Simplified Models for Vector Boson Scattering

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Alboteanu/Kilian/JRR, **JHEP 0811** (2008) 010;
Beyer/Kilian/Krstonošić/Mönig/JRR/Schmitt/Schröder, **EPJC 48** (2006), 353;
JRR/Kilian/Sekulla, arXiv:1307.8170 and in prep.

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  - Importance of longitudinal EW gauge bosons
- Deviations from the SM: where? what? how?
- Anomalous Triple Gauge Couplings: dibosons
- Anomalous Quartic Gauge Couplings: tribosons, VV scattering
- Hot topic: Snowmass BNL 04/13, SM@LHC Freiburg 04/13, LHCEWWG 04/13, Snowmass 07/13, Dresden workshop 10/13

Extensions of the SM

- Lagrangian of the EW SM (no fermions/QCD here):

$$\mathcal{L}_{EW} = -\frac{1}{2}\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

  with building blocks:

$$D_\mu = \partial_\mu + \frac{i}{2}g\tau^I W^I_\mu + \frac{i}{2}g' B_\mu$$

$$W_{\mu\nu} = \frac{i}{2}g\tau^I(\partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\epsilon_{IJK}W^J_\mu W^K_\nu)$$

$$B_{\mu\nu} = \frac{i}{2}g'(\partial_\mu B_\nu - \partial_\nu B_\mu)$$

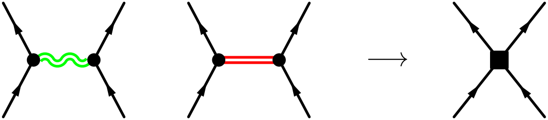
- Any EFT has higher-dimensional operators: Weinberg, 1979

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[\frac{a_i}{\Lambda}\mathcal{O}_i^{(5)} + \frac{c_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \frac{e_i}{\Lambda^4}\mathcal{O}_i^{(8)} \cdots\right]$$

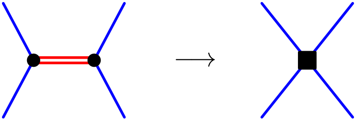
- without more fundamental theory $\Rightarrow$ no clue on the scale (neither on the coefficients)

Effective EW Dim. 6 Operators

Hagiwara/Hikasa/Peccei/Zeppenfeld, 1987; Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

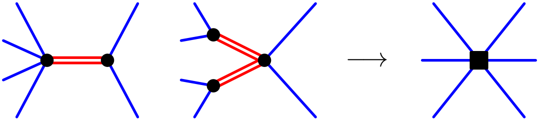


$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{\Lambda^2}\mathrm{tr}\left[J^{(I)} \cdot J^{(I)}\right]$$

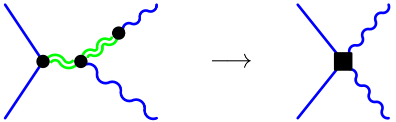


$$\mathcal{O}'_{h,1} = \frac{1}{F^2}\left((D\Phi)^\dagger\Phi\right) \cdot \left(h^\dagger(D^\Phi)\right) - \frac{v^2}{2}|D\Phi|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{\Lambda^2}(\Phi^\dagger\Phi - v^2/2)\,(D\Phi)^\dagger \cdot (D\Phi)$$



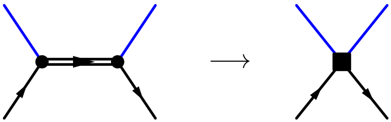
$$\mathcal{O}'_{h,3} = \frac{1}{\Lambda^2}\frac{1}{3}(\Phi^\dagger\Phi - v^2/2)^3$$



$$\mathcal{O}_{\Phi W} = -\frac{1}{\Lambda^2}\frac{1}{2}(\Phi^\dagger\Phi - v^2/2)\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

$$\mathcal{O}_B = \frac{1}{\Lambda^2}\frac{i}{2}(D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi)$$

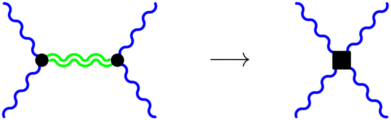
$$\mathcal{O}_{\Phi B} = -\frac{1}{\Lambda^2}\frac{1}{4}(\Phi^\dagger\Phi - v^2/2)B_{\mu\nu}B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{\Lambda^2}\bar{q}h(\not{D}h)q$$

Effective Dim. 8 Operators

Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993



$$\mathcal{O}_\lambda = \frac{i}{\Lambda^4} \mathrm{tr}\left[W_{\mu\nu} \times W^{\nu\rho} (\Phi^\dagger \frac{\vec{\sigma}}{2} \left[D_\rho, D^\mu \right] \Phi) \right.$$

$$\mathcal{O}_\kappa = (D^\mu \Phi)^\dagger (D^\nu \Phi)(\Phi^\dagger \left[D_\mu, D_\nu \right] \Phi)$$

- operators linked through e.o.m.
- SM: 59 independent operators (1 fermion gen.) Buchmüller/Wyler, 1986; Grzadkowski/Iskrzynski/Misiak/Rosiek, 2010
- Renormalization mixes operators
- Beware of power counting

Classification of Operators (I): Dim 6

(always v^2 subtracted)

- Dimension-6 operators (CP-conserving)

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu}W^{\nu\rho}W_{\rho}^{\mu}]$$
$$\mathcal{O}_{W} = (D_\mu\Phi)^\dagger W^{\mu\nu}(D_\nu\Phi)$$
$$\mathcal{O}_{B} = (D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi)$$
$$\mathcal{O}_{\partial\Phi} = \partial_\mu\left(\Phi^\dagger\Phi\right)\partial^\mu\left(\Phi^\dagger\Phi\right)$$
$$\mathcal{O}_{\Phi W} = \left(\Phi^\dagger\Phi\right)\text{Tr}[W^{\mu\nu}W_{\mu\nu}]$$
$$\mathcal{O}_{\Phi B} = \left(\Phi^\dagger\Phi\right)B^{\mu\nu}B_{\mu\nu}$$

- Dimension-6 operators (CP-violating)

$$\mathcal{O}_{\widetilde{W}W} = \Phi^\dagger\widetilde{W}_{\mu\nu}W^{\mu\nu}\Phi$$
$$\mathcal{O}_{\tilde{B}B} = \Phi^\dagger\tilde{B}_{\mu\nu}B^{\mu\nu}\Phi$$
$$\mathcal{O}_{\widetilde{W}WW} = \text{Tr}[\widetilde{W}_{\mu\nu}W^{\nu\rho}W_{\rho}^{\mu}]$$
$$\mathcal{O}_{\widetilde{W}} = (D_\mu\Phi)^\dagger\widetilde{W}^{\mu\nu}(D_\nu\Phi)$$

	ZWW	AWW	HWW	HZZ	HZA	HAA	WWWW	ZZWW	ZAWW	AAWW
$\mathcal{O}_{WWW}$	✓	✓					✓	✓	✓	✓
$\mathcal{O}_{W}$	✓	✓	✓	✓	✓		✓	✓	✓	
$\mathcal{O}_{B}$	✓	✓		✓	✓					
$\mathcal{O}_{\Phi d}$			✓	✓						
$\mathcal{O}_{\Phi W}$			✓	✓	✓	✓				
$\mathcal{O}_{\Phi B}$				✓	✓	✓				
$\mathcal{O}_{\tilde{W}WW}$	✓	✓					✓	✓	✓	✓
$\mathcal{O}_{\tilde{W}}$	✓	✓	✓	✓	✓					
$\mathcal{O}_{\tilde{W}W}$			✓	✓	✓	✓				
$\mathcal{O}_{\tilde{B}B}$				✓	✓	✓				

Classification of Operators (II): Dim 8

(always v^2 subtracted)

