# Top threshold at ILC

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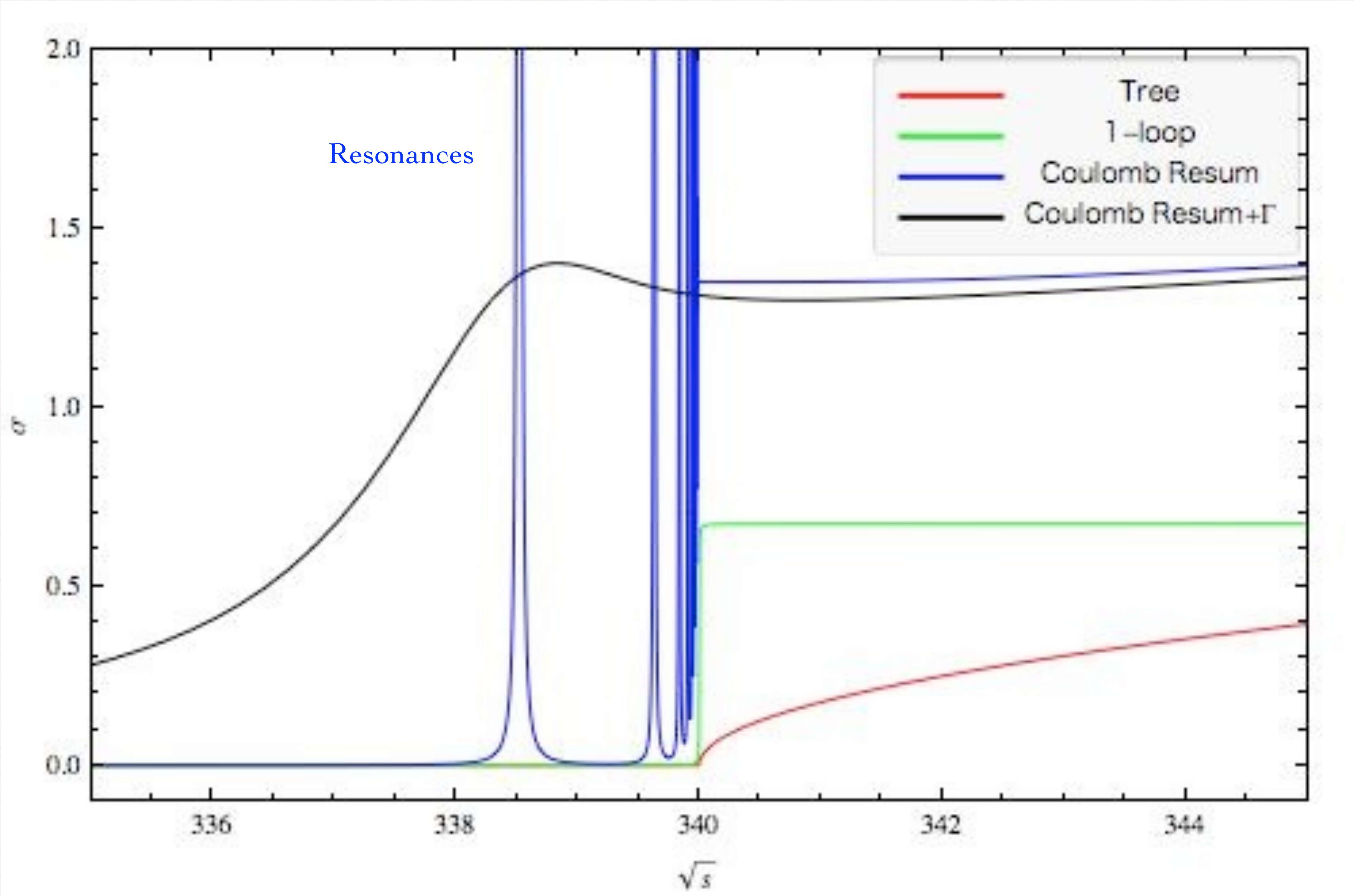
TTbarXSection: Bneke,YK, Schuller :
hep-ph.0501289, 0705.4518[hep-ph], 0801.3464[hep-ph], in preparation
Ultrasoft correction: Beneke, YK, Penin:
0804.4004[hep-ph], 0706.2733[hep-ph]

# $e^+e^- \rightarrow t\bar{t}$ near threshold



# Top threshold

- $m_t$ and $y_t$ measurement is important to test SM/BSM
- $(\Delta m_t)_{exp}$ <= 50MeV→ theory goal $\delta\sigma$<=3%
- NNLO result @2001

