Recent Developments in Event-Shapes

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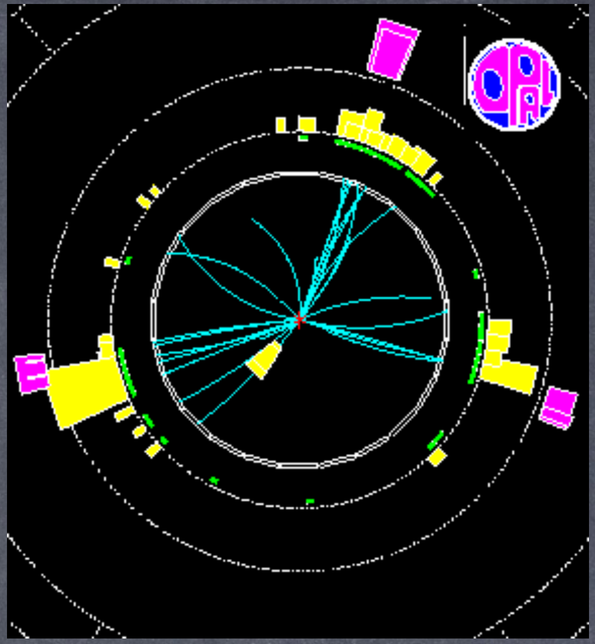
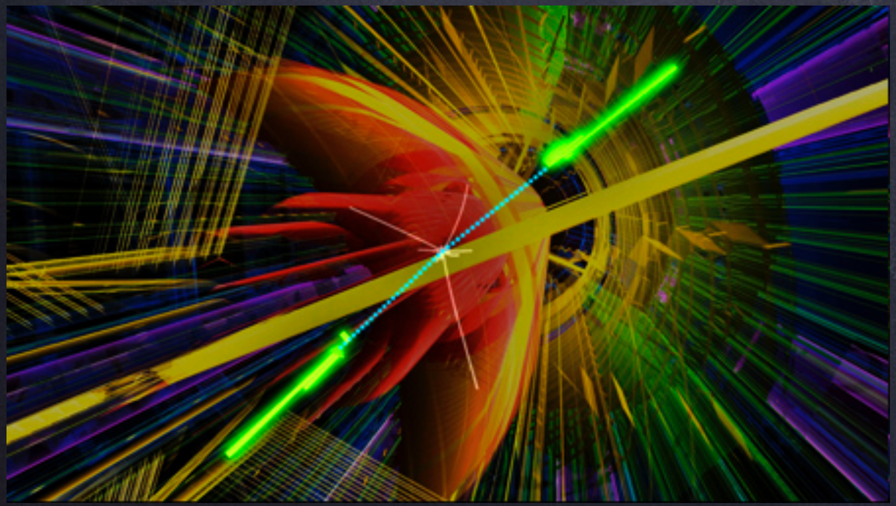
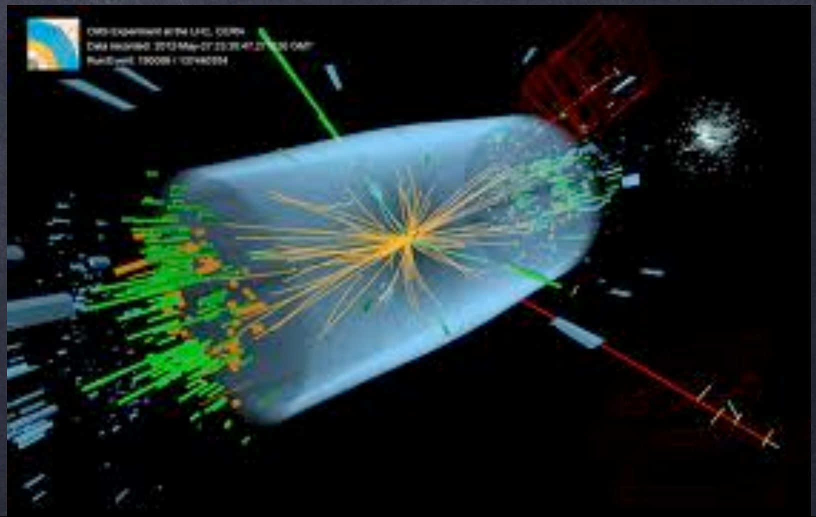
Outline

- Introduction
- Power corrections & Hadron Mass Effects
- Thrust, C-parameter and Heavy Jet Mass
- Consequences for an ILC
- Oriented Event Shapes
- Massive Event Shapes
- Conclusions and Outlook

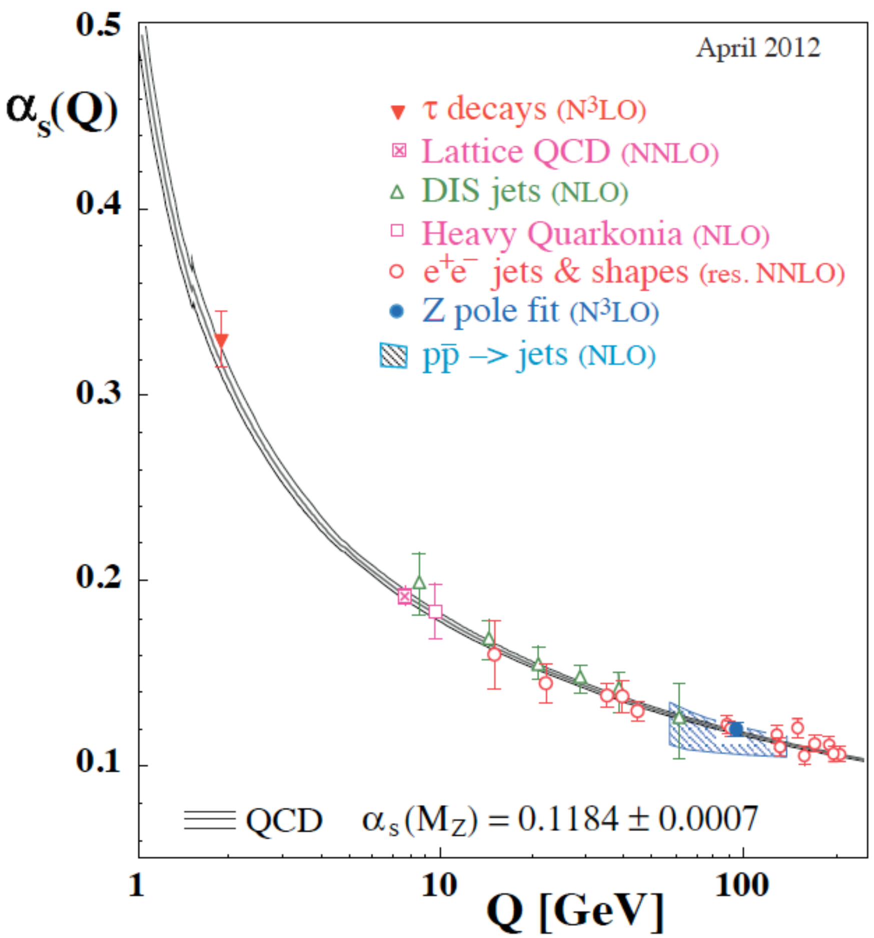


Introduction





Jets are essential to high-precision determination of α_s and the top mass



2004

asymptotic freedom

See talk of G. Luisoni and J. Mackenzie for update on recent determinations

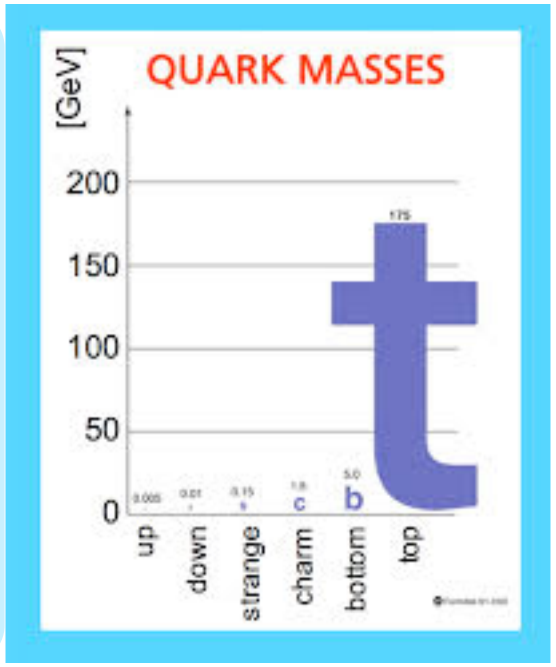
Jets are essential to high-precision determination of α_s and the top mass

Top is by far the heaviest quark

Its mass usually determined by reconstruction

Short distance scheme to achieve high precision

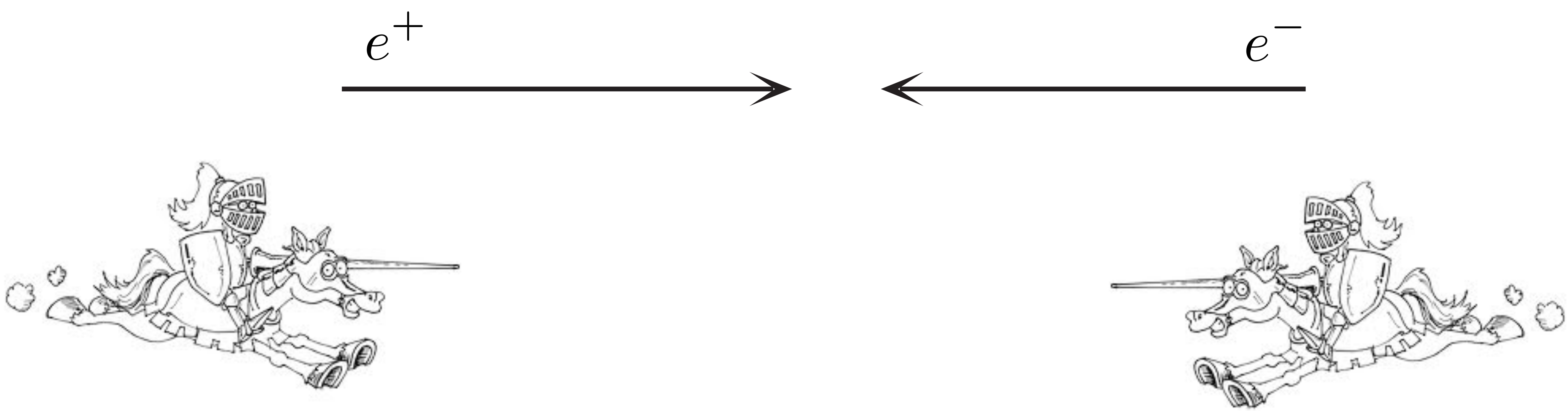
Jet physics play an essential role in achieving this goal



See talk by A. Hoang for top mass measurements at the ILC and LHC

Jet formation and evolution

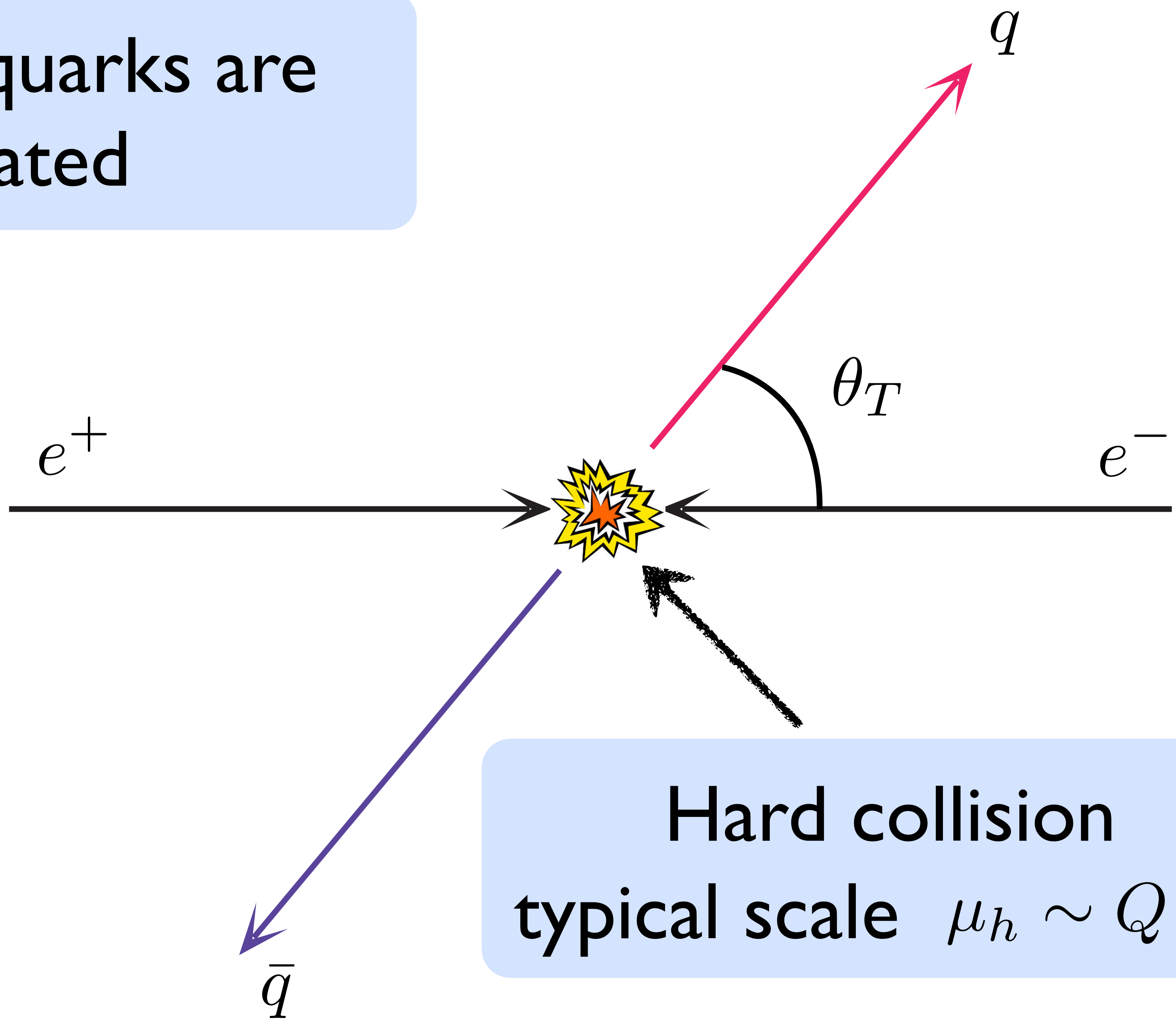
center of mass energy $Q = E_{e^+} + E_{e^-}$



Electron and positron collide at very high energy

Jet formation and evolution

primary quarks are created

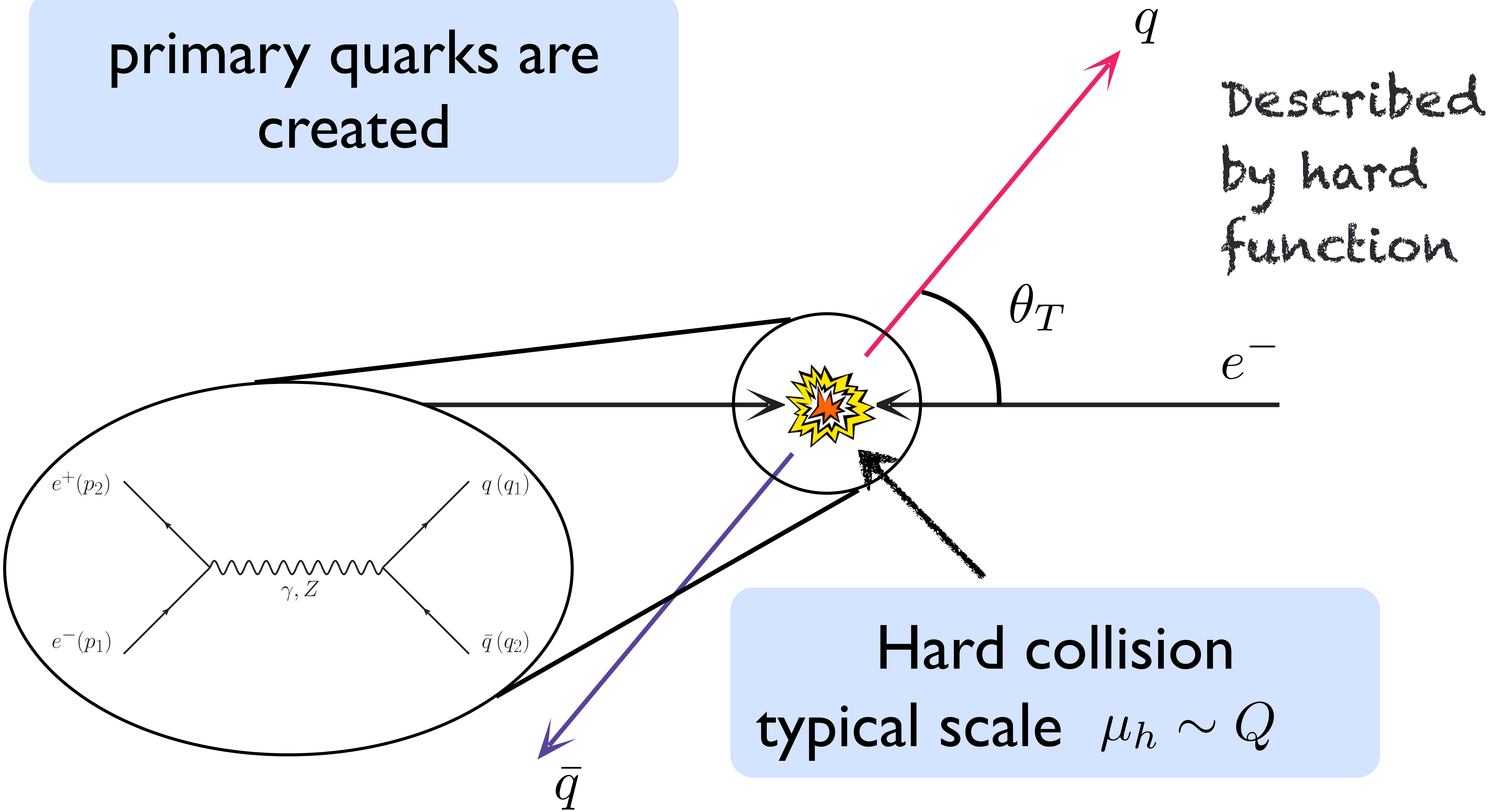


Hard collision
typical scale $\mu_h \sim Q$

At very short distance the hard interaction occurs

Jet formation and evolution

primary quarks are created

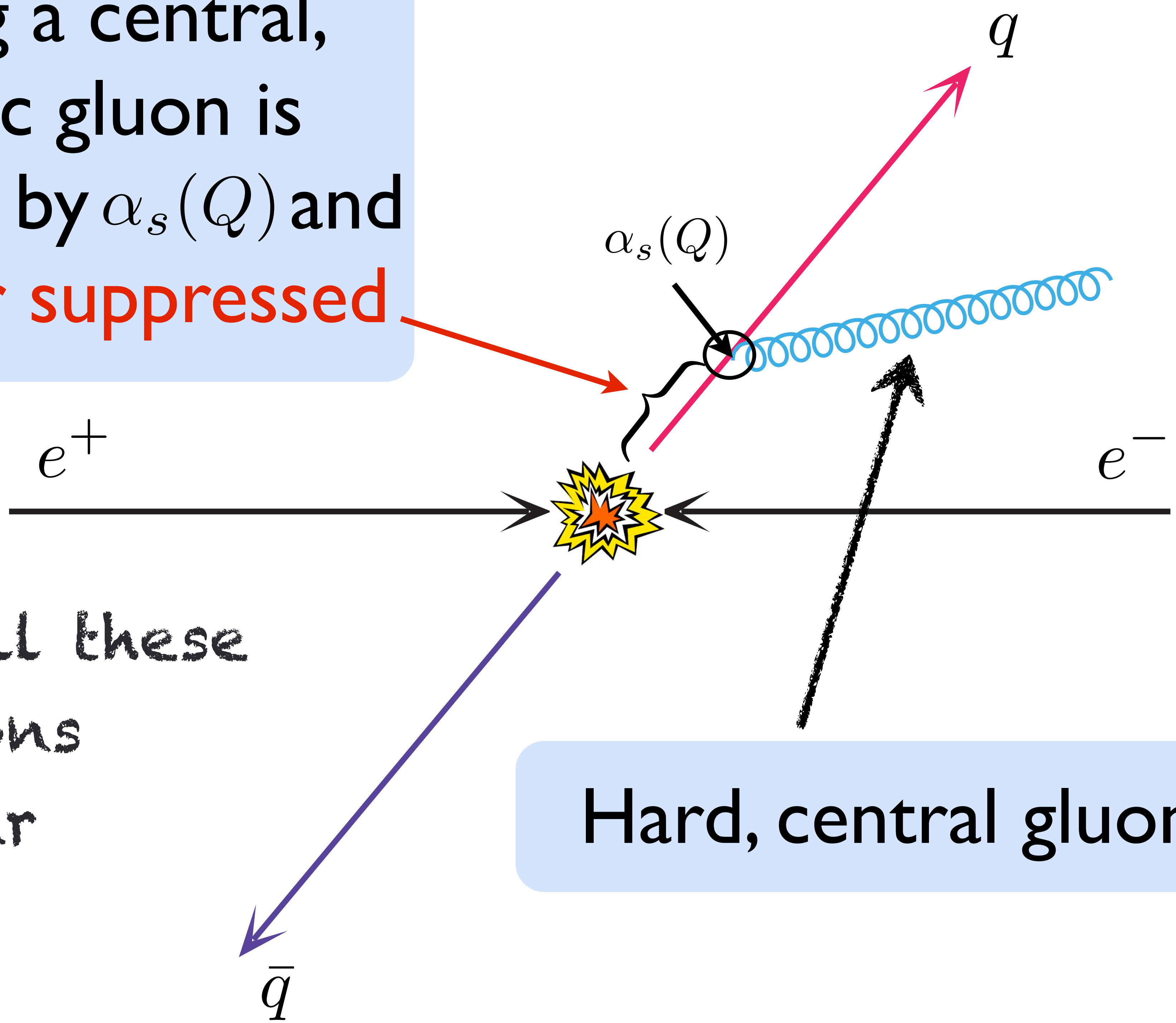


Hard collision
typical scale $\mu_h \sim Q$

At very short distance the hard interaction occurs

Jet formation and evolution

Radiating a central, energetic gluon is suppressed by $\alpha_s(Q)$ and also power suppressed



