

Reconstructing SUSY Contribution to Muon $g-2$ at ILC

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LCWS13, 11-15 November 2013, Tokyo

Status of Muon $g-2$

Precise result from Brookhaven E821 experiment

Dedicated studies on standard model prediction

[Hagiwara,Liao,Martin,Nomura,Teubner;Davier,Hoecker,Malaescu,Zhang]

$$a_{\mu}^{(\text{exp})} - a_{\mu}^{(\text{SM})} = \begin{cases} (26.1 \pm 8.0) \times 10^{-10} \\ (28.7 \pm 8.0) \times 10^{-10} \end{cases}$$

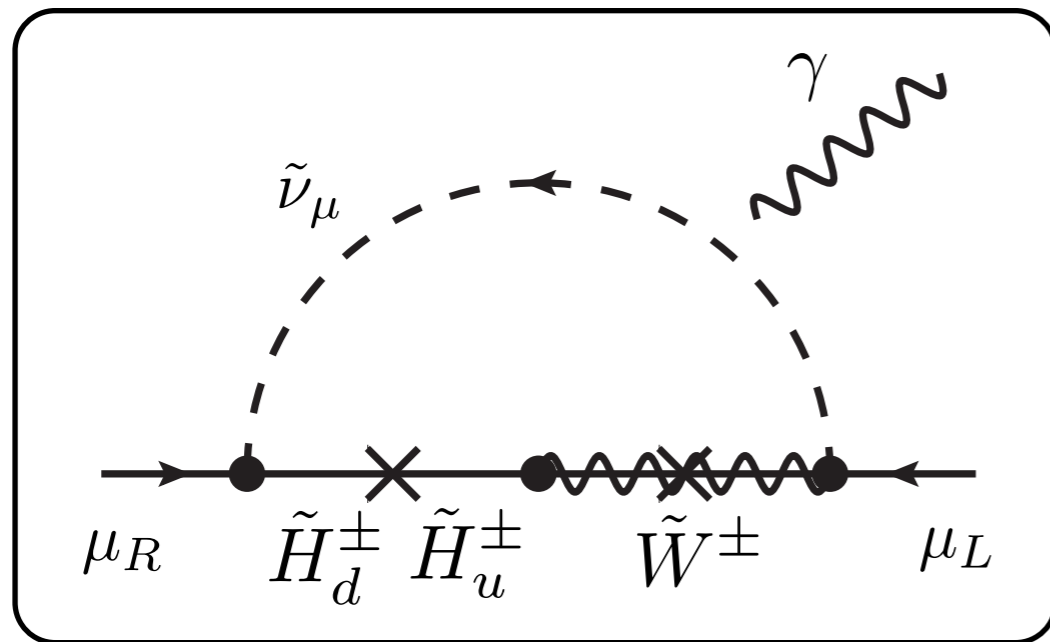
> 3σ deviation

(possibly) a signal of new physics

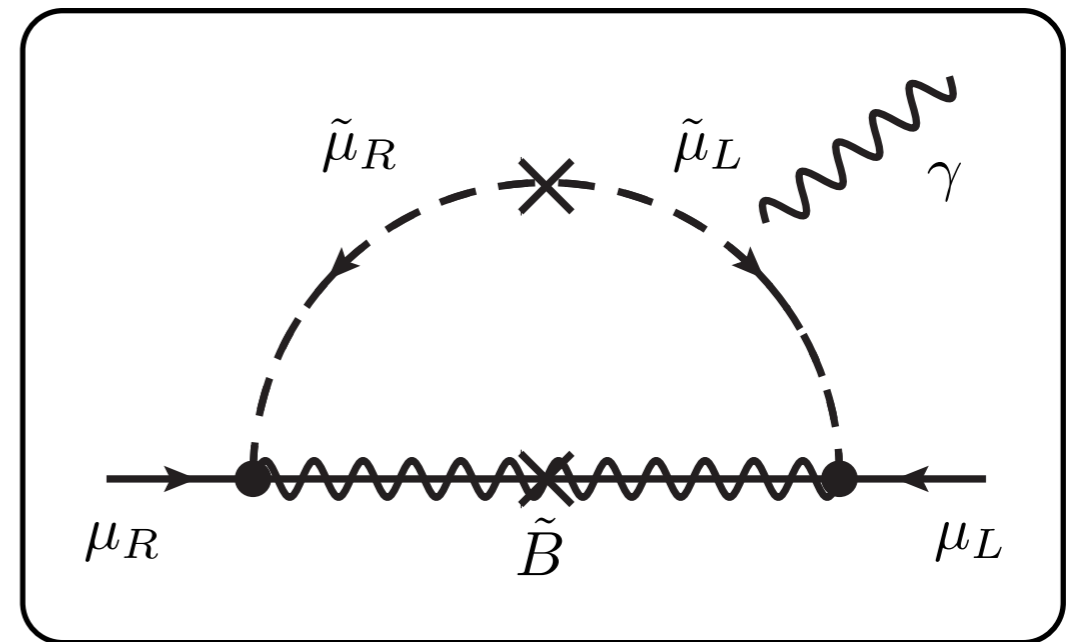
c.f. Electroweak: $a_{\mu}^{(\text{EW})} = (15.4 \pm 0.1) \times 10^{-10}$