- Dimension-8 operators (only $D_\mu\Phi$)

$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right] ,$$

- Dimension-8 operators (only field strength/mixed)

$$\mathcal{O}_{T,0} = \text{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \cdot \text{Tr}\left[W_{\alpha\beta}W^{\alpha\beta}\right] ,$$
$$\mathcal{O}_{T,1} = \text{Tr}\left[W_{\alpha\nu}W^{\mu\beta}\right] \cdot \text{Tr}\left[W_{\mu\beta}W^{\alpha\nu}\right] ,$$
$$\mathcal{O}_{T,2} = \text{Tr}\left[W_{\alpha\mu}W^{\mu\beta}\right] \cdot \text{Tr}\left[W_{\beta\nu}W^{\nu\alpha}\right] ,$$
$$\mathcal{O}_{T,5} = \text{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \cdot B_{\alpha\beta}B^{\alpha\beta} ,$$
$$\mathcal{O}_{T,6} = \text{Tr}\left[W_{\alpha\nu}W^{\mu\beta}\right] \cdot B_{\mu\beta}B^{\alpha\nu} ,$$
$$\mathcal{O}_{T,7} = \text{Tr}\left[W_{\alpha\mu}W^{\mu\beta}\right] \cdot B_{\beta\nu}B^{\nu\alpha} ,$$
$$\mathcal{O}_{T,8} = B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}$$
$$\mathcal{O}_{T,9} = B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha} .$$

$$\mathcal{O}_{M,0} = \text{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$\mathcal{O}_{M,1} = \text{Tr}\left[W_{\mu\nu}W^{\nu\beta}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$\mathcal{O}_{M,2} = \left[B_{\mu\nu}B^{\mu\nu}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,3} = \left[B_{\mu\nu}B^{\nu\beta}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,4} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}D^\mu\Phi\right] \cdot B^{\beta\nu} ,$$
$$\mathcal{O}_{M,5} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}D^\nu\Phi\right] \cdot B^{\beta\mu} ,$$
$$\mathcal{O}_{M,6} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}W^{\beta\nu}D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,7} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}W^{\beta\mu}D^\nu\Phi\right] ,$$

Classification of Operators (III)

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$\mathcal{O}_{S,0/1}$	✓	✓	✓						
$\mathcal{O}_{M,0/1/6/7}$	✓	✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{M,2/3/4/5}$		✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{T,0/1/2}$	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,5/6/7}$		✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,8/9}$			✓			✓	✓	✓	✓

- Dim. 8 operators generate aQGCs, but not aTGCs
- generate neutral quartics
- Redundancy of the operators:
  - Equations of motion: $D_\mu W^{\mu\nu} = \Phi^\dagger (D^\nu \Phi) - (D^\nu \Phi)^\dagger \Phi + \ldots$
  - Gauge symmetry structure: $[D_\mu, D_\nu]\, \Phi \propto W_{\mu\nu} \Phi$
  - Integration by parts (up to total derivatives)
  - Leads to relations like:

$$\begin{aligned} \mathcal{O}_B &= \mathcal{O}_{\tilde{W}} + \frac{1}{2}\mathcal{O}_{WW} - \frac{1}{2}\mathcal{O}_{BB} \\ \mathcal{O}_{BW} &= -2\,\mathcal{O}_W - \mathcal{O}_{WW} \\ \mathcal{O}_{\partial W} &= -4\,\mathcal{O}_{WWW} + \text{gauge-fermion operators} \end{aligned}$$

Classification of Operators (IV)

Originally for heavy Higgses or Higgsless models

$$\mathcal{L}_\chi \ni \beta_1 \mathcal{L}_0' + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v}\sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2}\sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \ldots \qquad \alpha_i^{(n)} = \frac{v^{n-4}}{\Lambda^{n-4}} C_i^n$$

$$\mathcal{L}_0' = \frac{v^2}{4}\mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]$$

$$\mathcal{L}_1 = \mathrm{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] \qquad \mathcal{L}_6 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\nu\right]$$

$$\mathcal{L}_2 = \mathrm{i}\,\mathrm{tr}\left[\mathbf{B}_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu]\right] \qquad \mathcal{L}_7 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\nu\right]$$

$$\mathcal{L}_3 = \mathrm{i}\,\mathrm{tr}\left[\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu]\right] \qquad \mathcal{L}_8 = \tfrac{1}{4}\mathrm{tr}\left[\mathbf{T}\mathbf{W}_{\mu\nu}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^{\mu\nu}\right]$$

$$\mathcal{L}_4 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}^\nu\right] \qquad \mathcal{L}_9 = \tfrac{\mathrm{i}}{2}\mathrm{tr}\left[\mathbf{T}\mathbf{W}_{\mu\nu}\right]\mathrm{tr}\left[\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu]\right]$$

$$\mathcal{L}_5 = \mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]\mathrm{tr}\left[\mathbf{V}_\nu\mathbf{V}^\nu\right] \qquad \mathcal{L}_{10} = \tfrac{1}{2}\left(\mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right)^2$$

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$$\mathcal{L}_0' = \frac{v^2}{4} \operatorname{tr}[\mathbf{T}\mathbf{V}_\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\mu]$$

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Indirect info on new physics in $\beta_1, \alpha_i, \dots$ (Flavor physics only in M)

Electroweak precision observables (LEP I/II, SLC):

$$\Delta S = -16\pi\alpha_1 \qquad \alpha_1 = 0.0026 \pm 0.0020$$

$$\Delta T = 2\beta_1/\alpha_{\text{QED}} \qquad \beta_1 = -0.00062 \pm 0.00043$$

$$\Delta U = -16\pi\alpha_8 \qquad \alpha_8 = -0.0044 \pm 0.0026$$

Classification of approaches

Remarks:

- EFT approach leads to new interaction vertices
- Coupling constants are EFT Lagrangian parameters
- Framework for higher-order corrections straightforward (though rarely needed)
- Threshold/soft-collinear resummation $\Rightarrow$ momentum-dependent couplings/form factors
- Anomalous couplings understood as effective vertices/vertex functions
- Nevertheless: Lagrangian for new physics reconstructable
- Parameterize new physics effects as new resonances/particles

Classification of approaches

- Switch diff. operator bases (dep. on vertex): Snowmass EW White Paper

for the WWWW-Vertex:

$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{8}$$

$$\alpha_4 + 2\cdot\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{8}$$

for the WWZZ-Vertex:

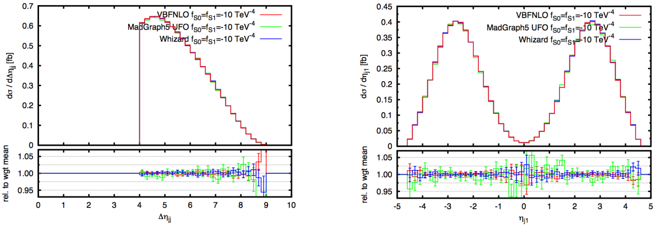
$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{16}$$

$$\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{16}$$

for the ZZZZ-Vertex:

$$\alpha_4 + \alpha_5 = \left(\frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,1}}{\Lambda^4}\right)\frac{v^4}{16}$$

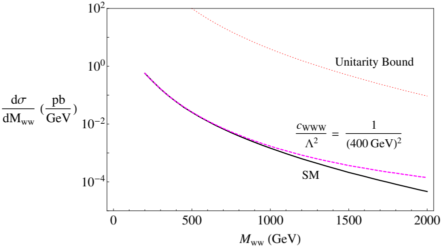
- Full agreement among generators: VBF@NLO, WHIZARD, Madgraph



Classification of approaches

- Translation between operator bases: Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993; Wudka, 1994; Degrande/Greiner/Kilian/Mattelaer/Mebane/Stelzer/Willenbrock/Zhang, 2012

$$\begin{aligned}
\Delta g_1^\gamma &= 0 \\
\Delta g_1^Z &= c_W \frac{m_Z^2}{2\Lambda^2} \\
\Delta\kappa_\gamma &= (c_W + c_B)\frac{m_W^2}{2\Lambda^2} \\
\Delta\kappa_Z &= \delta_Z + (c_W - c_B \tan^2\theta_W)\frac{m_W^2}{2\Lambda^2} \\
\lambda_\gamma &= \lambda_Z = c_{WWW}\frac{3g^2 m_W^2}{2\Lambda^2}
\end{aligned}$$



- Effective Field Theory description valid, if
  - $\hat{s} \ll \Lambda^2$: new physics out of direct LHC reach
  - Operator coefficients rather smallish, e.g. $c_{WWW} \lesssim 1$
  - No large logarithms in the game (resummation)
- Relation $\Delta g_1^Z = \Delta\kappa_Z + \tan^2\theta_W \Delta\kappa_\gamma$ invalidated by dim 8 operators

Unique way of operator assignment?

- Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
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- Incoherent sum of channels at LHC prevent eliminating operators!

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- Incoherent sum of channels at LHC prevent eliminating operators!
- Similar to B physics: observables process [decay] specific

(Integrating out) Resonances

Operator coefficients ⇒ new physics scale Λ: $\alpha_i = v^k/\Lambda^k$

- Operator normalization is arbitrary
- Power counting can be intricate

New physics in electroweak sector:

- Narrow resonances ⇒ particles
- Wide resonances ⇒ continuum

$SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge $g' \neq 0$ and fermion masses)

	$J = 0$	$J = 1$	$J = 2$
$I = 0$	σ^0 (Higgs ?)	ω^0 $(\gamma'/Z'\ ?)$	f^0 (Graviton ?)
$I = 1$	$\pi^\pm, \pi^0$ (2HDM ?)	$\rho^\pm, \rho^0$ $(W'/Z'\ ?)$	$a^\pm, a^0$
$I = 2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

- $I = 0$: resonant in W^+W^- and ZZ scattering
- $I = 1$: resonant in W^+Z and W^-Z scattering
- $I = 2$: resonant in W^+W^+ and W^-W^- scattering

accounts for weakly and strongly interacting models

Integrating out resonances

- Simplest example: scalar singlet σ:

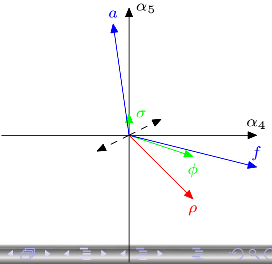
$$\mathcal{L}_\sigma = -\frac{1}{2}\left[\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v \sigma \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] - h_\sigma \mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right] \mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right]$$

- Effective Lagrangian $\mathcal{L}_\sigma^{\mathrm{eff}} = \frac{v^2}{8M_\sigma^2}\left[g_\sigma \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + h_\sigma \mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right] \mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right]^2$

- leads to anomalous quartic couplings (aQGCs)

$$\alpha_5 = g_\sigma^2 \left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_7 = 2 g_\sigma h_\sigma \left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_{10} = 2 h_\sigma^2 \left(\frac{v^2}{8M_\sigma^2}\right)$$

Resonance	σ	ϕ	ρ	f	a
$\Gamma[g^2 M^2/(64\pi v^2)]$	6	1	$\frac{4}{3}\left(\frac{v^2}{M^2}\right)$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$



Example: a Scalar Resonance

[Not counting the observed Higgs with $M = 126$ GeV.]

A higher-mass scalar resonance σ^0 will have

- Mass M_σ.
- Coupling to the Higgs sector (Higgs and longitudinal W/Z):

$$g_L^\sigma (D_\mu \Phi)^\dagger (D^\mu \Phi)\, \sigma$$

- Coupling to the gauge sector (transversal W/Z):

$$g_T^\sigma \mathrm{tr}\left[\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right]\, \sigma$$

The width Γ_σ gets contributions from both couplings.

Possible Origin: 2HDM isosinglet

A 2HDM is a renormalizable model (no higher-dimensional terms needed).
If revamped to the form of Minimal SM + higher-D operators:

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \qquad g_T^\sigma = O\left(\frac{1}{4\pi M_\sigma}\right) \quad \text{[loop]}$$

Possible Origin: something else

In a strongly interacting model, the hierarchy could be different:

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \qquad g_T^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}$$

Simplified Models for VBS (and VVV)

- Rise of amplitude (anomalous coupling) may be Taylor expansion of a resonance
- We have no idea which resonances exist and where they come from.
- Including a resonance in the model, there still may be further sources for anomalous couplings (further resonances, $A_{\text{nonres}}(s)$, deviation from the Breit-Wigner shape, etc.)
- Beyond the resonance, the amplitude may eventually rise and need unitarization again.

Consequence:

- Resonances in all accessible spin/isospin channels
- Couplings to the Higgs and gauge sectors are unrelated and arbitrary
- Still include anomalous couplings
- Unitarization (later)

Vector Boson Scattering

Beyer et al.,hep-ph/0604048

1 TeV, $1\,\mathrm{ab}^{-1}$, full $6f$ final states, 80 % e^-_R, 60 % e^+_L polarization, binned likelihood

Contributing channels: $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$, $ZZ \to ZZ$

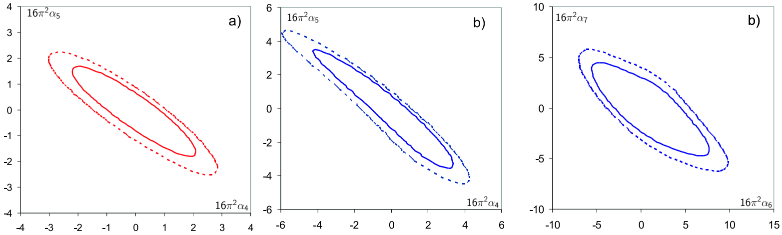
Process	Subprocess	σ [fb]
$e^+e^- \to \nu_e\bar\nu_e q\bar q q\bar q$	$WW \to WW$	23.19
$e^+e^- \to \nu_e\bar\nu_e q\bar q q\bar q$	$WW \to ZZ$	7.624
$e^+e^- \to \nu\bar\nu q\bar q q\bar q$	$V \to VVV$	9.344
$e^+e^- \to \nu e q\bar q q\bar q$	$WZ \to WZ$	132.3
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to ZZ$	2.09
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to W^+W^-$	414.
$e^+e^- \to b\bar b X$	$e^+e^- \to t\bar t$	331.768
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to W^+W^-$	3560.108
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to ZZ$	173.221
$e^+e^- \to e\nu q\bar q$	$e^+e^- \to e\nu W$	279.588
$e^+e^- \to e^+e^- q\bar q$	$e^+e^- \to e^+e^- Z$	134.935
$e^+e^- \to X$	$e^+e^- \to q\bar q$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55



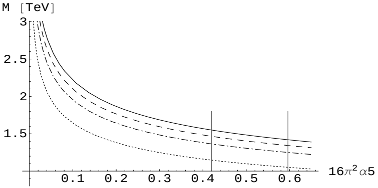
Interpretation as limits on resonances

Beyer et al.,hep-ph/0604048

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma / M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

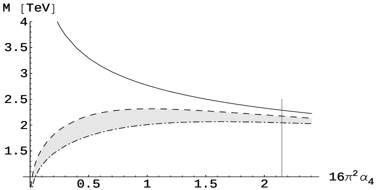


$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho^\pm} = v \left(\frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2(\alpha_4^\lambda)^2/(2c_w^2)} \right)^{\frac{1}{4}}$$



upper/lower limit from λ_Z, grey area: magnetic moments

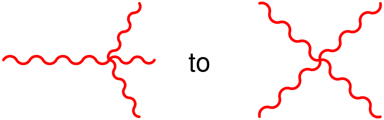
Final result:

Spin	$I=0$	$I=1$	$I=2$
0	1.55	–	1.95
1	–	2.49	–
2	3.29	–	4.30

Spin	$I=0$	$I=1$	$I=2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

And Triple Vector Boson Production?

relate to ??