Scale uncertainty about 20 %

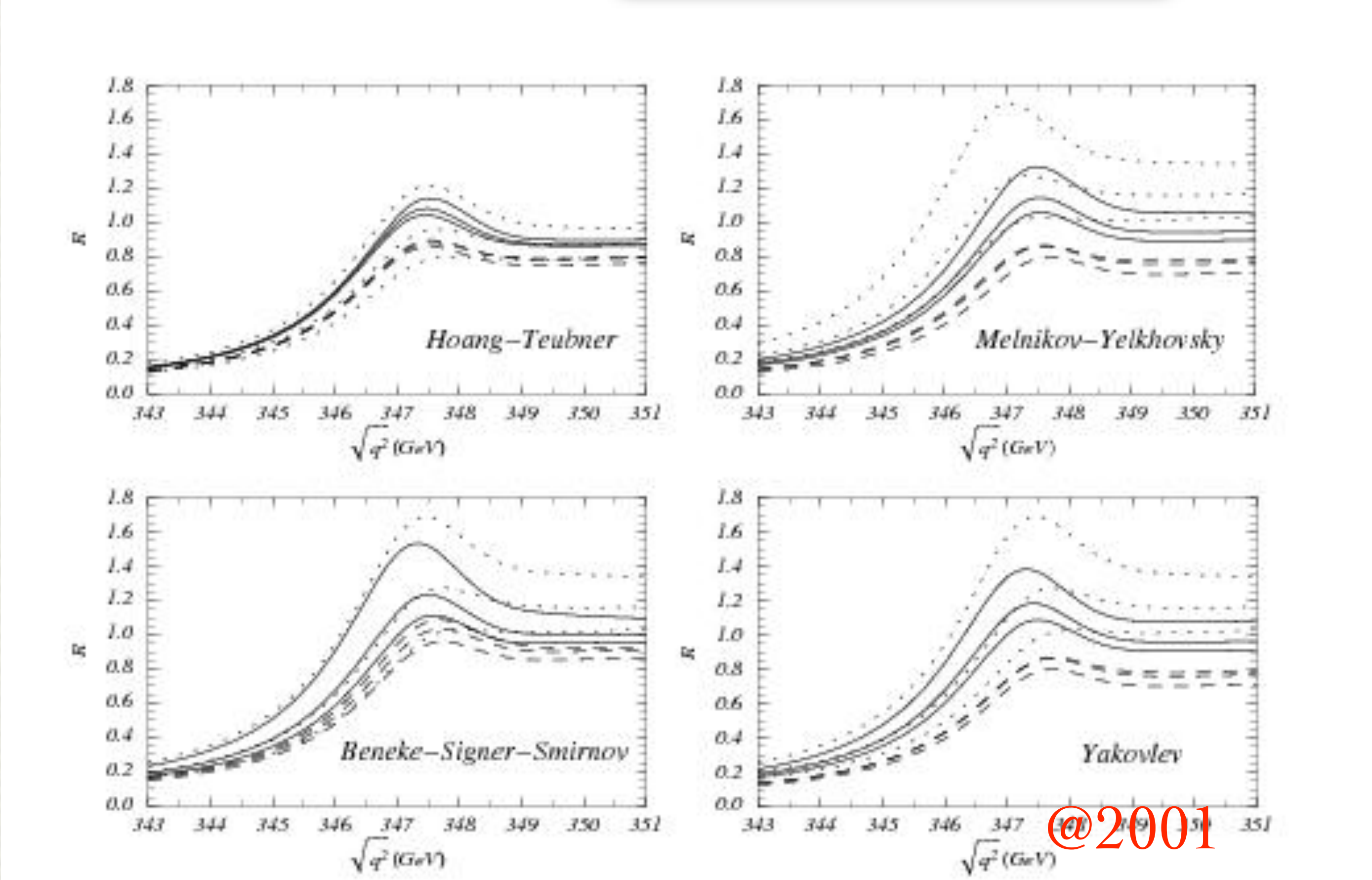



Figure 1: The total normalised photon-induced $t\bar{t}$ cross section

- We will advance the theory calculation to NNNLO

# Effective Field Theory

- Integrate out Hard (Caswell-Lepage('86))

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left( iD_0 + \frac{\vec{D}^2}{2m_t} \right) \psi + [\psi \to \chi] + \cdots$$

- Integrate out Soft/Potentia gluons

(Pineda-Soto('98); Luke-Manohar-Rothstein('99))

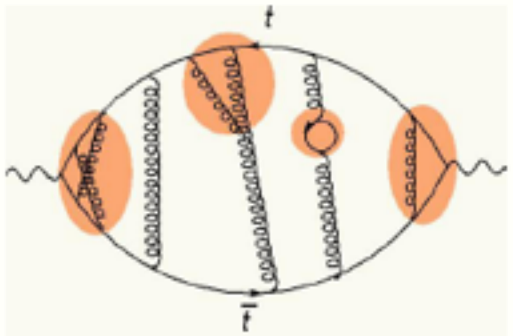
$$\mathcal{L}_{\mathrm{pNRQCD}} = \psi^\dagger \left( i\partial_0 + \frac{\vec{\nabla}^2}{2m_t} \right) \psi + \int d\vec{r} \, [\psi^\dagger \chi] \, V_{pot}(r) \, [\chi^\dagger \psi]$$

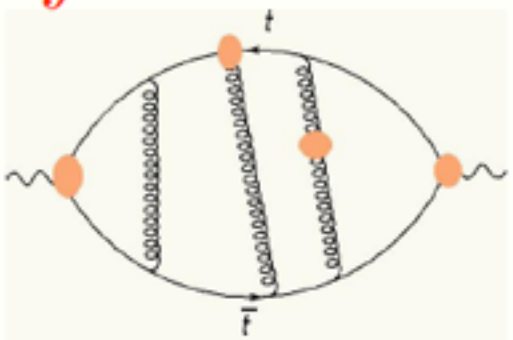
$$+ig \, \psi^\dagger [A_{0,us} + \frac{\nabla \vec{A}_{us}}{m}] \psi - \frac{1}{4} F_{us}^2 + \cdots$$

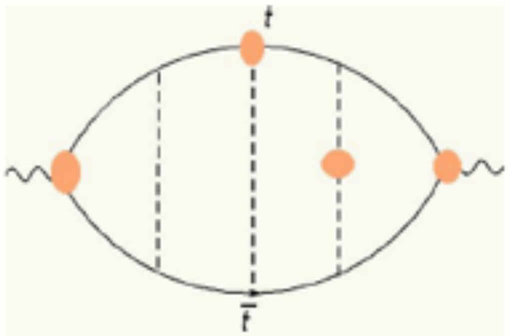
- Remaining Mode is Ultra Soft gluon: $k \sim m(v^2, \vec{v}^2)$

# Threshold XS in EFT

Principal quantity is $\Pi(q) = i\int d^4x e^{iqx}\langle 0|J^\mu(x)J_\mu(0)|0\rangle$




- Integrating hard mode $\longrightarrow$ NRQCD (Caswel-Lepage '86):

  $J^i(x) = \left[\bar{t}\gamma^i t\right] \quad \rightarrow \quad c_v\left[\psi^\dagger\sigma^i\chi\right]$

- Integrating soft/potential modes $\longrightarrow$ PNRQCD

  (Pineda-Soto '97/Luke - Manohar-Rothstein'99):

  $$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger\left[i\partial_0 + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} - g_s\mathbf{x}\mathbf{E}(t,\mathbf{0})\right]\psi + (\psi\leftrightarrow\chi)$$
  $$+\int d\mathbf{r}\left[\psi^\dagger\psi\right](x+\mathbf{r})\,V_{pot}(\mathbf{r})\left[\chi^\dagger\chi\right](x) + \cdots$$

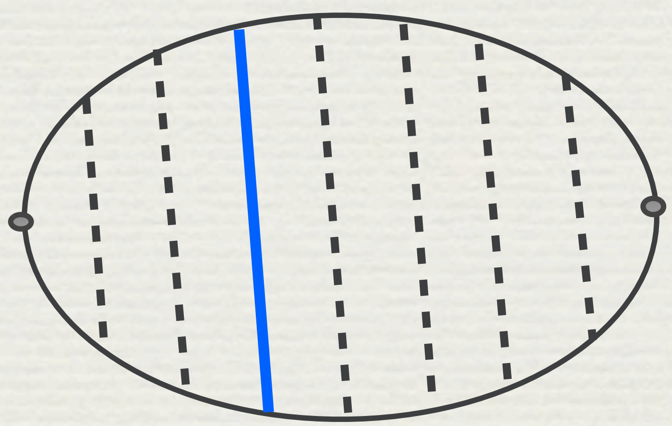
$$\Pi(q) = i\int d^4x e^{iEx}\, c_v^2\,\langle 0|\left[\psi^\dagger\sigma^i\chi\right](x)\ \left[\chi^\dagger\sigma_i\psi\right](0)|0\rangle$$

