frac{3}{2}\phi^{1/2}(\mu-\epsilon_z) \qquad (1-21)$$

In the low temperature limit, the current density is

$$J(F)=\frac{4\pi em}{h^3}\int_0^\infty d\epsilon_z(\mu-\epsilon_z)\exp\left[-8\pi\frac{\sqrt{2m}}{3heF}(E_0-\epsilon_z)\right]$$

$$=\frac{4\pi em}{h^3}\exp\left(\frac{-8\pi\sqrt{2m}}{3heF}\phi^{3/2}\right)\int_0^\infty d\acute{\epsilon}\,\acute{\epsilon}\exp\left[-4\pi\frac{\sqrt{2m}}{heF}\phi^{1/2}\acute{\epsilon}\right] \qquad (1-22)$$

where $\varepsilon'=\varepsilon_z-\mu$.

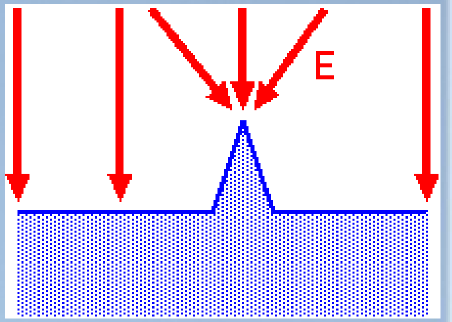
$$J=\frac{e^3F^2}{8\pi\phi}\exp\left(-\frac{8\pi\sqrt{2m}}{3heF}\phi^{3/2}\right) \qquad (1-23)$$

(Fowler-Nordheim formula)

Fowler-Nordheim Plot

Fowler-Nordheim formula with field enhancement factor κ

$$J = \frac{e^3 \kappa^2 F^2}{8 h \pi \phi} \exp\left(-\frac{8\sqrt{2m}}{3 h e \kappa F} \phi^{3/2}\right) \qquad (1-24)$$



κ: local field enhancement by surface condition,

Taking $\ln(J/F^2)$ and plotting as a function of 1/F,

$$\ln\left(J/F^2\right) = \ln\left(\frac{e^3 \kappa^2}{8 h \pi \phi}\right) - \left(\frac{8\sqrt{2m}}{3 h e \kappa} \phi^{3/2}\right) \frac{1}{F} \qquad (1-25)$$

The gradient gives information on the surface condition, κ.

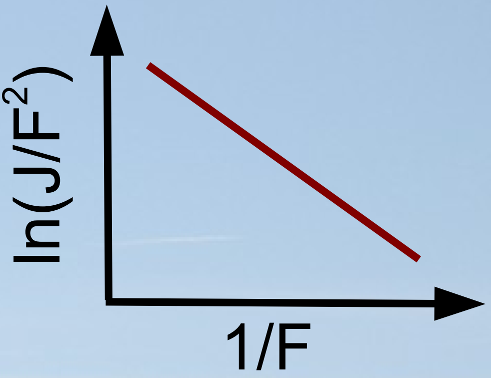
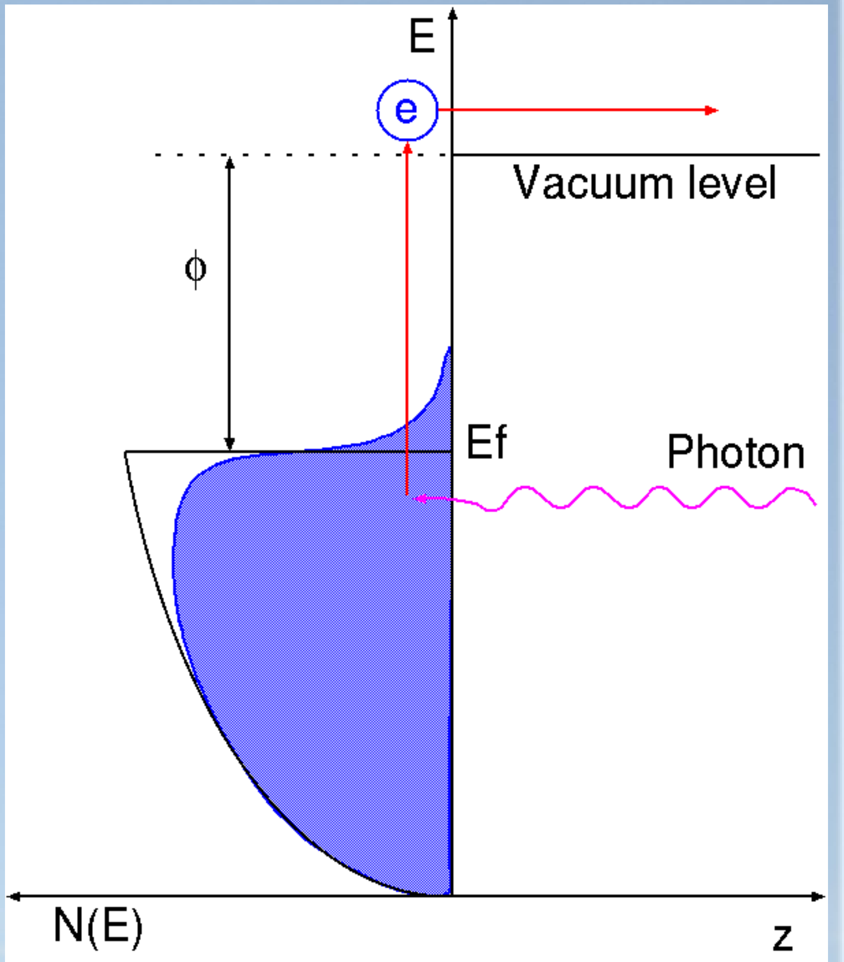


Photo-electron Emission

- Electron emission by photo-electron effect.
  - Photons excite electrons into higher energy states.
  - If the states are higher than the vacuum level, the electrons goes to vacuum.
  - Condition for photo-emission is approximately,

$$h\nu > \phi \qquad (1-26)$$



Emission Densiy (1)

Photo-electron current density is given by

$$J = \frac{4\pi emkT}{h^3} P \int_{E_0 - h\nu}^{\infty} d\epsilon_z \ln\left[1 + \exp\frac{(\mu - \epsilon_z)}{kT}\right] \qquad (1-27)$$

where P is transition probability by photon excitation. For further manipulation, replacing $y=(\varepsilon_z+h\nu-E_0)/kT$ and $\delta=h(\nu-\nu_0)/kT$,

$$J = \frac{4\pi emkT}{h^3} P \int_{E_0 - h\nu}^{\infty} d\epsilon_z \ln\left[1 + \exp\frac{(\mu - \epsilon_z)}{kT}\right]$$

$$= \frac{4\pi em k^2 T^2}{h^3} P \int_0^{\infty} dy \ln\left[1 + \exp(\delta - y)\right] \qquad (1-29)$$

Emission Density (2)

$$f(\delta)=\int_0^\infty dy \ln\left[1+e^{\delta-y}\right] \qquad (1-30)$$

(a) δ=h(ν-ν₀)/kT <0 (ph. energy is less than f):

$$f(\delta)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{n\delta}}{n}\int_0^\infty dy\, e^{-ny}$$

$$=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{n\delta}}{n^2} \qquad (1-31)$$

since

$$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} \qquad (1-32)$$

$$\ln(1+e^{\delta-y})=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{n(\delta-y)}}{n} \qquad (1-33)$$

Emission Density (3)

(b) $\delta = h(\nu - \nu_0)/kT > 0$ (ph. energy is more than f),

$$f(\delta) = \left(\int_0^\delta dy + \int_\delta^\infty dy \right) \left[\ln\left(1 + e^{\delta - y}\right) \right] \qquad (1-34)$$

(b-1) first integral, $w = \delta - y$

$$\int_0^\delta dy \ln\left(1 + e^{\delta - y}\right) = \int_0^\delta dw \ln\left(1 + e^{w}\right)$$

$$= \int_0^\delta dw \left\{ w + \ln\left(1 + e^{-w}\right) \right\}$$

$$= \left[\frac{w^2}{2} \right]_0^\delta + \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} \left[e^{-nw} \right]_0^\delta$$

$$= \frac{\delta^2}{2} + \frac{\pi^2}{12} + \sum_{n=1}^{\infty} (-1)^n \frac{e^{-n\delta}}{n^2} \qquad (1-35)$$

Emission Density (4)

(b-2) second integral, w=y-δ

$$\int_{\delta}^{\infty} dy \ln\left(1+e^{\delta-y}\right)=\int_{0}^{\infty} dw \ln\left(1+e^{-w}\right)$$

$$=\left[w \ln\left(1+e^{-w}\right)\right]_0^{\infty}+\int_{0}^{\infty} dw \frac{w}{1+e^{w}} \quad (1-36)$$

the first term of rhs is 0 and the second term is

$$\int_{0}^{\infty} dw \frac{w}{1+e^{w}}=\frac{\pi^2}{12} \quad (1-37)$$

Finally, sum of (b-1) + (b-2) gives f(δ)

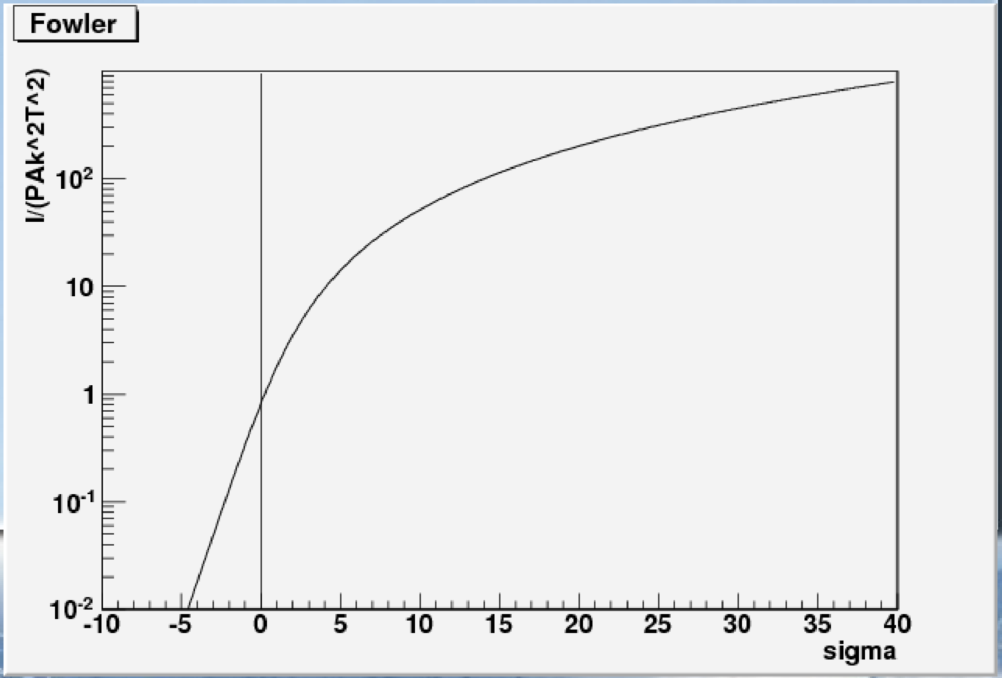
$$f(\delta)=\frac{\delta^2}{2}+\frac{\pi^2}{6}+\sum_{n=1}^{\infty}(-1)^n \frac{e^{-n\delta}}{n^2} \quad (1-38)$$

Fowler Equation

$$J = AT^2 P \begin{cases} \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{n\delta}}{n^2} & \delta < 0 \\ \displaystyle\frac{\delta^2}{2} + \frac{\pi^2}{12} + \sum_{n=1}^{\infty} (-1)^{n} \frac{e^{-n\delta}}{n^2} & \delta > 0 \end{cases} \qquad (1-39)$$

$$A = \frac{4\pi e m k^2}{h^3}$$

- Fowler equation gives photo-current spectrum.
- The absolute density is hard to estimate because *P* depends on the surface condition.



