Charm and Bottom Quark Masses from Non-Lattice Methods

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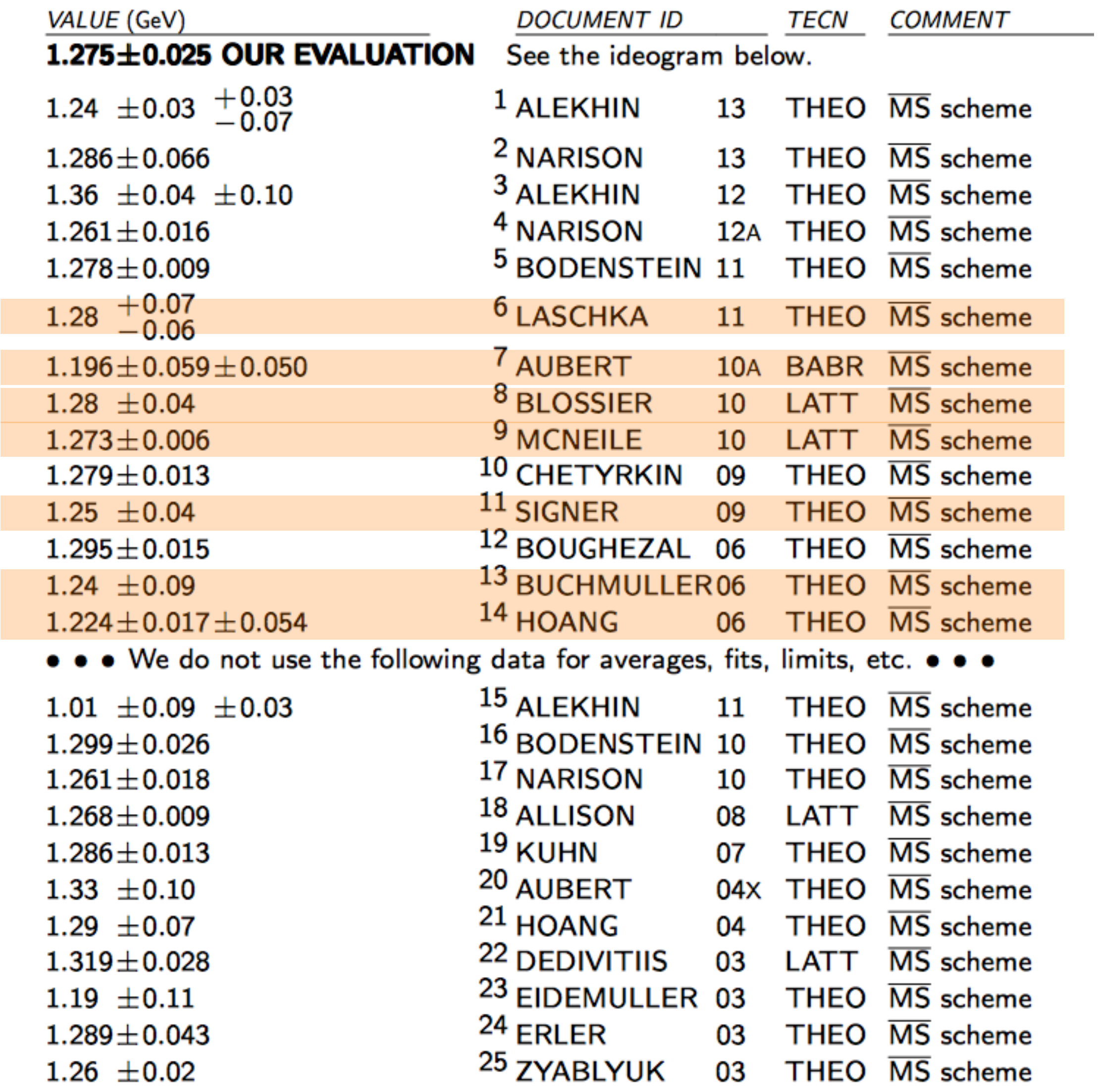
Outline

- Charm mass determinations
  - Deep inelastic scattering
  - Charmonium sum rules
- Bottom mass determinations
  - Inclusive (semileptonic) B decays
  - Upsilon sum rules
- Outlook and Conclusions

Charm Mass



VALUE (GeV)	DOCUMENT ID	TECN	COMMENT
1.275±0.025 OUR EVALUATION	See the ideogram below.		
1.24 ±0.03 $^{+0.03}_{-0.07}$	[1] ALEKHIN 13	THEO	$\overline{\text{MS}}$ scheme
1.286±0.066	[2] NARISON 13	THEO	$\overline{\text{MS}}$ scheme
1.36 ±0.04 ±0.10	[3] ALEKHIN 12	THEO	$\overline{\text{MS}}$ scheme
1.261±0.016	[4] NARISON 12A	THEO	$\overline{\text{MS}}$ scheme
1.278±0.009	[5] BODENSTEIN 11	THEO	$\overline{\text{MS}}$ scheme
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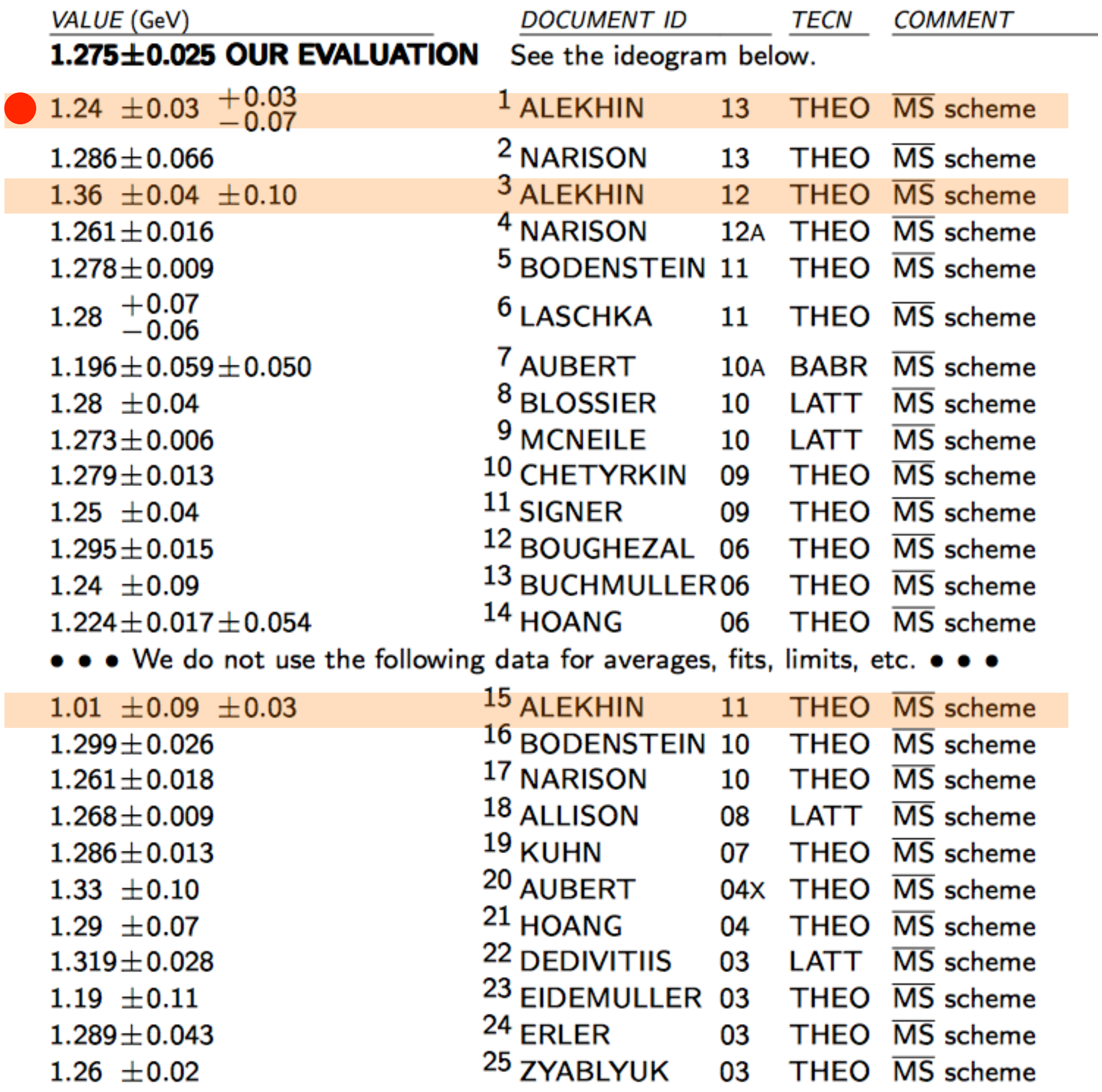
1.289±0.043	[24] ERLER 03	THEO	$\overline{\text{MS}}$ scheme
1.26 ±0.02	[25] ZYABLYUK 03	THEO	$\overline{\text{MS}}$ scheme

