Higgs pair production at LHC and ILC

Kunio Kaneta (Kavli IPMU)

Based on

- PLB718 (2013) 1441. Collaboration with N. Haba, Y. Mimura, R. Takahashi
- arXiv:1311.0067. Collaboration with N. Haba, Y. Mimura, E. Tsedenbaljir

LCWS @ Tokyo 14 Nov. 2013

Outline

1, Motivation

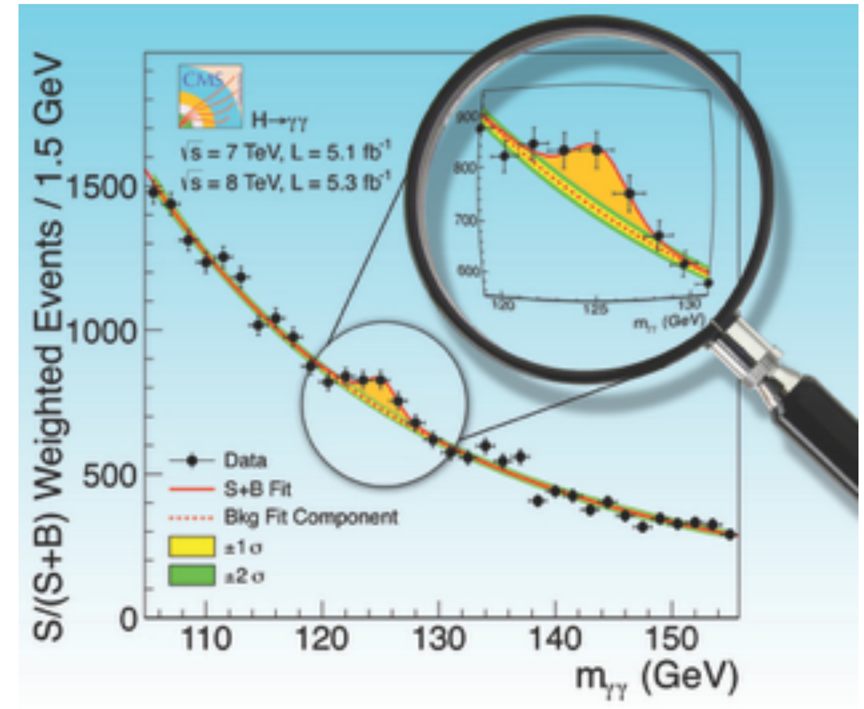
2, Higgs potential and the Higgs self-interaction

3, Non-perturbative Higgs model

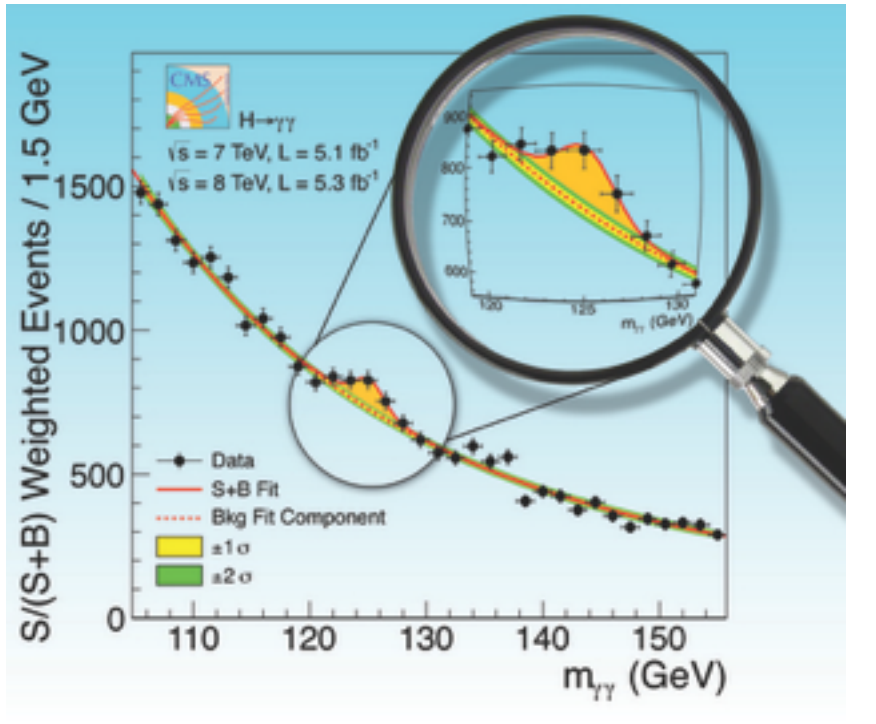
4, Summary

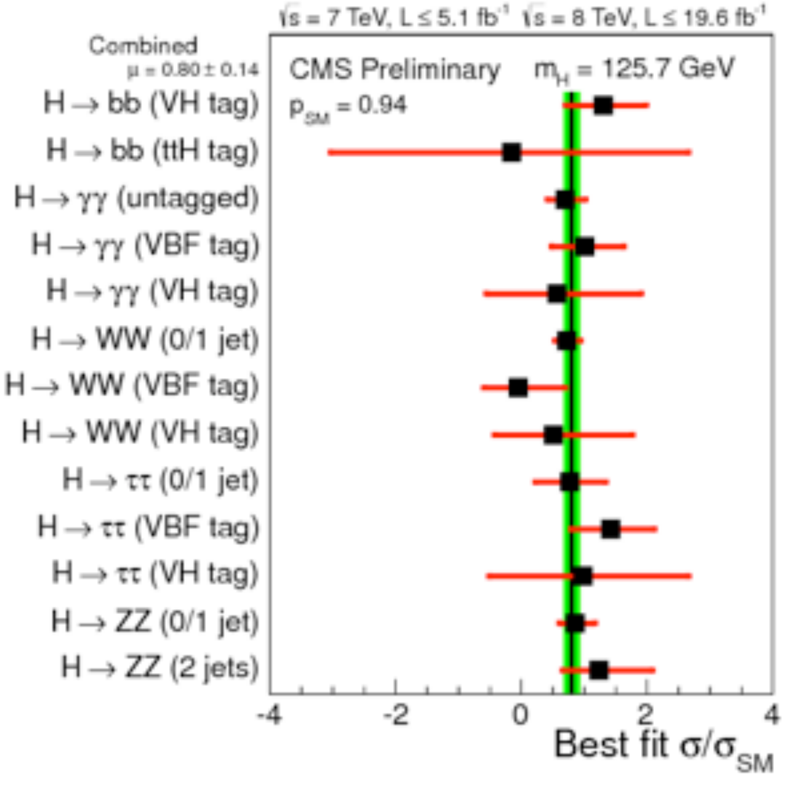
1, Motivation

☆ Higgs has been discovered at $m_h \simeq 126\ \mathrm{GeV}$



☆ Higgs has been discovered at $m_h \simeq 126\ \mathrm{GeV}$

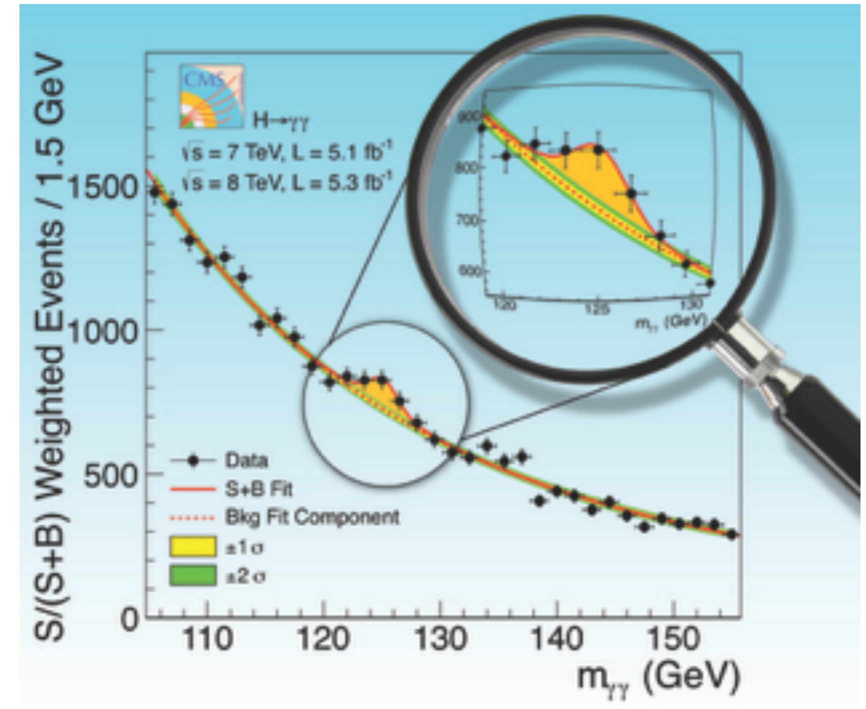


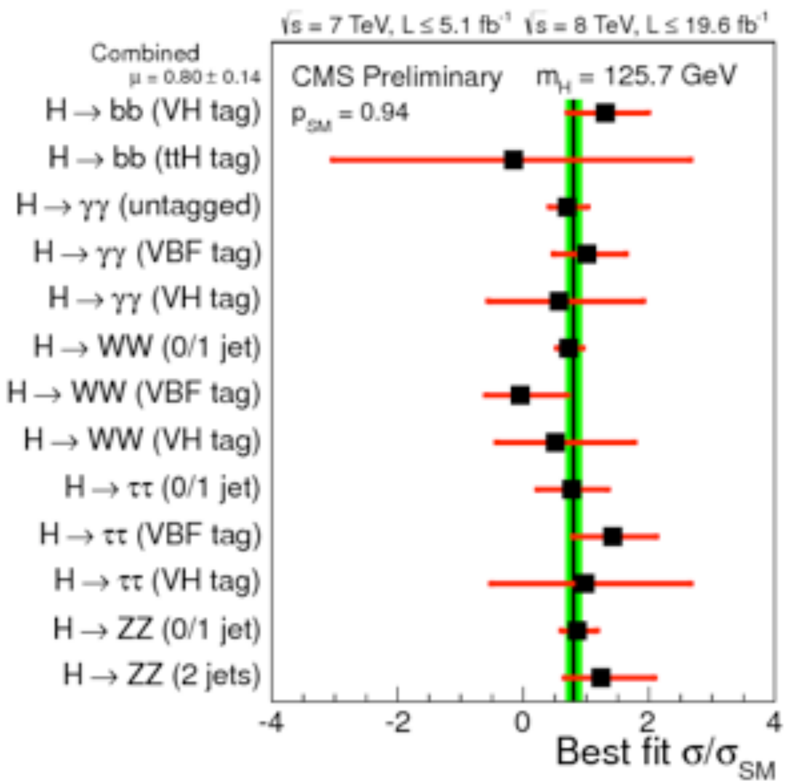


☆ Interactions of Higgs are now probing.

(consistent with SM)

☆ Higgs has been discovered at $m_h \simeq 126 \text{ GeV}$

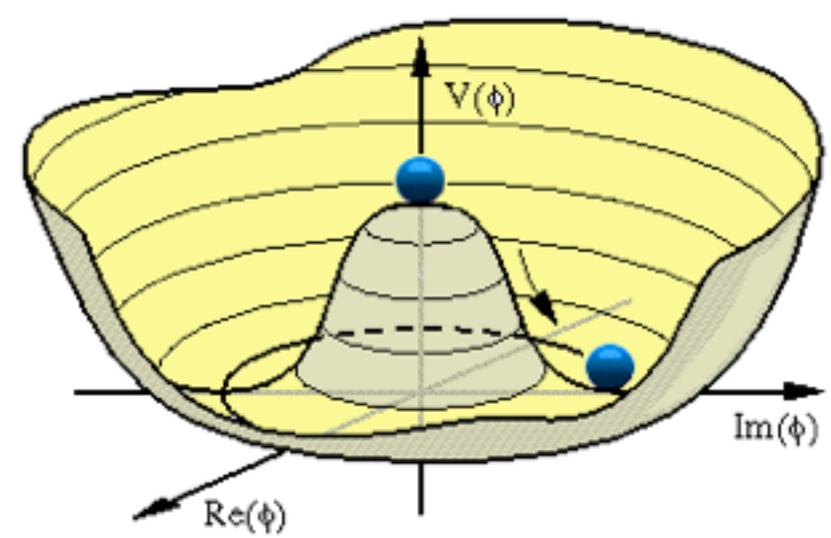




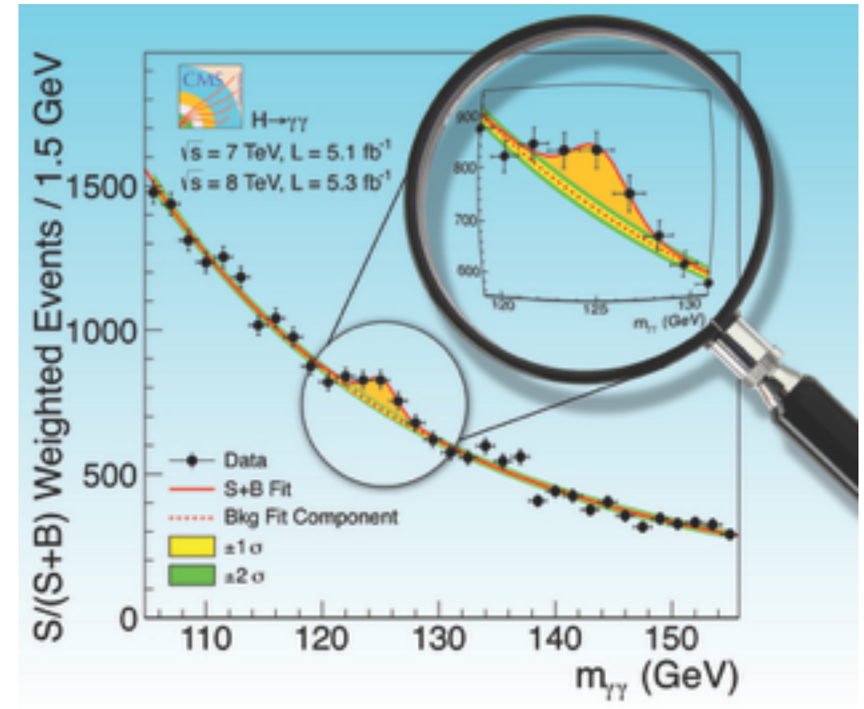
☆ Interactions of Higgs are now probing.

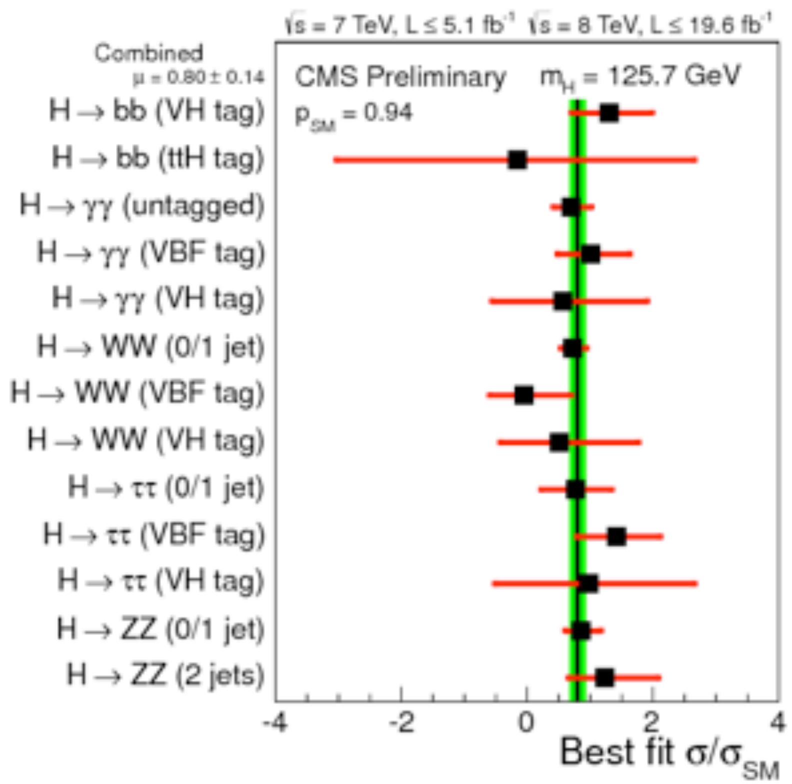
(consistent with SM)

☆ Higgs potential is still mystery.



☆ Higgs has been discovered at $m_h \simeq 126\ \mathrm{GeV}$



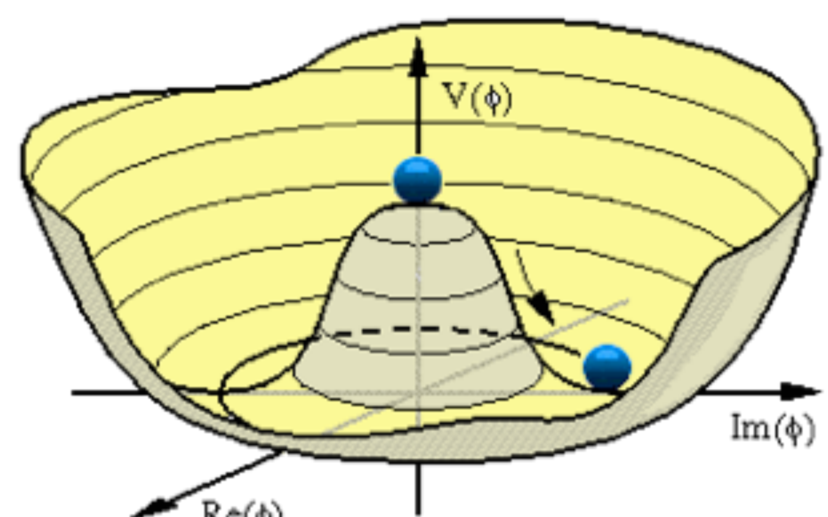


☆ Interactions of Higgs are now probing.

