# Full one-loop electroweak radiative corrections to the $t\bar{t}\gamma$ and $e^-e^+\gamma$ production at ILC with GRACE-Loop


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In collaboration with

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# Outline

1. **Introduction**
2. **The GRACE-Loop system**
3. **The recently development of GRACE-Loop**
4. **How to use GRACE-Loop:** $e^+e^- \to t\bar{t}\gamma$ *as a example.*
5. **The physical results of** $t\bar{t}\gamma, e^+e^-\gamma$ **production at ILC.**
6. **Conclusions**

# Introduction

1. **Thank to the achievements of the LHC: the discovery a new boson compatible with a SM Higgs [1,2].**
2. **We expect that the main goals of the ILC program are**
   - precise measurements of Higgs properties: Higgs Boson mass, Spin, CP, Higgs couplings;
   - precise measurements of the interaction of top quarks, gauge bosons, etc;
   - searches for physics beyond the Standard Model (BSM).

**$\Longrightarrow$Electroweak radiative corrections to the processes at $e^-e^+$ collisions will play an important role at the high precision program of ILC.**

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[1] **Physics Letters B 716 (2012) 30-61**
[2] **Phys.Lett. B716 (2012) 1-29**

# The luminosity determination at ILC

1. **At ILC, the integrated luminosity is calculated by**

$$\int \mathcal{L} dt = \frac{N_{obs}}{\sigma_{th}}$$

2. **Electroweak radiative corrections to Bhabha scattering are important.**
3. **One-loop electroweak corrections to $e^+e^- \to e^+e^-$ were finished by many authors**
   - K.Tobimatsu and Y.Shimizu: *Prog. Theor. Phys.* **74** *(1985), 567-575;* **75** *(1986), 905-913.*
   - M. Bohm et al: *Phys. Lett. B* **144** *(1984) 414; Nucl. Phys. B* **304** *(1988) 687.*
   - F. A. Berends et al:*Nucl. Phys. B* **304** *(1988) 712.*
4. **Two-loop QED correction to $e^+e^- \to e^+e^-$:** A.A. Penin: *PhysRevLett.95.010408.*

$\Longrightarrow$ **We present a full $\mathcal{O}(\alpha)$ electroweak radiative corrections to process $e^+e^- \to e^+e^-\gamma$ GRACE-Loop in this talk.**

# Top quark pair production at ILC

1. **The experimental results of CDF and D0 observed a large top quark forward-backward asymmetry.**
2. **In the future, the measurement will be performed at the ILC without QCD background**
3. **The top quark properties (such as top quark mass, $\Gamma_t$, $Y_t$) also will be measured by scanning the top pair threshold at ILC.**
   $\Longrightarrow$ The precise theoretical calculations of $t\bar{t}$ and $t\bar{t}\gamma$ productions at $e^+e^-$ collisions are mandatory.
4. **One-Loop EW corrections to $e^+e^- \to t\bar{t}$ were calculated by**
   - J. Fujimoto et al, Mod. Phys. Lett. 3A, 581 (1988);
   - J. Fleischer et al, Eur. Phys. J. C 31, 37 (2003).

$\Longrightarrow$ **The calculation of $e^+e^- \to t\bar{t}, t\bar{t}\gamma$ with GRACE-Loop is also presented in this talk.**

# GRACE-Loop system

**GRACE-Loop is a generic automated program for calculating High Energy Physics processes** [3].

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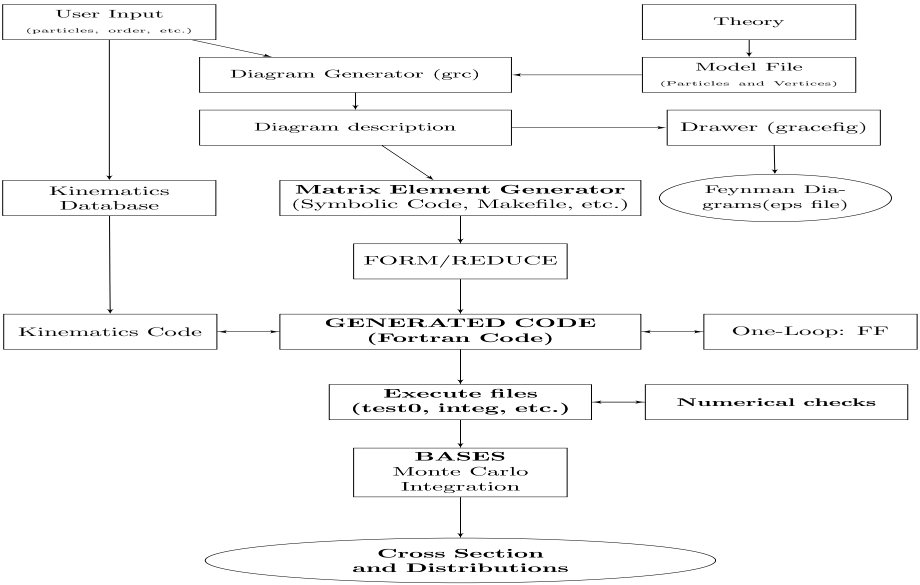
- *All Feynman diagrams for a given process at fixed order of perturbation theory.*
- *A FORM or REDUCE code.*
- *A Fortran code generated for amplitude calculations.*
- *Kinematic library.*
- *The multi-dimensional integration by BASES.*
- *Event generation by SPRING.*