SUSY Solution

chargino-muon sneutrino



neutralino-smuon



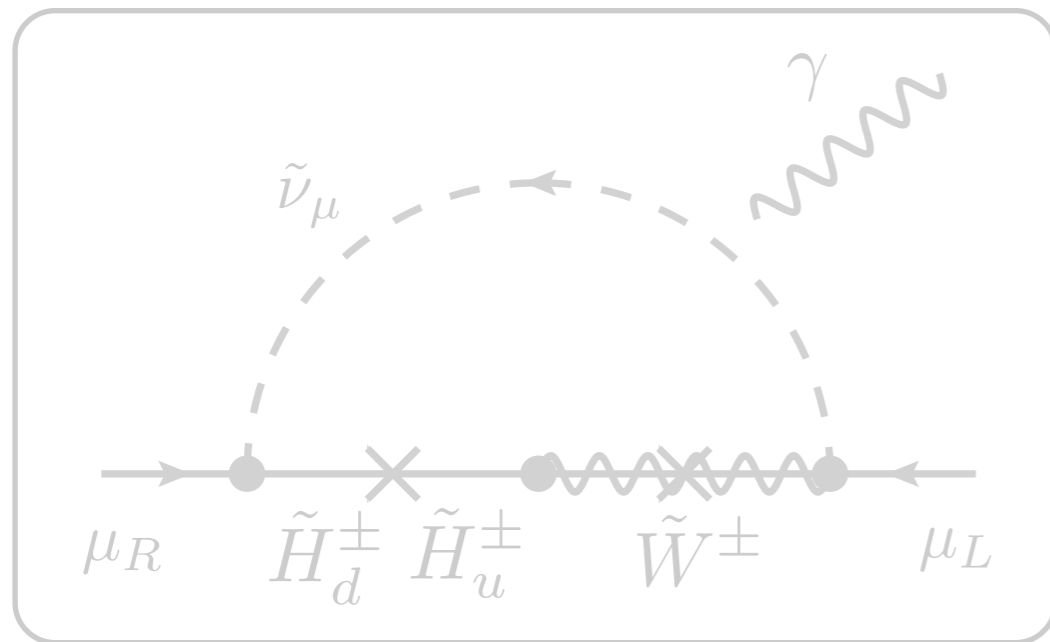
$$a_\mu(\text{SUSY}) \sim \frac{\alpha_2(\alpha_Y)}{4\pi} \frac{m_\mu^2}{m_{\text{soft}}^2} \tan \beta \quad (\tan \beta \sim 10)$$

→ Enhanced by $\tan \beta$

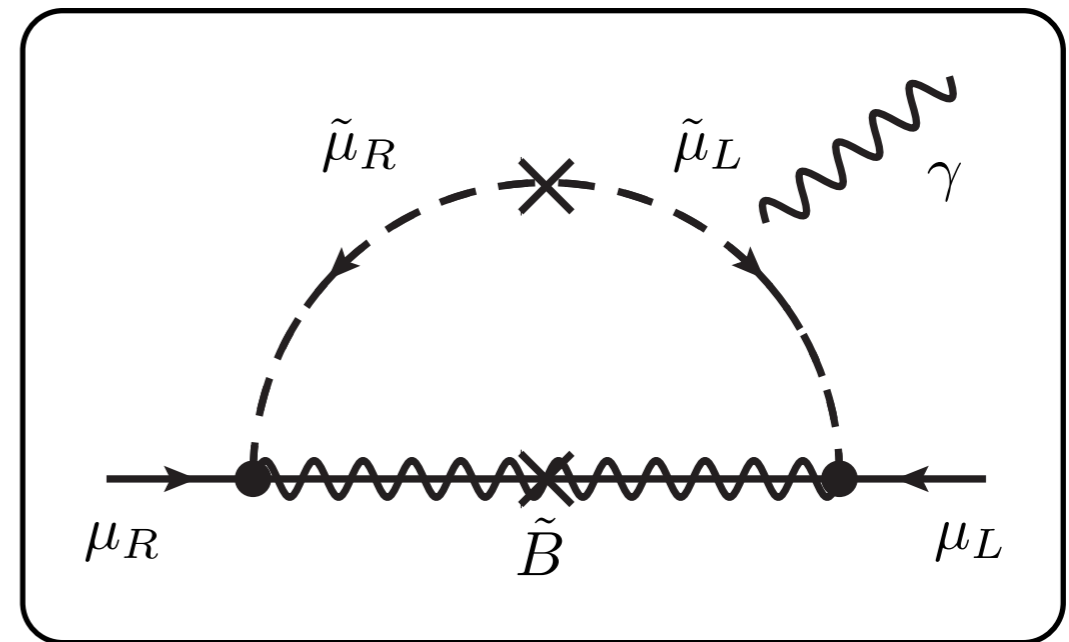
Question: How to test SUSY solution?

SUSY Solution

chargino-muon sneutrino



neutralino-smuon



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Question: How to test SUSY solution?

Test of SUSY Solution

Stage 1

discovery of SUSY particles necessary for
SUSY contribution to muon $g-2$

Stage 2

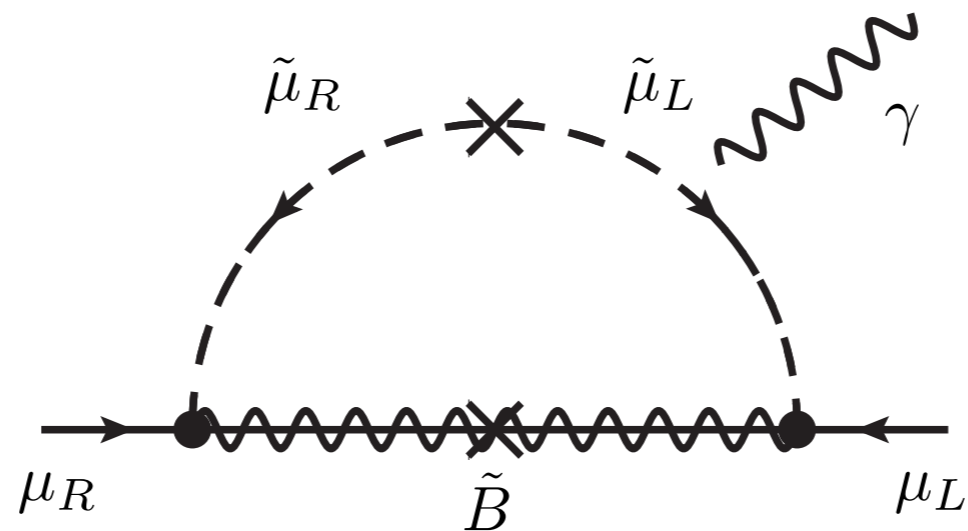
reconstruct SUSY contribution to muon $g-2$
by information available at colliders (ILC)