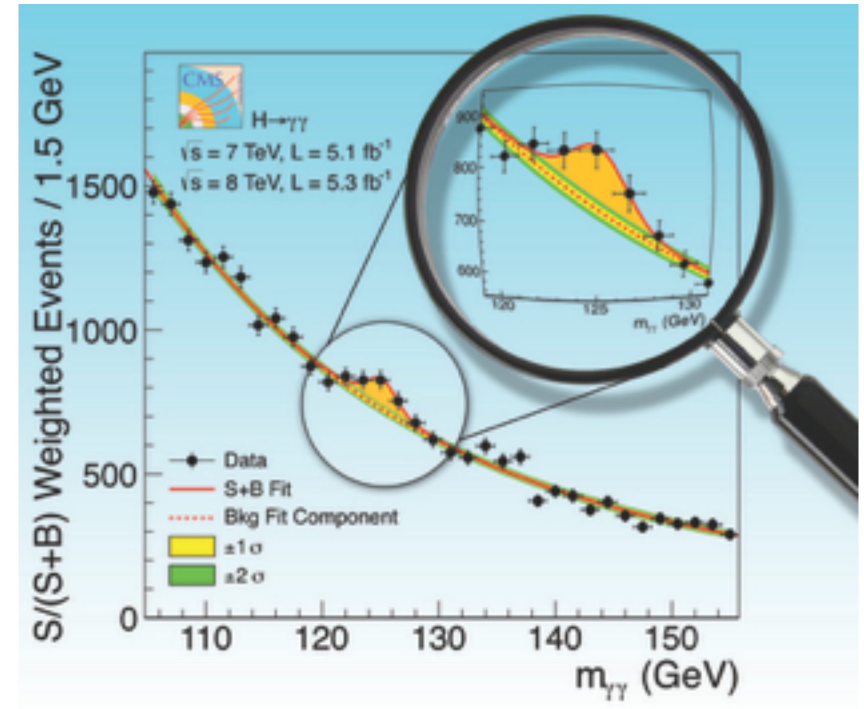
(consistent with SM)

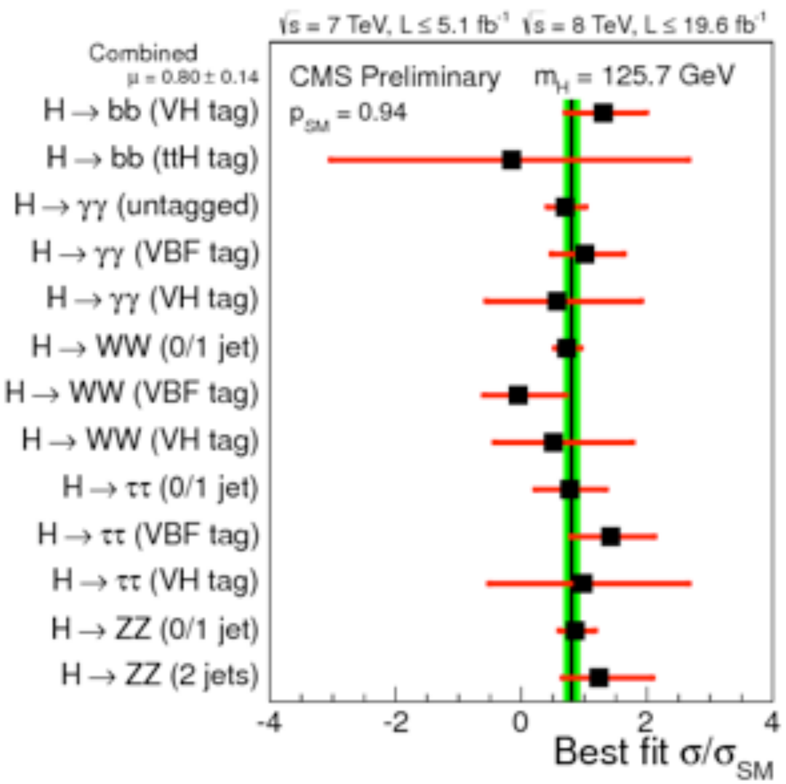
☆ Higgs potential is still mystery.

・How does the Higgs field acquire a VEV?



☆ Higgs has been discovered at $m_h \simeq 126\ \mathrm{GeV}$



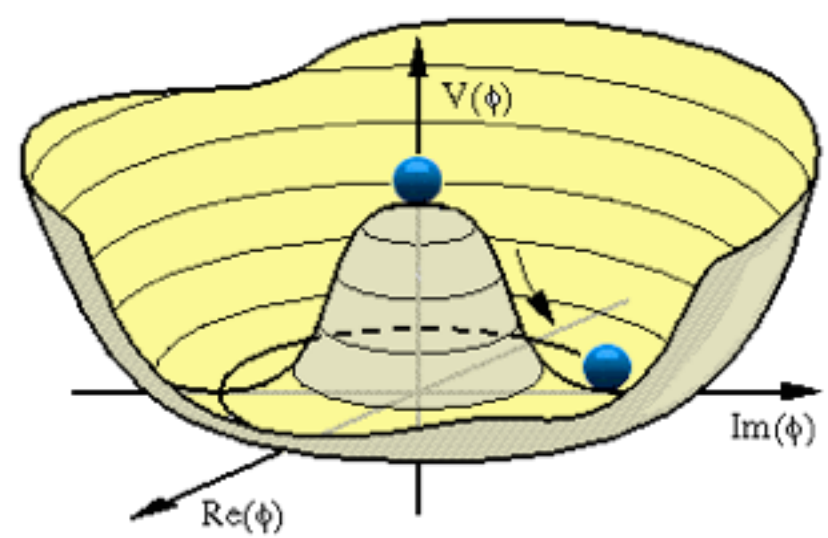


☆ Interactions of Higgs are now probing.

(consistent with SM)

☆ Higgs potential is still mystery.

- How does the Higgs field acquire a VEV?
- What kind of interaction works on there?



"Interactions in nature" (excepting Gravity)

Gauge forces

"Higgs forces"

"Interactions in nature" (excepting Gravity)

Gauge forces

☆ Gauge interaction stems from

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

☆ For fermions: $\bar{Q}\ i\gamma^\mu D_\mu\ Q$

For Higgs boson: $|D_\mu H|^2$

Self-interaction: $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$

☆ well-known

"Higgs forces"

"Interactions in nature" (excepting Gravity)

Gauge forces

☆ Gauge interaction stems from

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

☆ For fermions: $\bar{Q}\ i\gamma^\mu D_\mu\ Q$

For Higgs boson: $|D_\mu H|^2$

Self-interaction: $-\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$

☆ well-known

"Higgs forces"

☆ Higgs self-interaction gives

$$\langle H \rangle = \frac{v}{\sqrt{2}}$$

☆ Lagrangian

$$-\mathcal{L}_{\rm Higgs} = m^2 |H|^2 + \lambda (|H|^2)^2$$

(We really do not know)

(☆ Yukawa interaction) (out of my talk)

"Interactions in nature" (excepting Gravity)

Gauge forces

☆ Gauge interaction stems from

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

☆ For fermions: $\bar{Q}\ i\gamma^\mu D_\mu\ Q$

For Higgs boson: $|D_\mu H|^2$

Self-interaction: $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$

☆ well-known

"Higgs forces"

☆ Higgs self-interaction gives

$$\langle H \rangle = \frac{v}{\sqrt{2}}$$

☆ Lagrangian

$$-\mathcal{L}_{\text{Higgs}} = m^2|H|^2 + \lambda(|H|^2)^2$$

(We really do not know)

(☆ Yukawa interaction) (out of my talk)

key point is ...

- Higgs self-interaction is an origin of EW symmetry breaking.
- It is important to probe the Higgs self-coupling.

2, Higgs potential and the Higgs self-interaction

☆ Higgs potential as a function of $|H|^2$

$$V = V(|H|^2)$$

We know the Higgs field acquires a VEV. $H = \begin{pmatrix} \chi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad |H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-$

☆ Higgs potential as a function of $|H|^2$

$$V = V(|H|^2)$$

We know the Higgs field acquires a VEV. $H = \begin{pmatrix} \chi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad |H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-$

expansion around $\frac{v^2}{2}$

$$\begin{aligned} V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right) \\ + \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2 \\ + \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3 \\ \cdots \end{aligned}$$

☆ Higgs potential as a function of $|H|^2$

$$V = V(|H|^2)$$

We know the Higgs field acquires a VEV.

$$H = \begin{pmatrix} \chi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad |H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-$$

expansion around $\frac{v^2}{2}$

$$V = V\left(\frac{v^2}{2}\right) + \cancel{V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)}$$

vanished

$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$

$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$

$$\cdots$$

stationary condition

$$V'\left(\frac{v^2}{2}\right) = 0$$

☆ Higgs potential as a function of $|H|^2$

$$V = V(|H|^2)$$

We know the Higgs field acquires a VEV.

$$H = \begin{pmatrix} \chi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad |H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-$$

expansion around $\frac{v^2}{2}$

$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition

$$V'\left(\frac{v^2}{2}\right) = 0$$

physical Higgs mass

$$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$$

☆ Higgs potential as a function of $|H|^2$

$$V = V(|H|^2)$$

We know the Higgs field acquires a VEV.

$$H = \begin{pmatrix} \chi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad |H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-$$

expansion around $\frac{v^2}{2}$

$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

higher dimensional OPs

stationary condition

$$V'\left(\frac{v^2}{2}\right) = 0$$

physical Higgs mass

$$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$$

Beyond the SM contribution

☆ Higgs potential as a function of $|H|^2$

$$V = V(|H|^2)$$

We know the Higgs field acquires a VEV.

$$H = \begin{pmatrix} \chi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad |H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-$$

expansion around $\frac{v^2}{2}$

$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$

$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$

$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$

$$\cdots$$

higher dimensional OPs

stationary condition

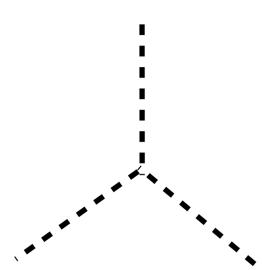
$$V'\left(\frac{v^2}{2}\right) = 0$$

physical Higgs mass

$$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$$

Beyond the SM contribution

cubic Higgs coupling



$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right]$$

☆ Higgs potential as a function of $|H|^2$

$$V = V(|H|^2)$$

We know the Higgs field acquires a VEV.

$$H = \begin{pmatrix} \chi^+ \\ (v+h+i\chi)/\sqrt{2} \end{pmatrix}, \quad |H|^2 = \frac{v^2}{2} + vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-$$

expansion around $\frac{v^2}{2}$

$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$

$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$

$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$

$$\cdots$$

stationary condition

$$V'\left(\frac{v^2}{2}\right) = 0$$

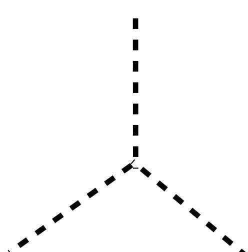
physical Higgs mass

$$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$$

Beyond the SM contribution

higher dimensional OPs

cubic Higgs coupling



$$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \quad \left(= \frac{m_h^2}{2v}\right)$$

$$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$$

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}(1 + C_h)$$

$C_h = 0$ for the SM Higgs potential

We focus on $C_h \neq 0$ case.

$$
\begin{aligned}
V = V\left(\frac{v^2}{2}\right) &+ V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right) \\
&+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2 \\
&+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3 \\
&\cdots
\end{aligned}
$$

stationary condition

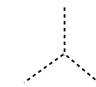
$$V'\left(\frac{v^2}{2}\right) = 0$$

physical Higgs mass

$$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$$

Beyond the SM contribution

cubic Higgs coupling



$$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \quad \left(= \frac{m_h^2}{2v}\right) \qquad C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$$

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}(1 + C_h)$$

$C_h = 0$ for the SM Higgs potential

We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ ***LHC*** (3 contributions)

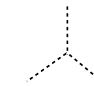
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition
$V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass
$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V''$ $\left(= \frac{m_h^2}{2v}\right)$

$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}\left(1 + C_h\right)$$

$C_h = 0$ for the SM Higgs potential

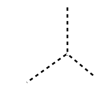
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V''$ $\left(= \frac{m_h^2}{2v}\right)$

$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}\left(1 + C_h\right)$$

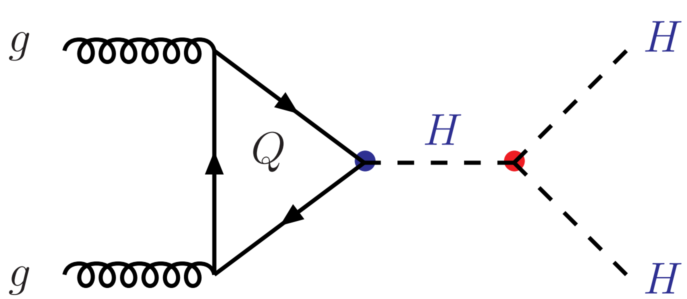
$C_h = 0$ for the SM Higgs potential

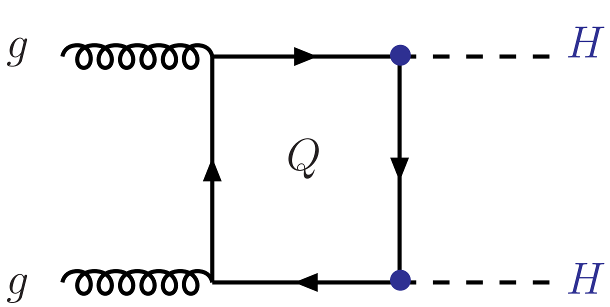
We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

- gluon-gluon fusion





We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

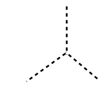
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



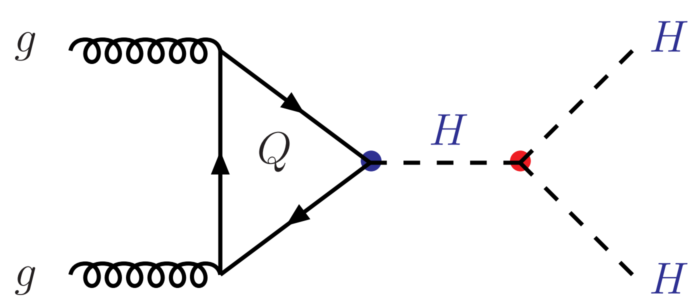
$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}\left(1 + C_h\right)$$

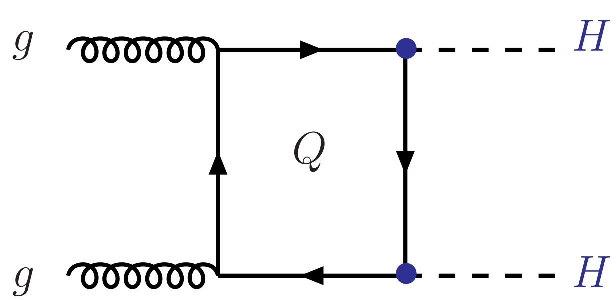
$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$

$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

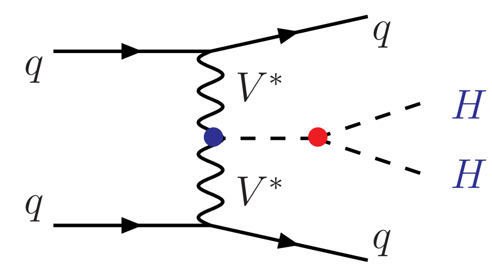
$C_h = 0$ for the SM Higgs potential

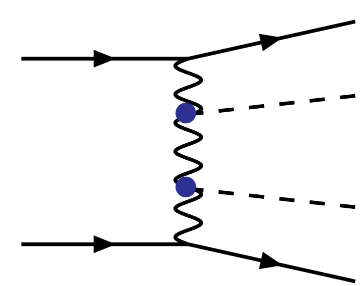
・gluon-gluon fusion

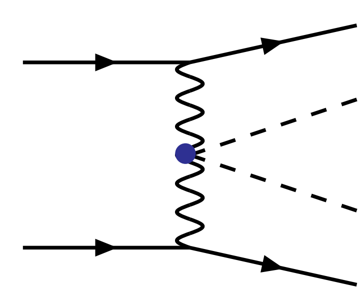




・Vector boson fusion







We focus on $C_h \neq 0$ case.

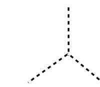
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$

$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

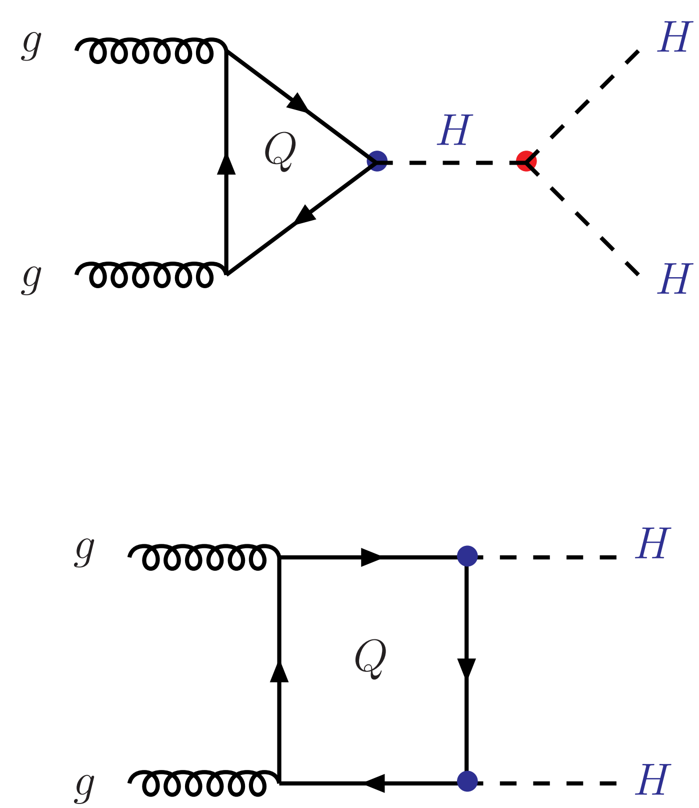
$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}(1 + C_h)$$

$C_h = 0$ for the SM Higgs potential

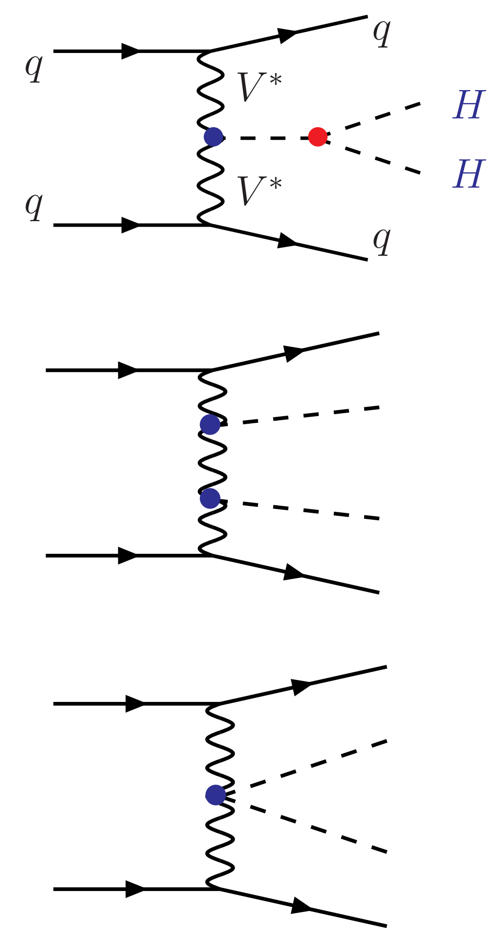
☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