# Potential Perturbation

$$V_{pot} = -\frac{C_F \alpha_s}{r} + \frac{C_2}{r^2} + C_3 \delta(\mathbf{r}) + \cdots$$

- Higher order corr to the potential (case of Coulomb pot)

$$\tilde{V}_C = -\frac{4\pi C_F \alpha_s(\mathbf{q})}{\mathbf{q}^2} \times \left[1 + \frac{\alpha_s(\mathbf{q})}{4\pi} a_1 + \left(\frac{\alpha_s(\mathbf{q})}{4\pi}\right)^2 a_2 + \left(\frac{\alpha_s(\mathbf{q})}{4\pi}\right)^3 \left[a_3 + 8\pi^2 C_A^3 \left(\frac{1}{3\epsilon} + \ln \frac{\mu_{US}^2}{\mathbf{q}^2}\right)\right]\right]$$

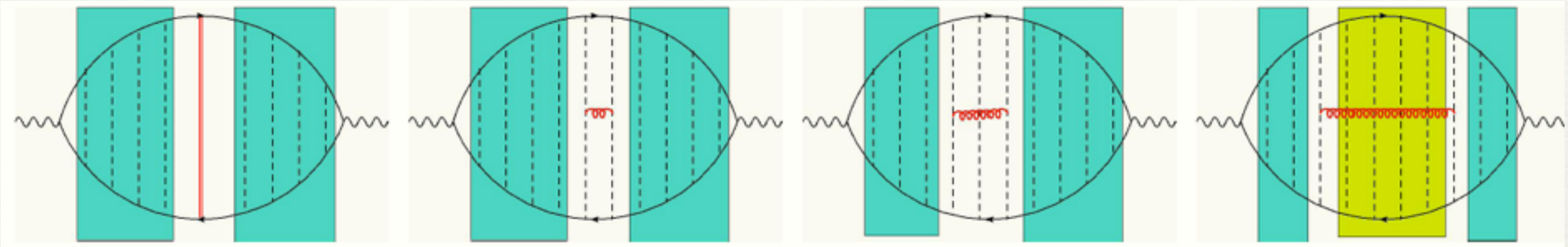


All order Coulomb ladder

We treat higher order potentials as perturbation to the all order resummation of LO Coulomb exchage.
→ We call Potetial Insertion

Single nsertion of a1, a2, a3 is NLO, NNLO and NNNLO, respectively

# Ultrasoft Correction

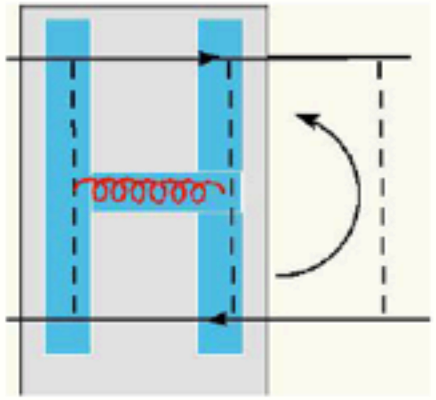


- quark-gluon vertex is $1/m$ suppressed; $\psi^\dagger(iD_0 + \frac{D^2}{2m})\psi$
- $n_g$, number of potential exchange $\sim \Delta t$; $n_g > 1 \Leftrightarrow$ UV finite
- ADM $1/\epsilon$ of the Coulomb pot is a counter term for us corr

$$\delta V_C^{\mathrm{ADM}} = \frac{1}{\epsilon}\frac{C_A^3 \alpha_s^4}{\mathbf{q}^2}$$

$$\mathrm{H}_0 = -\frac{\alpha_s^3 C_A^2}{\epsilon}\left(\frac{1}{mq} + \frac{2(p^2/m - E)}{\mathbf{q}^3} + \epsilon L_{\mathrm{Bethe}}\right) + O(\epsilon)$$

$$\mathrm{H}_1 = -\frac{1}{\epsilon}\frac{C_A^2(C_A - 2C_F)\alpha_s^4}{\mathbf{q}^2} + L_{\mathrm{Bethe}} + O(\epsilon)$$



- UV cancelation happens tricky way:
  $\frac{(p^2/m - E)}{\mathbf{q}^3} \Rightarrow \frac{C_F \alpha_s}{\mathbf{q}^2}$ (Eq. of motion)

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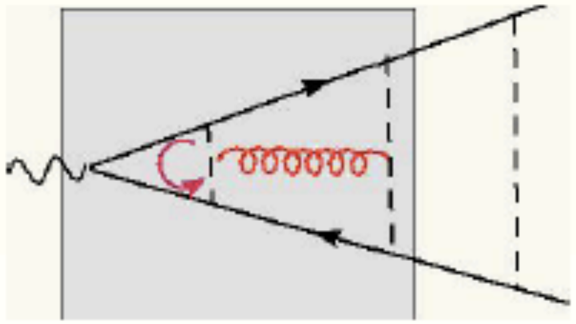
# US renormalization II: Vertex

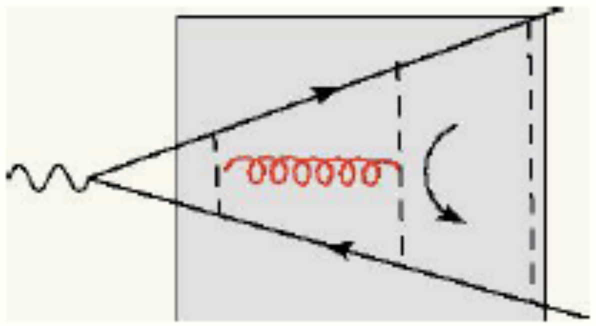
$$\delta V_C^{\rm ADM} = \frac{1}{\epsilon}\frac{C_A^3\alpha_s^4}{\mathbf{q}^2}$$

$$\mathrm{H}_0 =$$

$$-\frac{\alpha_s^3 C_A^2}{\epsilon}\left(\frac{1}{mq} + \frac{2(p^2/m-E)}{\mathbf{q}^3} + \epsilon L_{\rm Bethe}\right) + O(\epsilon)$$