Yes, the same Feynman graphs (in the SM), but...

TVBP: one external $W/Z/\gamma$ is always far off-shell. Unitarization has to proceed differently, and a different set of (anomalous) couplings contributes. This is particularly true for resonances.

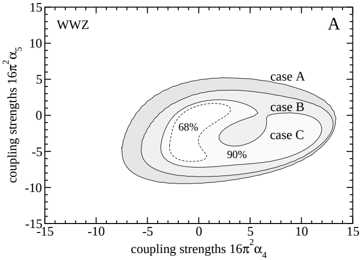
$\Rightarrow$ This is important physics which should be treated independently w.r.t. VBS processes. Don't just combine the results!

ILC Results: Triboson production

Beyer et al.,hep-ph/0604048

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4 + \alpha_6)$, $(\alpha_5 + \alpha_7)$, $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, $1\,\mathrm{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

$16\pi^2\times$	WWZ no pol.	WWZ e^- pol.	WWZ both pol.	ZZZ no pol.	best
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

Veto against $E^2_{\text{mis}} + p^2_{\perp,\text{mis}}$

No angular correlations yet

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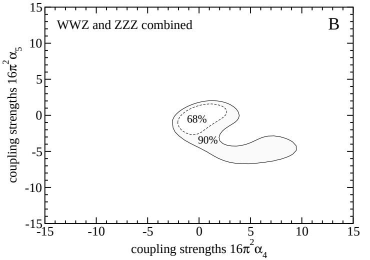

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Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section: $\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2$

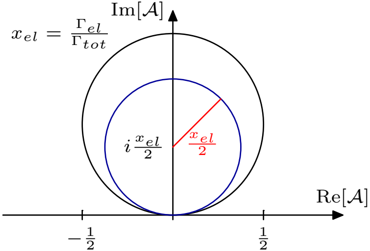
Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im}\left[\mathcal{M}_{ii}(t=0)\right]/s \qquad t = -s(1-\cos\theta)/2$$

Partial wave amplitudes: $\mathcal{M}(s,t,u) = 32\pi \sum_\ell (2\ell+1)\mathcal{A}_\ell(s) P_\ell(\cos\theta)$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_\ell \frac{32\pi(2\ell+1)}{s}|\mathcal{A}_\ell|^2 \overset{!}{=} \sum_\ell \frac{32\pi(2\ell+1)}{s}\text{Im}\left[\mathcal{A}_\ell\right] \quad \Rightarrow \quad \boxed{|\mathcal{A}_\ell|^2 = \text{Im}\left[\mathcal{A}_\ell\right]}$$



Argand circle $\boxed{\left|\mathcal{A}(s) - \frac{i}{2}\right| = \frac{1}{2}}$

Resonance: $\mathcal{A}(s) = \frac{-M\Gamma_{\text{el}}}{s - M^2 + iM\Gamma_{\text{tot}}}$

Counterclockwise circle, radius $\frac{x_{el}}{2}$

Pole at $s = M^2 - iM\Gamma_{\text{tot}}$

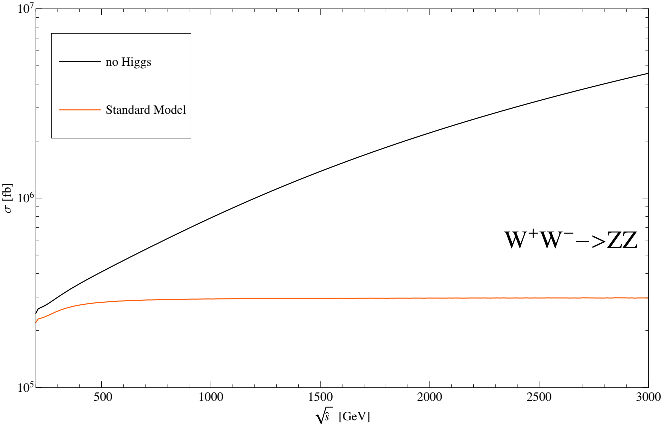
SM: VBS Amplitude unitarized by Higgs

Higgsless Model

$$\mathcal{A}_{WWZZ} = \frac{s}{v^2} + \mathcal{O}(1)$$

SM

$$\mathcal{A}_{WWZZ} = \frac{s}{v^2} - \frac{1}{v^2}\frac{s^2}{s - M_h{}^2} + \mathcal{O}(1)$$



Unitarity Bound for α_4 AQGC

Bounds for α_4

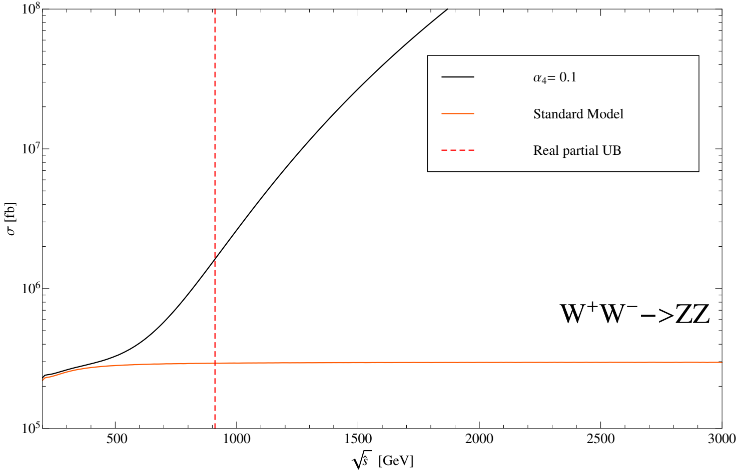
$$\ell = 0 : \sqrt{s} \leq \left(\frac{6\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.5\text{ TeV}}{\sqrt[4]{\alpha_4}}$$

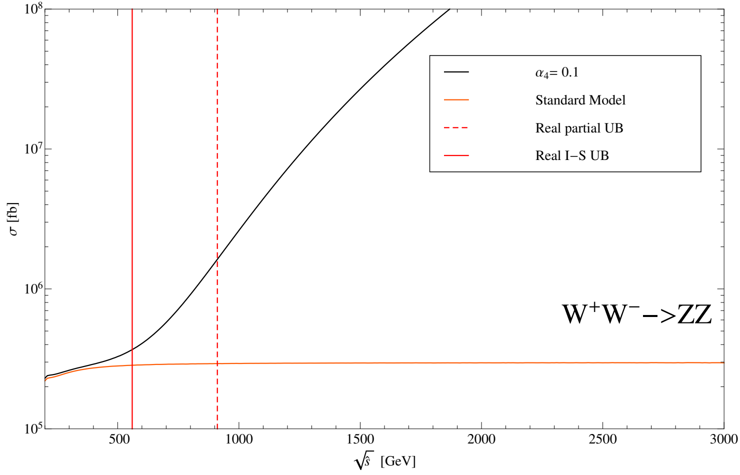
$$\ell = 2 : \sqrt{s} \leq \left(\frac{60\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.9\text{ TeV}}{\sqrt[4]{\alpha_4}}$$

