(Towards) the exact beamstrahlung radiation angle and spectrum

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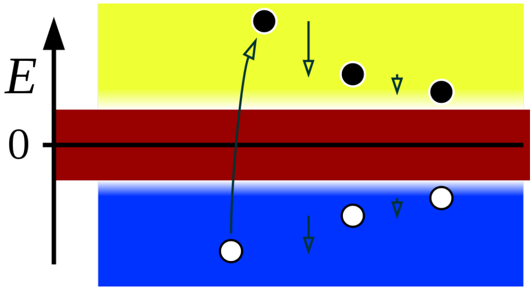
Preamble: Intense fields can polarise the vacuum



" In strong external fields the normal vacuum is unstable and decays into a new vacuum that contains real particles. "

Greiner and Muller, QED of Strong Fields

- The Schwinger limit ($E_{\text{cr}} = 10^{18}$ V/m)
- Particles in future linear colliders will see $E \to E_{\text{cr}}$
- How do we incorporate these vacuum changes in a QFT $\to$ phenomenology?

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1b. The next generation of linear colliders has strong fields at the IP

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4. The Furry picture predicts distinct phenomenology

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5b. Beamstrahlung process examined as a starting point

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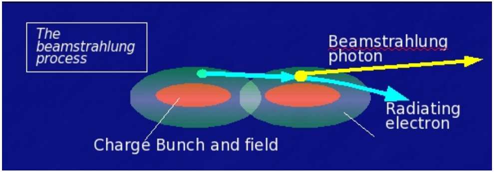
6. We are in an era of experimental tests of this phenomenology

7. A new strong field event generator is required

Strong fields at the collider Interaction Point



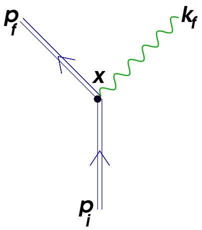
$\Upsilon \approx 1$ sets the strong field scale.

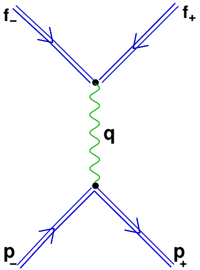
$$\Upsilon = \frac{e|\vec{a}|}{mE_{\text{cr}}}(k \cdot p)$$

- Υ depends on collider bunch parameters and the pinch effect
- Future linear colliders will have "strong" IP fields
- **All** collider processes are potentially "strong field processes"

Machine	LEP2	SLC	ILC	CLIC
E (GeV)	94.5	46.6	500	1500
$N(\times 10^{10})$	334	4	2	0.37
σ_x, σ_y (μm)	190, 3	2.1, 0.9	0.49, 0.002	0.045, 0.001
σ_z (mm)	20	1.1	0.15	0.044
Υ_{av}	0.00015	0.001	0.24	4.9

Strong field processes at Collider IP





- **1st order:**
  - Beamstrahlung & coherent pair production
  - pair backgrounds from beamstrahlung photons
  - beam-beam simulations (CAIN, Guinea-PIG)
  - basis of ISR/FSR simulations
  - 1-vertex permitted $p_i + rk - p_f - k_f = 0$

- **ALL** processes at the IP are ”strong field” processes

- **2nd order:**
  - Need exact solutions in fields of both bunches
  - Need to obtain the cross-section for a generic 2nd order process
  - crosscheck: ”normal processes” in limit $E \to 0$

Furry Picture

Furry Picture

$$\mathcal{L}_{\text{QED}}^{\text{Int}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}_{\text{QED}}^{\text{FP}} = \bar{\psi}^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}}A\!\!\!/\,\psi^{\text{FP}}$$

Equations of Motion

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

Wavefunction

$$\psi^{\text{FP}} = E_p\, e^{-ipx}\, u_p, \quad E_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(eA\!\!\!/^{\text{ext}} k\!\!\!/ + i2e(A^e p) - ie^2 A^{\text{ext}\,2}\right)\right]$$

Propagator

$$G^{\text{FP}} = \int \frac{d^4p}{(2\pi)^4}\, E_p(x)\, \frac{p\!\!\!/ + m}{p^2 - m^2}\, \bar{E}_p(x')\, e^{-ip\cdot x}\, u_p$$

(Volkov) Solution of the FP Dirac equation

Solution of the 2nd order Dirac equation with external 4-potential A_μ^{ext}

$$[D^2 + m^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}]\psi^{\text{FP}} = 0, \quad D_\mu = \partial_\mu + ieA_\mu^{\text{ext}}$$