For GRACE system, please visit website:

**http://minami-home.kek.jp/**

[3] **Phys. Rept. 430 (2006) 117**

# The GRACE-Loop system



# The GRACE-Loop system

The non-linear gauge fixing Lagrangian condition[4]

$$\begin{aligned}\mathcal{L}_{GF} &= -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} \\ &+\xi_W\frac{g}{2}(v+\tilde{\delta}H+i\tilde{\kappa}\chi_3)\chi^+|^2 \\ &-\frac{1}{2\xi_Z}(\partial.Z+\xi_Z\frac{g}{2c_W}(v+\tilde{\varepsilon}H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2 .\end{aligned}$$

- $\xi_W = \xi_Z = \xi_A = 1$: 'tHoof-Feynman gauge

$$\frac{1}{k^2 - M_W^2}\left[g_{\mu\nu} - (1-\xi_W)\frac{k^\mu k^\nu}{k^2 - \xi_W^2 M_W^2}\right]$$

- $\hookrightarrow$ the result must be independence of non-linear gauge parameters

[4] Phys. Rept. **430**, 117 (2006)

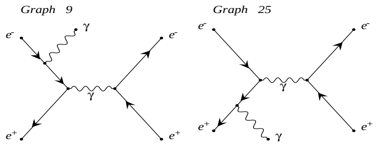
**The GRACE-Loop system has also been used to calculate**

- $2 \to 3$-body processes such as $e^+e^- \to ZHH$, $e^+e^- \to t\bar{t}H$, $e^+e^- \to \nu\bar{\nu}H$, etc.
- $2 \to 4$-body process as $e^+e^- \to \nu_\mu\bar{\nu}_\mu HH$.

**Recently the processes:**

- $e^+e^- \to t\bar{t}\gamma$ (Eur. Phys. J. C **73**, 2400 (2013)).
- $e^+e^- \to e^+e^-\gamma$ at ILC in preparation.
- $pp \to W^+W^- + 1$jet at LHC in progress.

1. **The large numerical cancellation problem.**




$$\sum_{\lambda=0}^{3}\epsilon_\lambda^\mu(q)\epsilon_\lambda^\nu(q) \to -g^{\mu\nu} + \frac{q^\mu n^\nu + q^\nu n^\mu}{n.q} - n^2\frac{q^\mu q^\nu}{(n.q)^2}$$

| Amplitude | Non-Axial Gauge | Axial Gauge |
|---|---|---|
| $\mathcal{M}_1^2 + \mathcal{M}_2^2$ | $0.1116212357 \cdot 10^{+13}$ | $0.3644158264 \cdot 10^{+02}$ |
| $2\mathcal{M}_1^*\mathcal{M}_2$ | $-0.1116212356 \cdot 10^{+13}$ | $0.1546482734 \cdot 10^{+03}$ |
| $\lvert\mathcal{M}_1 + \mathcal{M}_2\rvert^2$ | $0.1910871582 \cdot 10^{+03}$ | $0.1910898560 \cdot 10^{+03}$ |

2. **The Monte-Carlo integration step costs much in CPU time.**

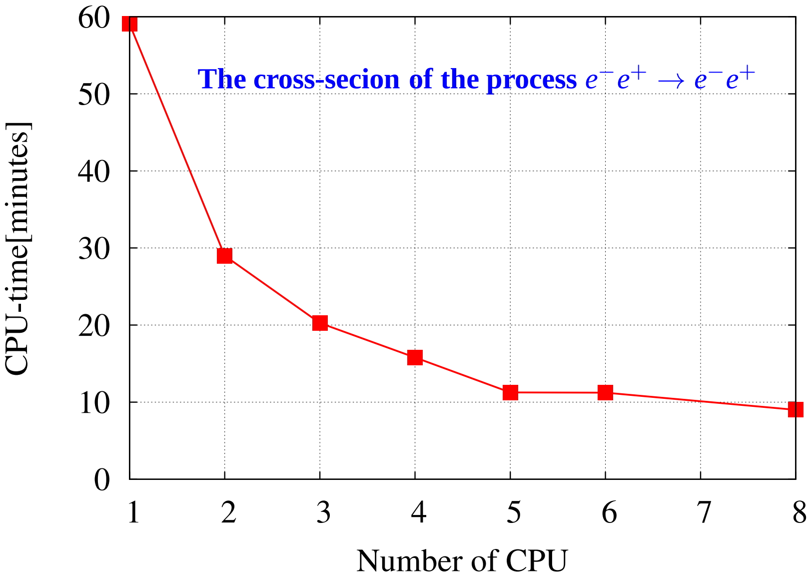
The process: $e^+e^- \to e^+e^-\gamma$

| CPU | Memory | CPU time |
|---|---|---|
| Intel(R) Xeon(R), X5660@2.80GHz | 49 GB | $\geq$3 months @$\sqrt{s}$. |