Hard, central gluon

We will call these contributions nonsingular

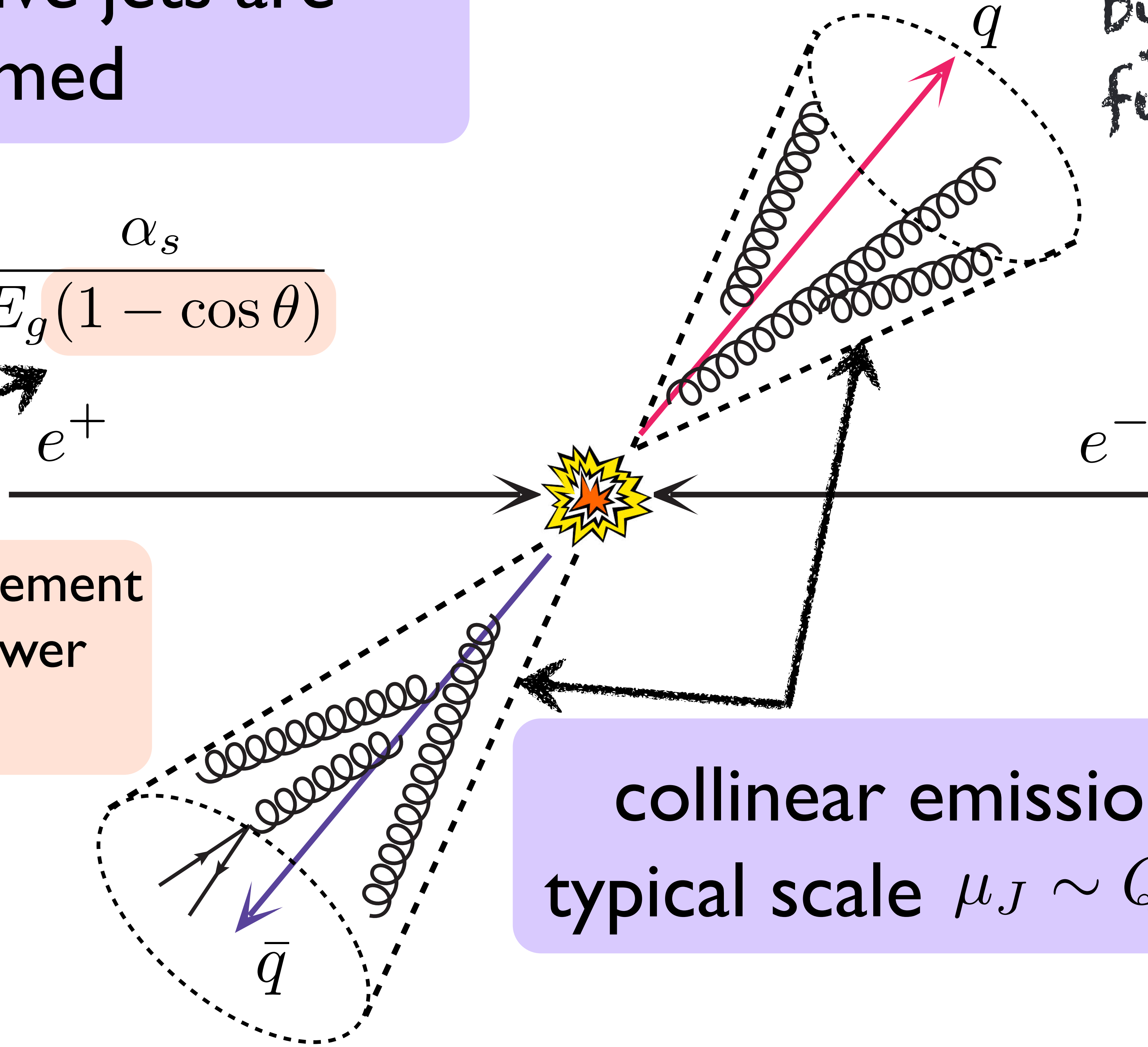
Creating a third jet is very unlikely

Jet formation and evolution

perturbative jets are formed

splitting probability $\sim \dfrac{\alpha_s}{E_g(1-\cos\theta)}$

collinear enhancement compensates power of $\alpha_s(Q)$



Described by jet function

collinear emissions typical scale $\mu_J \sim Q\,\Lambda_{\rm QCD}$

At longer distances collinear radiation off quarks occur

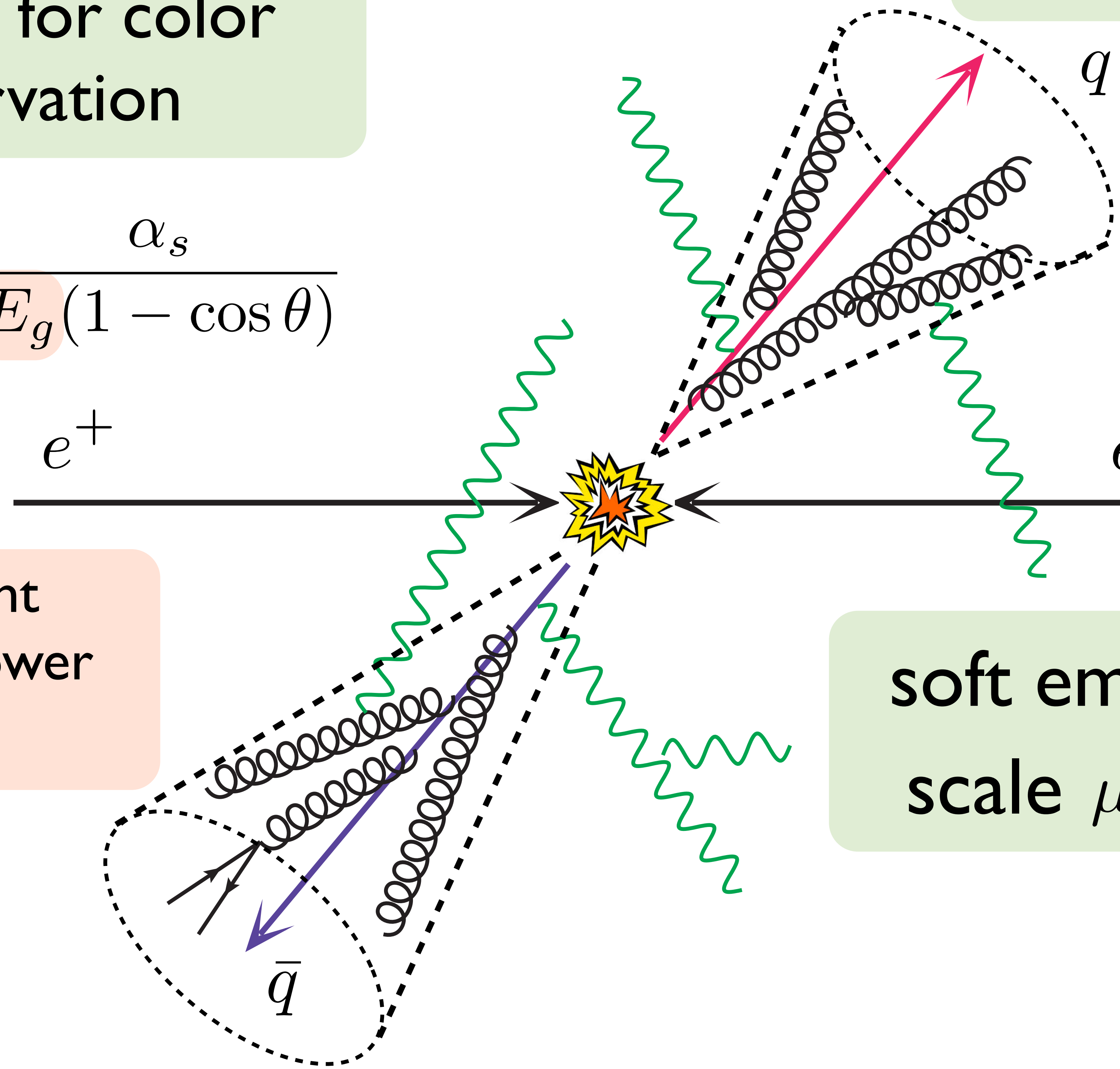
Jet formation and evolution

cross talk between jets necessary for color conservation

These are still perturbative gluons

splitting probability $\sim \frac{\alpha_s}{E_g(1-\cos\theta)}$

soft enhancement compensates power of $\alpha_s(Q)$



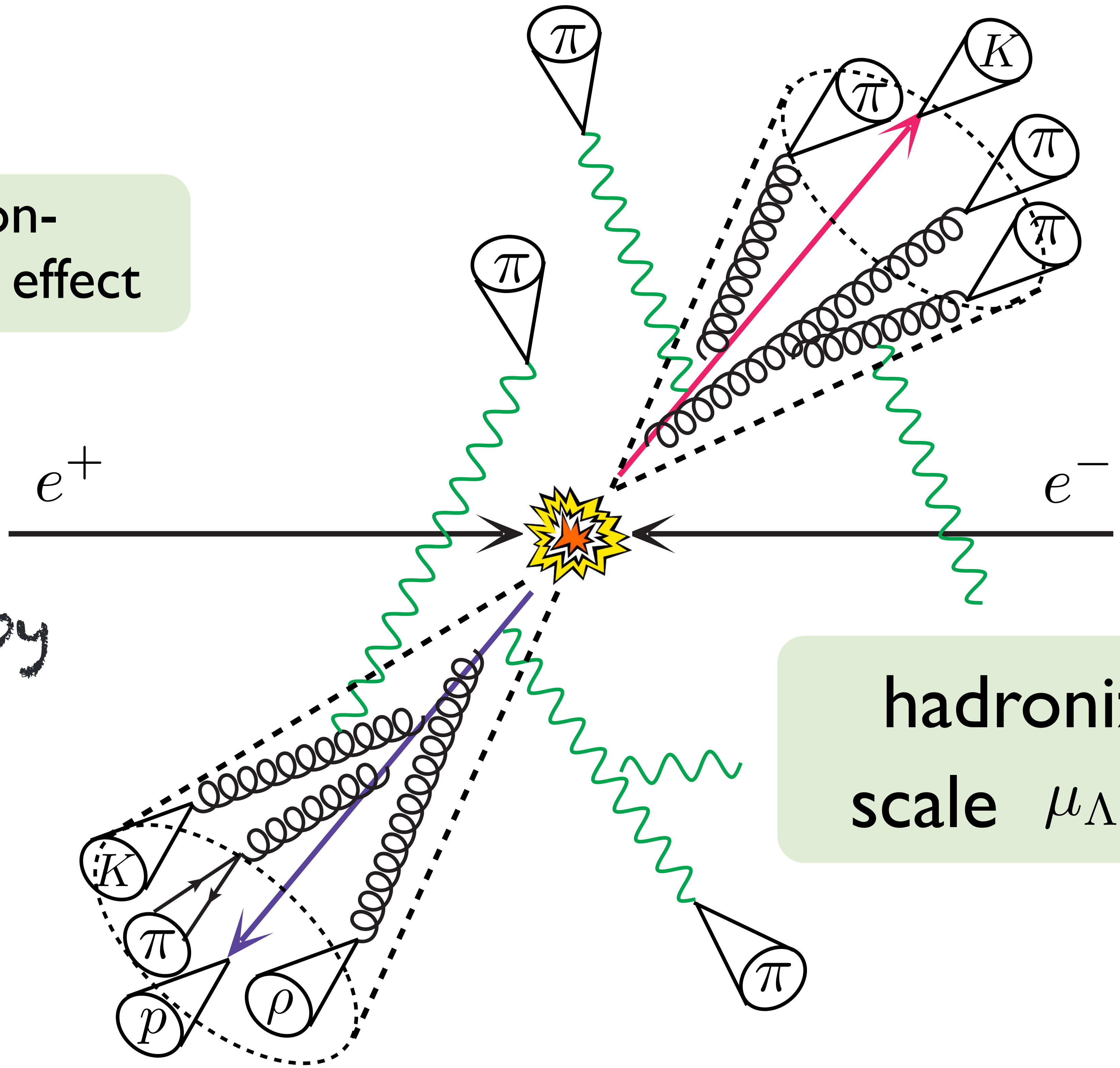
Described by soft function

soft emissions scale $\mu_S \gtrsim \Lambda_{\rm QCD}$

At long distance, large angle soft gluons are emitted

Jet formation and evolution

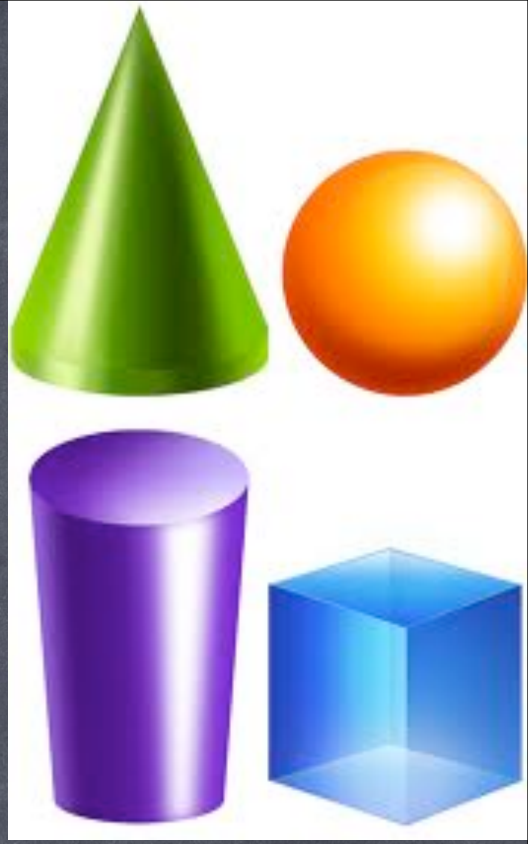
Purely non-perturbative effect



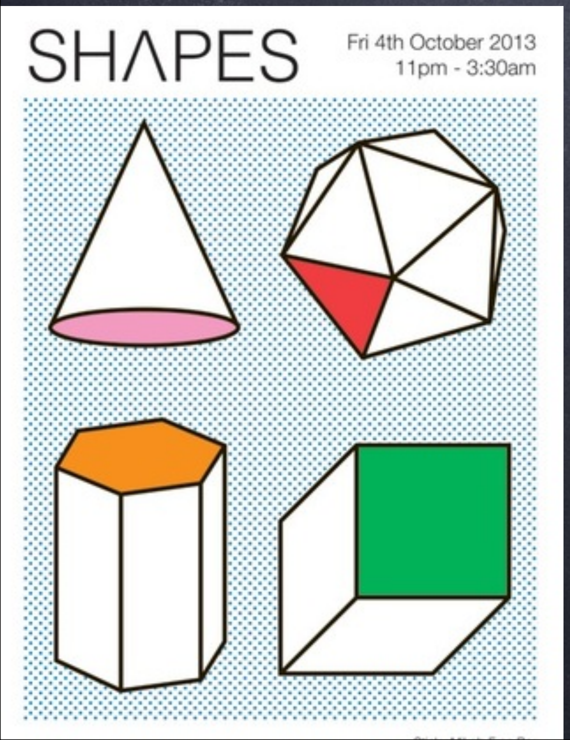
Described by the shape function

hadronization scale $\mu_\Lambda \sim \Lambda_{\rm QCD}$

At very long distances hadronization takes place



Event Shapes



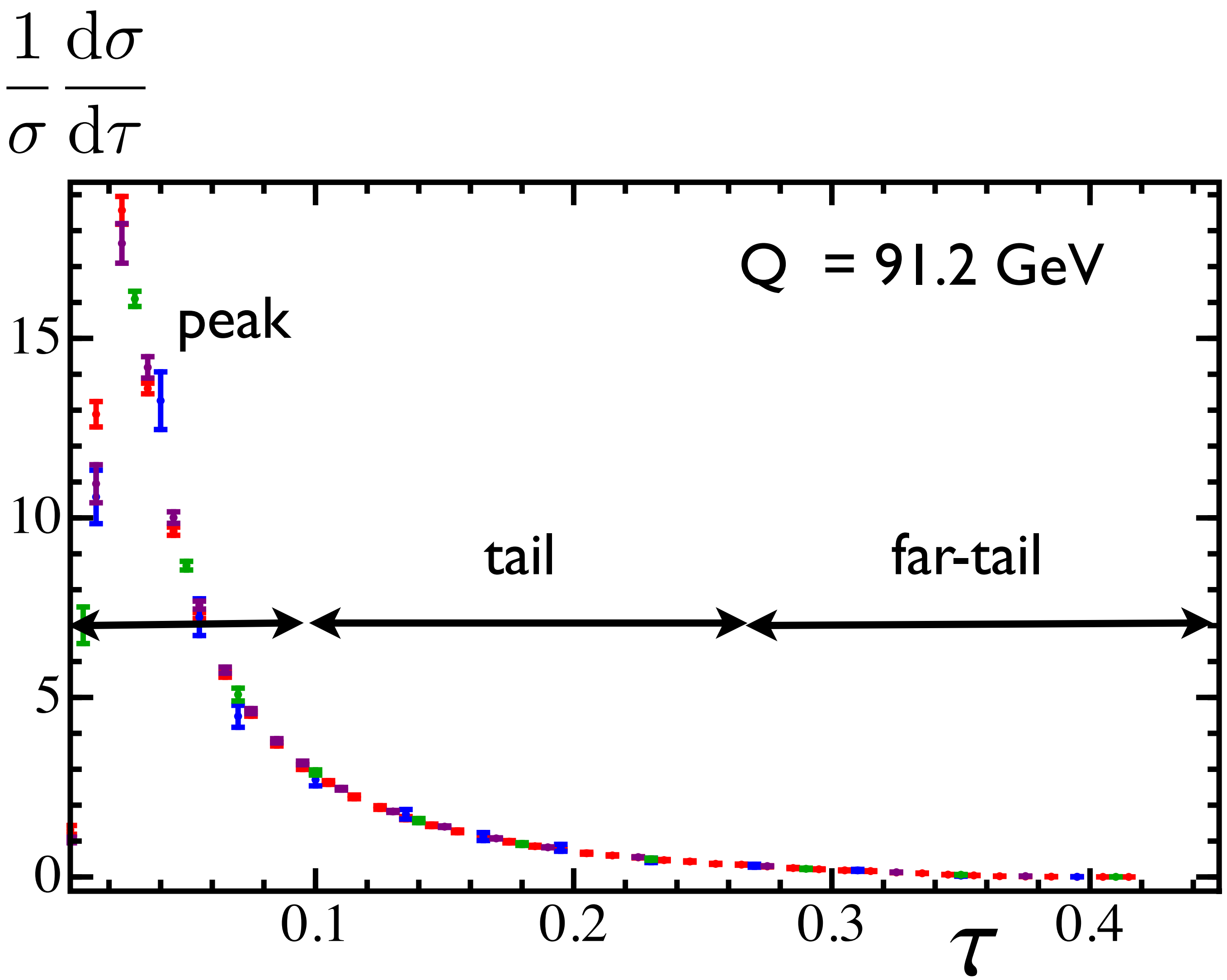
Event Shapes

Event shapes characterize in a geometrical way the distribution of hadrons in the final state

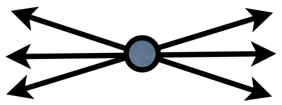
$$e^+e^- \to \text{jets}$$

Thrust $\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$

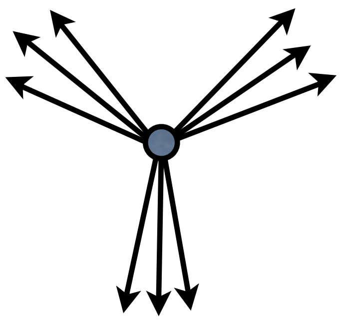
They are theoretically **more friendly than a Jet algorithm**



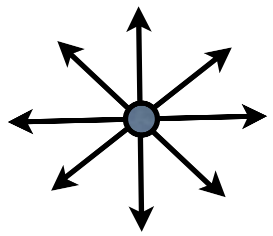
dijet $\tau \sim 0$



three jets $\tau \sim 0.3$



spherical $\tau \sim 0.5$



Continuous transition from 2-jet to 3-jet, ... multi-jet events

Most common Event shapes

- Thrust $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i\,|}$$ *[E. Farhi]*

- Angularities $$\tau_{(a)} = \frac{1}{Q} \sum_i E_i\,(\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$ *[Berger, Kucs, Sterman]*

- Jet Masses $$\rho_{\pm} = \frac{1}{Q^2} \Big(\sum_{i\in\pm} p_i\Big)^2$$ *[Clavelli]*
  [Chandramohan Clavelli]

- Jet Broadening $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$ *[Catani, Turnock, Webber]*

- C-parameter $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$ *[Parisi]*
  [Donoghue, Low, Pi]

- 2-Jettiness $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$ *[Stewart, Tackmann, Waalewijn]*

Most common Event shapes

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2-jet event shapes

$e \to 0$

↕

dijet configuration

Most common Event shapes

- Thrust $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i\,|}$$

- Angularities $$\tau_{(a)} = \frac{1}{Q} \sum_i E_i\,(\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$

Depend on a continuous parameter

- Jet Masses $$\rho_{\pm} = \frac{1}{Q^2} \Big(\sum_{i \in \pm} p_i \Big)^2$$

- Jet Broadening $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$

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- Jet Broadening $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$ Recoil sensitive

- C-parameter $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$

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- Jet Broadening $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$

double sum

- C-parameter $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$

does not require minimization procedure

- 2-Jettiness $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$

Most common Event shapes

• Thrust	$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i \lvert \vec{p}_i \cdot \vec{n} \rvert}{\sum \lvert \vec{p}_i \rvert}$$	Will show fits for α_s
• Angularities	$$\tau_{(a)} = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a (1 - \lvert \cos\theta_i \rvert)^{1-a}$$	there is no data
• Jet Masses	$$\rho_\pm = \frac{1}{Q^2} \Big(\sum_{i \in \pm} p_i \Big)^2$$	α_s fits are work in progress
• Jet Broadening	$$B = \frac{\sum_i \lvert \vec{p}_i \times \vec{n} \rvert}{\sum_i \lvert \vec{p}_i \rvert}$$	
• C-parameter	$$C = \frac{3}{2} \frac{\sum_{i,j} \lvert \vec{p}_i \rvert \lvert \vec{p}_j \rvert \sin^2(\theta_{ij})}{(\sum_i \lvert \vec{p}_i \rvert)^2}$$	Will show fits for α_s
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Resummation of large logarithms

Event shapes are not inclusive quantities

Large logs at small τ

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s}{3\pi}\frac{1}{\tau}\Big(3 + 4\log\tau + \ldots\Big)$$

Invalidates perturbative expression for small

One has to reorganize the expansion by considering $\alpha_s \lg(\tau) \sim \mathcal{O}(1)$

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Counting more clear in the exponent of cumulant

$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} \mathrm{d}\tau \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

$$\begin{aligned}\log\Sigma(\tau_c) = {} & \alpha_s(\log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^2\,(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^3\,(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^4\,(\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \ldots\end{aligned}$$

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LO

$$\alpha_s^2\,(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)$$

$$\alpha_s^3\,(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$

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$$\alpha_s^2\,(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)$$ NLO

$$\alpha_s^3\,(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$

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[Catani, Seymour] $$\alpha_s^2(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)$$ NLO

State of the art $$\alpha_s^3(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$ NNLO

$$\alpha_s^4(\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)$$

$\dots$

not known!