Quantum Efficiency

Quantum Efficiency, η, is practically used to qualify the photo-electron emission

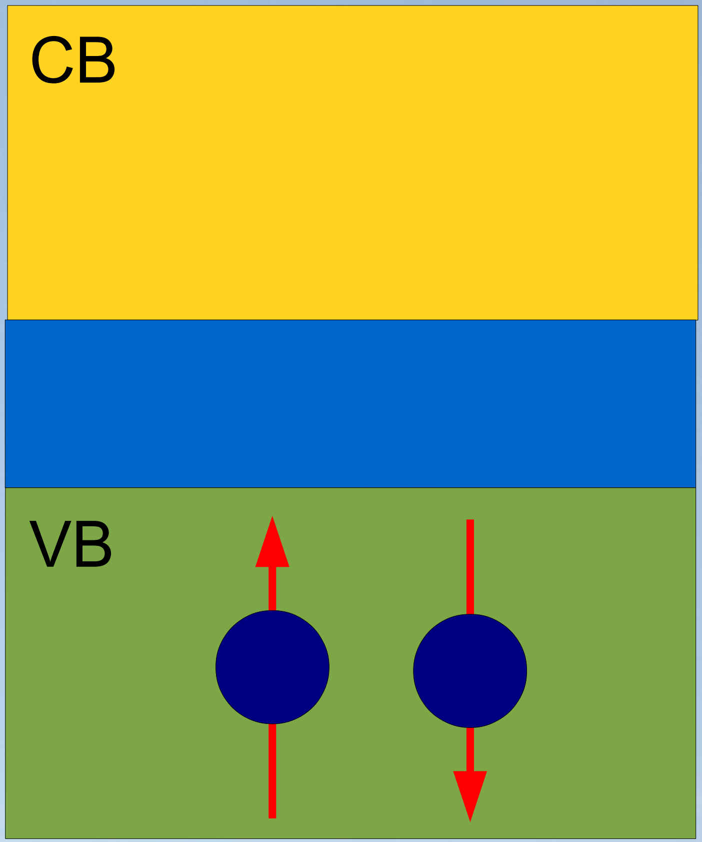
$$\eta = \frac{number\ of\ photoelectrons}{number\ of\ photons} \qquad (1-40)$$

With practical units,

$$\eta[\%] = 124 \frac{J[nA]}{P[\mu W]\lambda[nm]} \qquad (1-41)$$

Polarized electron

- Polarized Electron is generated by photo-emission with GaAs semiconductor cathode.
- It is essential for polarization that GaAs is direct transission type semiconductor.
- Transission from the valence band (VB) to conduction band (CB) by circularily polarized photon is spin dependent.



Excitation

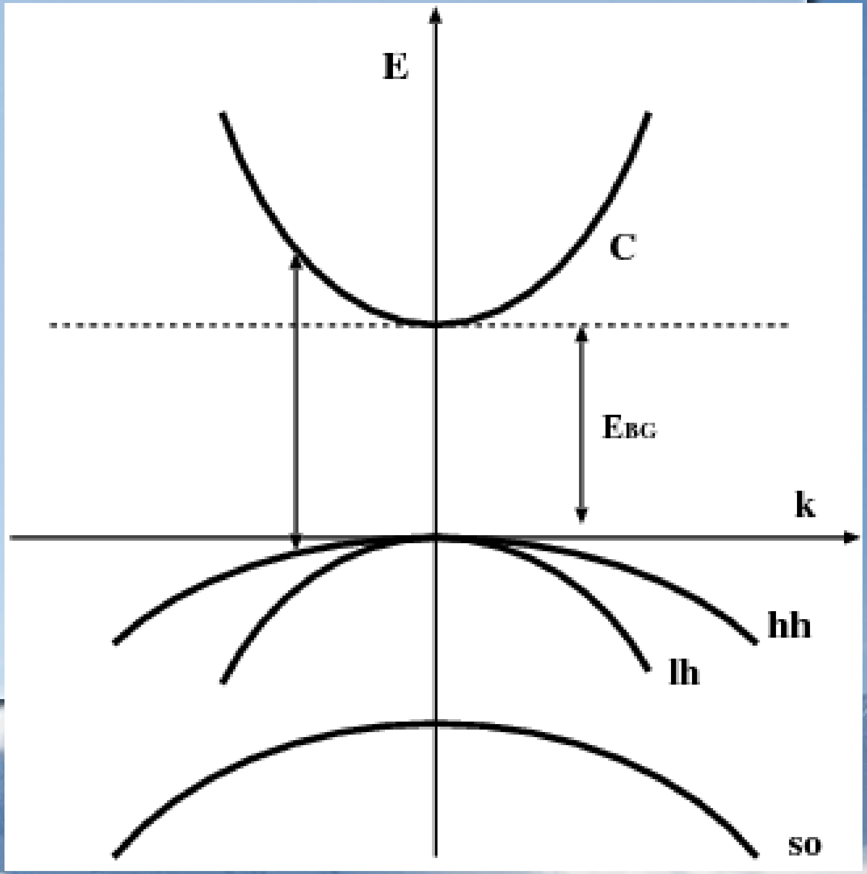
- Transition probability ~ Fermi's golden rule

$$W_{i\rightarrow f}=\frac{2\pi}{\hbar}M^2 D(\hbar\omega)f(E)$$

  - ***M***: Matrix element
  - ***D:*** joint density of states of ħω photon
  - ***f: f***ermi distribution function
- Considering only near the band gap, the transition probability is proportional to ***M*****.**

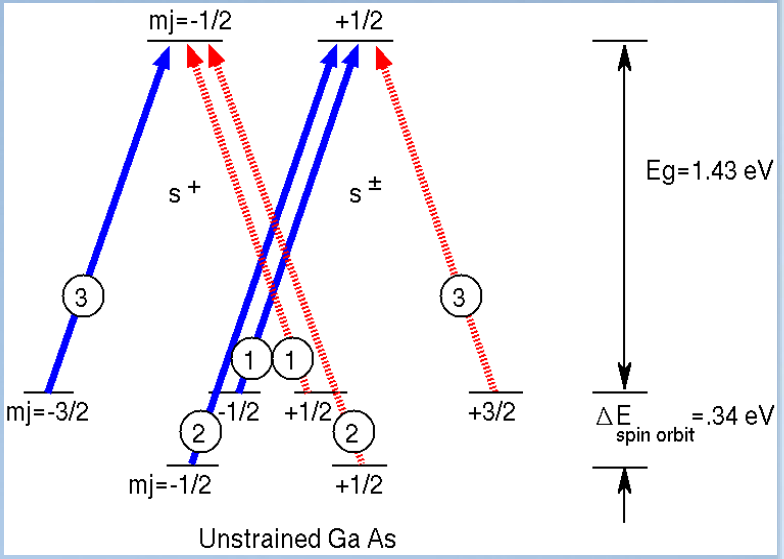
Matrix Element of GaAs

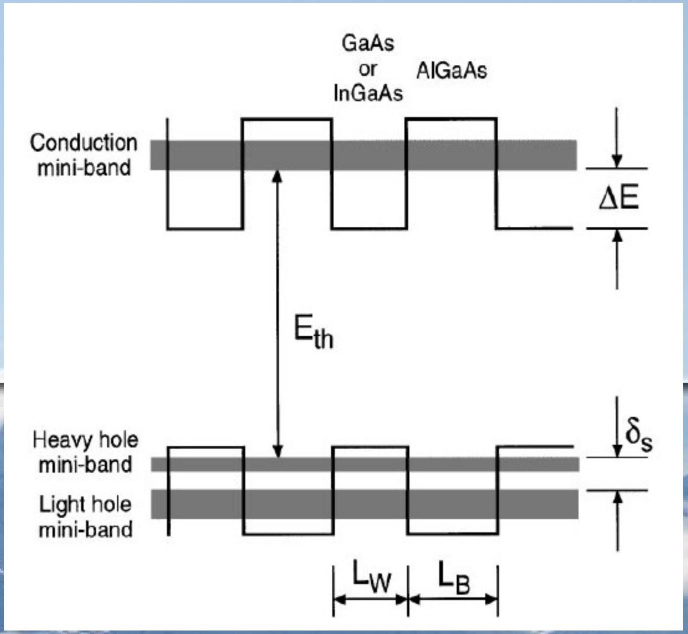
- Band gap of GaAs is Γ point (k=0).
  - VB:
    - J=|3/2,±3/2> (heavy hole)
    - J=|3/2, ±1/2> (light hole).
  - CB:
    - J=|1/2,±1/2>
- Matrix Element of transition (Clebsh-Gordon coef.)
  - Heavy hole: $\sqrt{3}/2$
  - Light hole: 1/2



Polarization

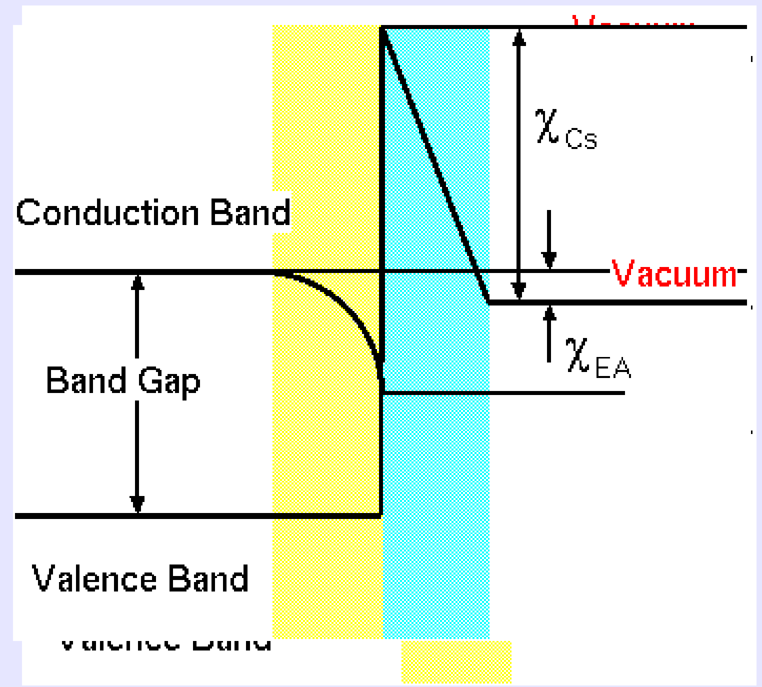
- Electron excited by circularly polarized photon is 50% polarized , 3:1.
- The polarization is enhanced by introducing energy selection.
    - Untied the degeneration by strained or super-lattice structure.
    - One of the transition is suppressed and the polarization can be up to 90%.





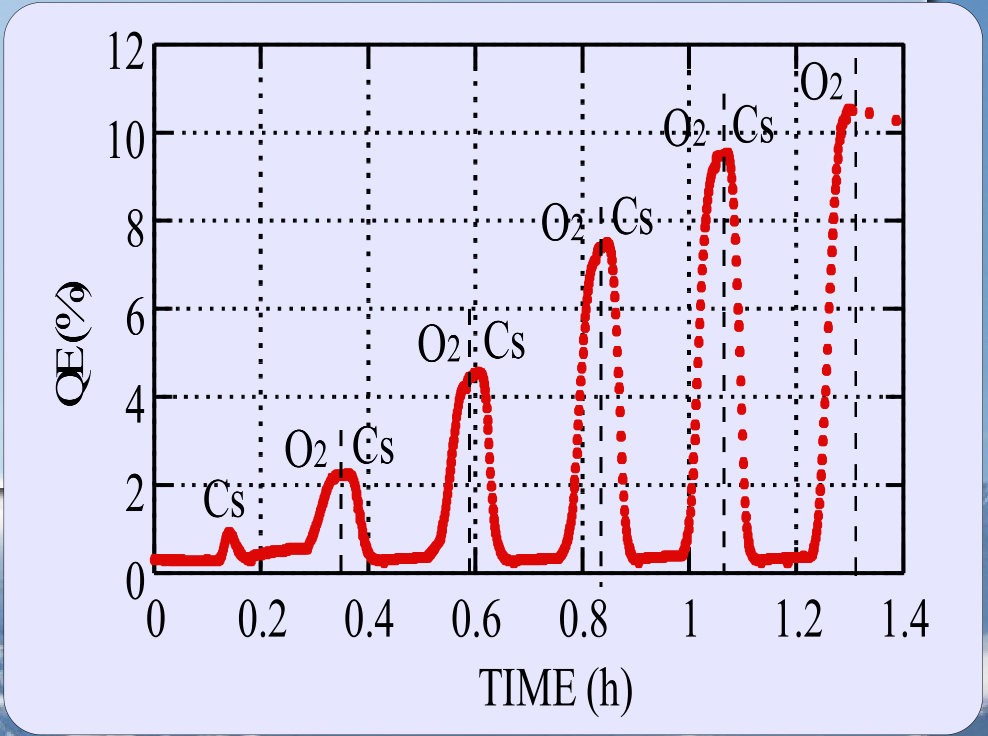
NEA surface (1)

- On NEA surface, the vacuum level is less than the lowest state of CB.
- NEA surface is important for polarized electron emission to the vacuum, since the polarized electrons are at the bottom of CB.
- NEA surface is made by artificial treatment.