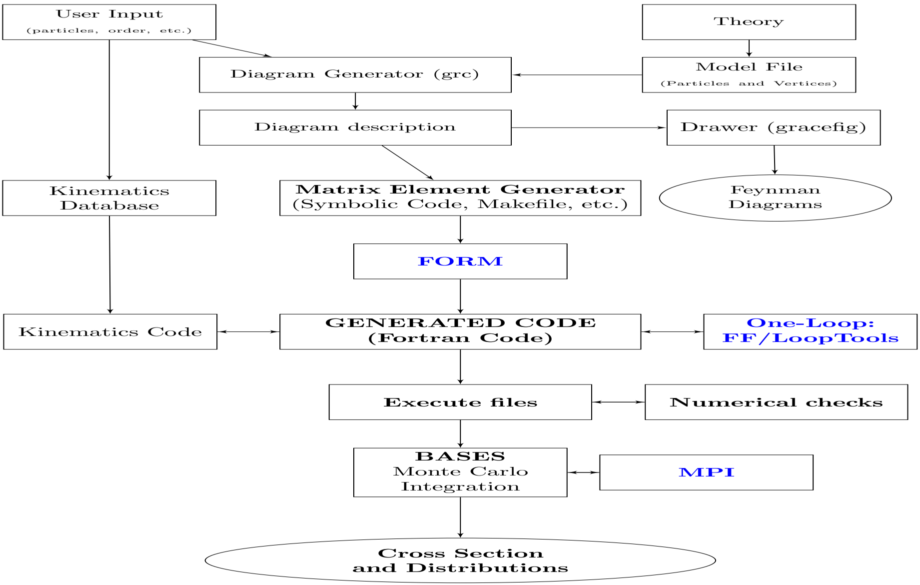
$\Longrightarrow$ **BASES with MPI**[5]

[5]**The Message Passing Interface:** ***http://www.mcs.anl.gov/research/projects/mpi***

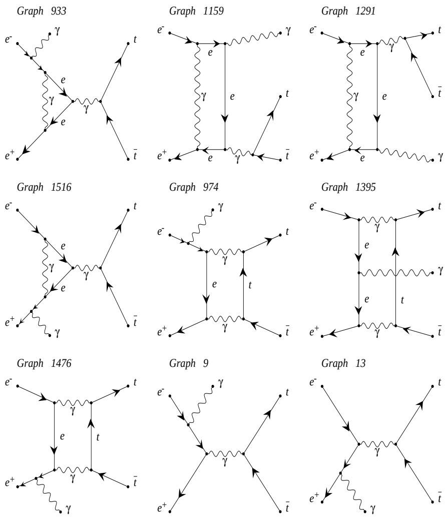
# The recently development of GRACE-Loop



# The recently development of GRACE-Loop



# How to use GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.




*produced by GRACEFIG*

**The process: $e^+e^- \to t\bar{t}\gamma$**

Model = "nlg2301.mdl";

Process;

ELWK = $\{5, 3\}$;

Initial = {electron, positron} ;

Final = {photon, t, t-bar} ;

Expand = Yes;

OPI = No;

Kinem = "2302";

Pend;

- 16 tree diagrams,
- 1814 one-loop diagrams.

# How to use GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.

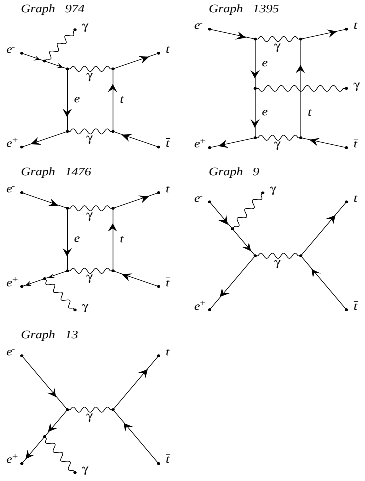
The total cross section is given by

$$\sigma_{tot} = \int_{\Omega_{2\to3}} d\sigma_T^{e^+e^-\to t\bar{t}\gamma} + \int_{\Omega_{2\to3}} d\sigma_V^{e^+e^-\to t\bar{t}\gamma}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda)$$
$$+\delta_{soft}(\lambda, E_\gamma < k_c) \int_{\Omega_{2\to3}} d\sigma_T^{e^+e^-\to t\bar{t}\gamma} + \int_{\Omega_{2\to4}} d\sigma_H^{e^+e^-\to t\bar{t}\gamma_S\gamma_H}(k_c)$$