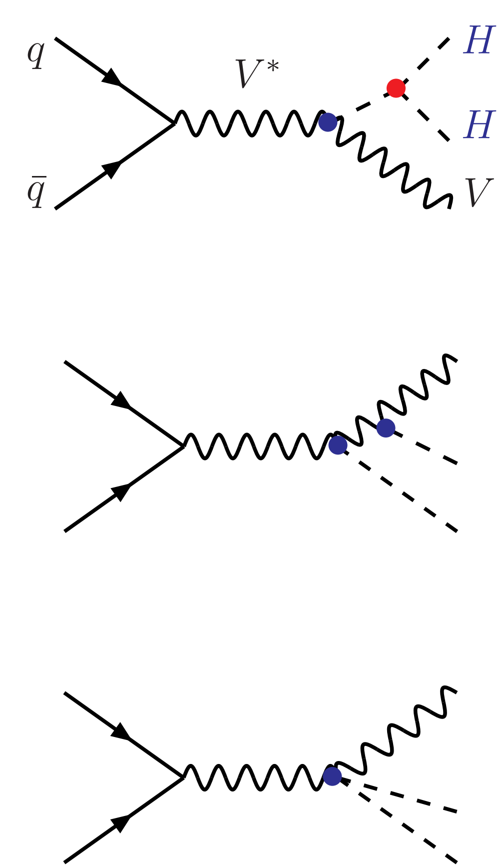
- gluon-gluon fusion



- Vector boson fusion



- Higgs-strahlung



We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

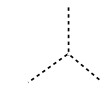
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right) + \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2 + \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3 \cdots$$

stationary condition
$V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass
$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



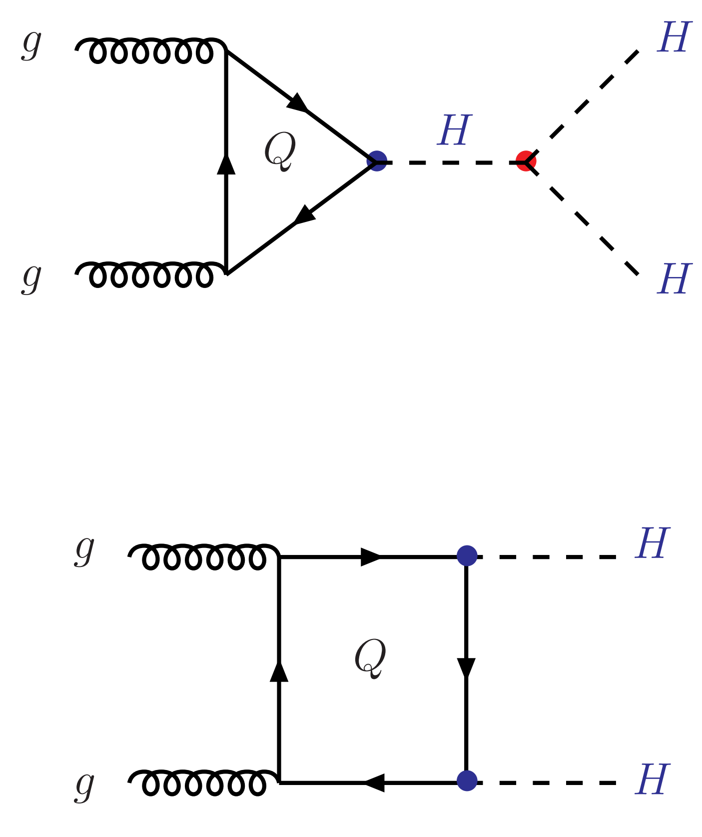
$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\text{SM}}\left(1 + C_h\right)$$

$\lambda_{hhh}^{\text{SM}} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$

$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$C_h = 0$ for the SM Higgs potential

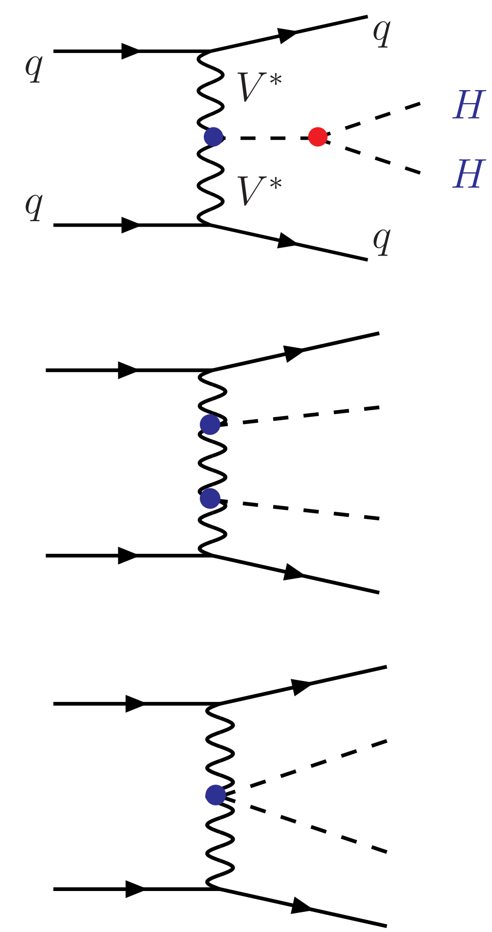
- gluon-gluon fusion



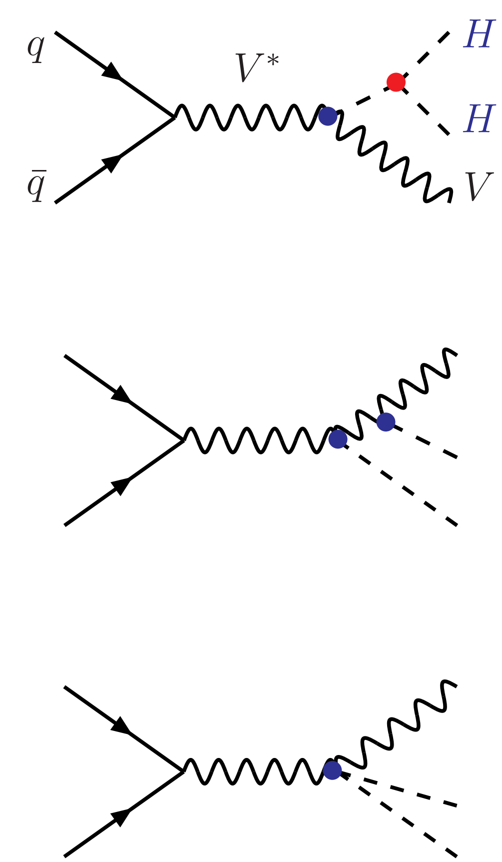
$$\sigma(pp \to gg \to hh)^{\text{LO}}_{\text{SM},14\text{TeV}} \sim 20 \text{ fb}$$

≫

- Vector boson fusion



- Higgs-strahlung



$$\sigma(pp \to hhjj)^{\text{LO}}_{\text{SM},14\text{TeV}} \sim 1.6 \text{ fb}$$

We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

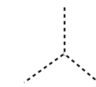
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition
$V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass
$m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



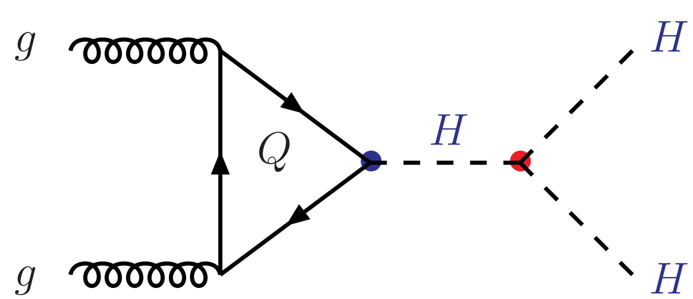
$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3 V'''\right] = -i6\lambda_{hhh}^{\rm SM}\left(1 + C_h\right)$$

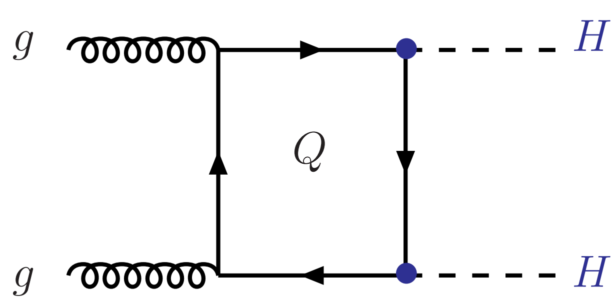
$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$

$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$C_h = 0$ for the SM Higgs potential

- gluon-gluon fusion





$$\sigma(pp \to gg \to hh)^{\rm LO}_{\rm SM,14TeV} \sim 20 \text{ fb}$$

In order to understand C_h contribution,
we utilize the following effective theory for gluon-Higgs interaction

$$\mathcal{L}_{\rm eff} = \frac{\alpha_s}{12\pi} G^a_{\mu\nu} G^{a\mu\nu} \ln\left(1 + \frac{h}{v}\right)$$
$$= \frac{\alpha_s}{12\pi} G^a_{\mu\nu} G^{a\mu\nu} \left(\frac{h}{v} - \frac{h^2}{2v} + \cdots\right)$$

(Hagiwara and Murayama)

Amplitude with C_h is obtained by

$$\mathcal{M}(gg \to hh) = \frac{\alpha_s}{3\pi v^2}\left(\frac{3m_h^2(1 + C_h)}{\hat{s} - m_h^2} - 1\right)$$

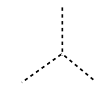
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$ $\quad C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}(1 + C_h)$$

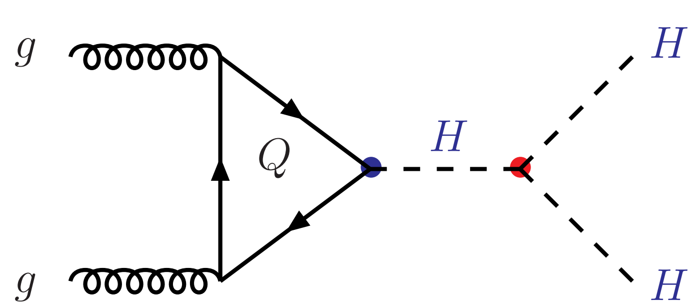
$C_h = 0$ for the SM Higgs potential

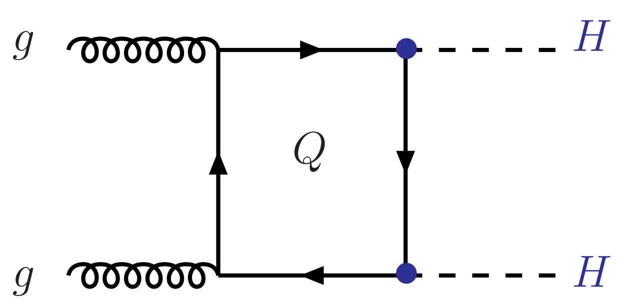
We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ ***LHC*** (3 contributions)

• gluon-gluon fusion





$$\sigma(pp \to gg \to hh)^{\rm LO}_{\rm SM,14TeV} \sim 20 \text{ fb}$$

In order to understand C_h contribution,
we utilize the following effective theory for gluon-Higgs interaction

$$\mathcal{L}_{\rm eff} = \frac{\alpha_s}{12\pi}G^a_{\mu\nu}G^{a\mu\nu}\ln\left(1 + \frac{h}{v}\right)$$
$$= \frac{\alpha_s}{12\pi}G^a_{\mu\nu}G^{a\mu\nu}\left(\frac{h}{v} - \frac{h^2}{2v} + \cdots\right)$$

(Hagiwara and Murayama)

Amplitude with C_h is obtained by

$$\mathcal{M}(gg \to hh) = \frac{\alpha_s}{3\pi v^2}\left(\frac{3m_h^2(1 + C_h)}{\hat{s} - m_h^2} - 1\right)$$

- $C_h > 0$, some (positive) $\hat{s}$ makes $\mathcal{M} = 0$
- $C_h < 0$, there is no cancellation at any $\hat{s} > 4m_h^2$

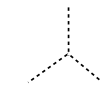
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right) + \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2 + \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3 \cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$ $\quad C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}(1 + C_h)$$

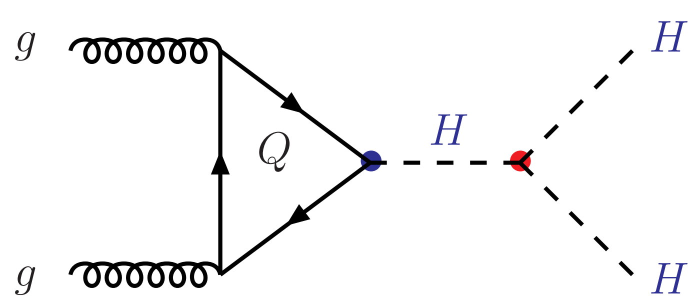
$C_h = 0$ for the SM Higgs potential

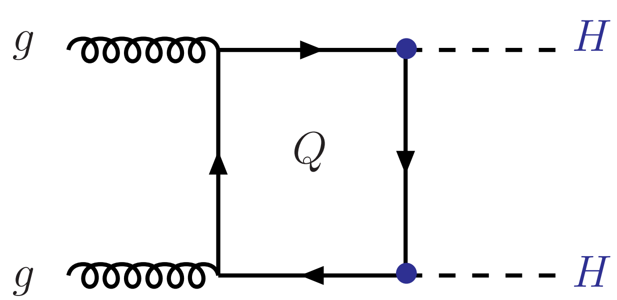
We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

• gluon-gluon fusion





$\sigma(pp \to gg \to hh)^{\rm LO}_{\rm SM, 14TeV} \sim 20$ fb

In order to understand C_h contribution,
we utilize the following effective theory for gluon-Higgs interaction

$$\mathcal{L}_{\rm eff} = \frac{\alpha_s}{12\pi}G^a_{\mu\nu}G^{a\mu\nu}\ln\left(1 + \frac{h}{v}\right) = \frac{\alpha_s}{12\pi}G^a_{\mu\nu}G^{a\mu\nu}\left(\frac{h}{v} - \frac{h^2}{2v} + \cdots\right)$$

(Hagiwara and Murayama)

Amplitude with C_h is obtained by

$$\mathcal{M}(gg \to hh) = \frac{\alpha_s}{3\pi v^2}\left(\frac{3m_h^2(1 + C_h)}{\hat{s} - m_h^2} - 1\right)$$

- $C_h > 0$, some (positive) $\hat{s}$ makes $\mathcal{M} = 0$
- $C_h < 0$, there is no cancellation at any $\hat{s} > 4m_h^2$

$C_h < 0$ enhances $gg \to hh$

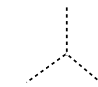
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$ $C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}(1 + C_h)$$

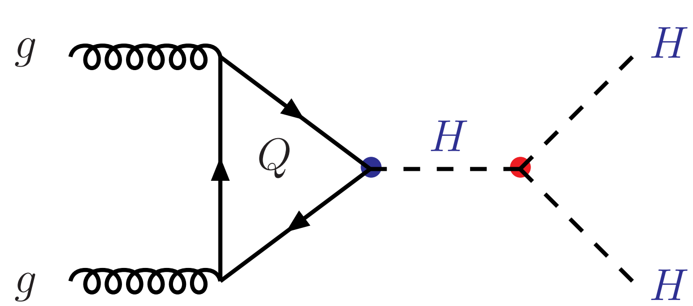
$C_h = 0$ for the SM Higgs potential

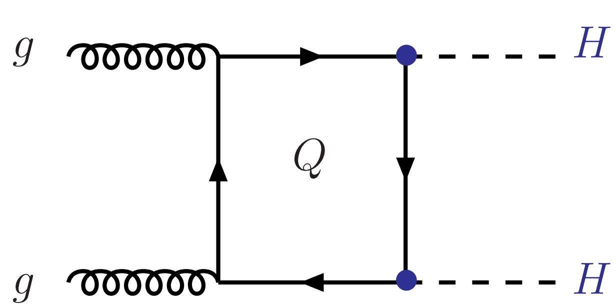
We focus on $C_h \neq 0$ case.

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

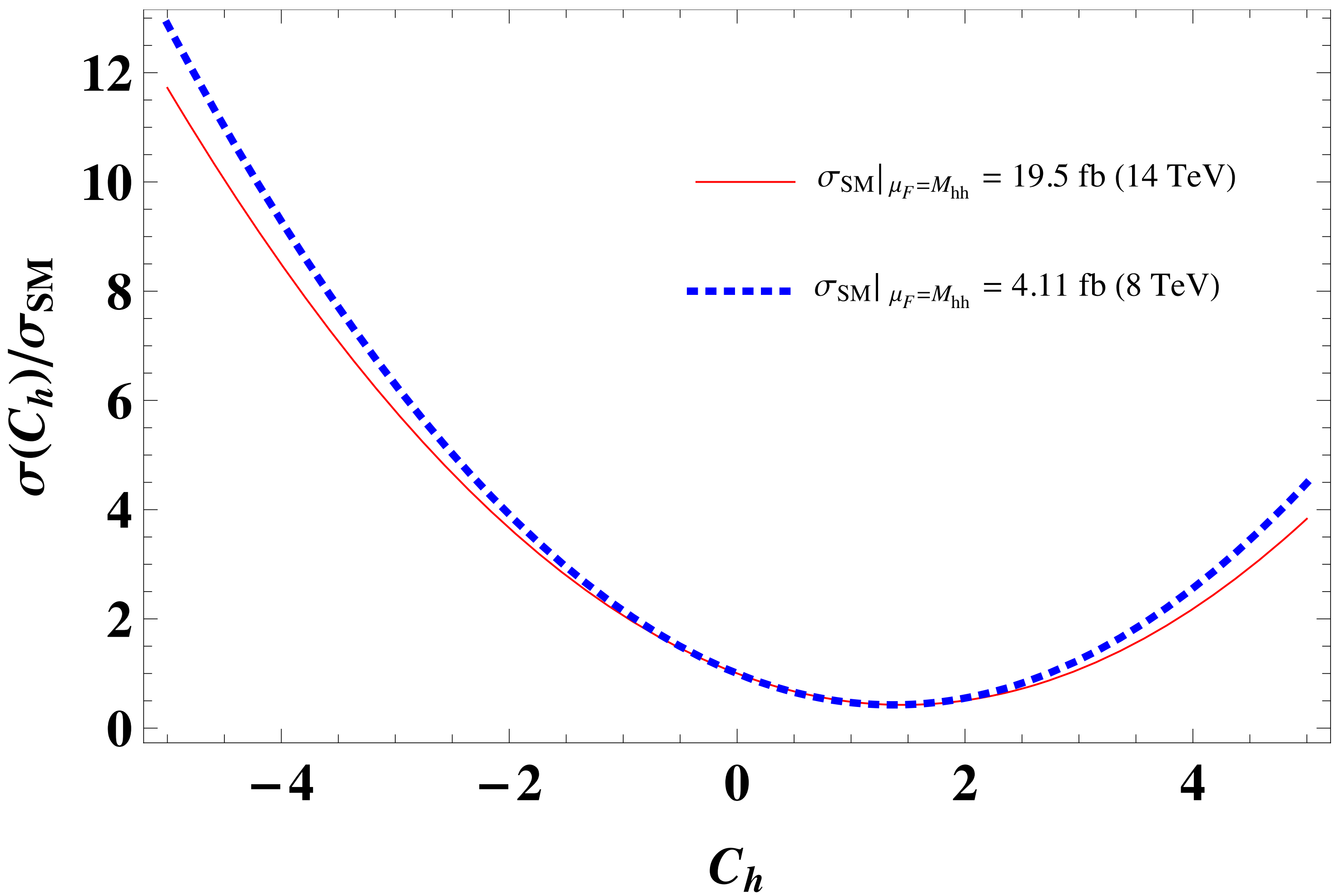
- gluon-gluon fusion





$\sigma(pp \to gg \to hh)^{\rm LO}_{\rm SM,14TeV} \sim 20$ fb

Ratio of $\sigma(C_h \neq 0)$ and $\sigma(C_h = 0) = \sigma({\rm SM})$



The crosse section can be factor 10 larger than $\sigma_{\rm SM}$

We focus on $C_h \neq 0$ case.

