

# **Radiative corrections to the Higgs boson coupling constants in multi Higgs models**

**Mariko Kikuchi (Univ. of Toyama)**

**Collaborators**

**Shinya Kanemura(Univ. of Toyama),**

**Kei Yagyu(National Central Univ.)**

**Phys.Rev. D87 (2013) 015012 M. Aoki, S. Kanemura, M. K., K. Yagyu**

**Paper in preparation S. Kanemura, M. K., K. Yagyu**

**LCWS2013, UNIV. OF TOKYO, NOVEMBER 11-15, 2013**

# Introduction

- ◆ Higgs boson( $h$ ) was discovered at LHC last year !

The data indicate that it is a SM-like Higgs boson.

$$m_h = 126 \text{ GeV}$$

- ◆ What is the shape of the Higgs sector?

No principle for minimal Higgs with one doublet

**SM-like  $\neq$  SM**

Second Higgs boson

- ◆ We have to construct the Higgs sector by the bottom-up approach.

- Direct searches
- Indirect searches

**We here evaluate  $h$ -couplings in 2 Higgs doublet models with radiative corrections.**

# Higgs as a probe of new physics

## Physics of $h$

Deviations in coupling constants of the  $h$   
due to heavy Higgs bosons (or NP particles)

There are two possibilities to change couplings of  $h$

- Mixing among scalar fields
- Radiative corrections

We can discriminate extended Higgs models by precisely measuring **the pattern of deviations** in  $h$ -couplings.

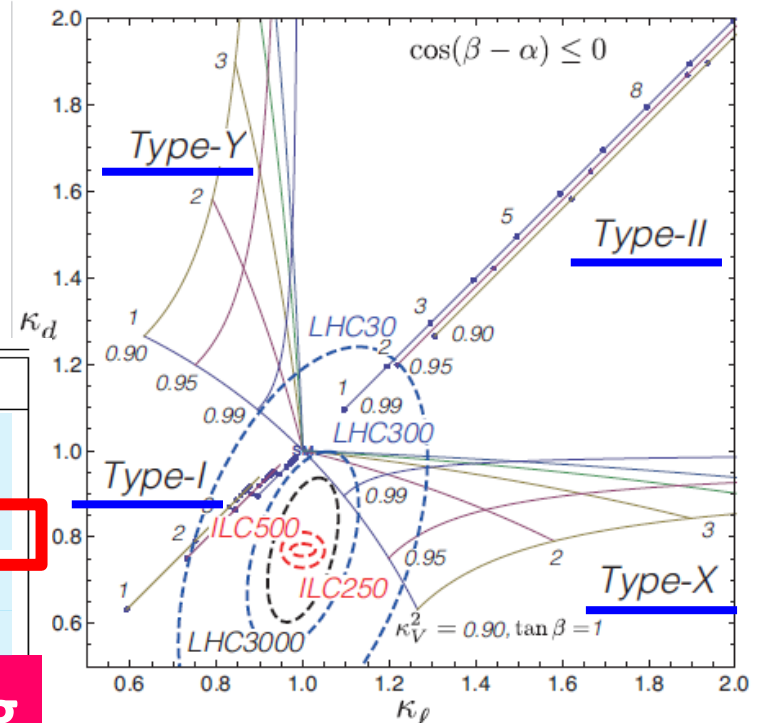
$$g_{hXX} = g_{hXX}^{SM} \times \kappa_{hXX}$$

$$\cos(\beta - \alpha) < 0$$

Model	$\mu$	$\tau$	$b$	$c$	$t$	$g_V$
Singlet mixing	↓	↓	↓	↓	↓	↓
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓

**Finger printing**

Kanemura, Tsumura, Yagyu, Yokoya  
in ILC HIGGS WHITE PAPER



# Future precision measurements

ILC 250–1000GeV

[D.M. Asner](#), [T. Barklow](#), [C. Calancha](#), [K. Fujii](#), [N. Graf](#),  
[H. E. Haber](#), [A. Ishikawa](#), [S. Kanemura](#), [S. Kawada](#), [M.](#)  
[Kurata](#), [A. Miyamoto](#), [H. Neal](#), [H. Ono](#), [C. Potter](#), [J.](#)  
[Strube](#), [T. Suehara](#), [T. Tanabe](#), [J. Tian](#), [J. Tsumura](#), [S.](#)  
[Watanuki](#), [G. Weiglein](#), [K. Yagyu](#), [H. Yokoya](#)

(%) ILC HIGGS WHITE PAPER

	cc	bb	tt	$\tau\tau$	$\mu\mu$	WW	ZZ	$\gamma\gamma$	gg	hhh
ILC(250)	6.8	5.3	-	5.7	91	4.8	1.3	17	6.4	-
ILC(250+500)	2.8	1.6	14	2.3	91	1.1	1.0	8.3	2.3	83
ILC (250+500+1000)	1.8	1.3	3.1	1.6	16	1.1	1.0	3.8	1.6	21
ILC(LumUp)	1.0	0.7	1.9	0.9	10	0.6	0.5	2.3	0.9	13

**$h$ -couplings can be measured typically by O(1) % !!**

**Calculating  $h$ -couplings accurately  
with radiative corrections is very important !**

**Precision measurements  
of Higgs couplings**

×

**Theoretical predictions  
at loop level**

# In this talk

We comprehensively evaluate  $h$ -coupling constants with radiative corrections assuming extended Higgs models, in order to find the pattern of deviations from the SM.

## ◆ Which models do we consider ?

- ① Singlet model ( $\Phi$ +S), ② 2HDM (Type-I, II, X, Y), ③ Higgs triplet model ( $\Phi$ + $\Delta$ )
- This time
- S: singlet field  
 $\Phi$ : doublet field  
 $\Delta$ : triplet field
- done
- done
- Aoki, Kanemura, Kikuchi,  
Yagyu, PRD87.015012 (2013)

## ◆ Which couplings do we consider ?

$hcc, hbb, h\tau\tau$

Yukawa

$hWW, hZZ$

Gauge

$h\gamma\gamma, hgg, h\gamma Z$

Loop

$hhh$

Self coupling

## ◆ We calculate renormalized $h$ -couplings

$$g_{hXX}^{\text{reno}} = g_{hXX}^{\text{tree}} + \delta g_{hXX} + g_{hXX}^{1PI}$$

$$\Delta g_{hXX} = \frac{g_{hXX} - g_{hXX}^{\text{SM}}}{g_{hXX}^{\text{SM}}}$$

# 2 Higgs Doublet Model (Model)

$\Phi_1, \Phi_2$  (2-Higgs doublets)

Dangerous!!

In general, in a multi-doublet structure, **FCNC** appears.

◆ To avoid FCNC, we impose  **$Z_2$  symmetry**

$$\Phi_1 \rightarrow +\Phi_1, \quad \Phi_2 \rightarrow -\Phi_2$$

**4 Types** of Yukawa interaction according to charge assignment of  $Z_2$

Barger, Hewett, Phillips(1990),  
Aoki, Kanemura, Tsumura, Yagyu(2009),  
Logan, Su, Haber, ....

<i>type</i>	$\Phi_1$	$\Phi_2$	$u_R$	$d_R$	$l_R$	$Q_L, L_L$
Type-I	+	−	−	−	−	+
Type-II	+	−	−	+	+	+
Type-X	+	−	−	−	+	+
Type-Y	+	−	−	+	−	+

**Phenomenology is different** in each type of Yukawa interaction.

# 2 Higgs Doublet Model (2HDM)

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)} \\ + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\ + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} \left[ (\Phi_1^\dagger \Phi_2)^2 + (\text{h.c.}) \right]$$

$$\Phi_1 \text{ and } \Phi_2 \Rightarrow \underbrace{h, H, A^0, H^\pm}_{\substack{\uparrow \quad \uparrow \quad \uparrow \text{charged} \\ \text{CPEven CPodd}}} \oplus \text{Goldstone bosons}$$

$$m_h^2 = v^2 \left( \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{\lambda}{2} \sin^2 2\beta \right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda) \sin^2 \beta \cos^2 \beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,$$

$$m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

$M_{\text{soft}}$ : soft breaking scale

$$\lambda = \lambda_3 + \lambda_4 + \lambda_5$$

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i = 1, 2)$$

**Field mixing**

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \quad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix} \\ \begin{bmatrix} w_{1\pm} \\ w_{2\pm} \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

$$\frac{v_2}{v_1} \equiv \tan \beta$$

$$M_{\text{soft}} \left( = \frac{m_3}{\sqrt{\cos \beta \sin \beta}} \right):$$

soft-breaking scale  
of the discrete symm.

# SM-like limit

	$\xi_h^u$	$\xi_h^d$	$\xi_h^e$
Type-I	$\frac{\cos \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$
Type-II	$\frac{\cos \alpha}{\sin \beta}$	$-\frac{\sin \alpha}{\cos \beta}$	$-\frac{\sin \alpha}{\cos \beta}$
Type-X	$\frac{\cos \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$-\frac{\sin \alpha}{\cos \beta}$
Type-Y	$\frac{\cos \alpha}{\sin \beta}$	$-\frac{\sin \alpha}{\cos \beta}$	$\frac{\cos \alpha}{\sin \beta}$

Scale factor  $\kappa_{hXX}$

$$g_{hXX} = g_{hXX}^{SM} \times \kappa_{hXX}$$

## SM-like limit

$$\kappa_{hVV} = \sin(\beta - \alpha) \rightarrow 1$$

$$\kappa_{hff} = \xi_h^f \rightarrow 1$$

Current LHC data seem to suggest that the Higgs is roughly SM-like within large errors

**SM-like  $\neq$  SM**



# Renormalizations

We calculate all  $h$ -couplings at one loop-level by on-shell renormalization.

Full EW loop corrections and extra Higgs loop corrections

◆ Gauge couplings	$hWW, hZZ$	} Kanemura, Okada, Senaha, Yuan (2004)
◆ Self coupling	$hhh$	
<b>NEW</b> ◆ Yukawa coupling	$hff$	Guasch, Hollik, Peñaranda(2001)

## Renormalized coupling

### 1. Parameter shift

$$\Gamma_{hff}^{\text{tree}} = -i \frac{m_f}{v} \xi_h^f,$$

$$\hat{\Gamma}_{hff}(p_1^2, p_2^2, q^2) = \Gamma_{hff}^{\text{tree}} + \delta\Gamma_{hff} + \Gamma_{hff}^{1\text{PI}}(p_1^2, p_2^2, q^2)$$

$$\delta\Gamma_{hff} = -i \frac{m_f}{v} \xi_h^f \left[ \frac{\delta m_f}{m_f} - \frac{\delta v}{v} + \delta Z_V^f + \frac{1}{2} \delta Z_h + \frac{\delta \xi_h^f}{\xi_h^f} + \frac{\xi_H^f}{\xi_h^f} (\delta C_h + \delta \alpha) \right].$$

### 2. Calculation of 1PI diagrams ( 2 point functions , 3 point functions)

### 3. On-shell conditions

Renormalization conditions   $\delta m_b = \Pi_{bb}^{1\text{PI}}[m_b],$

### 4. Evaluations of the deviations from SM predictions

$$\Delta\Gamma_{hff} \equiv \frac{\Gamma_{hff}^{\text{THDM}} - \Gamma_{hff}^{\text{SM}}}{\Gamma_{hff}^{\text{SM}}}$$

# Renormalized coupling ( $hbb$ in Typell)

$$\Gamma_{hbb} = \Gamma_{hbb}^{tree} + \delta\Gamma_{hbb} + \Gamma_{hbb}^{1PI}$$

## ◆ Counter-term

$$\delta\Gamma_{hbb} = -\frac{m_b}{v} \frac{c_\alpha}{c_\beta} \delta C_{Hh} + \frac{m_b}{v} \frac{s_\alpha}{c_\beta} \left( \frac{\delta m_b}{m_b} - \frac{\delta v}{v} + \frac{s_\beta}{c_\beta} \delta\beta + \delta Z_b + \frac{1}{2} \delta Z_h \right)$$

## ◆ Each counter-term

$$\delta m_b = \Pi_{bb}^{1PI}[m_b],$$

$$\frac{\delta v}{v} = \frac{1}{2} \left( \frac{s_W^2 - c_W^2}{s_W^2} \frac{\Pi_{WW}^{1PI}[m_W^2]}{m_W^2} + \frac{c_W^2}{s_W^2} \frac{\Pi_{ZZ}^{1PI}[m_Z^2]}{m_Z^2} - \frac{d}{dp^2} \Pi_{\gamma\gamma}^{1PI}[p^2] \Big|_{p^2=0} + \frac{2s_W}{c_W} \frac{\Pi_{\gamma Z}^{1PI}[0]}{m_Z^2} \right),$$

$$\delta C_{Hh} = \frac{1}{2(m_H^2 - m_h^2)} (\Pi_{Hh}^{1PI}[m_h^2] - \Pi_{Hh}^{1PI}[m_H^2]),$$

$$\delta\alpha = \frac{1}{2(m_H^2 - m_h^2)} \left\{ 2c_\alpha s_\alpha \left( -\frac{1}{c_\beta} \frac{\delta T_1}{v} + \frac{1}{s_\beta} \frac{\delta T_2}{v} \right) + \Pi_{Hh}^{1PI}[m_H^2] + \Pi_{Hh}^{1PI}[m_h^2] \right\},$$

$$\delta\beta = -\frac{1}{2m_A^2} \left( \Pi_{zA}^{1PI}[0] + \Pi_{zA}^{1PI}[m_A^2] - 2s_\beta \frac{\delta T_1}{v} + 2c_\beta \frac{\delta T_2}{v} \right),$$

$$\delta Z_h = -\frac{d}{dp^2} \Pi_{hh}^{1PI}[p^2] \Big|_{p^2=m_h^2},$$

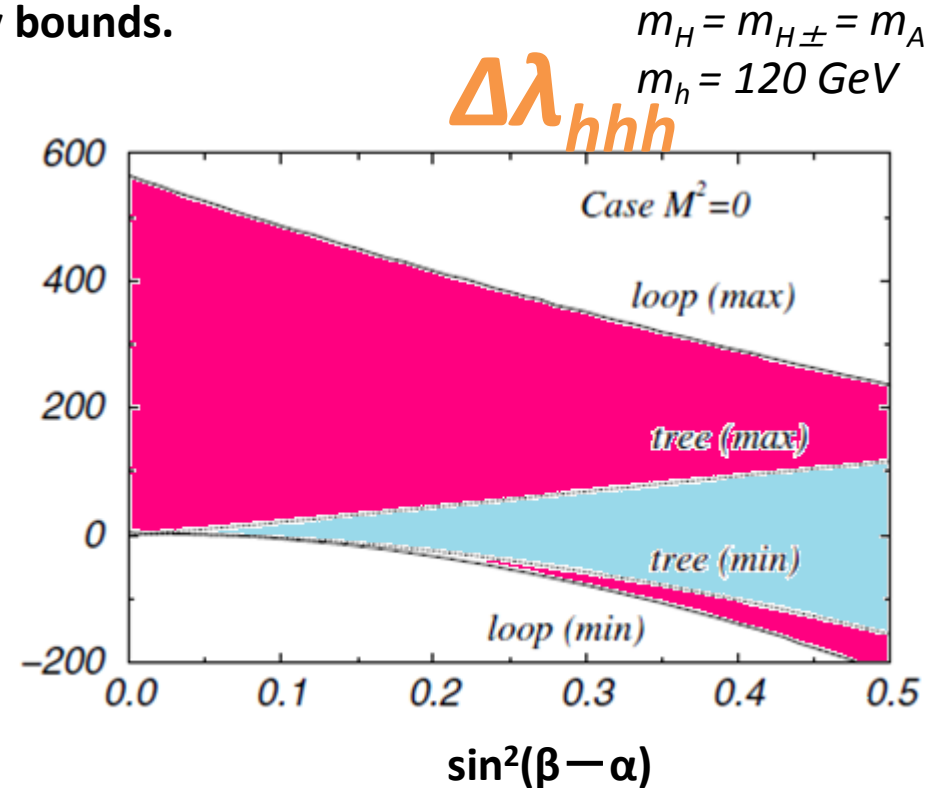
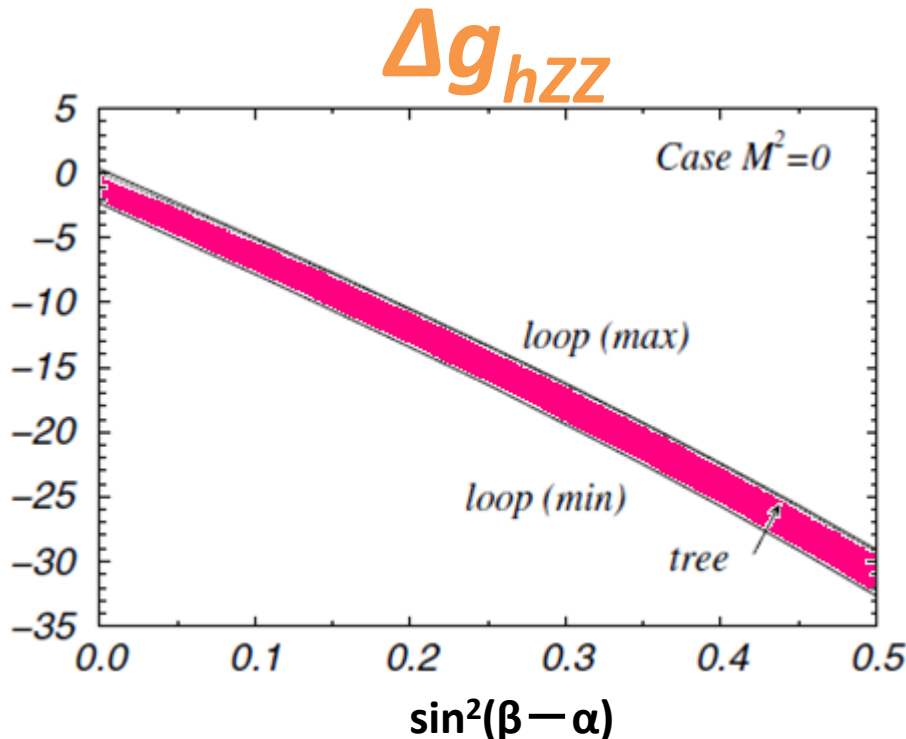
$$\delta Z_b = -\frac{d}{dp^2} \Pi_{bb}^{1PI}[p^2] \Big|_{p^2=m_b^2}.$$

# Deviations in $hZZ$ , $hhh$

Kanemura, Okada, Senaha, Yuan (2004)

## Mixing angle dependence

Allowed region of the deviation in the  $hZZ$ ,  $hhh$  coupling under the unitarity and the vacuum stability bounds.