**The result is**

1. **satisfied the Ward indentity.**
2. $C_{UV}$ **independent.**
3. **photon mass ($\lambda$) independent.**
4. **non linear gauge parameters independent.**
5. $k_c$ **independent.**
6. **in agreement with orther calculation.**

# How to use GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.




produced by GRACEFIG

## 1. Ward identity

| $\sum_{\lambda=0}^{3} \epsilon_\mu^\lambda \epsilon_\nu^\lambda$ | $2\mathcal{R}(\mathcal{M}_{Tree}^{+}\mathcal{M}_{Loop})$ |
|---|---|
| $-q_\mu q_\nu$ | $\mathcal{O}(10^{-35})$ |
| $-g_{\mu\nu}$ | 0.11082450191582567(4) |
| $-g_{\mu\nu} + \frac{q_\mu n_\nu + q_\nu n_\mu}{n.q}$ | 0.11082450191582567(3) |

## 2. Full one-loop diagrams of $e^+e^- \to t\bar{t}\gamma$

| $\sum_{\lambda=0}^{3} \epsilon_\lambda^\mu(q)\epsilon_\lambda^\nu(q)$ | $2\mathcal{R}(\mathcal{M}_{Tree}^{+}\mathcal{M}_{Loop})$ |
|---|---|
| $-q_\mu q_\nu$ | $\mathcal{O}(10^{-27})$ |
| $-g_{\mu\nu} + \frac{q_\mu n_\nu + q_\nu n_\mu}{n.q}$ | -47.189380$\cdots$ |

# How to use GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.

**Test on the calculation**

$$\sigma_{tot} = \int_{\Omega_{2\to3}} d\sigma_T^{e^+e^-\to t\bar{t}\gamma} + \int_{\Omega_{2\to3}} d\sigma_V^{e^+e^-\to t\bar{t}\gamma}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda) + \delta_{soft}(\lambda, E_\gamma < k_c) \int_{\Omega_{2\to3}} d\sigma_T^{e^+e^-\to t\bar{t}\gamma} + \int_{\Omega_{2\to4}} d\sigma_H^{e^+e^-\to t\bar{t}\gamma_S\gamma_H}(k_c)$$

## 1. $C_{UV}$ independent of the amplitude

| $C_{UV}$ | $2\mathcal{R}(\mathcal{M}_{Tree}^{+}\mathcal{M}_{Loop})$ |
|---|---|
| 0 | $-6.7575992336127728658083765531206107 \cdot 10^{-3}$ |
| $10^2$ | $-6.7575992336127728658083765531205867 \cdot 10^{-3}$ |
| $10^4$ | $-6.7575992336127728658083765531189308 \cdot 10^{-3}$ |

The result is stable over 30 digits in quadruple precision.

# How to use GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.

## 2. $\lambda$ independence of the result.

| $\lambda$ [GeV] | $2\mathcal{R}(\mathcal{M}^+_{Tree}\mathcal{M}_{Loop})$+soft contribution |
|---|---|
| $10^{-19}$ | $-1.674389236949202039765435422043876 6\cdot 10^{-3}$ |
| $10^{-21}$ | $-1.6743892369492020382892402083349623\cdot 10^{-3}$ |
| $10^{-23}$ | $-1.6743892369492020382744348901161470\cdot 10^{-3}$ |

The result are stable over 18 digits.

## 3. Gauge invariance of the amplitude check

| $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\epsilon})$ | $2\mathcal{R}(\mathcal{M}^+_{Tree}\mathcal{M}_{Loop})$ |
|---|---|
| $(0,0,0,0,0)\cdot 10^0$ | $-6.7575992336127728658083765531206\cdot 10^{-3}$ |
| $(1,2,3,4,5)\cdot 10^1$ | $-6.7575992336127728658083831456193\cdot 10^{-3}$ |
| $(1,2,3,4,5)\cdot 10^2$ | $-6.7575992336127728658090556378842\cdot 10^{-3}$ |

The result is stable over 21 digits in quadruple precision.

# How to use GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.

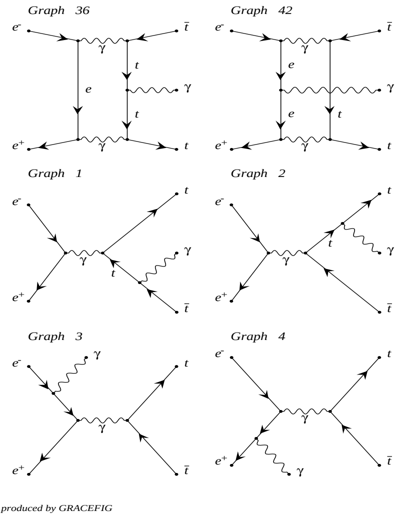
## Test on the calculation

$$\sigma_{tot} = \int_{\Omega_{2\to3}} d\sigma_T^{e^+e^-\to t\bar{t}\gamma} + \int_{\Omega_{2\to3}} d\sigma_V^{e^+e^-\to t\bar{t}\gamma}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda)$$
$$+\delta_{soft}(\lambda, E_\gamma < k_c) \int_{\Omega_{2\to3}} d\sigma_T^{e^+e^-\to t\bar{t}\gamma} + \int_{\Omega_{2\to4}} d\sigma_H^{e^+e^-\to t\bar{t}\gamma_S\gamma_H}(k_c)$$