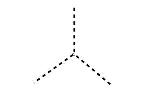
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}\left(1 + C_h\right)$$

$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$

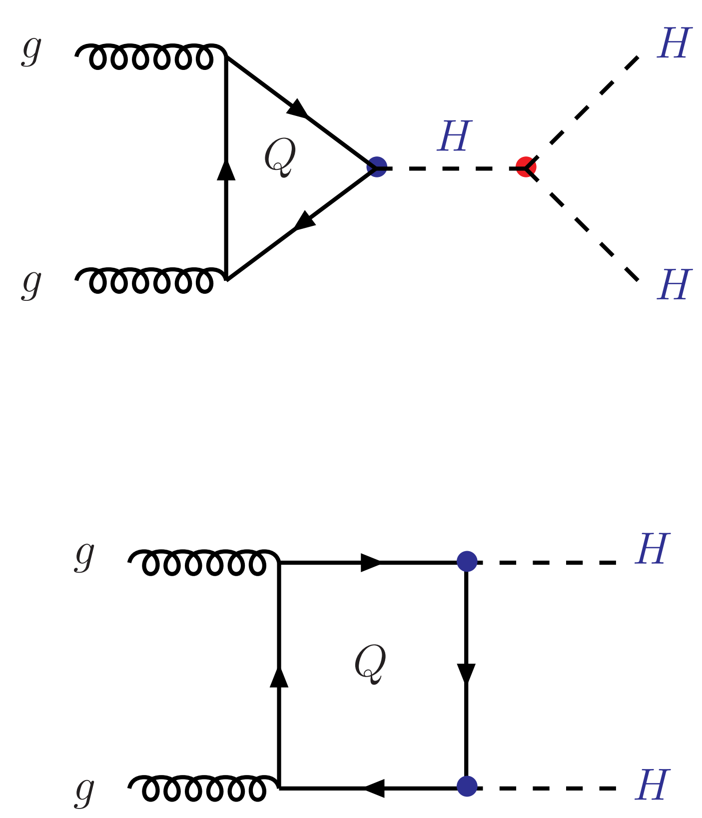
$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$C_h = 0$ for the SM Higgs potential

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

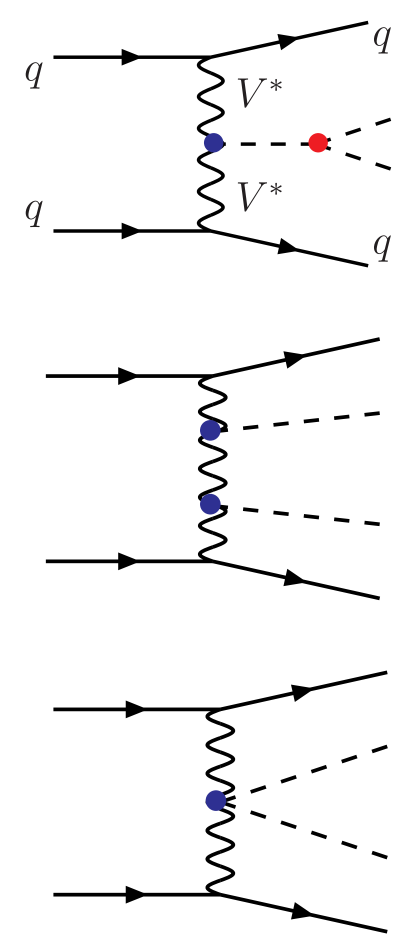
- gluon-gluon fusion



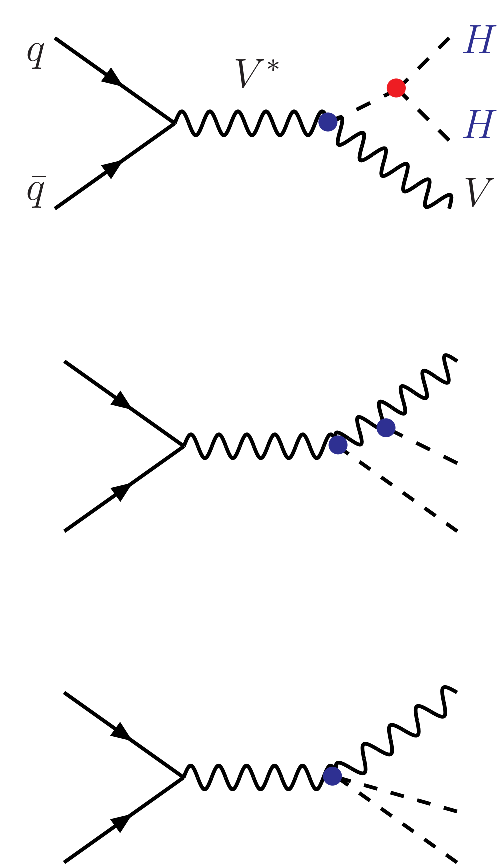
$$\sigma(pp \to gg \to hh)^{\rm LO}_{\rm SM,14TeV} \sim 20 \text{ fb}$$

≫

- Vector boson fusion



- Higgs-strahlung



$$\sigma(pp \to hhjj)^{\rm LO}_{\rm SM,14TeV} \sim 1.6 \text{ fb}$$

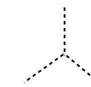
$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling



$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{\rm SM}(1 + C_h)$$

$\lambda_{hhh}^{\rm SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$ $\quad C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$C_h = 0$ for the SM Higgs potential

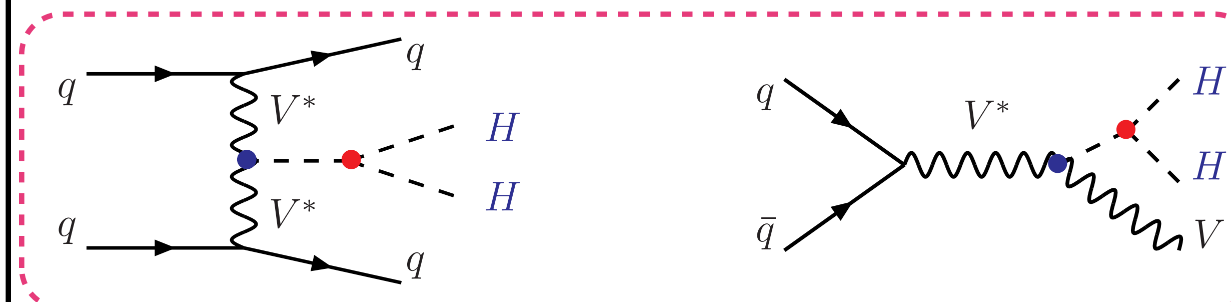
We focus on $C_h \neq 0$ case.

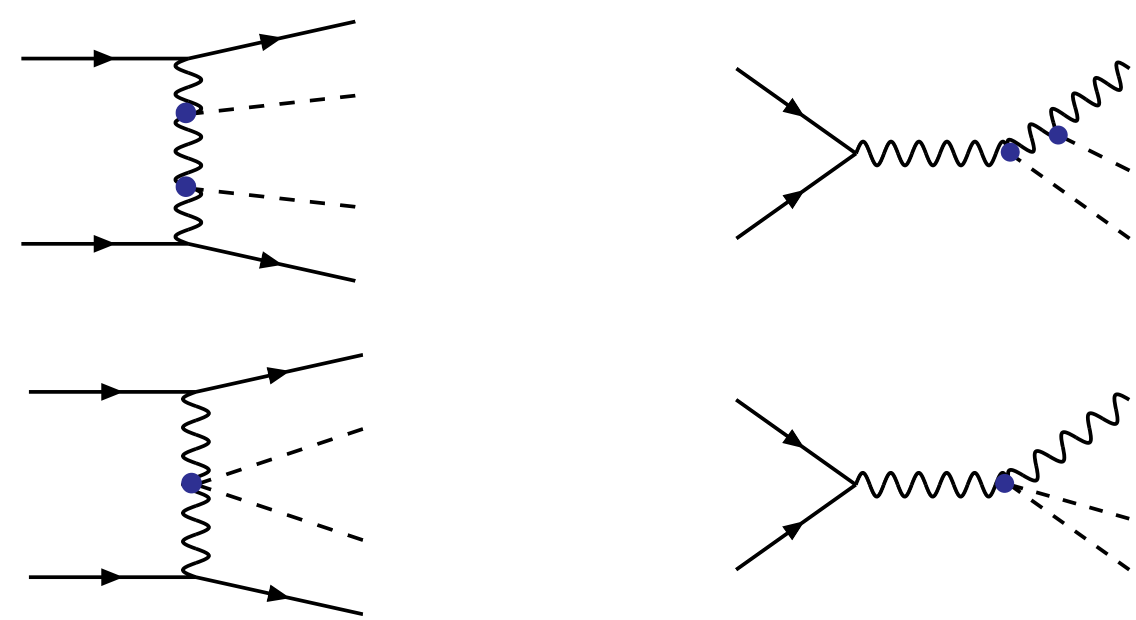
☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

How about these contributions?

- Vector boson fusion
- Higgs-strahlung





$$\sigma(pp \to hhjj)^{\rm LO}_{\rm SM,14TeV} \sim 1.6 \text{ fb}$$

We focus on $C_h \neq 0$ case.

$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)$$
$$+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3$$
$$\cdots$$

stationary condition $V'\left(\frac{v^2}{2}\right) = 0$

physical Higgs mass $m_h^2 \equiv v^2 V''\left(\frac{v^2}{2}\right)$

Beyond the SM contribution

cubic Higgs coupling

$$-i6\left[\frac{1}{2}vV'' + \frac{1}{6}v^3V'''\right] = -i6\lambda_{hhh}^{SM}\left(1 + C_h\right)$$

$\lambda_{hhh}^{SM} \equiv \frac{v}{2}V'' \left(= \frac{m_h^2}{2v}\right)$

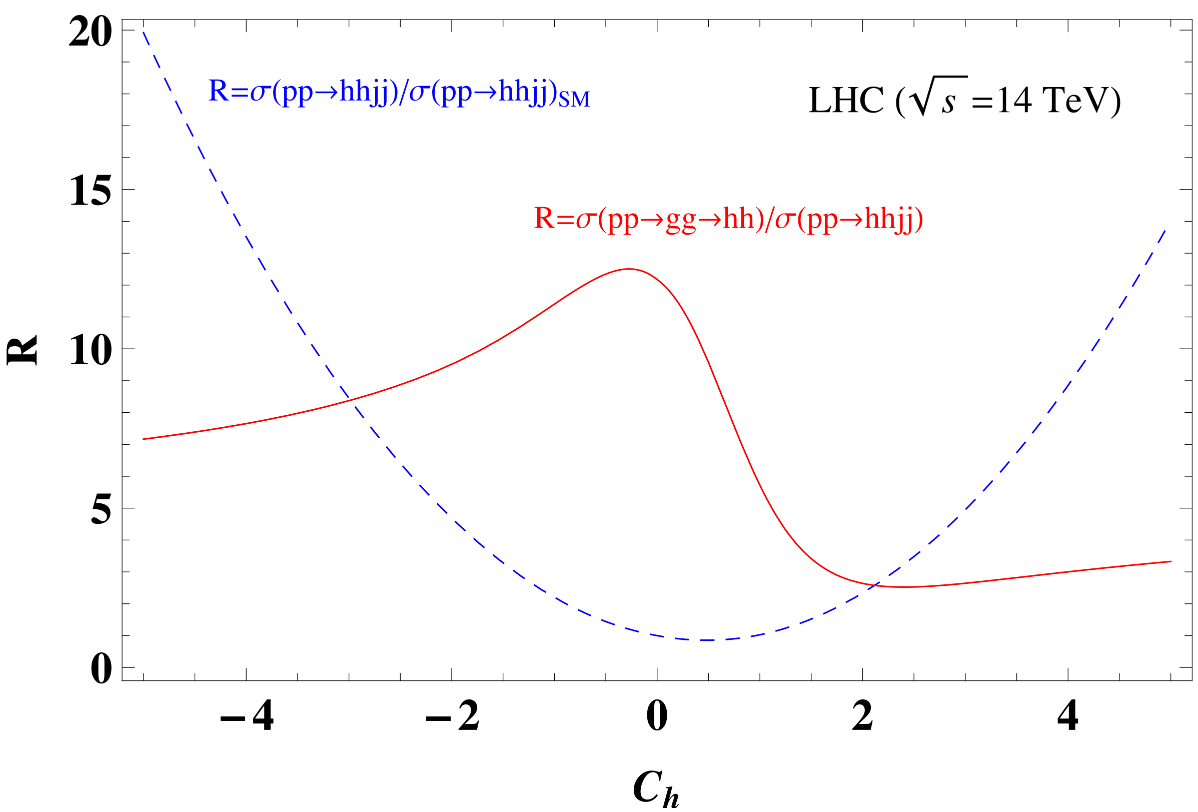
$C_h \equiv \frac{v^2}{3}\frac{V'''}{V''}$

$C_h = 0$ for the SM Higgs potential

☆ Possible enhancement of Higgs boson pair production

☆ $pp \to hhX$ @ **LHC** (3 contributions)

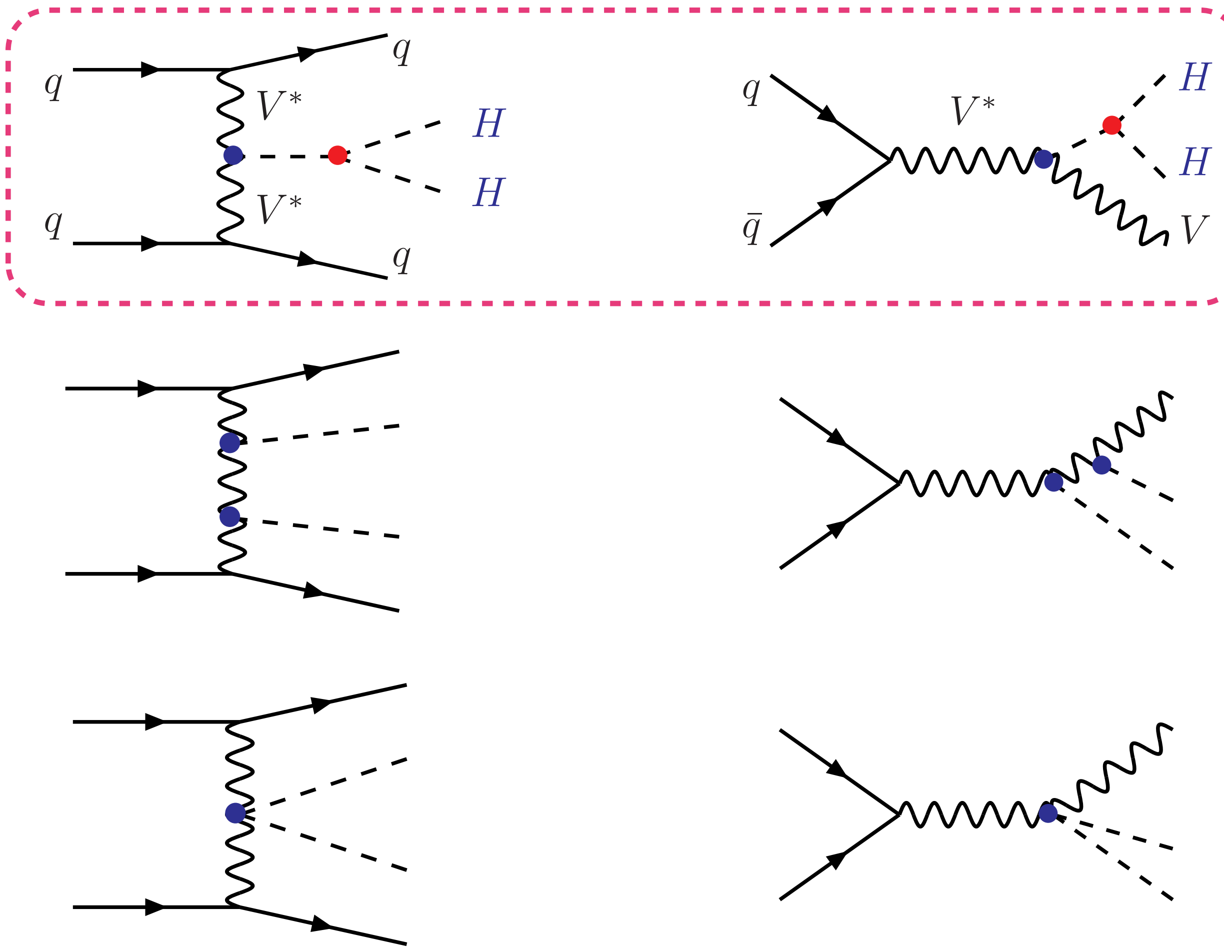
How about these contributions?



we can see

$gg \to hh$ always larger than VBF & Higgs-strahlung

- Vector boson fusion
- Higgs-strahlung



$\sigma(pp \to hhjj)^{LO}_{SM,14TeV} \sim 1.6$ fb

3, Non-perturbative Higgs model

3, Non-perturbative Higgs model

Now we have found that $C_h < 0$ easily enhance the Higgs pair production.

Now we have found that $C_h < 0$ easily enhance the Higgs pair production.