[Weinzierl]

[Gehrmann-De Rider, Gehrmann, Glover, Heinrich]

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\end{aligned}$$

LL

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LL NLL

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Event shapes are not inclusive quantities

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LL NLL N^2LL

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\end{aligned}$$

LL | NLL | N^2LL | N^3LL

Resummation of large logarithms

Event shapes are not inclusive quantities

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = -\frac{2\alpha_s}{3\pi}\frac{1}{\tau}\Big(3 + 4\log\tau + \ldots\Big)$$

Large logs at small τ

Invalidates perturbative expression for small

One has to reorganize the expansion by considering $\alpha_s \lg(\tau) \sim \mathcal{O}(1)$

Counting more clear in the exponent of cumulant

$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} d\tau \frac{1}{\sigma_0}\frac{d\sigma}{d\tau}$$

$$\begin{aligned}
\log\Sigma(\tau_c) = \alpha_s & (\log^2\tau_c + \log\tau_c + 1) \\
\alpha_s^2 & (\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1) \\
\alpha_s^3 & (\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1) \\
\alpha_s^4 & (\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1) \\
& \vdots
\end{aligned}$$

LL | NLL | N²LL | N³LL | not known!

State of the art

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

Factorization

Factorization theorem for event shapes

(Will focus on SCET but CSS is equivalent)

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

[Berger, Kuks, Sterman]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Factorization theorem for event shapes

(Will focus on SCET but CSS is equivalent)

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{QCD}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

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Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

$$S_e(e) = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

Soft Wilson lines

event shape operator

Leading power correction comes from soft function

Factorization theorem for event shapes

(Will focus on SCET but CSS is equivalent)

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

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Soft Wilson lines

event shape operator

Leading power correction comes from soft function

$$S_e = \hat{S}_e \otimes F_e$$

[Korchemsky, Sterman, Tafat]

perturbative

nonperturbative & [Korchemsky & Sterman]

perturbative [VM, Thaler, Stewart]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} \otimes F_e$$

Factorization theorem for event shapes

(Will focus on SCET but CSS is equivalent)

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{QCD}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

[Berger, Kuks, Sterman]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

Heavy Jet Mass is slightly more complicated than this (more on this later)

Large Log Resummation

Renormalization group evolution

The hierarchy among the scales depends on the position on the spectrum

hard scale $\mu_H \sim Q$ — $\log^n\left(\frac{Q}{\mu}\right)$

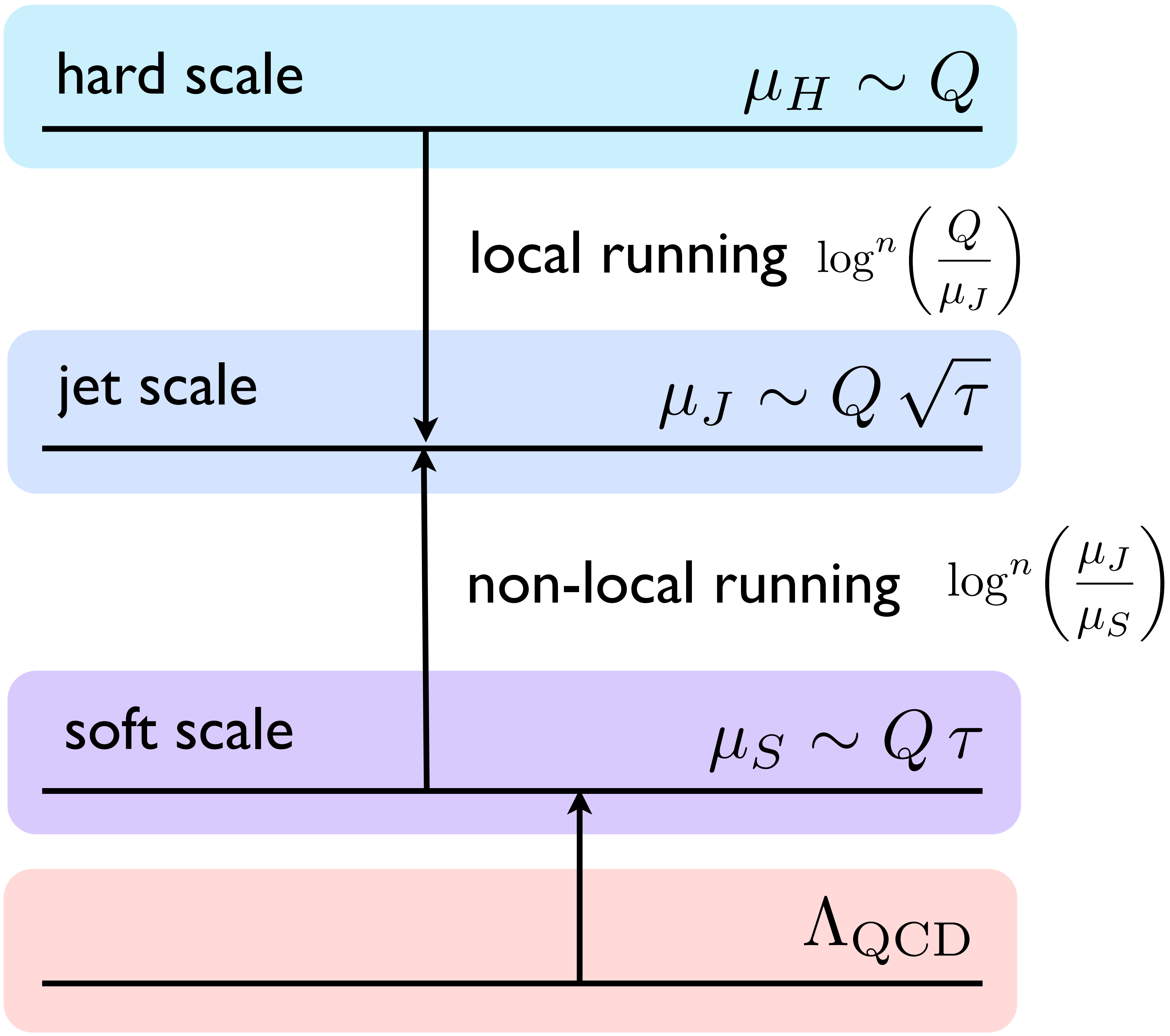
jet scale $\mu_J \sim Q\sqrt{\tau}$ — $\log^n\left(\frac{Q^2\tau}{\mu^2}\right)$

soft scale $\mu_S \sim Q\,\tau$ — $\log^n\left(\frac{Q\tau}{\mu}\right)$

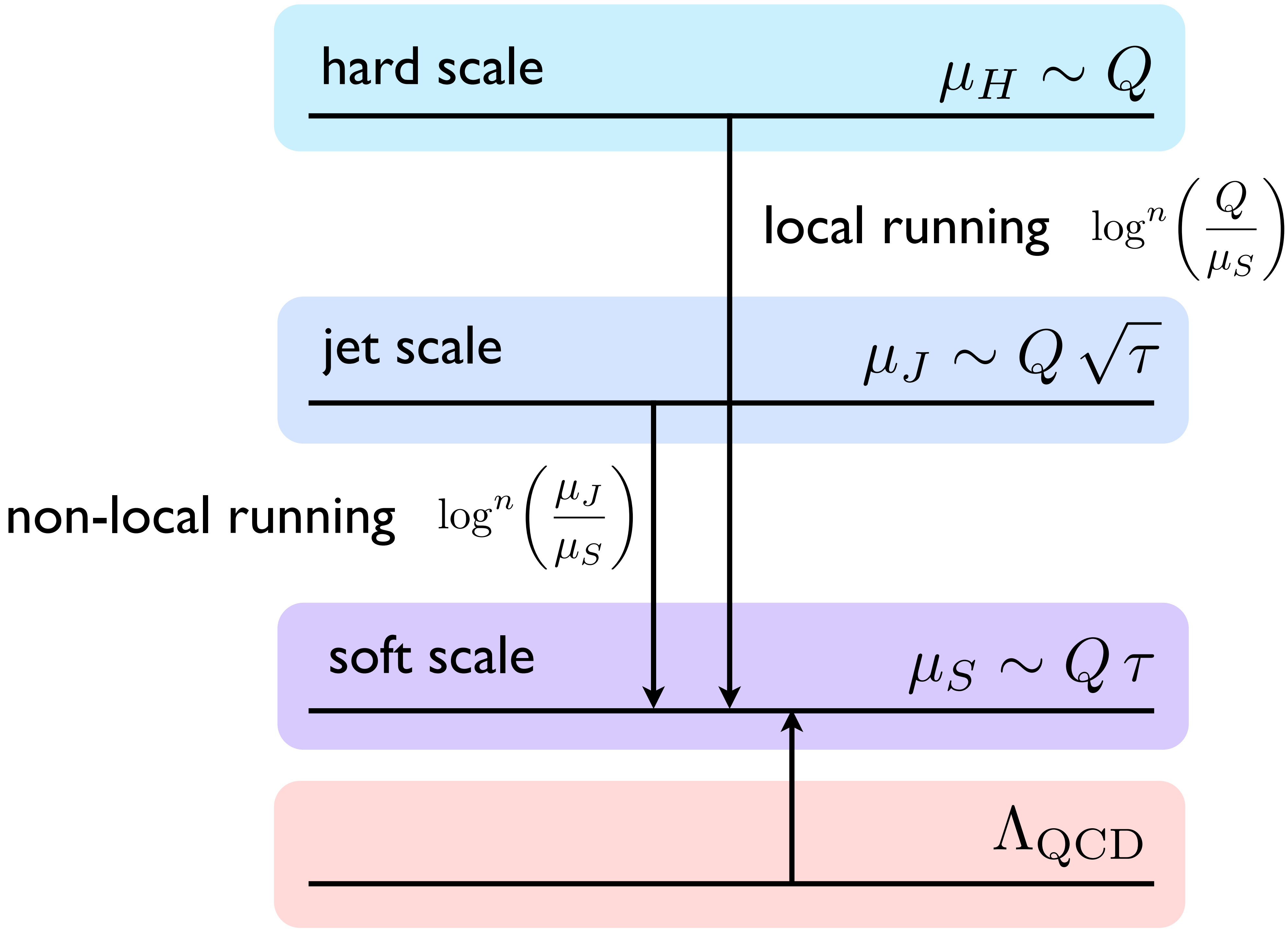
$\Lambda_{\rm QCD}$

large logs

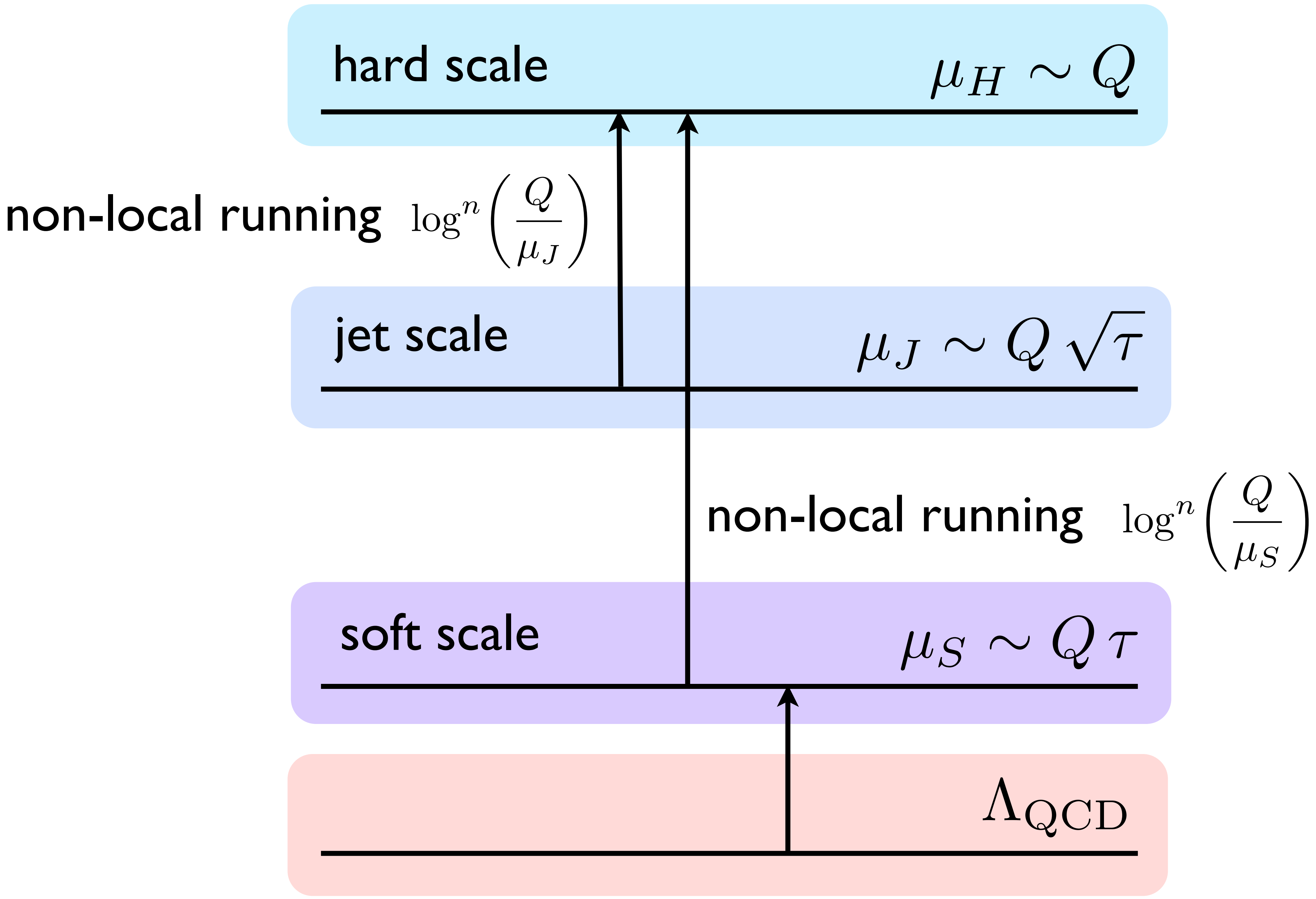
Renormalization group evolution



Renormalization group evolution



Renormalization group evolution



Power Corrections

OPE for non-perturbative corrections

$$S_e(e) = \langle\, 0 \,|\, \overline{Y}^\dagger_{\bar{n}} Y^\dagger_n \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$$

[Lee & Sterman]

For $e \gg \frac{\Lambda_{\rm QCD}}{Q}$

$$\delta(\ell - Q\hat{e}) \simeq \delta(\ell) - \delta'(\ell) Q\hat{e} + \ldots$$

Correct up to $\mathcal{O}(\alpha_s)$

Shape function can be expanded in the tail

$$F_e(\ell) \simeq \delta(\ell) - \Omega_1 \delta'(\ell)$$

$$\Omega_1 = \langle\, 0 \,|\, \overline{Y}^\dagger_{\bar{n}} Y^\dagger_n Q\hat{e} Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$$

OPE for non-perturbative corrections

$$S_e(e) = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

[Lee & Sterman]

For $e \gg \frac{\Lambda_{\mathrm{QCD}}}{Q}$

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$$\Omega_1 = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger Q\hat{e} Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} - \frac{\Omega_1}{Q} \frac{\mathrm{d}}{\mathrm{d}e} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} \simeq \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}\left(e - \frac{\Omega_1}{Q}\right) + \mathcal{O}\left[\left(\frac{\Lambda_{\mathrm{QCD}}}{Q\, e}\right)^2\right]$$

Universality: $\Omega_1^e = c_e\, \Omega_1^\rho$

Leading power corrections proportional to each other, calculable coefficient

Mass Effects in SCET

[VM, I.W. Stewart, J.Thaler] PRD87 (2013) 013025

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^{\perp}\, f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

$$y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right)$$ rapidity

$$r \equiv \frac{p^{\perp}}{m^{\perp}}$$ transverse velocity

All event shapes can be expressed in terms of these two variables

$$m^{\perp} = \sqrt{p_T^2 + m^2}$$ transverse mass

$$\eta = \ln\left(\frac{\sqrt{r^2+\sinh^2 y}+\sinh y}{r}\right)$$ pseudo-rapidity

$$v = \frac{\sqrt{r^2+\sinh^2 y}}{\cosh y}$$ velocity

massless limit

$$v = r = 1$$

$$y = \eta$$

$$m^{\perp} = p^{\perp}$$

Mass Effects in SCET

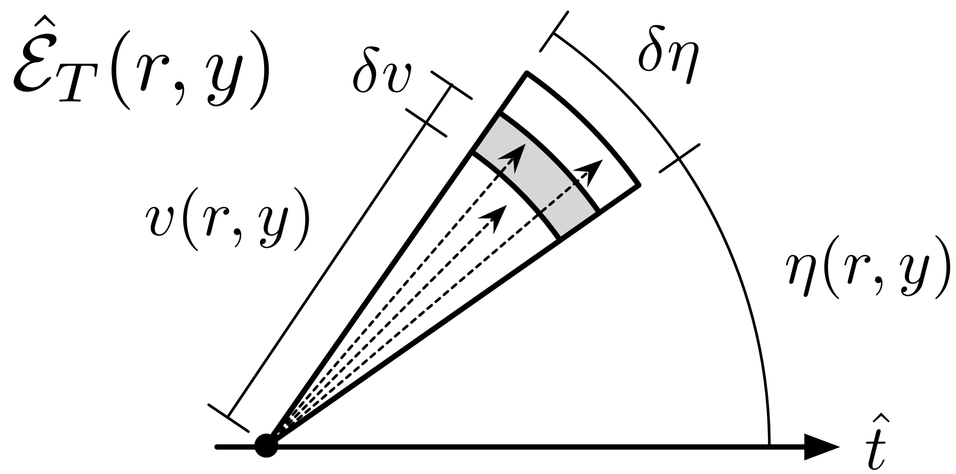
[VM, I.W. Stewart, J.Thaler] PRD87 (2013) 013025

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^\perp\, f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

Transverse velocity operator

$$\hat{\mathcal{E}}_T(r,y)|\,N\,\rangle = \sum_{i\in N} m_i^\perp \delta(r-r_i)\delta(y-y_i)|\,N\,\rangle$$



$$v = v(r,y)$$
$$\eta = \eta(r,y)$$

measures momenta of particles with given transverse velocity flowing at a given rapidity

$$\hat{e} = \frac{1}{Q}\int \mathrm{d}y\,\mathrm{d}r\,\mathcal{E}_T(r,y) f_e(r,y)$$

$$\hat{e}\,|\,N\,\rangle = e(N)\,|\,N\,\rangle$$

$$\mathcal{E}_T(v,\eta) = -\frac{v(1-v^2\tanh^2 y)^{\frac{3}{2}}}{\cosh\eta}\lim_{R\to\infty} R^3\int_0^{2\pi}\mathrm{d}\phi\,\hat{n}_i\,T_{0i}(R, v\,R\,\hat{n})$$

Mass Effects in SCET

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$$e(N) = \frac{1}{Q} \sum_{i \in N} m_i^{\perp} \, f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

$$\Omega_1^e = \langle\, 0 \,|\, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} Q \hat{e} Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$$

Mass Effects in SCET

[VM, I.W. Stewart, J.Thaler] PRD87 (2013) 013025

$$e(N) = \frac{1}{Q} \sum_{i\in N} m_i^{\perp} \, f_e(r_i, y_i)$$

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$$\Omega_1^e = \int \mathrm{d}r \, \mathrm{d}y \, f_e(r,y) \langle \, 0 \, | \, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r,y) Y_n \overline{Y}_{\bar{n}} \, | \, 0 \, \rangle$$

Mass Effects in SCET

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One has to generalize the transverse energy flow operator

$$\Omega_1^e = \int \mathrm{d}r\,\mathrm{d}y\, f_e(r,y) \langle 0 \,|\, \overline{Y}_{\bar n}^\dagger Y_n^\dagger \mathcal{E}_T(r,y) Y_n \overline{Y}_{\bar n} \,|\, 0 \rangle = c_e \int \mathrm{d}r\, g_e(r)\Omega_1(r)$$

Boost invariance requires this term is y-independent

Operator definition of power correction

$$\Omega_1(r) = \langle 0 \,|\, \overline{Y}_{\bar n}^\dagger Y_n^\dagger \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar n} \,|\, 0 \rangle$$