## 4. $k_c$ independence of the result.

| $k_c$[GeV] | $\sigma_H$ | $\sigma_S$ | $\sigma_{S+H}$ |
|---|---|---|---|
| $10^{-5}$ | $4.17272 \cdot 10^{-2}$ | $5.88546 \cdot 10^{-2}$ | 0.10058 |
| $10^{-3}$ | $2.92668 \cdot 10^{-2}$ | $7.13173 \cdot 10^{-2}$ | 0.10058 |
| $10^{-1}$ | $1.67899 \cdot 10^{-2}$ | $8.37731 \cdot 10^{-2}$ | 0.10056 |

# How to use GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.




Ref [1]: A. Denner and S. Dittmaier,
Nucl. Phys. B **734** (2006) 62.

| | $2\mathcal{R}(\mathcal{M}^+_{Tree}\mathcal{M}_{Loop})$ |
|---|---|
| GRACE | $-0.1203895107415245457335$ |
| Ref [1] | $-0.1203895107415245457333$ |

| | $\sigma_{2\mathcal{R}(\mathcal{M}^+_{Tree}\mathcal{M}_{Loop})}$ |
|---|---|
| GRACE | $-1.273651(\pm 0.001596)\cdot 10^{-02}$ |
| Ref [1] | $-1.273651(\pm 0.001596)\cdot 10^{-02}$ |

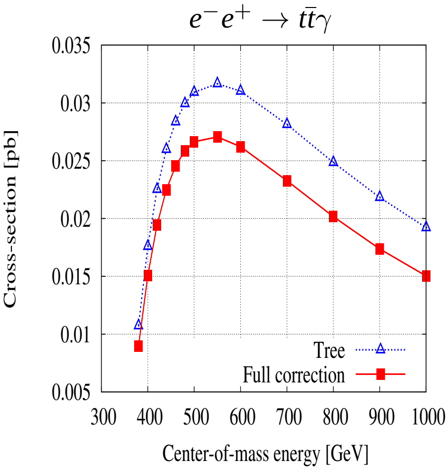
# The physical results of the process $e^+e^- \to t\bar{t}\gamma$

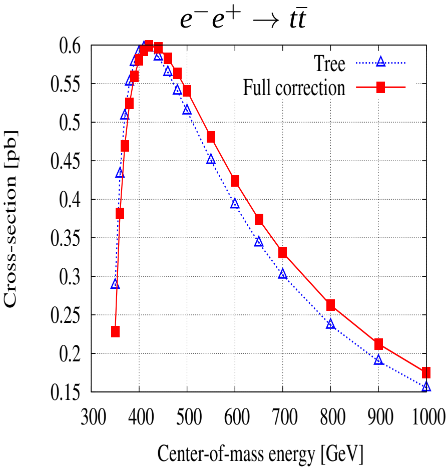
Our input parameters for the calculation are as follows.

| | | |
|---|---|---|
| $\alpha = 1/137.0359895$ | $G_\mu = 1.16639 \cdot 10^{-5}\ \mathrm{GeV}^{-2}$ | $M_Z = 91.187$ GeV |
| $M_W = 80.3759$ GeV | $M_H = 120$ GeV | $m_e = 0.51099$ MeV |
| $m_\tau = 1776.82$ MeV | $m_\mu = 105.6583$ MeV | $m_u = 1.7$ MeV |
| $m_d = 4.1$ MeV | $m_c = 1.27$ GeV | $m_s = 101$ MeV |
| $m_b = 4.19$ GeV | $m_t = 172.0$ GeV | |

We apply an energy cut of $E_\gamma^{cut} \geq 10$ GeV and an angle cut of $10^\circ \leq \theta_\gamma^{cut} \leq 170^\circ$ on the photon.