☆ An example for realizing $C_h < 0$

· Toy model

$$V(|H|^2) = m^2|H|^2 + \Lambda^{4-2a}(|H|^2)^a \qquad \begin{cases} m_h^2 = 2m^2(1-a) \\ C_h = \frac{v^2}{3}\frac{V'''}{V''} = \frac{2}{3}(a-2) \end{cases}$$

$$< 0 \qquad (\text{when } a < 0)$$

This kind of potential can be realized in context of SQCD.

Now we have found that $C_h < 0$ easily enhance the Higgs pair production.

☆ An example for realizing $C_h < 0$

・Toy model

$$V(|H|^2) = m^2|H|^2 + \Lambda^{4-2a}(|H|^2)^a$$

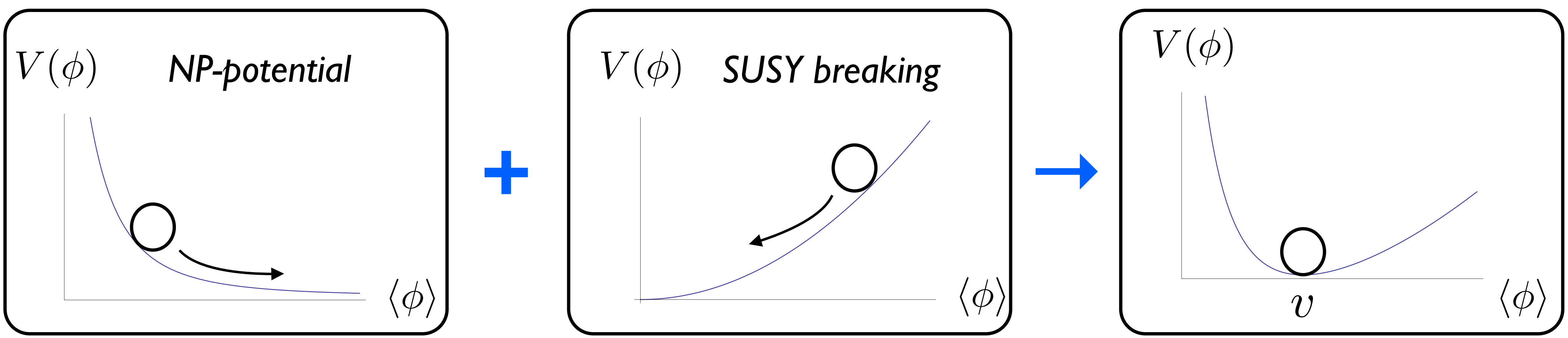
$$\begin{cases} m_h^2 = 2m^2(1-a) \\ C_h = \frac{v^2}{3}\frac{V'''}{V''} = \frac{2}{3}(a-2) \end{cases}$$

$$< 0 \qquad (\text{when } a < 0)$$

This kind of potential can be realized in context of SQCD.

・Non-perturbative Higgs model (SQCD)

$$W_{np} = \Lambda^3 \left(\frac{\Lambda^2}{H_u H_d}\right)^\kappa \qquad C_h \simeq -\frac{5}{3} - \frac{4}{3}\kappa \quad < 0 \ (\kappa > 0)$$



(D'Hoker-Mimura-Sakai, Haba-Okada)

3, Non-perturbative Higgs model

Let us consider other possibility of new physics effect.

Let us consider other possibility of new physics effect.

☆ Non-canonical kinetic term

$$\boxed{\mathcal{L}_{\rm kin} = F\left(\frac{|H|^2}{v^2/2}\right) D_\mu H^\dagger D^\mu H}$$

i.e. non-canonical Kahler potential

$F(x) = 1$ in SM

(Chivukula and Koulovassilopoulos, ...)

Let us consider other possibility of new physics effect.

☆ Non-canonical kinetic term

i.e. non-canonical Kahler potential

$$\boxed{\mathcal{L}_{\rm kin} = F\left(\frac{|H|^2}{v^2/2}\right) D_\mu H^\dagger D^\mu H}$$

$F(x) = 1$ in SM

(Chivukula and Koulovassilopoulos, ...)

$G(x) \equiv xF(x)$ and expand around $x = 1$

$$\Rightarrow \quad (M_W^2 W^+W^- + \frac{M_Z^2}{2}Z^2)\left(1 + G'(1)\frac{2h}{v} + (G'(1) + 2G''(1))\frac{h^2}{v^2} + \cdots\right)$$

Let us consider other possibility of new physics effect.

☆ Non-canonical kinetic term

$$\boxed{\mathcal{L}_{\rm kin} = F\left(\frac{|H|^2}{v^2/2}\right) D_\mu H^\dagger D^\mu H}$$

i.e. non-canonical Kahler potential

$F(x) = 1$ in SM

(Chivukula and Koulovassilopoulos, ...)

$G(x) \equiv xF(x)$ and expand around $x = 1$

$$\Rightarrow \quad (M_W^2 W^+ W^- + \frac{M_Z^2}{2} Z^2)\left(1 + G'(1)\frac{2h}{v} + (G'(1) + 2G''(1))\frac{h^2}{v^2} + \cdots\right)$$

$G'(1) = 1$ $\quad G''(1) = 0$ corresponds to SM

Let us consider other possibility of new physics effect.

☆ Non-canonical kinetic term

$$\boxed{\mathcal{L}_{\rm kin} = F\left(\frac{|H|^2}{v^2/2}\right) D_\mu H^\dagger D^\mu H}$$

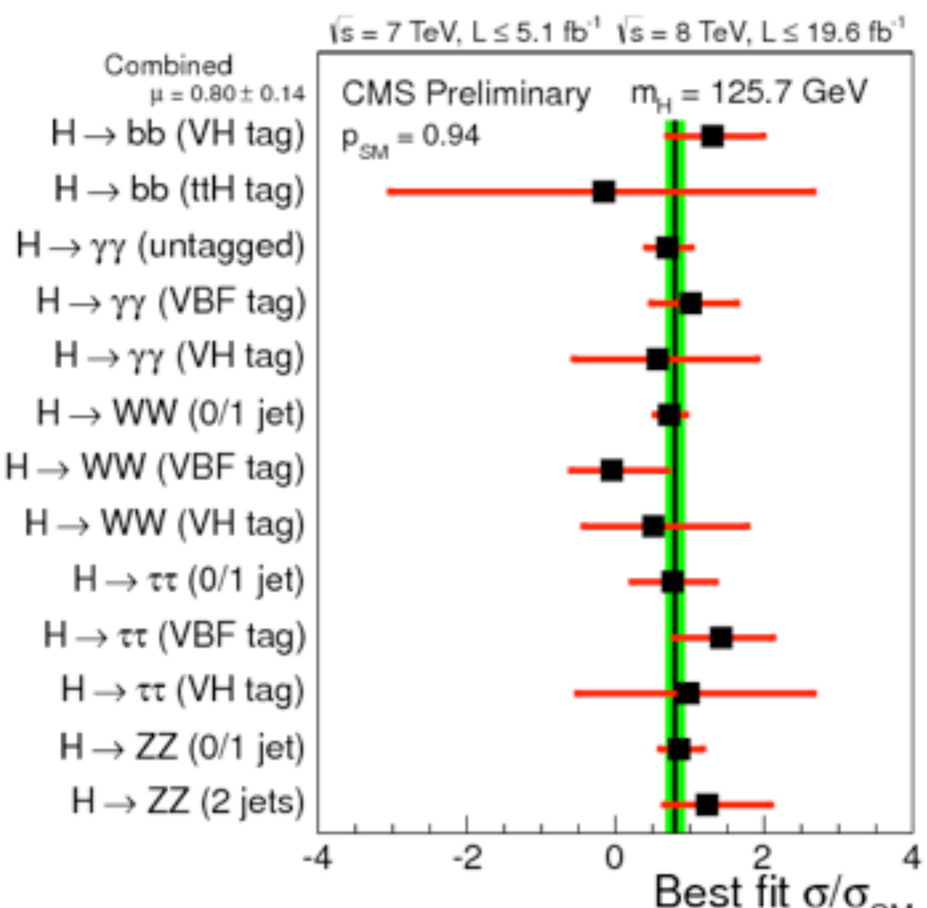
i.e. non-canonical Kahler potential

$F(x) = 1$ in SM

(Chivukula and Koulovassilopoulos, ...)

$G(x) \equiv xF(x)$ and expand around $x = 1$

$$\Rightarrow (M_W^2 W^+W^- + \frac{M_Z^2}{2}Z^2)\left(1 + G'(1)\frac{2h}{v} + (G'(1) + 2G''(1))\frac{h^2}{v^2} + \cdots\right)$$



considering the Higgs coupling measurements,
(at least)

Higgs to W^+W^-/ZZ coupling is consistent with SM.

$$G'(1) \simeq 1$$

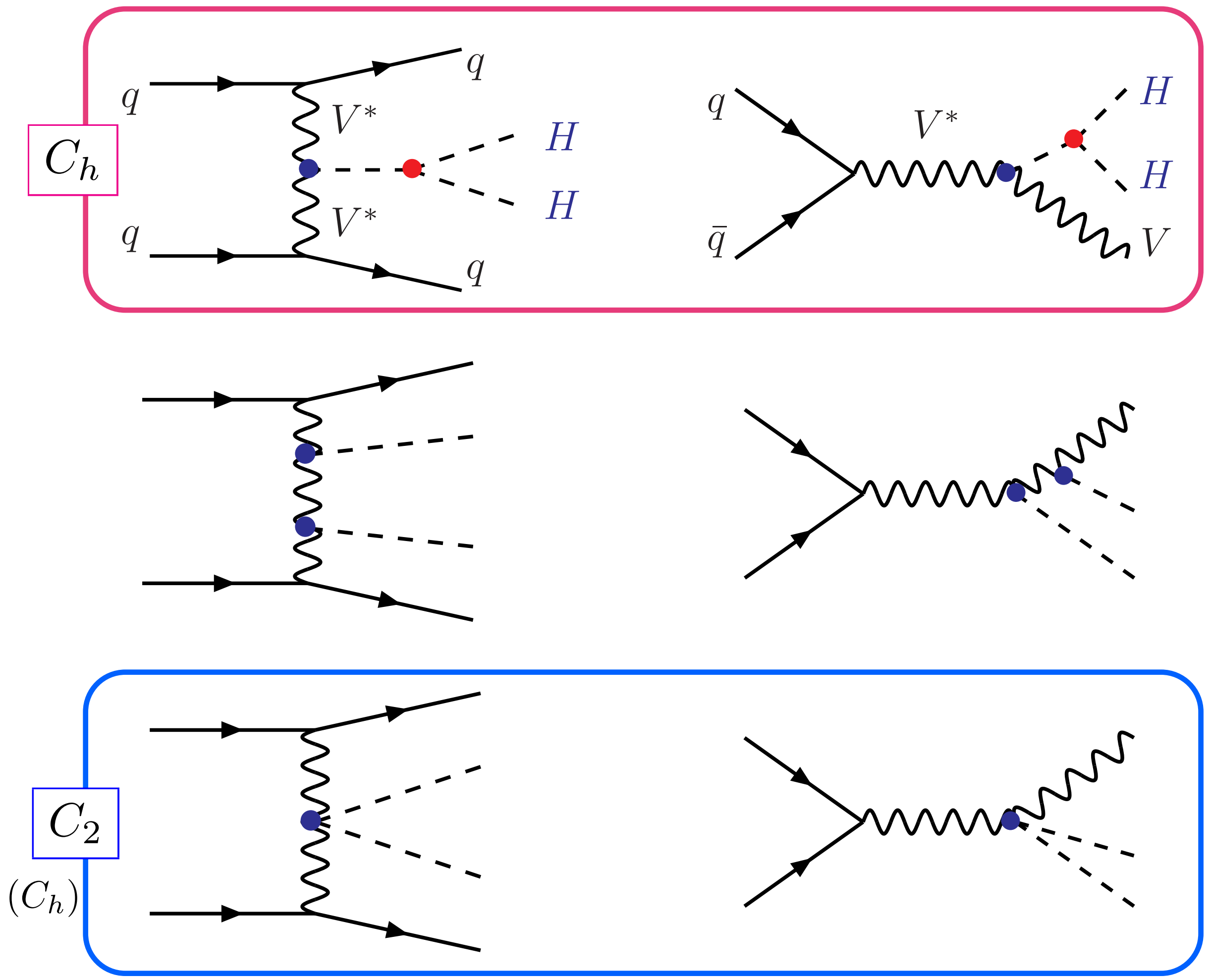
Here we have two parameters related to New Physics

C_h and $C_2 \equiv 2G''(1)$

→ How can these two parameters change Higgs pair production?