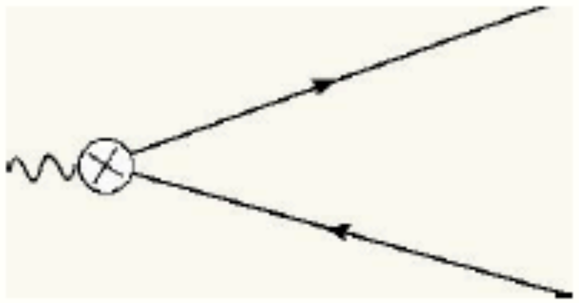
$$\mathrm{H}_1 = -\frac{1}{\epsilon}\frac{C_A^2(C_A-2C_F)\alpha_s^4}{\mathbf{q}^2} + L_{\rm Bethe} + O(\epsilon)$$





- Loop near photon vertices are more singular
  $\Leftrightarrow$ Vertex Renormalization
- $1/\epsilon$ cancelation does not happen exactly anymore
  - due to vertex divergence, (3+1) dimensional Eq. of motion is invalid
  - diagrams with different loop order get mis-match of Eq. of motion

Needs external current renormalization



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- all the logarithmic parts obtained analytically
- there are uncanceled iΓ/ε, which in turn produces scale dependence -> unstable top EFT

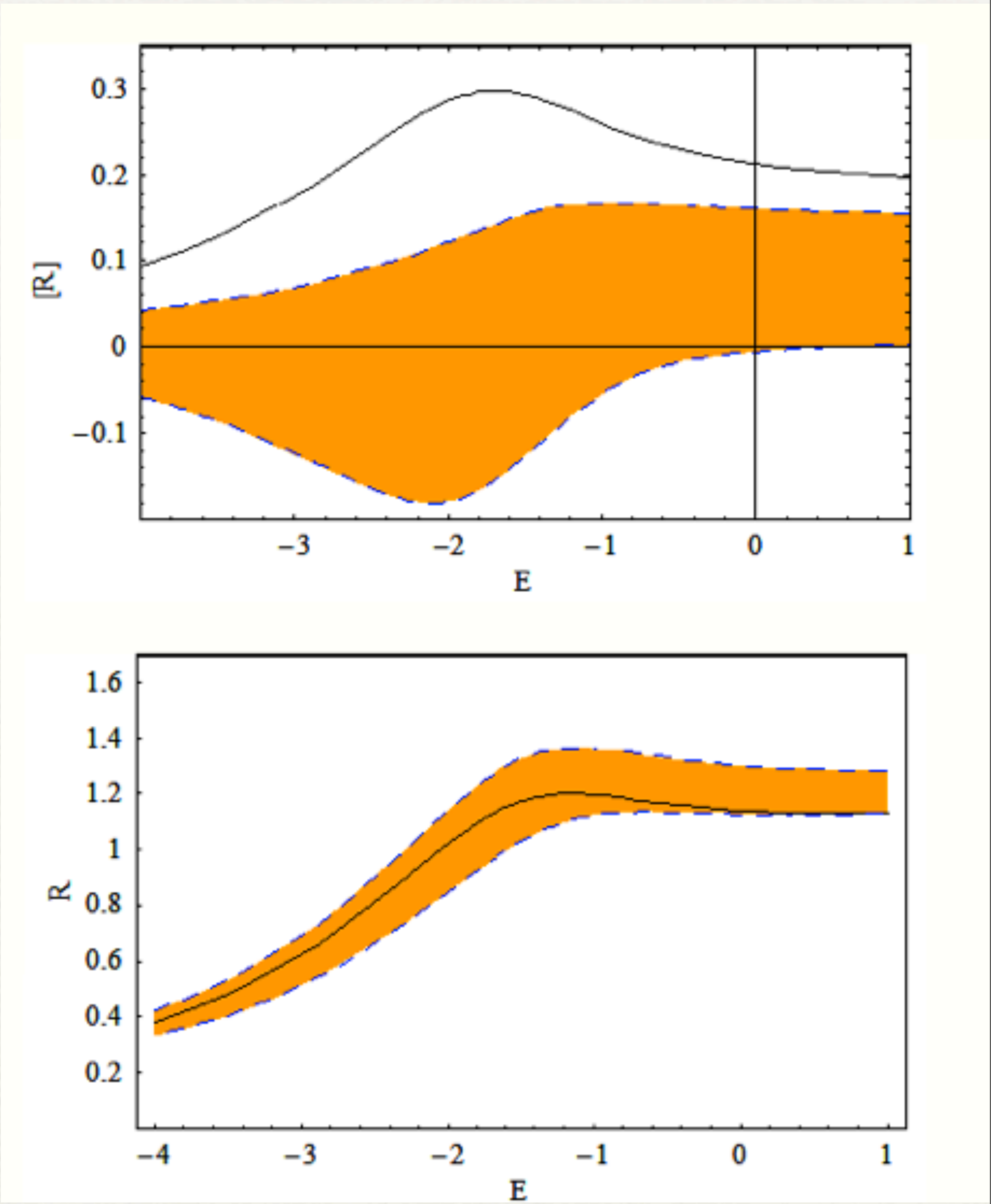
$$\delta^{us}G(E) = \frac{2m^2\alpha_s^4}{9\pi^2}\Bigg\{\left[\frac{17\,i\hat{\Gamma}_t}{24}+\frac{527\,\hat{G}_C}{72}\right]\frac{1}{\epsilon^2}+\left[\frac{17\,i\hat{\Gamma}_t}{12}+\frac{221\,\hat{G}_C}{36}\right]\frac{L_\mu}{\epsilon}+\left[\left(\frac{19}{12}\ln 2-\frac{91}{72}\right)i\hat{\Gamma}_t\right.$$
$$\left.+\left(-\frac{119}{12}\ln 2+\frac{2059}{108}\right)\hat{G}_C\right]\frac{1}{\epsilon}+\left[-\frac{34\,i\hat{\Gamma}_t}{3}-\frac{595\,\hat{G}_C}{9}\right]L_{\alpha_s}^2+\left[-\frac{17\,i\hat{\Gamma}_t}{12}-\frac{833\,\hat{G}_C}{36}\right]L_\mu^2$$
$$+\left[\frac{34\,i\hat{\Gamma}_t}{3}+\frac{748\,\hat{G}_C}{9}\right]L_{\alpha_s}L_\mu+\left[\frac{2380\,\mathcal{P}^2}{27}+\left(\frac{272\ln 2}{9}-\frac{23483}{162}+\frac{2380}{27\lambda}+\frac{272}{27\lambda^2}\right)\mathcal{P}\right.$$
$$\left.+\left(\frac{27\lambda}{2}-\frac{16}{3\lambda}\right)\psi'+\frac{64}{27\lambda^3}+\frac{4\,(-1331+306\ln 2)}{81\lambda}+\frac{4\,(-199+114\ln 2)}{81\lambda^2}\right]L_{\alpha_s}$$
$$+\left[-\frac{1496\,\mathcal{P}^2}{27}+\left(-\frac{34\ln 2}{3}+\frac{5065}{72}-\frac{1496}{27\lambda}-\frac{136}{27\lambda^2}\right)\mathcal{P}+\left(\frac{8}{3\lambda}-\frac{81\lambda}{8}\right)\psi'\right.$$
$$\left.-\frac{32}{27\lambda^3}+\frac{163-114\ln 2}{27\lambda^2}+\frac{271-51\ln 2}{9\lambda}\right]L_\mu+\delta^{us}(\hat{E})\Bigg\},$$

$$L_\mu=\ln\frac{\mu}{m_t},\quad L_{\alpha_s}=\ln\alpha_s,\quad \lambda=\frac{C_F}{2\sqrt{-\hat{E}}},\quad \mathcal{P}=\ln\left(\frac{C_F}{\lambda}\right)+\gamma_E+\psi(1-\lambda),$$