α_4 AQGC contribution to $WW \to ZZ$

$$\mathcal{A}(s,t,u) = 4\alpha_4 \frac{t^2 + u^2}{v^4}$$

- Bound depends on coupling α_4
- Use strongest bound


10^8
10^7
10^6
10^5
σ [fb]
500
1000
1500
2000
2500
3000
$\sqrt{\hat{s}}$ [GeV]
α_4= 0.1
Standard Model
Real partial UB
$W^+W^- -> ZZ$



10^8
10^7
10^6
10^5
σ [fb]
500
1000
1500
2000
2500
3000
$\sqrt{\hat{s}}$ [GeV]
α_4= 0.1
Standard Model
Real partial UB
Real I–S UB
$W^+W^- -> ZZ$

Cut-Off Method (a.k.a. “Event Clipping”)

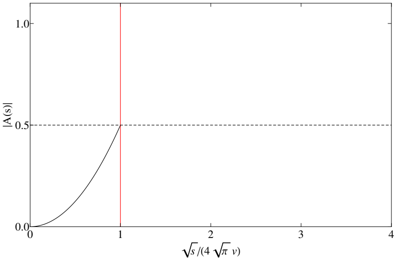
Cut-Off function

$$\Theta\left(\Lambda_C^2 - s\right)$$

- Naive prevention of Unitarity violation
- No continuous transition at Λ_C
- Ignore any interesting physics above Unitary bound
- Better: Use observables, which do not conflict unitarity condition

Cut-Off energy Λ_C

Λ_C equates unitarity bounds (often 0th partial wave)



Form Factor

Form Factor

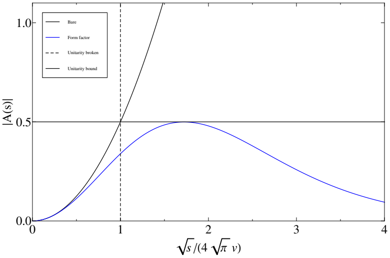
$$\frac{1}{\left(1+\frac{s}{\Lambda_{FF}^2}\right)^n}$$

Parameters

n Chosen to prevent breaking of Unitarity

Λ_{FF} Calculate highest possible value that satisfy real Unitarity bound ($0th$ partial wave)

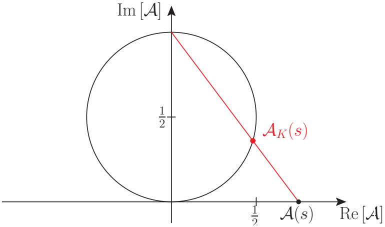
- Use Form Factor to suppress breaking of unitarity
- Can be generally used for arbitrary anomalous operator
- Need "Fine Tuning"



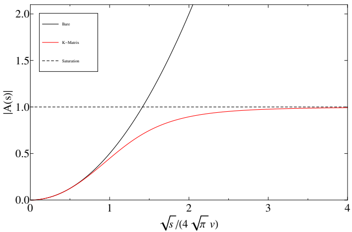
K-Matrix

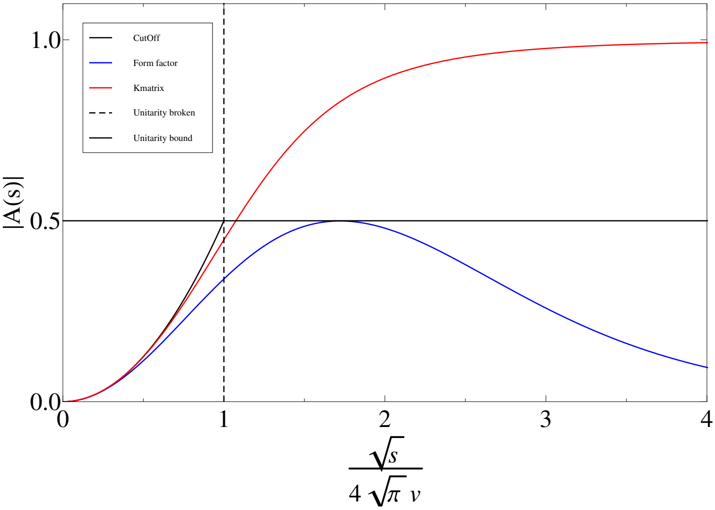
K-Matrix Unitarisation

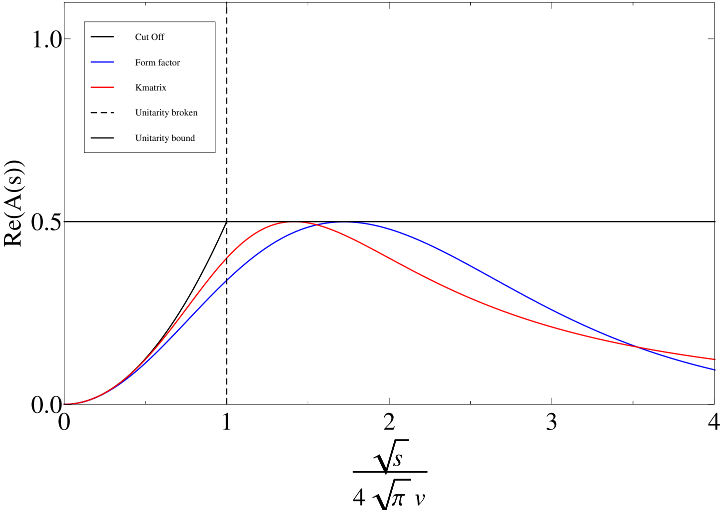
$$\mathcal{A}_K(s) = \frac{1}{\mathrm{Re}(\frac{1}{\mathcal{A}(s)}) - \mathrm{i}} = \frac{\mathcal{A}(s)}{1 - \mathrm{i}\mathcal{A}(s)} \quad \text{if } \mathcal{A}(s) \in \mathbb{R}$$



- Projection of elastic amplitudes onto Argand-Circle
- At high energies the amplitude saturises
- Is usable for complex amplitudes
- Not dependent on additional parameters




CutOff
Form factor
Kmatrix
Unitarity broken
Unitarity bound
|A(s)|
1.0
0.5
0.0
0
1
2
3
4
$\frac{\sqrt{s}}{4\sqrt{\pi}\,v}$


Cut Off
Form factor
Kmatrix
Unitarity broken
Unitarity bound
1.0
0.5
0.0
Re(A(s))
0
1
2
3
4
$\frac{\sqrt{s}}{4\sqrt{\pi}\,v}$

"Comparison"

- Which Unitarisation scheme provides the best description?

$\rightarrow$ All of them:
Unitarisation schemes are an arbitrary way to guarantee Unitarity

Form Factor

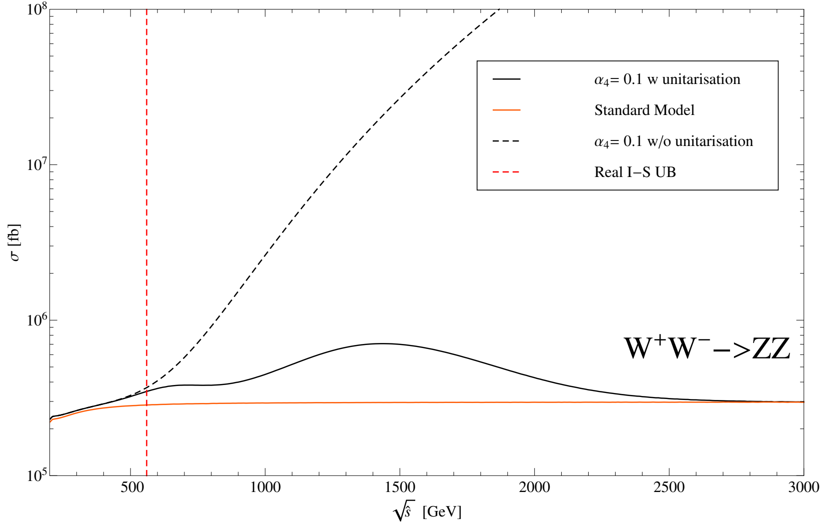
- Suppression of amplitude to get below Unitarity bound