# The physical results of the process $e^+e^- \to t\bar{t}\gamma$




$E_\gamma \geq 10GeV, 0^0 \leq \theta_\gamma \leq 170^0.$

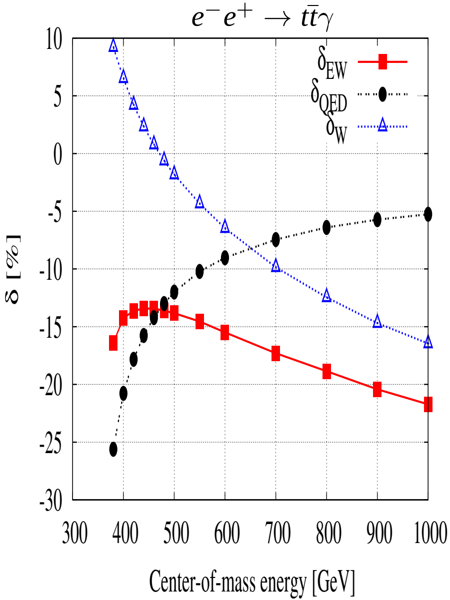
$P.H.Khiem, Y.Kurihara, et\ al,$

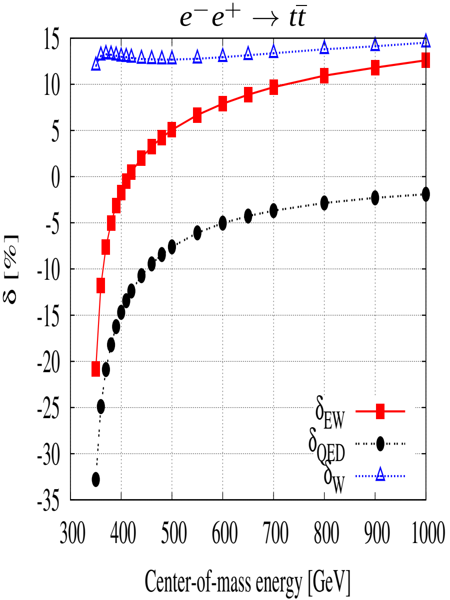
$Eur.\ Phys.\ J.\ C\mathbf{73}, 2400(2013)$

$Mod.\ Phys.\ Lett.\ \mathbf{3A}, 581(1988);$

$Eur.\ Phys.\ J.\ C\mathbf{31}, 37(2003).$

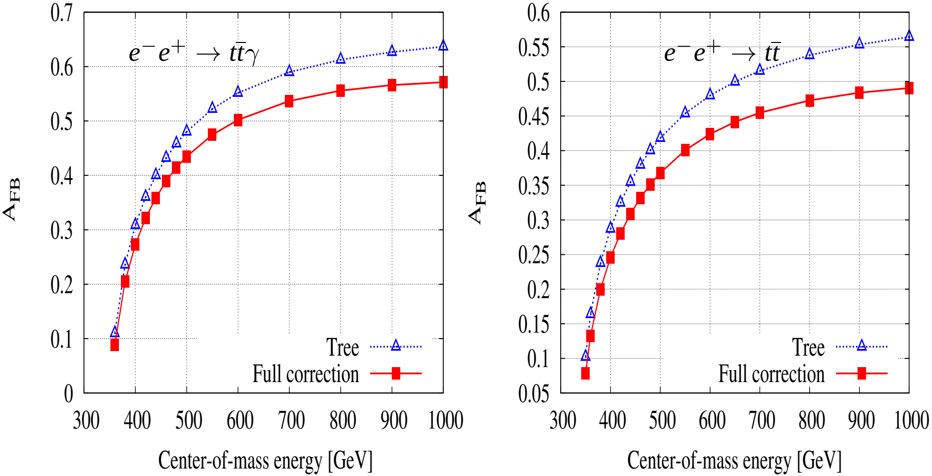
# The physical results of the process $e^+e^- \to t\bar{t}\gamma$



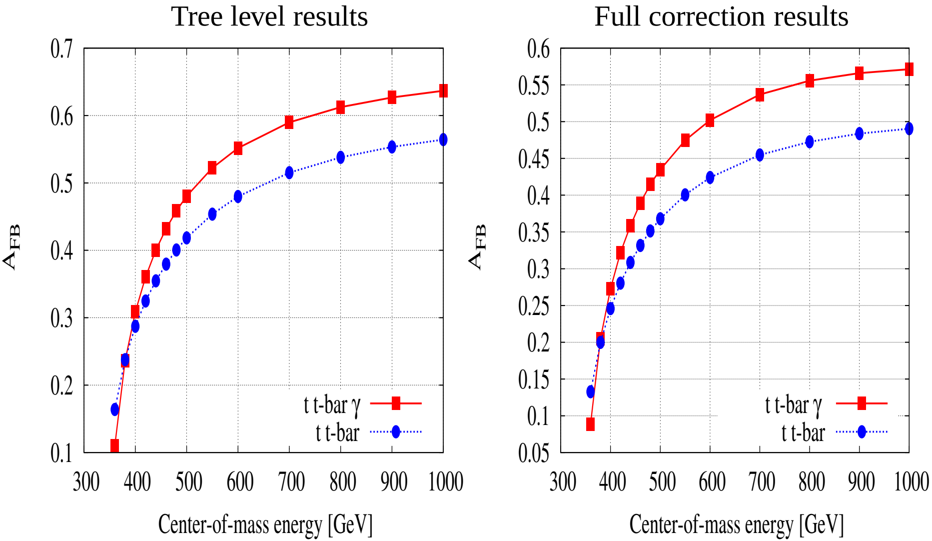
# The physical results of the process $e^+e^- \to t\bar{t}\gamma$

$$A_{FB} = \frac{\sigma(0^0 \le \theta_t \le 90^0) - \sigma(90^0 \le \theta_t \le 180^0)}{\sigma(0^0 \le \theta_t \le 90^0) + \sigma(90^0 \le \theta_t \le 180^0)}$$