Beneke-YK 08

# US corr

- Ultrasoft correction alone; constant(solid), log +const. orange band
- $R_{LO}+R_{us}$




Ultrasoft correction is scale dependent and not physical

# Bulding blocks

$$J^i = c_v \psi^\dagger \sigma^i \chi + d_v \frac{1}{6m_t^2} \psi^\dagger \sigma^i \mathbf{D}^2 \chi$$

$c^{(2)}$: Beneke-Signer-Smirnov('97),
Czanecki-Melnikov('97)

$c^{(3)}_{n_f}$: Marquard-Piclum- Seidel-Steinhauser(06)

$d_v^{(1)}$: Luke-Savage('97)

$$\mathcal{L}_{\rm QCD} \Leftrightarrow \mathcal{L}_{\rm PNRQCD}$$

$a_2$: Schröder('98)

$a_{3,\mathrm{pade}}$: Chishtie-Elias (01)
(New: $a_{3,\,n_f}$
Smirnov-Smirnov-Steinhauser (Sep.08))

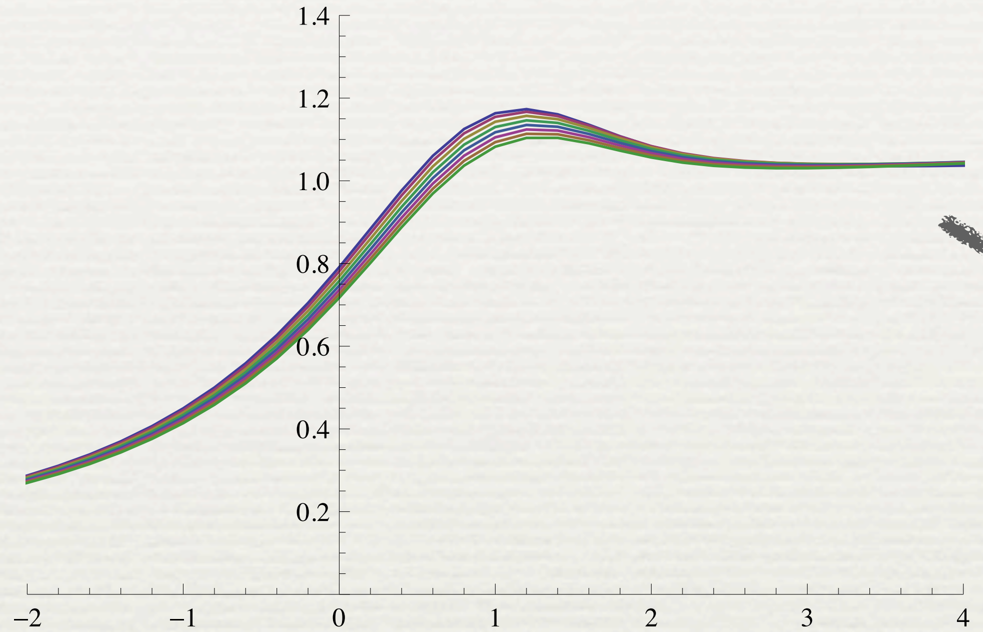
Anzai-YK-Sumino, Smirnov-Smirnov-Steinhauser (2010)

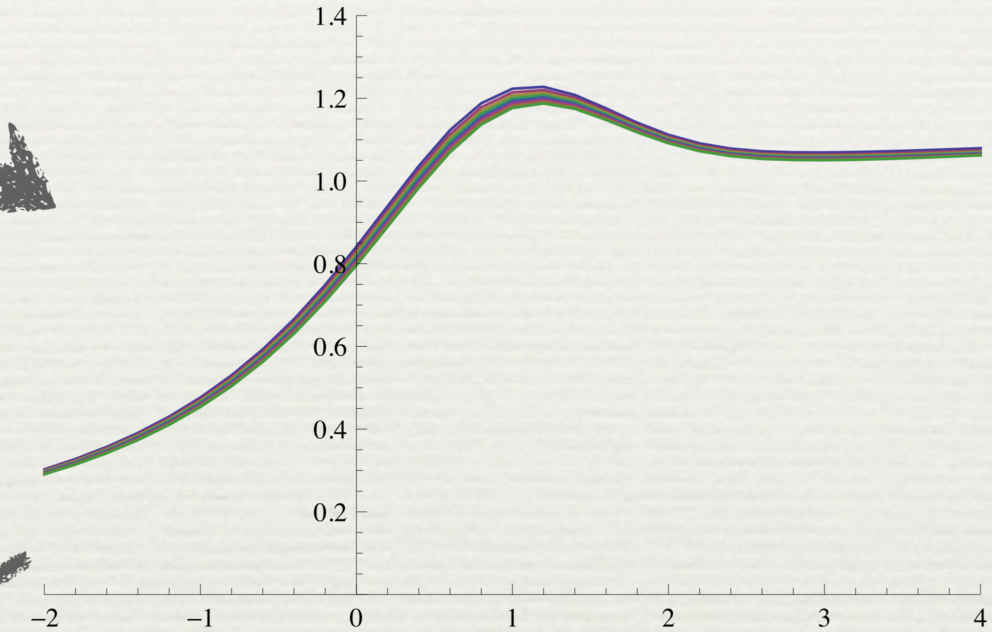
$\delta\mathcal{L}^{(1)}$: Manohar('97),
Beneke-Signer-Smirnov('99),
Wüster-Schuller('03)