NEA surface (2)

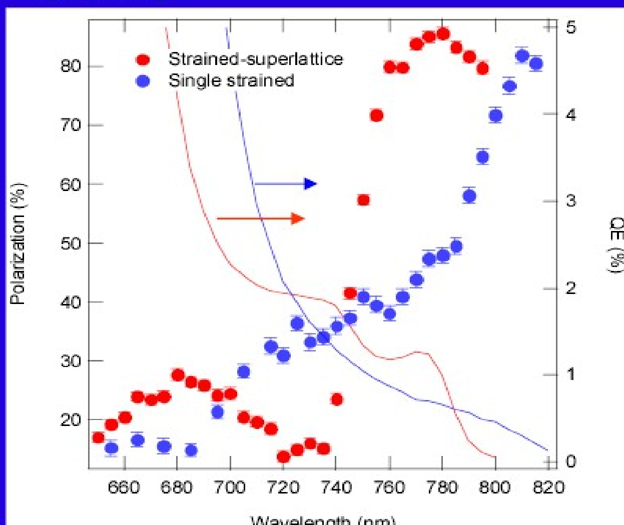
- NEA surface is made by evaporation of Cs and O_2 on conditioned GaAs.
- GaAs conditioning: chemical etching by H_2SO_4 and treatment by HCl-Isopropanol solution followed by heat cleaning.
- Alternating deposition of Cs and O_2.
- The process should be made in extremely low vacuum pressure, <5.0E-9Pa.



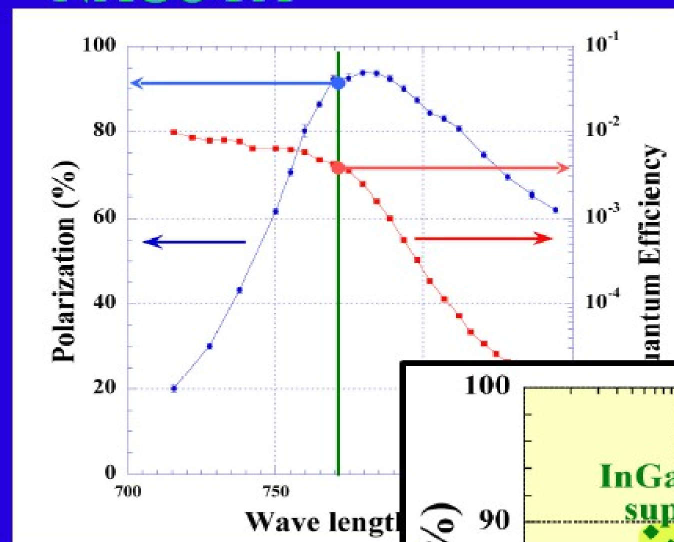
Polarized Electron (3)

Performance of GaAs/GaAsP superlattice

SLAC

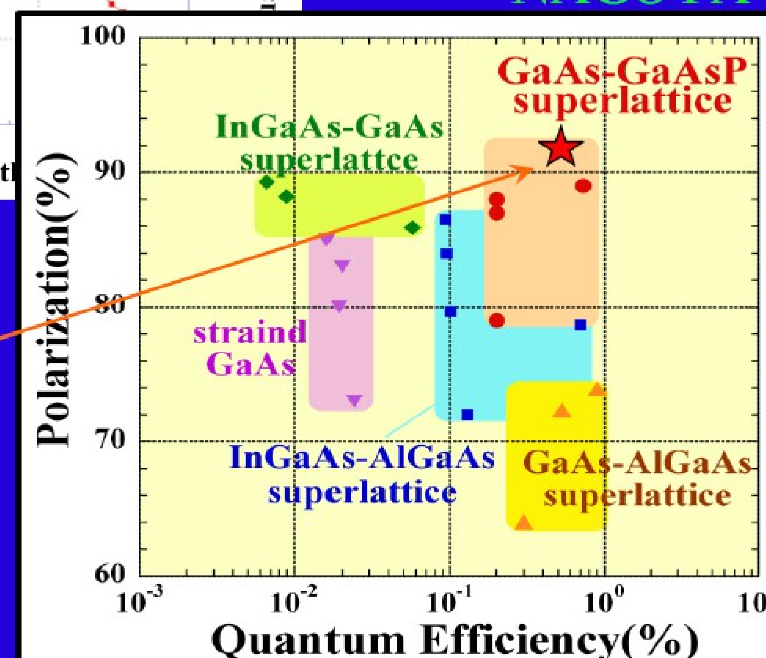


NAGOYA



T. Nishitani,
M. Yamamoto

NAGOYA



GaAs-GaAsP superlattice shows
the best performance !

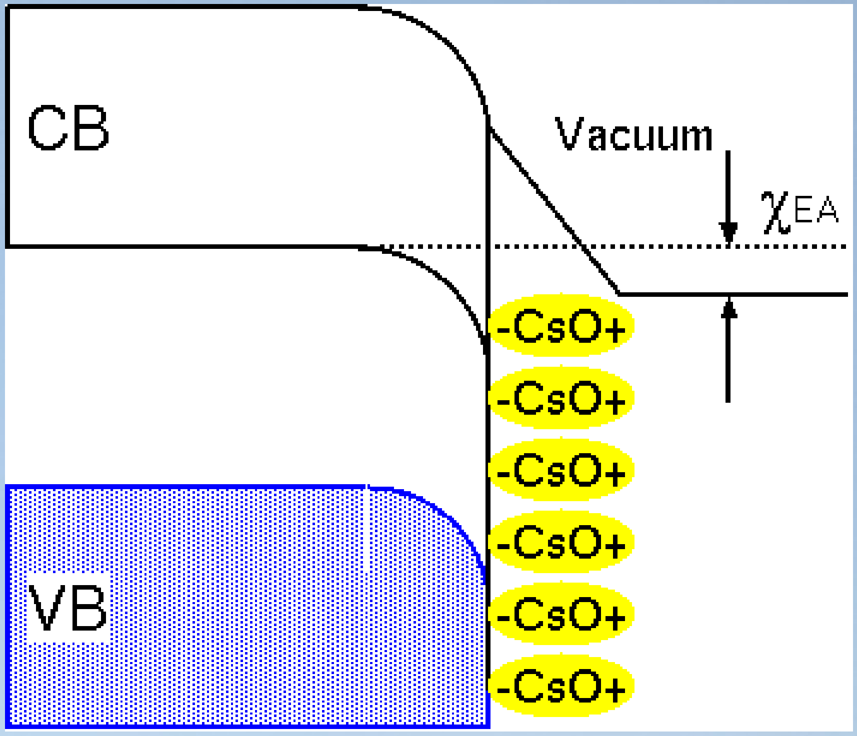
@778nm

Polarization ~ 90%

Q.E. ~ 0.5%

NEA model (1)

- There is no established model for NEA surface. There are two main candidates.
- **Cs-O electric dipole model**
  - Composition of Cs-Ox forms electric dipole on the surface.
  - The vacuum potential is effectively decreased by the dipole potential.



NEA model (2)

- **Hetero-junction model**
  - III-V semiconductor + Cs_xO_{1-x} hetero-junction is made at the surface of GaAs.
  - Bulk Cs_2O is n-type semi-conductor, φ=0.8eV and electron affinity χ=0.55 eV.
  - In GaAs and Cs_2O hetero-junction, the vacuum level becomes below the conduction band in GaAs.

C. A. Sanford, J. Vac. Sci. Tech. B7(6), 1989

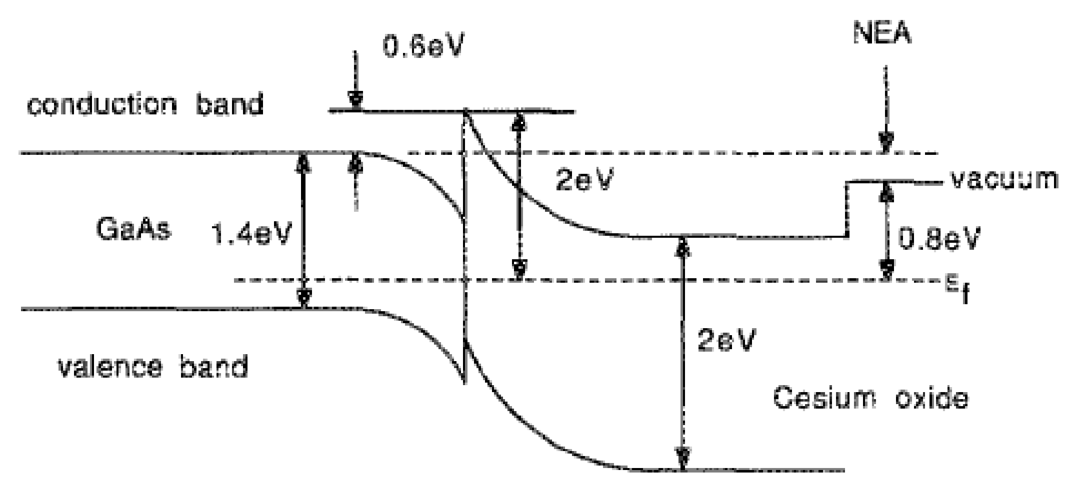


FIG. 1. GaAs/cesium oxide heterojunction band diagram.

Related Physics Process

Roll of the field

- Electrons in cathode is tightly bond by the potential.
- The external field on the cathode surface is important not only to lead the beam, but also to extract the beam from the cathode.
- Surface field modifies the work function of cathode. (Schottky effect)
- The emittable current density is limited by the coulomb potential of the beam and the external field. (Space charge limitation)

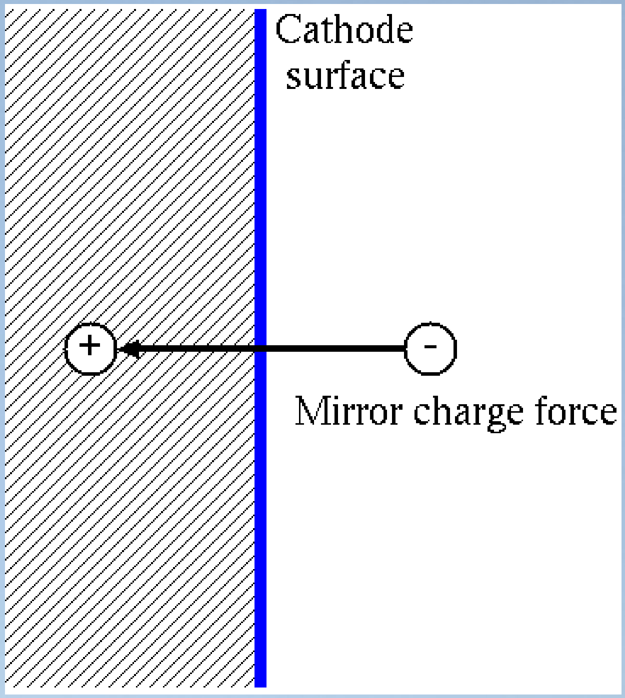
Schottky Effect (1)

Force by mirror charge

$$F_m(z) = -\frac{1}{4\pi\epsilon}\frac{e^2}{(2z)^2} \qquad (2-1)$$

Potential of the mirror charge

$$V_m(z) = -\frac{1}{4\pi\epsilon}\int_z^\infty \frac{e}{4z'^2}dz' = -\frac{e^2}{16\pi\epsilon z} \qquad (2-2)$$



Schottky Effect (2)

Mirror charge potential and external field give

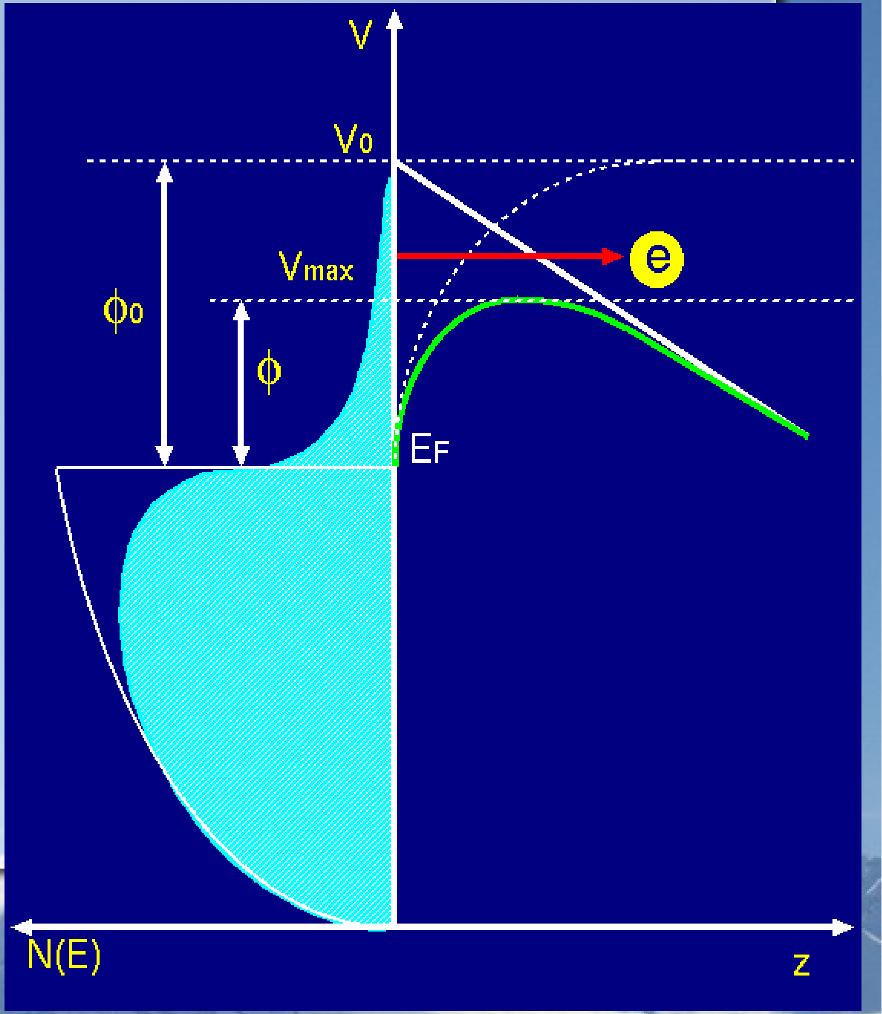
$$V(z)=\phi_0-\frac{e^2}{16\pi\epsilon z}-eEz \qquad (2-3)$$