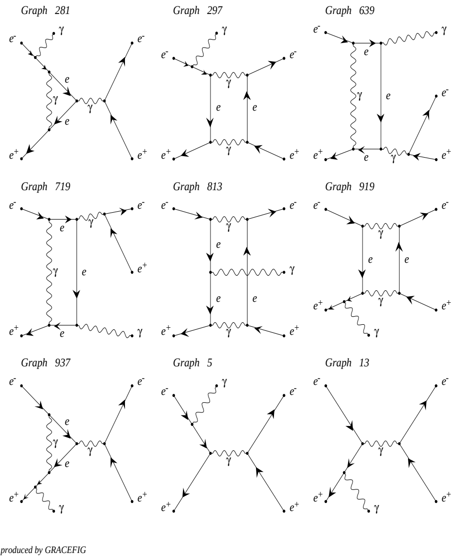


# The physical results of the process $e^+e^- \to t\bar{t}\gamma$

$A_{FB}$ in $t\bar{t}\gamma$ and $t\bar{t}$ production



# The process $e^+e^- \to e^+e^-\gamma$ with GRACE-Loop.




**The process: $e^+e^- \to e^+e^-\gamma$**

Model = ”nlg2301.mdl”;
Process;
ELWK = $\{5, 3\}$;
Initial = {electron, positron} ;
Final = {photon, electron, positron}
Expand = Yes;
OPI = No;
Kinem = ”2302”;
Pend;

- 32 tree diagrams,
- 3456 one-loop diagrams.

# The physical results of process $e^+e^- \to e^+e^-\gamma$

Our input parameters for the calculation are as follows.

| | | |
|---|---|---|
| $\alpha = 1/137.0359895$ | $G_\mu = 1.16639 \cdot 10^{-5}\ \mathrm{GeV}^{-2}$ | $M_Z = 91.187$ GeV |
| $M_W = 80.3759$ GeV | $M_H = 125$ GeV | $m_e = 0.51099$ MeV |
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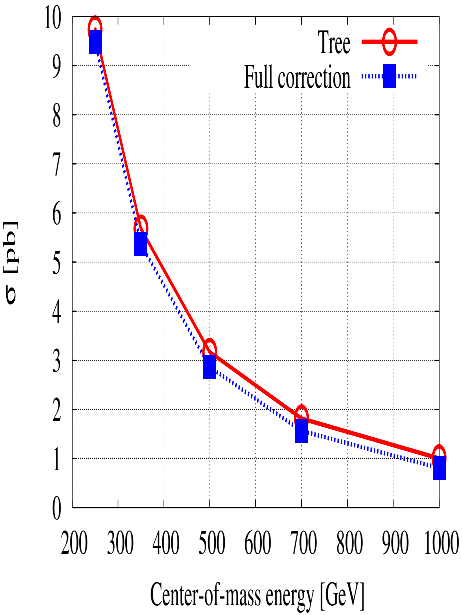
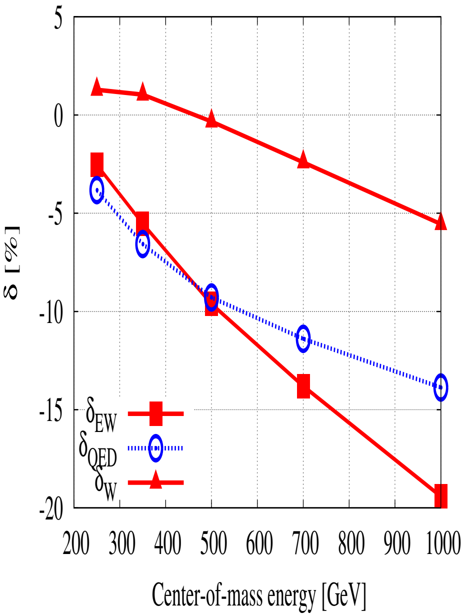
| $m_d = 4.1$ MeV | $m_c = 1.27$ GeV | $m_s = 101$ MeV |
| $m_b = 4.19$ GeV | $m_t = 172.0$ GeV | |