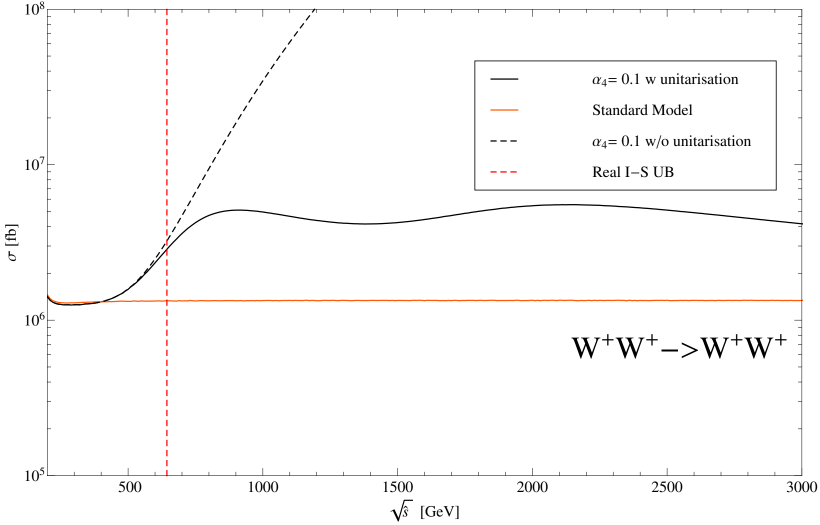
MC Generate less events than possible

K-Matrix

- Saturation of amplitude to achieve Unitarity

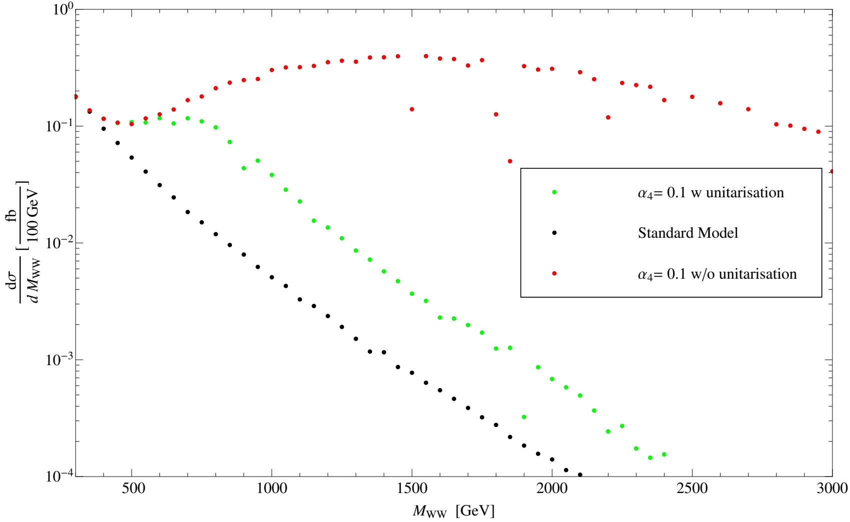
MC Generate maximal possible number of events


α_4= 0.1 w unitarisation
Standard Model
α_4= 0.1 w/o unitarisation
Real I−S UB
$W^+W^- \to ZZ$
σ [fb]
$\sqrt{\hat{s}}$ [GeV]
10^5
10^6
10^7
10^8
500
1000
1500
2000
2500
3000

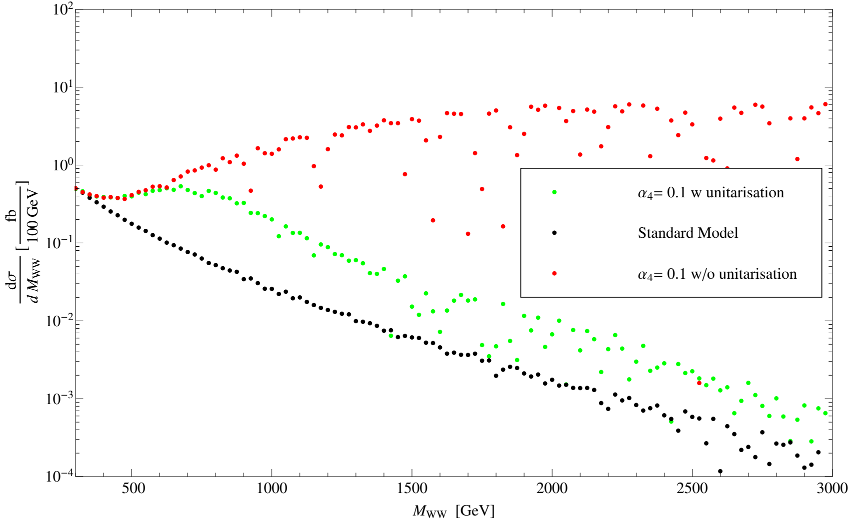


α_4= 0.1 w unitarisation
Standard Model
α_4= 0.1 w/o unitarisation
Real I−S UB
$W^+W^+ \rightarrow W^+W^+$
σ [fb]
$\sqrt{\hat{s}}$ [GeV]
10^8
10^7
10^6
10^5
500
1000
1500
2000
2500
3000

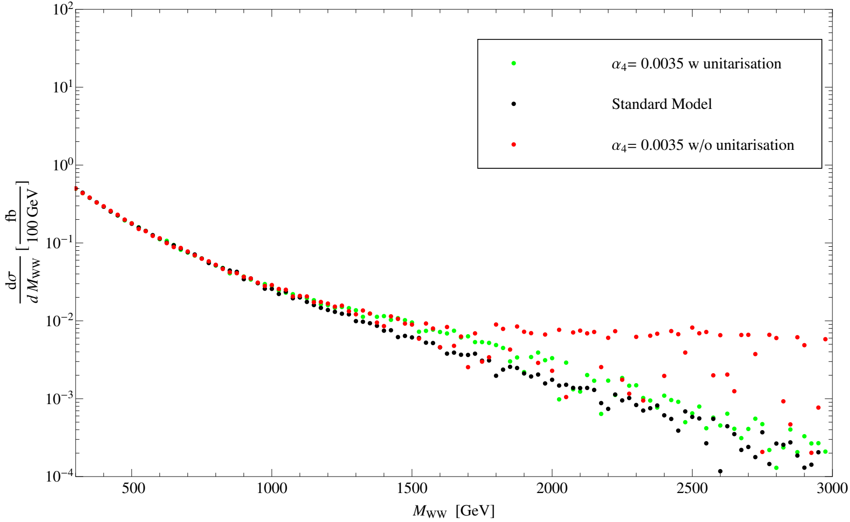
$PP \to e^+e^+\nu_e\nu_e + jj$ at $s = 8$ TeV



$PP \to e^+e^+\nu_e\nu_e + jj$ at $s = 14$ TeV



$PP \to e^+e^+\nu_e\nu_e + jj$ at $s = 14$ TeV



Summary/Conclusions

- Triple/Quartic gauge couplings measured either
  - via diboson production
  - via triple boson production
  - via vector boson scattering
- Unify LHC and ILC descriptions
- SM deviations in EW effective Lagrangian (SM + higher-dim. op.)
- Want to set model independent limits AQGC (α_4, $\frac{f_{S,0}}{\Lambda_{NP}^4}$)
- We do not know at which scale Λ_{NP} the EFT breaks down
- But: Energy range for testing AQGC is bound by Unitarity
- Unitarisation scheme:
  Number of events generated by MC fulfill unitarity condition
- Unitarisation introduces model dependece
- Unified description for different channels difficult/impossible

Summary/Conclusions

- VBS Simplified Models: EFT approach for low-energy regime, unitarized by form factors in resonance scheme at high energies
- interpreted as resonances coupled to EW bosons
- Approach includes/generalizes standard EFT ansatz
- Issue of unitarity (PDFs help – but kill energy reach)
- Photon-induced processes: better sensitivity, but higher constraints!
- Sensitivity rises with number of intermediate states:
  - LHC sensitivity limited in pure EW sector: $\sim 1 - X$ TeV (???)
  - ILC : $1.5 - 6$ TeV

Always get the correct ellipses...