☆ $pp \to hhjj$ @ **LHC**

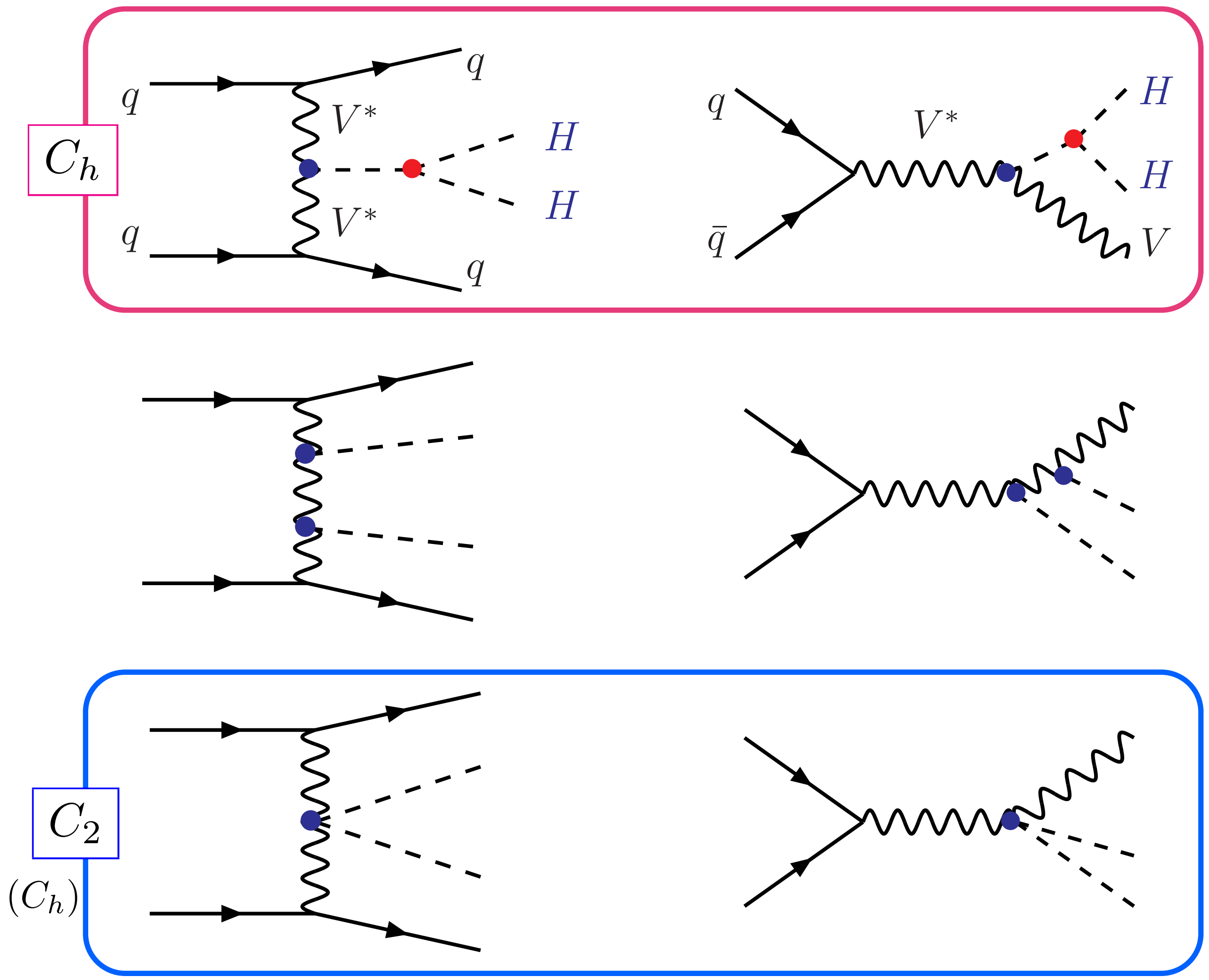
• VBF and Higgs-strahlung



$$\sigma(pp \to hhjj)^{\mathrm{LO}}_{\mathrm{SM,14TeV}} \sim 1.6 \text{ fb}$$

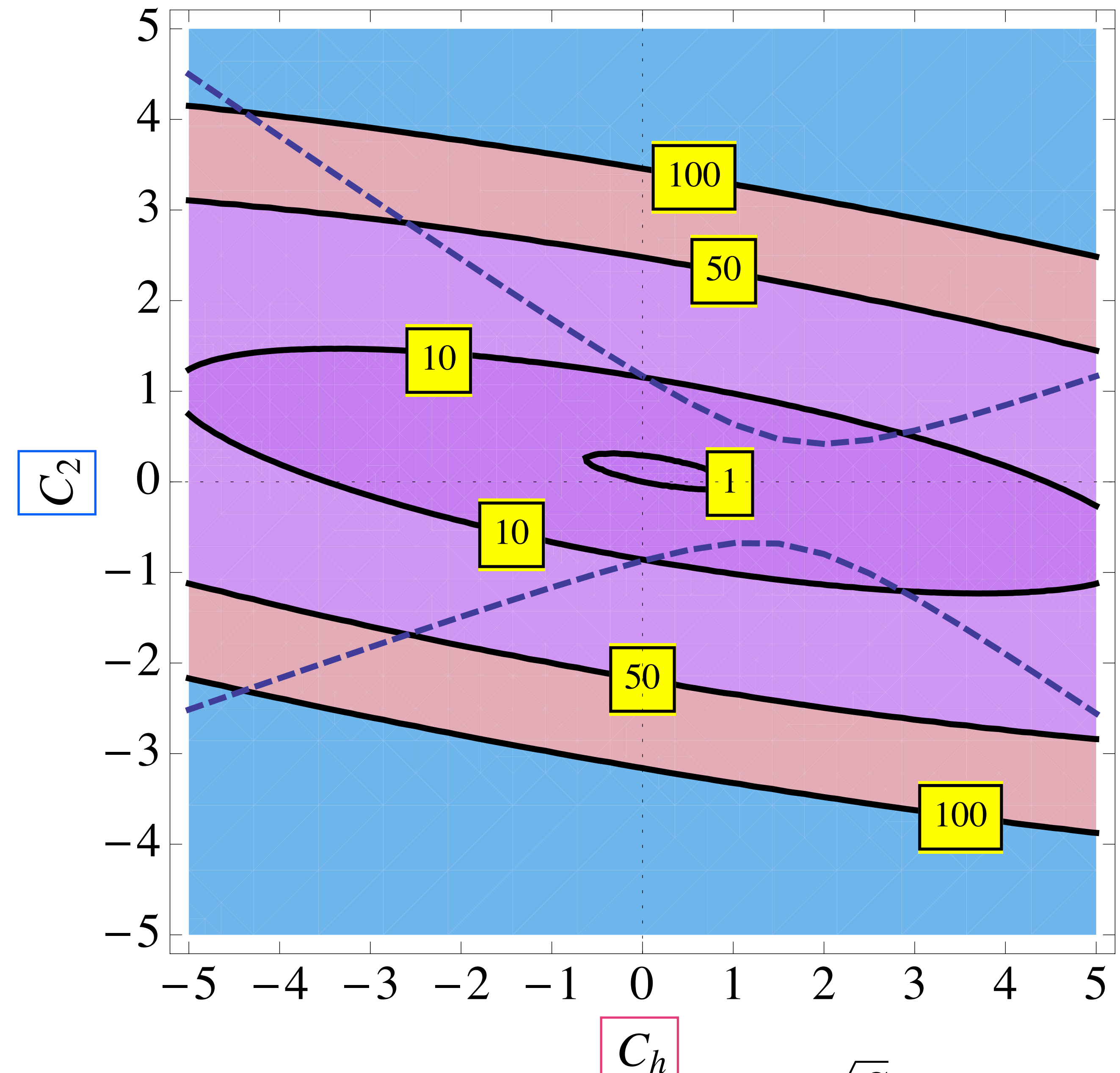
☆ $pp \to hhjj$ @ **LHC**

• VBF and Higgs-strahlung



$\sigma(pp \to hhjj)^{\mathrm{LO}}_{\mathrm{SM,14TeV}} \sim 1.6$ fb

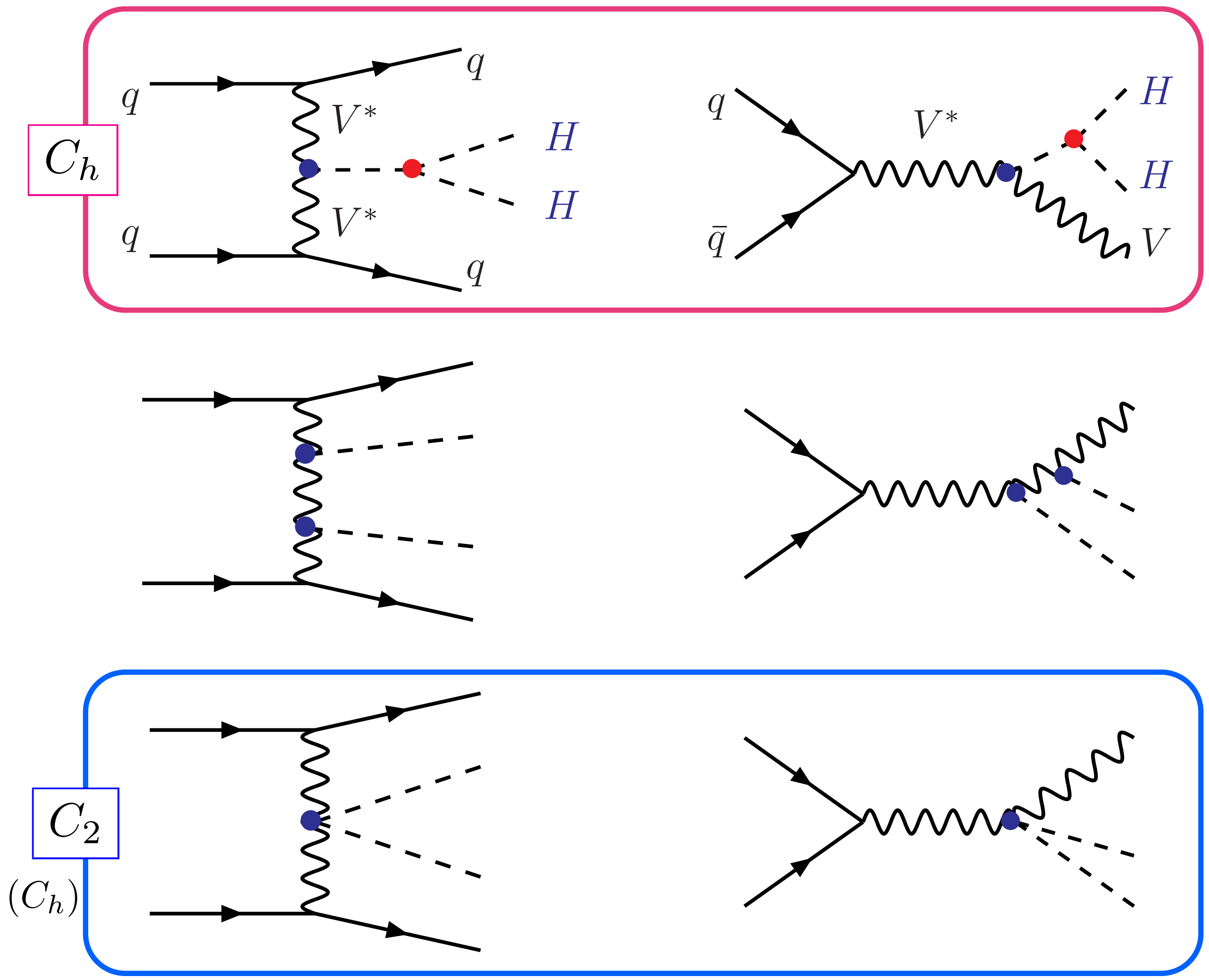
$\sigma(pp \to hhjj)/\sigma(pp \to hhjj)_{\mathrm{SM}}$



$\sqrt{S} = 14$ TeV

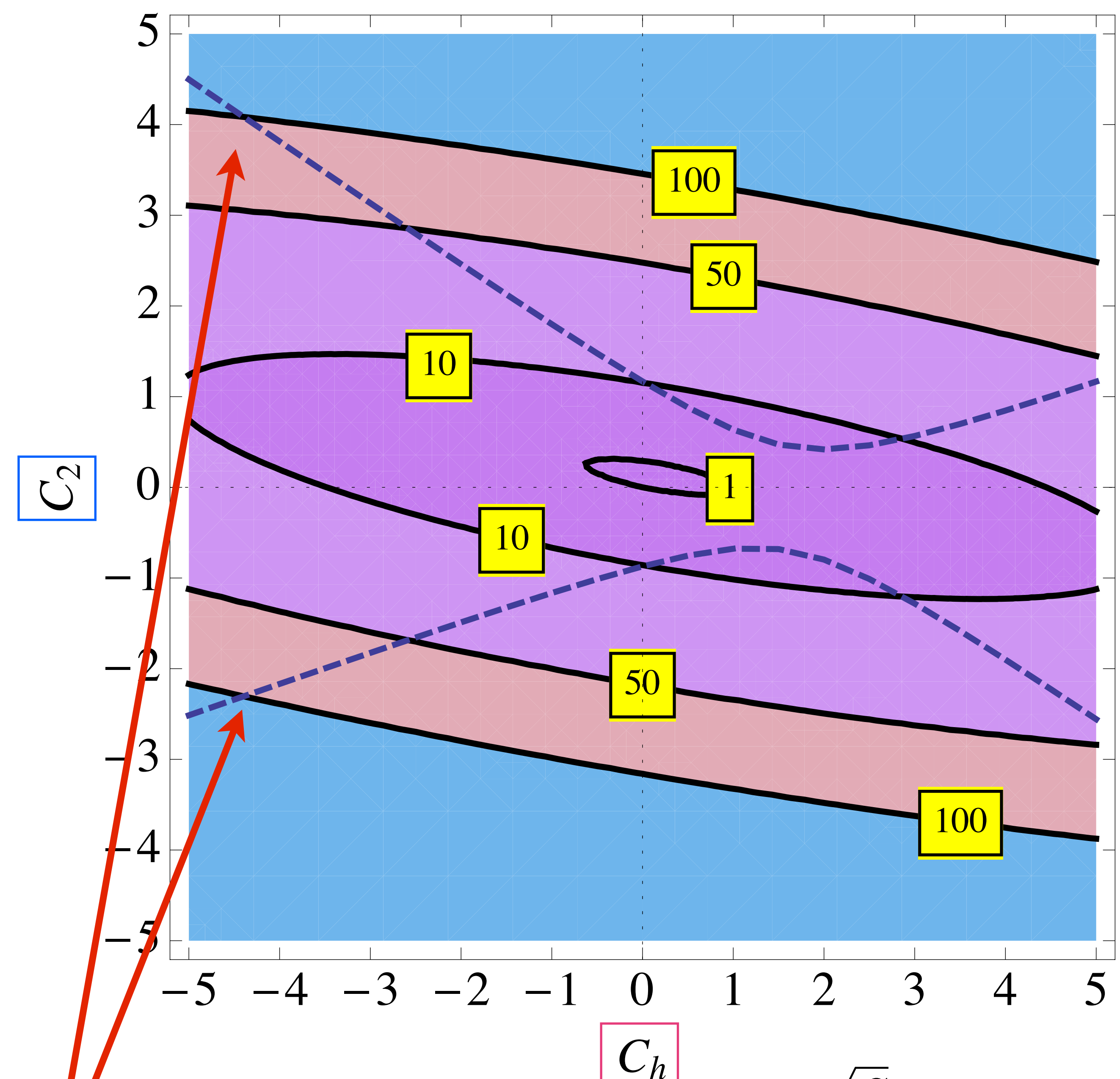
☆ $pp \to hhjj$ @ **LHC**

- VBF and Higgs-strahlung



$$\sigma(pp \to hhjj)^{\rm LO}_{\rm SM,14TeV} \sim 1.6 \text{ fb}$$

$$\sigma(pp \to hhjj)/\sigma(pp \to hhjj)_{\rm SM}$$



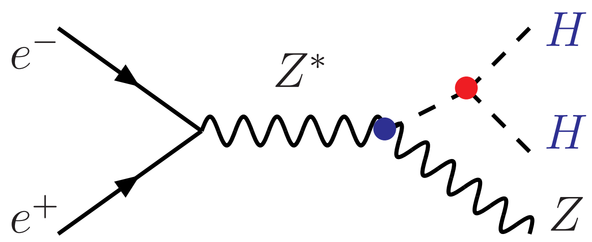
$\sqrt{S} = 14$ TeV

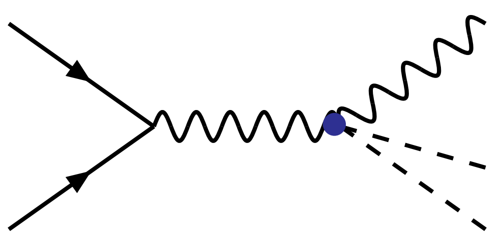
$$\sigma(pp \to hhjj) = \sigma(pp \to gg \to hh)$$