Will not discuss:

- Non-relativistic charmonium sum rules
- $B \to X_c \, l \, \nu$ (see bottom part)
- Lattice

Deep Inelastic Scattering



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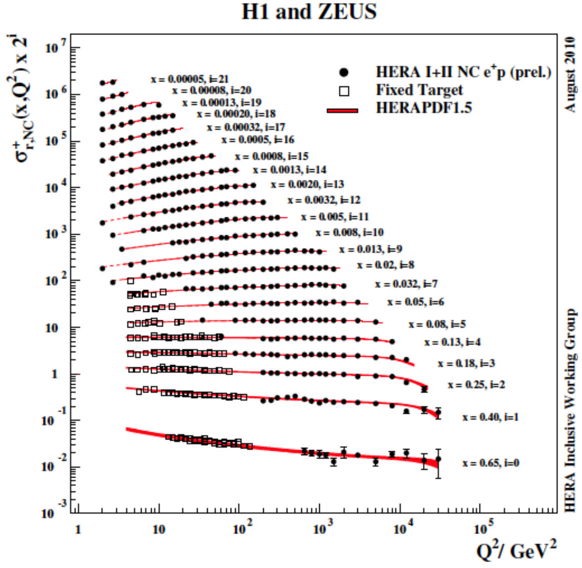
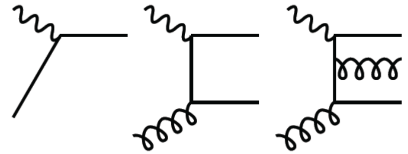
Method:

- Charm production in photon-induced in DIS
- “inclusive” F_2^c
- Tremendous amount of data
- Very precise theory



● consistent and NNLO

Deep Inelastic Scattering



Theoretical issues:

- Different “schemes” to implement charm mass:
    - Variable vs. fixed-flavor number schemes: charm pdf vs. pdf without charm
    - Different version of variable flavor numbers VFN schemes (ACOT-type schemes)
    - Correlation to fits of pdf’s and value of α_s
    - Related to ways to sum logarithms of the charm mass

- Intrinsic charm: non-perturbative non-zero charm pdf
- Different technical implementations of VFN (technical, conceptual)
- Issues small for bottom, but significant for charm

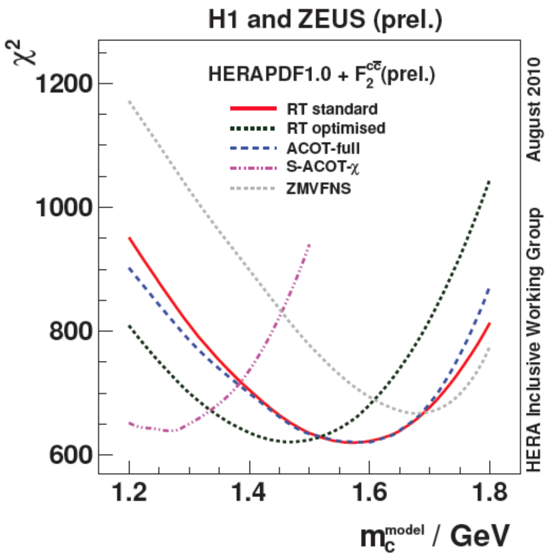
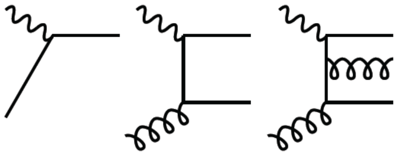


Figure 9: Comparison of χ^2 for HERA I + $F_2^{c\bar{c}}$ fits using different heavy flavor schemes as a function of the charm quark mass parameter m_c^{model}. *(Figure from H1prelim-10-143 & ZEUS-prel-10-019)*

- Significant dependence of the charm mass on method used
- No unanimous agreement of best scheme
- In principle very powerful method
- **Sorting out problems probably more important for jet physics itself than for charm mass.** (→ SCET version: w.i.p., “mass modes”)
- Not complete coherent picture at this time

Deep Inelastic Scattering



Alekhin etal. (2013):

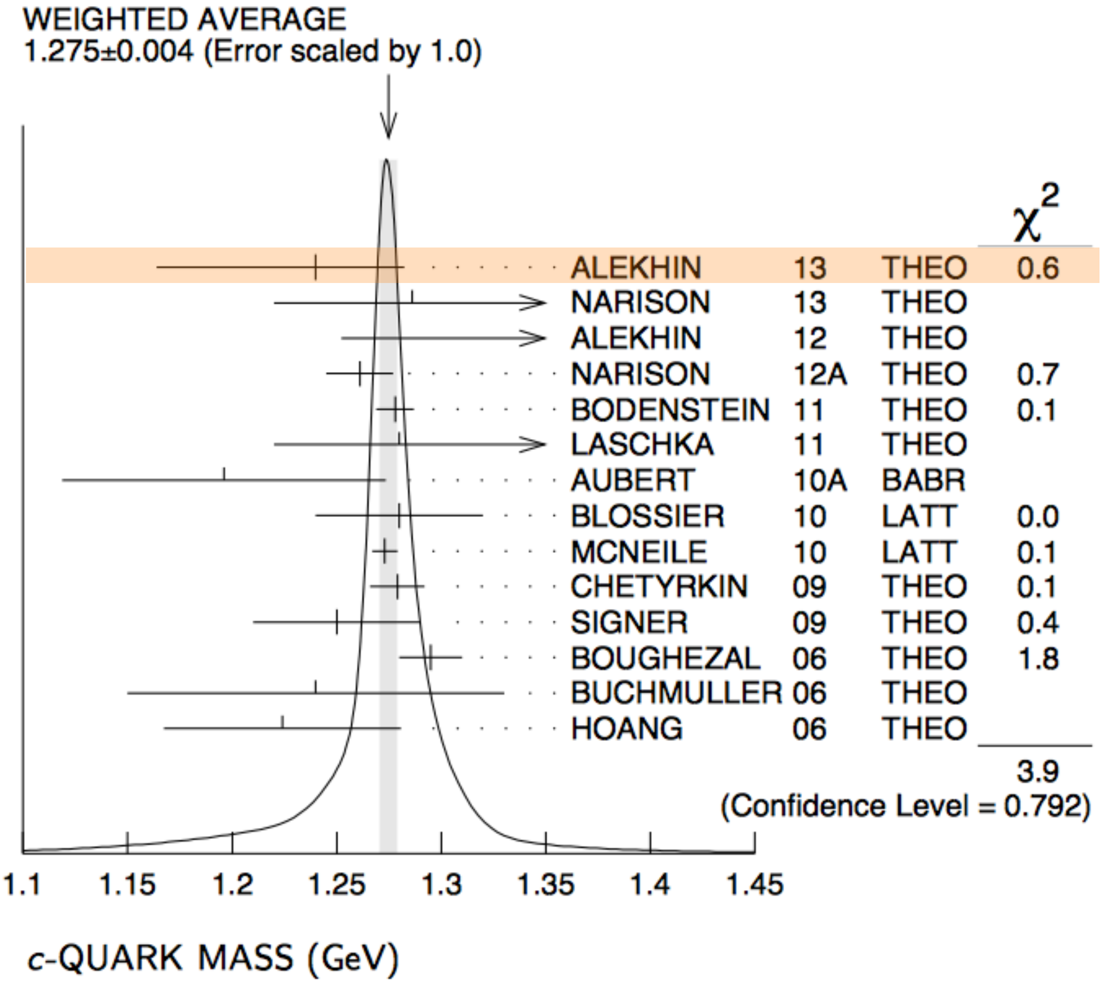
- Fixed-Fermion-Number scheme (n_f=3 strictly)
  - Only non-charm (u,d,s) pdf
  - All charm dynamics inside the short-distance calculation
  - No summation of logarithms related to charm mass $\ln^n(m_c/Q)$
- Combined and correlated analysis of
  - Fixed-order input at $\text{NNLO}_{\text{approx}}$
  - Fitting of pdf's to all data (incl. non-charm, non-DIS)
  - Fitting of α_s ($\alpha_s(M_Z)$ = 0.114 xxxx)

$$m_c(m_c) = 1.24 \pm 0.03(\text{exp}) \, {}^{+0.03}_{-0.02}(\text{scale}) \, {}^{+0.00}_{-0.07}(\text{theory}) \text{ GeV}$$