Backup Slides:

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = \mathrm{ie}\left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\,\nu} W_{\nu\rho}^+ A^{\rho\mu}\right]$$
$$+ \mathrm{ie}\frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\,\nu} W_{\nu\rho}^+ Z^{\rho\mu}\right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Anomalous triple and quartic gauge couplings

$$\begin{aligned}\mathcal{L}_{QGC} = {} & e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W^-_\mu W^+_\nu - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W^+_\nu \right] \\ & + e^2 \frac{c_w}{s_w} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W^-_\mu W^+_\nu + W^+_\mu W^-_\nu \right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W^+_\nu \right] \\ & + e^2 \frac{c_w^2}{s_w^2} \left[g_1^{ZZ} Z^\mu Z^\nu W^-_\mu W^+_\nu - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W^+_\nu \right] \\ & + \frac{e^2}{2 s_w^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W^-_\mu W^+_\nu - g_2^{WW} \left(W^{-\mu} W^+_\mu \right)^2 \right] + \frac{e^2}{4 s_w^2 c_w^4} h^{ZZ} (Z^\mu Z_\mu)^2\end{aligned}$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 \qquad \Delta g_1^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) - g^2 (\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2 \left[\alpha_4 + \alpha_5 + 2 (\alpha_6 + \alpha_7 + \alpha_{10}) \right]$$

Parameters

$$\mathcal{L}_\sigma = -\frac{g_\sigma v}{2} \text{Tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] \sigma$$
$$\mathbf{V}_\mu = -\mathrm{i} g \mathbf{W}_\mu + \mathrm{i} g' \mathbf{B}_\mu$$
$$\mathbf{W}_\mu = W^a_\mu \frac{\tau^a}{2}$$
$$\mathbf{B}_\mu = W^a_\mu \frac{\tau^3}{2}$$

$$\mathcal{L}_\phi = \frac{g_\phi v}{4} \text{Tr}\left[\left(\mathbf{V}_\mu \otimes \mathbf{V}^\mu - \frac{\tau^{aa}}{6} \text{Tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\right) \phi\right]$$
$$\phi = \sqrt{2}\left(\phi^{++}\tau^{++} + \phi^{+}\tau^{+} + \phi^{0}\tau^{0} + \phi^{-}\tau^{-} + \phi^{--}\tau^{--}\right)$$
$$\tau^{++} = \tau^{+} \otimes \tau^{+}$$
$$\tau^{+} = \frac{1}{2}\left(\tau^{+} \otimes \tau^{3} + \tau^{3} + \tau^{+}\right)$$
$$\tau^{0} = \frac{1}{\sqrt{6}}\left(\tau^{3} \otimes \tau^{3} - \tau^{+} \otimes \tau^{-} - \tau^{-} + \tau^{+}\right)$$

1

SM Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{min}} = & -\frac{1}{2}\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] && W^\pm,\ Z \\ & + (\partial_\mu\phi)^\dagger\,\partial^\mu\phi - V(\phi) && h \\ & + \frac{v^2}{4}\text{tr}\left[(\mathbf{D}_\mu\Sigma)^\dagger(\mathbf{D}^\mu\Sigma)\right] && w^\pm,\ z \\ & - \frac{g_h v}{2}\text{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] h\end{aligned}$$

Vector Bosons

$$\mathbf{W}_{\mu\nu} = \partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu + \mathrm{i}g\left[\mathbf{W}_\mu, \mathbf{W}_\nu\right]$$

$$\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{B}_\nu - \partial_\nu\mathbf{B}_\mu$$

$$\mathbf{W}_\mu = W^a_\mu\frac{\tau^a}{2} \qquad \mathbf{B}_\mu = B_\mu\frac{\tau^3}{2}$$

$$\mathbf{D}_\mu = \partial_\mu + \mathrm{i}g\mathbf{W}_\mu - \mathrm{i}g'\mathbf{B}_\mu$$

Higgs Sector

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix}$$

$$\Sigma = \exp\left[-\frac{\mathrm{i}}{v}w^a\tau^a\right]$$

$$\mathbf{V}_\mu = \Sigma\left(\mathbf{D}_\mu\Sigma\right)$$

Unitary Gauge

- Goldstone bosons are absorbed by vector bosons as longitudinal degrees of freedom
- $w^a \equiv 0 \to \Sigma \equiv 1$
- $\mathbf{D}_\mu = \partial_\mu - \mathbf{V}_\mu = \partial_\mu + \frac{\mathrm{i}g}{2}\left(\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_w} Z\tau^3\right)$

$$\mathcal{L}_{\text{min}} = -\frac{1}{2}\mathrm{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\mathrm{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + \underbrace{(\partial_\mu\phi)^\dagger\,\partial^\mu\phi - \frac{v^2}{4}\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] - \frac{g_h v}{2}\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] h}_{\widehat{=}_{g_h=1}(\mathbf{D}_\mu\phi)^\dagger\mathbf{D}^\mu\phi} - V(\phi)$$

- Coincides with known SM parametrisation

Gaugeless Limit

- $g \to 0,\ g' \to 0$
- $\mathbf{D}_\mu = \partial_\mu$
- $\mathbf{V}_\mu = \Sigma\,(\partial_\mu \Sigma)$

$$\begin{aligned}\mathcal{L}_{\text{min}} = &- \frac{1}{2}\mathrm{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\mathrm{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] \\ &+ (\partial_\mu \phi)^\dagger\,\partial^\mu \phi - V(\phi) \\ &+ \frac{v^2}{4}\mathrm{tr}\left[(\partial_\mu \Sigma)^\dagger(\partial^\mu \Sigma)\right] - \frac{g_h v}{2}\mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] h\end{aligned}$$

- Decoupling of Higgs Sector

Isospin decomposition

- Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4}\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + \alpha_4 \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right] + \alpha_5 \left(\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\right)^2$$

- Leads to the following amplitudes: $s = (p_1 + p2)^2 \quad t = (p1 - p3)^2 \quad u = (p1 - p4)^2$

$$\boxed{\mathcal{A}(s,t,u) =:} \quad \mathcal{A}(w^+w^- \to zz) = \frac{s}{v^2} + 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(w^+z \to w^+z) = \frac{t}{v^2} + 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2+u^2}{v^4}$$

$$\mathcal{A}(w^+w^- \to w^+w^-) = -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5)\frac{s^2+t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4}$$