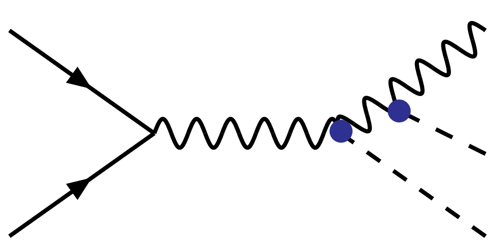
☆ $e^+e^- \to hhX$ @ **ILC**

☆ $e^+e^- \to hhX$ @ ***ILC***

- Higgs-strahlung

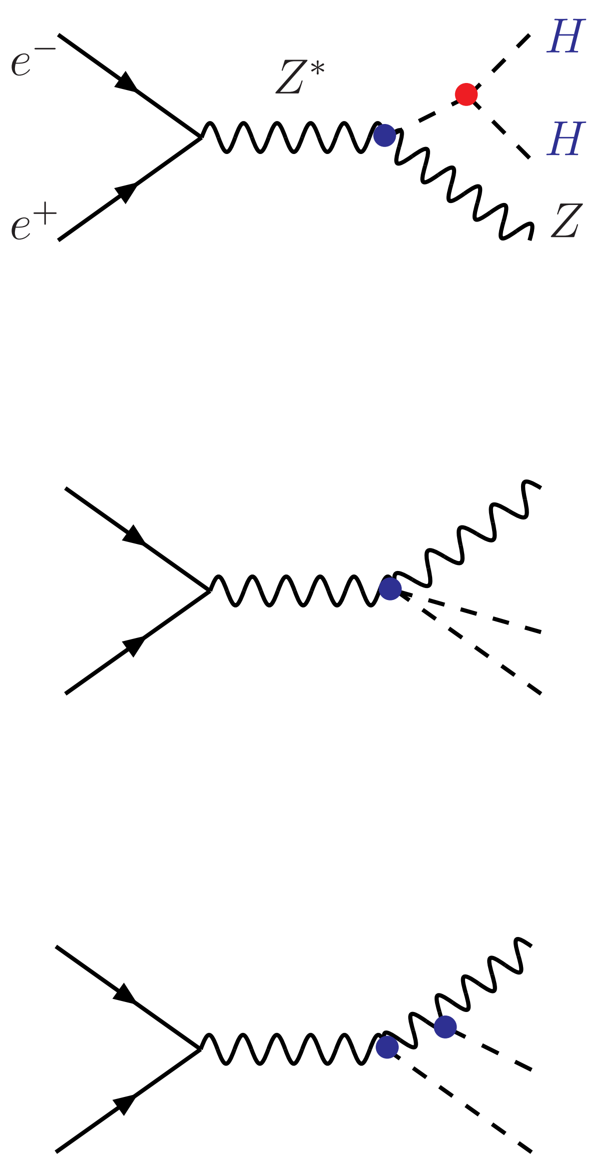




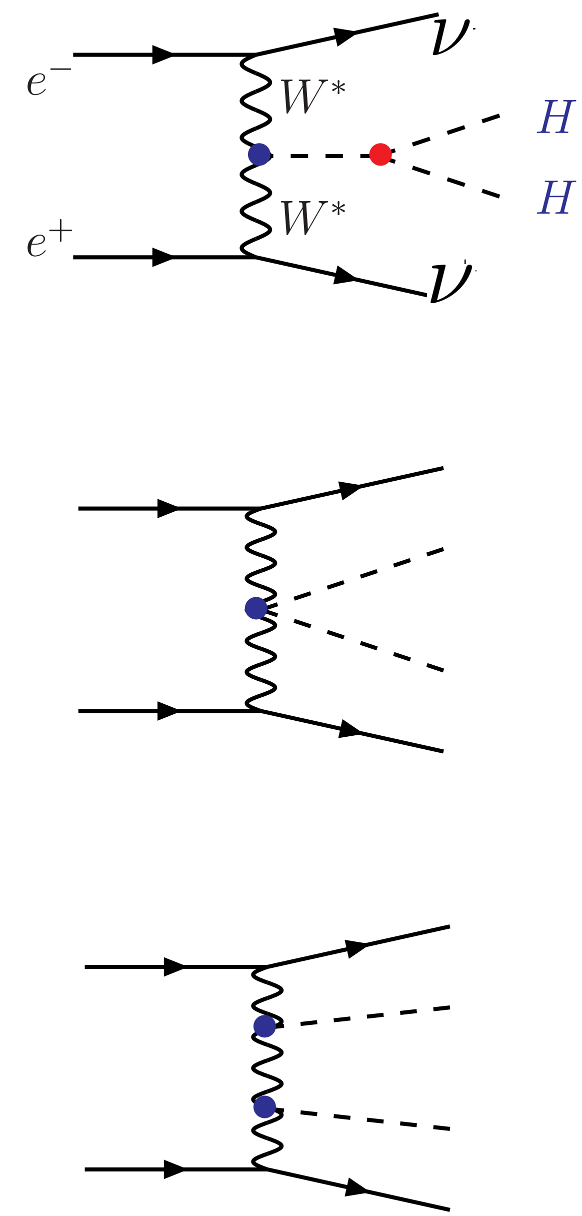


☆ $e^+e^- \to hhX$ @ ***ILC***

- Higgs-strahlung

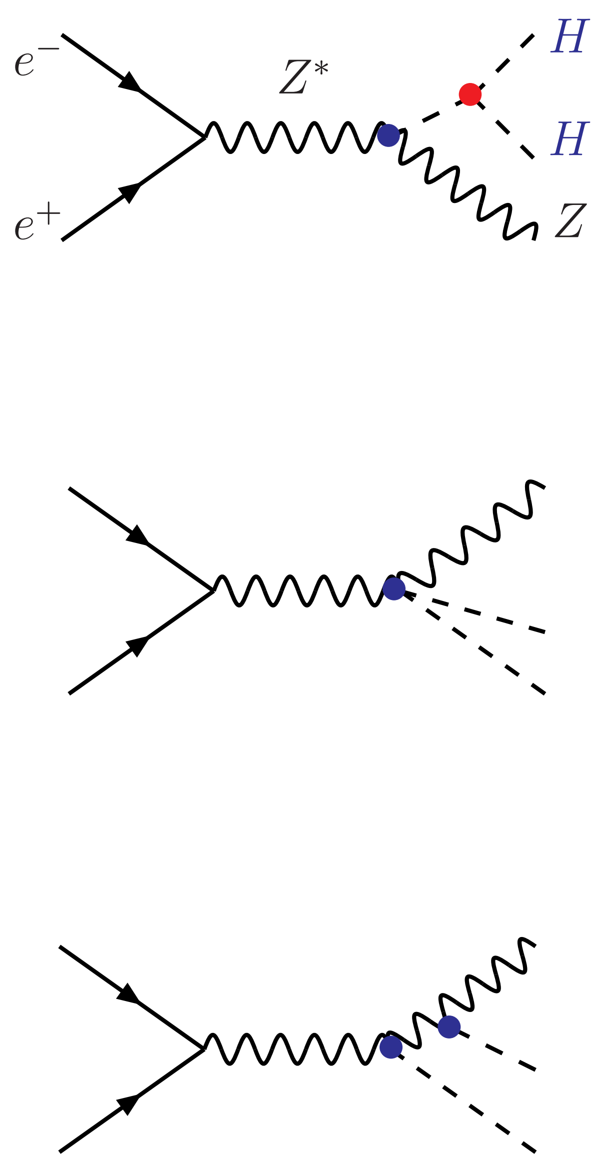


- Vector boson fusion



☆ $e^+e^- \to hhX$ @ *ILC*

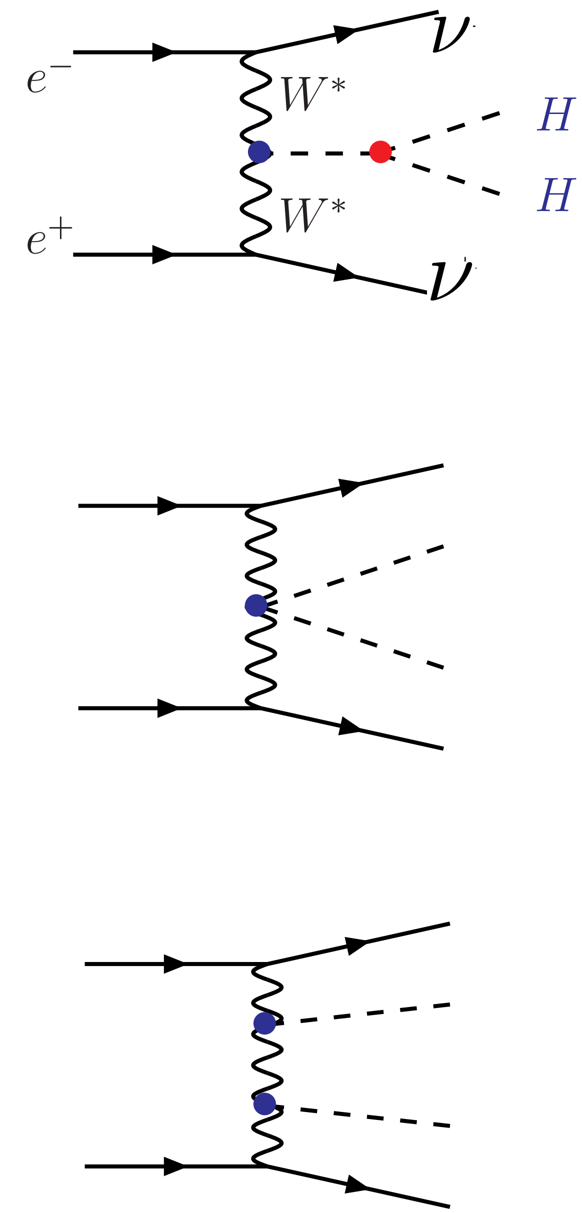
- Higgs-strahlung



$$\sigma(e^+e^- \to Zhh)_{\mathrm{SM,500GeV}} \sim 0.16 \text{ fb}$$

$\gg$

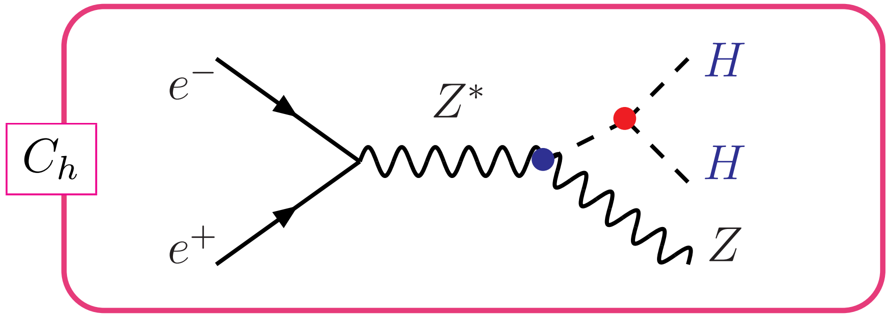
- Vector boson fusion

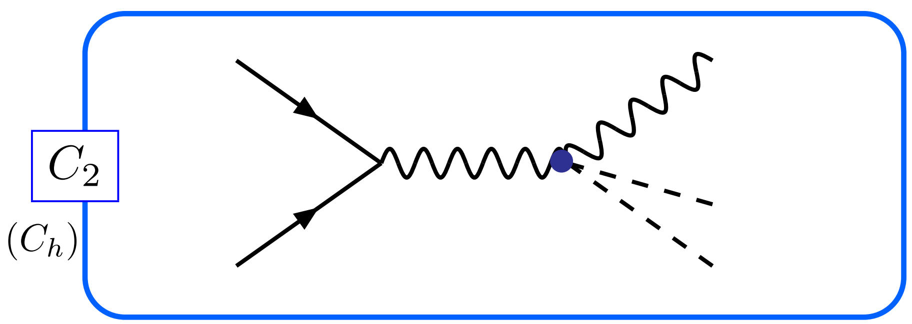


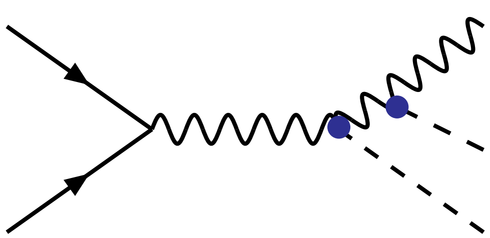
$$\sigma(e^+e^- \to \nu\bar{\nu}hh)_{\mathrm{SM,500GeV}} \sim 0.033 \text{ fb}$$

☆ $e^+e^- \to hhX$ @ *ILC*

- Higgs-strahlung

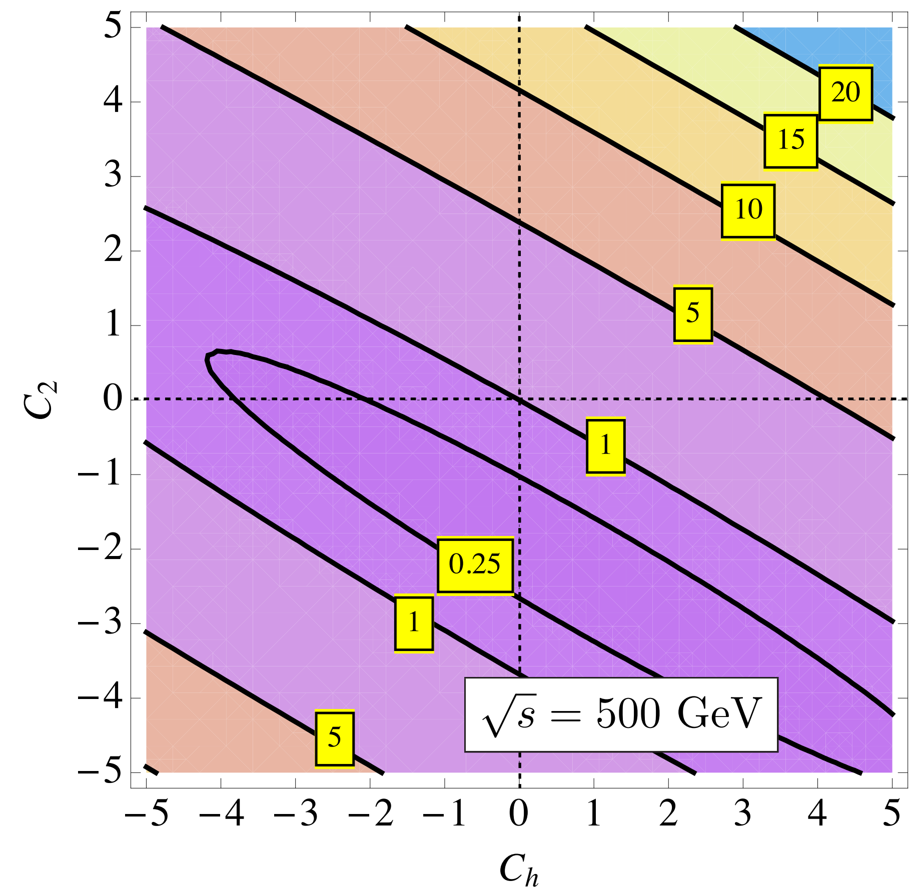


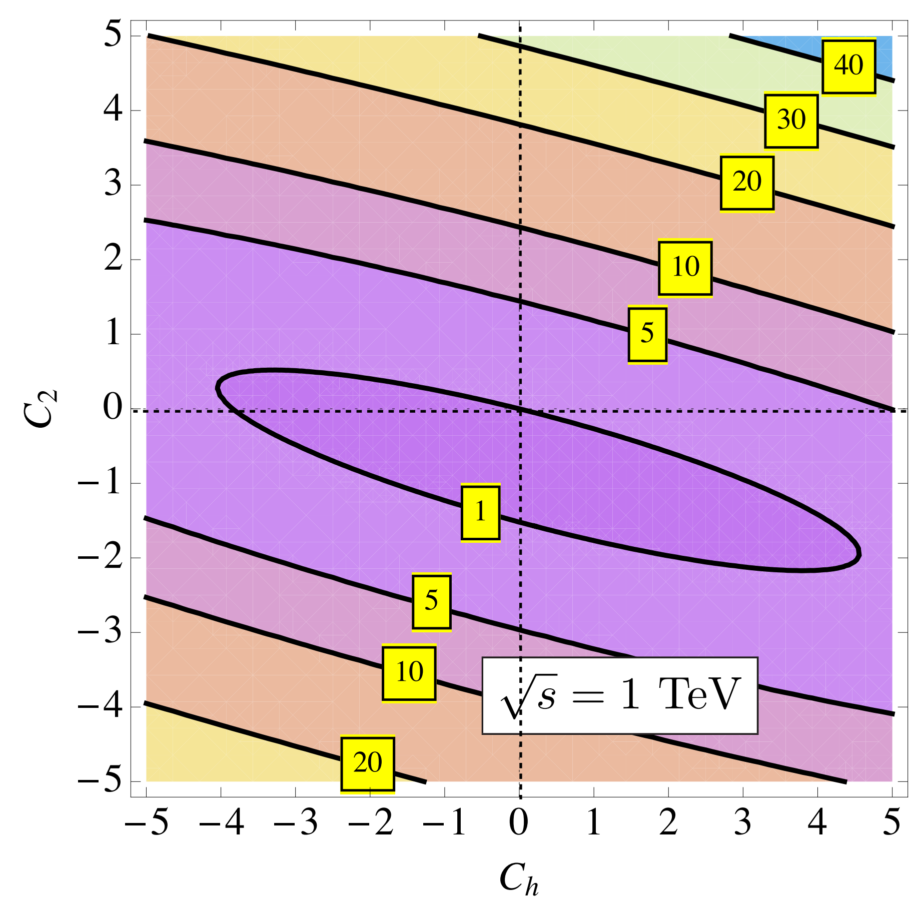




$$\sigma(e^+e^- \to Zhh)_{\mathrm{SM,500GeV}} \sim 0.16 \text{ fb}$$

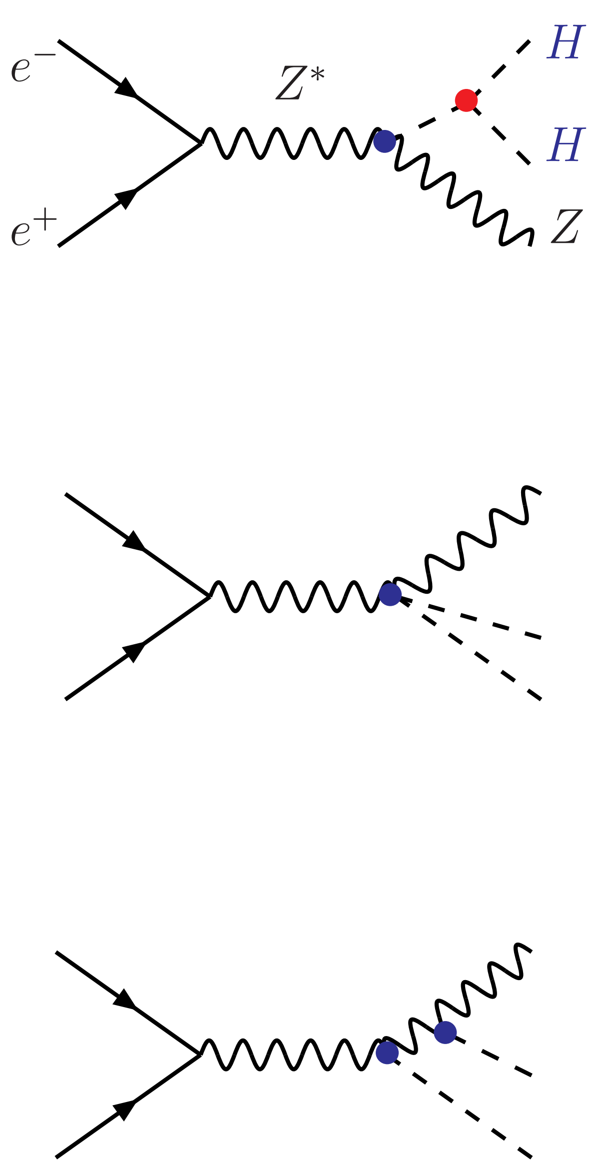
$$\sigma(e^+e^- \to Zhh)/\sigma(\mathrm{SM})$$





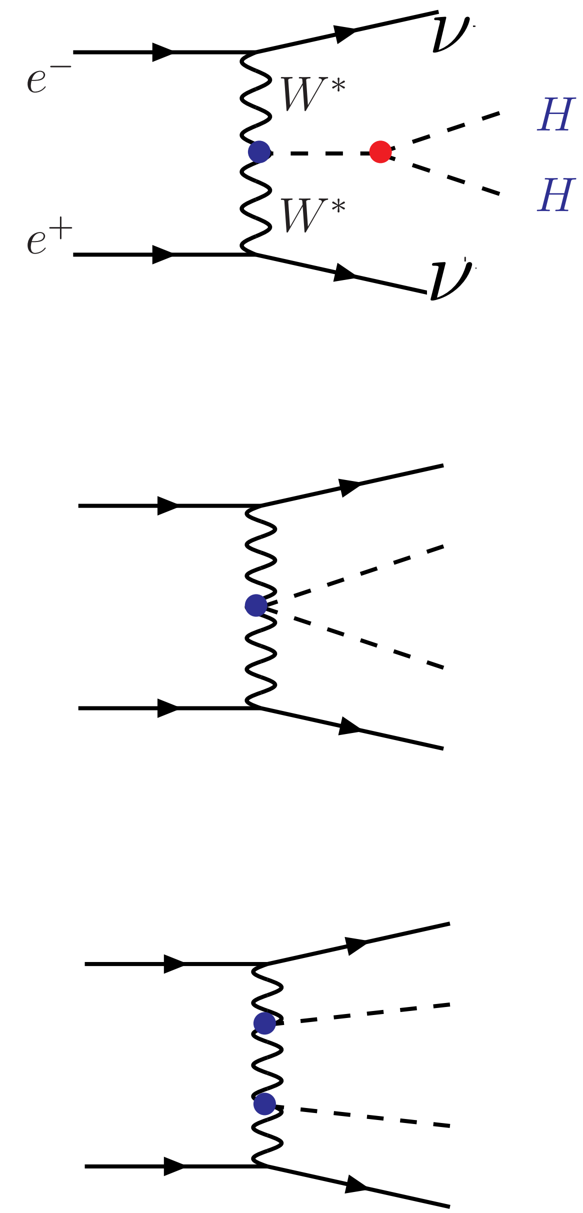
☆ $e^+e^- \to hhX$ @ *ILC*

- Higgs-strahlung



$$\sigma(e^+e^- \to Zhh)_{\mathrm{SM,500GeV}} \sim 0.16 \text{ fb}$$

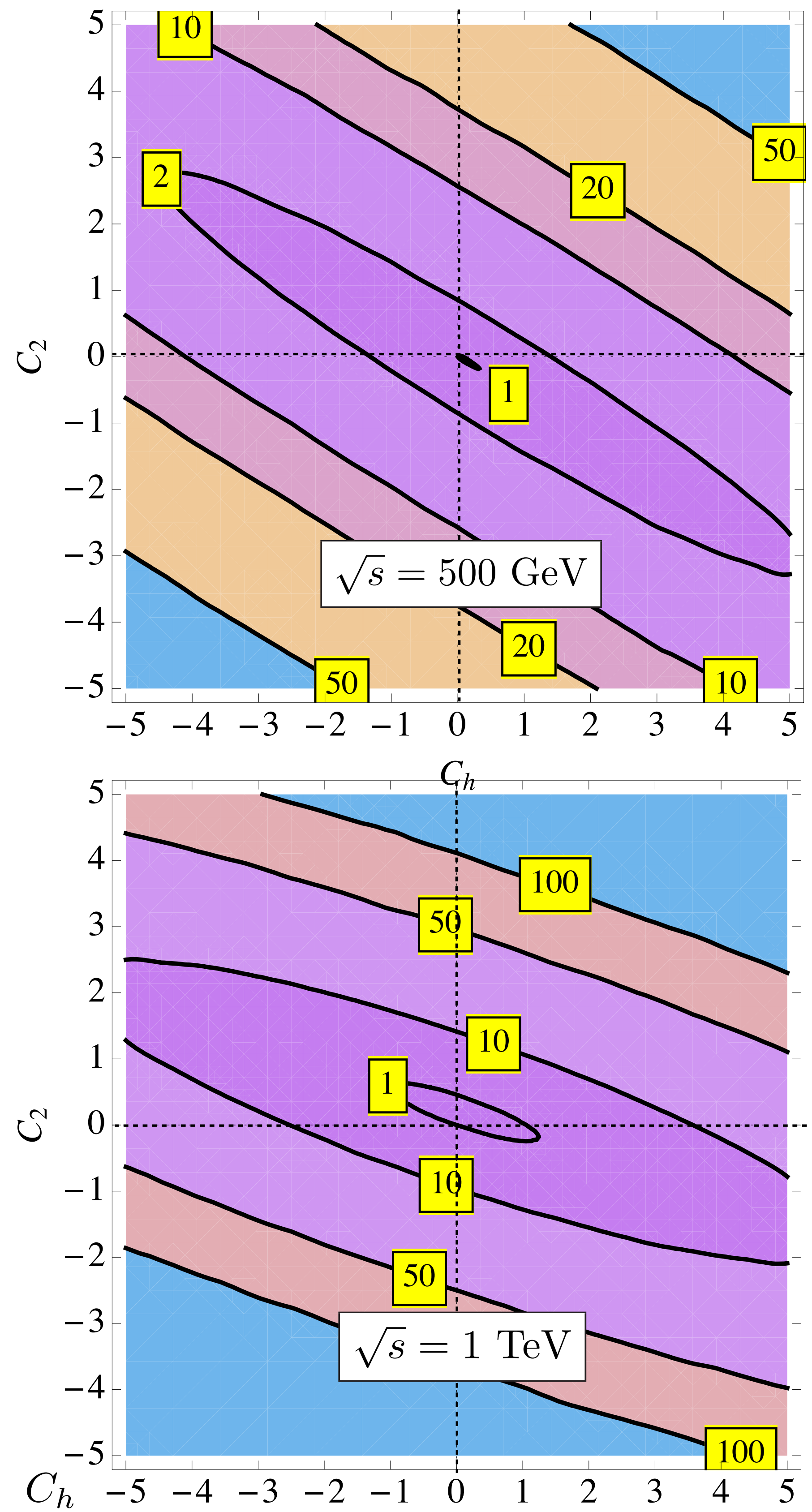
- Vector boson fusion



$$\sigma(e^+e^- \to \nu\bar{\nu}hh)_{\mathrm{SM,500GeV}} \sim 0.033 \text{ fb}$$

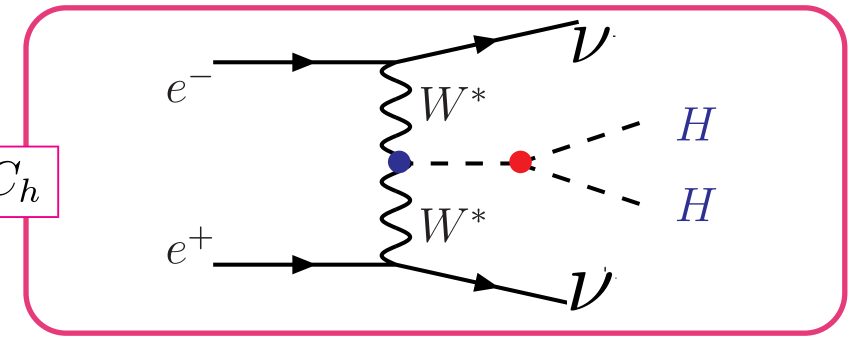
☆ $e^+e^- \to hhX$ @ ***ILC***

$\sigma(e^+e^- \to \nu\bar{\nu}hh)/\sigma(\mathrm{SM})$

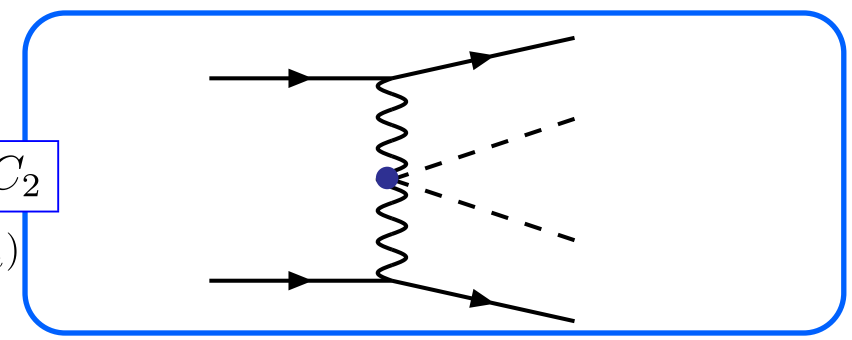


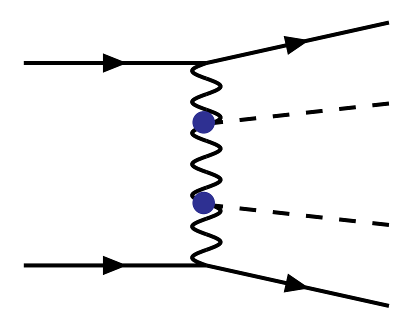
- Vector boson fusion

C_h



C_2
(C_h)





$$\sigma(e^+e^- \to \nu\bar{\nu}hh)_{\mathrm{SM,500GeV}} \sim 0.033 \text{ fb}$$

Summary

☆ Higgs self-interaction is still unknown.