Neutralino Contribution

Contribution becomes sizable

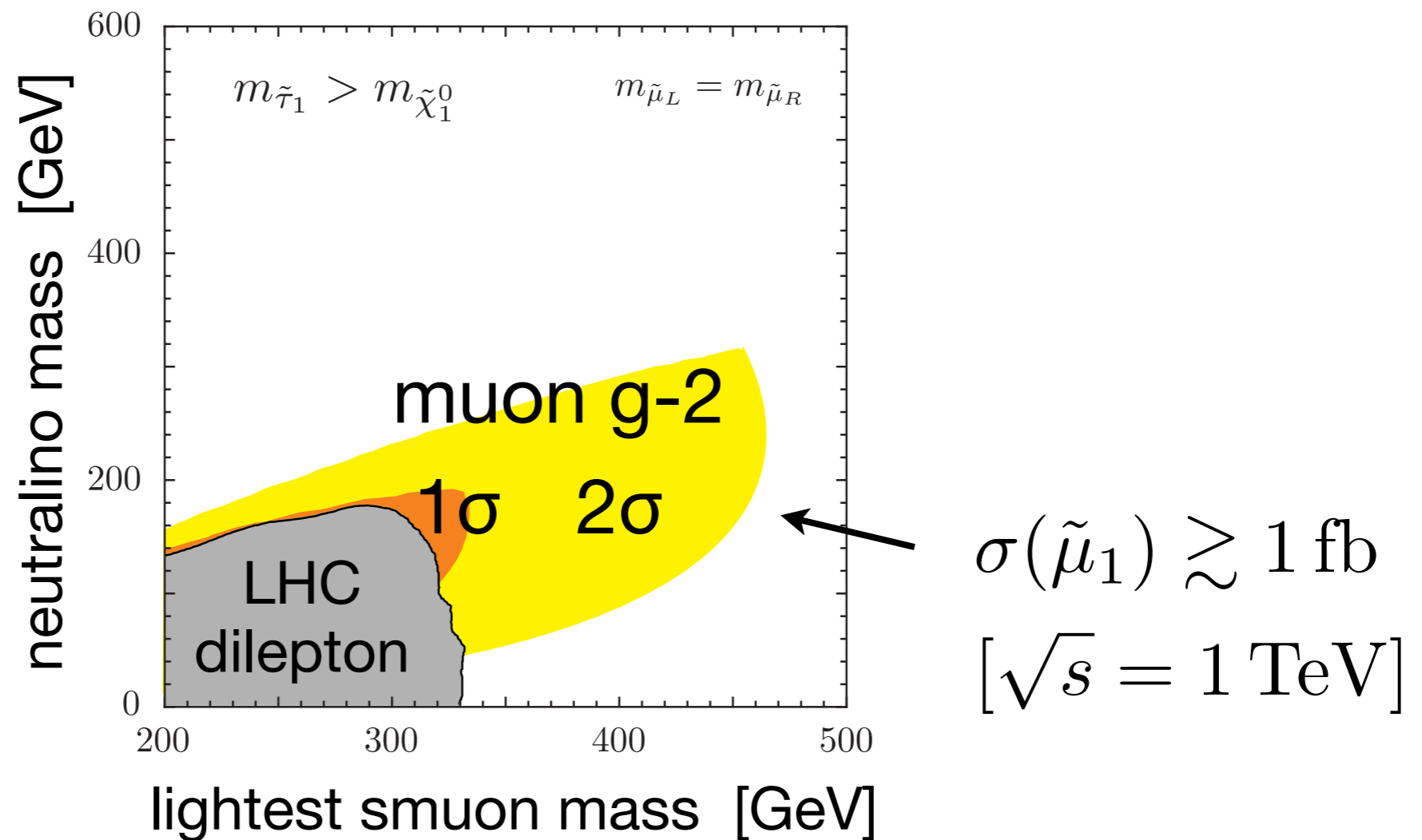
- ✓ Light: left- and right-handed smuons, Bino
- ✓ large smuon LR mixing parameter ($\propto \mu \tan\beta$)
 - ▶ too large LR mixing spoils vacuum stability

Upper bounds on masses of SUSY particles



Mass Region

(Lightest) smuon and Bino are within kinematical reach of ILC at $\sqrt{s} = 1$ TeV [Endo, Hamaguchi, Kitahara, Yoshinaga]



ILC can *probe* neutralino contribution to muon g-2

Test of SUSY Solution

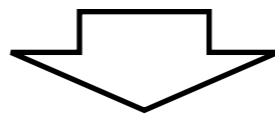
Stage 1

discovery of SUSY particles necessary for
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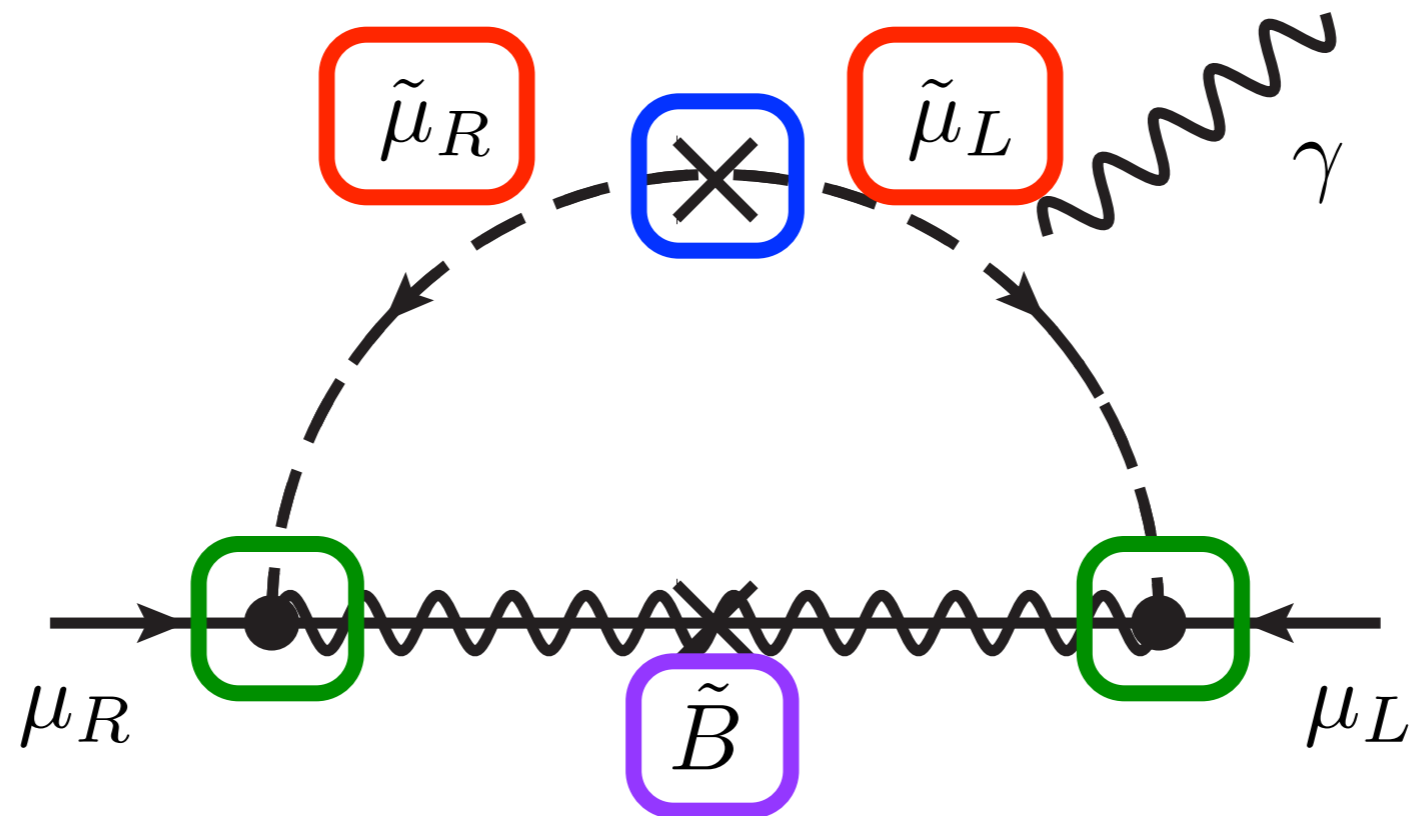