Wishlist for charm mass VFN schemes:

- Improvement on scheme issues of Variable-Flavor-Number schemes
- Confirmation of FFN scheme result by VFN schemes (correlated analyses)
- Issue of VFN schemes (disadvantage? – do not exist in FFN scheme)
  - Implementations cannot be unique (mass/Q variable scale)
  - More renormalization scales (μ, mass matching scale)

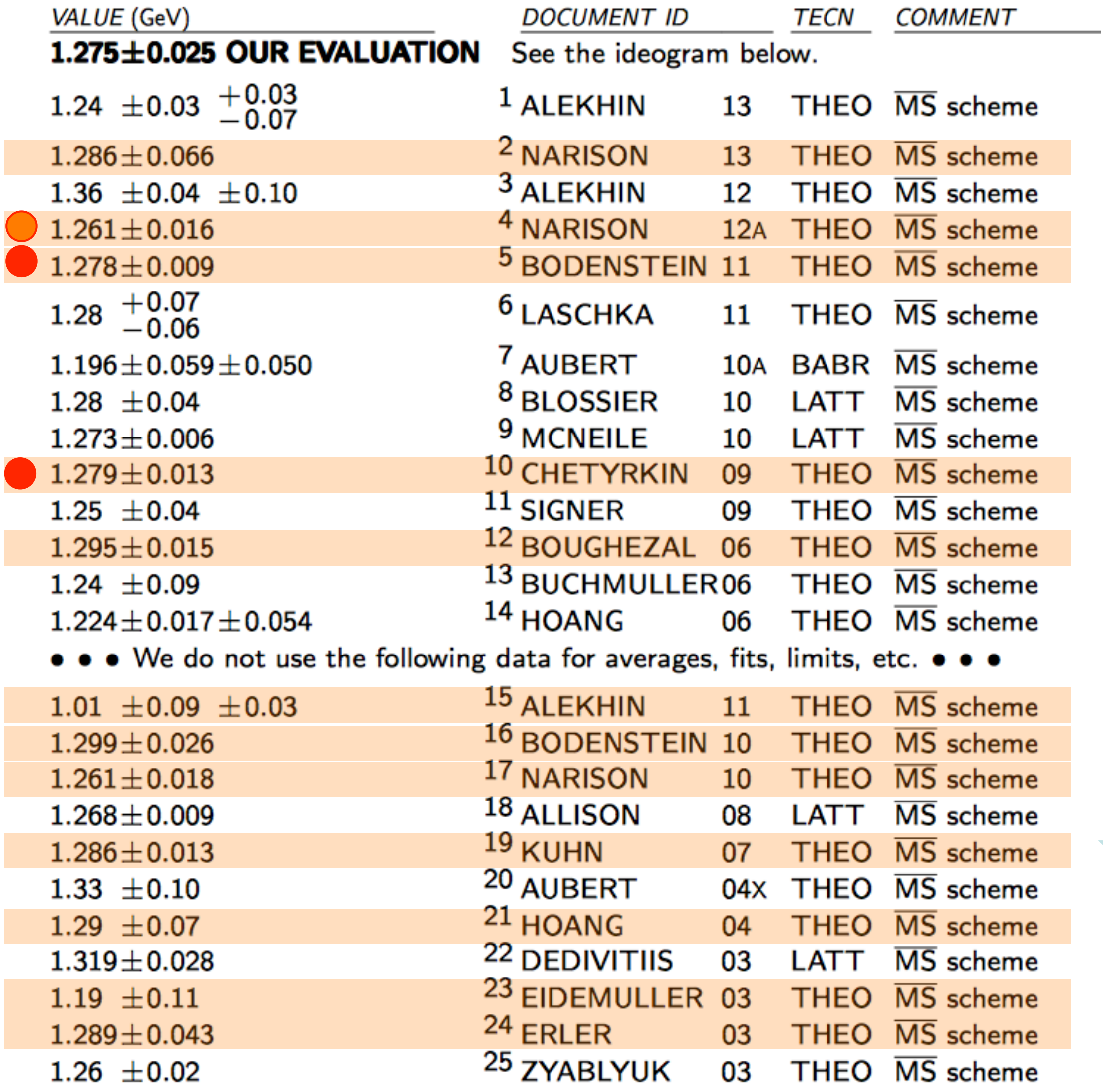
Inclusive B-Decays



			χ^2
ALEKHIN	13	THEO	0.6
NARISON	13	THEO	
ALEKHIN	12	THEO	
NARISON	12A	THEO	0.7
BODENSTEIN	11	THEO	0.1
LASCHKA	11	THEO	
AUBERT	10A	BABR	
BLOSSIER	10	LATT	0.0
MCNEILE	10	LATT	0.1
CHETYRKIN	09	THEO	0.1
SIGNER	09	THEO	0.4
BOUGHEZAL	06	THEO	1.8
BUCHMULLER	06	THEO	
HOANG	06	THEO	
			3.9

(Confidence Level = 0.792)

Charmonium Sum Rules



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Method:

- Moments of the $R_c(e^+e^- \to \text{charm})$ cross section
- Moments computed from pQCD
- Non-perturbative corrections small (a la SVZ)