- $\Delta g_{hZZ} \leq 0$
- $hZZ$  can deviate several % by radiative correction.

- $hhh$  can deviate O(100)% by radiative correction.

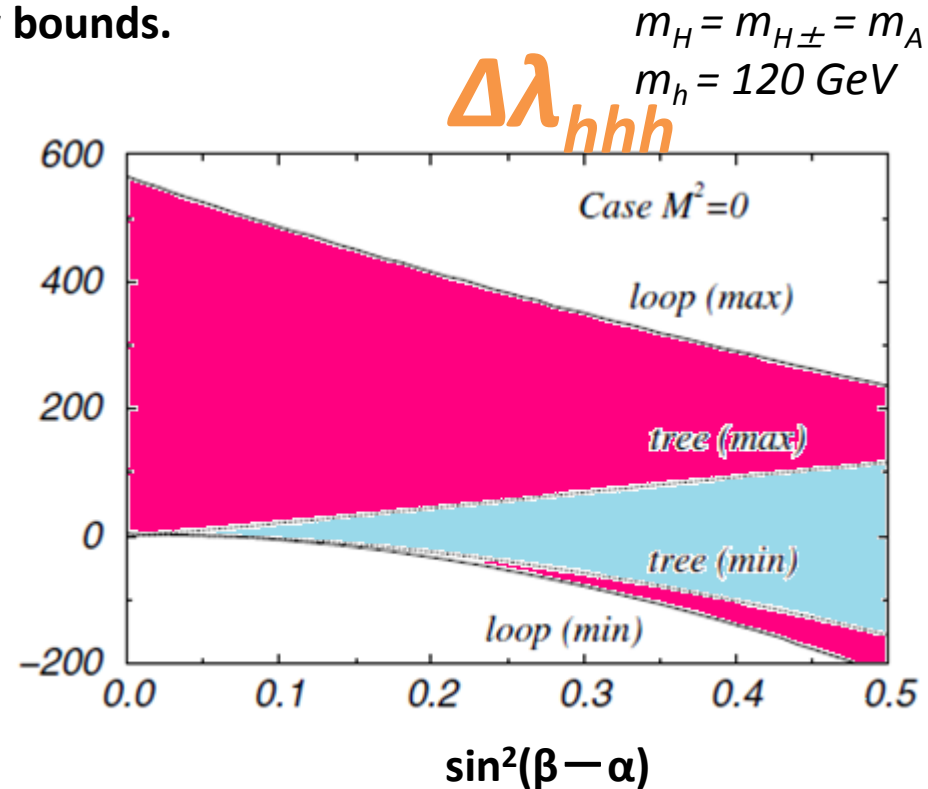
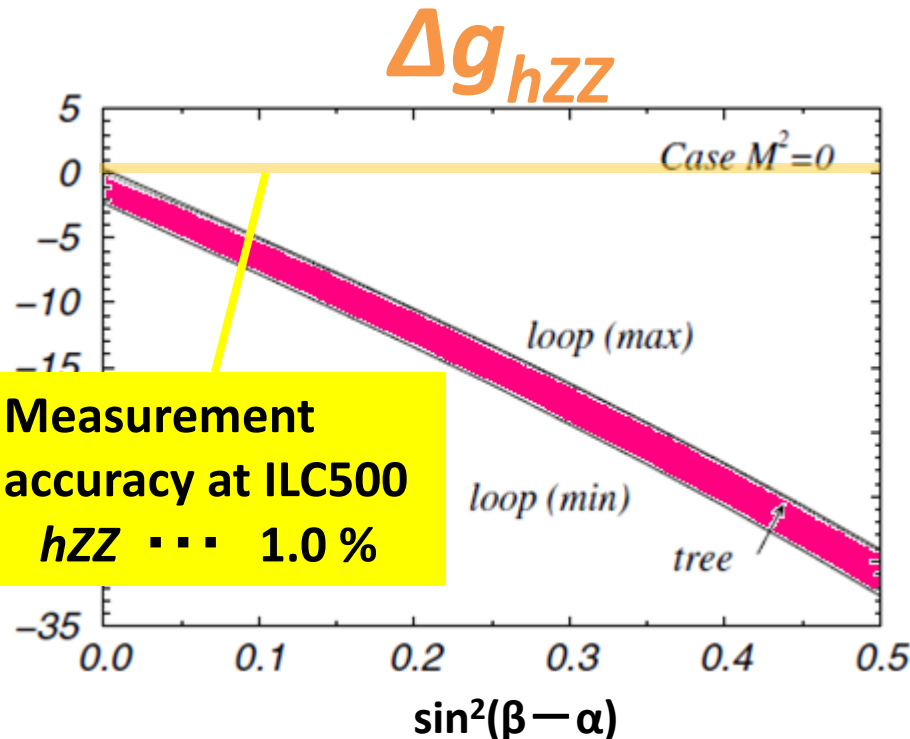
**Non-decoupling loop effect**

# Deviations in $hZZ$ , $hhh$

Kanemura, Okada, Senaha, Yuan (2004)

## Mixing angle dependence

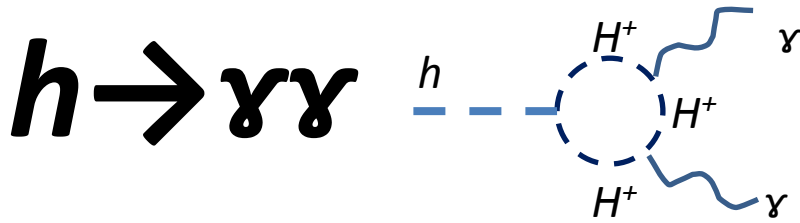
Allowed region of the deviation in the  $hZZ$ ,  $hhh$  coupling under the unitarity and the vacuum stability bounds.



→ We can determine  $\sin^2(\beta - \alpha)$  with high precision.

- $hhh$  can deviate  $O(100)\%$  by radiative correction.

**Non-decoupling loop effect**



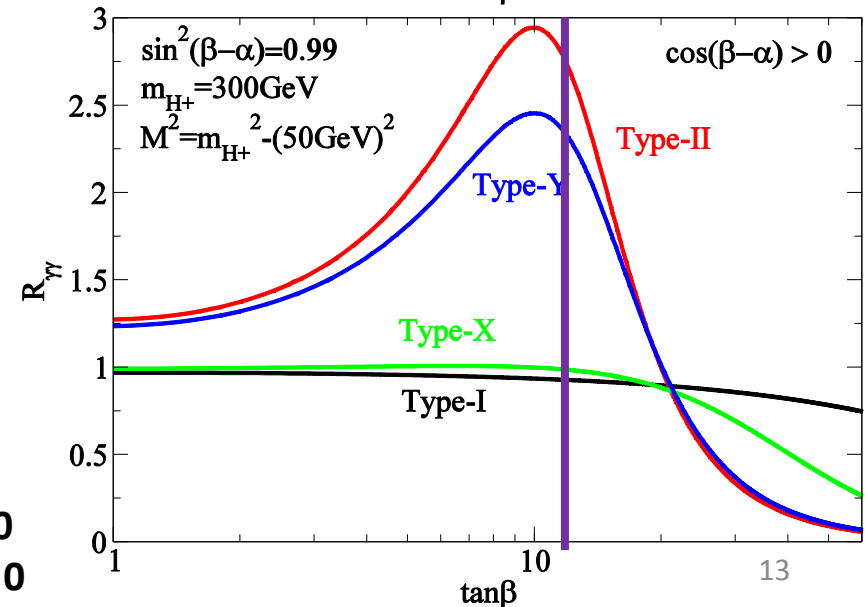
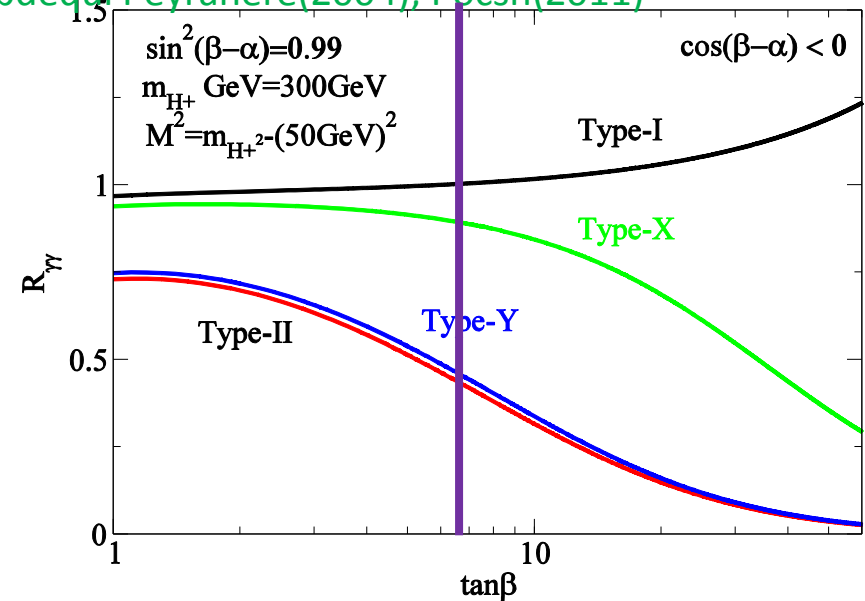
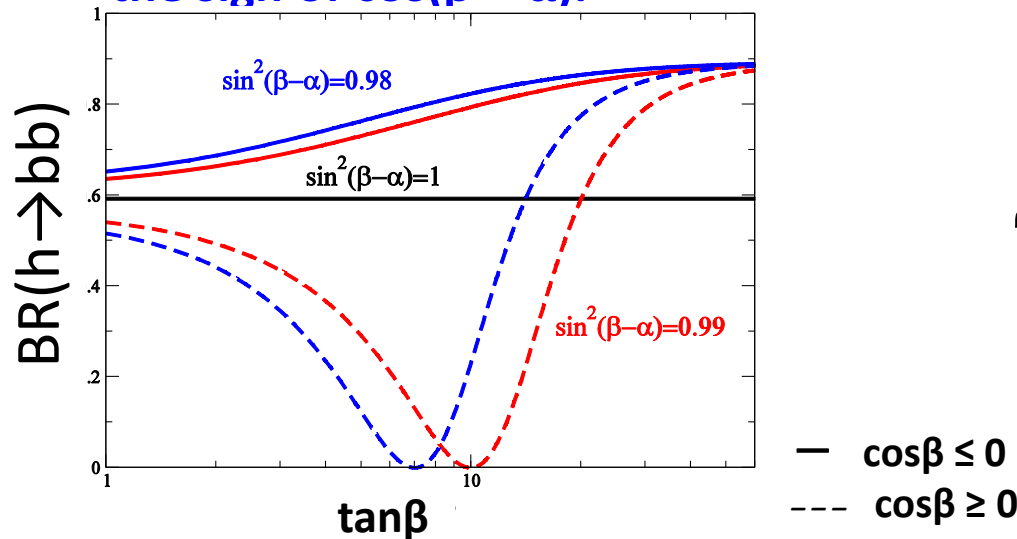
Zeppenfeld, Kinnunen, Nikitenko, Richter-Was(2000),  
Ginzburg, Krawczyk, Osland(2001), Spira, Djouadi,  
Graudenz, Zerwas(1995), Arhrib, Hollik, Penaranda,  
Capdequi Peyranere(2004), Pocsh(2011)

$$R_{\gamma\gamma} \equiv \frac{\sigma(gg \rightarrow h)_{2HDM} \times BR(h \rightarrow \gamma\gamma)_{2HDM}}{\sigma(gg \rightarrow h)_{SM} \times BR(h \rightarrow \gamma\gamma)_{SM}}$$

ATLAS 1.2 – 1.8 , CMS 0.3 – 1.3

◆  $R_{\gamma\gamma}$  depends on types of Yukawa interaction.

◆ Magnitudes of  $R_{\gamma\gamma}$  are different by the sign of  $\cos(\beta - \alpha)$ .



# Deviations in $hff$ (1)

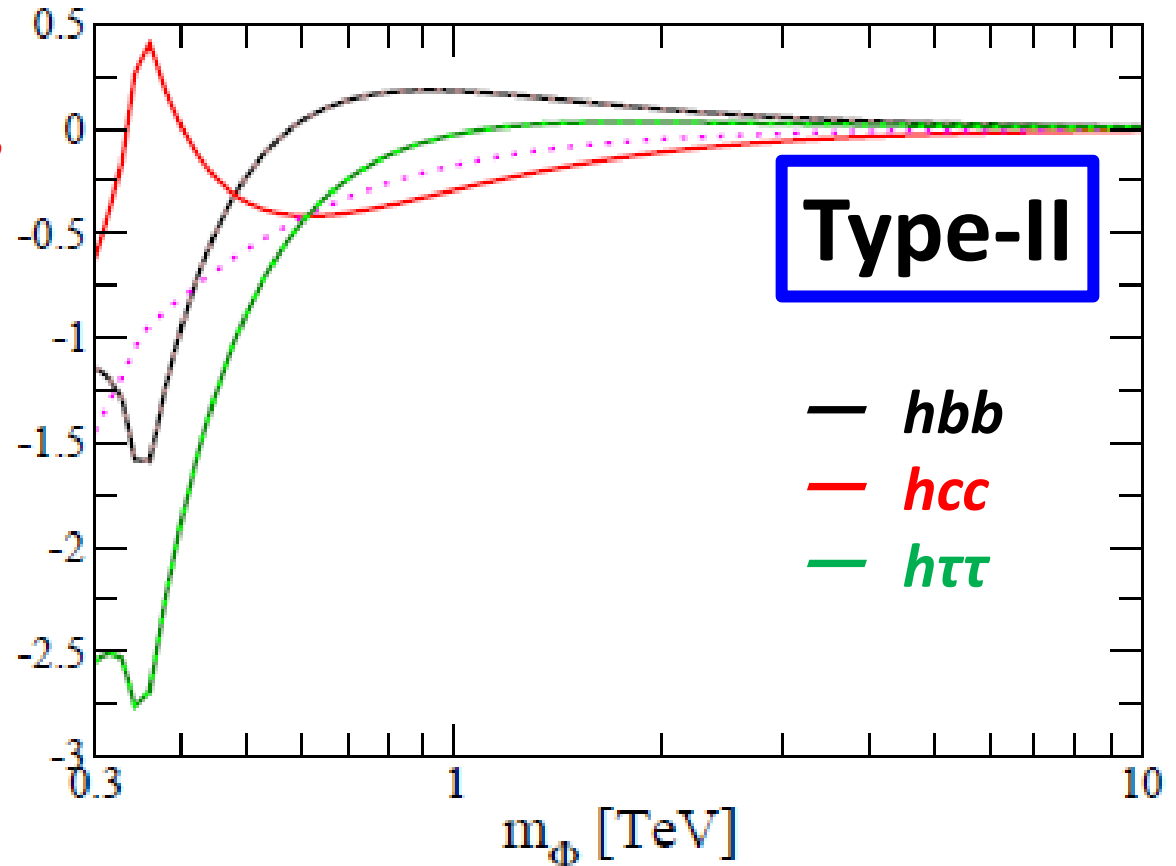
$$\Delta\Gamma_{hff} \equiv \frac{\Gamma_{hff}^{\text{THDM}} - \Gamma_{hff}^{\text{SM}}}{\Gamma_{hff}^{\text{SM}}}$$

$$\sin^2(\beta - \alpha) = 1$$

—  $\tan\beta=1$   
 ---  $\tan\beta=3$

$$m_\phi^2 = M^2 + (300\text{GeV})^2$$

$\Delta\Gamma_{hff}$   
 (%)



We have checked  
 consistency of our  
 calculation.

# Deviations in $hff$ (2)

$$\Delta\Gamma_{hff} \equiv \frac{\Gamma_{hff}^{\text{THDM}} - \Gamma_{hff}^{\text{SM}}}{\Gamma_{hff}^{\text{SM}}}$$

$$\sin^2(\beta - \alpha) = 1$$

**Type-II**

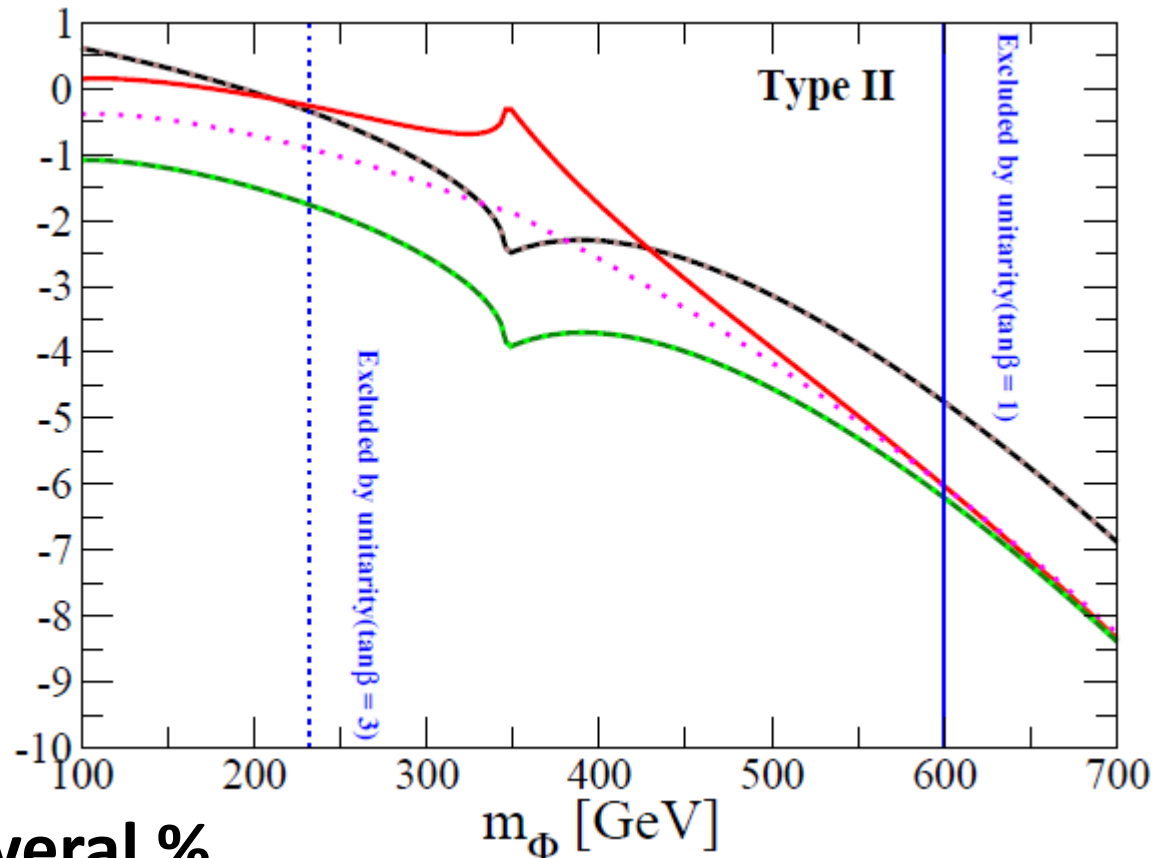
Radiative corrections  
are maximum by  
Non-decoupling effect.