Maximum at $z_{max}=\frac{1}{4}\sqrt{\frac{e}{\pi\epsilon E}}$

$$V_{max}=V_0-e\sqrt{\frac{eE}{4\pi\epsilon}} \qquad (2-4)$$

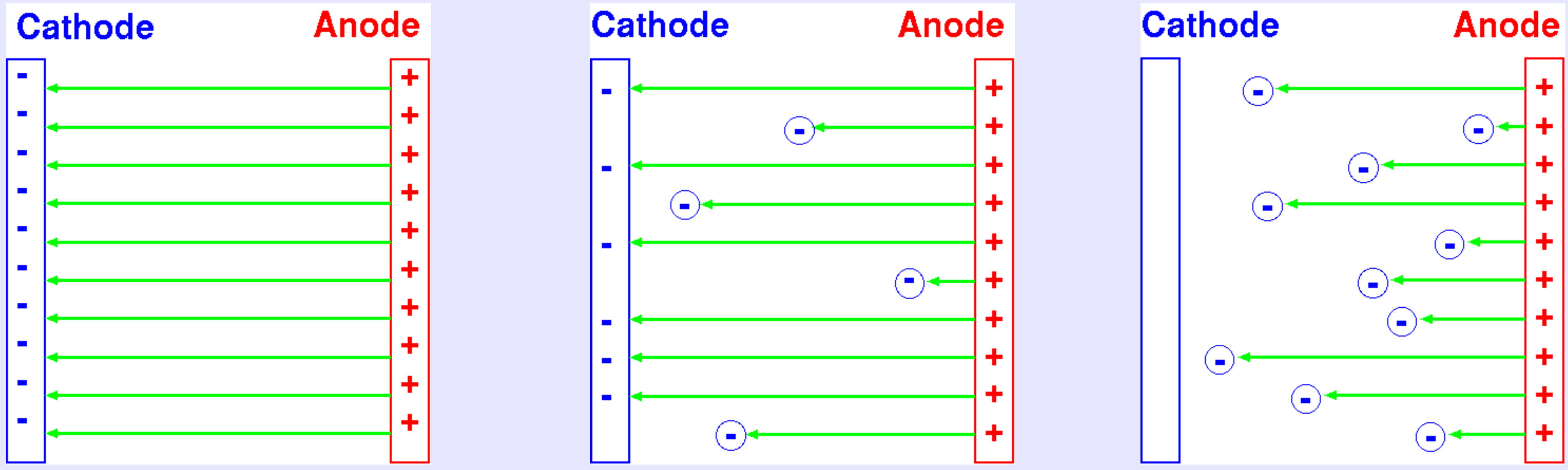
Effective work function

$$\phi(E)=V_{max}-\mu=\phi_0-e\sqrt{\frac{eE}{4\pi\epsilon}} \qquad (2-5)$$



Space Charge Limit

- Electron terminates the electric flux (Gauss's law).
- Electric field is weakened by the space charge.
- When all flux is terminated by the charge, the field at the cathode surface is disappeared .



Space Charge Limit (2)

Poisson equation is

$$\frac{d^2V(z)}{dz^2}=-\frac{\rho(z)}{\epsilon_0} \quad (2-7)$$

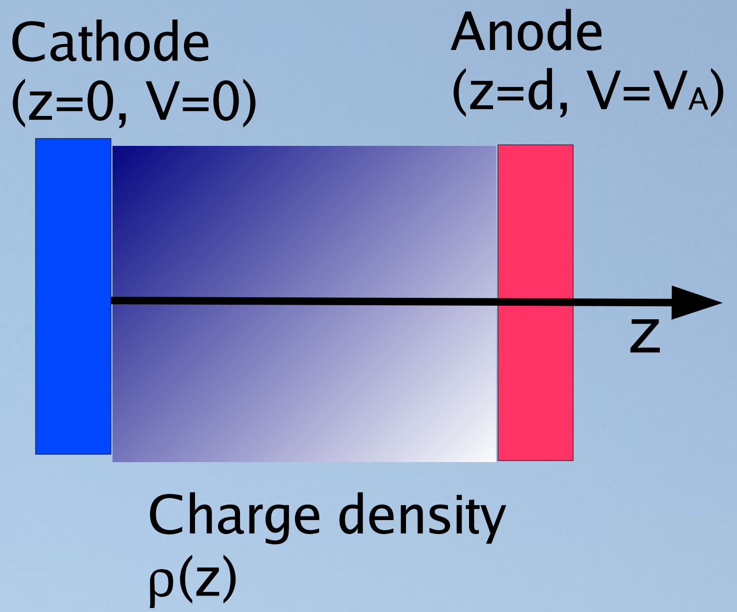
The current density J is given by the charge density ρ and velocity v,

$$J=-\rho(z)v(z) \quad (2-8)$$

According energy conservation,

$$\frac{1}{2}mv(z)^2=eV(z) \quad (2-9)$$

$$\frac{d^2V(z)}{dz^2}=\frac{J}{\epsilon_0}\sqrt{\frac{m}{2e}}V(z)^{-1/2} \quad (2-10)$$



SC Limited Current (2)

Multiplying *2(dV/dz)* and integrating both sides,

$$\left(\frac{dV(z)}{dz}\right)^2 = \frac{4J}{\epsilon_0}\sqrt{\frac{m}{2e}}\,V(z)^{1/2} \qquad (2-11)$$

Taking square root of both sides and integrate it again,

$$\frac{4}{3}V^{3/4} = \sqrt{\frac{4J}{\epsilon_0}}\sqrt[4]{\frac{m}{2e}}\,z \qquad (2-12)$$

Extract *J*

$$J = \frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V(z)^{3/2}}{z^2}$$

$$= 2.33\times 10^{-6}\frac{V(z)^{3/2}}{z^2} \qquad (2-13)$$

SC Limited Current (3)

Substituting the anode conditions, the space charge limited current density is obtained as

$$J(V_A, d) = 2.33\times 10^{-6}\frac{V_A^{3/2}}{d^2} \qquad (2-14)$$

V(z), E(z) , ρ(z) are expressed as a function of z

$$V(z) = V_A\left(\frac{z}{d}\right)^{3/4} \qquad (2-15)$$

$$E(z) = -\frac{dV(z)}{dz} = -\frac{4}{3}\frac{V_A}{d^{4/3}}z^{1/3} \qquad (2-16)$$

$$\rho(z) = -\frac{4\epsilon_0}{9}\frac{V_A}{d^{4/3}}z^{-2/3} \qquad (2-17)$$

Child-Langmuir Law

If the cathode emission density is more than the space charge limit, the current is given by C-L law

$$I = 2.33 \times 10^{-6} \frac{S V^{3/2}}{d^2} = P V^{3/2} (A) \qquad (2-18)$$

V and d : voltage and distance between two electrodes.

S : cathode area

P : perveance defined as;

$$P = 2.33 \times 10^{-6} \frac{S}{d^2} (A V^{-3/2}) \qquad (2-19)$$

DC Gun design (2D SC Limited Flow)

2D case solution for SC limited flow;

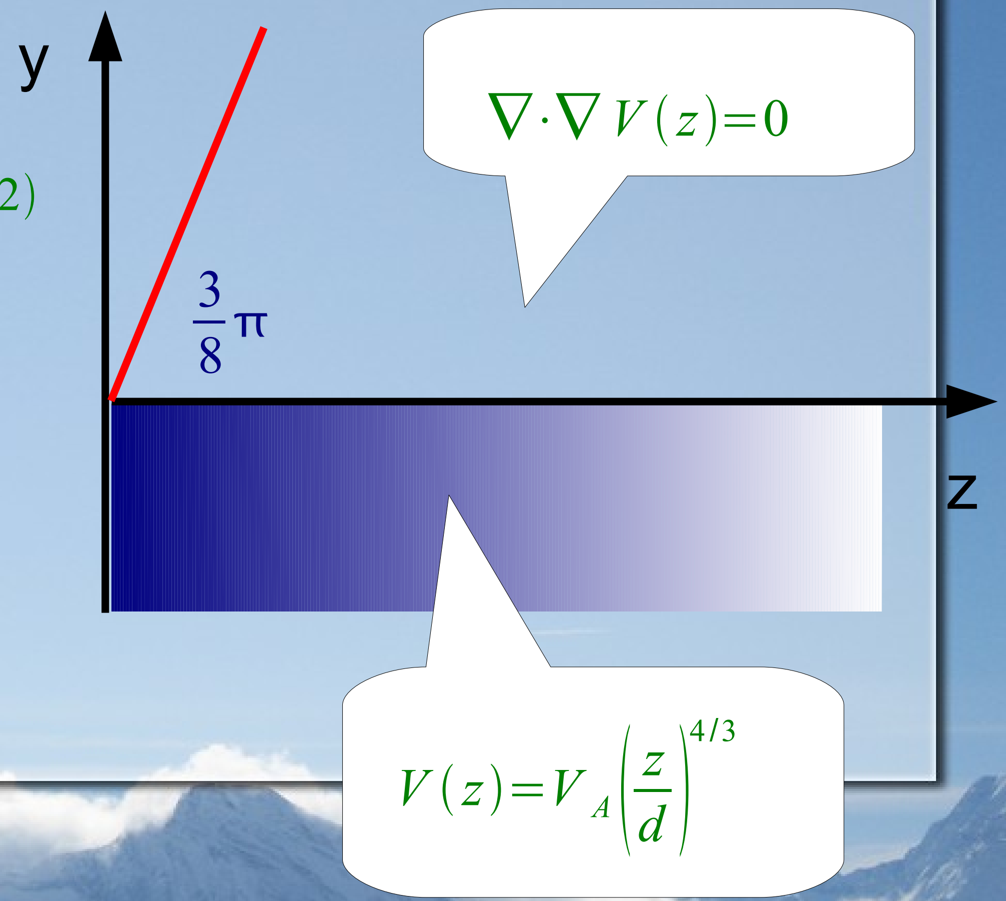
$$V(z,y)=V_A\frac{\Re\left[(z+iy)^{4/3}\right]}{d^{4/3}}$$

$$=V_A(z^2+y^2)^{2/3}\cos\frac{4}{3}\theta \quad (2-22)$$

V=0 equi-potential line:

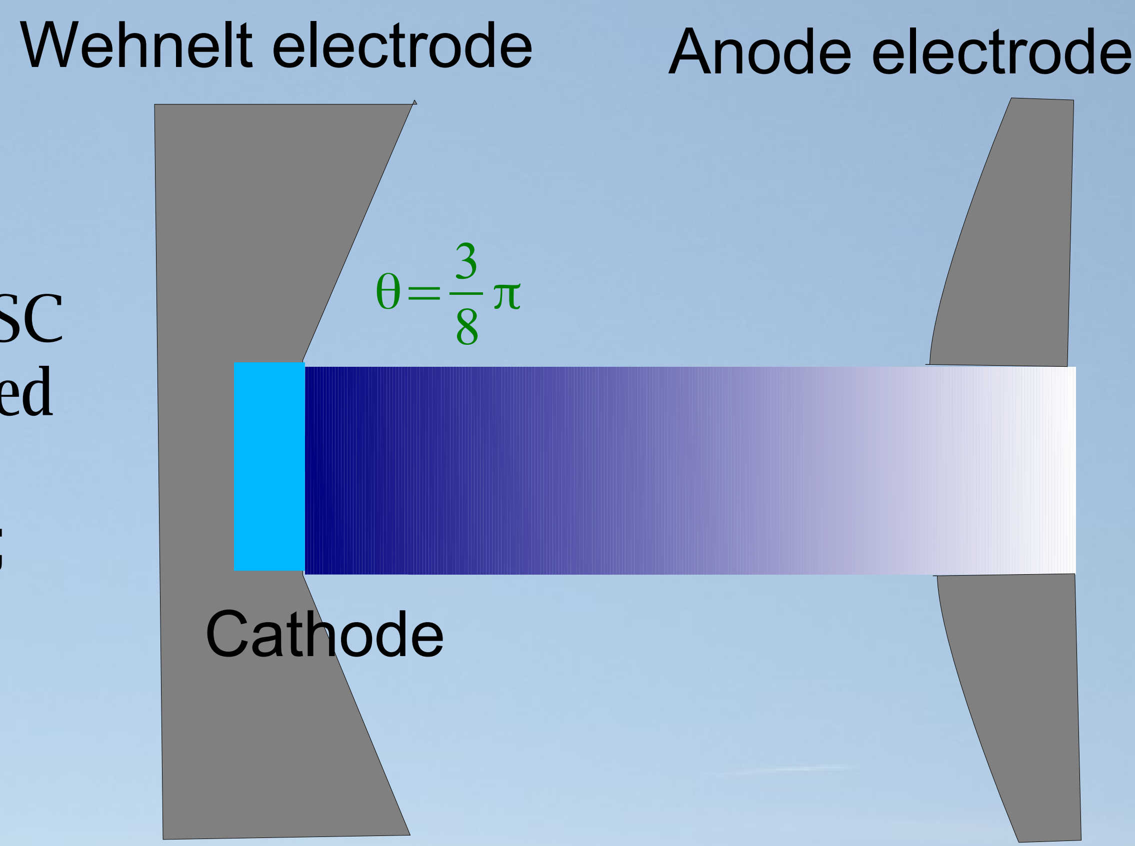
$$\cos\frac{4}{3}\theta=0\rightarrow\theta=\frac{3}{8}\pi$$

By setting an electrode (Wehnelt) with this angle, SCL flow is produced.



DC Gun design : Real geometry

- By setting Wehnelt and anode electrodes to reproduce the potential, SC limited current is extracted from the cahode.
- This is Pierce type gun;
    - Conventional type,
    - DC bias voltage,
    - Thermionic cathode,
    - Continous beam.



Space Charge Force (1)

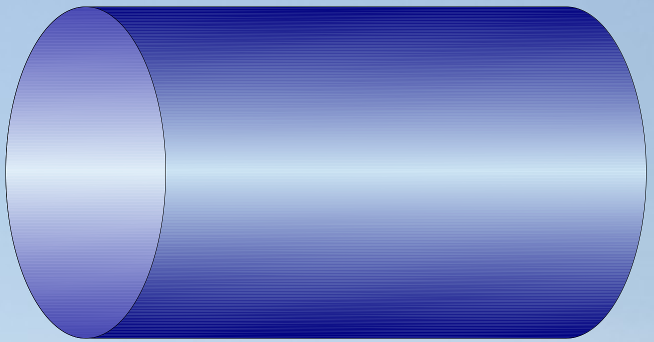
Charged particle beam has repulsion force by its own charge. The space charge force causes various beam quality degradations, e.g. bunch lengthening, emittance growth, tune shift, etc. The effect is suppressed by acceleration because it scaled as $1/\gamma^2$.

Consider a cylindrical beam with a constant density.

$$E_r = \frac{N e}{2\pi a^2 \epsilon_0} r \qquad (2-23)$$

magnetic flux density by the current,

$$B(r) = \frac{\mu_0}{r} \int_0^r r' J(r') dr' \qquad (2-24)$$



Space Charge Force (2)

Current density is

$$J(r) = \frac{Ne}{\pi a^2} \beta c \qquad (2-25)$$

The magnetic flux is given as

$$B(r) = \frac{\mu_0 N e \beta c}{2\pi a^2} r \qquad (2-26)$$

The Lorentz force to electron is

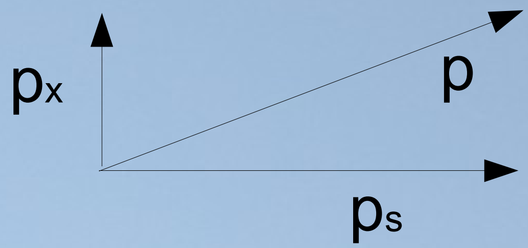
$$\boldsymbol{F} = e\boldsymbol{E} + e\beta c \boldsymbol{B} = \frac{Ne^2 r}{2\pi a^2 \epsilon_0}(1-\beta^2)\vec{e}_r$$

$$= \frac{Ne^2 r}{2\pi a^2 \epsilon_0 \gamma^2}\vec{e}_r \qquad (2-27)$$

which is scaled as $1/\gamma^2$.

Beam Emittance

Emittance is defined as area in the phase space where particles occupy. The phase space is defined x and x'=dx/ds



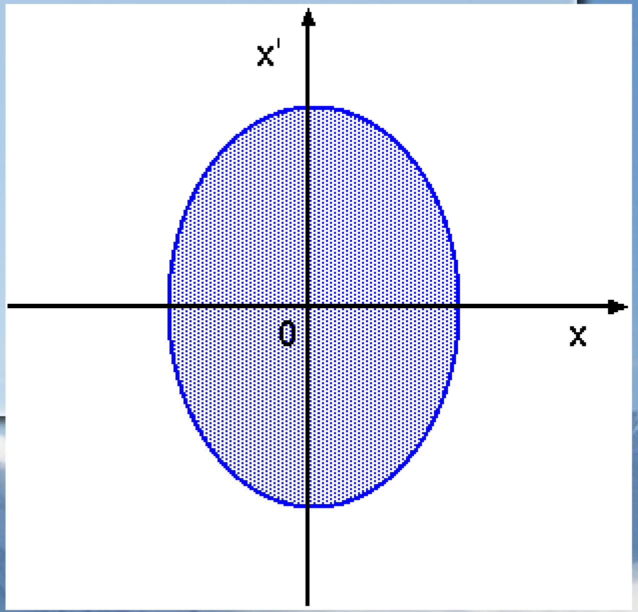
$$x' = \frac{dx}{ds} = \frac{v_x}{v_s} = \frac{p_x}{p_s} \sim \frac{p_x}{p} \qquad (2-28)$$

In general, RMS emittance is given as

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2} \qquad (2-29)$$

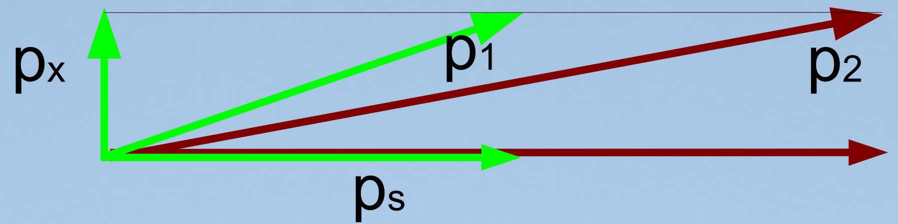
If there is no correlation between x and x',

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle} \qquad (2-30)$$



Normalized emittance

In acceleration, transverse momentum p_x is conserved, but p is scaled as



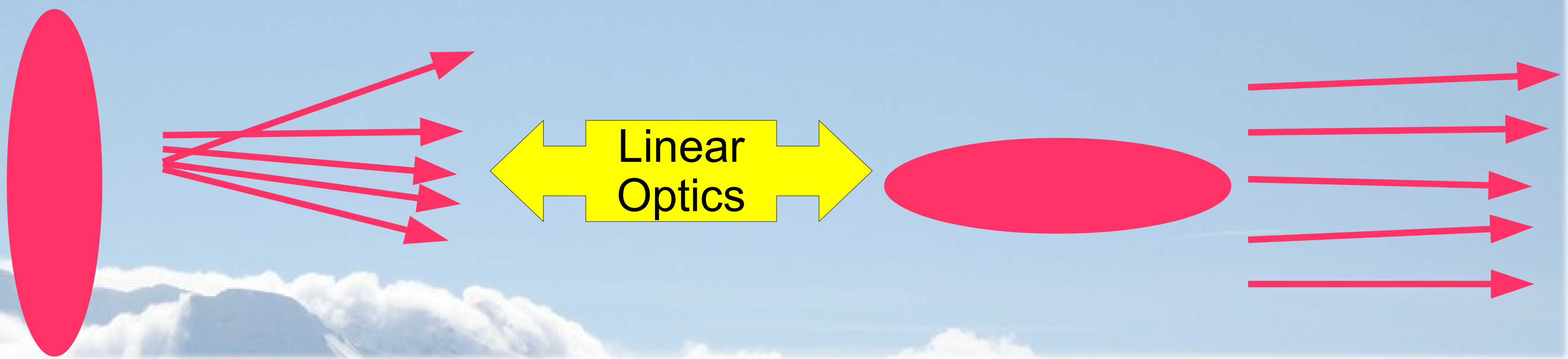
$$p_s = \gamma \beta mc \qquad (2-31)$$

The emittance is inversely scaled. To avoid the energy dependence ($\gamma\beta$) on the emittance, the normalized emittance is defined

$$\epsilon_{nx} = \gamma \beta \epsilon_x$$
$$= \gamma \beta \frac{R}{2} \sqrt{\left\langle \left(\frac{p_x}{\gamma \beta m c} \right)^2 \right\rangle} = \frac{R}{2mc} \sqrt{\langle p_x^{\ 2} \rangle} \qquad (2-32)$$

What is the matter on Emittance?

- Emittance shows the quality of the beam.
- Small emittance beam can be focused down to a small spot size.
- Small emittance beam can be extremely parallel.
- The shape of the beam depends on the optics, but the emittance is invariant in the frame of linear optics.



What is the fundamental limit on the emittance?

- Everbody wants small emittance beam, but what is the limit?
- One of the limit is the intrinsic emittance which the emitted beam from the cathode already has.
- The source of the intrinsic emittance of cathode is thermal energy and laser energy (photo-cathode case).

Emittance of Beam from Thermionic Cathode (1)

Thermionic electron emission density is already obtained

$$N=\frac{4\pi m}{h^3}k^2T^2\exp\left(-\frac{\phi}{kT}\right) \qquad (2-33)$$

Total transverse energy of emitted electron is obtained with a similar calculation as

$$E_t=\frac{4\pi m}{h^3}\int_{\mu+\phi}^{\infty}d\epsilon_z\int_0^{\infty}d\epsilon_t\,\epsilon_t\exp\left(-\frac{\epsilon_z+\epsilon_t-\mu}{kT}\right)$$

$$=\frac{4\pi m}{h^3}k^3T^3\exp\left(-\frac{\phi}{kT}\right) \qquad (2-34)$$

The average transverse energy per electron is

$$\langle\epsilon_t\rangle=\frac{E_t}{N}=kT \qquad (2-35)$$

Emittance of Beam from Thermionic Cathode (2)

Thermal energy is,

$$\langle \epsilon_x \rangle = \frac{kT}{2} \qquad (2-36)$$

The transverse emittance is

$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle} = \frac{1}{\gamma \beta m c} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle} \qquad (2-37)$$

Substituting the thermal energy $\frac{\langle p_x^2 \rangle}{2\mathrm{m}} = \langle \epsilon_x \rangle = \frac{kT}{2}$, emittance is

$$\varepsilon_x = \frac{1}{\gamma \beta} \sqrt{\langle x^2 \rangle \frac{kT}{mc^2}} = \frac{1}{\gamma \beta} \frac{R}{2} \sqrt{\frac{kT}{mc^2}} \qquad (2-38)$$

$$\varepsilon_{nx} = \gamma \beta \varepsilon_x = \frac{R}{2} \sqrt{\frac{kT}{mc^2}} \qquad (2-39)$$

Emittance of Beam from Photo-cathode (1)

Transverse energy from photo-emission is

$$E_t = \frac{4\pi m}{h^3} \int_{\mu+\phi-h\nu}^{\infty} d\epsilon_z \int_0^{\infty} d\epsilon_t \epsilon_t \left[\exp\left(\frac{\epsilon_z + \epsilon_t - \mu}{kT} \right) + 1 \right]^{-1} \qquad (2-40)$$

With T=0 approximation,

$$E_t = \frac{4\pi m}{h^3} \int_{\mu+\phi-h\nu}^{\mu} d\epsilon_z \int_0^{\mu-\epsilon_z} d\epsilon_t \epsilon_t$$

$$= \frac{4\pi m}{h^3} \frac{(h\nu-\phi)^3}{6} \qquad (2-41)$$

Emittance of Beam from Photo-cathode (2)