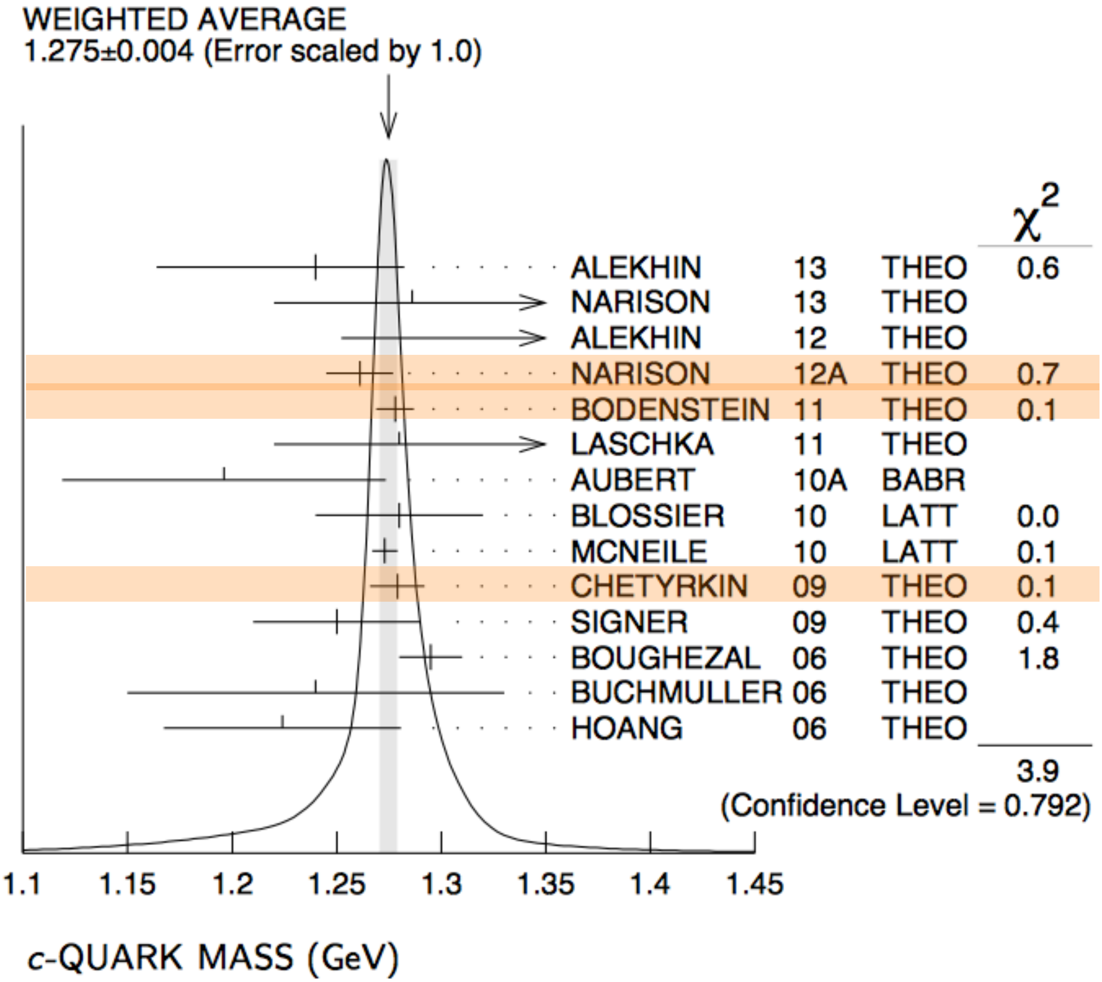
● 2009 CLEO data included and $O(\alpha_s^3)$

Outdated:

Lower order

Updated analyses

Charmonium Sum Rules



Charmonium Sum Rules

Regular moment method:

Total hadronic cross section

Moments of the cross section

$$R(s)=\frac{\sigma\left(e^+e^-\to\text{hadrons}\right)}{\sigma\left(e^+e^-\to\mu^+\mu^-\right)}\quad\Longrightarrow\quad M_n=\int_{4m^2}^{\infty}\frac{\mathrm{ds}}{\mathrm{s}^{n+1}}R(s)\overset{z=\frac{s}{4m^2}}{=}\frac{1}{\left(4m^2\right)^n}\int_1^{\infty}\frac{\mathrm{dz}}{z^{n+1}}R(z)$$

Vacuum polarization function

Vector current (electromagnetic)

$$\left(g_{\mu\nu}-q_\mu q_\nu\right)\Pi(q^2)=-i\int\mathrm{d}x\,e^{ix\cdot q}\left\langle 0\middle|\mathrm{T}\left\{J_\mu(x)J_\nu(x)\right\}\middle|0\right\rangle\qquad J_\mu(x)=\bar{q}(x)\gamma_\mu q(x)$$

electric charge

$$R(s)=12\pi\boxed{Q^2}\,\mathrm{Im}\,\Pi(s+i0^+)\quad\Longrightarrow\quad\Pi(q^2)-\Pi(0)=\frac{q^2}{12\pi^2Q^2}\int_{4m^2}^{\infty}\mathrm{d}s\,\frac{R(s)}{s(q^2-s)}$$

$$\Pi(q^2\approx 0,m)=\frac{1}{12\pi^2Q^2}\sum_{n=0}^{\infty}M_n\,q^{2n}\quad\Longrightarrow\quad M_n=6\pi\,i\,Q^2\oint\mathrm{d}s\,\frac{\Pi(s)}{s^{n+1}}$$

Duality bound: $n < m_c/\mathrm{Lambda}_{\mathrm{QCD}} \sim 3\text{-}4$

Charmonium Sum Rules

Finite energy sum rules:

$$\int_0^{s_0} p(s)\,\frac{1}{\pi}\,Im\,\Pi(s)\,ds \;=\; -\frac{1}{2\pi i}\oint_{C(|s_0|)} p(s)\,\Pi(s)\,ds \;+\; \mathrm{Res}[\Pi(s)\,p(s), s=0]$$

- Motivated by tau analyses
- Cut off high energy continuum where there is not data for R_c
- Modify weight resonances vs. non-resonance data
- Consistency requires: $p(s_0) = 0$ (polynomial in s^n , $n=0,\pm1,\ldots$)
- Same results expected as with regular moments:
  - High energy continuum model taken from theory anyway
  - Like linear combinations of different finite-energy moments

Theory status:

- Fixed-order moments known to $O(\alpha_s^3)$ Chetyrkin etal, etc
- R-ratio for massive quarks known to $O(\alpha_s^2)$ [$O(\alpha_s^3)$ (approximate)]
- Non-perturbative condensates (a la SVZ) small e.g. Pade

⇨ Both methods essentially with theory input at $O(\alpha_s^3)$

Charmonium Sum Rules

Finite energy: $\overline{m}_c(\overline{m}_c) = 1.278 \pm 0.009$ Bodenstein etal.

2 GeV < μ < 4 GeV (Δμ=35%)

Regular: $\overline{m}_c(\overline{m}_c) = 1.261 \pm 0.016$ Narison

$\overline{m}_c(\overline{m}_c) = 1.279 \pm 0.013$ Chetyrkin etal.

2 GeV < μ < 4 GeV

→ Continuum models = pQCD prediction

→ Not all available data used

→ All analyses used same renormalization scale for α_s(μ) and m(μ) and the regular fixed-order expansion.

Charmonium Sum Rules

n	$m_c(3\,\mathrm{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

Chetyrkin, Kuhn, Meier, Meierhofer, Marquard
Steinhauser (2009)

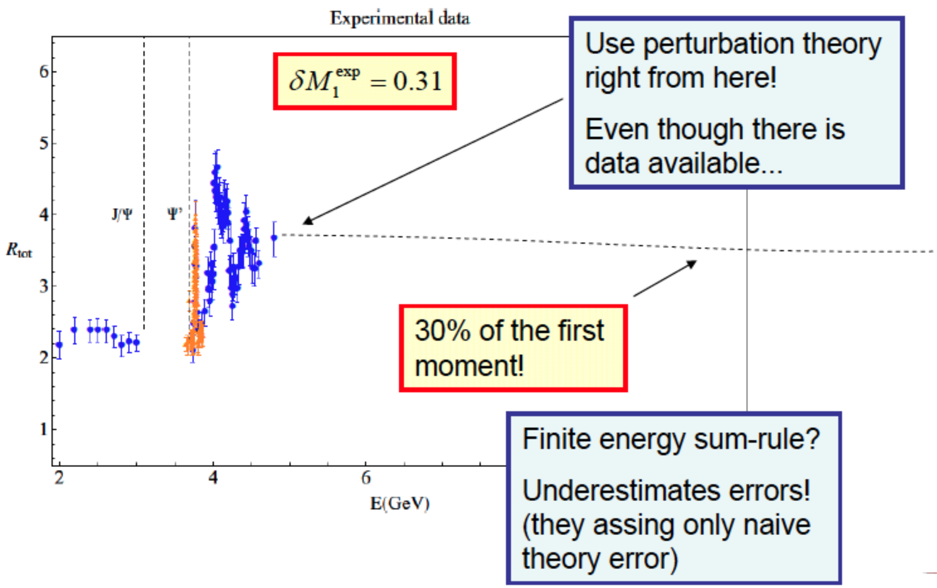
- $m_c(3\,\mathrm{GeV}) = 986 \pm 13\,\mathrm{MeV}$
- $m_c(m_c) = 1279 \pm 13\,\mathrm{MeV}$

2 GeV < μ < 4 GeV

Data used:

Data used in Kühn et al (2004, 2005, ...)