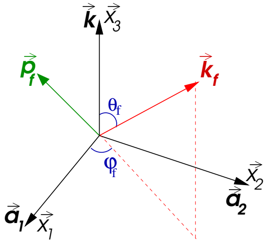
$$\psi^{\text{FP}} = e^{-i[p\cdot x + \$^p(k\cdot x)]}\, u_p$$

$$\$^p(k\cdot x) = \frac{1}{2(k\cdot p)}\int^{kx} 2eA^{\text{ext}}\cdot p - e^2 A^{\text{ext}\,2} - e\not{A}^{\text{ext}}\not{k}$$

Volkov phase

Volkov spinor

Lorenz gauge with condition
$A^0 = 0 \implies \vec{a}_1 \perp \vec{a}_2 \perp \vec{k}$



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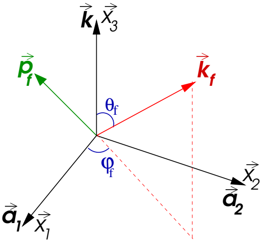


Orthornormality and Completeness of Volkov solutions **[Ritus, Ann Phys 69 552 (1971), Bergou and Varro, J Phys A 13 2823 (1980), Zakowicz JMathPhys 46 032304 (2005)]**

$$\int \frac{d^4x}{(2\pi)^4}\; e^{i[S^p(kx) - S^q(kx)]} = \delta^{(4)}(q-p)$$

$$\int \frac{d^4p}{(2\pi)^4}\; e^{i[S^p(kx) - S^p(ky)]} = \delta^{(4)}(x-y)$$

Volkov-type solutions

known solutions

- Single plane wave field **[Volkov, Z Phys 1935]**
- Circ/Linearly polarised field, constant field **[Nikishov and Ritus, JETP 1964]**
- Elliptically polarised field **[Lyulka, JETP 40 p815 1975]**
- 2 collinear orthogonal fields **[Lyulka 1975, Pardy 2004]**
- Coulomb fields + combinations **[Bagrov Gitman, Exact sols of Rel wave eqns 1990]**

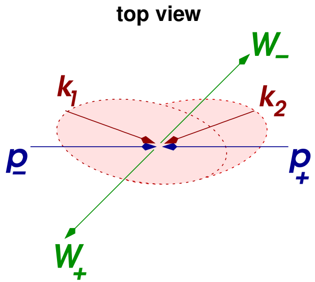
General procedure

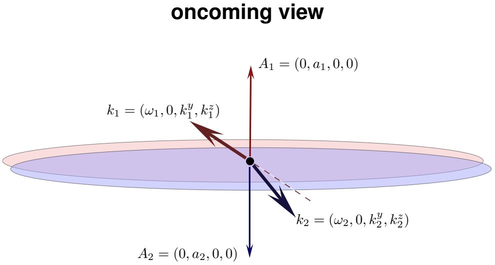
Klein-Gordon: $\left(D^2 + m^2\right) \phi_e = 0$ $\rightarrow$ Volkov phase

2nd order Dirac: $\left(D^2 + m^2 \pm \frac{ie}{2} F^{\mu\nu} \sigma_{\mu\nu}\right) \psi_e = 0$ $\rightarrow$ Volkov spinor

Dirac: $\left(i\not{D} - m\right) \psi_e = 0$ $\rightarrow$ particular solution

Solution of the FP Dirac equation in two fields





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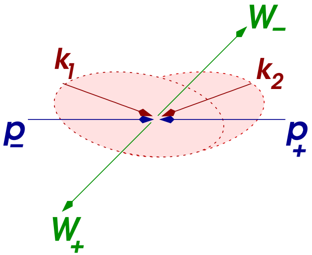

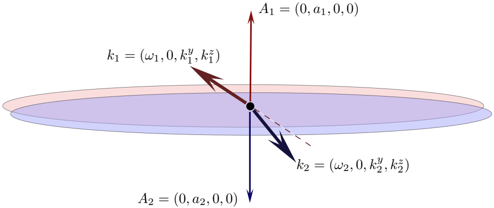

- Transition probabilities are covariant, so choose collinear $\vec{k}_1 || \vec{k}_2$ reference frame
- external field is a superposition; rewrite as orthogonal components

$$A_\mu = A_{1\mu}(k_1 \cdot x) + A_{2\mu}(k_2 \cdot x) \rightarrow A_{+\mu} + A_{-\mu} \quad \text{where} \quad A_+ \cdot A_- = 0$$