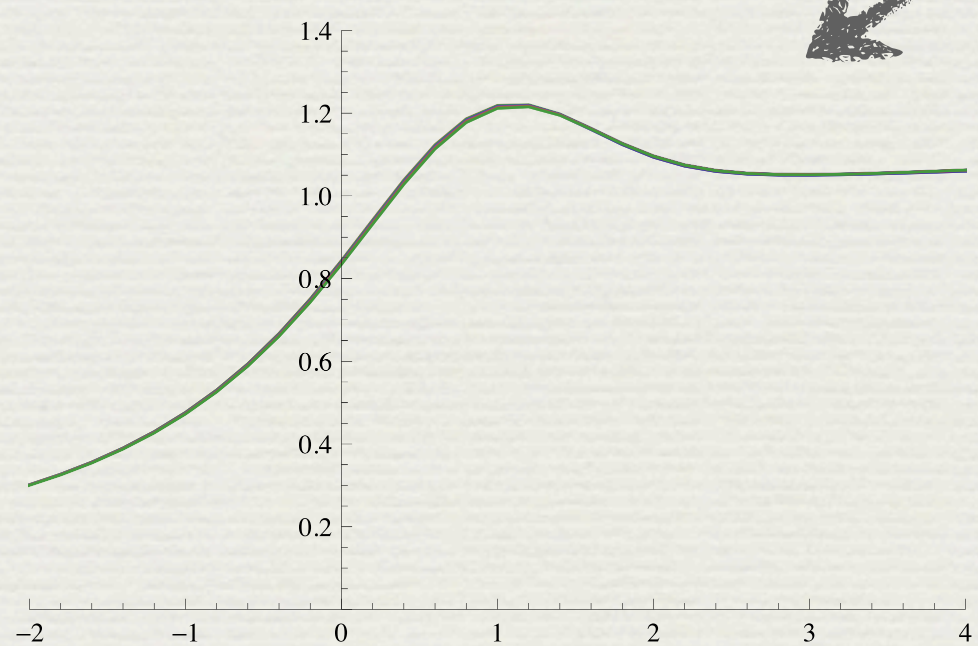
$\delta\mathcal{L}^{(2)}$: Kniehl-Penin- Smirnov-Steinhauser(02)
($\delta\mathcal{L}^{(2)} = \mathcal{O}(\epsilon)$ not known)

$\delta\mathcal{L}^{(us)}$: Brambilla-Pineda-Soto-Vairo('99),
Kniehl-Penin- Smirnov-Steinhauser(02)