Data used in Bodenstein et al



$\delta M_1^{\rm exp} = 0.31$

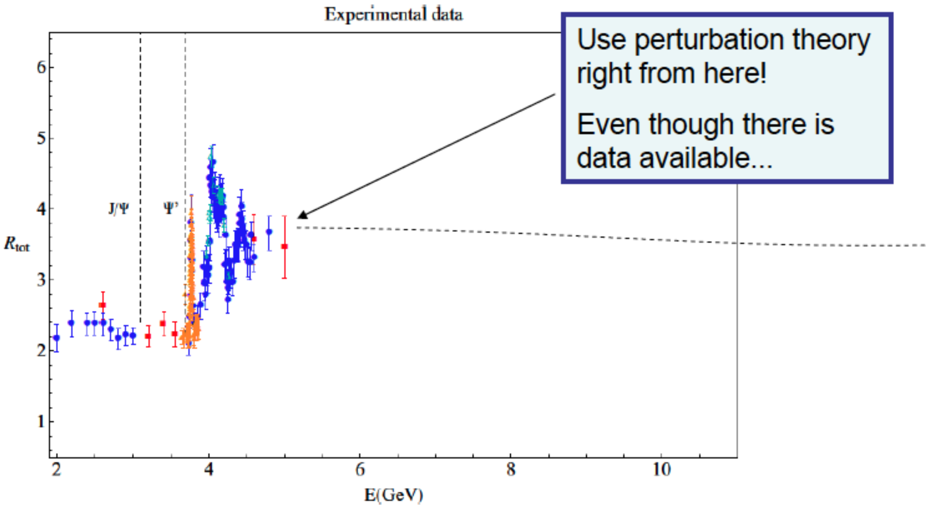
Use perturbation theory right from here!

Even though there is data available...

30% of the first moment!

Finite energy sum-rule?

Underestimates errors! (they assing only naive theory error)



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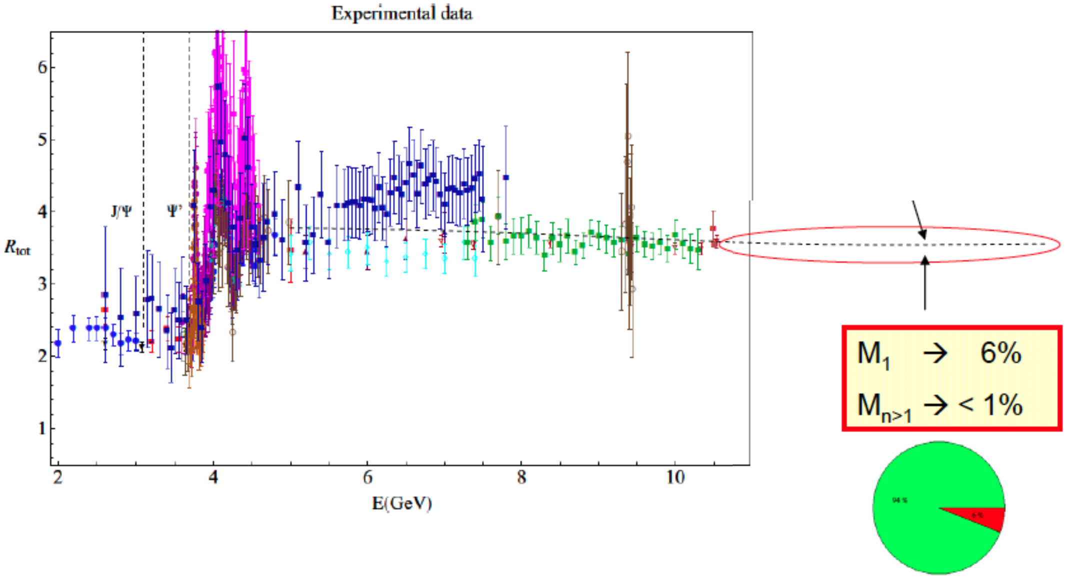

$M_1 \rightarrow$ 6%

$M_{n>1} \rightarrow$ < 1%

Charmonium Sum Rules

→ Different expansions: all equally qualified in pQCD.

Standard fixed order:

⇦ Used by previous analyses

$$M_n^{\rm pert} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Linearized:

$$\left(M_n^{\rm th,pert}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Iterative linearized:

⇦ Our default

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}}\tilde{C}_{n,0}^{0,0}$$

$$\overline{m}_c^{(1)}(\mu_m) = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}}\left\{\tilde{C}_{n,0}^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi}\left[\tilde{C}_{n,1}^{0,0} + \tilde{C}_{n,1}^{1,0}\ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\right]\right\}$$

Always has solutions!

Contour improved:

$$M_n^{\rm c,pert} = \frac{6\pi Q_c^2}{i}\int_c \frac{{\rm d}s}{s^{n+1}}\Pi(q^2,\alpha_s(\mu_\alpha^c(s,\overline{m}_c^2)),\overline{m}_c(\mu_m),\mu_\alpha^c(s,\overline{m}_c^2),\mu_m)$$

$$(\mu_\alpha^c)^2(s,\overline{m}_c^2) = \mu_\alpha^2\left(1-\frac{s}{4\overline{m}_c^2(\mu_m)}\right)$$

Charmonium Sum Rules

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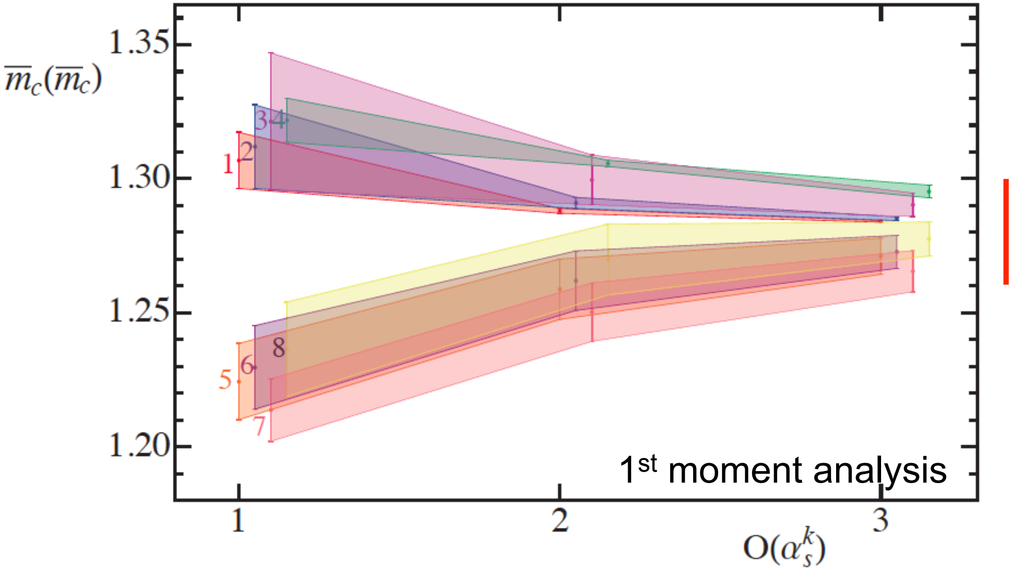
- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1279 \pm 13\,\text{MeV}$

2 GeV < μ < 4 GeV

Our check of different expansions: → first check: pert. error considerably larger



1st moment analysis

± 20 MeV

Dehnadi, AH, Mateu, Zebarjad (2011)