$M^2 = 0$

$\Delta\Gamma_{hff}$   
(%)

—  $hbb$   
—  $hcc$   
—  $h\tau\tau$

—  $\tan\beta=1$   
---  $\tan\beta=3$



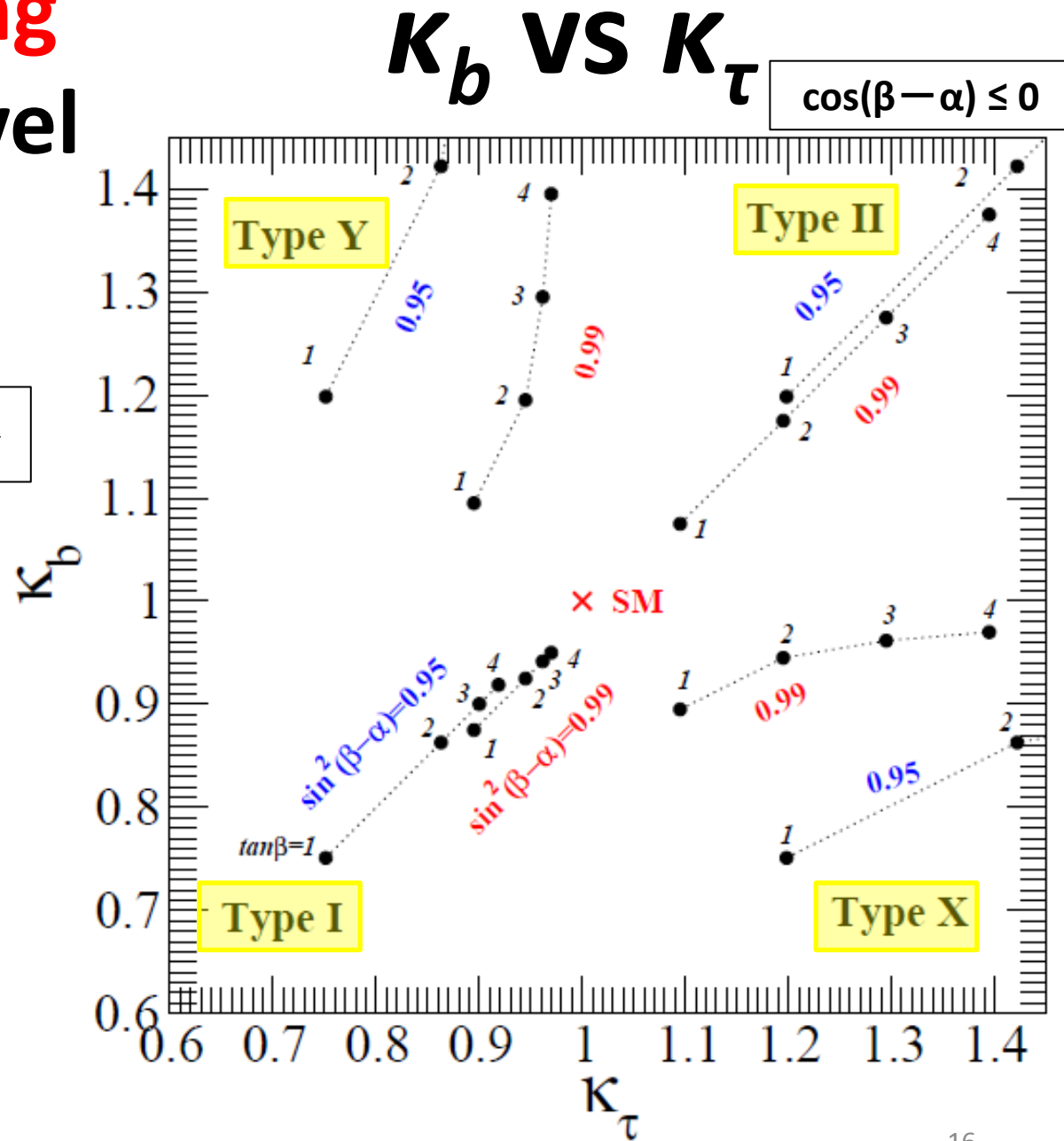
Deviation can be several % .

Kanemura, Kikuchi, Yagyu(2013)

# Finger printing at the tree level

$$g_{hXX} = g_{hXX}^{SM} \times \kappa_{hXX}$$

How about one-  
loop level ?





# Finger printing at the 1-loop level for each $\tan\beta$

◆ We perform a scan analysis for  $m_\phi$  and  $M$ .

$$100 \text{ GeV} \leq m_\phi \leq 1000 \text{ GeV}$$

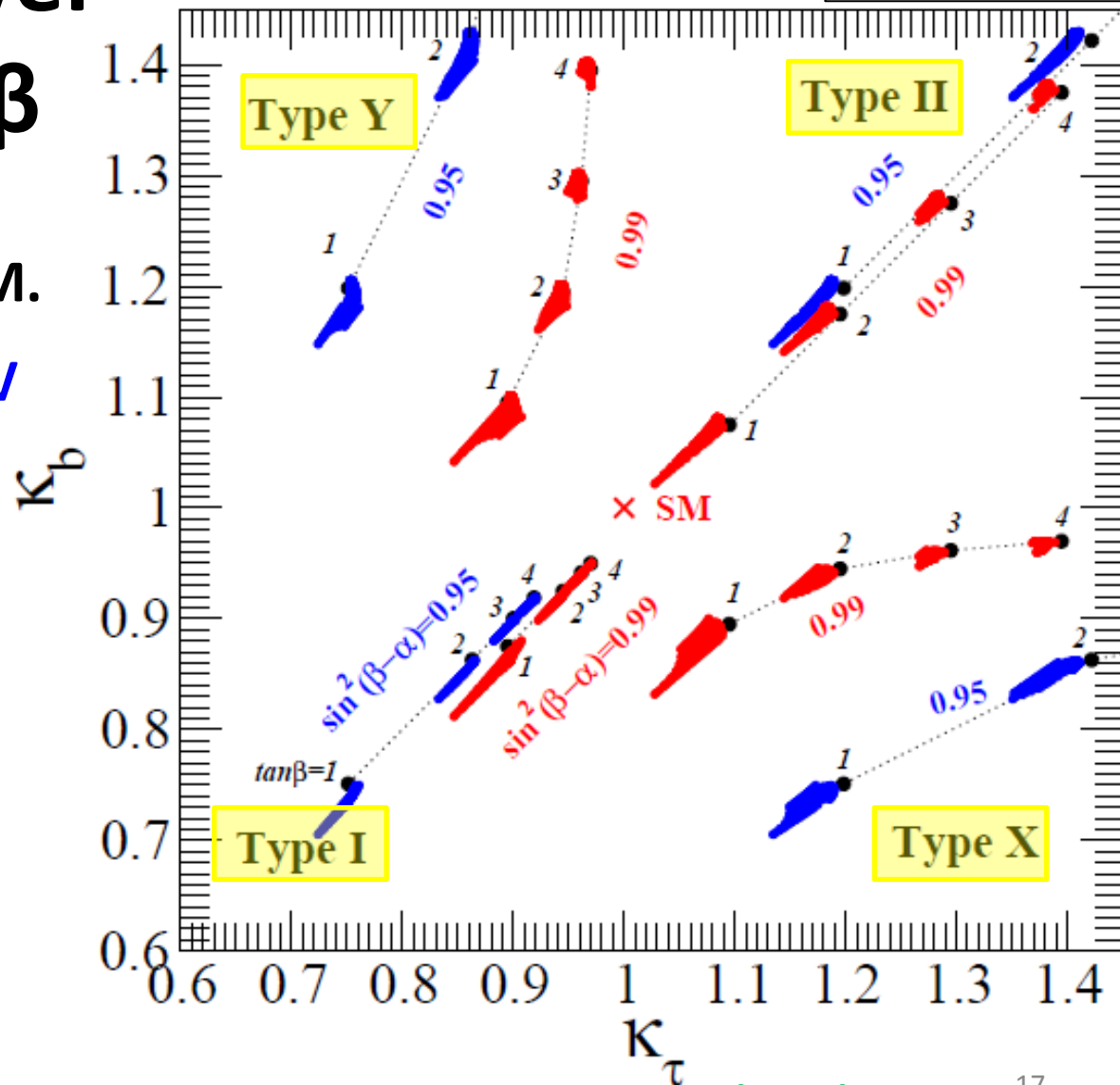
$$0 \leq M \leq m_\phi$$

We take into account constraints from **the unitarity** and **the vacuum stability**.

In the maximum radiative correction case  
 $\rightarrow$  deviation become prediction with smaller  $\tan\beta$  at the tree level

## $K_b$ VS $K_\tau$

$$\cos(\beta - \alpha) \leq 0$$



Kanemura, Kikuchi, Yagyu(2013)

# Finger printing at the 1-loop level

- ◆ We perform a scan analysis for  $m_\phi$ ,  $M$  and  $\tan\beta$ .

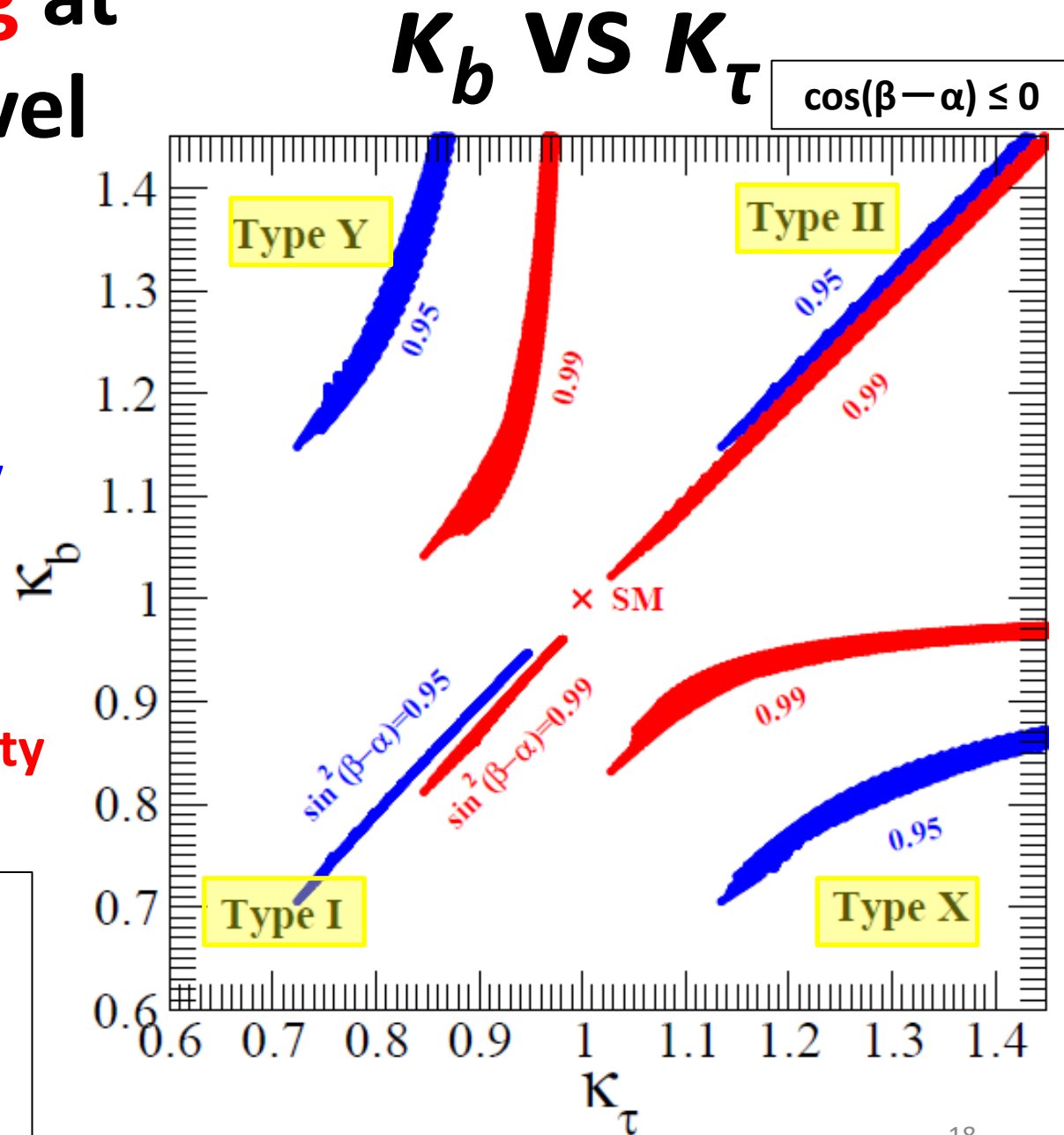
$$100 \text{ GeV} \leq m_\phi \leq 1000 \text{ GeV}$$

$$0 \leq M \leq m_\phi$$

$$1 \leq \tan\beta$$

We take into account constraints from **unitarity** and **vacuum stability**.

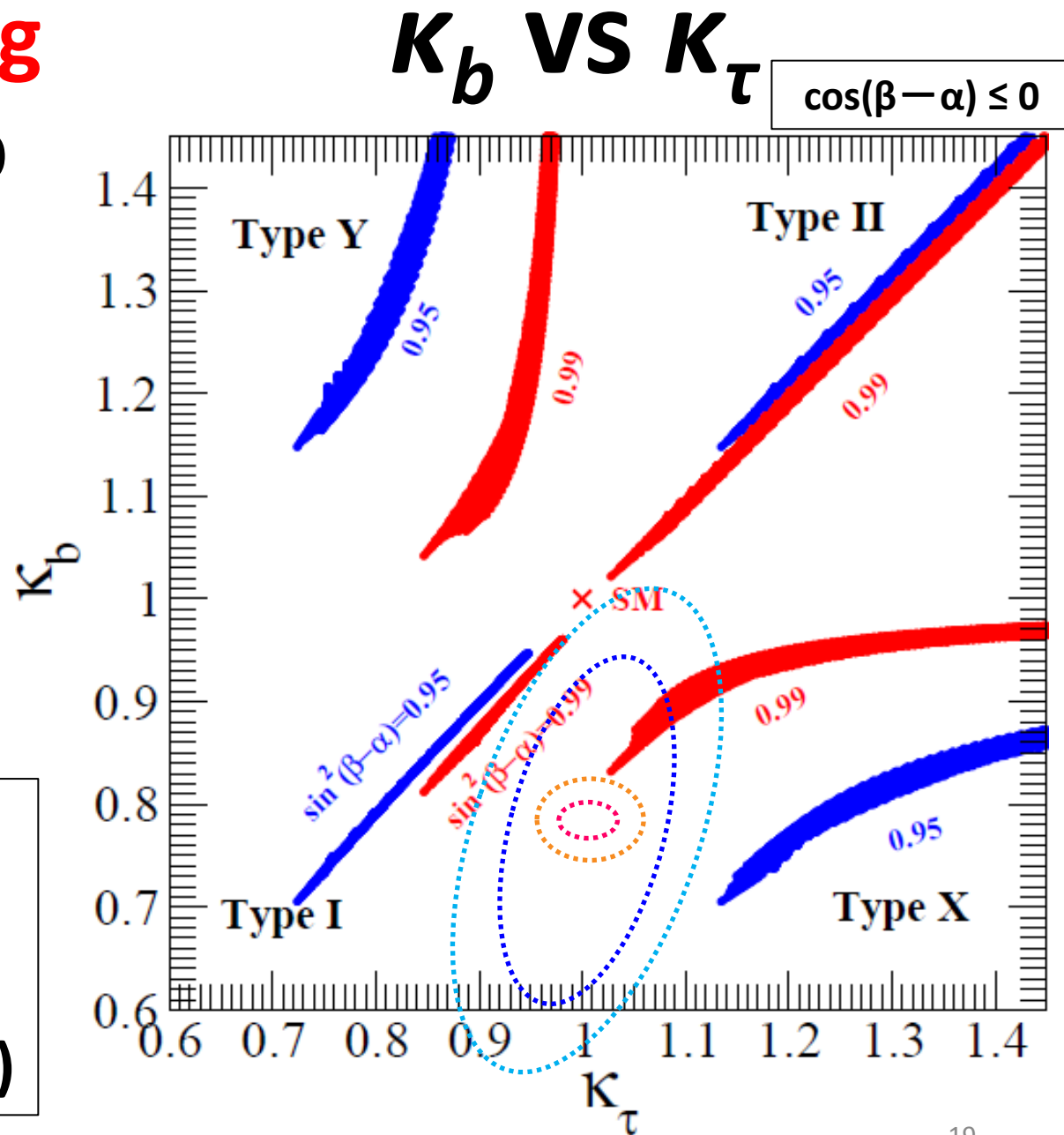
We can discriminate all type in the case with the maximum radiative corrections.



# Finger printing at the 1-loop

- - - - LHC300  
 - - - - LHC3000  
 - - - - ILC250  
 - - - - ILC500  
 (68% C.L.)