"Universality" coefficient

$$c_e = \int_{-\infty}^{\infty} \mathrm{d}y\, f_e(1,y)$$

$$g_e(r) = \frac{1}{c_e}\int \mathrm{d}y\, f_e(r,y)$$

encodes all mass effects

each $g_e(r)$ defines a universality class of events with same power correction

Event shapes considered

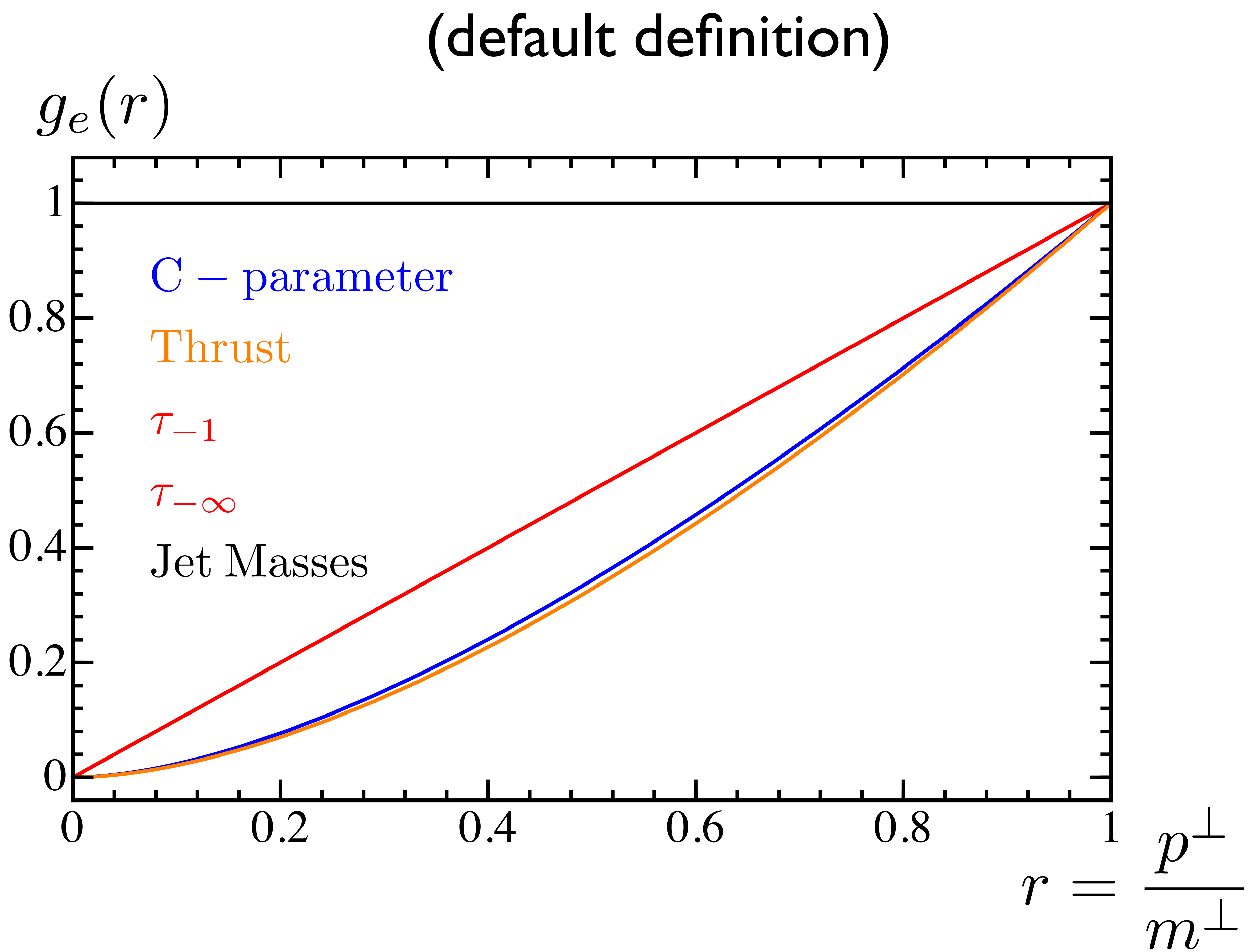
Thrust

Jet Masses

C-parameter

Angularities

2-Jettiness



Same color means same power correction

Event shapes considered

Thrust

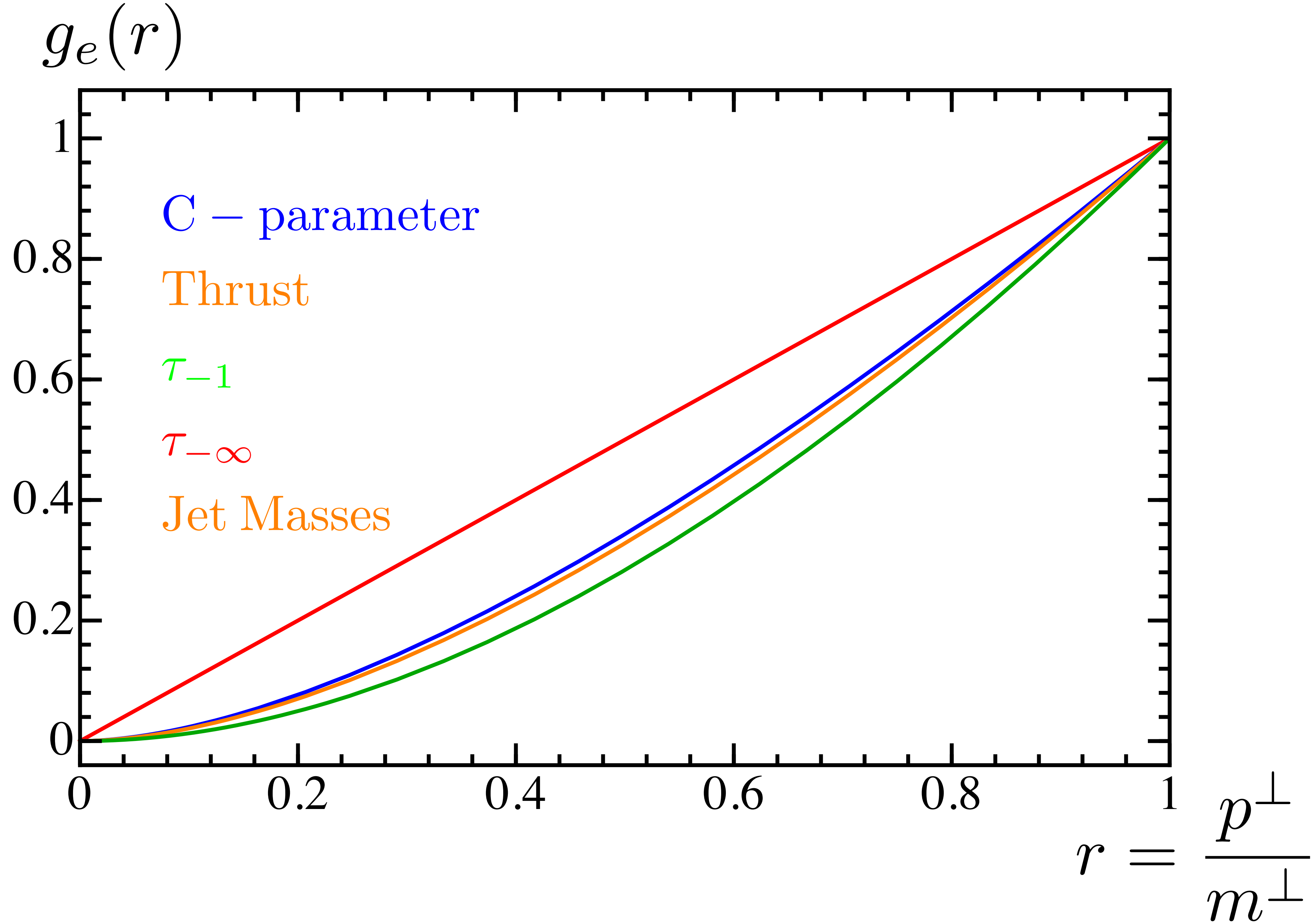
Jet Masses

C-parameter

Angularities

2-Jettiness

P-scheme $E \to |\vec{p}|$

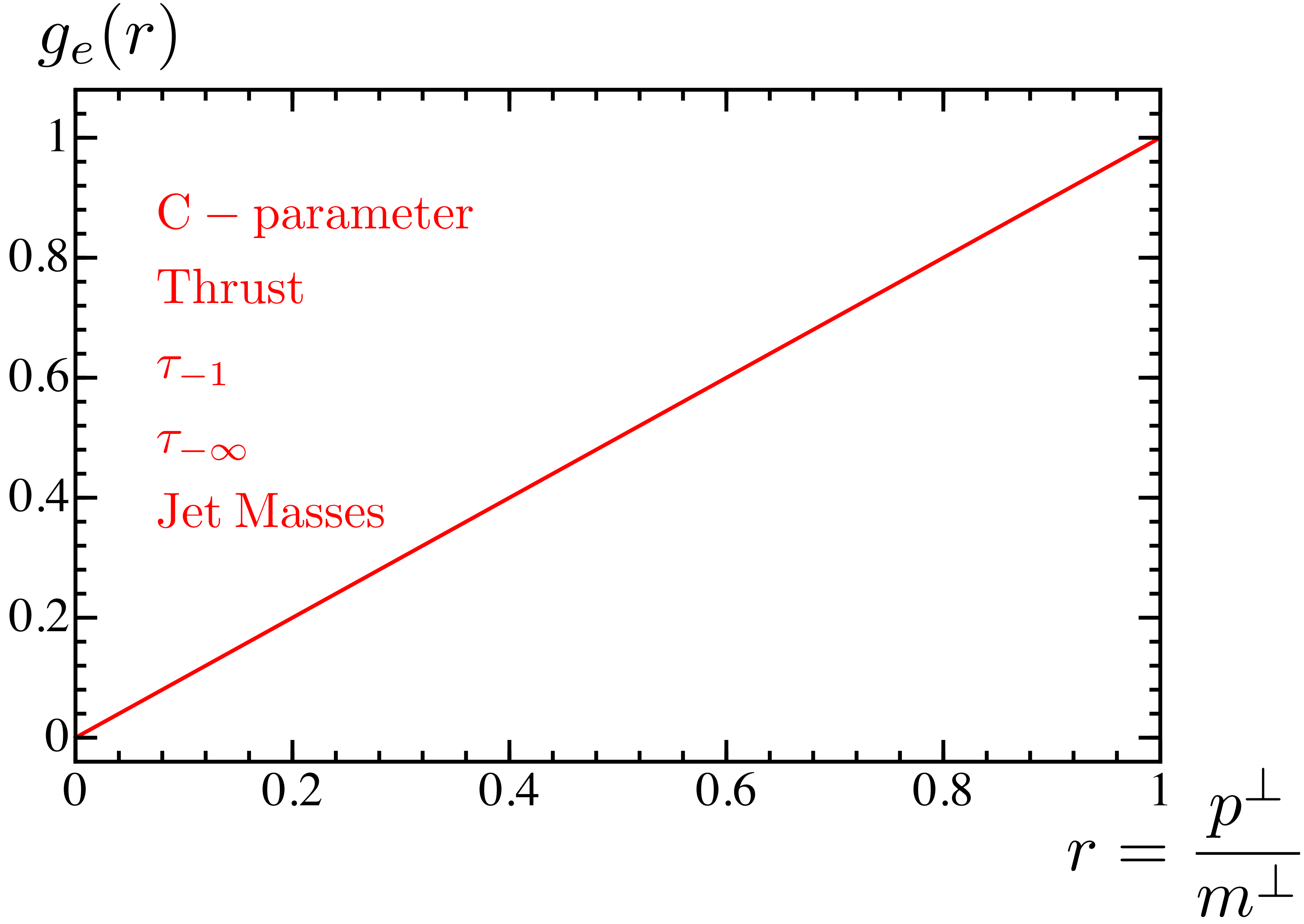


→ Scheme changes event shape definition

Event shapes considered

E-scheme $|\vec{p}| \to E$

- Thrust
- Jet Masses
- C-parameter
- Angularities
- 2-Jettiness



→ Scheme changes event shape definition

Thrust

Theoretical knowledge

$H(Q,\mu)$ Hard function known at 3 loops

$J_n(s,\mu)$ Jet function known at two loops
Running known at three loops

$S_\tau(\ell,\mu)$ Soft function known at two loops
Running known at three loops

Fixed-order predictions known at three loops

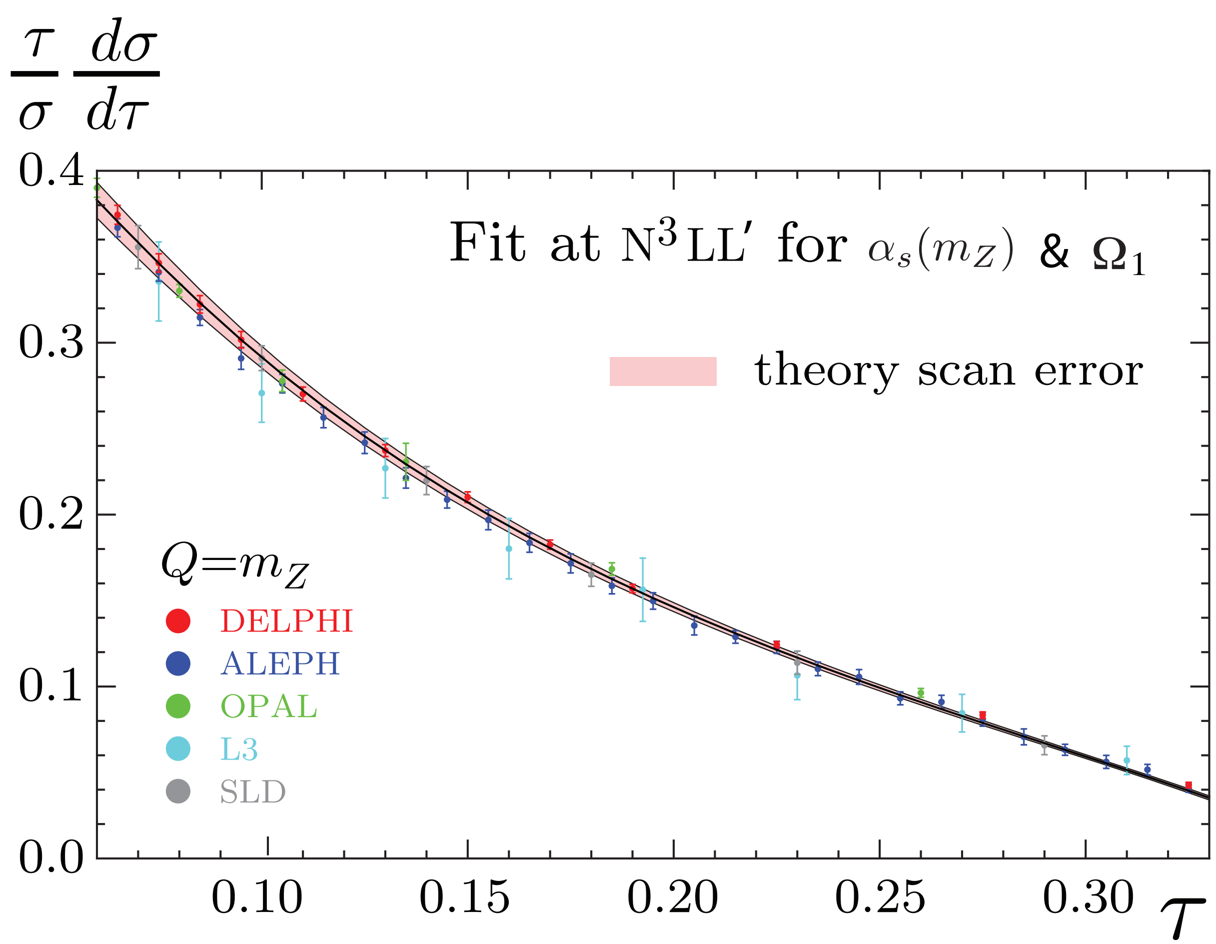
Mass corrections known at N^2LL and two loops

α_s determination: Thrust tail fits

[Abbate, Fickinger, Hoang, VM Stewart 1006.3080]

- N^3LL resummation, NNLO matrix elements
- Fits to Q > 34 GeV, global fit
- Thrust analysis only
- Power corrections OPE
- QED and bottom mass effects, axial singlet contribution
- Renormalon subtraction

Reproduce experimental data very accurately



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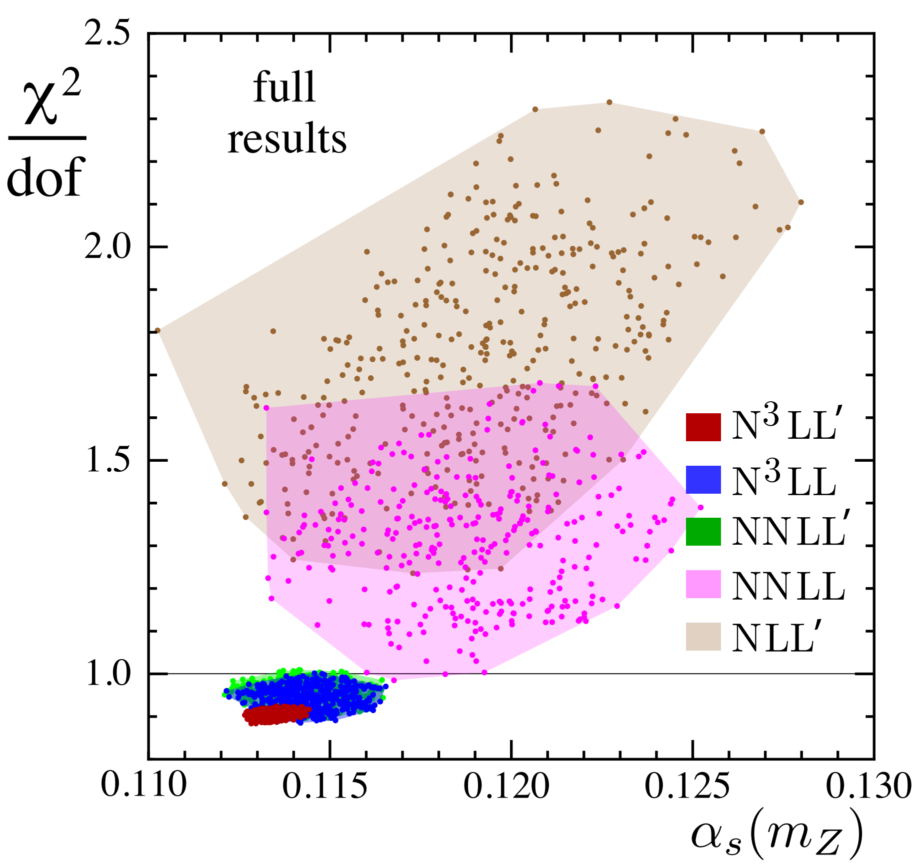
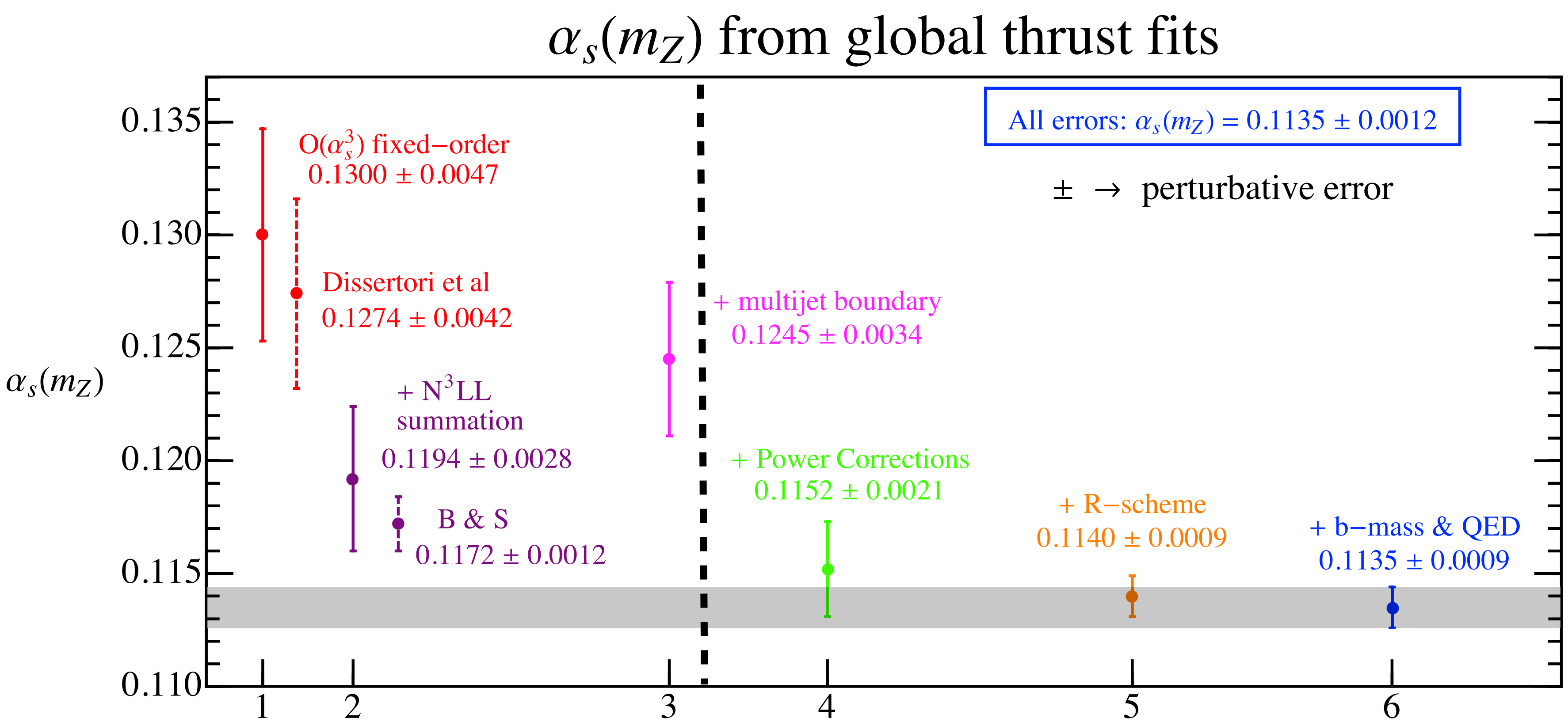
$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$





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$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$

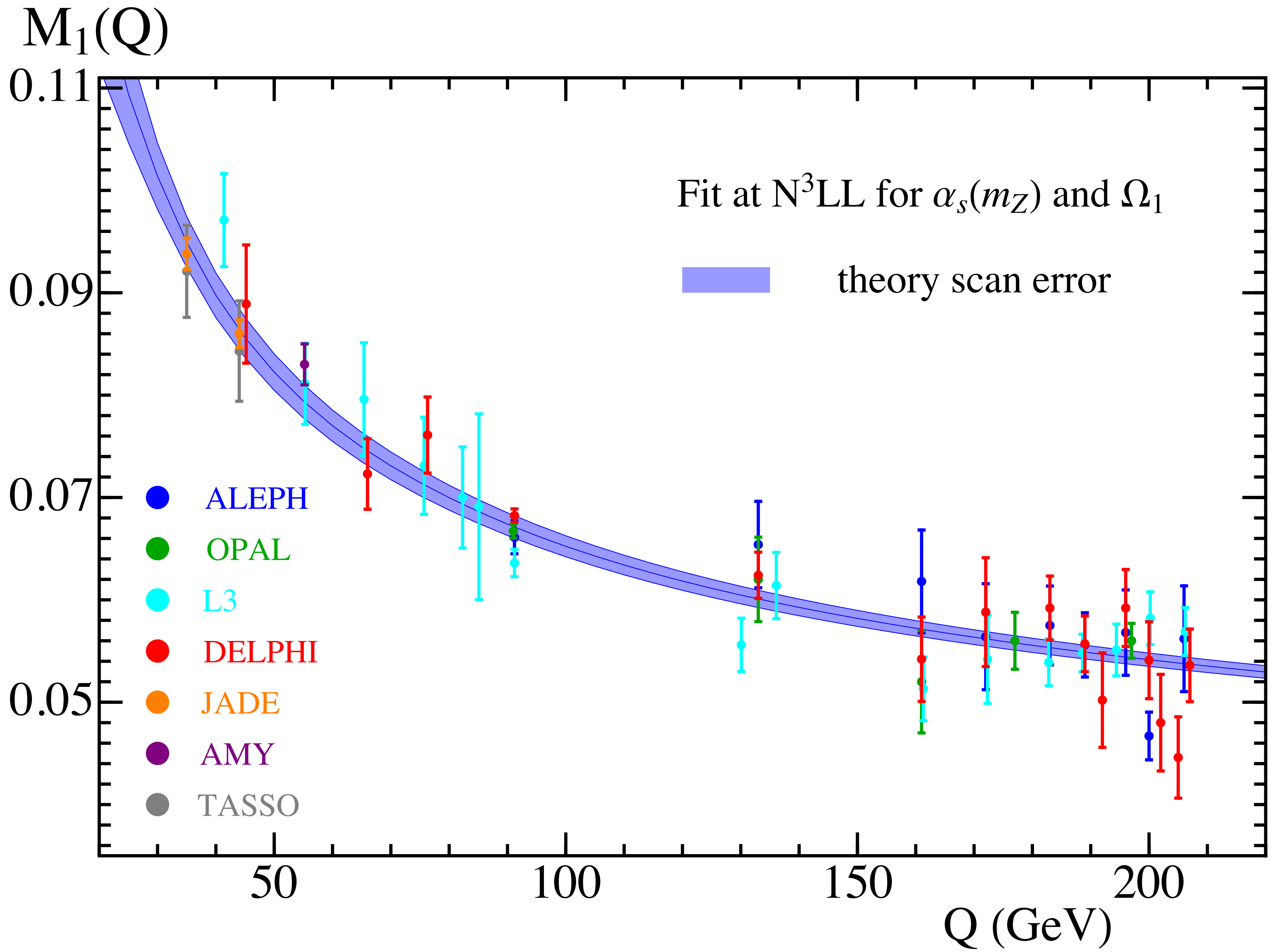
error includes conservative estimates of effects coming from higher order power corrections not included in the fit