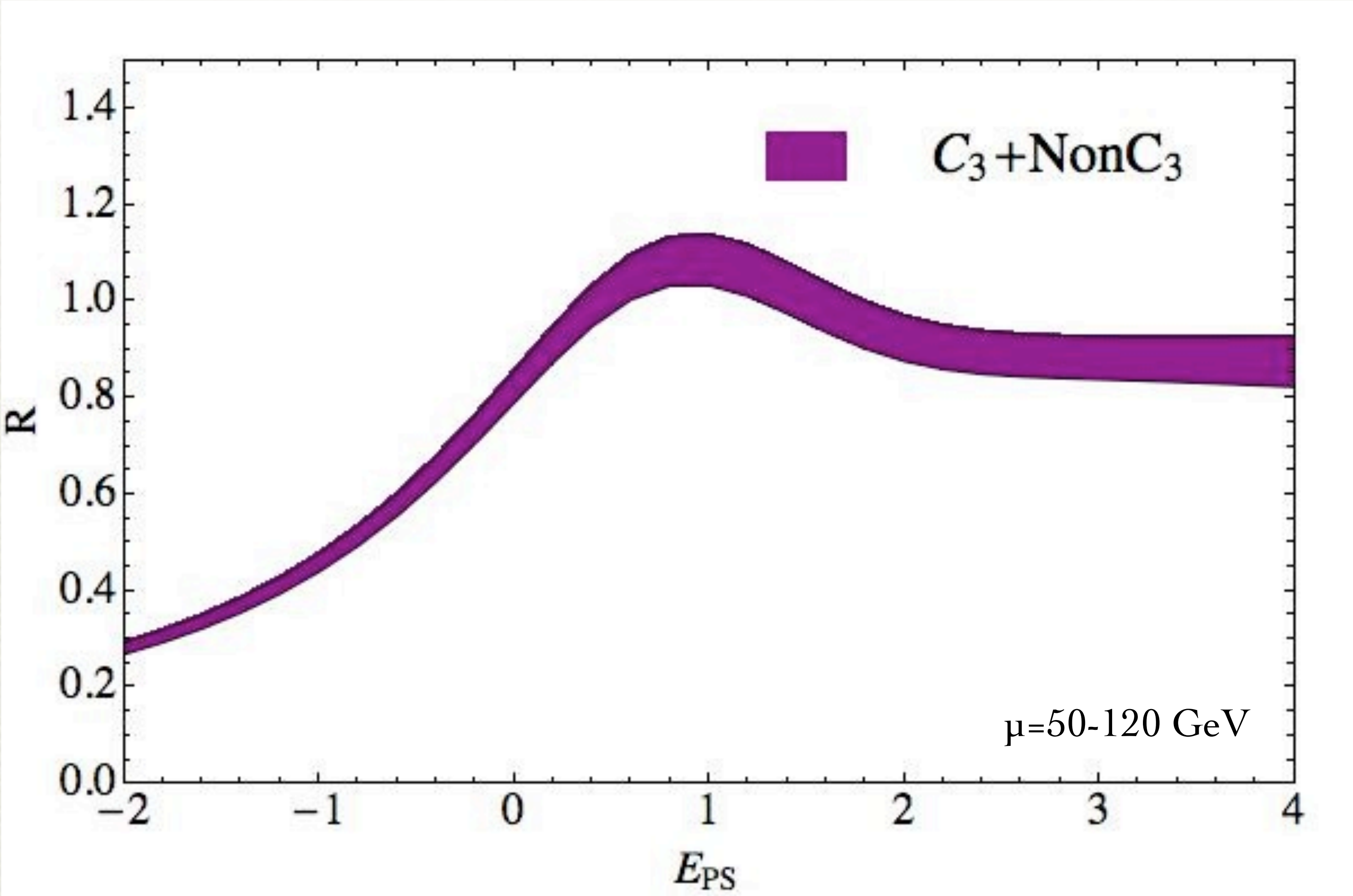
# Coulomb Corr



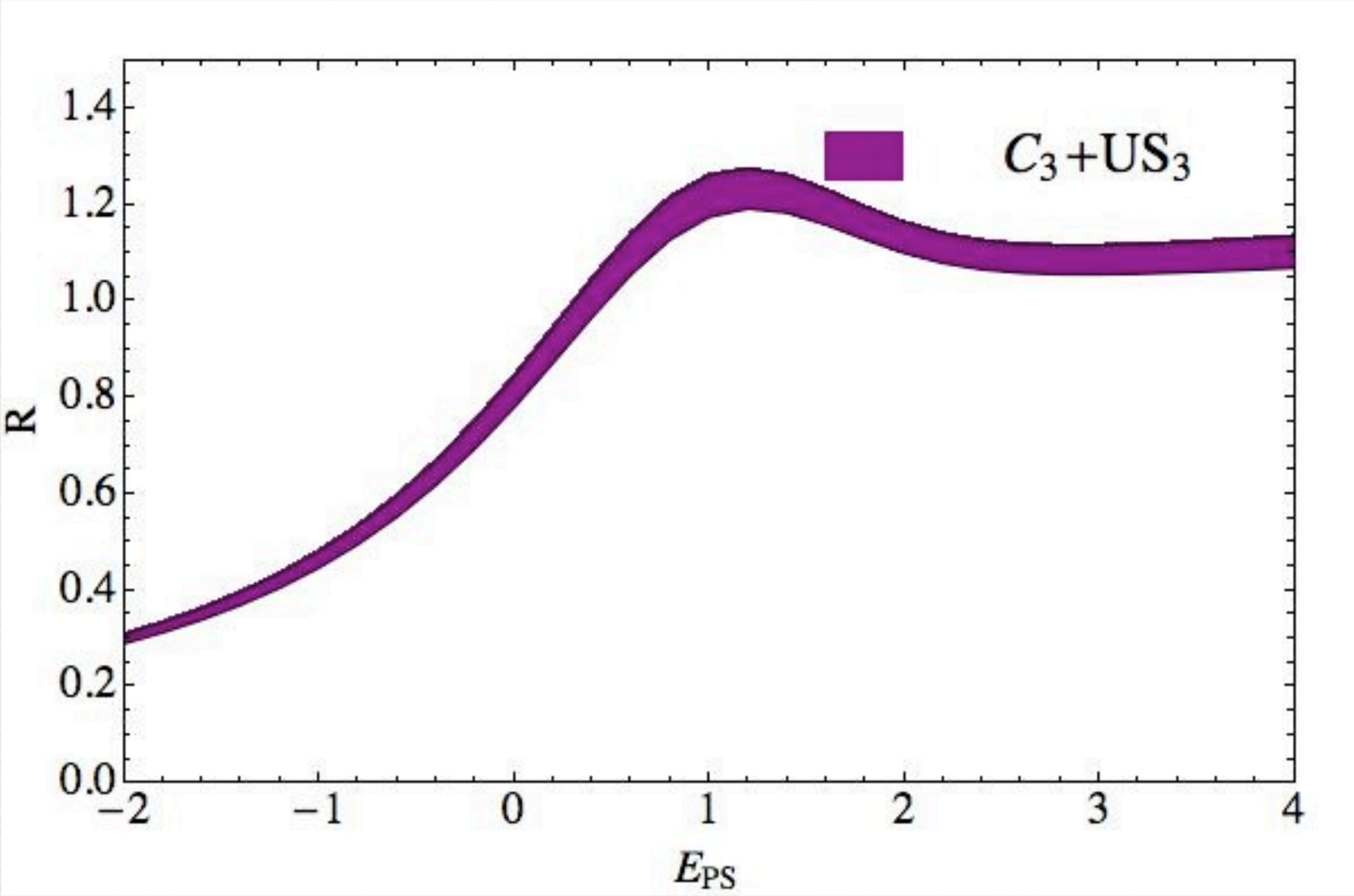



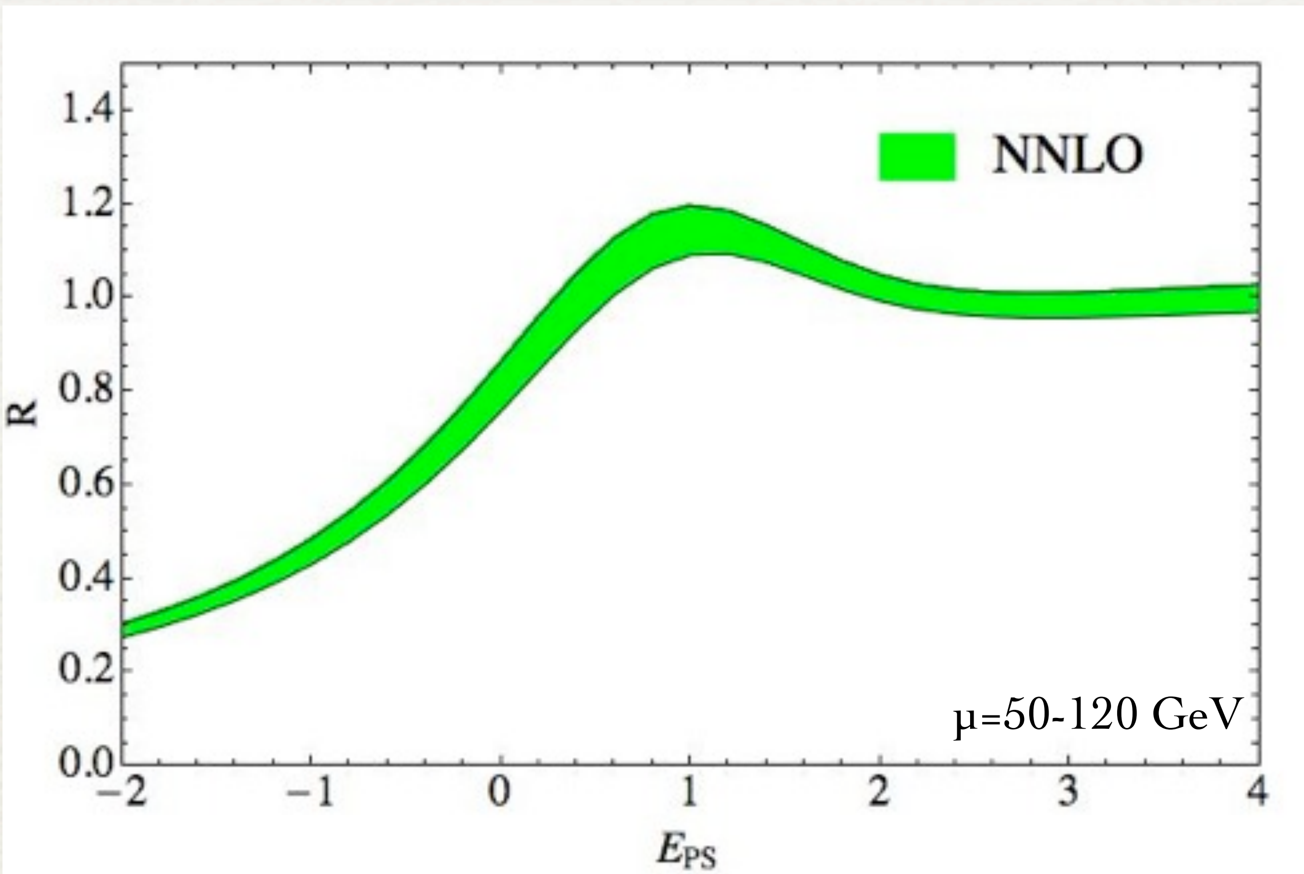
# NonCoulomb Corr

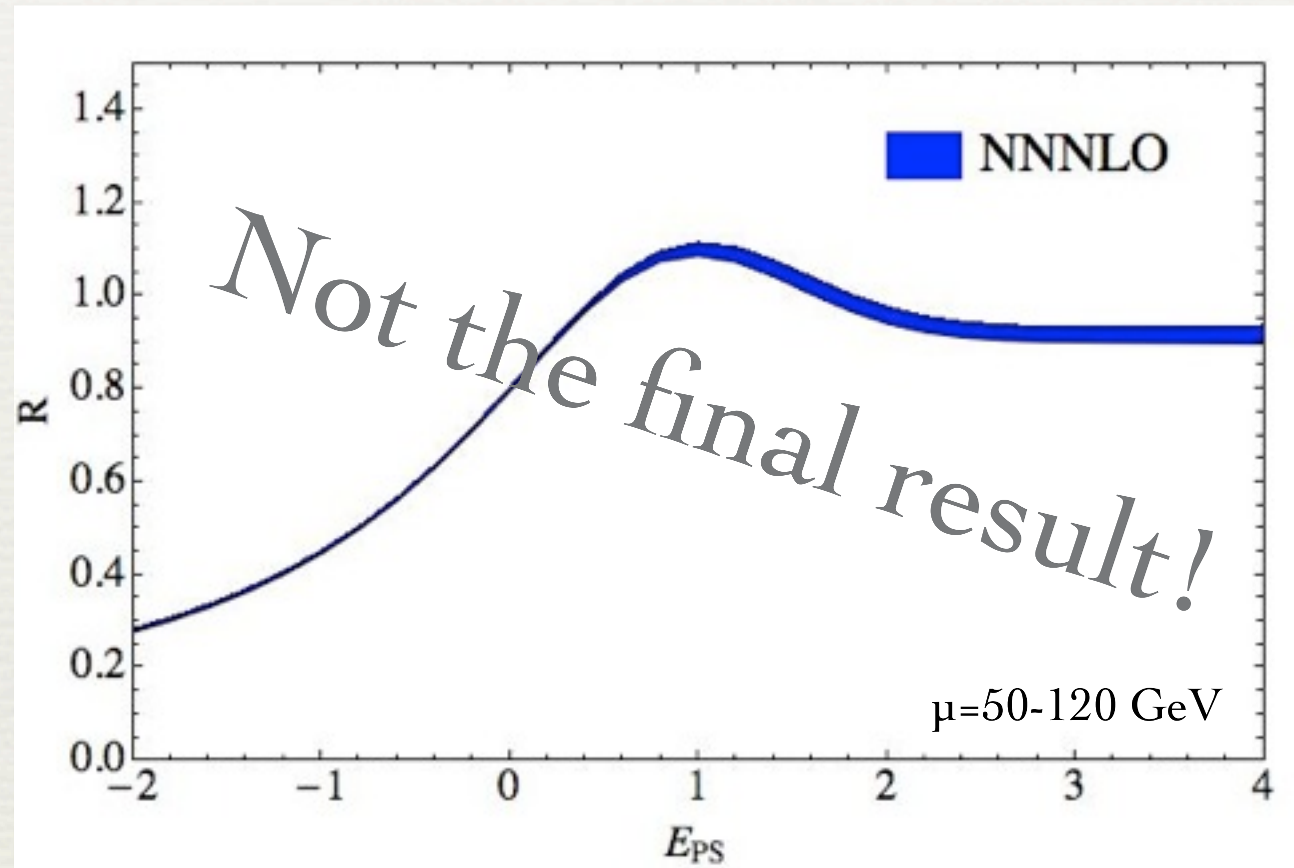


# Ultrasoft Corr



# $N^2LO$ and $N^{3'}LO$




Beneke-YK-Schuller in progress

Inputs:
$m_{PS}$(20GeV)=170 GeV,
$\Gamma_t$=1.4 GeV,
$\alpha_s(M_Z)$= 0.1180

# Issue of unstable top

- There are several different effects concerning to unstable nature of top quarks.

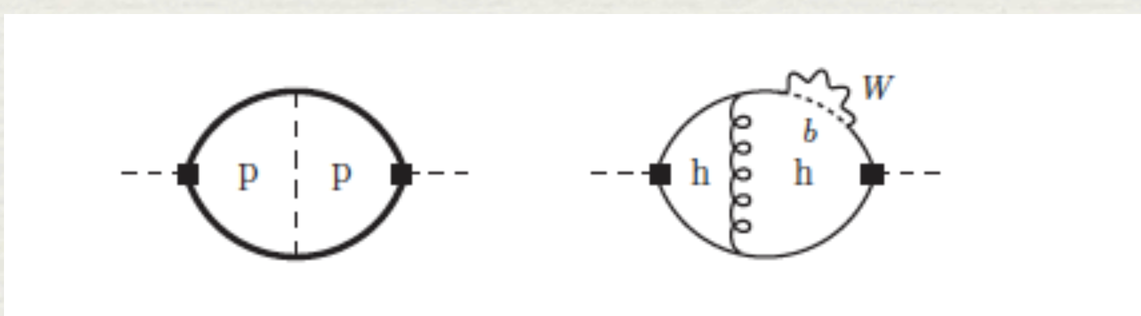
  → talk by A. Hoang, and P. Ruiz

- We have taken into account one of unstable top effect by E-> E+i$\Gamma_t$, We have introduced some uncanceled scale dependence.

$$\delta^{(2)} G_{div} = \boxed{\frac{(2\pi)^2}{3\epsilon\alpha_s} C_F(2C_F + 3C_A)} G_C(E) + \boxed{\frac{\Gamma}{\epsilon}\frac{2\pi C_F m}{\alpha_s}}$$

multiplicative Z-factor

need to renormalized counter term by unstable top EFT




It was started to seriously think this issue few years ago
[ Hoang-Reisser-Ruiz, Beneke-Falgari-Schwinn-Signer-Zanderighi, Jatzen-Ruiz, Penin-Piclum,....].
We have started to implement a prescription in our code, to fix the scale dependence, but not yet completed.

# Summary

- e+e- →tt cross section near threshold computed at NNNLO →sizable correction, reduction of scale dep.
- Still few missing parts→ c3,...
- Implementation of some of unstable effects started