We can measure  
 detailed deviations  
 at the ILC  
 (E=250GeV,500GeV)



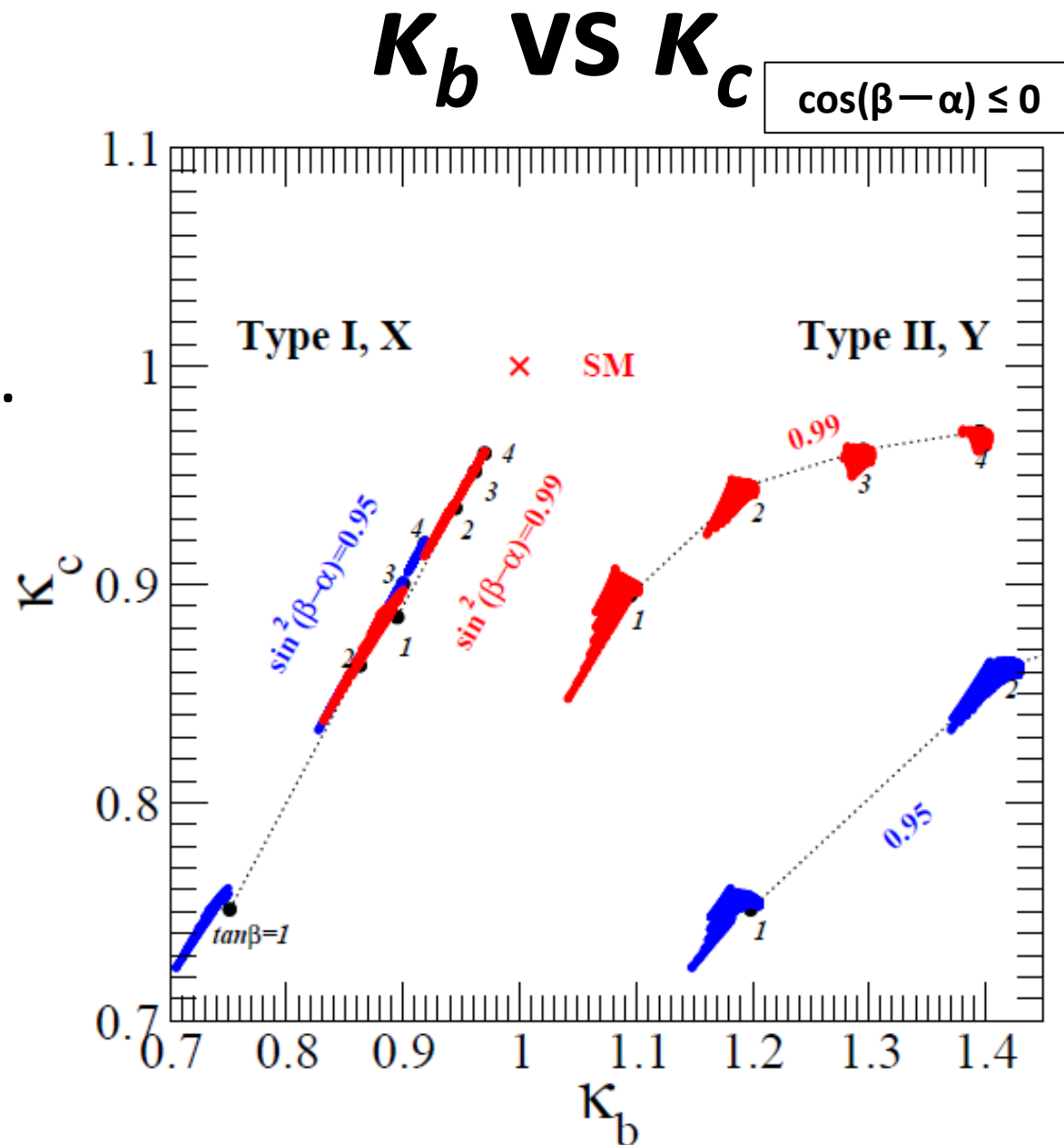
# 1-loop level for each $\tan\beta$

◆ We perform a scan  
analysis for  $m_\phi$  and  $M$ .

$$100 \text{ GeV} \leq m_\phi \leq 1000$$

$$0 \leq M \leq m_\phi$$

We take into account  
constraints from **the**  
**unitarity** and **the vacuum**  
**stability**.



# 1-loop level

- ◆ We perform a scan analysis for  $m_\phi$ ,  $\tan\beta$  and  $M$ .

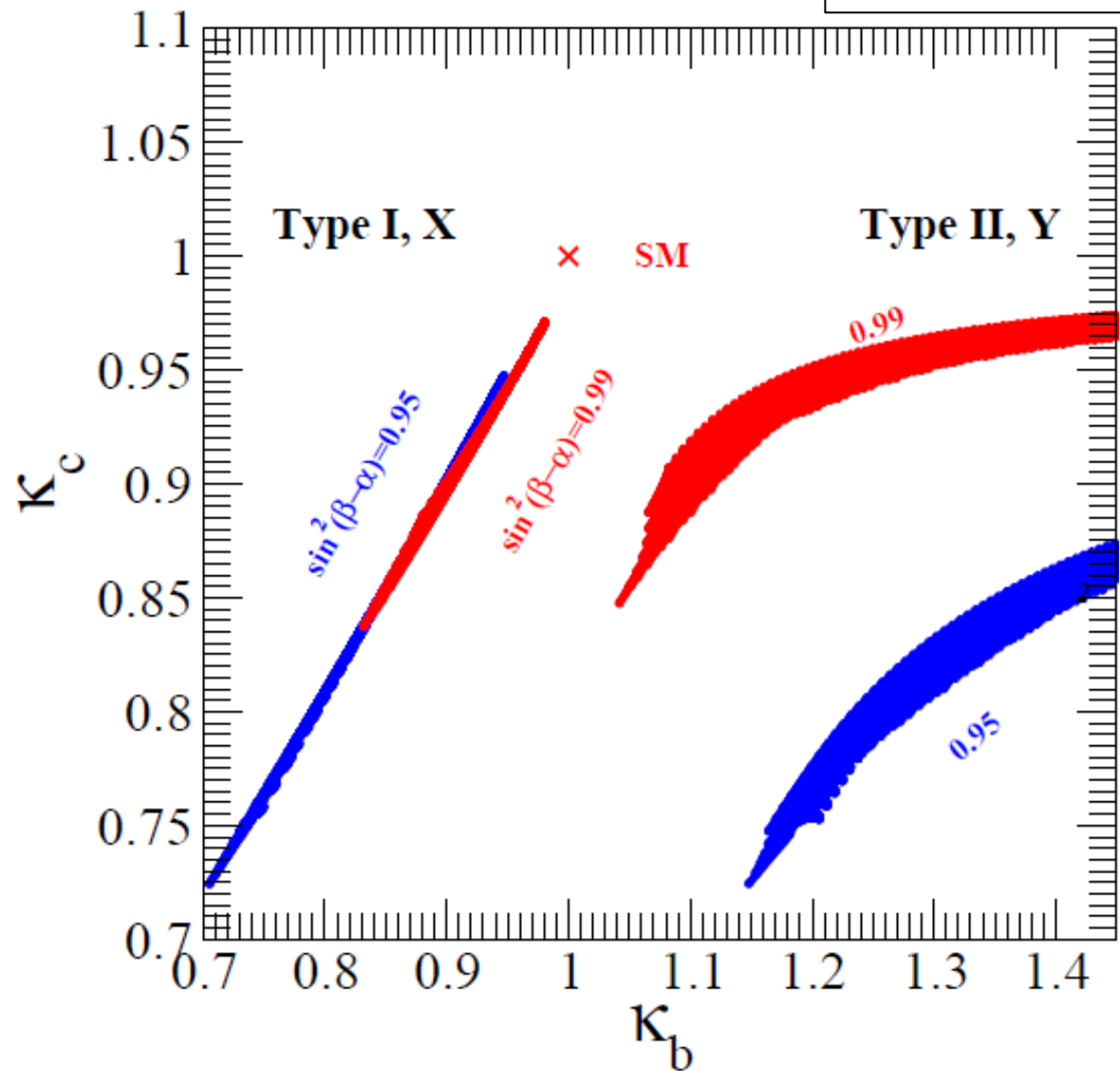
$$100 \text{ GeV} \leq m_\phi \leq 1000$$

$$0 \leq M \leq m_\phi$$

$$1 \leq \tan\beta$$

We take into account constraints from **the unitarity** and **the vacuum stability**.

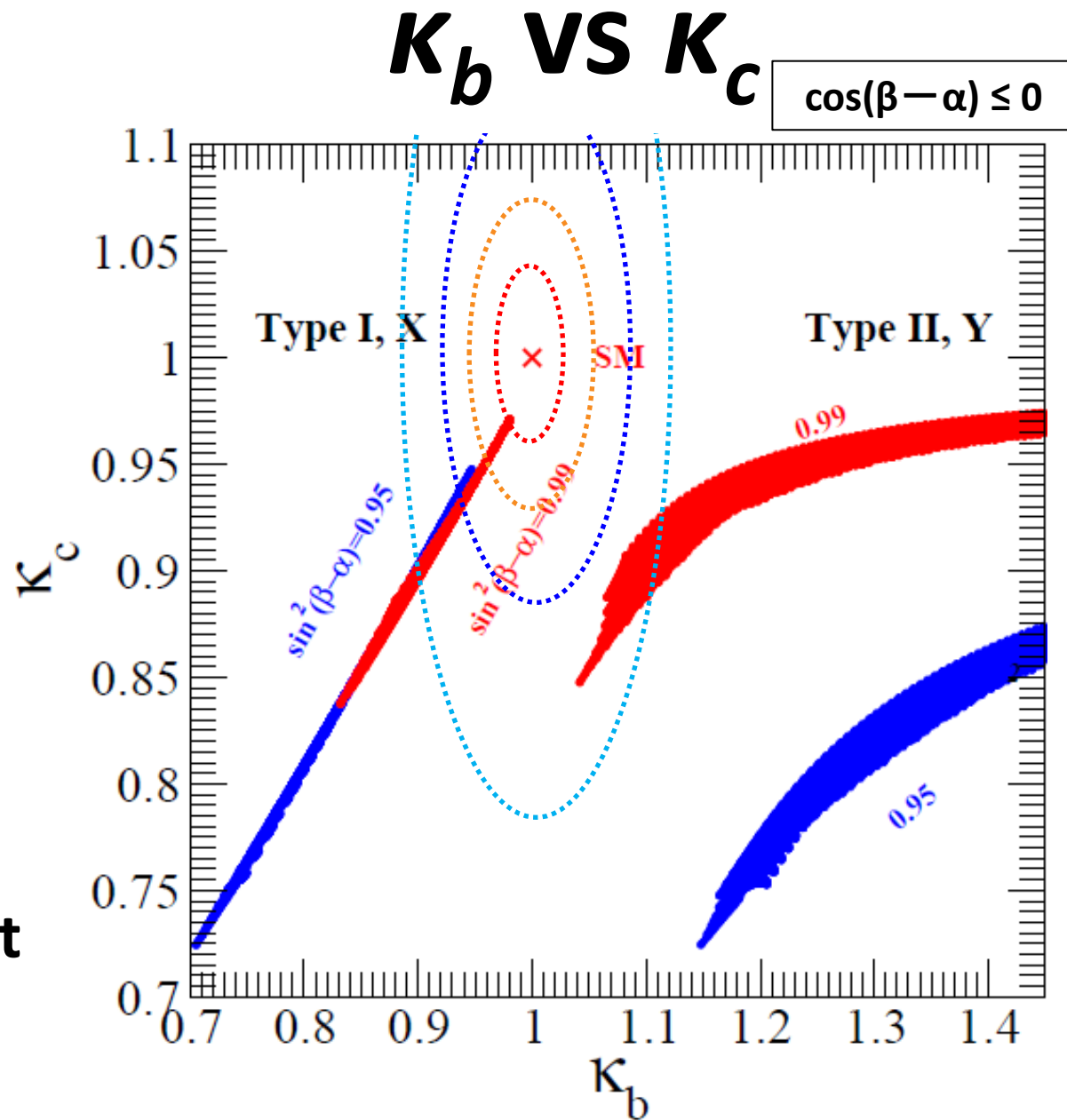
$$K_b \text{ VS } K_c \quad \cos(\beta - \alpha) \leq 0$$



# 1-loop level

----- *LHC300*  
----- *LHC3000*  
----- *ILC250*  
----- *ILC500*  
(68%C.L.)

We can measure  
detailed deviations at  
the ILC  
(E=250GeV,500GeV).



# Summary

- There is no principle for the minimal SM Higgs sector. Extended Higgs models can deviate  $h$ -couplings from the SM
- At the ILC, couplings of the SM-like Higgs boson ( $h$ ) will be measured quite precisely
- For comparison with such future precision data, we evaluate EW radiative corrections to the  $h$ -couplings in 2HDMs with 4 types of Yukawa interactions

$hcc, hbb, h\tau\tau, hWW, hZZ, h\chi\chi, hgg, h\chi Z, hhh$

- Even in the case with maximal non-decoupling loop effects, 4 types of Yukawa interactions can be well discriminated at the ILC, as long as the  $hVV$  coupling slightly differs from the SM value ( $\kappa_V < 0.99$  or more close to 1, but  $\neq 1$ )
- We can make **fingerprinting** of new physics models at the ILC

**Higgs is a probe of new physics!!!**

**Back UP**



# Renormalization

## ➤ Kinetic term

- Parameters in Lagrangian  $\dots g, g', v$
- Physical parameters  $\dots m_W, m_Z, \sin\theta_W, G_F, \alpha_{em}$ .

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1$$

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2},$$

$$G_F = \frac{\pi \alpha_{em}}{\sqrt{2} m_W^2 \sin^2 \theta_W}$$

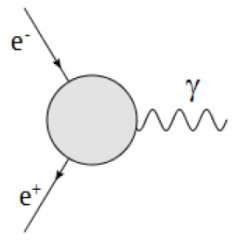
- Counter-terms  $\dots \delta m_W, \delta m_Z, \delta s_W, \delta G_F, \delta \alpha_{em}$ ,
- Renormalized conditions  $\dots \text{Re} \Pi_{WW}(p^2)|_{p^2=m_W^2} = 0,$

$$\delta m_W^2 = \text{Re} \Pi_{WW}^{1PI}(m_W^2),$$

$$\text{Re} \Pi_{ZZ}(p^2)|_{p^2=m_Z^2} = 0,$$

$$\delta m_Z^2 = \text{Re} \Pi_{ZZ}^{1PI}(m_Z^2),$$

On-shell  
conditions



$$= -ie\gamma^\mu$$

$$\frac{\delta \alpha_{em}}{\alpha_{em}} = \frac{d}{dp^2} \Pi_{\gamma\gamma}^{1PI}(p^2)|_{p^2=0} - \frac{2s_W}{c_W} \frac{\Pi_{\gamma Z}^{1PI}(0)}{m_Z^2}$$

- Counter term of  $v$

$$v^2 = \frac{m_W^2 \sin^2 \theta_W}{\pi \alpha_{em}}$$



$$\frac{\delta v}{v} = \frac{1}{2} \left( \frac{\delta m_W^2}{m_W^2} - \frac{\delta \alpha_{em}}{\alpha_{em}} + \frac{\delta s_W^2}{s_W^2} \right)$$

# Deviations in $hff$

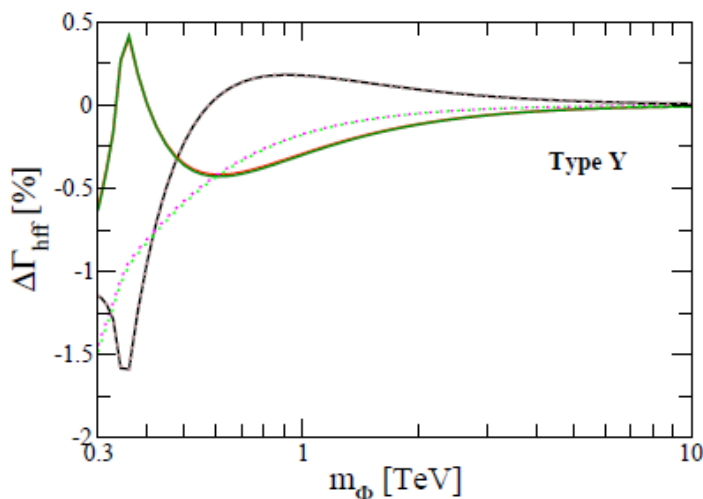
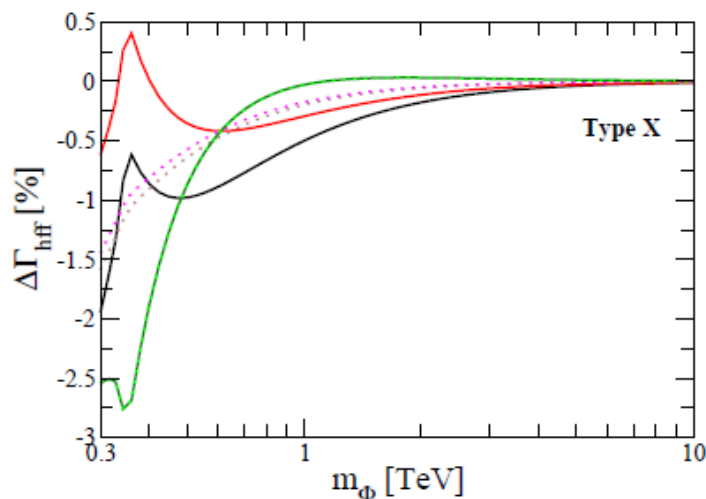
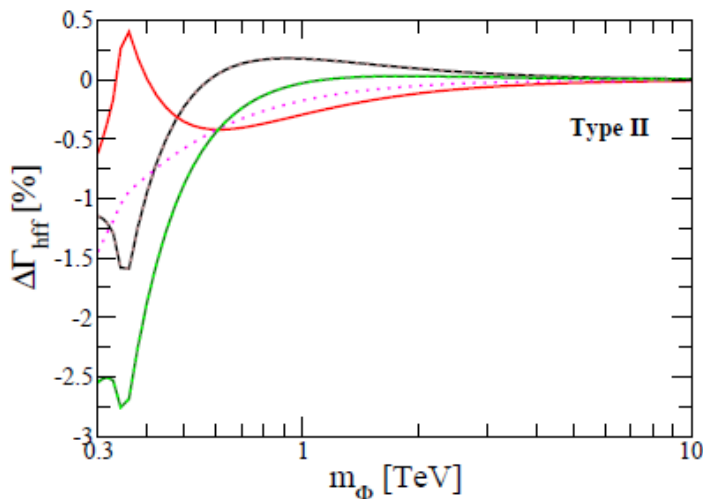
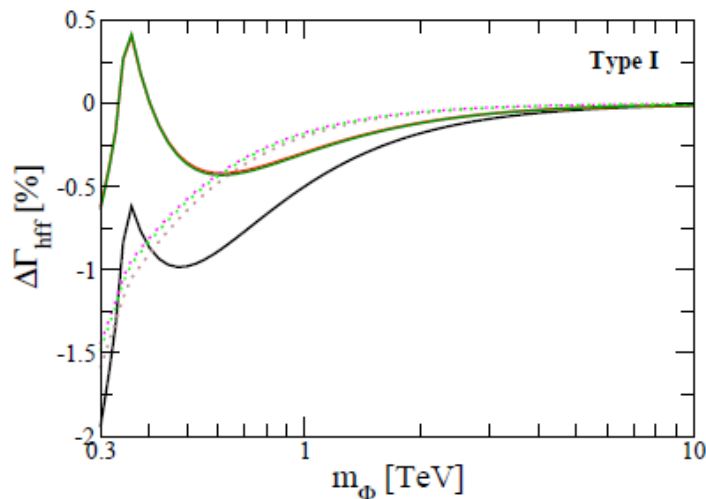
Kanemura, Kikuchi, Yagyu(2013)