α_s determination: Thrust moment fits

[Abbate, Fickinger, Hoang, VM, Stewart]

$$M_n = \frac{1}{\sigma}\int \mathrm{d}\tau\, \tau^n \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

Only first moment of thrust
Used N^3LL code, with power corrections and renormalon subtraction
Different levels of theoretical sophistication

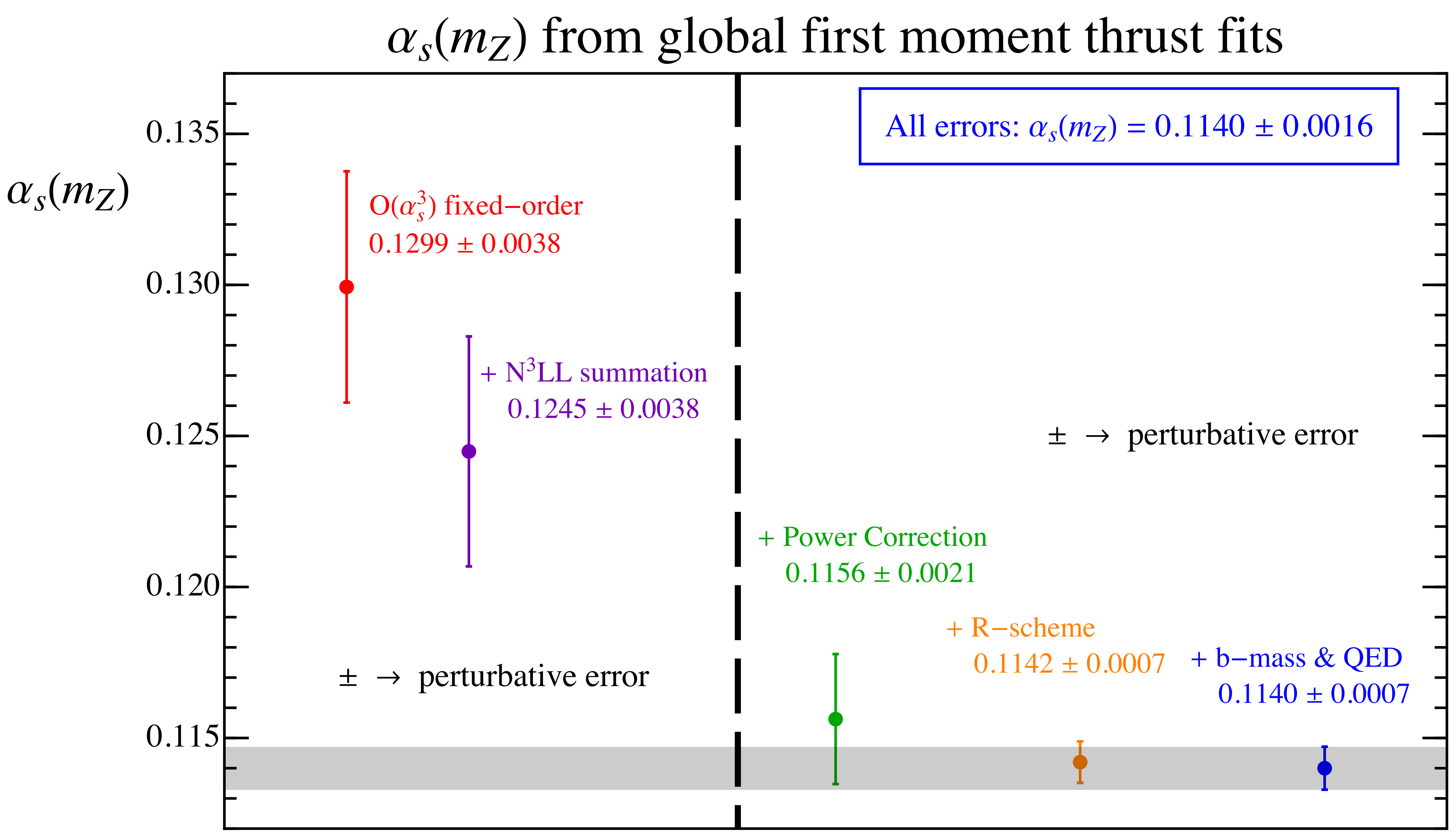


α_s determination: Thrust moment fits

[Abbate, Fickinger, Hoang, VM, Stewart]

$$M_n = \frac{1}{\sigma}\int d\tau\, \tau^n \frac{d\sigma}{d\tau}$$

Only first moment of thrust
Used N³LL code, with power corrections and renormalon subtraction
Different levels of theoretical sophistication

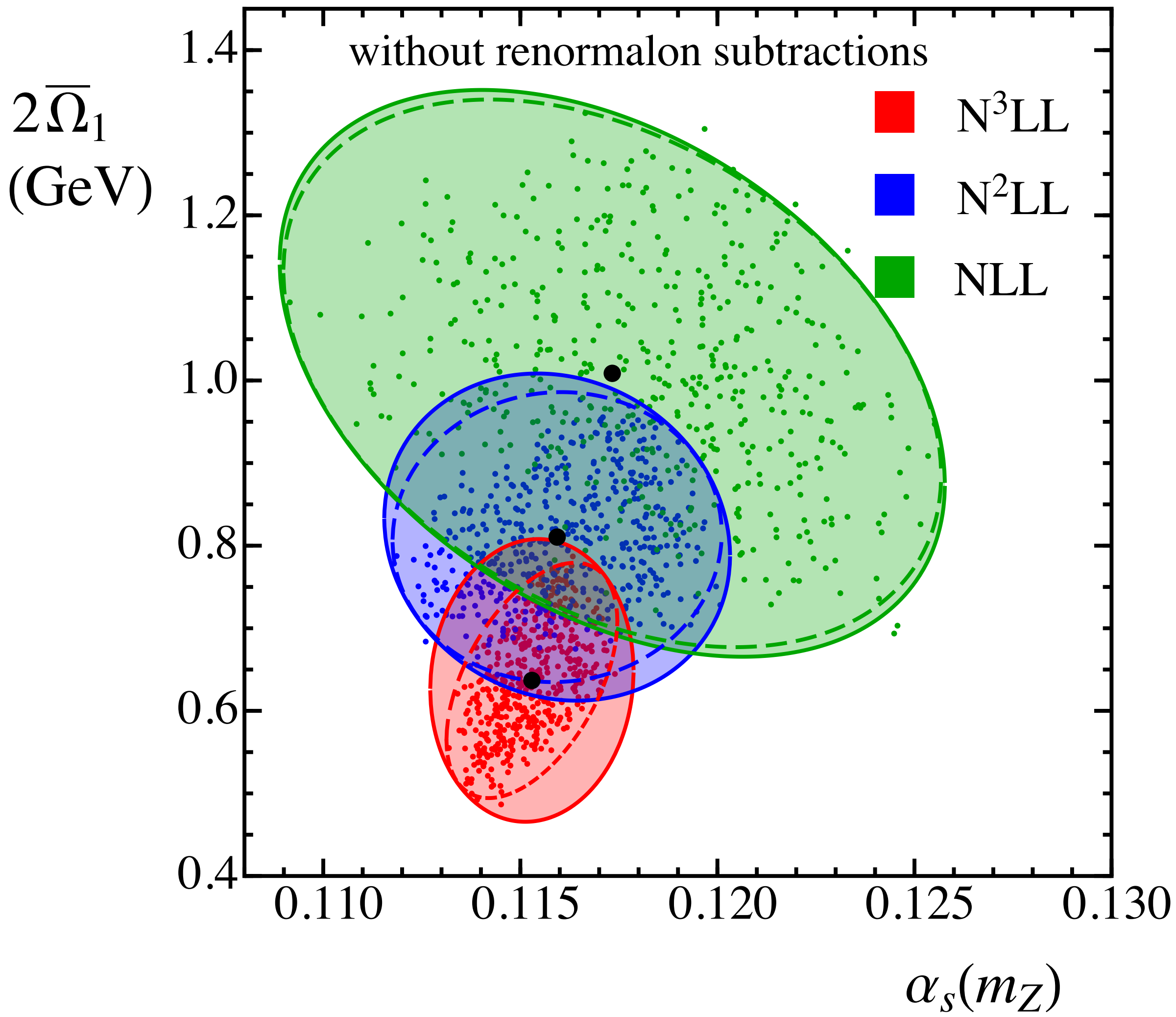


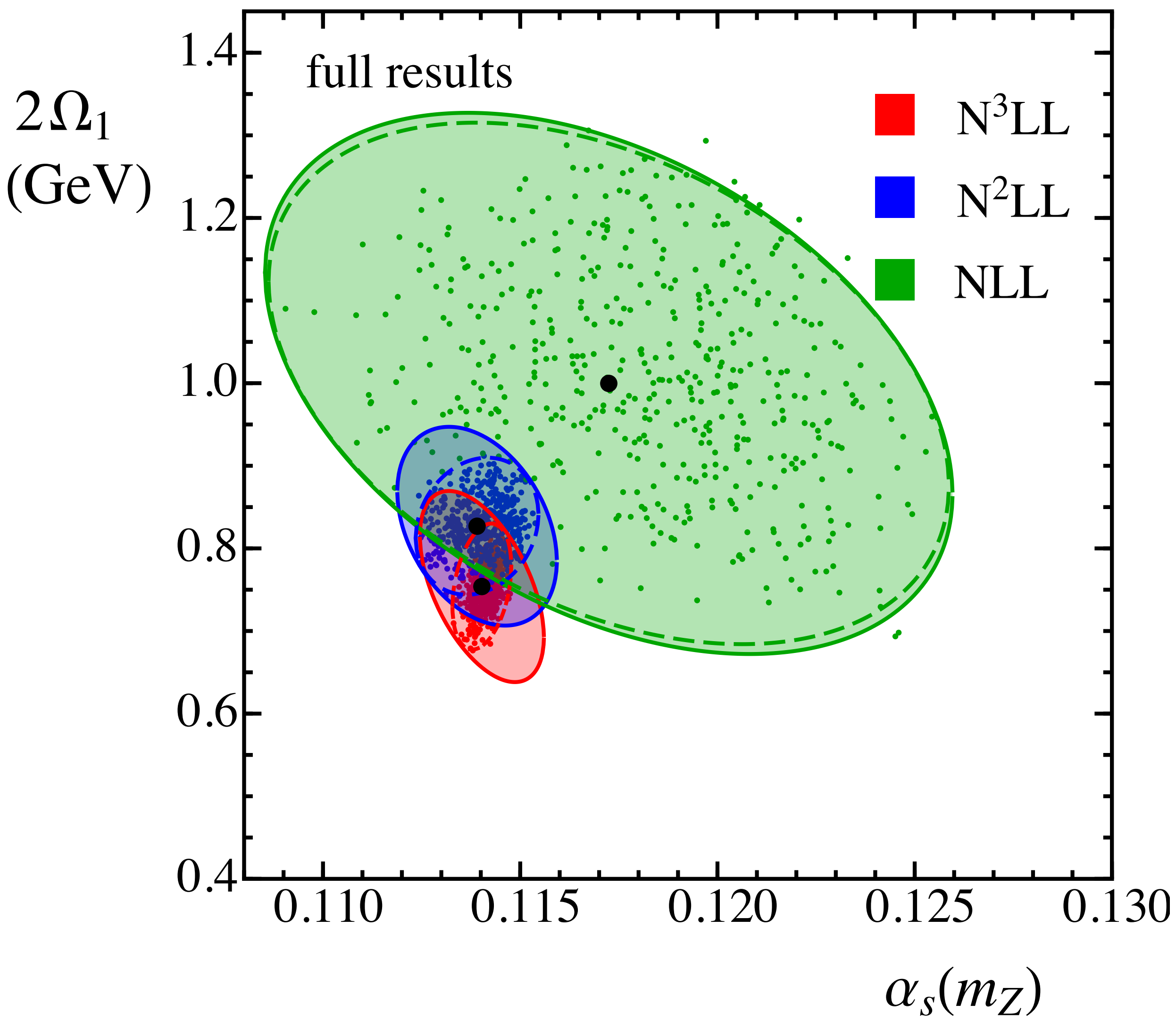
α_s determination: Thrust moment fits

[Abbate, Fickinger, Hoang, VM, Stewart]

$$M_n = \frac{1}{\sigma}\int \mathrm{d}\tau\,\tau^n\,\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

Only first moment of thrust
Used N^{3}LL code, with power corrections and renormalon subtraction
Different levels of theoretical sophistication
Significant error reduction when renormalon is removed





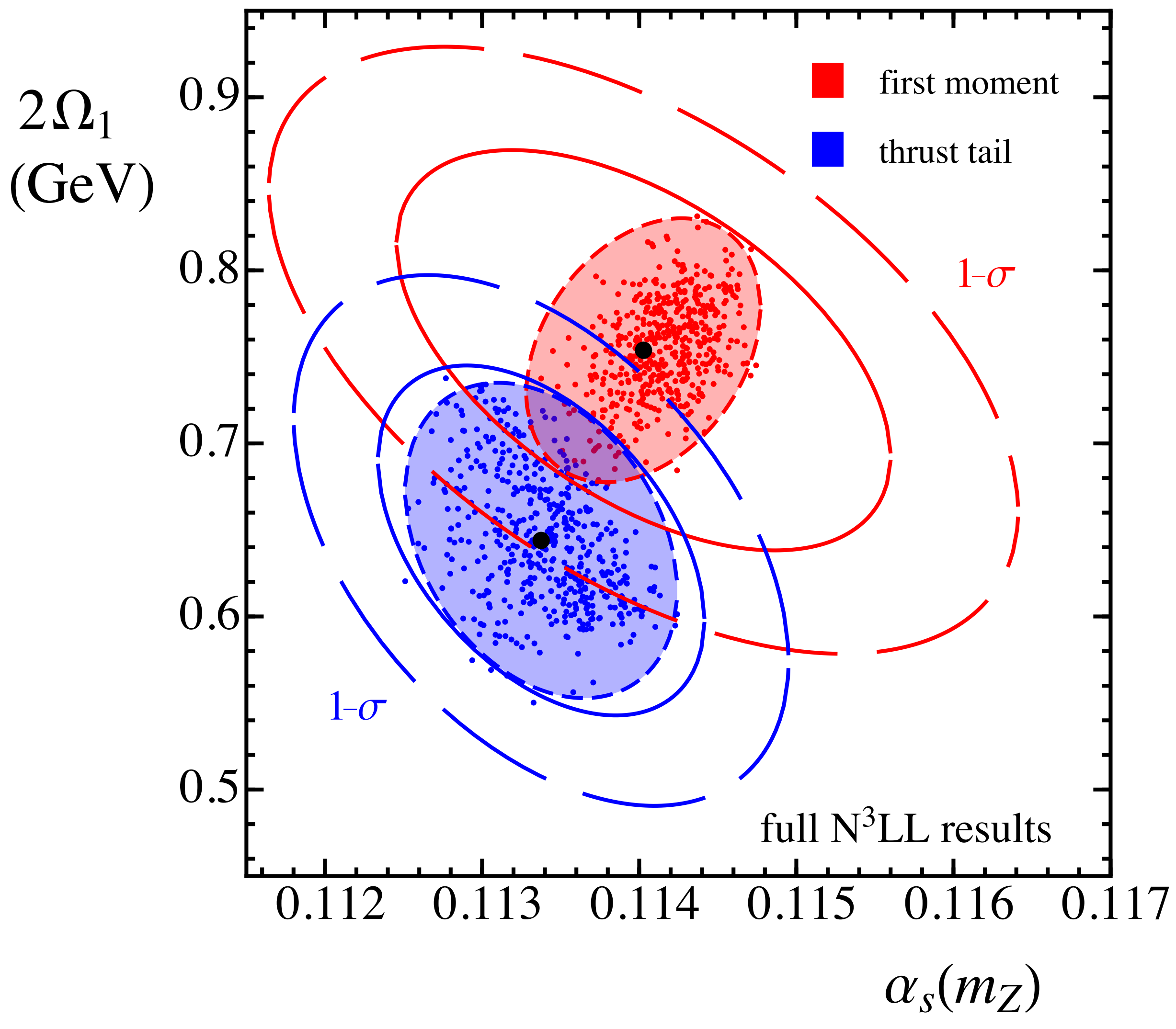
α_s determination: Thrust moment fits

[Abbate, Fickinger, Hoang, VM, Stewart]

$$M_n = \frac{1}{\sigma}\int \mathrm{d}\tau\,\tau^n \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

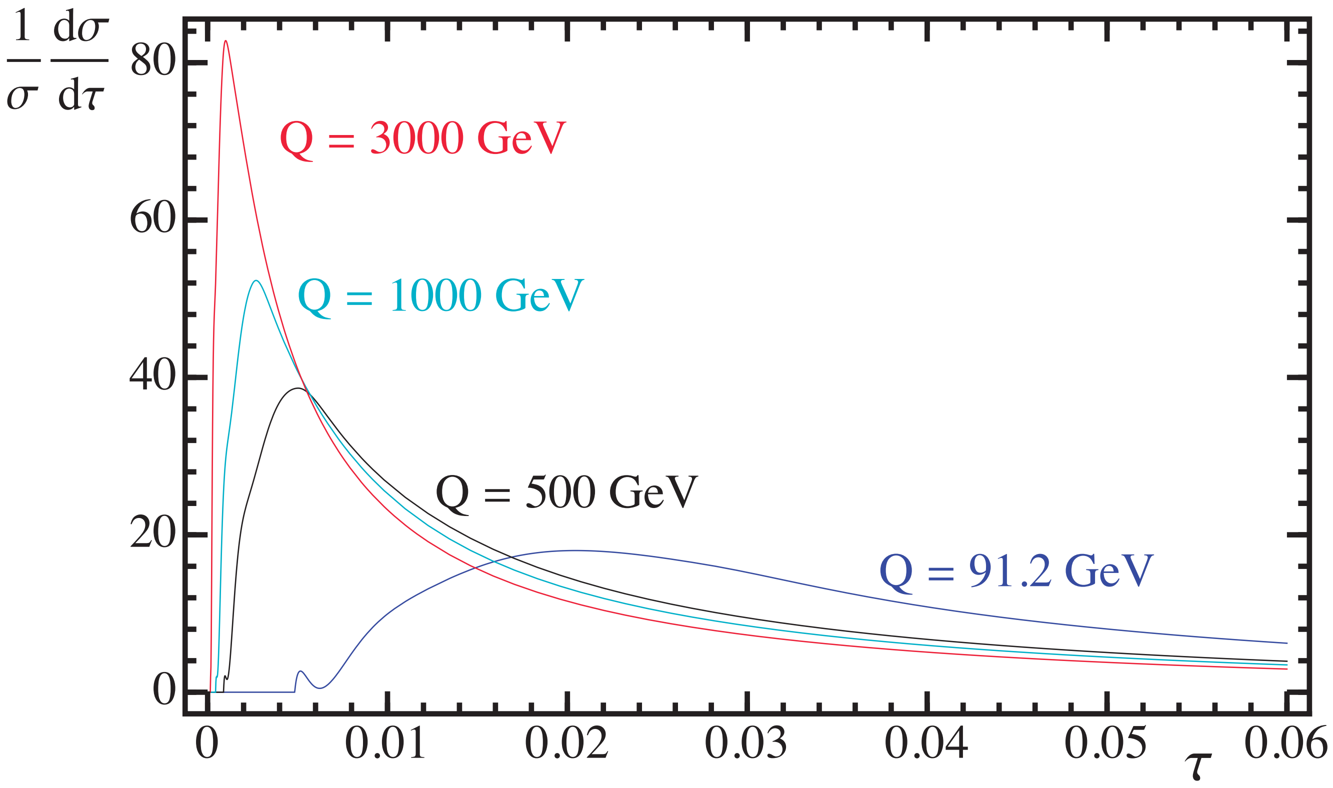
Only first moment of thrust
Used N^{3}LL code, with power corrections and renormalon subtraction
Different levels of theoretical sophistication
Significant error reduction when renormalon is removed

Good agreement with tail fits



Consequences for ILC

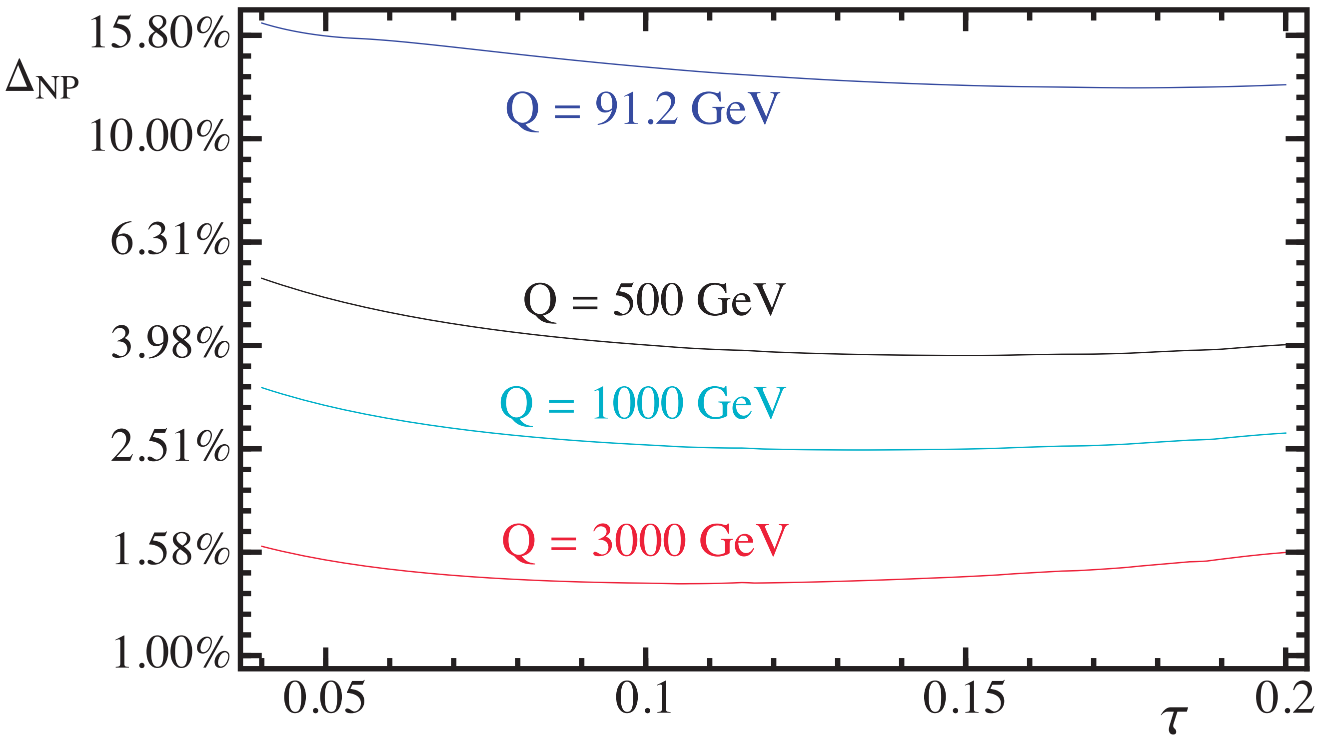
α_s determination: adding ILC data



<u>For increasing Q</u>

- Peak moves toward smaller
- Events tend to accumulate at very small region
- The tail regions becomes longer but less populated