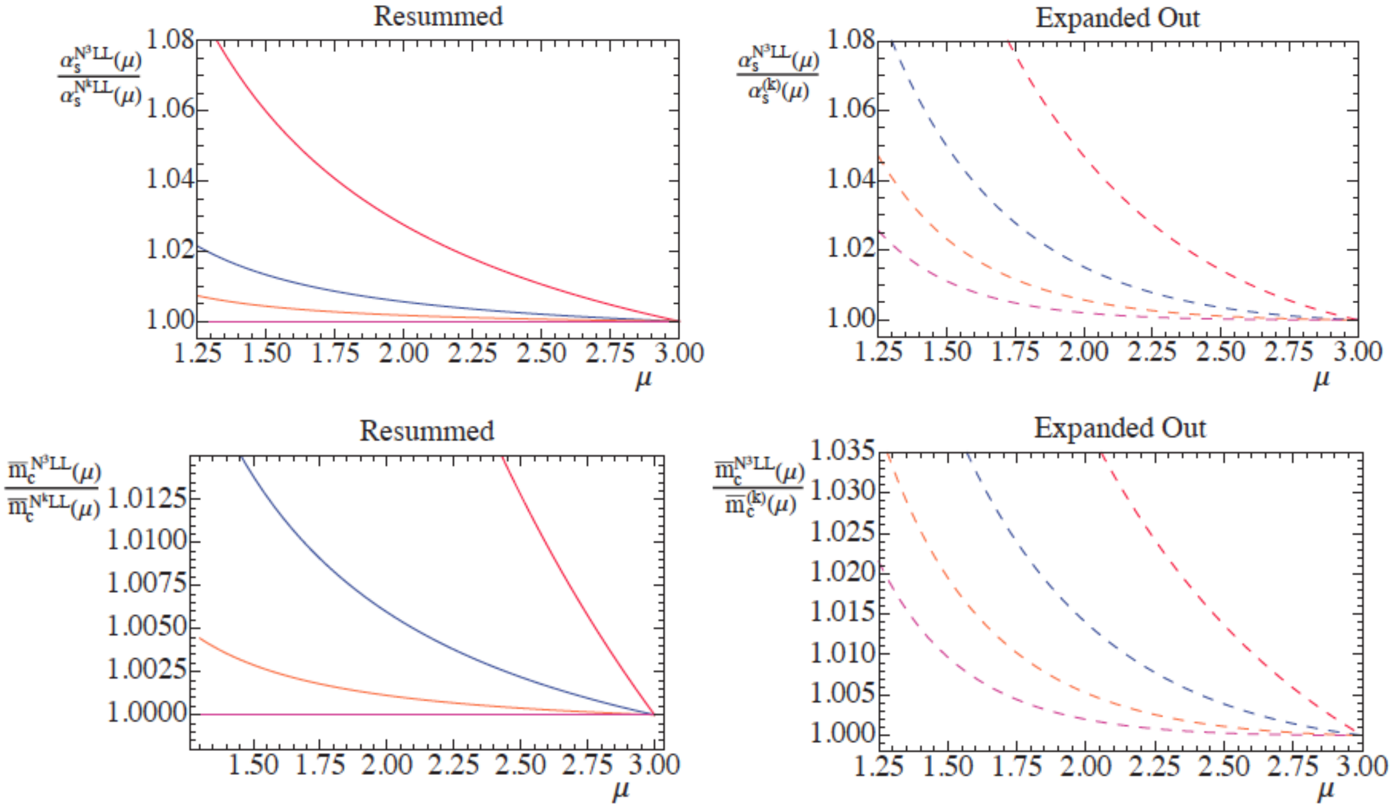
Figure 4. Results for $\overline{m}_c(\overline{m}_c)$ at various orders, for methods a (graphs 1 and 5), b (2,6), c (3,7), and d (4,8), setting $\mu_\alpha = \mu_m$ (graphs 1-4) and setting $\mu_m = \overline{m}_c(\overline{m}_c)$ (5-8). The shaded regions arise from the variation $2\,\text{GeV} \le \mu_\alpha \le 4\,\text{GeV}$.

Charmonium Sum Rules

Check of pQCD at μ = charm mass:

Search for instabilities of pQCD.

Dehnadi, AH, Mateu, Zebarjad (2011)



- Good convergence of pQCD observed at the charm mass scale
- No instability visible: $\mu = m_c$ viable choice
- But perturbative expansion has O(10 MeV) deviations from resummed results at $O(\alpha_s^3)$.

Charmonium Sum Rules

Dehnadi, AH, Mateu, Zebarjad (2011)

Aims of our analysis:

- Include all experimental data
- Define proper scale variation (so that using different expansions does not matter)

Standard fixed-order expansion:

For $\mu_\alpha = \mu_m$ along a countour line

Strong cancellation between RG evolution of mass and strong coupling

Behavior different for other expansion versions

1st moment analysis at $O(\alpha_s{}^3)$

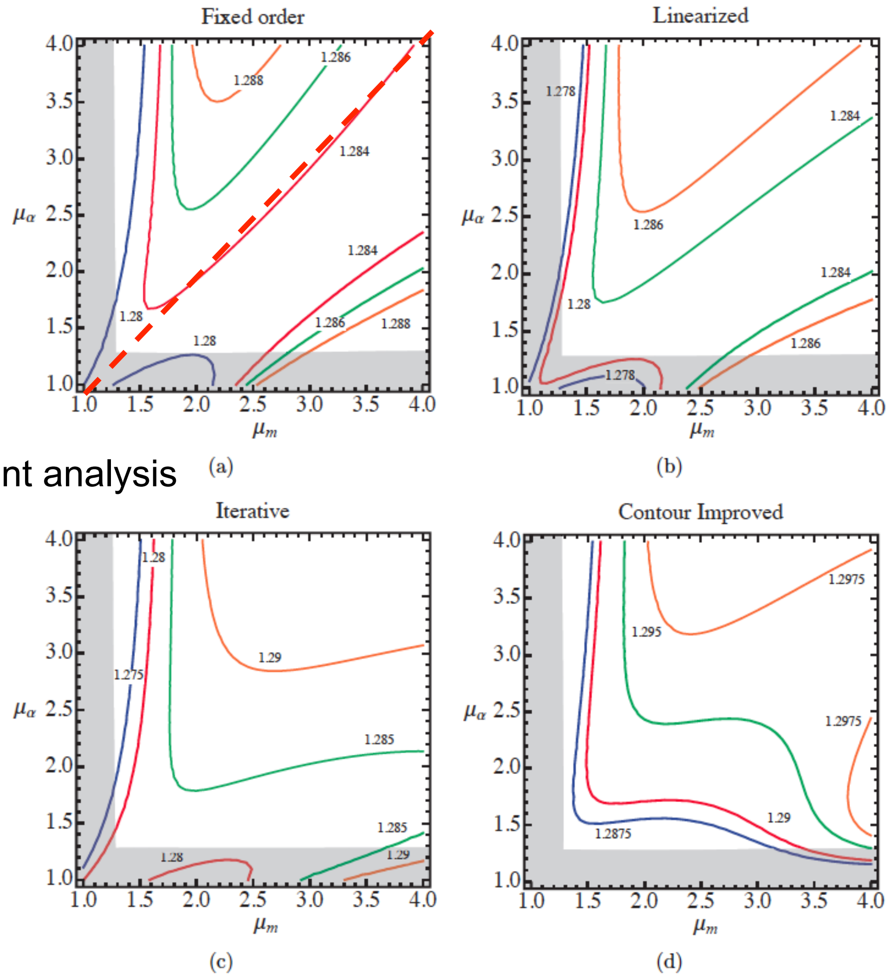
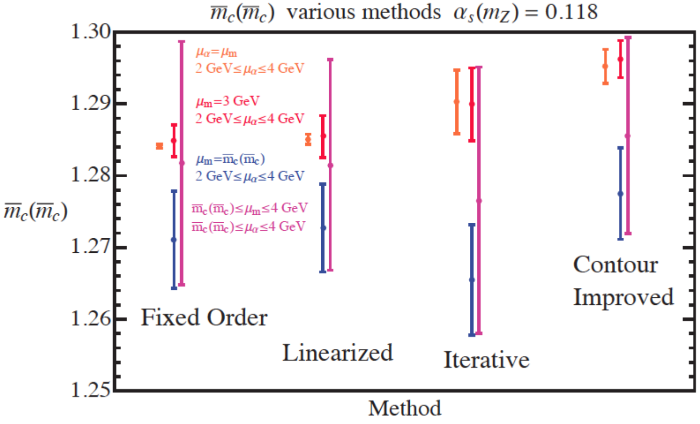


Figure 6. Contour plots for $\overline{m}_c(\overline{m}_c)$ as a function of μ_α and μ_m at $\mathcal{O}(\alpha_s^3)$, for methods (a)–(d). The shaded areas represent regions with $\mu_m, \mu_\alpha < \overline{m}_c(\overline{m}_c)$, and are excluded of our analysis.