$$\sin^2(\beta - \alpha) = 1$$

—  $\tan\beta=1$

---  $\tan\beta=3$

$$M^2 = m_\phi^2 - (300\text{GeV})^2$$



# Deviations in $hff$

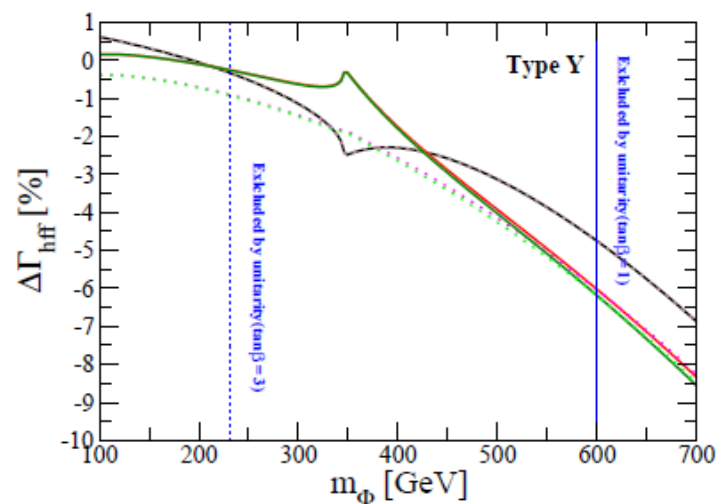
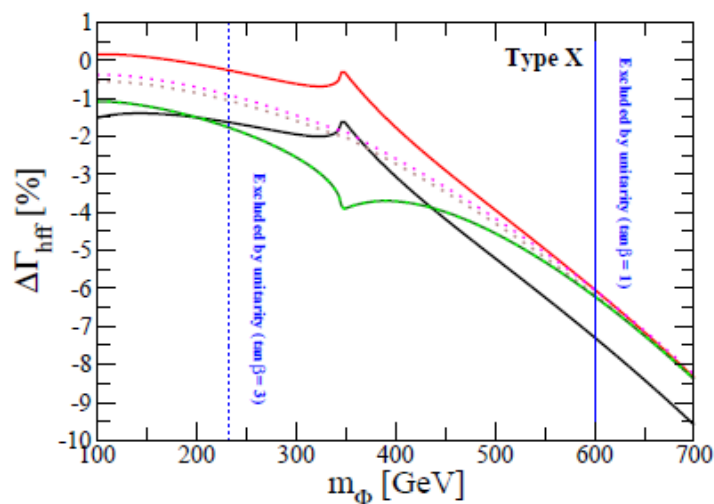
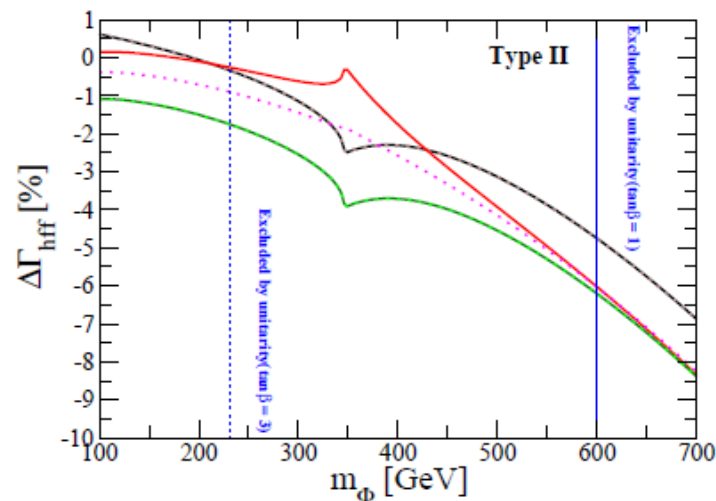
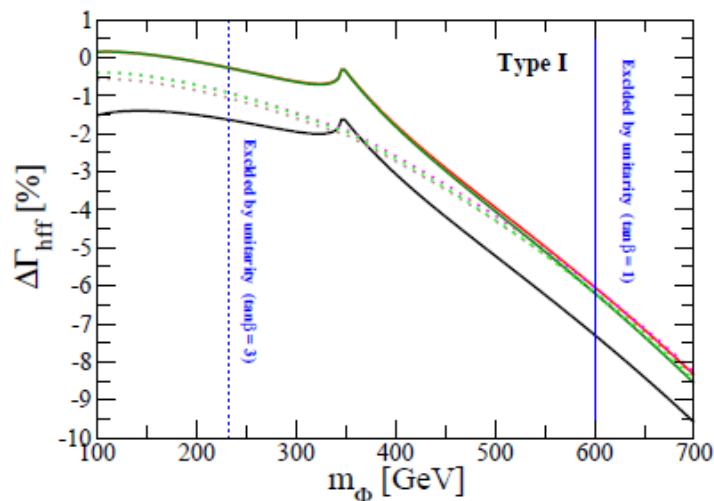
Kanemura, Kikuchi, Yagyu(2013)

$$\sin^2(\beta - \alpha) = 1$$

—  $\tan\beta = 1$

---  $\tan\beta = 3$

$M = 0$



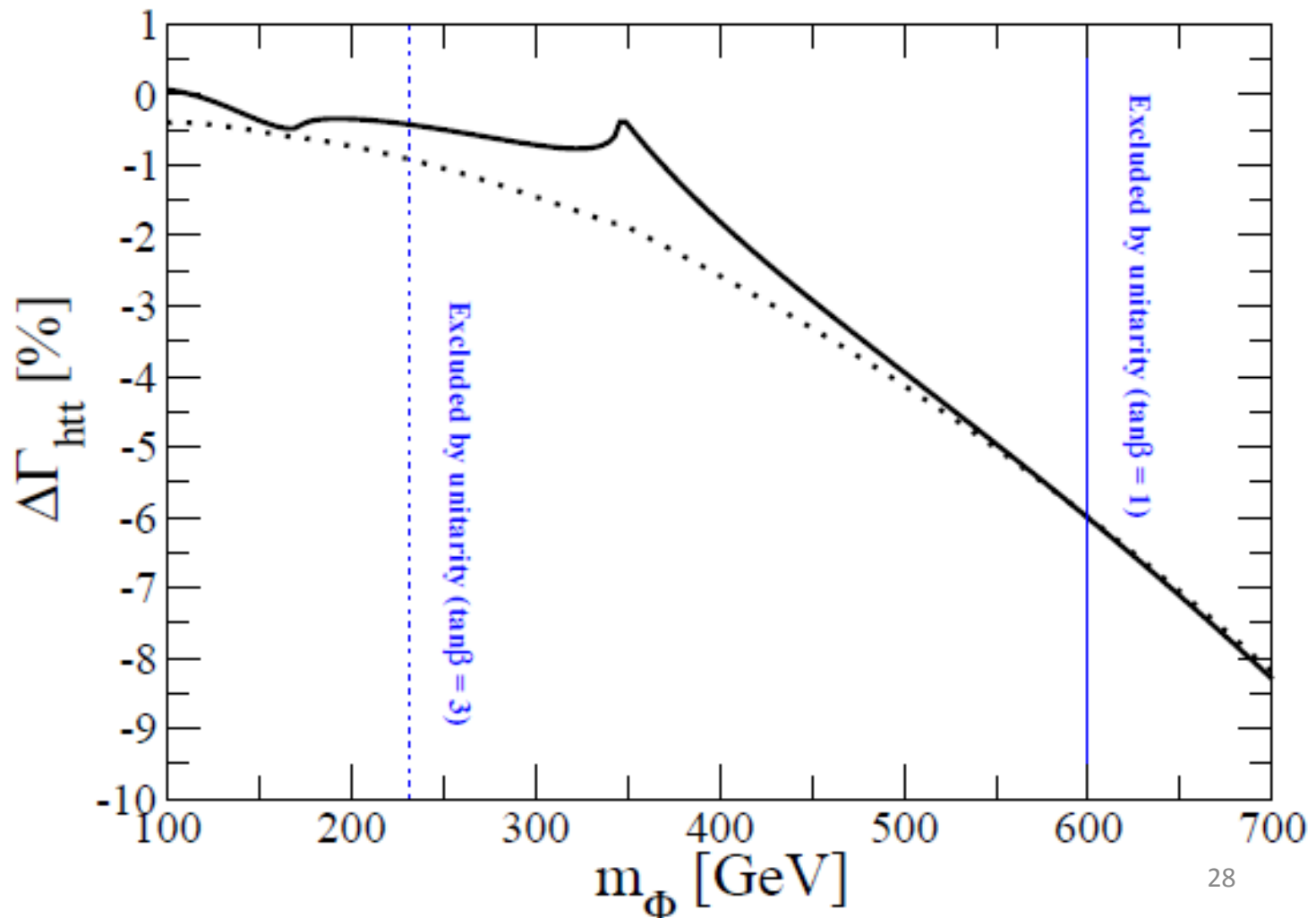
# Deviations in $htt$

$$\sin^2(\beta - \alpha) = 1$$

—  $\tan\beta=1$   
---  $\tan\beta=3$

$M = 0$

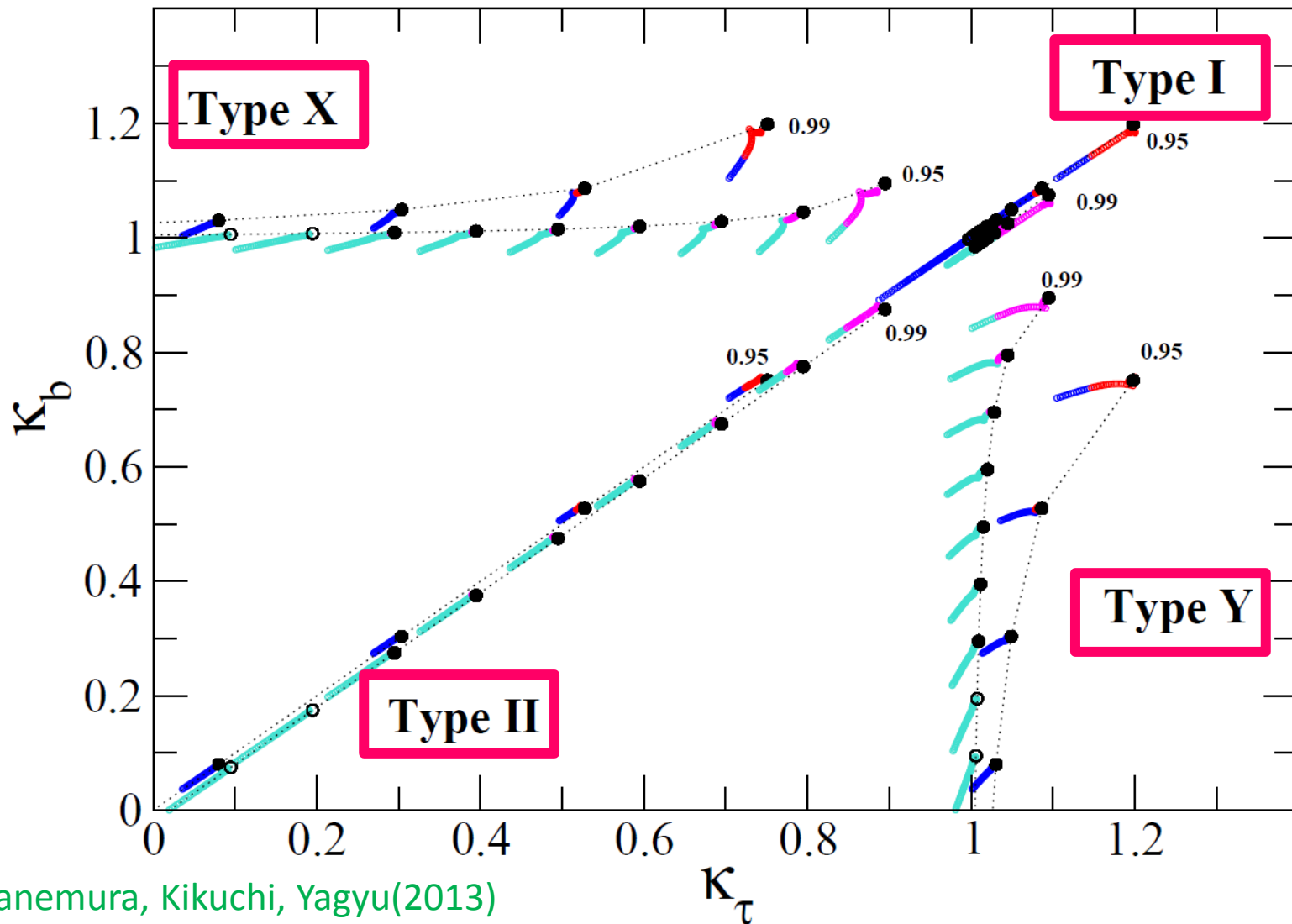
Kanemura, Kikuchi, Yagyu(2013)