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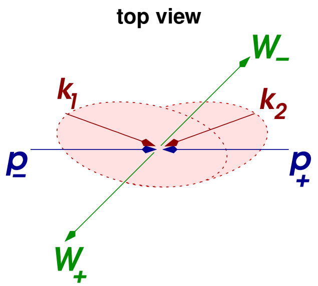

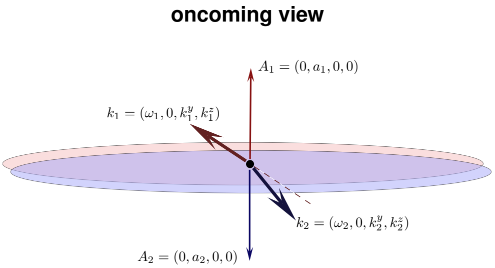

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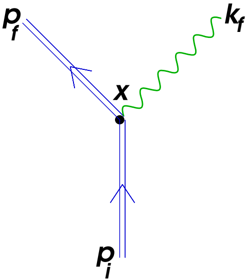
- solution is a product of Volkov solutions

$$\left[i\partial\!\!\!/ - eA\!\!\!/_+ - eA\!\!\!/_- - m\right]\psi^{\mathsf{FP}} = 0 \implies \psi^{\mathsf{FP}} = e^{-i\left[p\cdot x + \$^p_+ + \$^p_-\right]}u_r(p)$$

$$\text{where} \quad \$^p_+ = \int \frac{2eA_+(\phi)\cdot p - e^2A_+(\phi)^2 - eA\!\!\!/_+(\phi)k\!\!\!/_1}{2k_1\cdot p}\, d\phi$$

1st order Furry picture process and dressed vertex



FP Feynman diagrams only require a dressed vertex

$$\gamma_\mu^{\mathrm{FP}}(p_f,p_i) = e^{i\left[\not{S}_+^{p_f}+\not{S}_-^{p_f}\right]}\gamma_\mu\, e^{-i\left[\not{S}_+^{p_i}+\not{S}_-^{p_i}\right]}$$

$$\gamma_\mu^{\mathrm{FP}}(p_f,p_i) \to \int dr_1 dr_2\ \mathcal{F}^{-1}\left[\gamma_\mu^{\mathrm{FP}}(p_f,p_i)\right]\ e^{i(r_1k_1+r_2k_2)x}$$

contribution r_1k_1, r_2k_2 from external field enters
into the conservation of momentum, allowing 1 vertex process

$$\delta^4(p_f+k_f-p_i-r_1k_1-r_2k_2)$$

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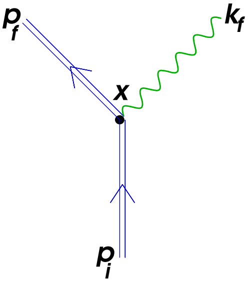

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two constant crossed fields leads to BesselK functions

$$A_\mu^{\text{ext}} = a_{1\mu}(k_1\cdot x)+a_{2\mu}(k_2\cdot x):\quad \mathcal{F}^{-1}\left[\gamma_\mu^{\text{FP}}(p_f,p_i)\right] \propto K_{\frac{1}{3},\frac{2}{3}}(z)$$

Traces are more complicated, and integration over final states needs care **[Hartin and Moortgat-Pick EPJC (2011)]**

$$\frac{|M_{fi}|^2}{VT} = -e^2\int dr_1dr_2\ \text{Tr}[..r_1..r_2..]\ \frac{d\vec{p}_f\,d\vec{k}_f}{4\omega_f\epsilon_f}\ \delta^{(4)}(p_f+k_f-p_i-r_1k_1-r_2k_2)$$

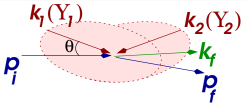
Beamstrahlung total transition probability

We get a modification to the standard beamstrahlung transition probability

$$W = -\frac{e^2 m}{2\epsilon_i}\int_0^\infty \frac{du}{(1+u)^2}\left[\int dz + \frac{1+(1+u)^2}{1+u} X \frac{d}{dz}\right]\mathrm{Ai}(z), \quad u = \frac{k_{1,2}.k_f}{k_{1,2}.(p_i - k_f)}$$