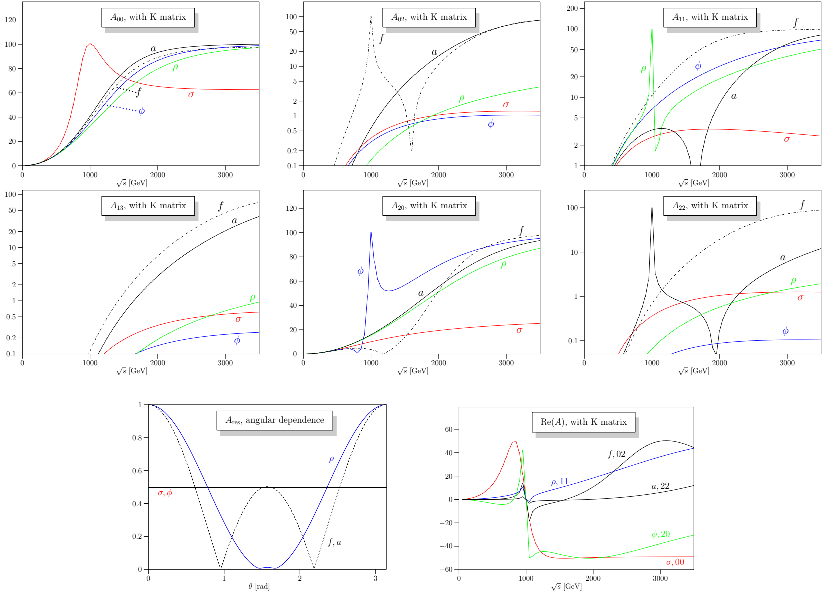
$$\mathcal{A}(w^+w^+ \to w^+w^+) = -\frac{s}{v^2} + 8\alpha_4 \frac{s^2}{v^4} + 4\left(\alpha_4 + 2\alpha_5\right)\frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(zz \to zz) = 8\left(\alpha_4 + \alpha_5\right)\frac{s^2+t^2+u^2}{v^4}$$

- (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

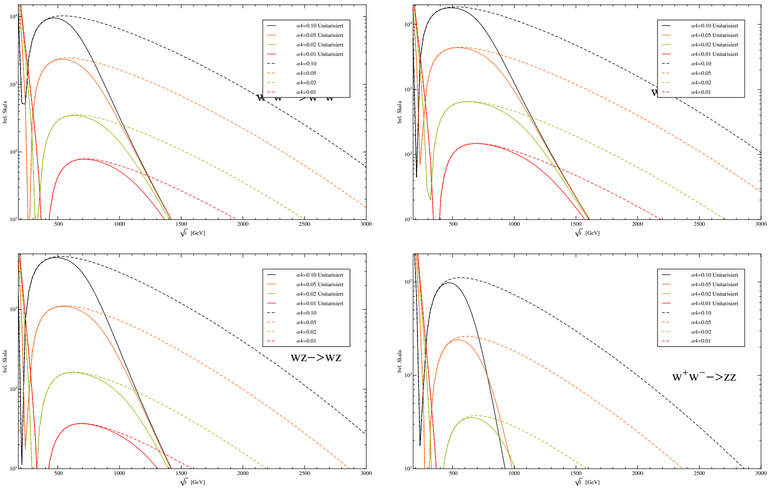
$$\begin{aligned}
\mathcal{A}(I=0) &= 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t) \\
\mathcal{A}(I=1) &= \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t) \\
\mathcal{A}(I=2) &= \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)
\end{aligned}$$

Eigenamplitudes



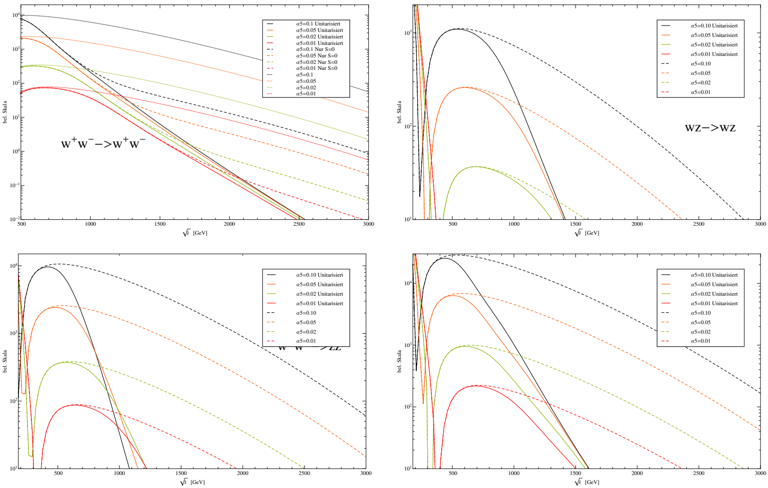
aQGCs: α_4

Kilian/JRR/Sekulla, 2013

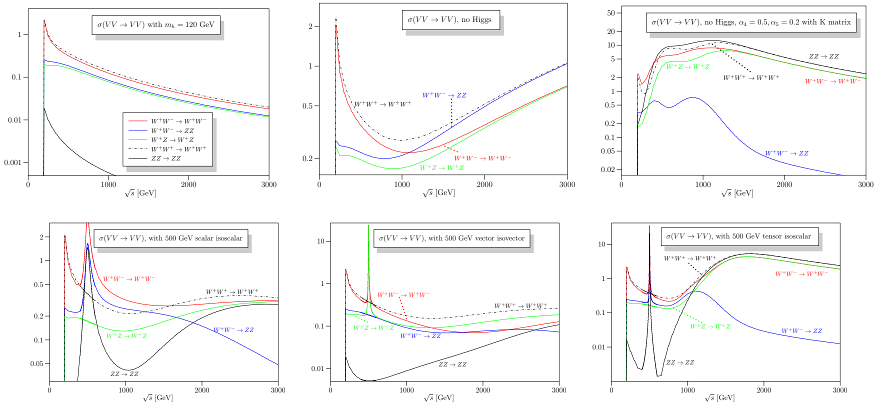


aQGCs: α_5

Kilian/JRR/Sekulla, 2013



“Partonic” cross sections

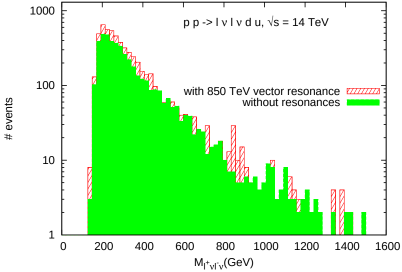


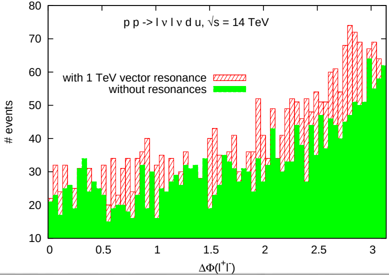
- $\sigma(\mathcal{VV} \to \mathcal{VV})$ in nb $\qquad M_R = 500$ GeV
- all amplitudes K-matrix unitarized
- Cut of 15° around the beam axis

LHC Example: Vector Isovector

Alboteanu/Kilian/JRR, 2008

- Example: 850 GeV vector resonance, coupling $g_\rho = 1$
- (Theory) Cuts:
  - $p_\perp(\ell\nu) > 30$ GeV
  - $|\delta R(\ell\nu)| < 1.5$
  - $\theta(u/d) > 0.5°$
- Integrated luminosity: 225 fb^{-1}
- Discriminator: angular correlations $\Delta\phi(\ell\ell)$
- Ongoing ATLAS study
  - More kinematic observables
  - Comparison and validation phase
  - first reproduce SM
  - then anom. couplings/BSM resonances





More about K-Matrix

Example

$$\mathcal{A}(s) = \frac{-c}{s - M^2} \qquad \to \frac{-c}{s - M^2 + \mathrm{i} M \Gamma_1}$$

$$\mathcal{A}(s) = -\frac{M}{v^2}\frac{s}{s - M^2} \to -\frac{M}{v^2}\frac{s}{s - M^2 + \mathrm{i} M \Gamma_2 \frac{s}{M^2}}$$

- Amplitudes satisfying Unitarity are invariant under Kmatrix-Unitarisation
- Transforms simple pole amplitude into Breit-Wigner form

$\Rightarrow$ Alternate Implementation of Dyson resummation for s-particle exchange

- Unitarisation of Pade and IAM (inverse-amplitude method) coincides with K-matrix Unitarisation