# $\kappa\tau$ vs $\kappa b$

$$\cos(\beta - \alpha) \geq 0$$

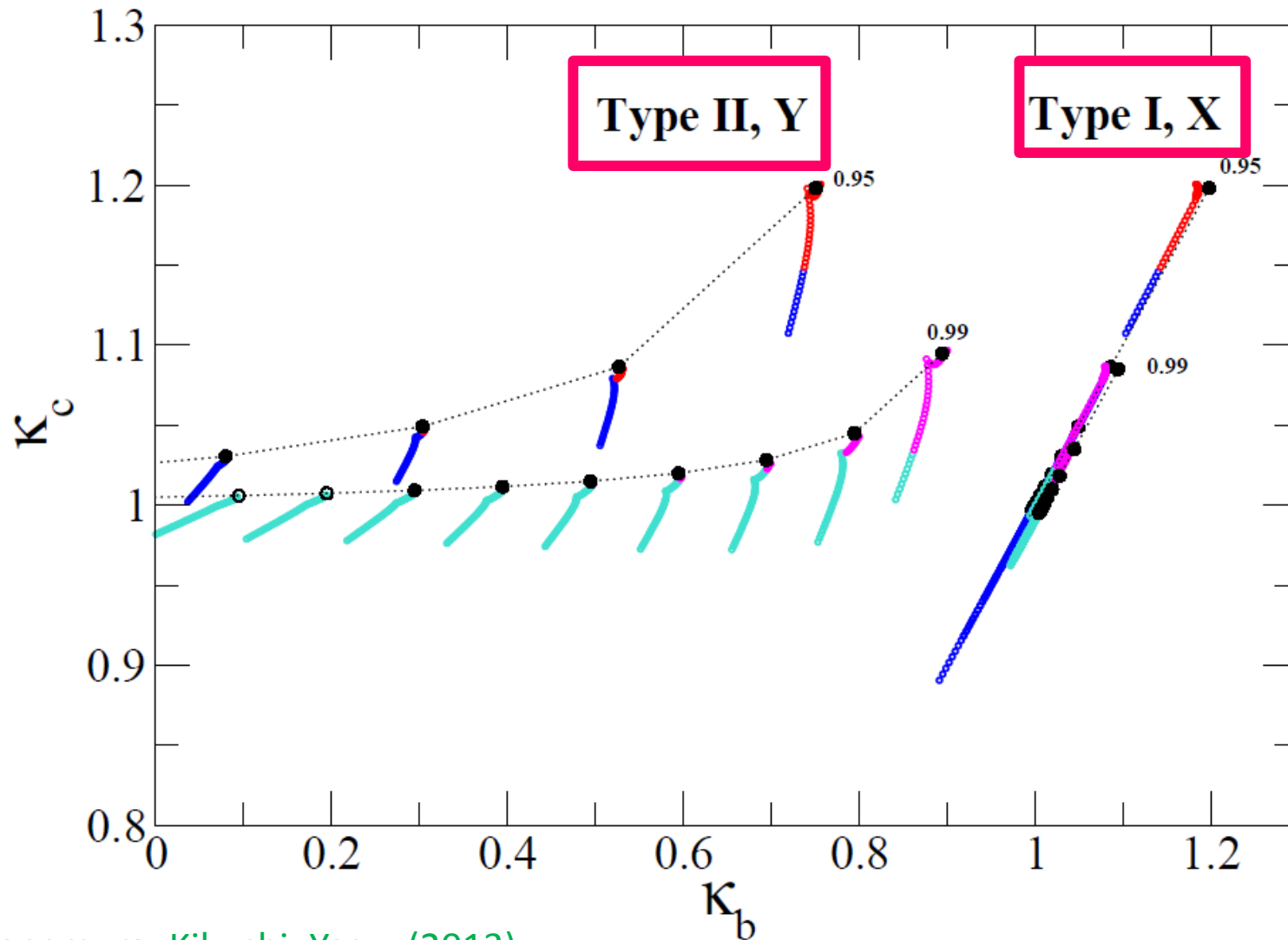
$$M = 0$$



# $\kappa c$ vs $\kappa b$

$$\cos(\beta - \alpha) \geq 0$$

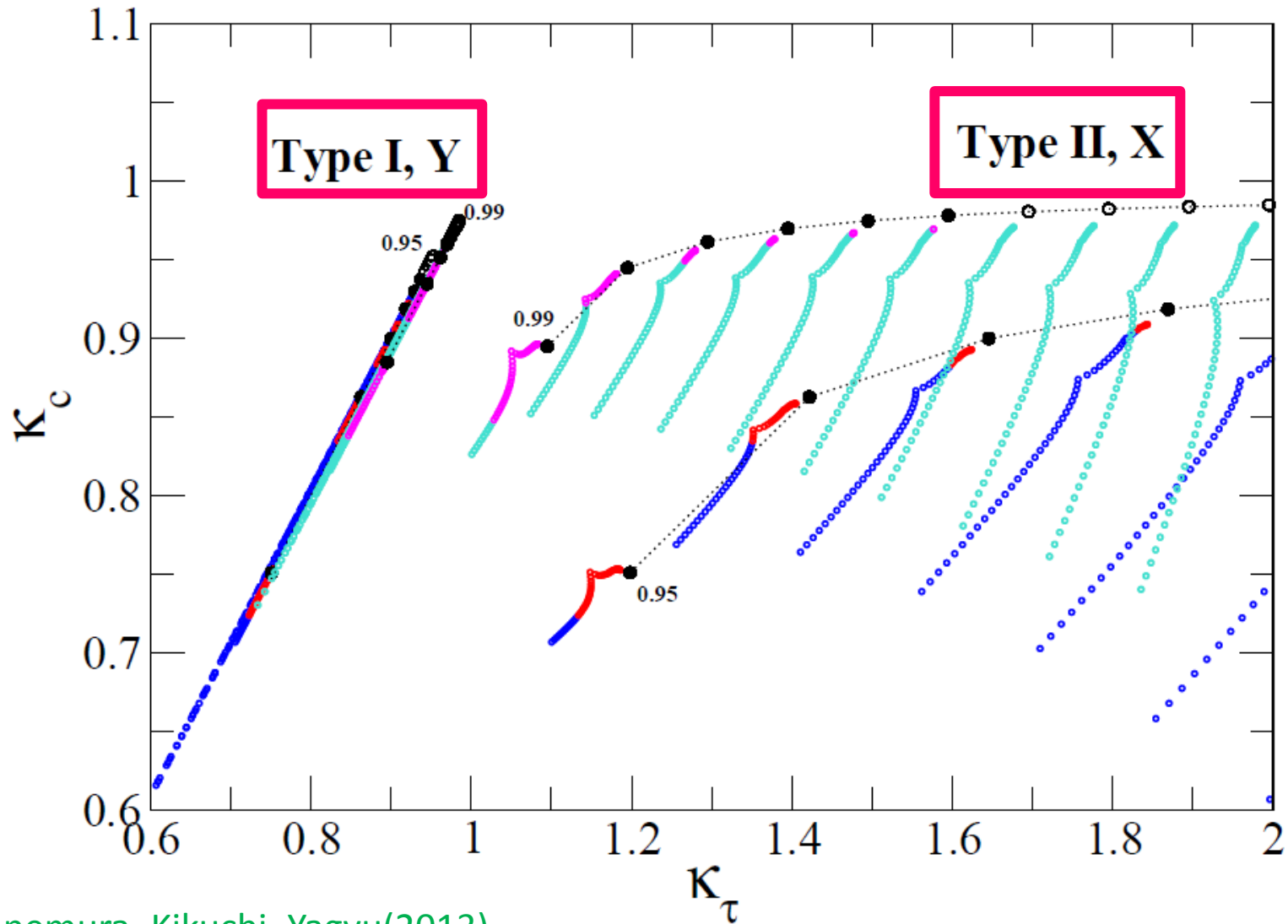
$$M = 0$$



# $KT$ VS $KC$

$$\cos(\beta - \alpha) \leq 0$$

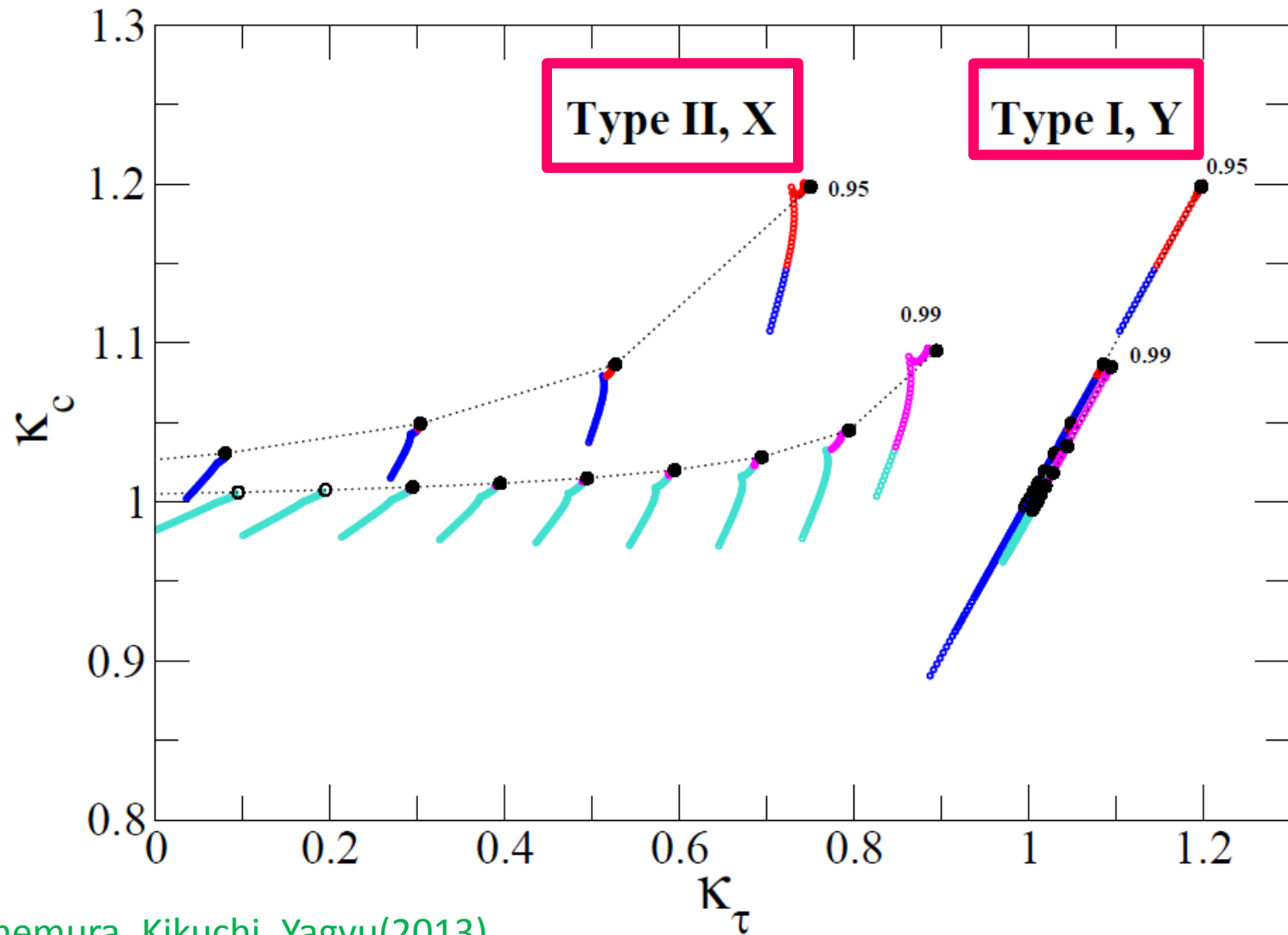
$$M = 0$$



# $KT$ VS $KC$

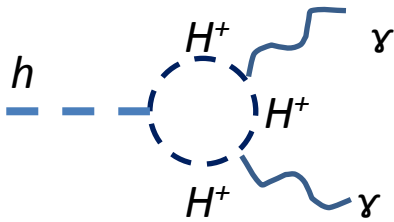
$$\cos(\beta - \alpha) \geq 0$$

$$M = 0$$

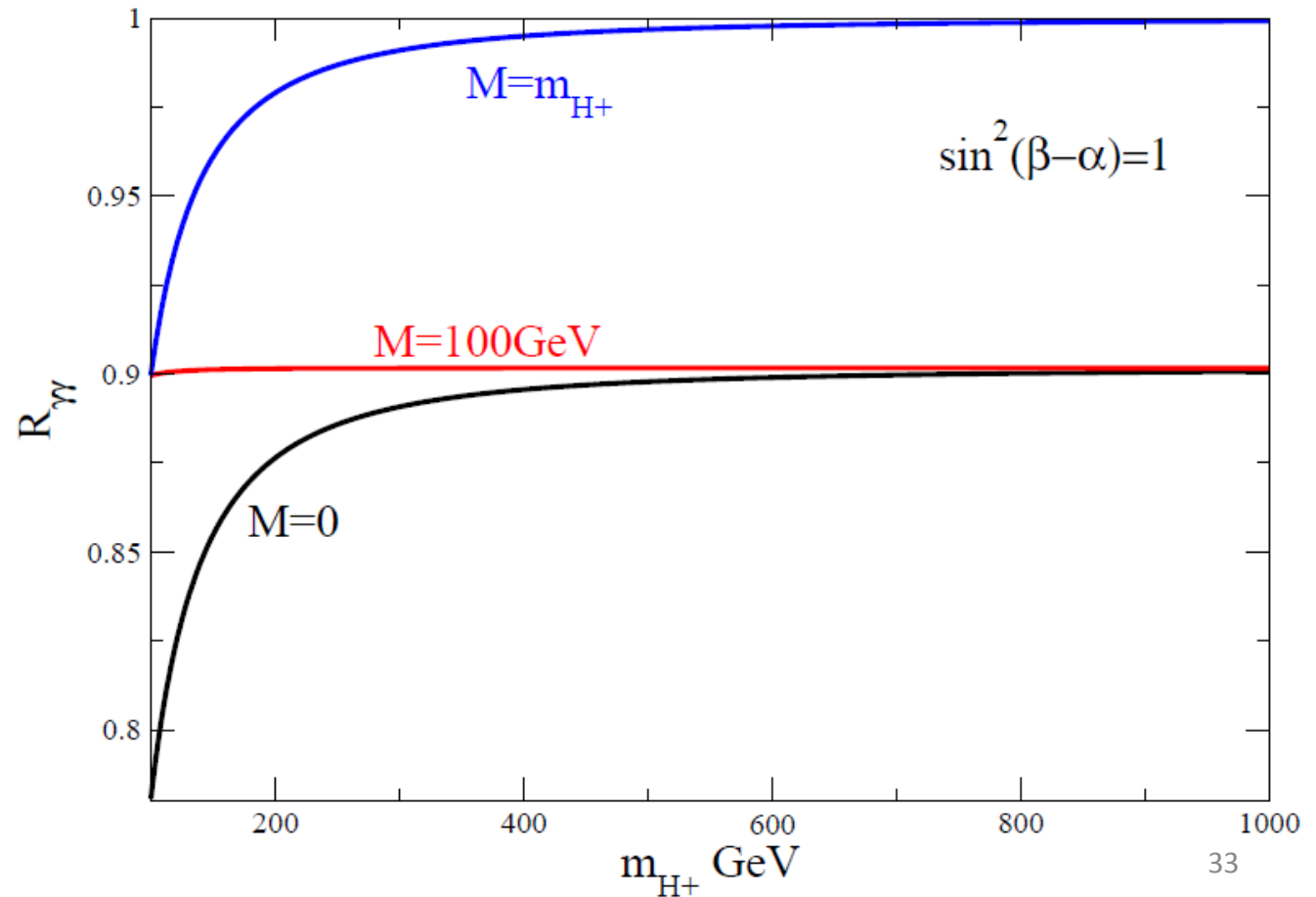




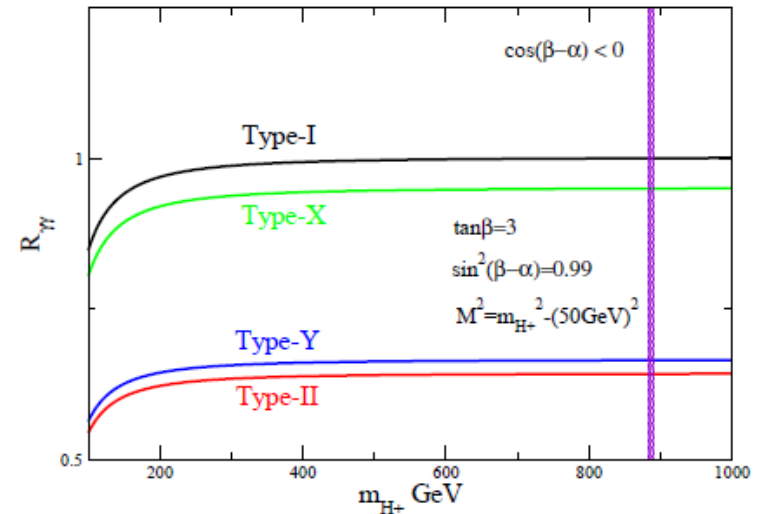
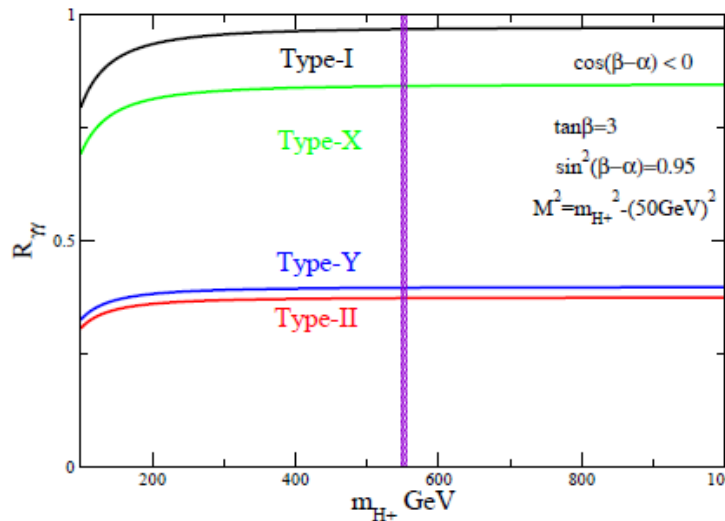
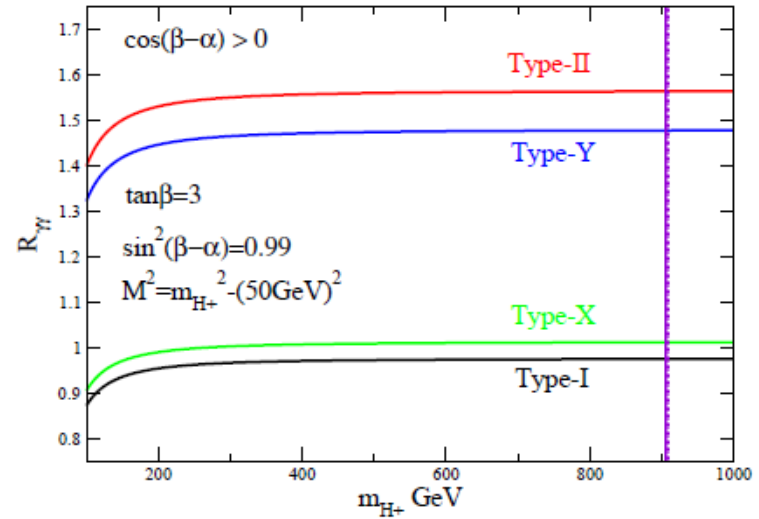
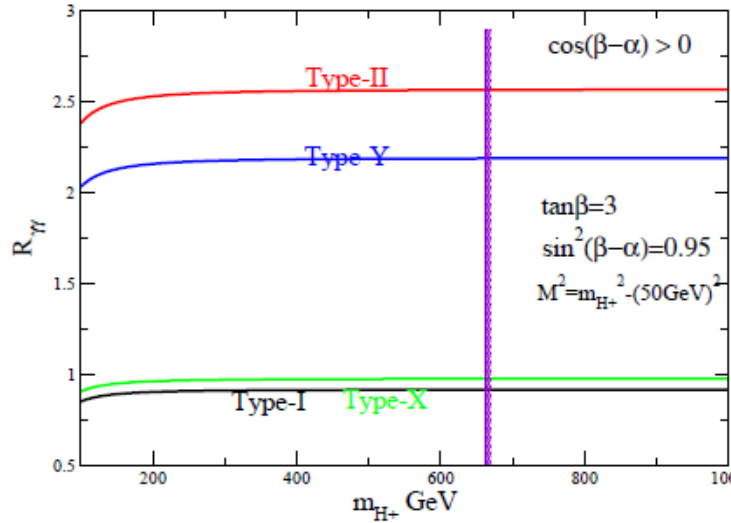
# $R_{\gamma\gamma}$ in $\sin^2(\beta-\alpha)=1$