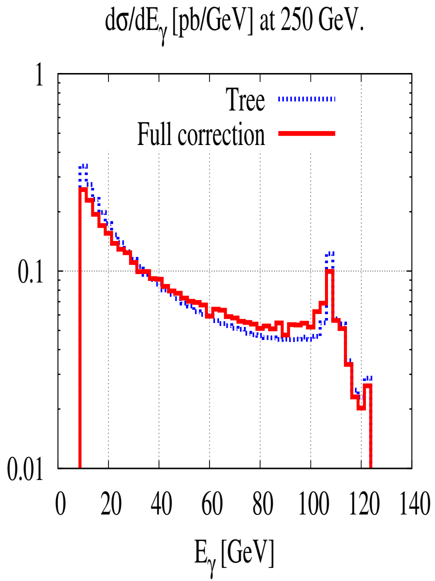
- $E^{\mathbf{cut}}_{\gamma,e^+,e^-} \geq 10$ GeV, $\theta^{\mathbf{cut}}_{\gamma,e^+,e^-} = 10^\circ$;
- $\theta^{\mathbf{cut}}_{\gamma/\{e^+,e^-\}} = 10^\circ$, $\theta^{\mathbf{cut}}_{e^+/e^-} = 10^\circ$.

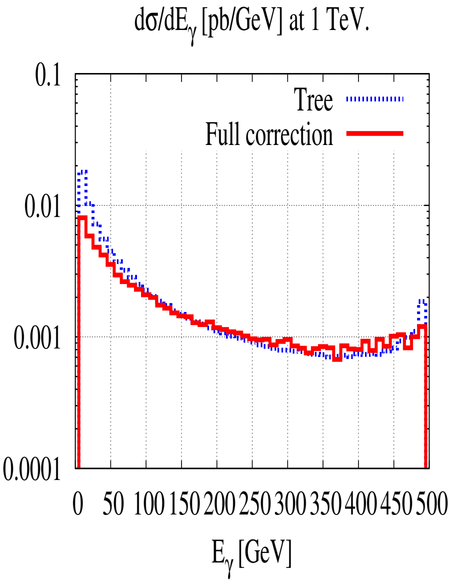
# The physical results of process $e^+e^- \to e^+e^-\gamma$



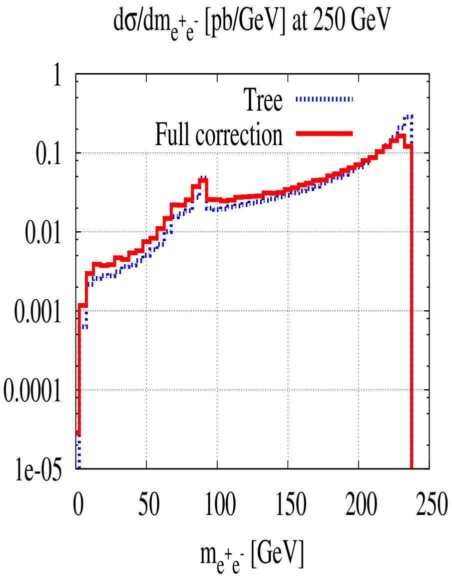



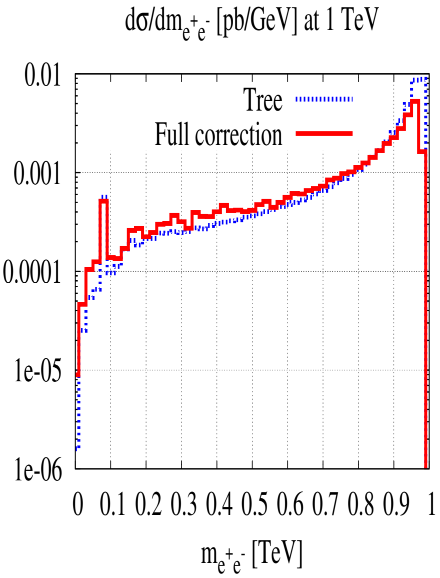
# The physical results of process $e^+e^- \to e^+e^-\gamma$



# The physical results of process $e^+e^- \to e^+e^-\gamma$



# Conclusions

- We introduced to the GRACE-Loop system which is a generic automated program for calculating High Energy Physics processes.
- The full $\mathcal{O}(\alpha)$ electroweak radiative corrections $e^+e^- \to t\bar{t}\gamma, e^+e^-\gamma$ at ILC were calculated sucessfully with GRACE-Loop.

# Conclusions

**The physical results of the process $e^+e^- \to t\bar{t}\gamma$**

- We find that the numerical value of the weak corrections varies from $10\%$ to $-16\%$ in the range of center-of-mass energy from 360 GeV to 1TeV.
- We also obtain a large value for the top quark forward-backward asymmetry in the $t\bar{t}\gamma$ process as compared with the one in $t\bar{t}$ production.

# Conclusions

**The physical results of the process $e^+e^- \to e^+e^-\gamma$**

- We find that the numerical value of the full electroweak radiative corrections varies from $-2\%$ to $-20\%$ in the range of center-of-mass energy from 250 GeV to 1TeV.
- This contribution is sizable. The full electroweak correction to the process play important role for the determination luminosity at ILC in the future.

**Thank you very much for your attention!**