1 field: $z = \frac{u^{2/3}}{(k_2\cdot p_i)^{2/3}}, \quad X = \frac{(k_2\cdot p_i)^{2/3}}{u^{2/3}}, \quad k_2 \equiv \Upsilon_2 \hat{k}_2$

2 fields: $z = \frac{u^{2/3}}{[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{1/3}}, \quad X = \frac{(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2 + 2a_1\cdot a_2 (k_1\cdot p_i)(k_2\cdot p_i)}{u^{2/3}\,[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{2/3}}$



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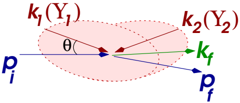
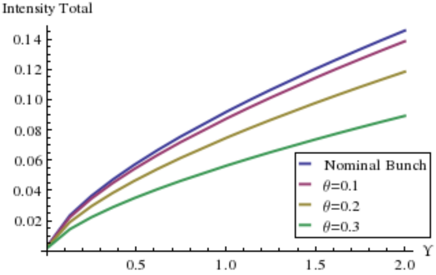
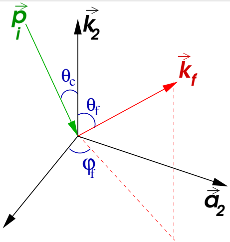


- Total intensity depends on field strength and angles
- θ depends on bunch disruption
- for ultra-relativistic bunches θ small
- Expt test with laser fields where θ can be large
- **Radiation angle needs closer examination!**



Beamstrahlung differential transition probability



- Return to an earlier stage of the calculation **[Nikishov, Trudy FIAN 111 p152 1979]**
- Divergence when $u = \dfrac{\omega_f(1-\cos\theta_f)}{2\epsilon_i - \omega_f(1-\cos\theta_f)} \to 0$
- i.e. **IR** condition ($\omega_f = 0$) and **backscattering** ($\theta_f = 0$). Why?

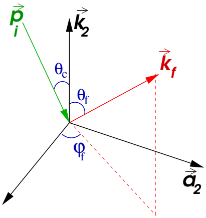
$$\frac{dW}{d\omega_f\, d\Omega} = -\frac{e^2 m}{2\epsilon_i}\frac{\omega_f \sin\theta_f}{1-\cos\theta_f}\left[X + \frac{1+(1+u)^2}{1+u}X^2\frac{d^2}{dz^2}\right]\mathrm{Ai}(z)^2, \quad u = \frac{k_2.k_f}{k_2.p_i - k_2.k_f}$$

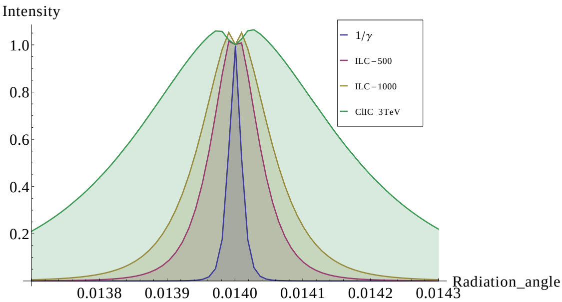
$$z = \frac{u^{2/3}}{(k_2\cdot p_i)^{2/3}}, \quad X = \frac{(k_2\cdot p_i)^{2/3}}{u^{2/3}}, \quad k_2 \equiv \Upsilon_2 \hat{k}_2$$

- modification to LSZ to accommodate Furry picture, means that the field has been acting since $t = -\infty$. **We have to limit the action**
- formation length can provide a limit to the action
- Can formally provide a limit using light cone slices **[Neville & Rohrlich, Phys Rev D, Trudy FIAN 3(8) p1692 1971]**

Beamstrahlung radiation angle (preliminary)

- examine plane of radiation orthogonal to field (x-z plane)
- 14 mrad crossing angle, normalised radiation intensity
- distribution peaked around direction of incoming electron
- vanishing external field coincides with classical result
- order of magnitude spread for ILC-1000
- will feed into greater angular spread for pair backgrounds