It is possible to *reconstruct* at ILC under some conditions

Reconstruction

Muon g-2 parameters

$$\underline{m_{\tilde{\mu}1}, m_{\tilde{\mu}2}}, \underline{m_{\tilde{\mu}LR}^2}, \underline{m_{\tilde{\chi}_1^0}}, \underline{\tilde{g}_{1,L}^{(eff)}}, \underline{\tilde{g}_{1,R}^{(eff)}}$$



Reconstruction

Muon g-2 parameters

$$m_{\tilde{\mu}1}, m_{\tilde{\mu}2}, m_{\tilde{\mu}LR}^2, m_{\tilde{\chi}_1^0}, \tilde{g}_{1,L}^{(\text{eff})}, \tilde{g}_{1,R}^{(\text{eff})}$$

Reconstruct neutralino contribution:

$$a_\mu = \frac{1}{16\pi^2} \frac{m_\mu^2}{m_{\tilde{\mu}}^2} \left[-\frac{1}{12} \left[(\hat{N}_L^\mu)^2 + (\hat{N}_R^\mu)^2 \right] F_1^N(x) - \frac{m_{\tilde{\chi}_1^0}}{3m_\mu} \hat{N}_L^\mu \hat{N}_R^\mu F_2^N(x) \right]$$

$$(\hat{N}_L^\mu)_i = \frac{1}{\sqrt{2}} \tilde{g}_{1,L}^{(\text{eff})} (U_{\tilde{\mu}})_{iL}, \quad (\hat{N}_R^\mu)_i = -\sqrt{2} \tilde{g}_{1,R}^{(\text{eff})} (U_{\tilde{\mu}})_{iR},$$

* Winos and Higgsinos are decoupled

Reconstruction

Parameter	Process	Result
$m_{\tilde{\mu}1}, m_{\tilde{\mu}2}, m_{\tilde{\chi}_1^0}$		
$m_{\tilde{\mu}LR}^2$		
$\tilde{g}_{1,L}^{(\text{eff})}, \tilde{g}_{1,R}^{(\text{eff})}$		

Setup

Sample point

Parameters	$m_{\tilde{\ell}_1}$	$m_{\tilde{\ell}_2}$	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^0}$	$\sin \theta_{\tilde{\mu}}$	$\sin \theta_{\tilde{\tau}}$	$a_{\mu}^{(\text{ILC})}$
Values	126	200	108	210	90	0.027	0.36	2.6×10^{-9}

$$(\tilde{\ell} = \tilde{e}, \tilde{\mu})$$

* other SUSY particles [Wino,Higgsino,colored] are decoupled.

- All of selectrons, smuons and staus are within kinematical reach of ILC at $\sqrt{s} = 500 \text{ GeV}$
- Close to SPS1a('): [left-handed sleptons are lighter]
 - avoid LHC/LEP limits
 - previous studies of ILC can be applied

Mass Measurement

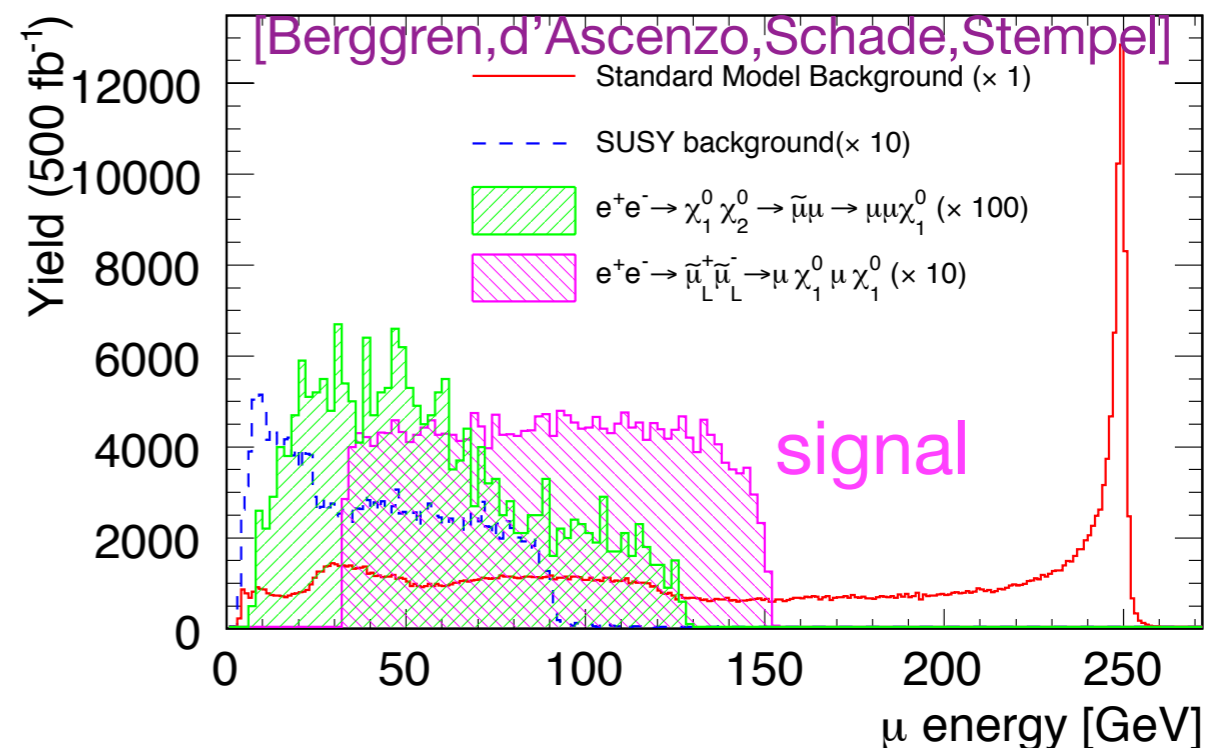
Smuon and neutralino masses are measured precisely by studying endpoints or by threshold scans

$$e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-, \quad e^+e^- \rightarrow \tilde{e}^+\tilde{e}^-$$