α_s determination: adding ILC data



<u>For increasing Q</u>

- Size of nonperturbative effects decreases with Q
- They scale as 1/Q.
- At very high Q, nonperturbative effects become smaller than expt. errors **may be neglected**

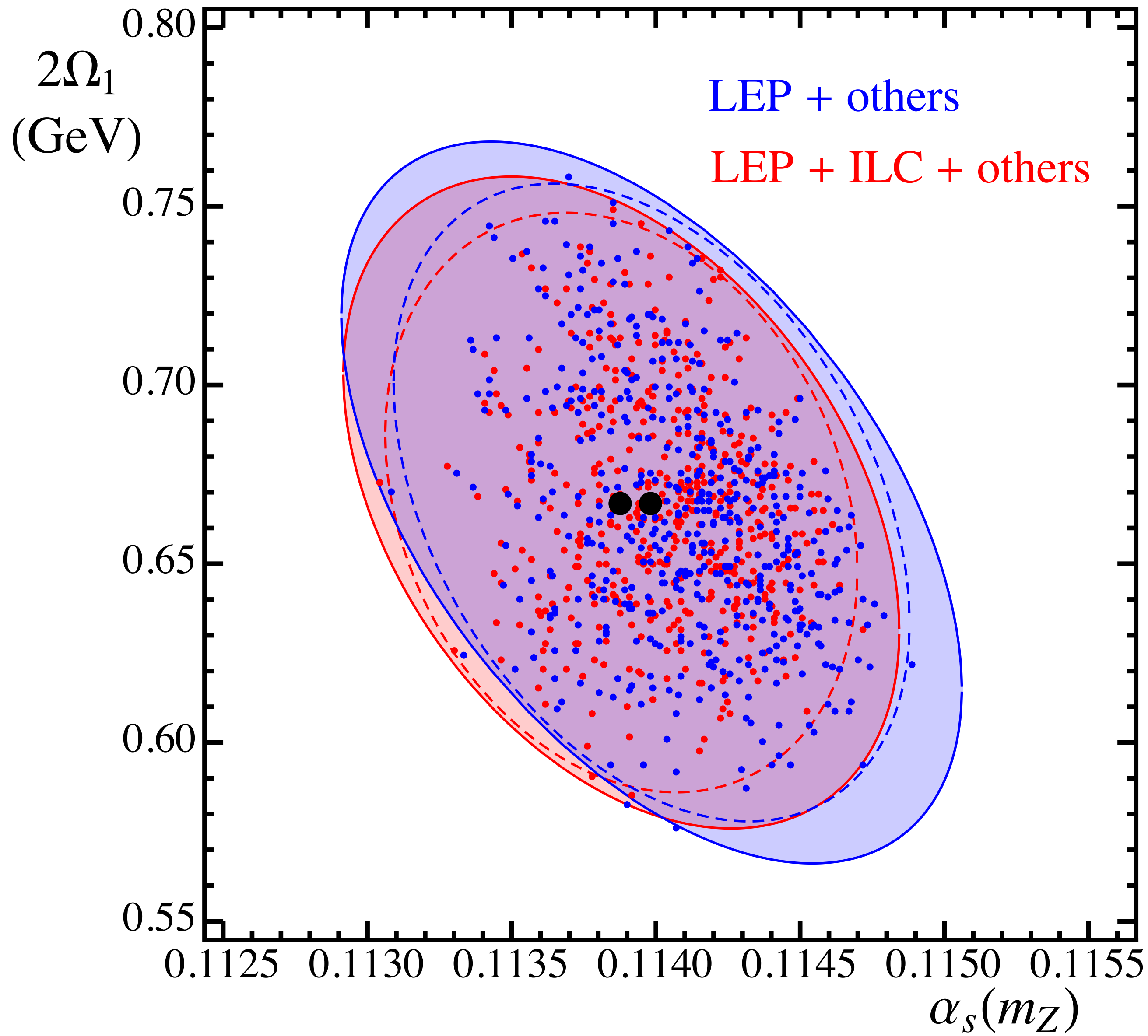
α_s determination: adding ILC data

Simple exercise: make up ILC data at 500 GeV

Assume 1% statistical and 1% systematic errors

Add this "ILC" data to LEP and other colliders data

Repeat fits



Unfortunately there is not much gain...

C-parameter

Theoretical knowledge

$H(Q,\mu)$ Hard function: same as thrust

$J_n(s,\mu)$ Jet function: same as thrust

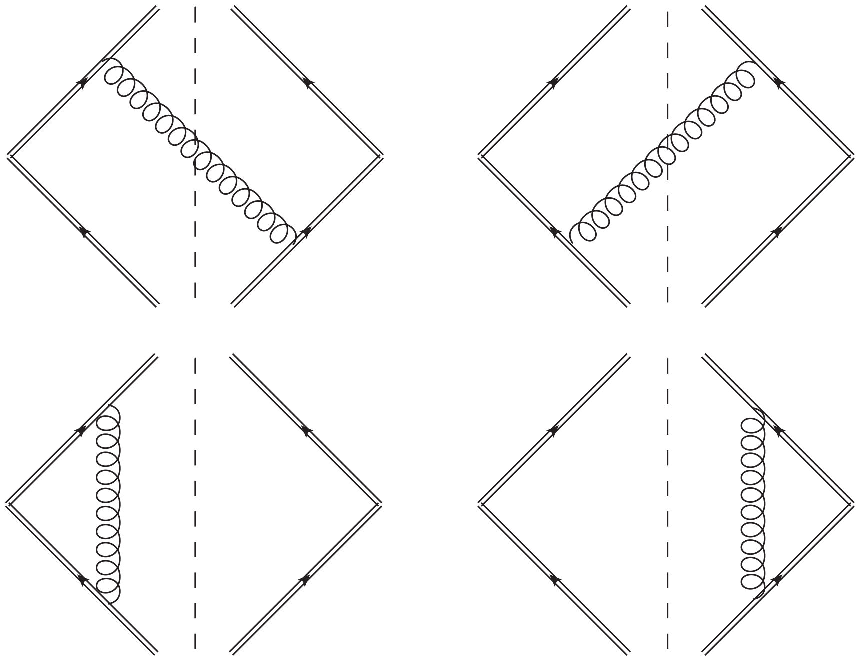
$S_C(\ell,\mu)$ Soft function known analytically at one loop, numerically at two loops

Running known at three loops

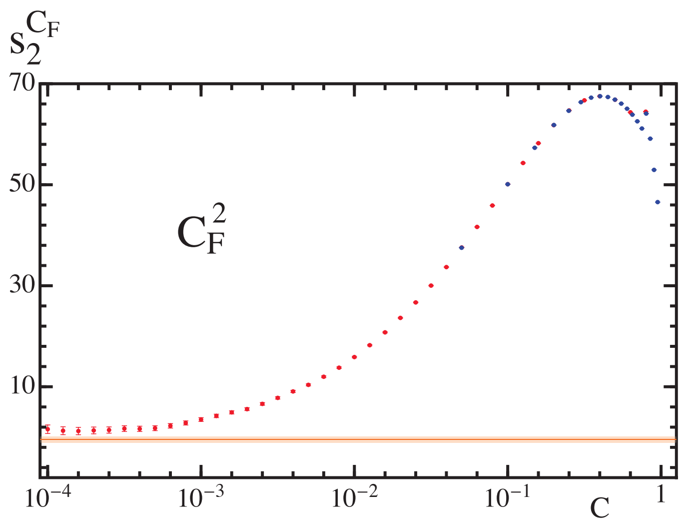
Fixed-order predictions known at three loops

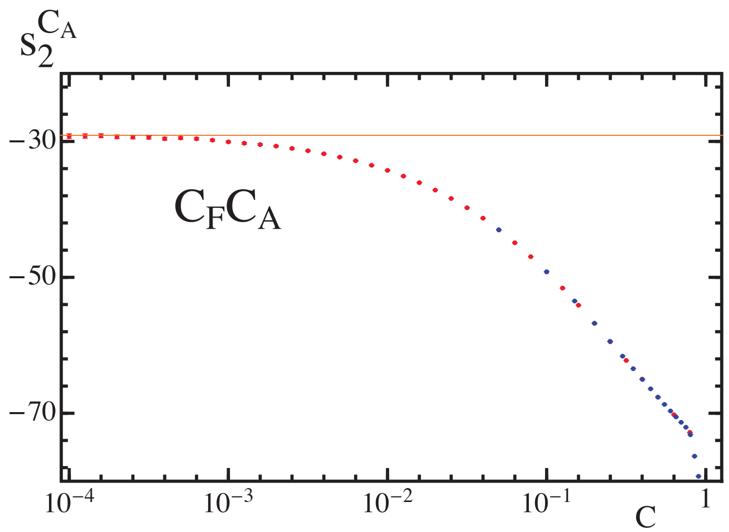
Mass corrections known at N^2LL and two loops

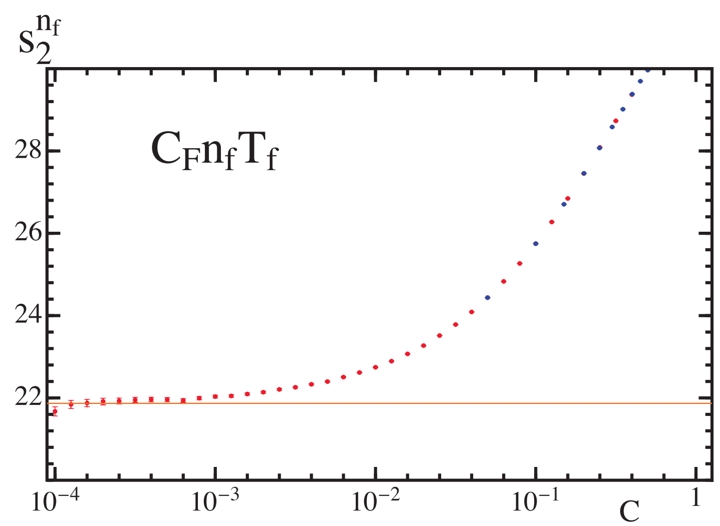
α_s determination: C-parameter tail fits



Analytic computation of soft function at 1-loop

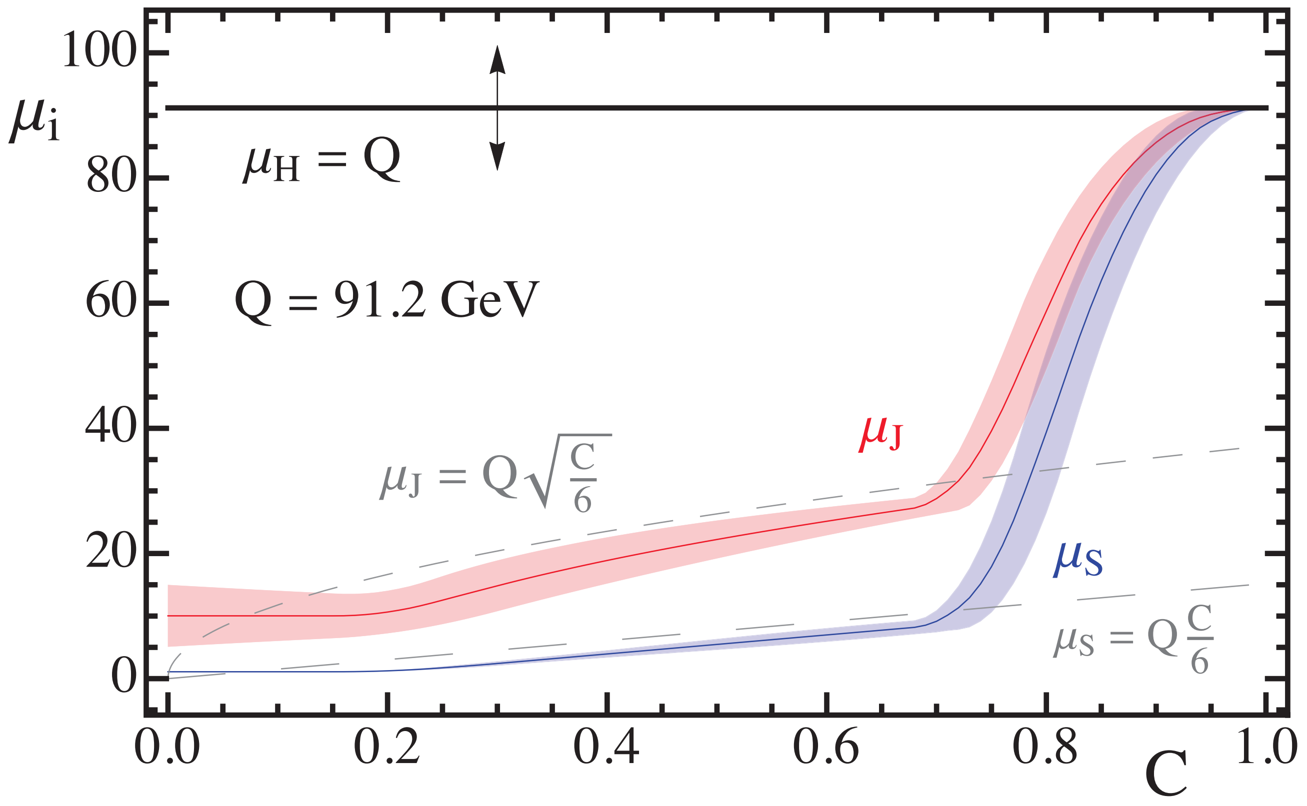






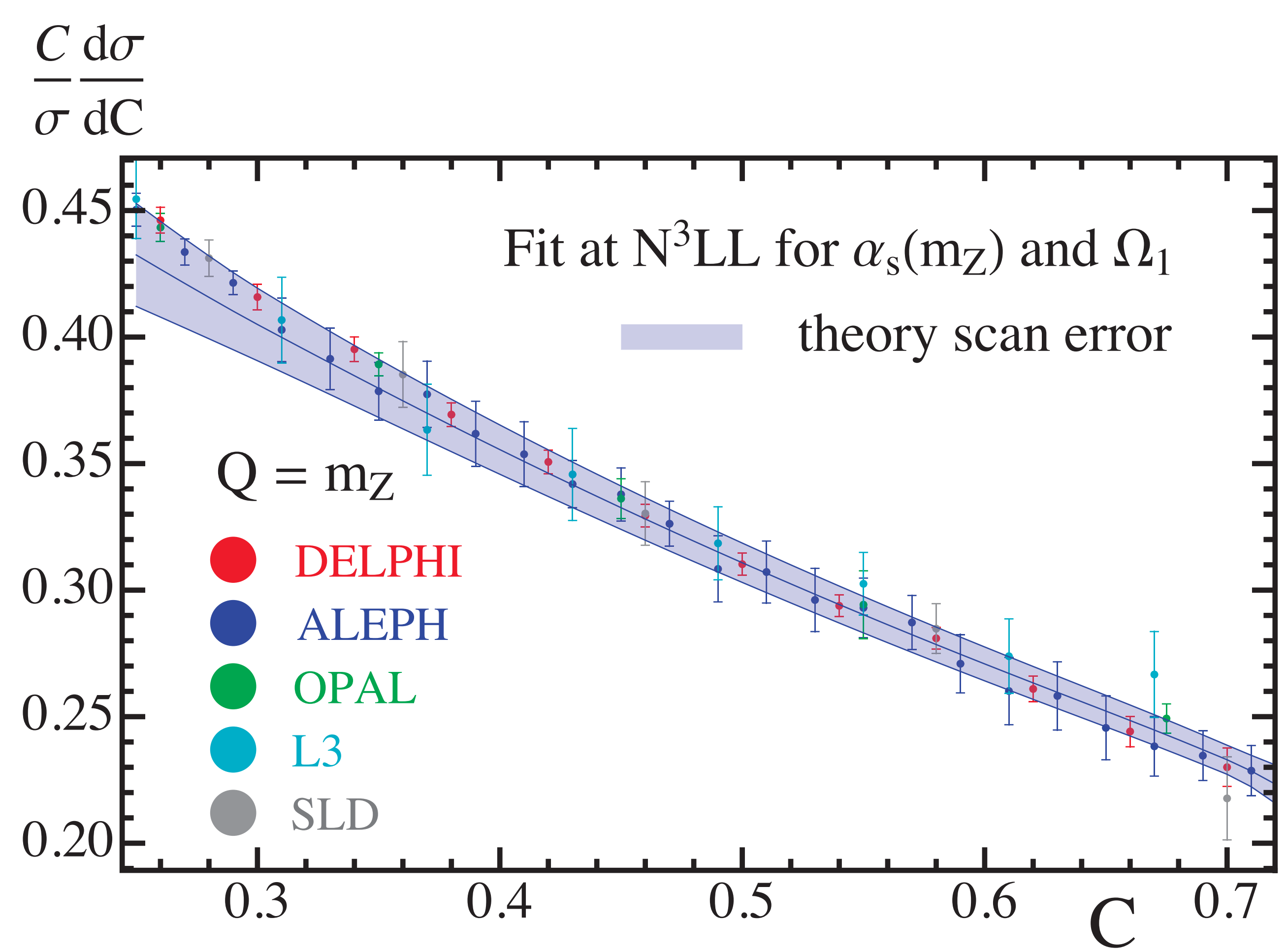
Numerical determination at 2-loops

α_s determination: C-parameter tail fits



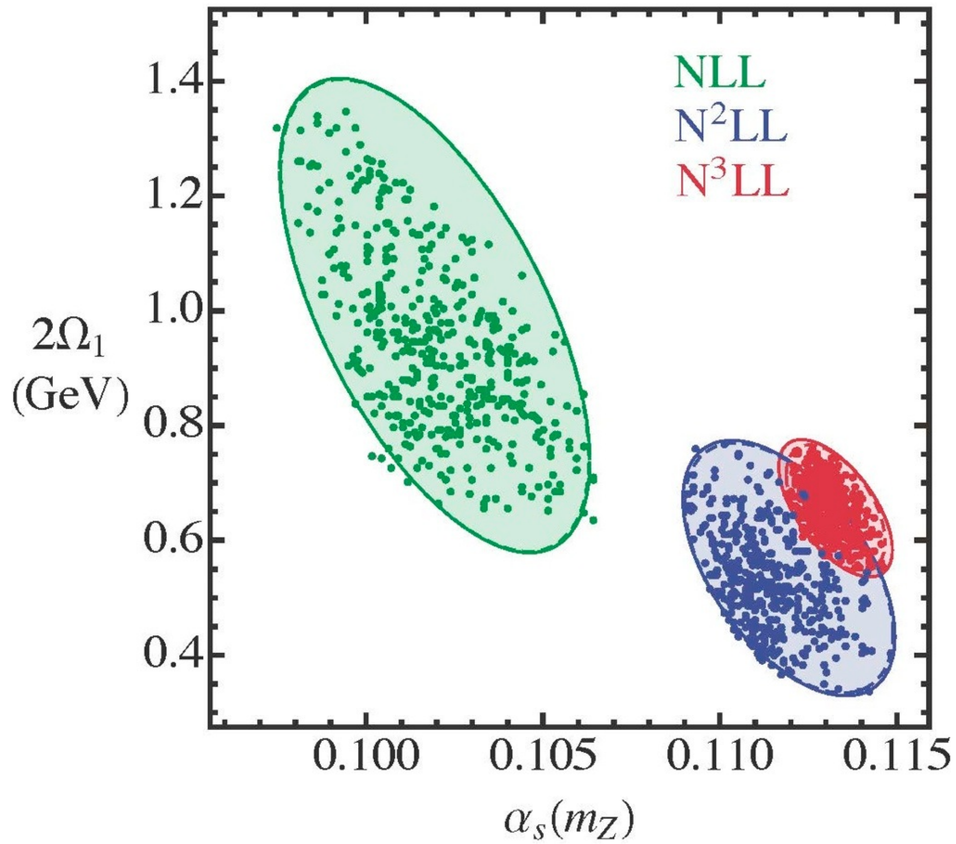
Complicated-looking renormalization scales