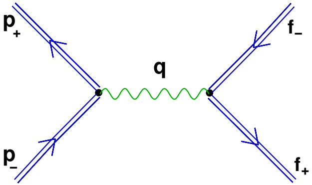




(e.g.) Generic two vertex Furry picture S channel

$$M_{fi} = g_1 g_2 \int dr_1 dr_2 ds_1 ds_2 \; \bar{v}_{p_+} \gamma^{\text{FP}\mu} \, u_{p_-} \, \bar{\epsilon}_{f_+} \gamma^{\text{FP}}_{\mu} \, \epsilon_{f_-} \; \frac{\delta(F-I-(r_1+s_1)k_1+(r_2+s_2)k_2)}{(I+r_1 k_1+r_2 k_2)^2}$$

- final states momentum $F \equiv f_- + f_+$ initial state momentum $I \equiv p_- + p_+$
- spin and polarisation sums as usual
- two dressed vertices γ^{FP}
- r_1, r_2, s_1, s_2 momentum contribution from two external fields at two vertices
- Phase integral not (much) more complicated than for 1 vertex process



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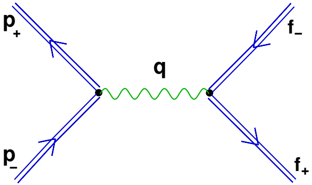

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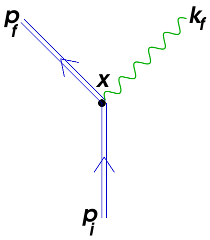
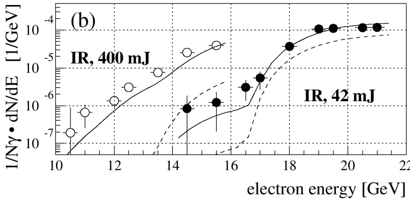
The pole structure depends on r_1, r_2 and is not standard need careful consideration of loops

Experimental tests - SLAC E144 - 1990s

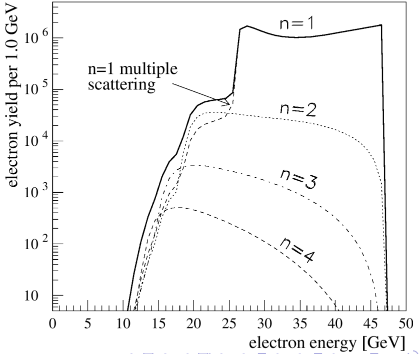




- Collided intense laser (10^{18} W/cm^2) with 46.6 GeV electrons
- effective momentum $q = p - \frac{e^2 a^2}{2k \cdot p} k$

$$(\sum_n) \quad q_i + nk \to q_f + k_f$$

- Compton-like scattering (HICS)
- Compton edge shifted by multiphoton effects



A strong field experiment at the ILC?

The actual proposal:

That we use some part of the extraction line to interact a terawatt LASER with the spent electron beam to do strong field physics

What we would like to measure/discover

- The mass shift, multiphoton effects to higher precision
- dependence of nonlinear effects on radiation angle, polarisation
- Discover higher order resonances
- Draw conclusions with expected primary IP effects

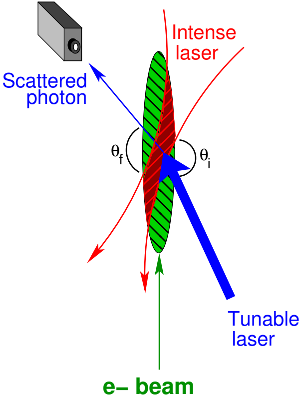
Issues/benefits

- Ideally be at a post IP beam focus
- We will have to think about possible backgrounds - situating of detectors
- Should be no interference with current extraction line diagnostics
- Dont need primary IP collisions
- Data collection time at the level of days for basic strong field phenomena

Reaching the critical field at the ILC

Experiment	$\lambda(\mu m)$	E_{laser}	focus	pulse	I(W/cm^2)	E_{e-} (GeV)	Υ
E144 (SLAC)	1	2 J	60 μm^2	1.5 ps	$\approx 10^{18}$	46.6	0.27
ILC (E144 laser)	1	2 J	60 μm^2	1.5 ps	$\approx 10^{18}$	125-500	0.72-2.9
ILC (PL 9000)	1	3 J	40 μm^2	0.5 ps	6.75×10^{18}	125-500	1.87-7.54

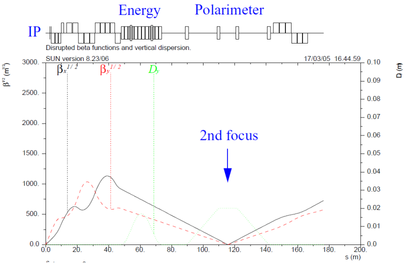
$$I = \frac{E_{\text{laser}}}{\text{spot} \times \text{pulse}}$$