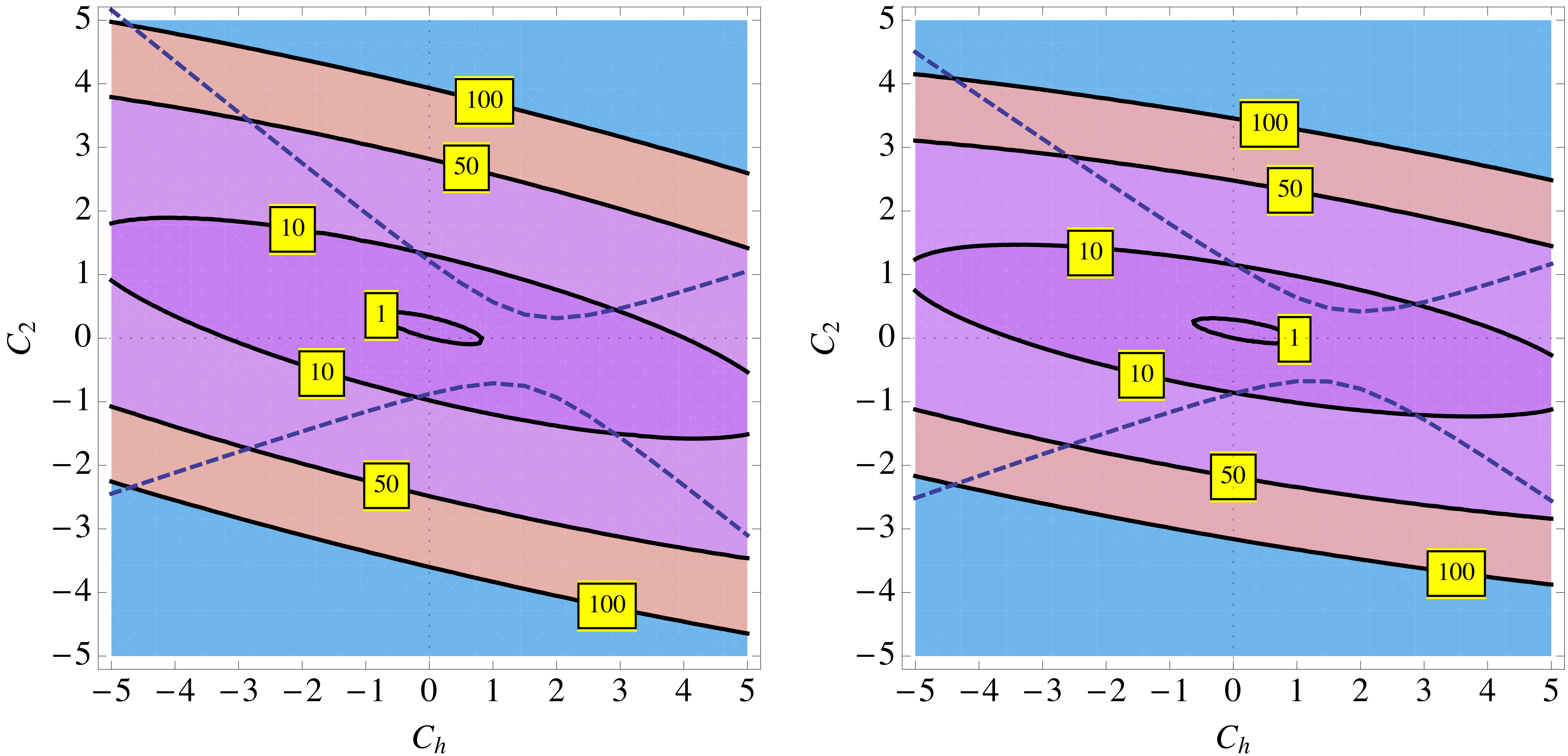
☆ Self-coupling measurement is important to understand how the Higgs field acquires a VEV.

☆ Non-perturbative Higgs model gives a characteristic Higgs potential.

☆ We pointed out that sizable enhancement of Higgs pair production is induced by non-zero C_h and C_2 .

Thank you for your attention

$$\sigma(pp \to hhjj)/\sigma(pp \to hhjj)_{\rm SM}$$

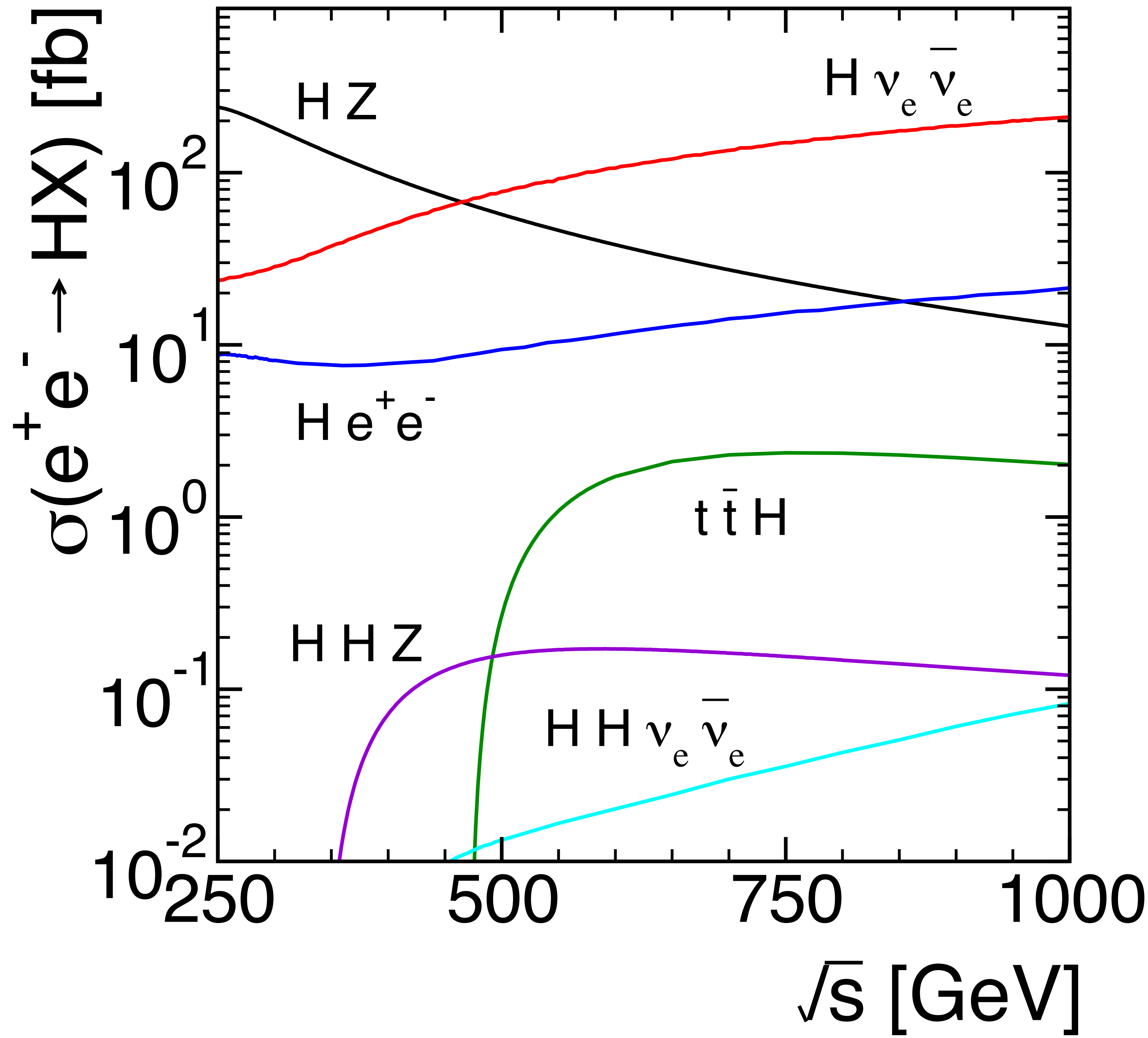


8TeV LHC

14TeV LHC

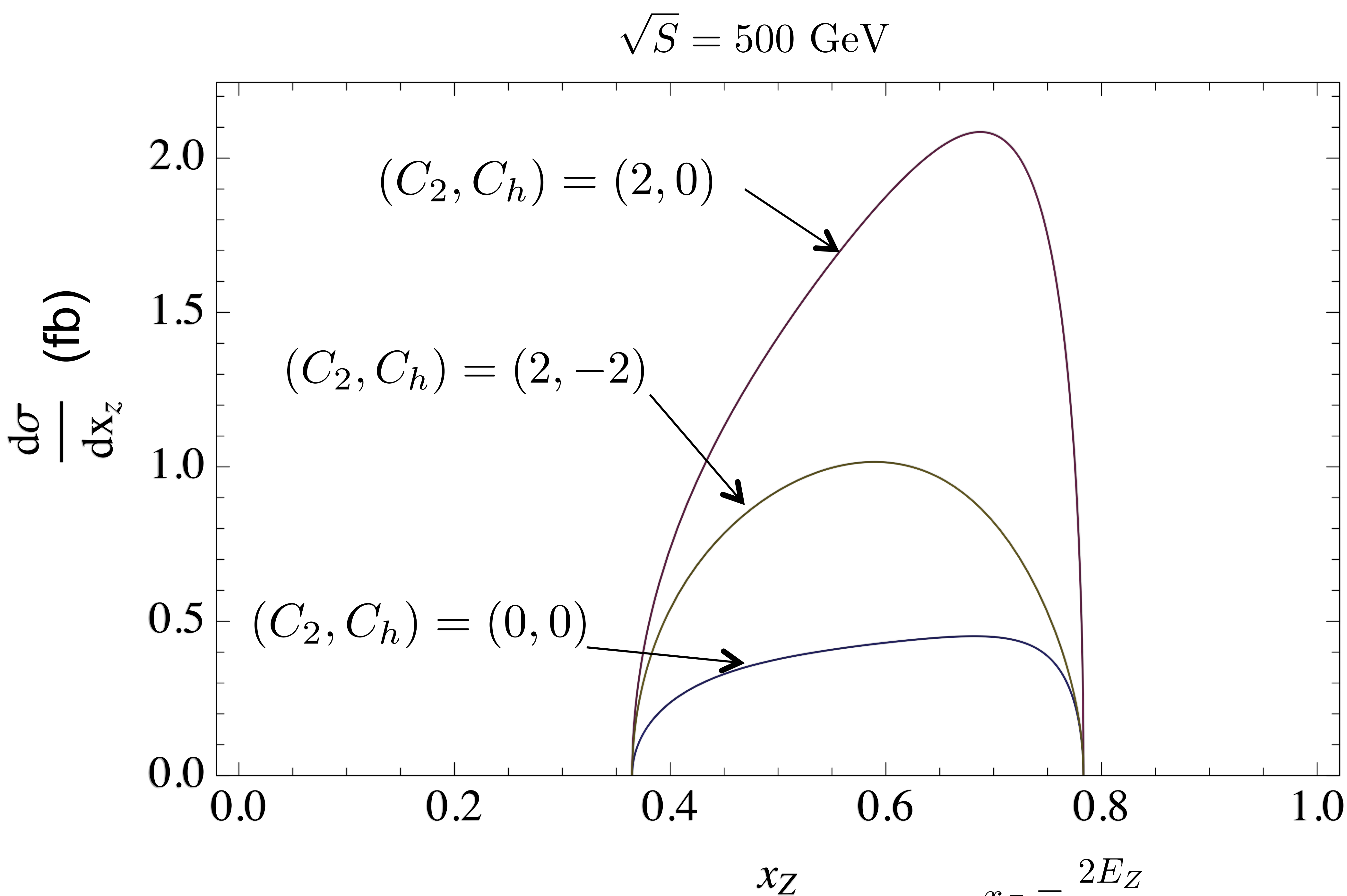
☆ $e^+e^- \to hhX$ @ ***ILC***

(ILC Higgs White Paper)



☆ $e^+e^- \to hhX$ @ ILC

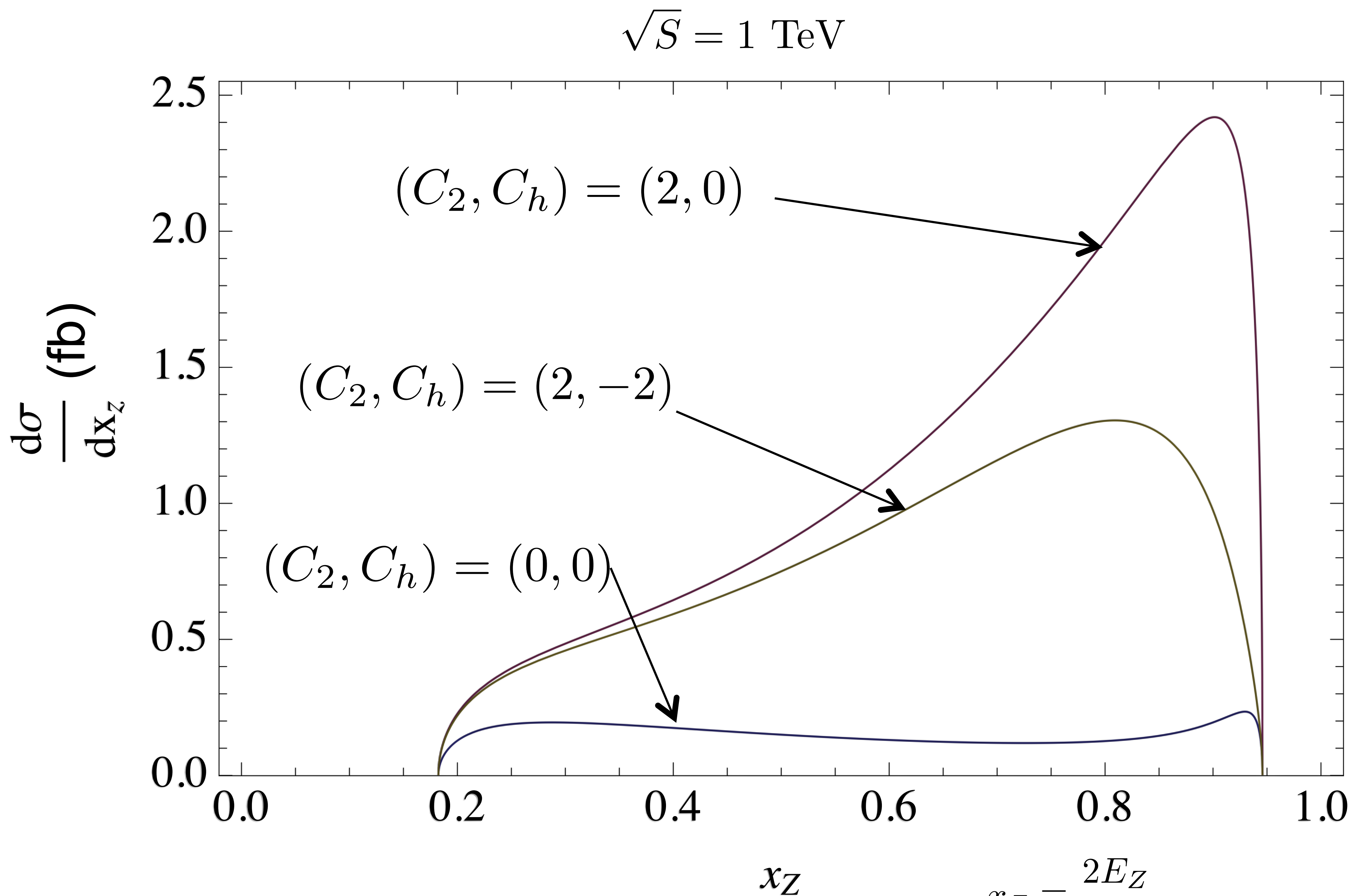
Energy distribution of Z in $e^+e^- \to hhZ$



$x_Z \equiv \frac{2E_Z}{\sqrt{S}}$

☆ $e^+e^- \to hhX$ @ ***ILC***

Energy distribution of Z in $e^+e^- \to hhZ$



$x_Z \equiv \frac{2E_Z}{\sqrt{S}}$