Previous studies: SPS1a('), $\sqrt{s} = 400, 500 \text{ GeV}$, $\mathcal{L} = 200\text{--}500 \text{ fb}^{-1}$

ILC can provide

$$\left\{ \begin{array}{l} \delta m_{\tilde{\mu}1} \sim 200 \text{ MeV} \\ \delta m_{\tilde{\mu}2} \sim 200 \text{ MeV} \\ \delta m_{\tilde{\chi}_1^0} \sim 100 \text{ MeV} \end{array} \right. \text{ or better}$$



Reconstruction

Parameter	Process	Result
$m_{\tilde{\mu}1}, m_{\tilde{\mu}2}, m_{\tilde{\chi}_1^0}$	$e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^-$ ($\tilde{\ell} = \tilde{e}, \tilde{\mu}$)	very precise
$m_{\tilde{\mu}LR}^2$	<div style="border: 1px solid black; border-radius: 15px; padding: 10px; display: inline-block;"> too small to measure directly </div>	
$\tilde{g}_{1,L}^{(\text{eff})}, \tilde{g}_{1,R}^{(\text{eff})}$		

Stauon LR Mixing

stauon LR mixing parameter is measured by the relation:

$$m_{\tilde{\mu}LR}^2 = \frac{m_{\mu}}{m_{\tau}} m_{\tilde{\tau}LR}^2, \quad m_{\tilde{\tau}LR}^2 = \frac{1}{2} (m_{\tilde{\tau}1}^2 - m_{\tilde{\tau}2}^2) \sin 2\theta_{\tilde{\tau}}$$

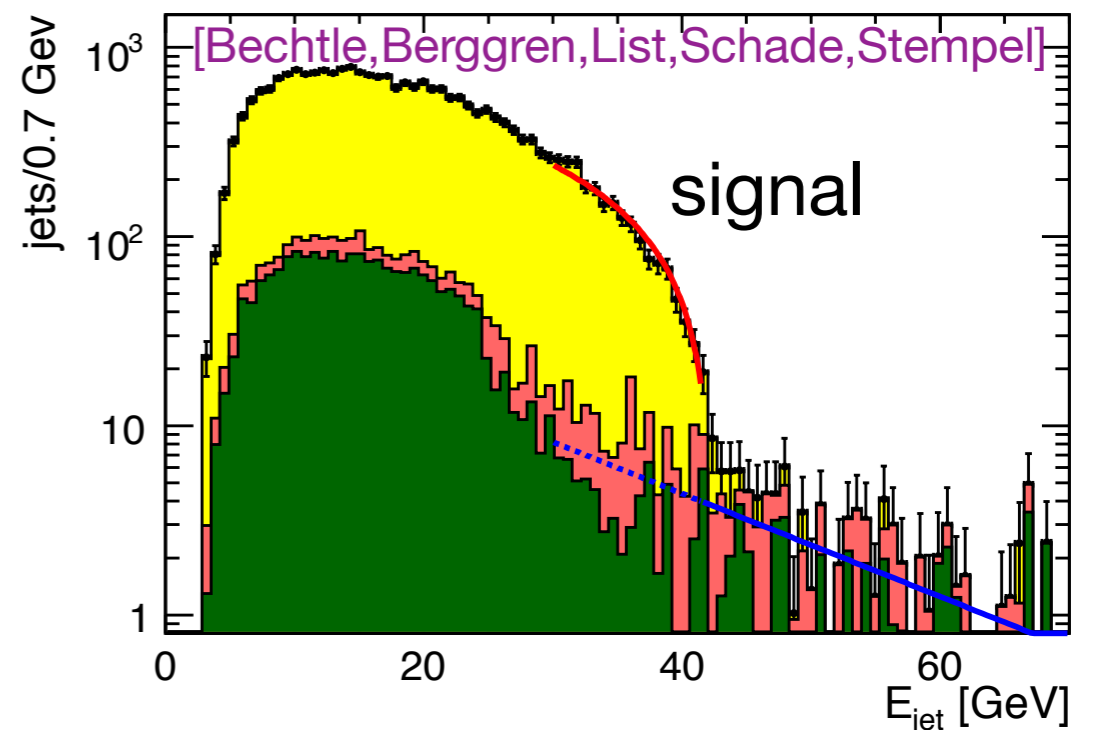
stau masses: endpoint of tau (-jet) energy

$$e^+ e^- \rightarrow \tilde{\tau}_1^+ \tilde{\tau}_1^-, \quad e^+ e^- \rightarrow \tilde{\tau}_2^+ \tilde{\tau}_2^-$$

$$\text{ILC} \begin{cases} \delta m_{\tilde{\tau}1} / m_{\tilde{\tau}1} \sim 0.1\% \\ \delta m_{\tilde{\tau}2} / m_{\tilde{\tau}2} \sim 3\% \end{cases}$$

Based on detailed study at SPS1a'

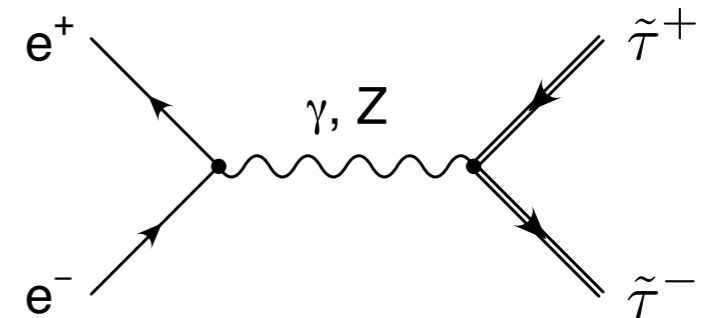
with $\sqrt{s} = 500 \text{ GeV}$, $\mathcal{L} = 500 \text{ fb}^{-1}$



Stauon LR Mixing contd.