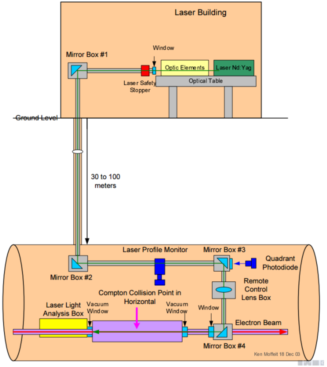


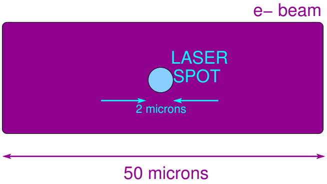
Powerlite™ DLS 9000

- Onset of detectable FP Compton scattering effects from $\nu^2 = 0.1$
- Can detect with today's technology
- The ratio of photon energies, incident, scattered angles is what's important
- Works too for two photon pair production, no e- beam, 2 tunable lasers

Possible intense LASER IP in the extraction line







- At secondary focus, illuminate centre of spent beam
- analysing magnets and detector
- Run parasitically with downstream polarimeter?
- Avoid damaging optical elements

Simulation of strong field effects

REQUIREMENTS:

To simulate a charge bunch collision and calculate the field strength at each point of production

To have a finely scaled simulation in order to accurately model disruption, hour glass effect etc.

To perform a relatively complex cross-section calculation at each point of production

To have full spin tracking

To be flexible enough to include new higher order processes

SOLUTION:

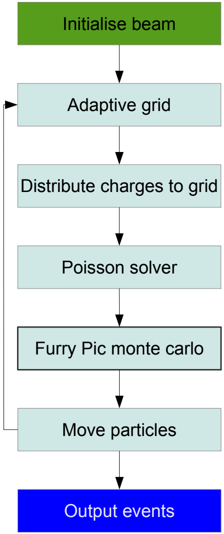
A PIC code using an efficient method for modeling the electrodynamics – crosscheck with CAIN/GP

MPI using openMPI or GPU programming

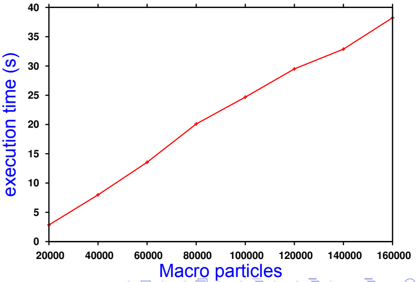
T-BMT with higher order corrections to AMM, Sokolov-Ternov and higher order helicity amplitudes

Allow new processes to be loaded externally

IPstrong - towards a strong field event generator



- Fortran 2003 with openMPI (Fortran 2008 has inbuilt gpu)
- 3D electrostatic poisson solver (MPI)
- Furry picture processes replace all other processes
- output in multiple formats (stdhep, lcio)
- cross-checks with existing programs



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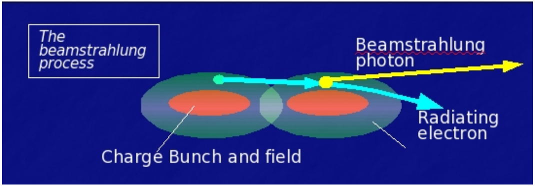
- opportunities to experimentally test FP effects becoming available
- laser IP in extraction line at ILC would test strong field effects in situ
- Need new event generator for FP monte carlo during real bunch collision

Backup Slides

Collider strong field physics



" Strong field processes are physics processes calculated simultaneously in the normal perturbation theory as well as exactly with respect to a strong electromagnetic field. "

" Such calculations are necessary when the external field seen by a particle approaches or exceeds E_{cr}. "

(W.H.) Furry Picture

- Separate gauge field into external A_μ^{ext} and quantum A_μ parts

$$\mathcal{L}_{\text{QED}}^{\text{Int}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}_{\text{QED}}^{\text{FP}} = \bar{\psi}^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}}\,A\!\!\!/\,\psi^{\text{FP}}$$

- Euler-Lagrange equation $\rightarrow$ new equations of motion requires exact (w.r.t. A^{ext}) solutions ψ^{FP}