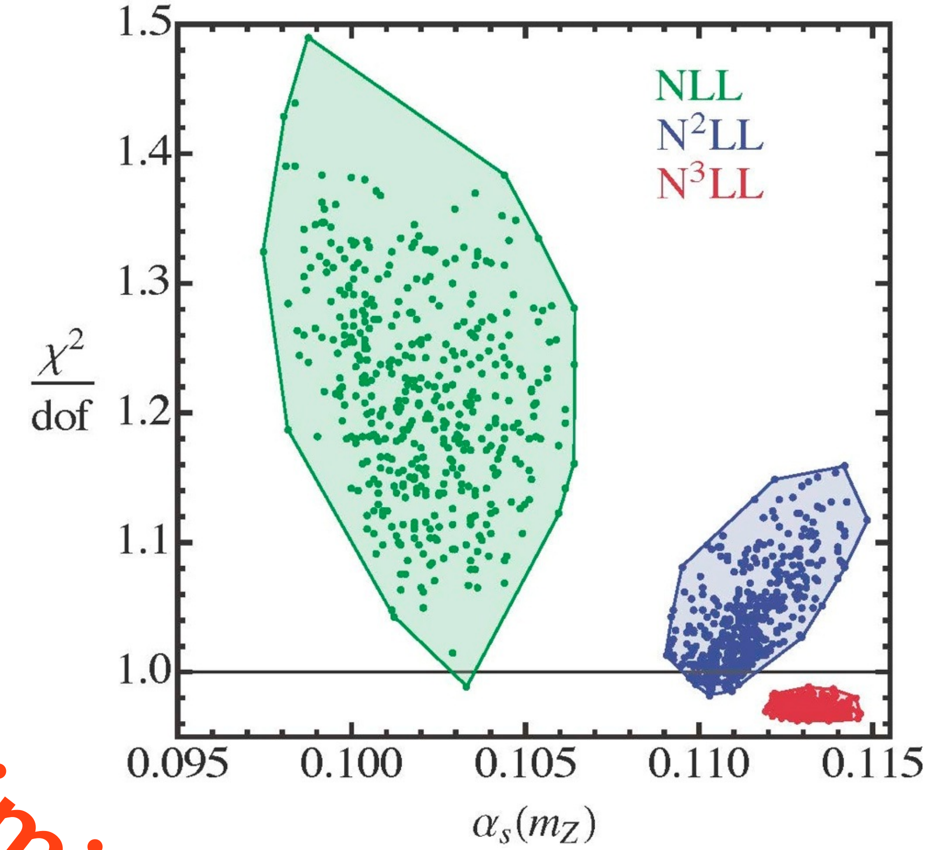
Estimate of theory uncertainties

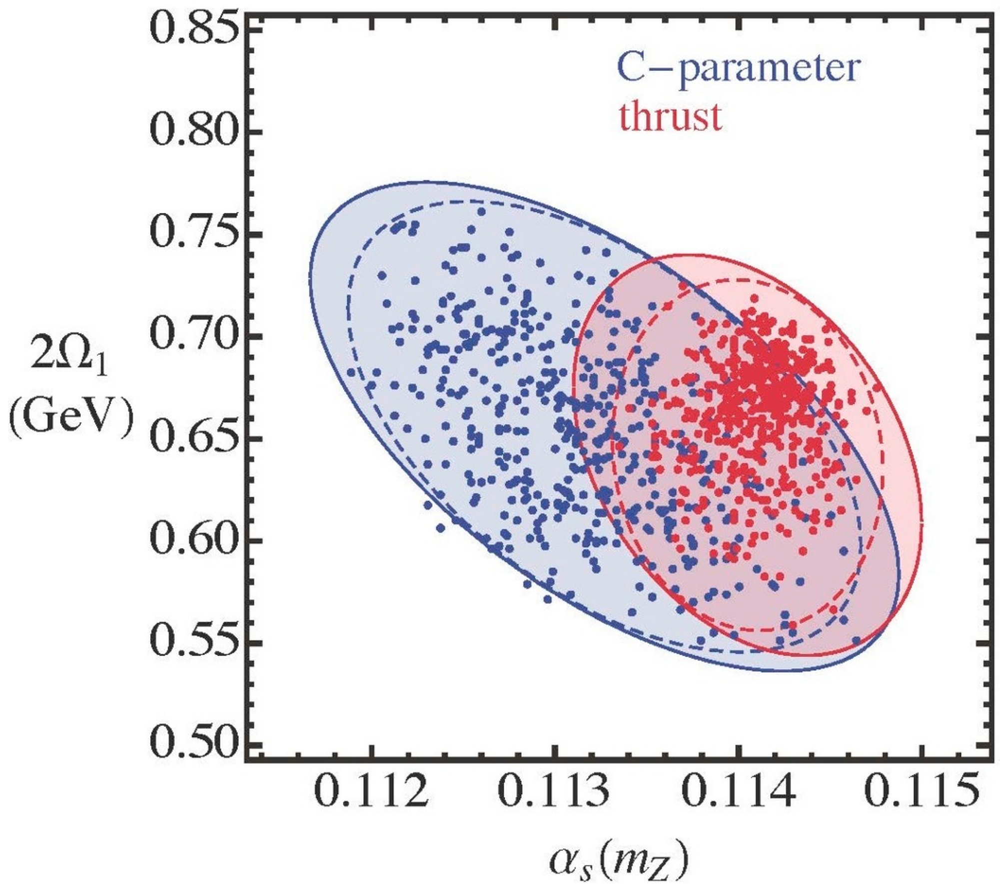


Very good description of experimental data

α_s determination: C-parameter tail fits







Preliminary results

$$\alpha_s(m_Z) = 0.1133 \pm 0.0016$$

Heavy Jet Mass

Theoretical knowledge

$H(Q,\mu)$ Hard function: same as thrust

$J_n(s,\mu)$ Jet function: same as thrust

$S(\ell_1,\ell_2,\mu)$ Soft function known analytically at two loops. Complicated non-global structure
Running known at three loops

Fixed-order predictions known at three loops

Mass corrections known at N²LL and two loops

Factorization Theorem

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}s_1\mathrm{d}s_2} = H(Q^2,\mu)\int \mathrm{d}k_1\,\mathrm{d}k_2\,J(s_1 - Qk_1,\mu)\,J(s_2 - Qk_2,\mu)\,S(k_1,k_2,\mu)$$

Factorization Theorem

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}s_1\mathrm{d}s_2} = H(Q^2,\mu)\int \mathrm{d}k_1\,\mathrm{d}k_2\,J(s_1 - Qk_1,\mu)\,J(s_2 - Qk_2,\mu)\,S(k_1,k_2,\mu)$$

double convolution

Factorization Theorem

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two-dimensional soft function

$$S(\ell_1,\ell_2,\mu) = \int \mathrm{d}k_1\,\mathrm{d}k_2\,S^{\mathrm{part}}(\ell_1 - k_1,\ell_2 - k_2,\mu)\,F(k_1,k_2)$$

Factorization Theorem

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}s_1\mathrm{d}s_2} = H(Q^2,\mu)\int \mathrm{d}k_1\,\mathrm{d}k_2\,J(s_1 - Qk_1,\mu)\,J(s_2 - Qk_2,\mu)\,S(k_1,k_2,\mu)$$

two-dimensional soft function

$$S(\ell_1,\ell_2,\mu) = \int \mathrm{d}k_1\,\mathrm{d}k_2\,S^{\mathrm{part}}(\ell_1 - k_1,\ell_2 - k_2,\mu)\,F(k_1,k_2)$$

two dimensional perturbative soft function

two dimensional non-perturbative shape function

Factorization Theorem

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}s_1\mathrm{d}s_2} = H(Q^2,\mu)\int \mathrm{d}k_1\,\mathrm{d}k_2\, J(s_1 - Qk_1,\mu)\, J(s_2 - Qk_2,\mu)\, S(k_1,k_2,\mu)$$

two-dimensional soft function

$$S(\ell_1,\ell_2,\mu) = \int \mathrm{d}k_1\,\mathrm{d}k_2\, S^{\mathrm{part}}(\ell_1 - k_1, \ell_2 - k_2,\mu)\, F(k_1,k_2)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = 2\,Q^2\int_0^{Q^2\rho} \mathrm{d}s_1 \left(\frac{\mathrm{d}\sigma}{\mathrm{d}s_1\mathrm{d}s_2}\right)_{s_2=Q^2\rho}$$

Heavy Jet Mass projection

Factorization Theorem

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}s_1\mathrm{d}s_2} = H(Q^2,\mu)\int \mathrm{d}k_1\,\mathrm{d}k_2\,J(s_1 - Qk_1,\mu)\,J(s_2 - Qk_2,\mu)\,S(k_1,k_2,\mu)$$

two-dimensional soft function

$$S(\ell_1,\ell_2,\mu) = \int \mathrm{d}k_1\,\mathrm{d}k_2\,S^{\mathrm{part}}(\ell_1 - k_1,\ell_2 - k_2,\mu)\,F(k_1,k_2)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = 2\,Q^2\int_0^{Q^2\rho}\mathrm{d}s_1\left(\frac{\mathrm{d}\sigma}{\mathrm{d}s_1\mathrm{d}s_2}\right)_{s_2=Q^2\rho} = \int_0^{Q\rho}\mathrm{d}k\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\rho}\left(\rho - \frac{k}{Q}\right)F_\rho(k)$$

Heavy Jet Mass projection

Factorization Theorem

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}s_1\mathrm{d}s_2} = H(Q^2,\mu)\int \mathrm{d}k_1\,\mathrm{d}k_2\,J(s_1 - Qk_1,\mu)\,J(s_2 - Qk_2,\mu)\,S(k_1,k_2,\mu)$$

two-dimensional soft function

$$S(\ell_1,\ell_2,\mu) = \int \mathrm{d}k_1\,\mathrm{d}k_2\,S^{\mathrm{part}}(\ell_1 - k_1, \ell_2 - k_2, \mu)\,F(k_1,k_2)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = 2\,Q^2\int_0^{Q^2\rho}\mathrm{d}s_1\left(\frac{\mathrm{d}\sigma}{\mathrm{d}s_1\mathrm{d}s_2}\right)_{s_2=Q^2\rho} = \int_0^{Q\rho}\mathrm{d}k\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\rho}\left(\rho - \frac{k}{Q}\right)F_\rho(k)$$

Heavy Jet Mass projection

Operator Product Expansion

$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i F(k_1, k_2)$$

Thrust power corrections

Operator Product Expansion

$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i F(k_1, k_2) \; = 2^i \, \Omega_{0,i}$$

Thrust power corrections

$$\Omega_{i,j} = \Omega_{j,i} = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i \, (k_2)^j \, F(k_1, k_2)$$

HJM power corrections

Operator Product Expansion

$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i F(k_1, k_2) \; = 2^i \, \Omega_{0,i}$$

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HJM power corrections

$$\Upsilon_{i,j} = \int_0^\infty \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1 - k_2) \, \theta(k_1 - k_2)(k_1)^i (k_2)^j \, F(k_1, k_2)$$

HJM moment

power corrections

Operator Product Expansion

$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i F(k_1, k_2) \; = 2^i \, \Omega_{0,i}$$

Thrust power corrections

$$\Omega_{i,j} = \Omega_{j,i} = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i \, (k_2)^j \, F(k_1, k_2)$$

HJM power corrections

$$\Upsilon_{i,j} = \int_0^\infty \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1 - k_2) \, \theta(k_1 - k_2)(k_1)^i (k_2)^j \, F(k_1, k_2)$$

HJM moment power corrections

Operator product expansion in the tail

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat\sigma}{\mathrm{d}\rho} - \frac{\Omega_{1,0}}{Q} \frac{\mathrm{d}^2\hat\sigma}{\mathrm{d}\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{\mathrm{d}^3\hat\sigma}{\mathrm{d}\rho^3} + 2 \, \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{\mathrm{d}^2\hat\sigma}{\mathrm{d}s_1 \mathrm{d}s_2} \right)_{s_1 = s_2 = Q^2 \rho}$$

Operator product expansion for thrust

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{\mathrm{d}\hat\sigma}{\mathrm{d}\rho} + \sum_n^\infty (-2)^n \, \frac{\Omega_{n,0}}{Q^n} \, \frac{\mathrm{d}^n\hat\sigma}{\mathrm{d}\tau^n}$$

Operator Product Expansion

$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2\,(k_1)^i F(k_1,k_2) \;=2^i\,\Omega_{0,i}$$

Thrust power corrections

$$\Omega_{i,j} = \Omega_{j,i} = \int \mathrm{d}k_1 \mathrm{d}k_2\,(k_1)^i\,(k_2)^j\,F(k_1,k_2)$$

HJM power corrections

$$\Upsilon_{i,j} = \int_0^\infty \mathrm{d}k_1 \mathrm{d}k_2\,(k_1-k_2)\,\theta(k_1-k_2)(k_1)^i(k_2)^j\,F(k_1,k_2)$$

HJM moment power corrections

Operator product expansion in the tail

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat\sigma}{\mathrm{d}\rho} - \frac{\Omega_{1,0}}{Q}\frac{\mathrm{d}^2\hat\sigma}{\mathrm{d}\rho^2} + \frac{1}{2}\frac{\Omega_{2,0}}{Q^2}\frac{\mathrm{d}^3\hat\sigma}{\mathrm{d}\rho^3} + 2\,\frac{\Omega_{1,1}-\Omega_{2,0}}{Q^2}\left(Q^6\frac{\mathrm{d}^2\hat\sigma}{\mathrm{d}s_1\mathrm{d}s_2}\right)_{s_1=s_2=Q^2\rho}$$

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{\mathrm{d}\hat\sigma}{\mathrm{d}\rho} + \sum_n^\infty (-2)^n\frac{\Omega_{n,0}}{Q^n}\frac{\mathrm{d}^n\hat\sigma}{\mathrm{d}\tau^n}$$

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$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i F(k_1, k_2) \; = 2^i \, \Omega_{0,i}$$

Thrust power corrections

$$\Omega_{i,j} = \Omega_{j,i} = \int \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1)^i \, (k_2)^j \, F(k_1, k_2)$$

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$$\Upsilon_{i,j} = \int_0^\infty \mathrm{d}k_1 \mathrm{d}k_2 \, (k_1 - k_2) \, \theta(k_1 - k_2) (k_1)^i (k_2)^j \, F(k_1, k_2)$$

HJM moment power corrections

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\rho} - \frac{\Omega_{1,0}}{Q} \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{\mathrm{d}^3\hat{\sigma}}{\mathrm{d}\rho^3} + 2 \, \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}s_1 \mathrm{d}s_2} \right)_{s_1 = s_2 = Q^2 \rho}$$

universal* leading power correction

non-universal subleading power correction

* modulo hadron mass effects

Operator Product Expansion

$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2\, (k_1)^i F(k_1, k_2) \;= 2^i\, \Omega_{0,i}$$

Thrust power corrections

$$\Omega_{i,j} = \Omega_{j,i} = \int \mathrm{d}k_1 \mathrm{d}k_2\, (k_1)^i\, (k_2)^j\, F(k_1, k_2)$$

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$$\Upsilon_{i,j} = \int_0^\infty \mathrm{d}k_1 \mathrm{d}k_2\, (k_1 - k_2)\, \theta(k_1 - k_2) (k_1)^i (k_2)^j\, F(k_1, k_2)$$

HJM moment power corrections

Operator product expansion in the tail

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\rho} - \frac{\Omega_{1,0}}{Q} \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{\mathrm{d}^3\hat{\sigma}}{\mathrm{d}\rho^3} + 2\, \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}s_1 \mathrm{d}s_2} \right)_{s_1 = s_2 = Q^2 \rho}$$

Operator product expansion for tree-level moments

$$M^{\rho}_{n,\mathrm{tree}} = \frac{1}{Q^n} \left(\Omega_{n,0} + \sum_{k=0}^{n-1} \Upsilon_{n-1-k,k} \right)$$

Operator Product Expansion

$$\Omega_i = \int \mathrm{d}k_1 \mathrm{d}k_2 \,(k_1)^i F(k_1,k_2) \; = 2^i\, \Omega_{0,i}$$

Thrust power corrections

$$\Omega_{i,j} = \Omega_{j,i} = \int \mathrm{d}k_1 \mathrm{d}k_2 \,(k_1)^i \,(k_2)^j \, F(k_1,k_2)$$

HJM power corrections

$$\Upsilon_{i,j} = \int_0^\infty \mathrm{d}k_1 \mathrm{d}k_2 \,(k_1 - k_2)\,\theta(k_1-k_2)(k_1)^i (k_2)^j \, F(k_1,k_2)$$

HJM moment power corrections

Operator product expansion in the tail

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = \frac{\mathrm{d}\hat\sigma}{\mathrm{d}\rho} - \frac{\Omega_{1,0}}{Q}\frac{\mathrm{d}^2\hat\sigma}{\mathrm{d}\rho^2} + \frac{1}{2}\frac{\Omega_{2,0}}{Q^2}\frac{\mathrm{d}^3\hat\sigma}{\mathrm{d}\rho^3} + 2\,\frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2}\left(Q^6 \frac{\mathrm{d}^2\hat\sigma}{\mathrm{d}s_1 \mathrm{d}s_2}\right)_{s_1=s_2=Q^2\rho}$$

Operator product expansion for tree-level moments

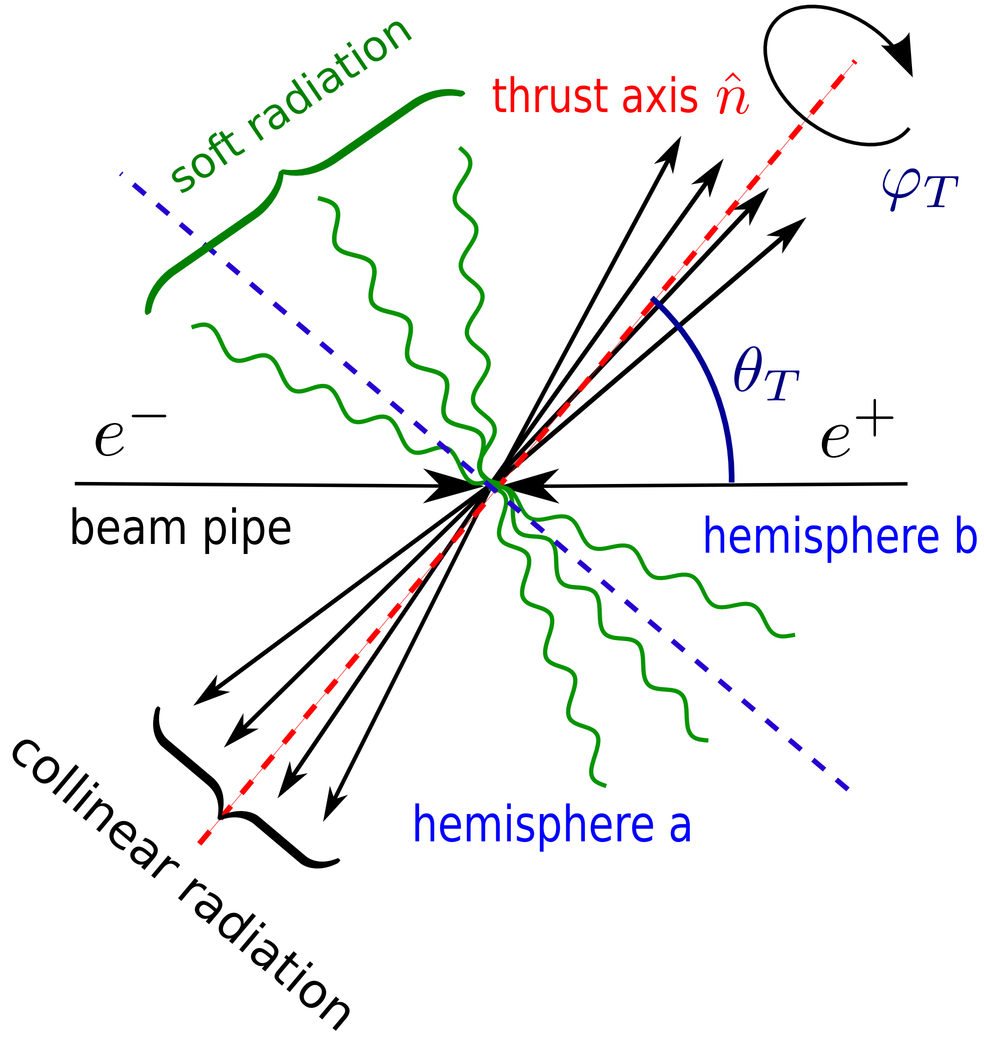
OPE parameters

$$M^\rho_{n,\mathrm{tree}} = \frac{1}{Q^n}\left(\Omega_{n,0} + \sum_{k=0}^{n-1} \Upsilon_{n-1-k,k}\right)$$

these moments do not show up in tail OPE !!!

Oriented Event Shapes

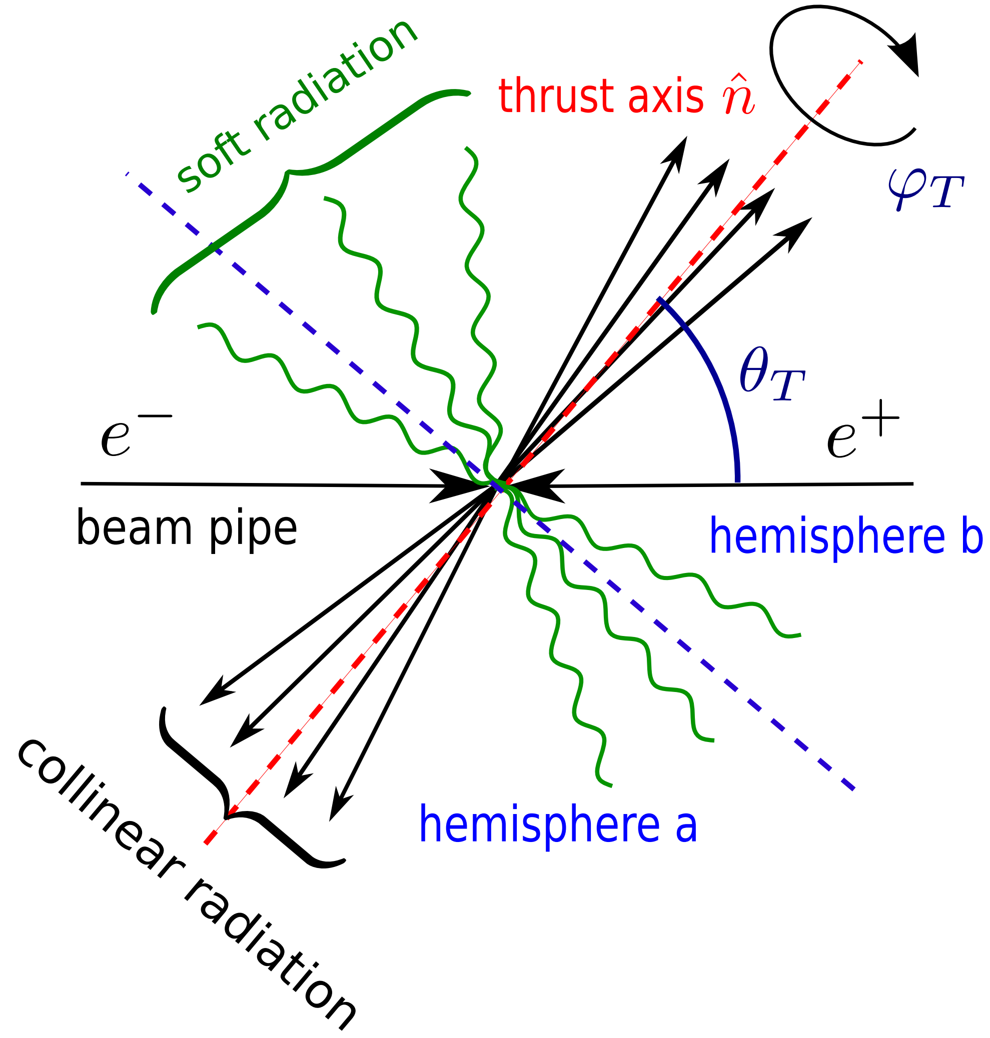
Oriented event shapes



$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e\,\mathrm{d}\cos\theta_T} = f(e,\cos\theta_T)$$