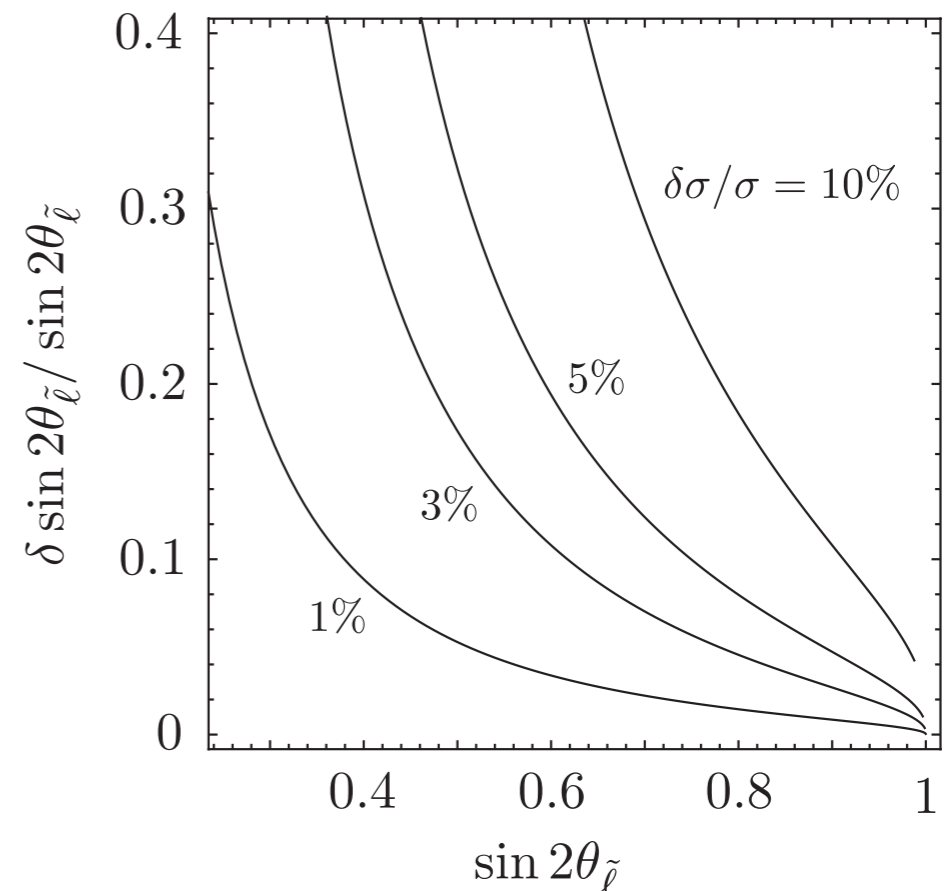
stau mixing angle: stau production cross section

Cross section depends on the angle via s-channel Z exchange



$$\sigma(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_1^*) = \underbrace{\alpha}_{\gamma} + \underbrace{\beta \cos 2\theta_{\tilde{\tau}}}_{\text{Z-exchange}} + \dots$$

Mixing angle is determined, once the cross section and the model point are given.



Stauon LR Mixing contd.

stau mixing angle: stau production cross section

Based on detailed study at SPS1a' w/. $\sqrt{s} = 500 \text{ GeV}$, $\mathcal{L} = 500 \text{ fb}^{-1}$
and with some discussions [Bechtle, Berggren, List, Schade, Stempel]

$$\delta\sigma(\tilde{\tau}_1)/\sigma(\tilde{\tau}_1) = 3.4\%$$

$$\longrightarrow \delta \sin 2\theta_{\tilde{\tau}} / \sin 2\theta_{\tilde{\tau}} = 9\% \quad \text{for } \sin 2\theta_{\tilde{\tau}} = 0.67$$

As a result, stau production processes yield $[500\text{fb}^{-1}]$

$$m_{\tilde{\mu}LR}^2 = \frac{m_{\mu}}{m_{\tau}} m_{\tilde{\tau}LR}^2 \longrightarrow \delta m_{\tilde{\mu}LR}^2 / m_{\tilde{\mu}LR}^2 = 12\%$$

Reconstruction

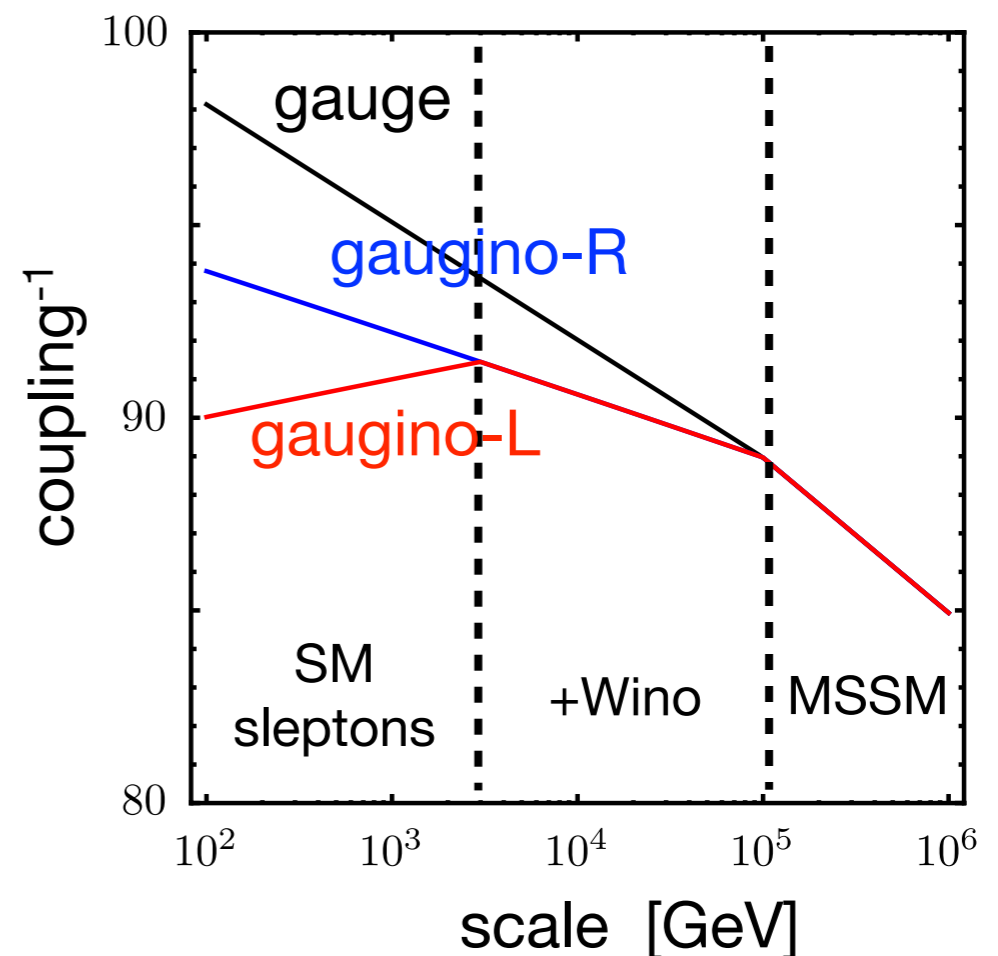
Parameter	Process	Result
$m_{\tilde{\mu}1}, m_{\tilde{\mu}2}, m_{\tilde{\chi}_1^0}$	$e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^-$ ($\tilde{\ell} = \tilde{e}, \tilde{\mu}$)	very precise
$m_{\tilde{\mu}LR}^2$	$e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$ [$\tilde{\tau}_1^+\tilde{\tau}_1^-$, $\tilde{\tau}_2^+\tilde{\tau}_2^-$]	12% at model point
$\tilde{g}_{1,L}^{(\text{eff})}, \tilde{g}_{1,R}^{(\text{eff})}$		