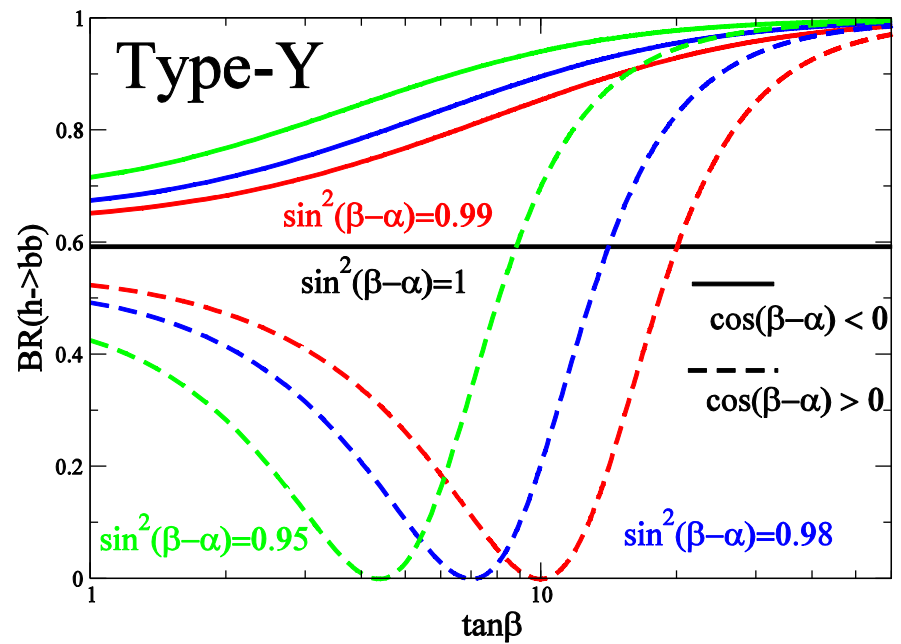
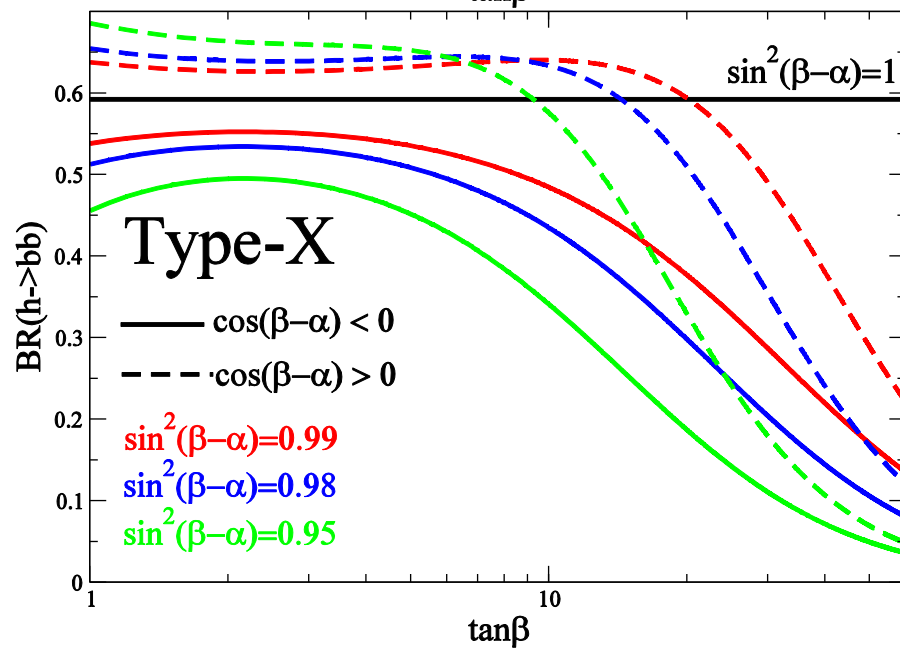
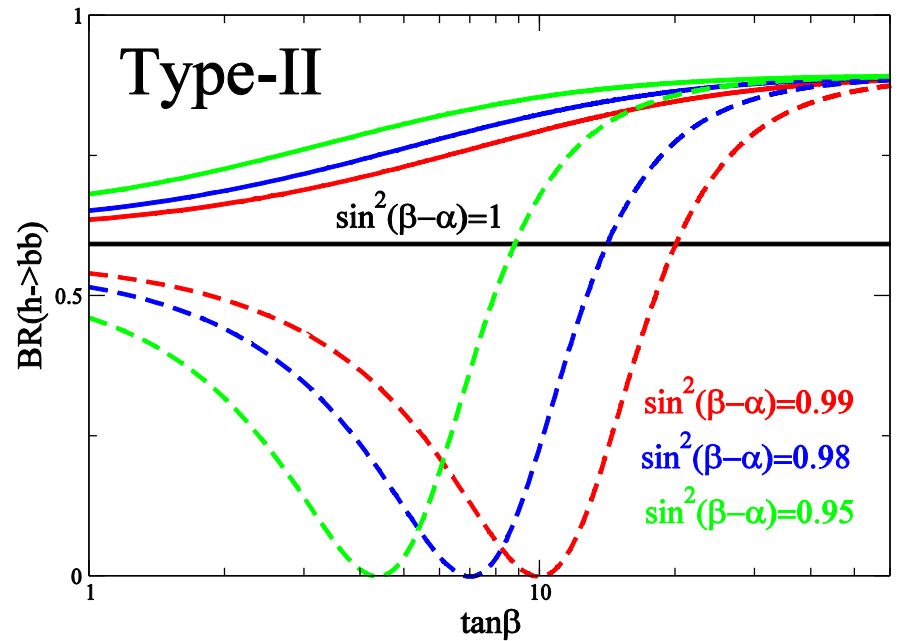
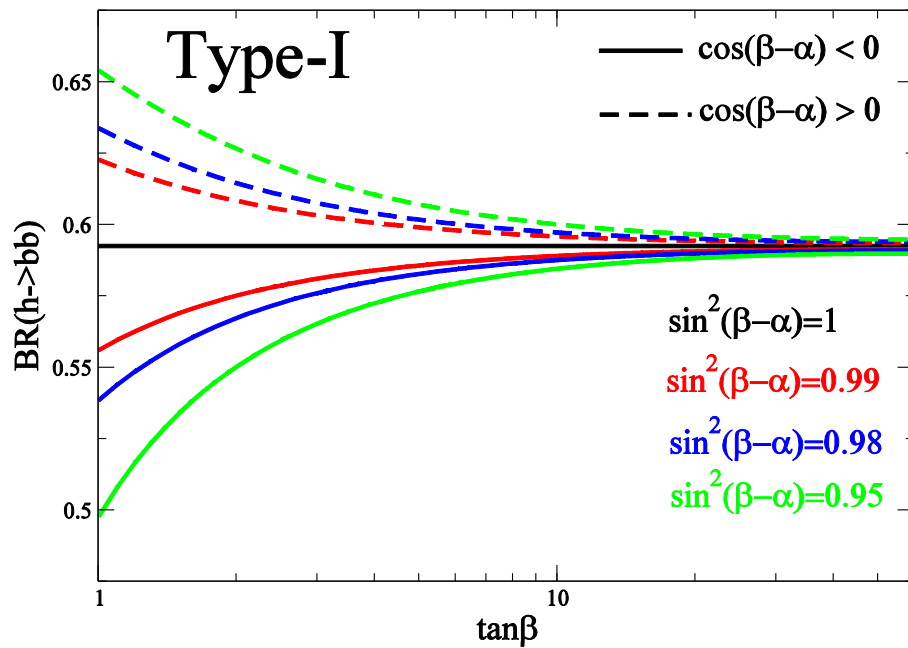


$$\lambda_{hH^+H^-} = M^2 + v^2\lambda'$$



# Mass dependence of $R_{\gamma\gamma}$



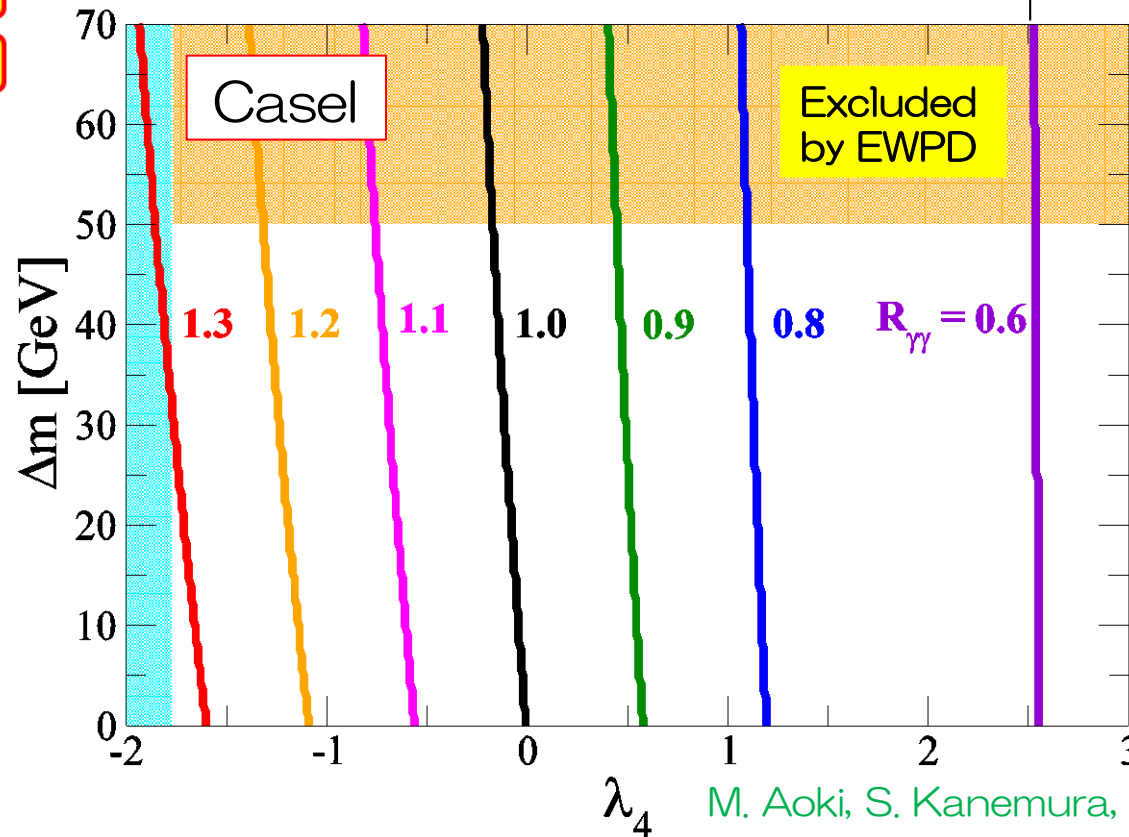


# Ratio of the event rate for $h \rightarrow \gamma \gamma$

A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka (2012);  
A. G. Akeroyd, S. Moretti (2012);

$$m_{H^{++}} = 300 \text{ GeV}, v_{\Delta} = 1 \text{ MeV}$$

$$\Gamma(h \rightarrow \gamma \gamma)_{HTM} = \left[ \begin{array}{c} t \\ \text{triangle} \end{array} + \begin{array}{c} W \\ \text{box} \end{array} + \begin{array}{c} H^{++} \\ \text{triangle} \end{array} + \begin{array}{c} H^{+} \\ \text{triangle} \end{array} \right]^2$$



$R_{\gamma\gamma}$  depends on  $\lambda_4$ .

$$\lambda_{hH^{++}H^{--}} \approx -\lambda_4 v$$

$$\lambda_{hH^+H^-} \approx -\left(\lambda_4 + \frac{\lambda_5}{2}\right)v$$

M. Aoki, S. Kanemura, M. K, K. Yagyu (2013)

$R_{\gamma\gamma}$  can be enhanced or reduced depending on the sign of  $\lambda_{hH^{++}H^{--}}$ .

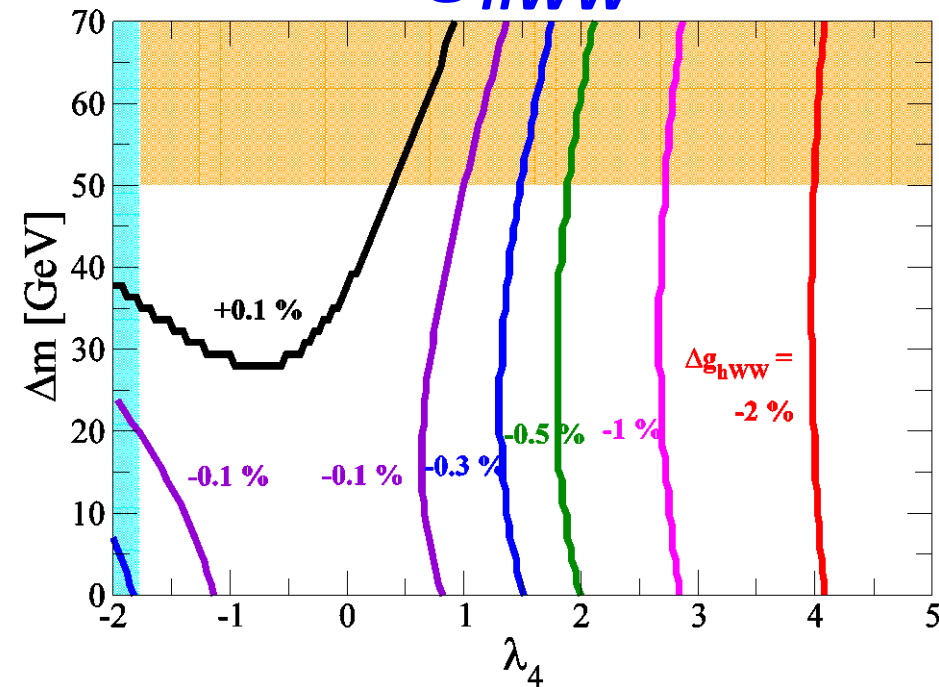
# Deviations for $hVV$

$$\Delta g_{hVV} \equiv \frac{\text{Re}M_1^{hVV} - \text{Re}M_1^{hVV}(\text{SM})}{\text{Re}M_1^{hVV}(\text{SM})}$$

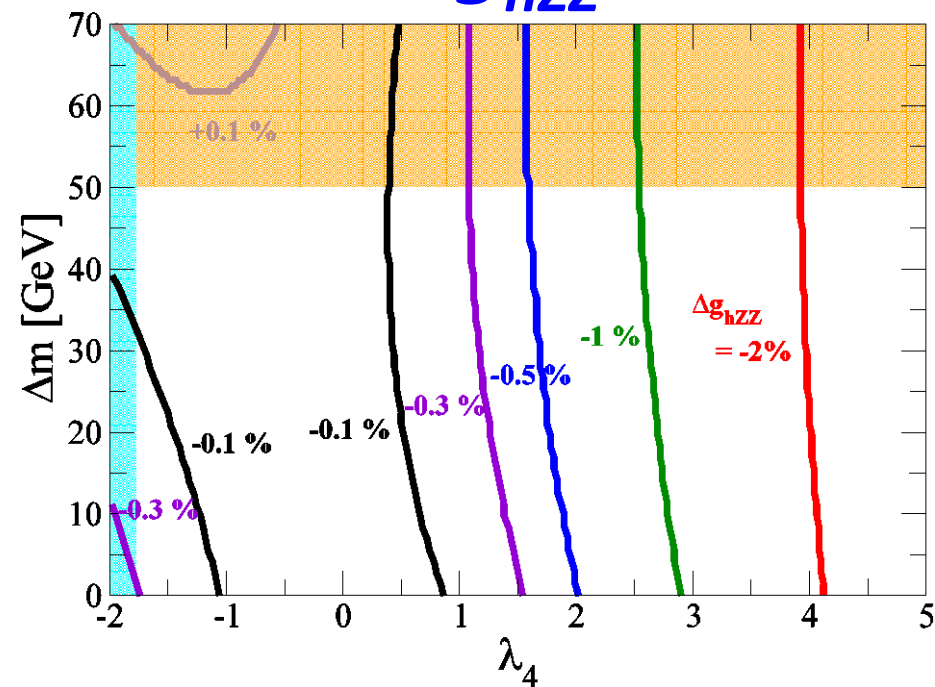
Case1

$m_{H^{++}}=300\text{GeV}, v_{\Delta}=1\text{MeV}$

$\Delta g_{hWW}$



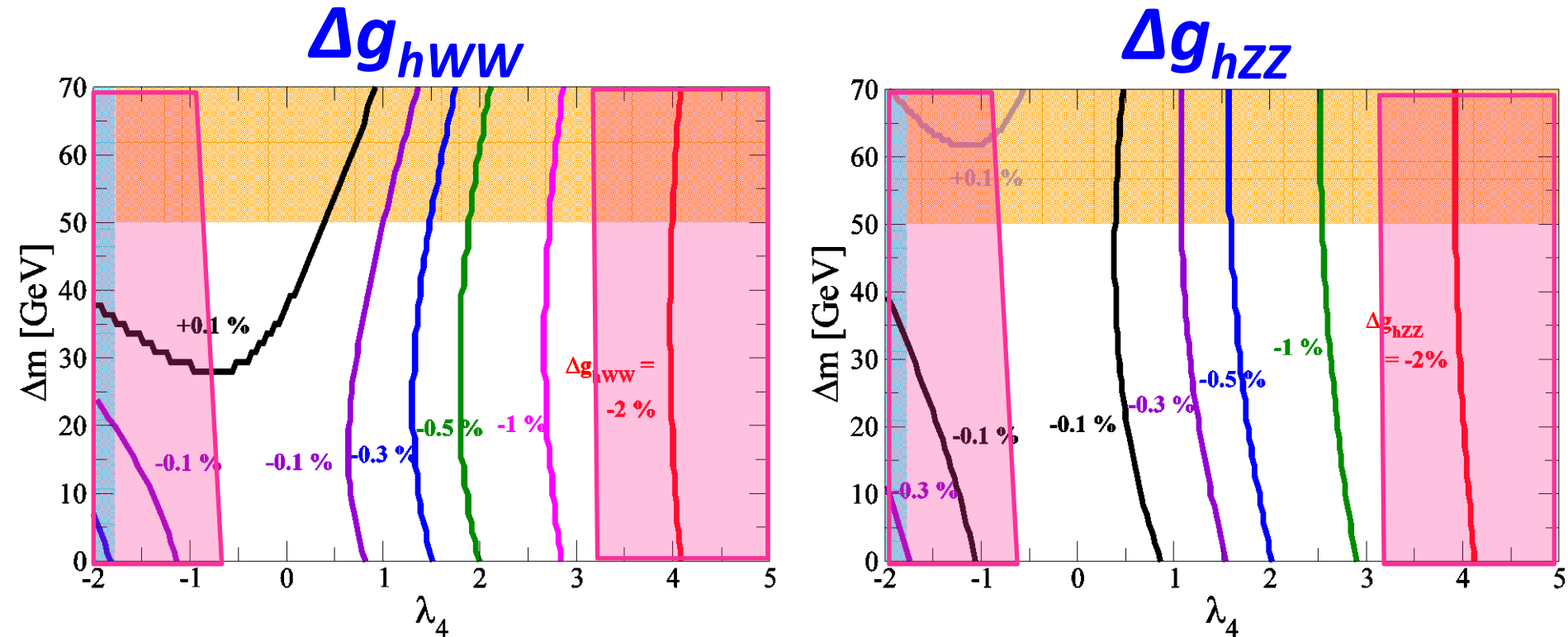
$\Delta g_{hZZ}$



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Deviations for  $hWW$  from the SM predictions can be several %.