Average of the transverse energy is

$$N=\frac{4\pi m}{h^3}\int_{\mu+\phi-h\nu}^{\mu} d\epsilon_z \int_0^{\mu-\epsilon_z} d\epsilon_t \;=\frac{4\pi m}{h^3}\frac{(h\nu-\phi)^2}{2} \qquad (2-42)$$

$$\epsilon_{x,y}=\frac{E_t}{2N}=\frac{h\nu-\phi}{6} \qquad (2-43)$$

The momentum is

$$\langle p_x^2\rangle=2\mathrm{m}\,\epsilon_{x,y}=m\frac{h\nu-\phi}{3} \qquad (2-44)$$

Emittance is

$$\varepsilon_x=\frac{1}{\gamma\beta}\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3\mathrm{mc}^2}} \qquad (2-45) \qquad \varepsilon_{nx}=\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3\mathrm{mc}^2}} \qquad (2-46)$$

Emittance of Beam from Photo-cathode (3)

Acounting thermal energy, the transverse energy becomes

$$\epsilon_{x,y}=\frac{E_t}{2N}=\frac{h\nu-\phi}{6}+\frac{kT}{2} \qquad (2-47)$$

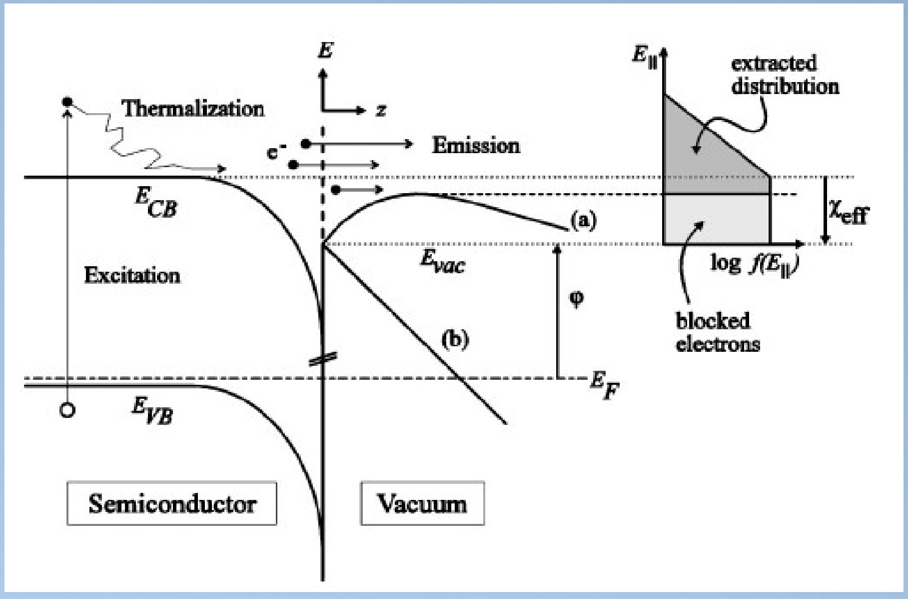
The transverse emittance is

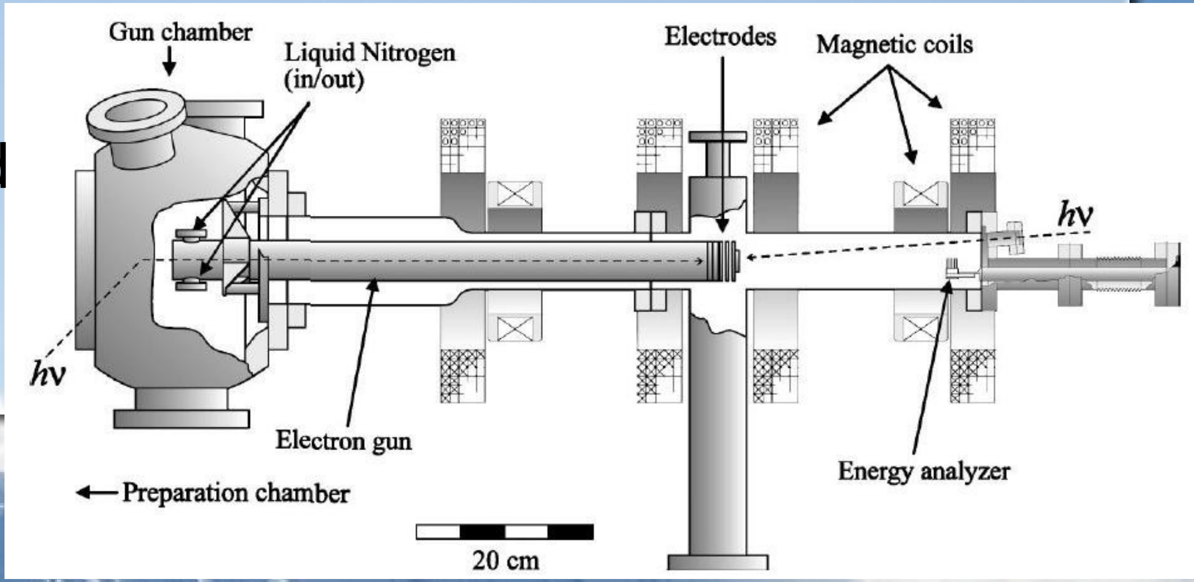
$$\varepsilon_x=\frac{1}{\gamma\beta}\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3mc^2}+\frac{kT}{mc^2}} \qquad (2-48)$$

$$\varepsilon_{nx}=\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3mc^2}\frac{kT}{mc^2}} \qquad (2-49)$$

Emittance measurement 1-1

- Energy spread from GaAs photo-cathode is directly measured by blocked electrode.
- Only electrons above the block potential barrier, is observed.
- Cathode is placed longitudinal B field (immerse).





S. Pastuszka, JAP, 88(11), 6788-6800 (2000)

Emittance measurement 1-2

Adiabatic condition

$$\frac{\lambda}{B}\left|\frac{dB}{dz}\right| \leqslant 1 \qquad (2-50)$$

ratio of transverse energy E⊥ and magnetic flux B is an adiabatic constant,

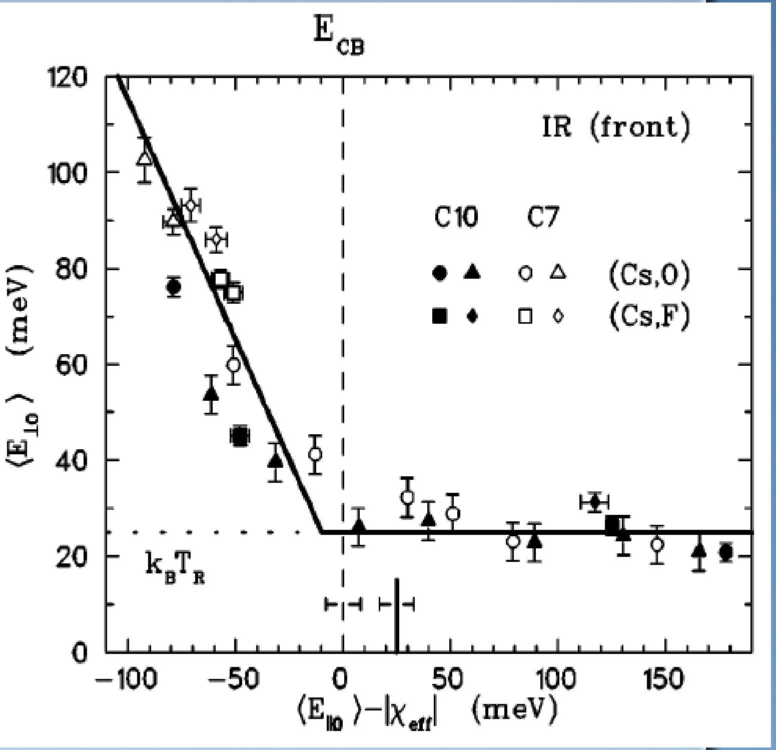
$$\frac{E_\perp}{B} = const \qquad (2-51)$$

From the energy conservation

$$E_{\parallel f} = E_{\parallel i} + \left(1 - \frac{B_f}{B_i}\right) E_{\perp i} \qquad (2-52)$$

The initial transverse energy is obtained as

$$\langle E_{\perp i} \rangle = \frac{d\langle E_{\parallel f} \rangle}{d\alpha} \qquad (2-53)$$

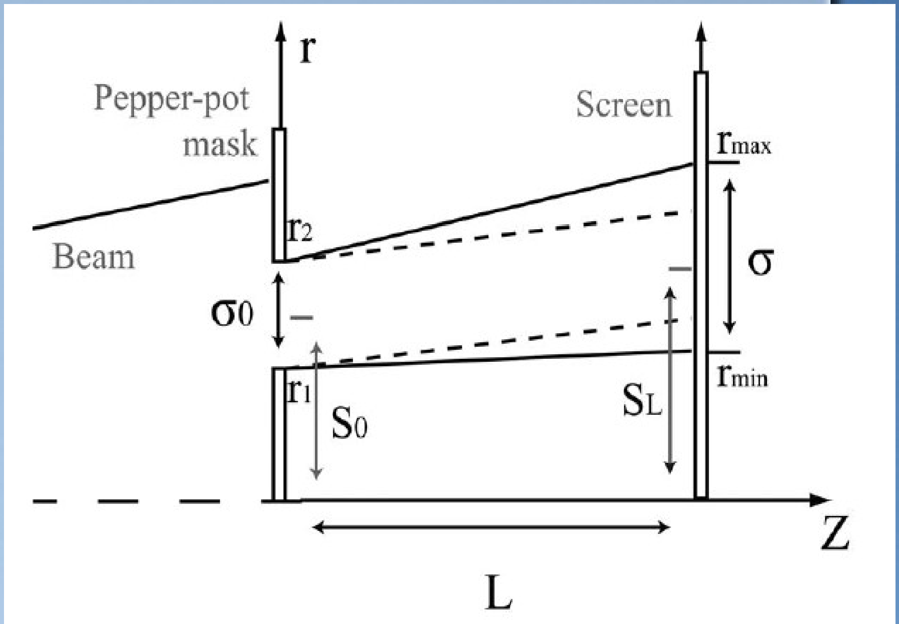


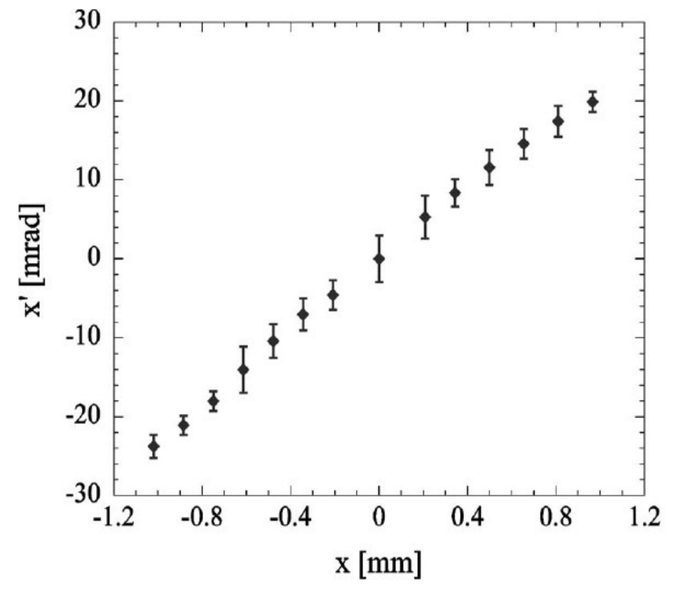
$E_{\parallel i}$ = 25 meV is confirmed.

S. Pastuszka, JAP, 88(11), 6788-6800 (2000)

Emittance measurement 2-1

- Beam emittance from SL GaAs photocathode is measured by pepper-pot method.
- The beam image passing small holes (pepper-pot) are observed.
- The phase-space distribution is reconstructed from the image.