a priory one expects a general function of e and cos

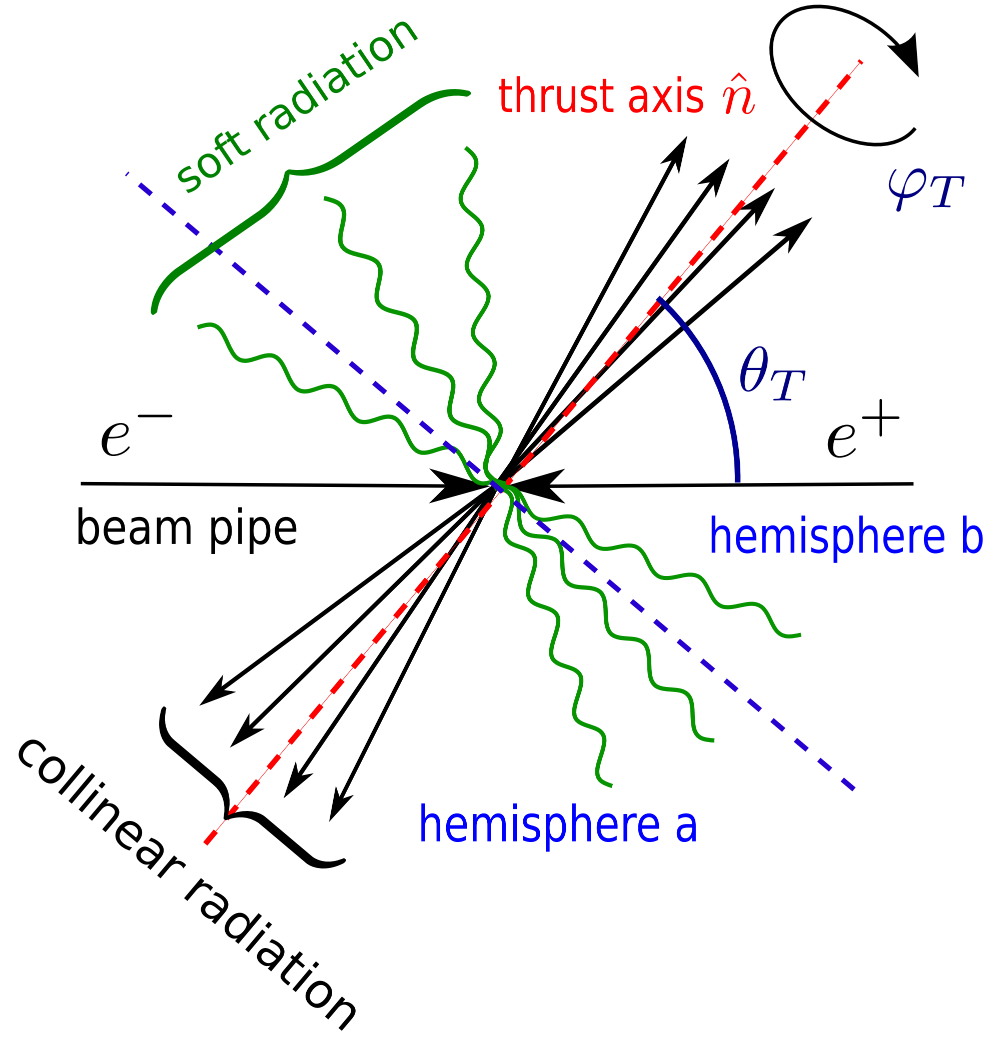
Oriented event shapes



General proof: [VM and G. Rodrigo JHEP11(2013)030]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e\,\mathrm{d}\cos\theta_T} = \frac{3}{8}\left(1+\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} + \left(1-3\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{ang}}}{\mathrm{d}e}$$

Oriented event shapes



General proof: [VM and G. Rodrigo JHEP11(2013)030]

$$\frac{1}{\sigma_0}\frac{d\sigma}{de\,d\cos\theta_T} = \frac{3}{8}\left(1+\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{d\sigma}{de} + \left(1-3\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{d\sigma_{\rm ang}}{de}$$

singular terms here

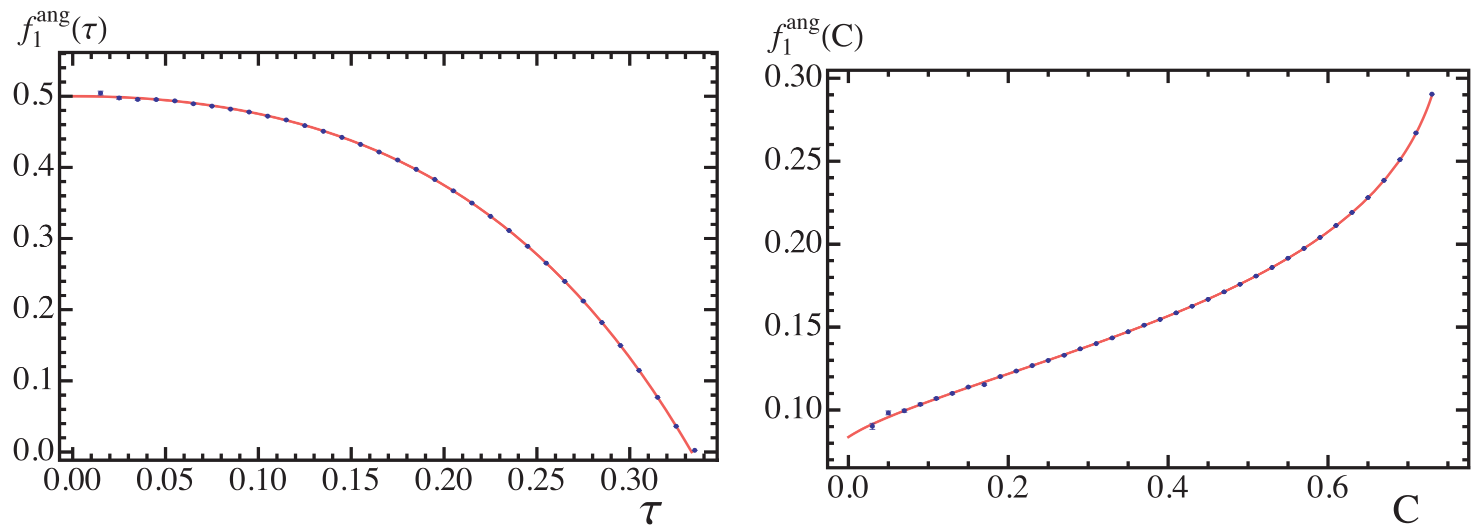
non-singular terms here

Oriented event shapes

[VM and G. Rodrigo 1307.3513]

Analytic computation at LO

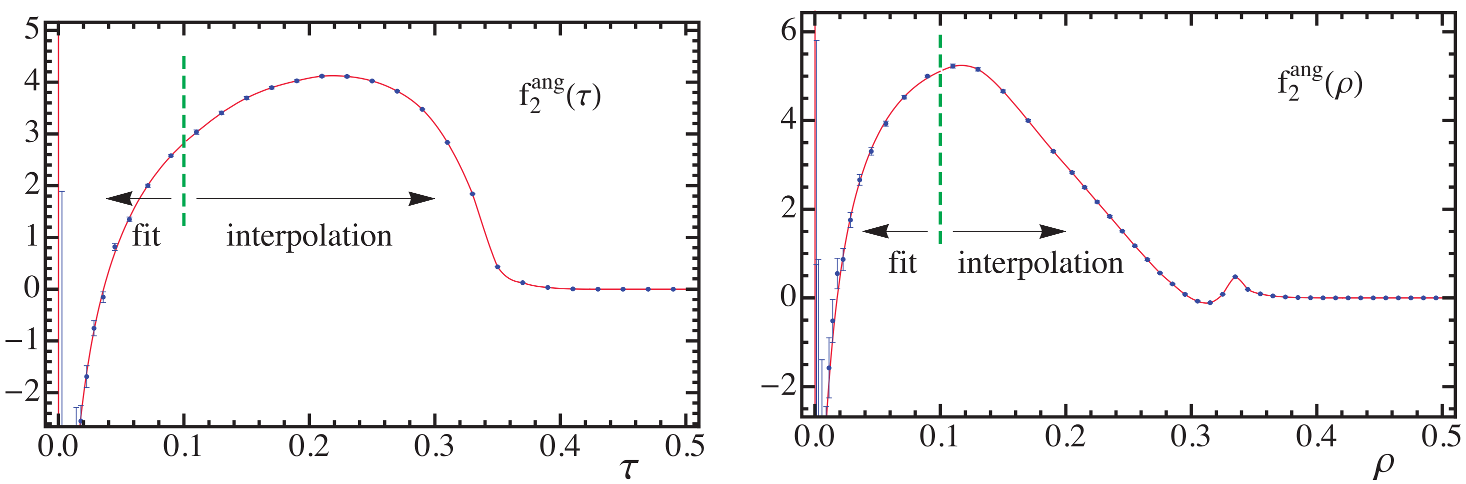
Agreement with EVENT2



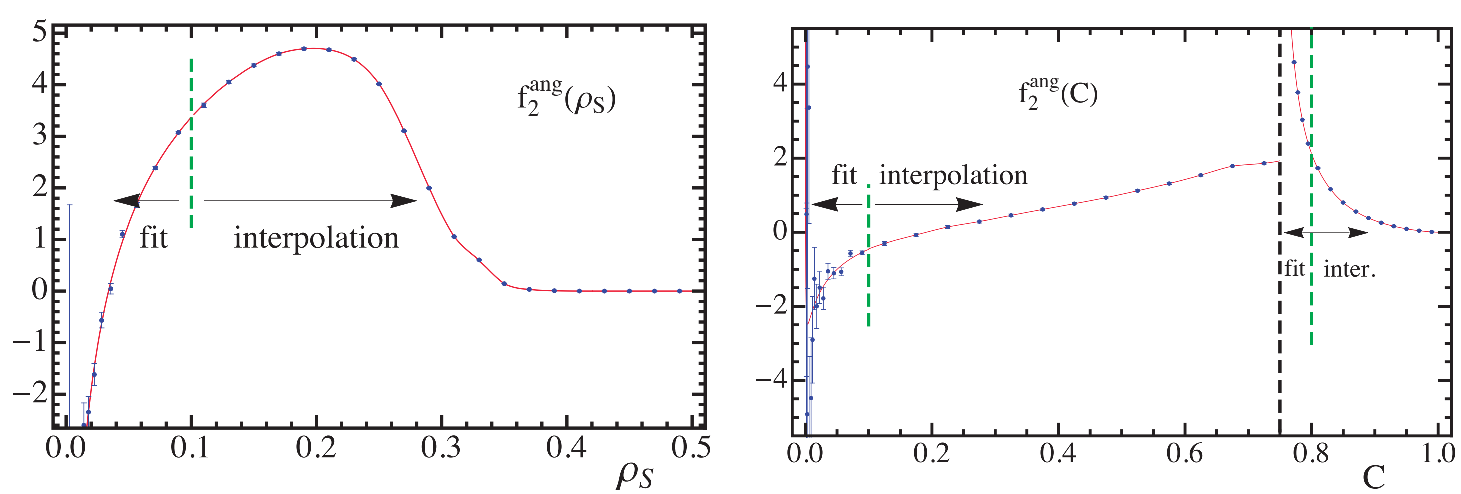
[VM and G. Rodrigo 1307.3513]

Numerical determination at NLO

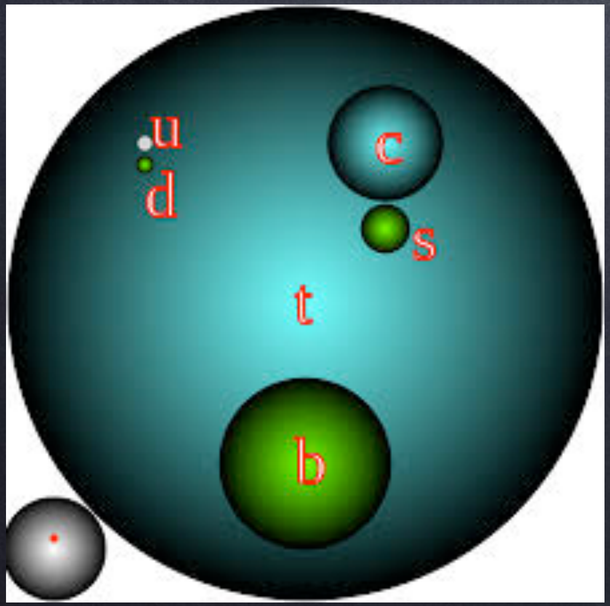
Using EVENT2



(a) (b)

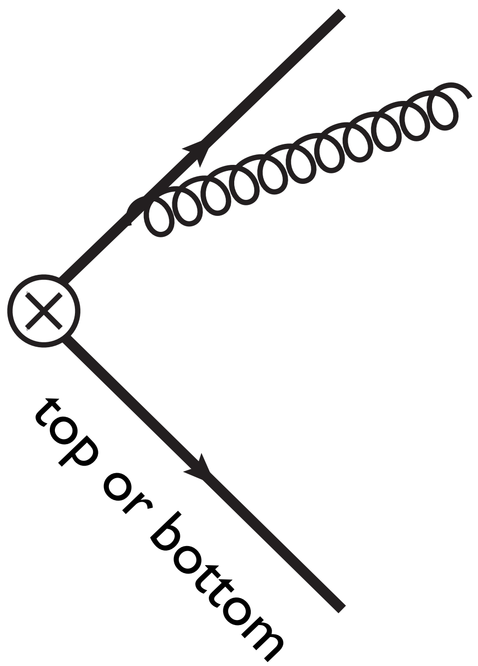


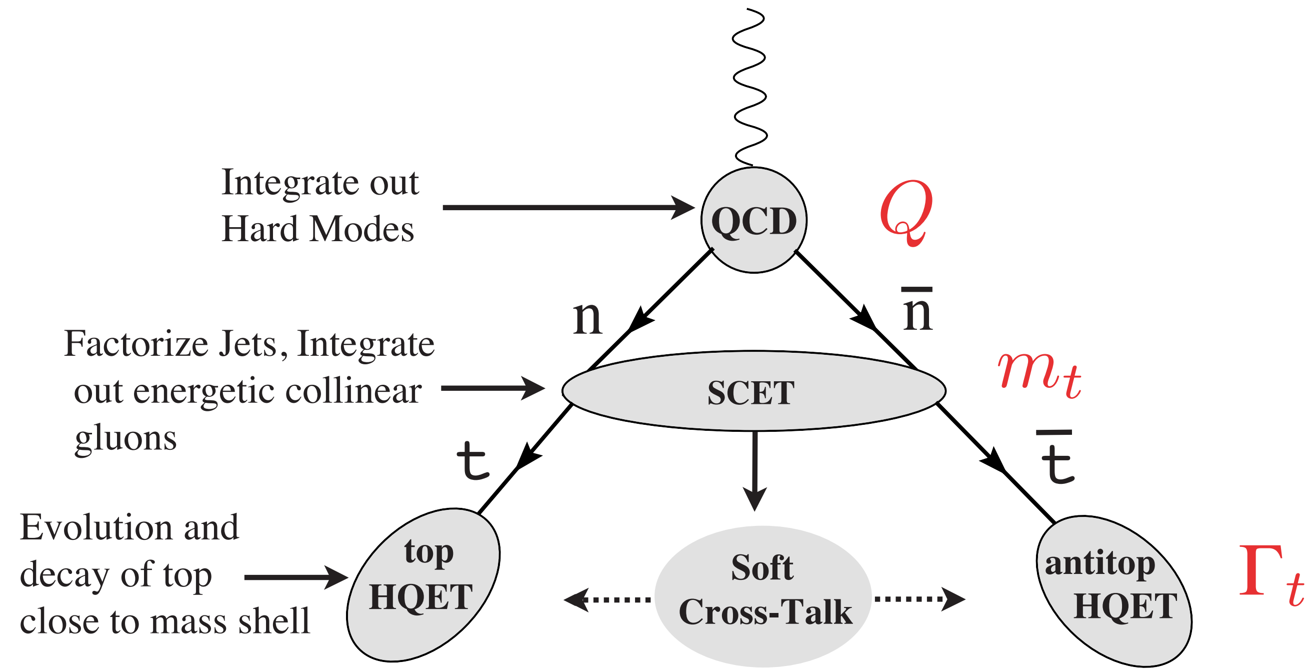
Massive Event Shapes



Primary mass effects

[S. Fleming, S. Mantry, A.H. Hoang, I.W. Stewart]





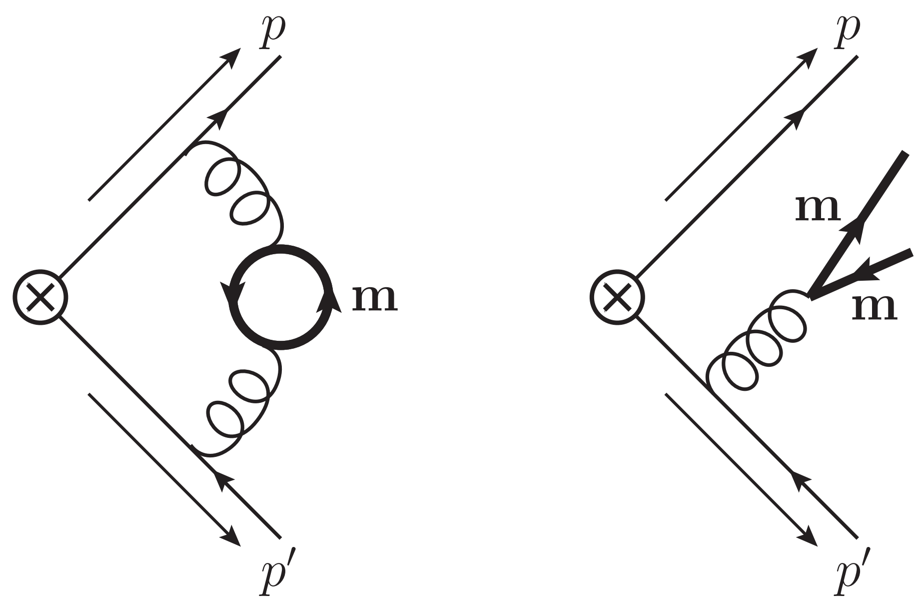
In the tail of the distribution only jet function is modified at N^2LL

When mass of jet very similar to mass of quark there appears a new hierarchy along with new large logs $\log^n\left(\frac{s-m^2}{m^2}\right)$

One has to match SCET to boosted HQET to sum them up

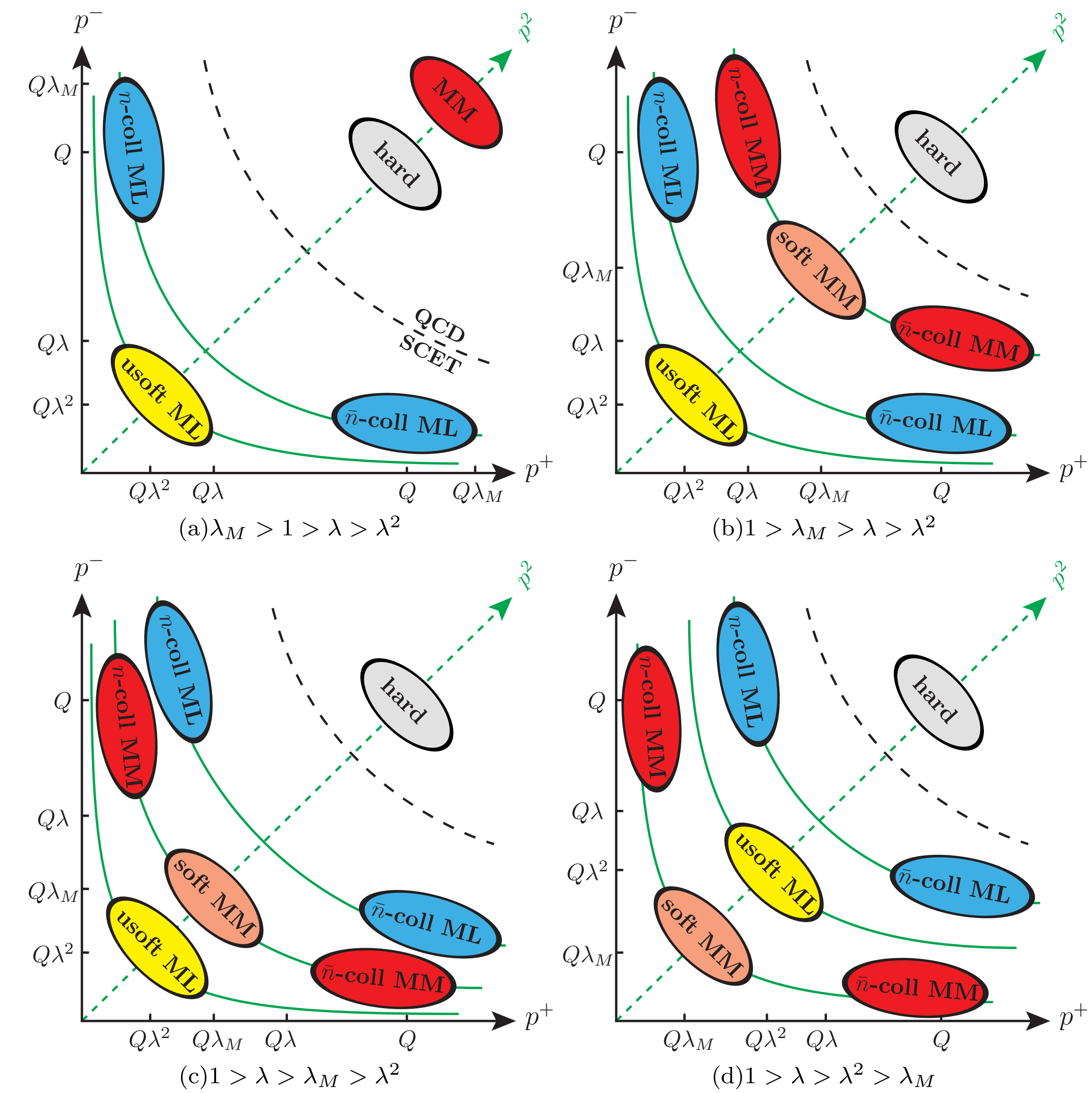
In this way one can also treat finite width effects

Secondary mass effects



[S. Gritschacher, A. H. Hoang, I. Jemos, P. Pietrulewicz]

Different scenarios



Appear at NNLO

Mass modes generate rapidity logs which enter at N^2LL

Depending on where the mass scale is sitting one has different theoretical set-ups

Conclusions & Outlook

Conclusions

- Precision measurements with jets are essential.
- Event Shapes are an ideal tool, great theoretical properties.
- Understanding power corrections mandatory for accurate theoretical predictions. Hadron mass effects cannot be neglected.
- C-parameter and HJM results on the way
- Oriented event shapes give extra handle on jets.
- Paving the way for precision top mass determination!