If we take into account the CMS data for  $R_{\gamma\gamma}$ , pink regions are excluded.



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

$\Delta g_{hVV}$  can be 1%.

# hhh

➤ Deviations for  $hhh$  from the SM predictions



$$\Delta\Gamma_{hhh} \equiv \frac{\text{Re}\Gamma_{hhh} - \text{Re}\Gamma_{hhh}^{\text{SM}}}{\text{Re}\Gamma_{hhh}^{\text{SM}}}$$

$$\Delta\Gamma_{hhh} \simeq \frac{-v}{48\pi^2 m_h^2} \left( \frac{\lambda_{H++H--h}^3}{m_{H++}^2} + \frac{\lambda_{H+H-h}^3}{m_{H+}^2} + \frac{4\lambda_{hAA}^3}{m_A^2} + \frac{4\lambda_{hHH}^3}{m_H^2} \right)$$

$$\simeq \frac{v^4}{48m_h^2\pi^2} \left[ \frac{\lambda_4^3}{m_{H++}^2} + \frac{\left(\lambda_4 + \frac{\lambda_5}{2}\right)^3}{m_{H+}^2} + \frac{(\lambda_4 + \lambda_5)^3}{2m_A^2} + \frac{(\lambda_4 + \lambda_5)^3}{2m_H^2} \right]$$

Deviations in  $hhh$  coupling from the SM prediction can be  $-10\% \sim +150\%$

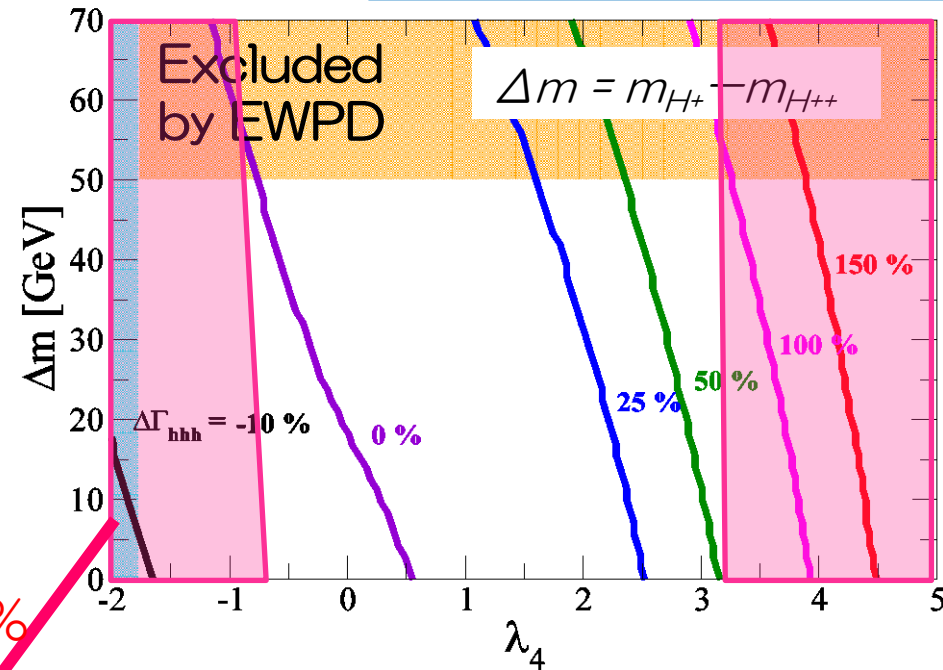
If we take into account results of  $R_{\gamma\gamma}$ , pink region is excluded.

Case I

$$\lambda_5 \geq 0$$

$$m_{H++} = 300 \text{ GeV}, v_\Delta = 1 \text{ MeV}$$

Not excluded



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

$\Delta\Gamma_{hhh}$  can be 50%.



Deviations are detectable at ILC!

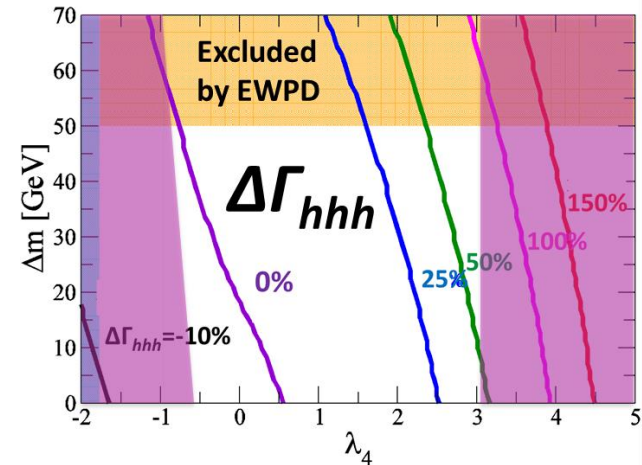
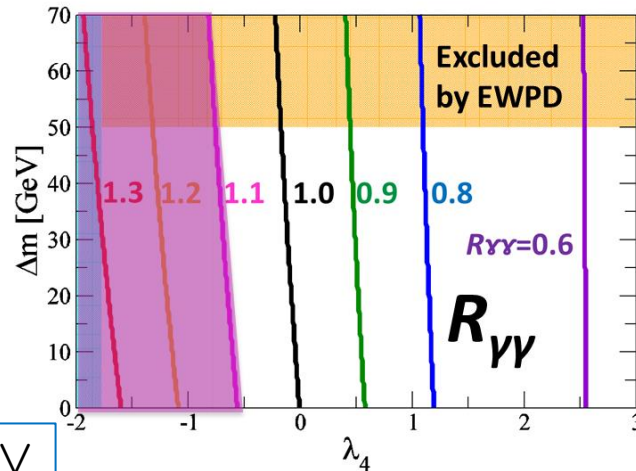
# Correlations of the deviation

Contributions to  $R_{\gamma\gamma}$  is opposite to one of  $\Delta\Gamma_{hhh}$ .

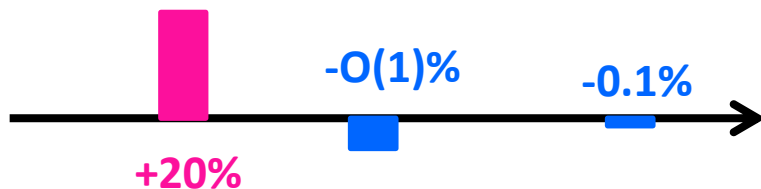
## Case-I

$$\Delta m = m_{H^+} - m_{H^{++}}$$

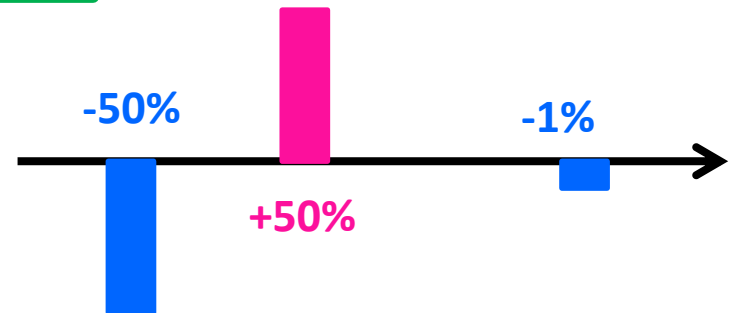
$$m_{\text{lightest}} = 300 \text{ GeV}, v_{\Delta} = 1 \text{ MeV}$$



$$\lambda_4 = -0.5$$

 $R_{\gamma\gamma}$ 
 $\Delta\Gamma_{hhh}$ 
 $\Delta g_{hWW}$ 
 $\Delta g_{hZZ}$ 


$$\lambda_4 = 3$$

 $R_{\gamma\gamma}$ 
 $\Delta\Gamma_{hhh}$ 
 $\Delta g_{hWW}$ 
 $\Delta g_{hZZ}$ 


There is a correlation in deviations in  $h\gamma\gamma$  and  $hhh$ .

By detecting this, we can discriminate the model from the others.



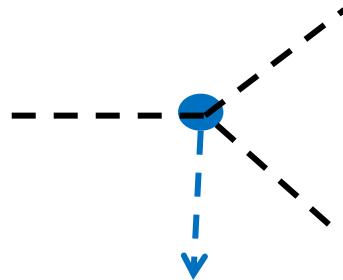
# hhh

$$\begin{aligned} m_{H^{++}}^2 &\simeq M^2 + \frac{1}{2} \lambda_4 v^2 \\ m_{H^\pm}^2 &\simeq M^2 + \left( \frac{1}{2} \lambda_4 + \frac{1}{4} \lambda_5 \right) v^2 \\ m_A^2 &\simeq m_{H^\pm}^2 \simeq M^2 + \frac{1}{2} (\lambda_4 + \lambda_5) v^2 \end{aligned}$$

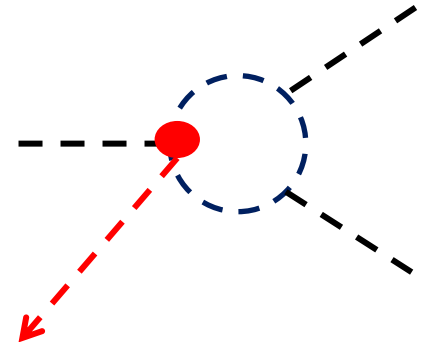
$$\begin{aligned} \lambda_{hhh}^{eff}(THDM) = & \frac{3m_h^2}{v} \left\{ 1 + \frac{m_H^4}{12\pi^2 m_h^2 v^2} \left( 1 - \frac{M^2}{m_H^2} \right)^3 + \frac{m_A^4}{12\pi^2 m_h^2 v^2} \left( 1 - \frac{M^2}{m_A^2} \right)^3 \right. \\ & \left. + \frac{m_{H^\pm}^4}{6\pi^2 m_h^2 v^2} \left( 1 - \frac{M^2}{m_{H^\pm}^2} \right)^3 - \frac{N_{ct} m_t^4}{3\pi^2 m_h^2 v^2} + \mathcal{O} \left( \frac{p_i^2 m_\Phi^2}{m_h^2 v^2}, \frac{m_\Phi^2}{v^2}, \frac{p_i^2 m_t^2}{m_h^2 v^2}, \frac{m_t^2}{v^2} \right) \right\}, \end{aligned}$$

Coupling parameters of the loop diagrams are different from the one of the tree diagram.

So, deviations by loop correction can be large.



$$\lambda_{hhh} \simeq -\lambda_1 v$$



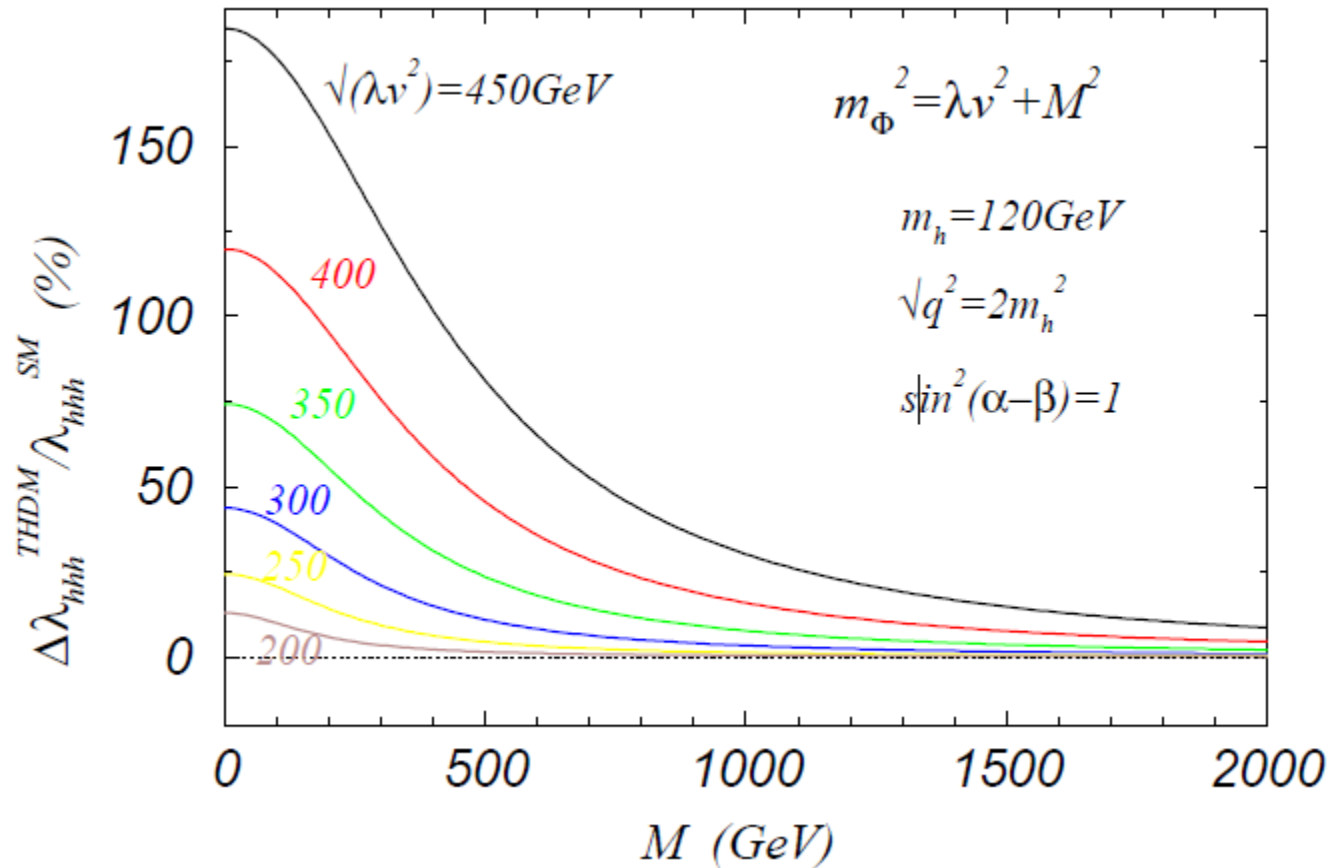
$$\lambda_{hH^{++}H^{--}} \simeq -\lambda_4 v$$

$$\lambda_{hH^+H^-} \simeq -\left( \lambda_4 + \frac{1}{2} \lambda_5 \right) v$$

$$\lambda_{hAA} \simeq \lambda_{hH^+H^-} \simeq -\frac{1}{2} (\lambda_4 + \lambda_5) v$$

# 2HDM の $hhh$ 結合

Kanemura, Okada, Senaha, Yuan (2004)



$$m_\Phi^2 = \lambda v^2 + M^2$$

# FCNC Suppression

The mechanism suppressed FCNC is required in multi Higgs models.

ex) 2 Higgs doublet models:

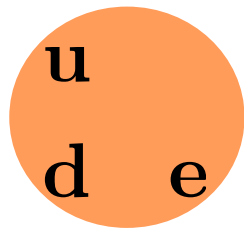
- discrete symmetry

$$\Phi_1 \rightarrow +\Phi_1, \quad \Phi_2 \rightarrow -\Phi_2$$

Barger, Hewett, Phillips

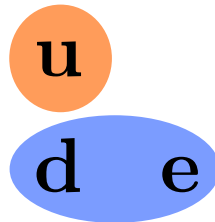
type	$\Phi_1$	$\Phi_2$	$u_R$	$d_R$	$l_R$	$Q_L, L_L$
typeI	+	-	-	-	-	+
typeII	+	-	-	+	+	+
typeX	+	-	-	-	+	+
typeY	+	-	-	+	-	+

Four types of Yukawa interaction



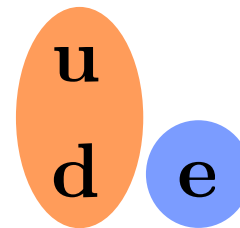
**Type-I**

Fermio-phobic 2HDM  
Neutrino-philic 2HDM



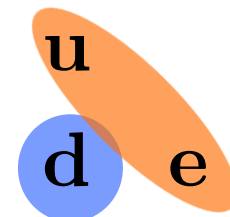
**Type-II**

MSSM,  
NMSSM



**Type-X**

Lepton-specific 2HDM,  
Radiative seesaw



**Type-Y**

Flipped 2HDM

# Characteristic of couplings

	2HDM	Higgs triplet model																				
<div><math>hff</math></div> <div>Yukawa couplings</div>	<div><table><tr><th></th><th>c</th><th>b</th><th><math>\tau</math></th></tr><tr><td>I</td><td>↓</td><td>↓</td><td>↓</td></tr><tr><td>II</td><td>↓</td><td>↑</td><td>↑</td></tr><tr><td>X</td><td>↓</td><td>↓</td><td>↑</td></tr><tr><td>Y</td><td>↓</td><td>↑</td><td>↓</td></tr></table></div> <div><math>\cos(\beta-\alpha)&lt;0</math></div> <div>Each type has a different pattern of deviations.</div>		c	b	$\tau$	I	↓	↓	↓	II	↓	↑	↑	X	↓	↓	↑	Y	↓	↑	↓	<div>① <math>v_{\Delta} / v_{\phi} \ll 1</math> → Mixing is very small.</div> <div>② Fermion don't couple to <math>\Delta</math>.</div> <div>Deviations are very <b>small</b>.</div>
	c	b	$\tau$																			
I	↓	↓	↓																			
II	↓	↑	↑																			
X	↓	↓	↑																			
Y	↓	↑	↓																			
<div><math>hVV</math></div> <div>Gauge couplings</div>	<div>&lt; Multi-doublet model &gt;</div> <div><math>g_{hVV} = g_{hVV}^{SM} \times \kappa_{hVV}</math></div> <div><math>\kappa_{hVV} = -\sin\alpha \cos\beta + \cos\alpha \sin\beta</math></div> <div><math>\kappa_{hVV} \leq 1</math></div>	<div>Gauge couplings can be larger than SM predictions because of CG coefficient.</div> <div><math>\kappa_{hWW} = \cos\beta \cos\alpha + \sqrt{2} \sin\beta \sin\alpha</math></div> <div><math>\kappa_{hVV} \geq 1</math> is possible</div>																				

44