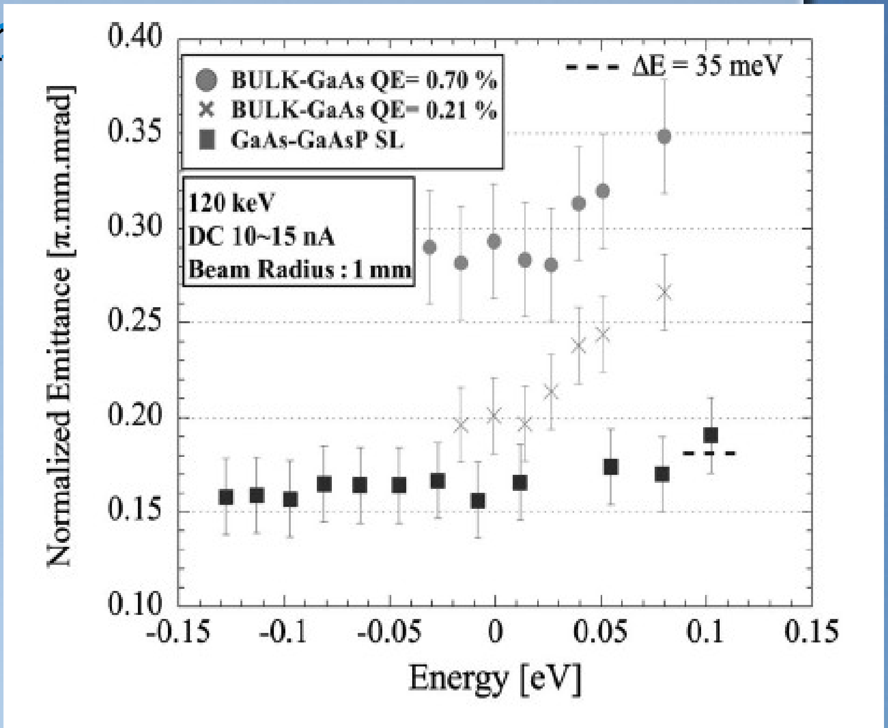


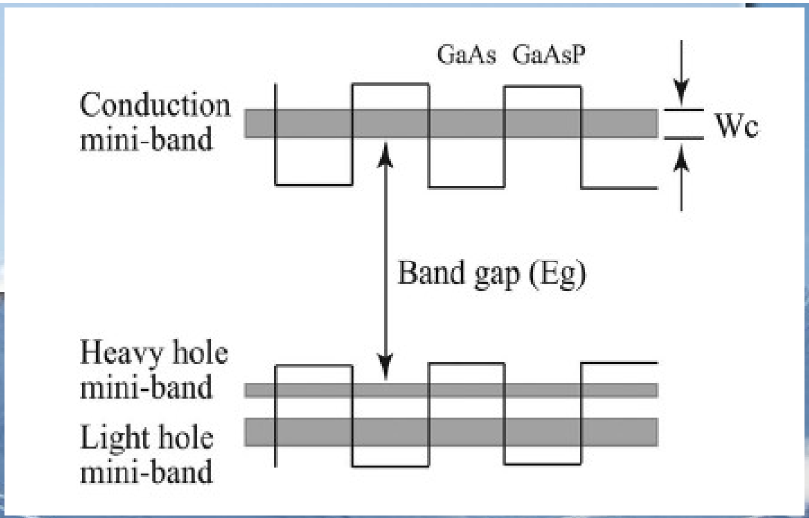


N. Yamamoto, JAP(102) 024904(2007)

Emittance measurem

- Emittance is measure as a function of laser wave length.
- Comparing Super-lattice GaAs and bulk GaAs, SL has smaller emittance, especially for shorter wave length.
- It can be considered due to confinement of the excited electrons in the conduction mini-band.
- ε_x~0.16 mm.mrad is confirmed.

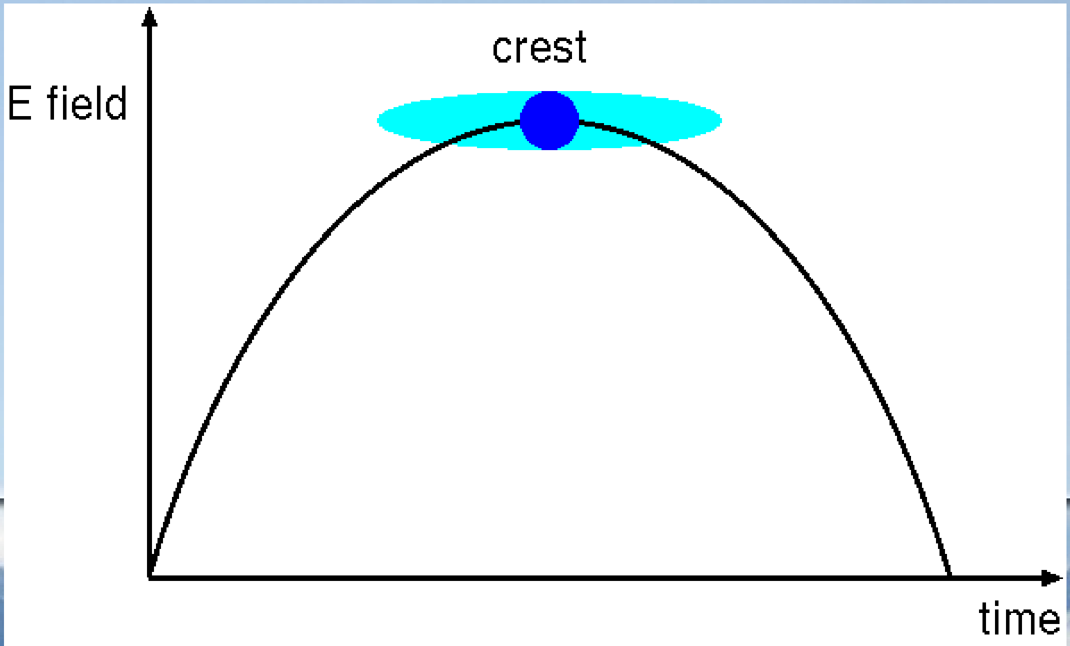




N. Yamamoto, JAP(102) 024904(2007)

Bunch Compression (1)

- In any RF accelerators, the beam should be concentrated in a short period of longitudinal space for small energy spread;
  - $E=E_0\cos(\omega t-ks)$, $k\delta_s \ll 1$ for efficient acceleration.
- Bunch compressor(buncher) shorten the bunch length down to an adequate size for acceleration.



Bunch Compression (2)

- There are two ways for bunch compression:
  - Velocity Bunching
  - Magnetic Bunching
- Velocity bunching is effective only for low energy;
  - Some particle source can generate only long bunch or continuous beam.
  - It should be bunched for RF acceleration.
- Magnetic bunching is effective for all energy region.
  - It is employed sometimes to get extremely short buhc after acceleration.
  - It is also used to compensate the bunch lengthning in DR for Linear colliders.

Velocity Bunching (1)

- Bunch compression is performed by velocity modulation within a bunch;
  - Bunch head is decelerated.
  - Bunch tail is accelerated.
- Velocity is modulated by energy modulation according to

$$c\beta = c\sqrt{1-\frac{1}{\gamma^2}} \qquad (2-54)$$

- Velocity is saturated to c at $\gamma >> 1$. Then, it works only for low energy particle ($\beta < 1$).

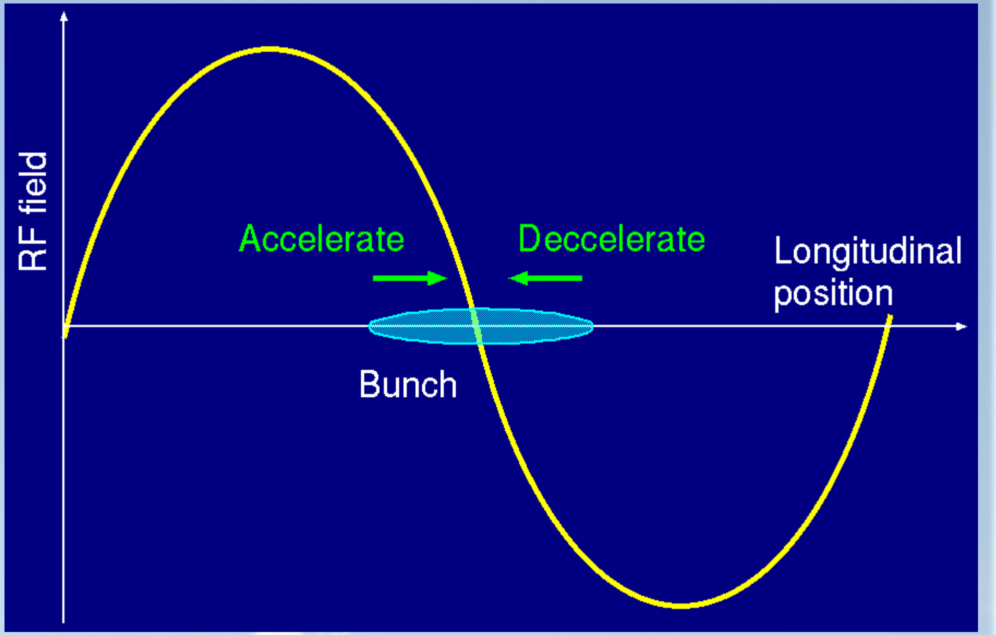
Velocity Bunching (2)

Energy modulation by RF cavity,

$$dE = -eV_0 \frac{d\sin(\omega t)}{dt} dt \qquad (2-55)$$

In linear approximation,

$$\frac{dE}{E_0} \sim \frac{-eV_0}{E_0} \omega\, dt \qquad (2-56)$$



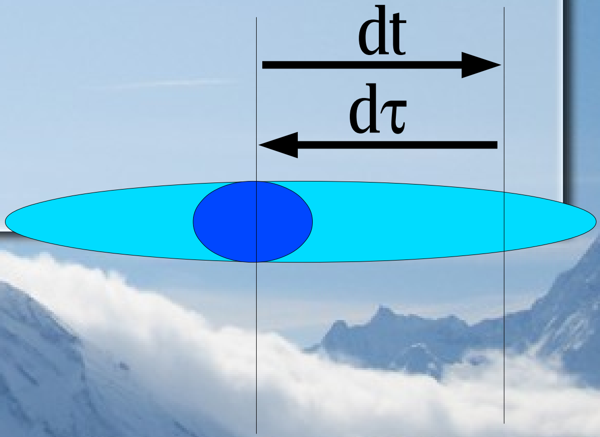
Velocity Bunching (3)

- In drift space L, Time delay ($d\tau$) with the energy modulation (dE) is
- If $d\tau$ equals to -dt, all particles are gathered at the bunch center, bunched.
- Because all electrons concentrate at t=0 position, RF phase of bunching determines the bunch longitudinal position.

$$\tau = \frac{L}{c\beta} \qquad (2-57)$$

$$d\tau = -\frac{L}{c\gamma^2\beta^3}\frac{dE}{E}$$

$$\sim -\frac{L}{c\gamma^2\beta^3}\frac{eV_0\omega}{E}dt \qquad (2-58)$$



Magnetic Bunching (1)

- Bunch compression is performed by energy modulation with dispersive path length difference.
  - Chicane, Wiggler, Arc, etc.
- A path length difference by a dispersive section, Δz is

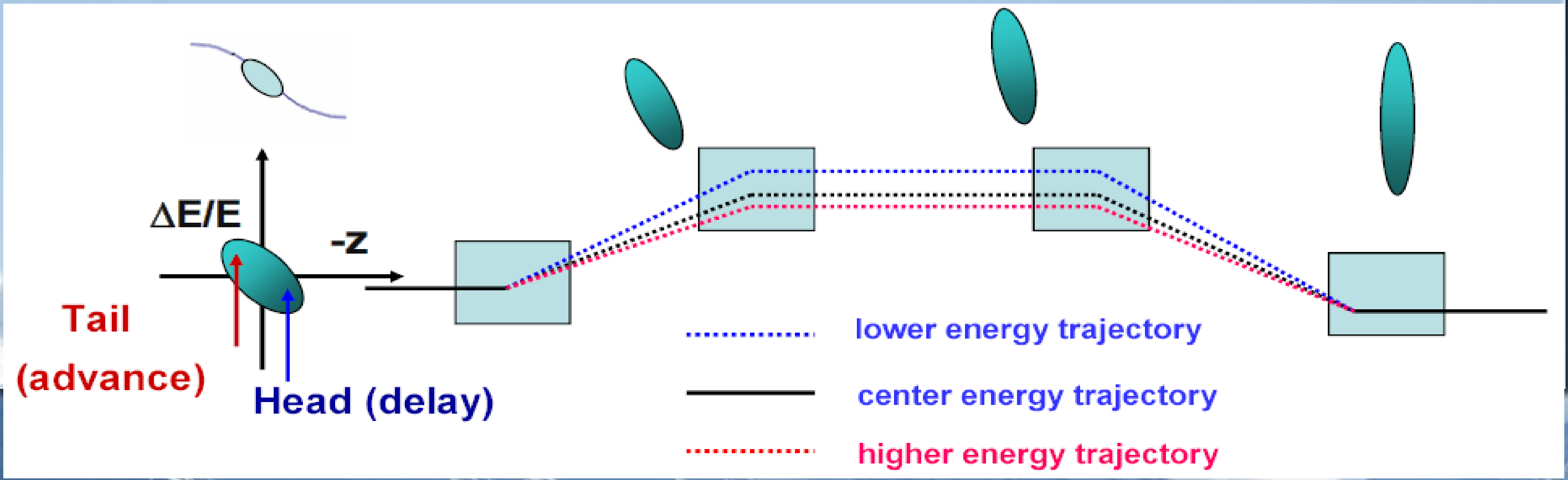
$$\Delta z = \eta_l \frac{\Delta P}{P} \qquad (2-59)$$

$$\eta_l = \int_L ds \frac{\eta}{\rho} \qquad (2-60)$$

- It works well for any energy particle.