Our conclusions:

Independent variation of μ_α and μ_m.

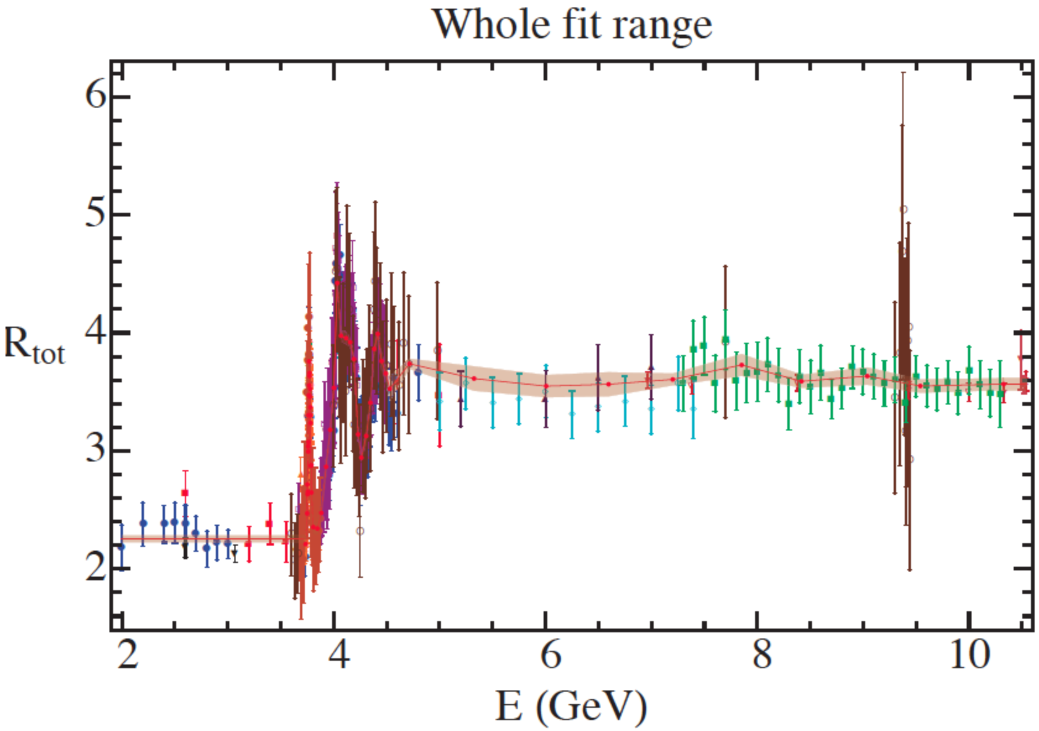
Reasonable choice: $m_c < \mu_\alpha$, $\mu_m < 4$ GeV



Charmonium Sum Rules

Experimental data:

Dehnadi, AH, Mateu, Zebarjad (2011)



- Data combination rebinning and reclustering of different data sets (motivated by approach used in g-2 R(had) data
- New: include subtraction of non-charm background
- Excellent agreement of reclustered data with pQCD above ~ 9 GeV
- 10% “experimental error” above 10.5 GeV (irrelevant as lower-E real data dominates)
- Moments M_n with complete experimental correlation

Hagiwara etal.

Charmonium Sum Rules

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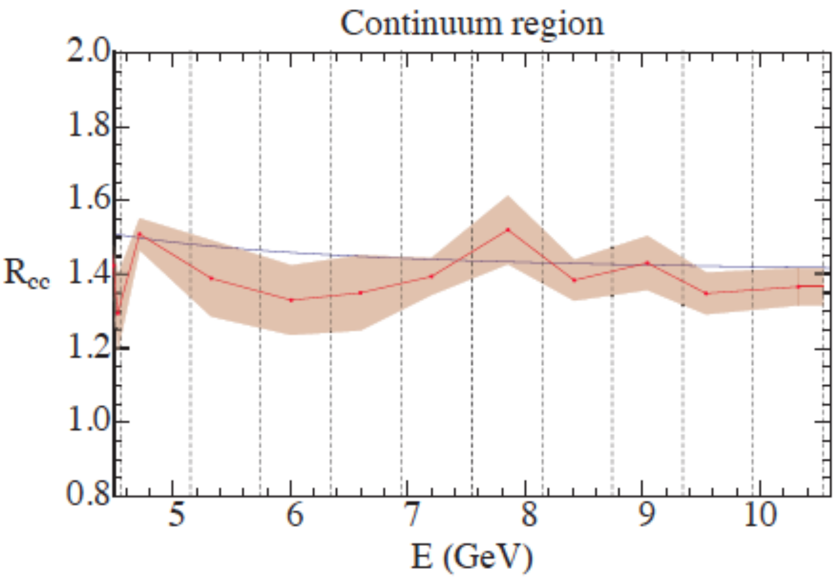

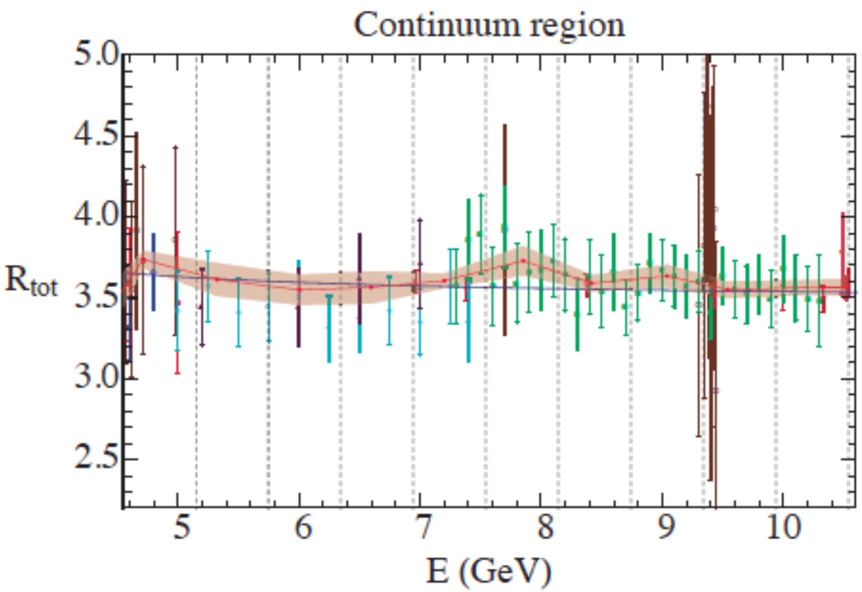

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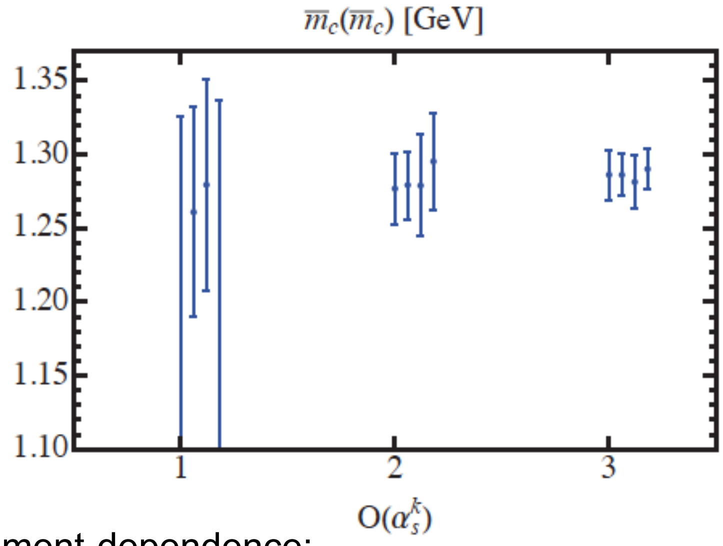
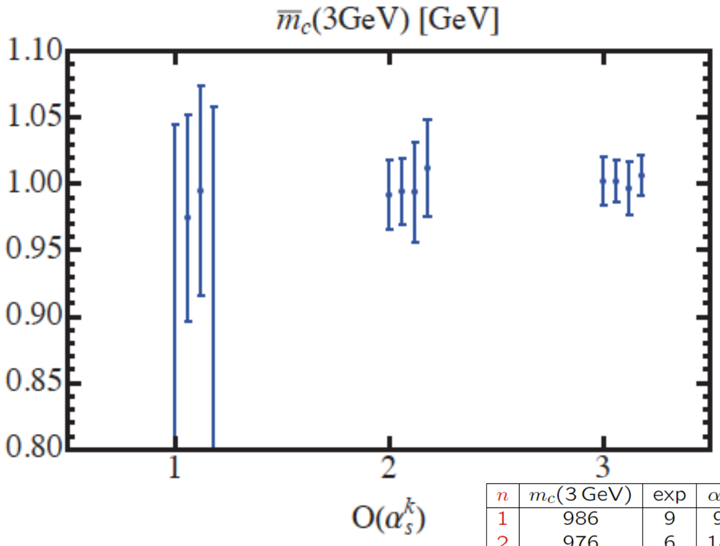
Charmonium Sum Rules