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

- For certain classes of external fields (plane waves, Coloumb fields and combinations) exact solutions exist **[Volkov Z Physik 94 250 (1935), Bagrov and Gitman Exact solutions of Rel wave equations (1990)]**
- A QFT which is non-perturbative wrt external gauge field A^{ext} and perturbative wrt ψ^{FP}, A

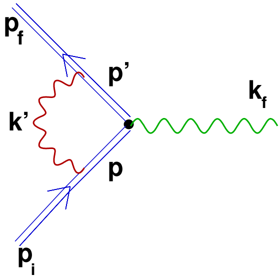
theoretical aspects of the Furry Picture

- External field makes space-time inhomogeneous so propagator depends on separate space-time points rather than on the difference between them **[Berestetski Lifshitz Pitaevski, QED $\S 109$]**
- Normalised IN and OUT states can be formed and LSZ extended to include such states **[Meyer, J Math Phys 11 312 (1970)]**
- Vanishing field strength at $t = \pm\infty \rightarrow$ stable vacuum
- Vacuum can be polarised so must include tadpole diagrams **[Schweber Relativistic QFT $\S$15g]**
- Operator and path integral representations for generating functional **[Fradkin, QED in an unstable vacuum]**
- Anomalous magnetic moment (one-loop) in a const crossed field varies from $\frac{\alpha}{2\pi}$ **[Ritus, JETP 30 1181 (1970)]**

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi} \int_0^\infty \frac{2\pi\, dx}{(1+x)^3} \left(\frac{x}{\Upsilon}\right)^{1/3} \mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{1/3}$$

Vertex function in (one) external field

$$\Gamma^{FP} = 2ie^2 \int drdsdl \int \frac{d^4k'}{k'^2}\, \gamma^{FP\nu}\, \frac{\not p' + m}{(q_f - k' - rk)^2 - m_*^2}\, \gamma_\mu^{FP}\, \frac{\not p + m}{(q_i - k' + sk)^2 - m_*^2}\, \gamma_\nu^{FP}\, \delta(q_f + k_f - q_i - lk)$$



- Examine pole structure of the vertex function

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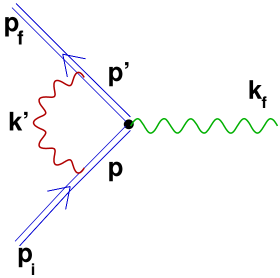

- Examine pole structure of the vertex function

- We combine denominators using Feynman parameters as normal,

$$\int \frac{d^4k'}{k'^2[(q_f - k' - rk)^2 - m_*^2][(q_i - k' + sk)^2 - m_*^2]} = \int_0^1 dxdydz \frac{d^4k'}{(k'^2 - \Delta)^3} \delta(x + y + z - 1)$$

- Numerator more complicated than the usual case - need new tricks, but apart from the usual divergences we end up with additional poles in the residual

$$\frac{1}{\Delta(r, s, x, y, z)}$$

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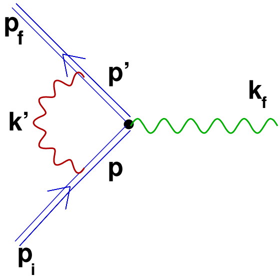

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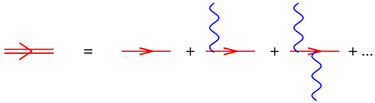
$$\frac{1}{\Delta(r, s, x, y, z)}$$

- Additional poles in the residue which match those in the tree level FP processwa
- Vertex function can be same order as tree-level diagram - must include!

Equiv Photon Approx and Perturbation expansion

- decompose external field into n equivalent photons
- sum the series to desired order of accuracy



$$G^e = G + G\hat{V}G + G\hat{V}G\hat{V}G + \ldots$$

$$G = (p^2 - m^2)^{-1}$$

$$\hat{V} = 2eA^e \cdot p - e^2 A^{e\,2}$$

- within certain constraints:
  - scalar particle
  - monochromatic photons

  the summation can be performed (Reiss Eberly 1966)

- Can the entire summation be performed in general ?

- The alternative is the Furry/Feynman method...