Magnetic Bunching (2)

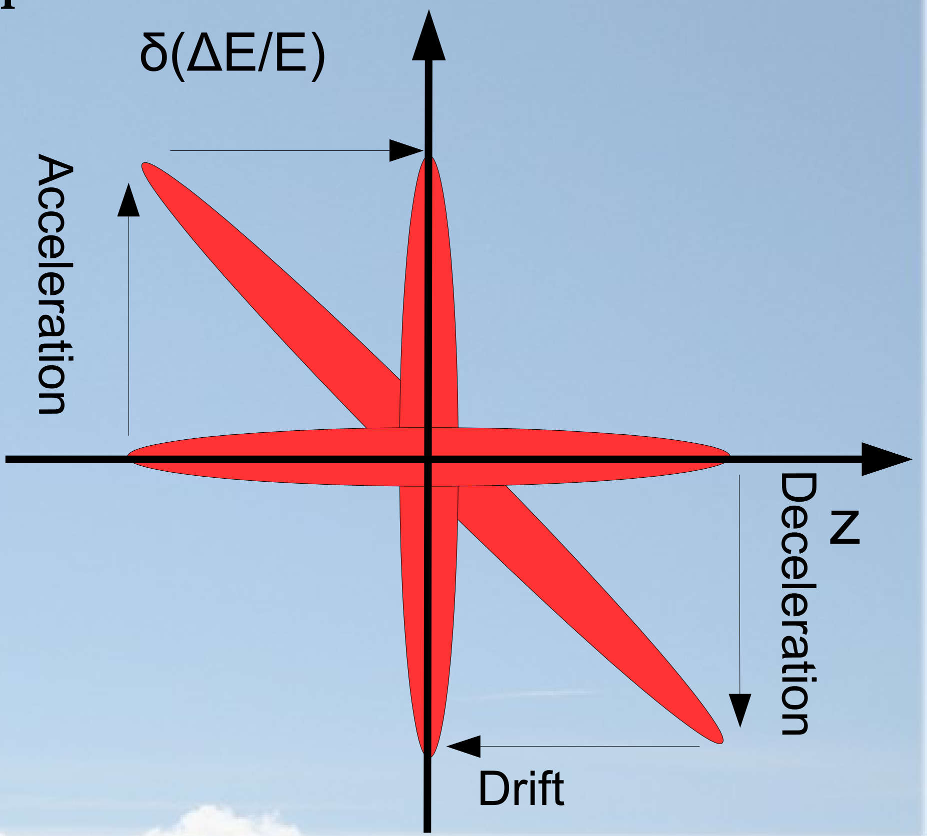
- Energy Modulation : RF cavity.
- Dispersive section : different path for different energy.
- Bunch head (tail) travels longer (shorter) path and bunch length becomes shorter.



By E.S. Kim

Common formalism (1)

- Bunching can be formalized with transfer matrix in linear approximation.
- Energy modulation is made by RF (acc- and deceleration).
- Drift space (velocity bunching) or drift through a dispersive section (magnetic bunching) rotates the beam in phase space.
- The bunch rotates 90 deg.



Common formalism (2)

R-matrices

Drift space:

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-60)$$

$$R_{56} = -\frac{L}{\gamma^2 \beta^2} \quad (2-61)$$

Dispersive section:

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-62)$$

$$R_{56} = \eta_l = \int ds \frac{\eta}{\rho} \quad (2-63)$$

RF Energy modulation

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-64)$$

$$R_{65} = \frac{1}{z} \frac{\Delta E}{E} \sim \pm \frac{eV_0}{E} \frac{\omega}{\beta c} \quad (2-65)$$

Common formalism (3)

Total Transfer Matrix of BC section.

$$\begin{bmatrix} z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix}$$

$$= \begin{bmatrix} 1+R_{56}R_{65} & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} \quad (2-66)$$

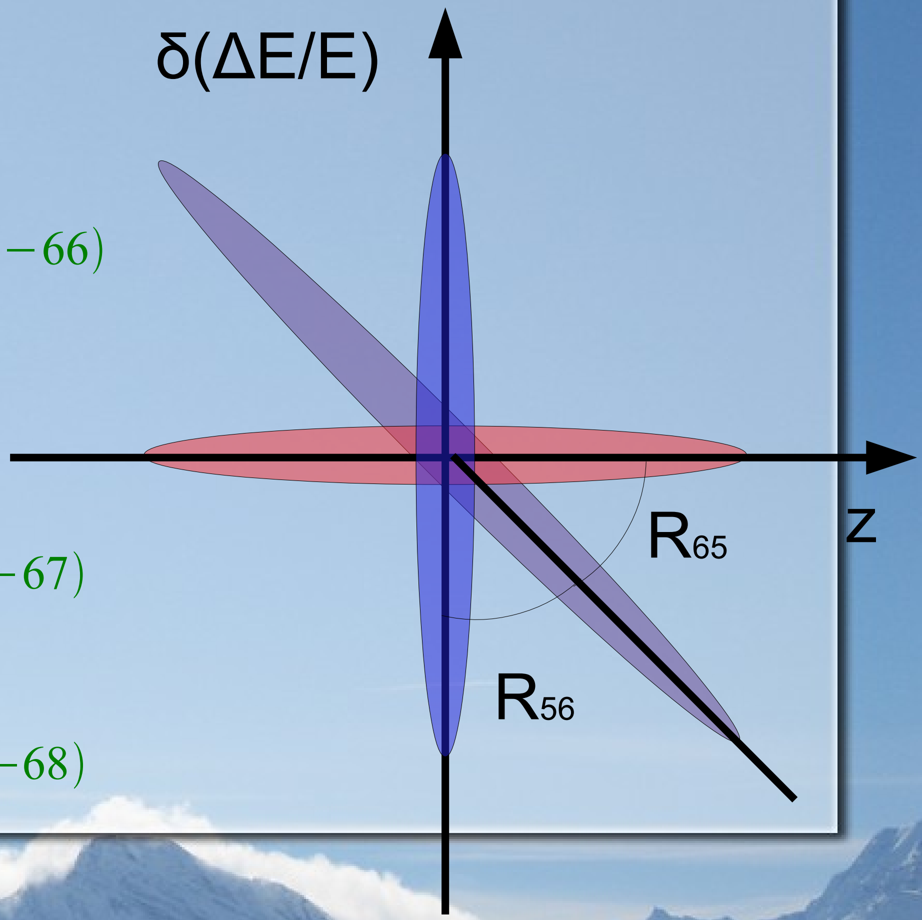
Bunching condition : $1+R_{56}R_{65}=0$

Velocity bunching:

$$1+R_{56}R_{65} = 1 - \frac{L}{\gamma^2\beta^2}\frac{eV_0}{E}\frac{\omega}{\beta c} = 0 \quad (2-67)$$

Magnetic bunching:

$$1+R_{56}R_{65} = 1 + \eta_l \frac{eV_0}{E}\frac{\omega}{\beta c} = 0 \quad (2-68)$$

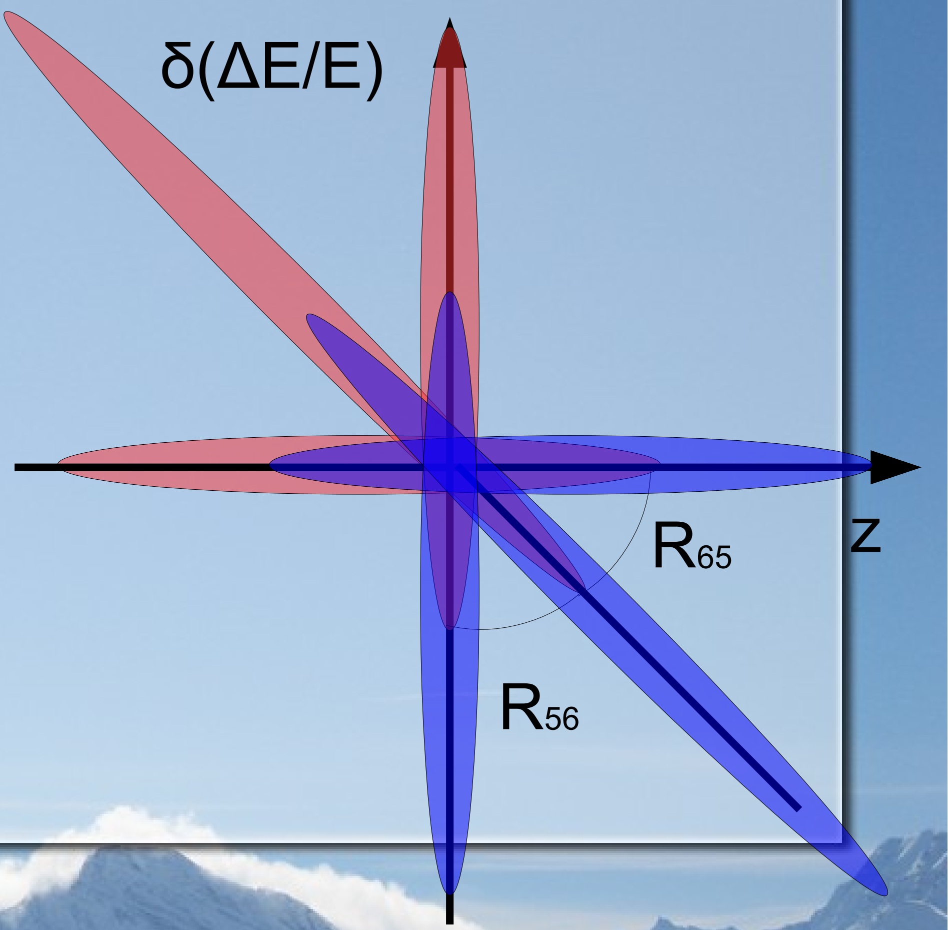


Common formalism (4)

- When the bunching condition is satisfied,

$$\begin{bmatrix} z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} 0 & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} \quad (2-68)$$

- The position $z(s_2)$ does not depends on $z(s_0)$.
- This is a good mechanism to stabilize the bunch position.



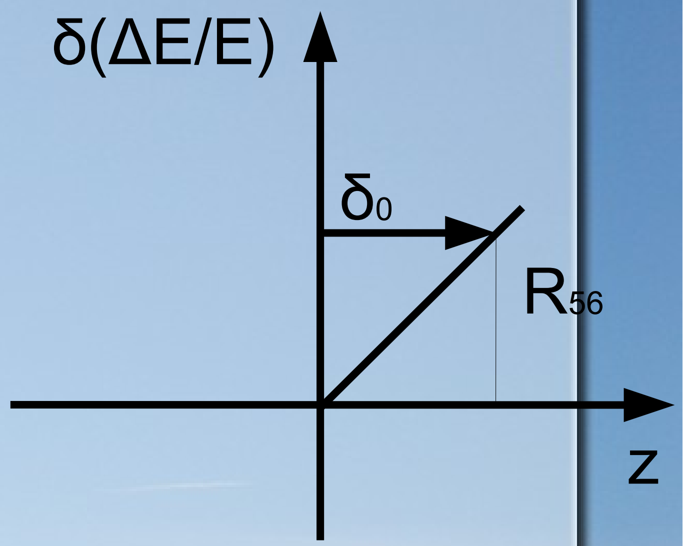
Common formalism (5)

Final bunch length after an optimized BC section ($1+R_{56}R_{65}=0$) is determined by the initial energy spread;

$$z_2 = R_{56}\delta_0 \qquad (2-69)$$

It can be understood by considering the transport of a reference point.

$$\begin{bmatrix} R_{56}\delta_0 \\ \delta_0 \end{bmatrix} = \begin{bmatrix} 0 & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \delta_0 \end{bmatrix} \qquad (2-70)$$



The actual bunch length is also limited by non-linearity in optics.

Energy Compression

Energy compression is a reverse process of the bunch compression.
Bean transfere by dispersive section (R_{56}) and energy modulation (R_{65}) is

$$\begin{bmatrix} z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & R_{56} \\ R_{65} & 1 + R_{56} R_{65} \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} \qquad (2-71)$$

Matching condition for energy compression is

$$1 + R_{56} R_{65} = 0 \qquad (2-72)$$

The final energy spread is

$$\delta(s_2) = z(s_0) R_{65} \qquad (2-73)$$