Gaugino Couplings

$$\mathcal{L}_{\text{int}} = \bar{\chi}_1^0 (N_L P_L + N_R P_R) \ell \tilde{\ell}^\dagger + \text{h.c.}$$

$$N_L \equiv \frac{1}{\sqrt{2}} \tilde{g}_{1,L}^{(\text{eff})} (U_{\tilde{\ell}})_{iL}, \quad N_R \equiv -\sqrt{2} \tilde{g}_{1,R}^{(\text{eff})} (U_{\tilde{\ell}})_{iR}$$

- LO is approximated by $U(1)_Y$ gauge coupling
 - Deviate due to mixing with (heavy) Winos/ Higgsinos and by radiative corrections
- **$O(1-10)\%$ correction**



Gaugino Couplings

Selectron productions involve neutralino contributions

Right-handed coupling: $\tilde{g}_{1,R}^{(\text{eff})}$

$$\sigma(e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^-) \sim (\tilde{g}_{1,R}^{(\text{eff})})^4$$

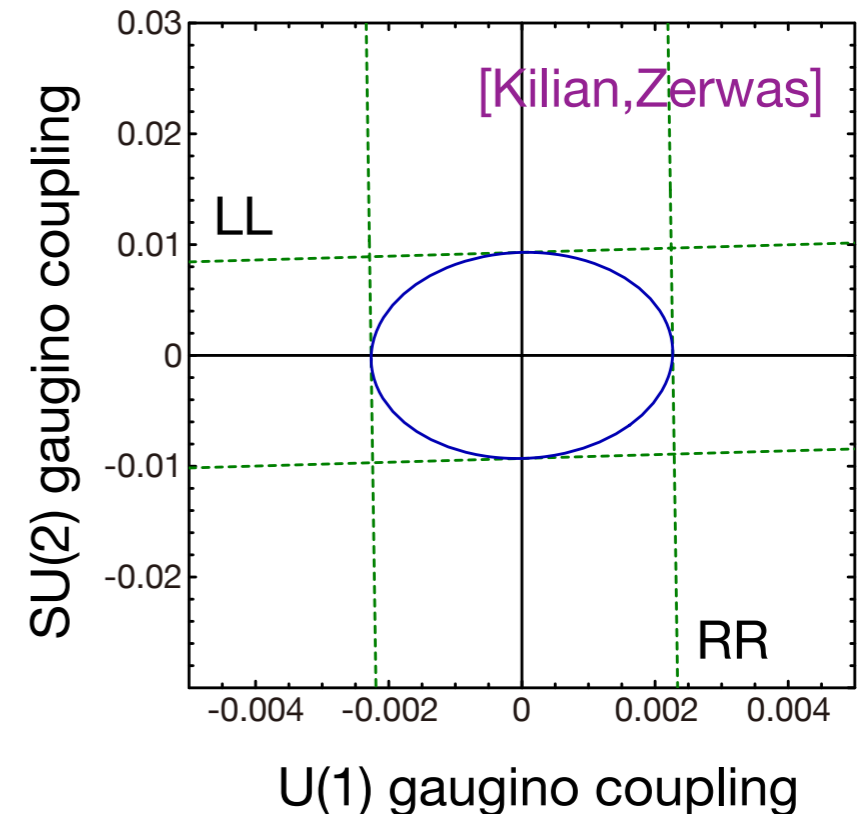
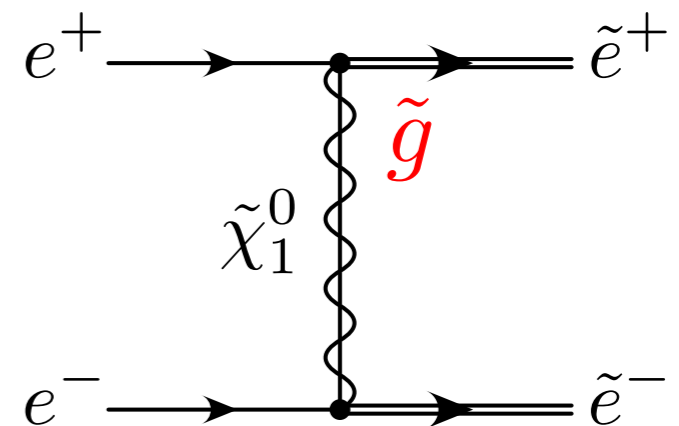
Based on previous study at SPS1a

with $\sqrt{s} = 500 \text{ GeV}$, $\mathcal{L} = 500 \text{ fb}^{-1}$

with discussions [Freitas,Manteuffel,Zerwas]

$$\delta\tilde{g}_{1,R}^{(\text{eff})} / \tilde{g}_{1,R}^{(\text{eff})} \lesssim 1\%$$

including contaminations from
(unobserved) Winos and Higgsinos



Gaugino Couplings

Left-handed coupling: $\tilde{g}_{1,L}^{(\text{eff})}$

$$\sigma(e^+e^- \rightarrow \tilde{e}_L^+ \tilde{e}_R^-) \sim (\tilde{g}_{1,L}^{(\text{eff})})^2 (\tilde{g}_{1,R}^{(\text{eff})})^2$$