Infinite momentum frame

- QED can be formulated in a Lorentz frame moving at the limit of the speed of light (Kogut & Soper Phys Rev D 1(10) 2901 (1970))
- regular coordinates (t, x, y, z) can be expressed in light cone coordinates $x_\pm = \frac{1}{2}(t \pm z)$; $x_\perp = (x, y)$
- light cone dirac matrices separate into sub-algebras whose members anti-commute $\gamma_\pm \gamma_\perp = -\gamma_\perp \gamma_\pm$
- light cone scalar products are $a.b = 2a_+ b_- + 2a_- b_+ - a_\perp . b_\perp$

Strong fields at the collider IP

- moving charge has longitudinal length contraction

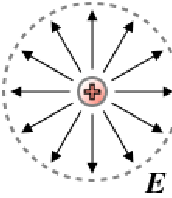
- relativistic charge bunch produces constant crossed plane wave field

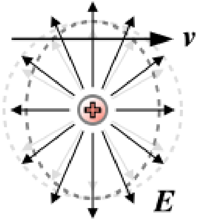
$$A_\mu = a_{1\mu}(k \cdot x)$$

$$a_{1\mu} = (0, \vec{a})$$

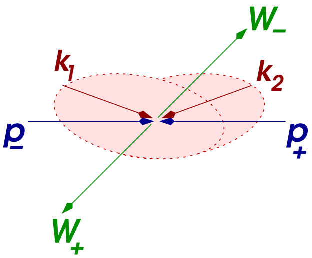
- particle p sees a field strength parameter Υ

$$\Upsilon = \frac{e|\vec{a}|}{mE_{\text{cr}}}(k \cdot p)$$





Volkov-type solutions in two external fields



- both incoming bunches contribute external fields
- external field wavevectors are generally anti-collinear
- Need new Volkov-type solution

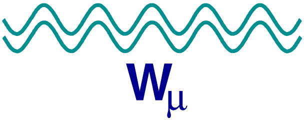
- strategy is to first solve Klein-Gordon equation $(D^2 + m_W^2)\phi_e^\pm$

$$\phi_e^\pm = \frac{1}{\sqrt{2\epsilon_p V}} \int dr \, \exp\left[-ib\, p \cdot x - ireA_e - \frac{(r-f)^2}{2|z|} \right]$$

- For constant crossed field Dirac equation solution proceeds from the Klein-Gordon solution

W boson Volkov Solution

- Equation of motion for the W boson



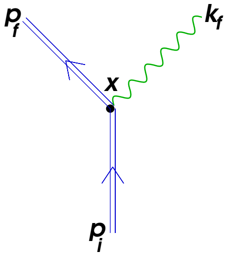
$$(D^2 + m_W^2)W_\nu + i2eF^\mu{}_\nu W_\mu = 0, \quad D^\mu W_\mu = 0$$

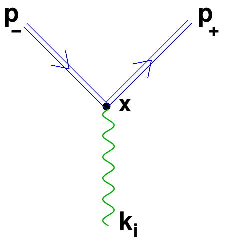
- with solution $W_\mu = E_p^W \, e^{-ip\cdot x} \, w_p$ where

$$E_p^W = \left(g_{\mu\nu} + \frac{e}{k\cdot p}\int F_{\mu\nu} - \frac{e^2}{2(k\cdot p)^2} A^{e2} k_\mu k_\nu \right)$$
$$\cdot \exp\left[-\frac{i}{2(k\cdot p)} \left(2e(A^e\cdot p) - e^2 A^{e2} \right) \right]$$

- similar solutions can be found for other particles that couple to A^e

Beamstrahlung, incoherent/coherent pair production

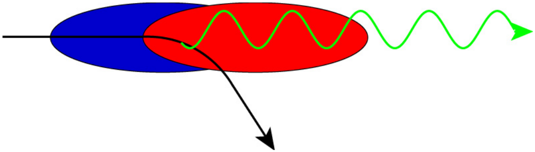




- IP beam-beam simulators - CAIN, Guinea-Pig
- beamstrahlung & coherent pair production calculated via quasi-classical approx
- incoherent pairs calculated with beamstrahlung photon and equivalent photon approx (EPA)
- **more exactly** these are 1st and 2nd order Furry picture processes

bkgd pairs	current	proposed
coherent	quasi-classical	1 vertex Furry picture
incoherent	EPA	2 vertex Furry picture

Formation length



" distance travelled by a charged particle while a radiated photon moves one wavelength in front of it "

A bad argument: *" If the bunch is sufficiently short we dont need to worry about strong field effects"*

- classical argument that only applies to the beamstrahlung
- strong field propagator integrated over all length scales