- Previous studies ignore the differences between left- and right-handed gaugino couplings.
- No info. of selectron LR production cross section.

$$\delta \tilde{g}_{1,L}^{(\text{eff})} / \tilde{g}_{1,L}^{(\text{eff})} = \text{a few \% (exp)} + 1 \% (\text{th})$$

exp: measurement of cross section
th: (unobserved) Winos, Higgsinos

For instance, $\delta\sigma(\tilde{e}_L\tilde{e}_R)/\sigma(\tilde{e}_L\tilde{e}_R) = 4\% \rightarrow \delta\tilde{g}_{1,L}^{(\text{eff})}/\tilde{g}_{1,L}^{(\text{eff})} = 2\%$

Reconstruction

Parameters	Processes	Result
$m_{\tilde{\mu}1}, m_{\tilde{\mu}2}, m_{\tilde{\chi}_1^0}$	$e^+ e^- \rightarrow \tilde{\ell}^+ \tilde{\ell}^-$ ($\tilde{\ell} = \tilde{e}, \tilde{\mu}$)	very precise
$m_{\tilde{\mu}LR}^2$	$e^+ e^- \rightarrow \tilde{\tau}^+ \tilde{\tau}^-$ [$\tilde{\tau}_1^+ \tilde{\tau}_1^-, \tilde{\tau}_2^+ \tilde{\tau}_2^-$]	12% at model point
$\tilde{g}_{1,L}^{(\text{eff})}, \tilde{g}_{1,R}^{(\text{eff})}$	$e^+ e^- \rightarrow \tilde{e}^+ \tilde{e}^-$ [$\tilde{e}_R^+ \tilde{e}_R^-, e_L^+ \tilde{e}_R^-$]	O(1)%

Reconstruction at ILC

Neutralino contribution to muon g-2 is reconstructed by measuring all the sleptons

$$\delta a_{\mu}^{(\text{ILC})} / a_{\mu}^{(\text{ILC})} \simeq 13 \%$$

at the sample point with $\sqrt{s} = 500 \text{ GeV}$, $\mathcal{L} \sim 500 \text{ fb}^{-1}$

[ME, Hamaguchi, Iwamoto, Kitahara, Moroi]

Comments

- **Effects of (heavy) Winos and Higgsinos**
 - Direct contributions to muon $g-2$
 - No signals in future LHC can reduce uncertainties
- **Largest uncertainty is from smuon LR mixing**
 - Relation between smuon and stau LR mixings can be affected by slepton trilinear couplings (A-terms)
 - a few percents if A-term \sim slepton masses
 - $\sigma(\tilde{\tau}_1 \tilde{\tau}_2)$ is very sensitive to the stau mixing angle, though there are no such studies.

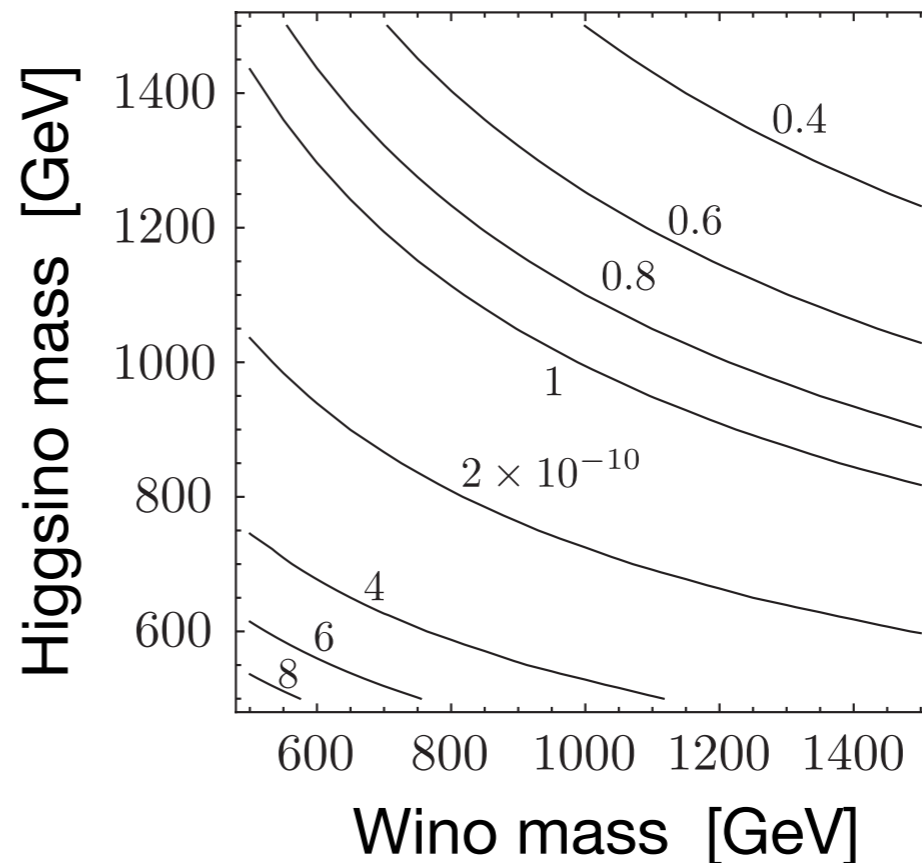
Comments

- Effects of (heavy) Winos and Higgsinos
 - Direct contributions to muon $g-2$
 - No signals in future LHC can reduce uncertainties

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Extra contribution
to muon $g-2$, Δa_μ
[Winos, Higgsinos]



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Comments

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Summary

- Muon $g-2$ has $>3\sigma$ deviation between SM prediction and experimental value.
- SUSY is a good candidate to solve the anomaly.
- We discussed how to test the SUSY contributions.
 - ✓ Neutralino contribution can be probed at ILC.
 - Sleptons are within kinematical reach.
 - It is possible to reconstruct the contribution, if all the sleptons (selectrons, smuons and staus) are measured.

$$\longrightarrow \delta a_{\mu}^{(\text{ILC})} / a_{\mu}^{(\text{ILC})} \simeq 13 \% \quad \text{at the sample point}$$