Dehnadi, AH, Mateu, Zebarjad (2011)

Our results:

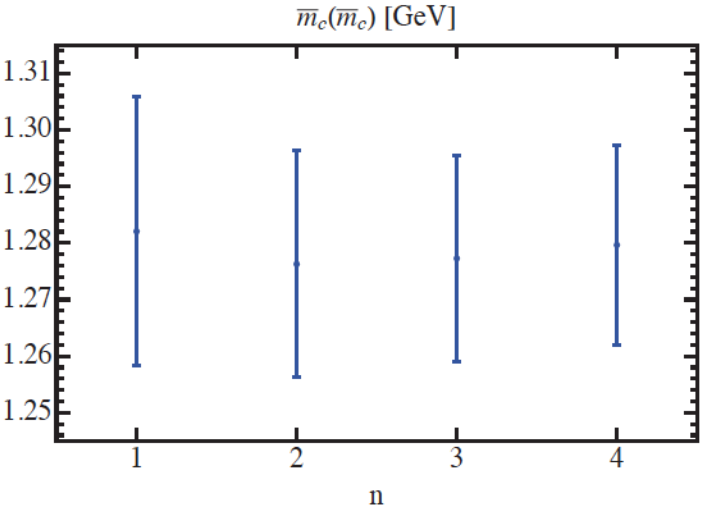
Order-dependence:





n	$m_c(3\,\text{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

Moment-dependence:



$$\overline{m}_c(\overline{m}_c) = 1.282 \pm (0.006)_{\text{stat}} \pm (0.009)_{\text{syst}} \pm (0.019)_{\text{pert}} \pm (0.010)_{\alpha_s} \pm (0.002)_{\langle GG\rangle}\ \text{GeV}\,,$$

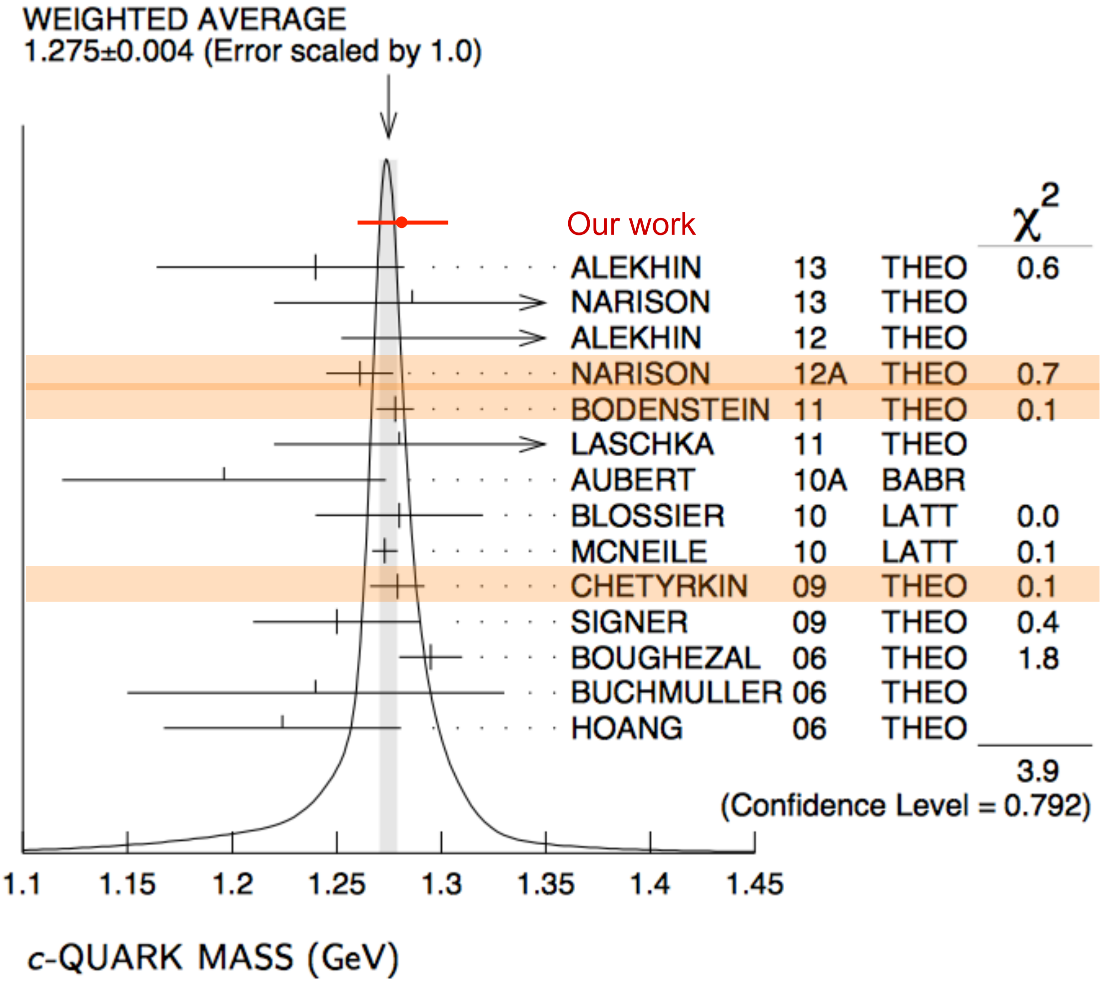
$$\overline{m}_c(3\ \text{GeV}) = 0.994 \pm (0.006)_{\text{stat}} \pm (0.009)_{\text{syst}} \pm (0.021)_{\text{pert}} \pm (0.010)_{\alpha_s} \pm (0.002)_{\langle GG\rangle}\ \text{GeV}\,,$$

Exp error : 11 MeV

Pert. Error: 20 MeV

$$\overline{m}_c(\overline{m}_c) = 1.282 \pm 0.024\ \text{GeV}$$
$$\overline{m}_c(3\ \text{GeV}) = 0.994 \pm 0.026\ \text{GeV}$$

Charmonium Sum Rules



Charm Mass Sum Rules

Dehnadi, AH, Mateu, w.i.p.

Our plans:

Reanalysis with our method:

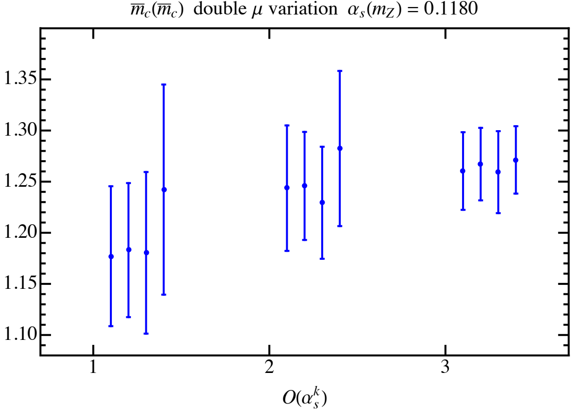
- Charm mass using lattice input for pseudo scalar correlator moments
- Finite energy sum rules

[taken from Allison etal. (2008)]

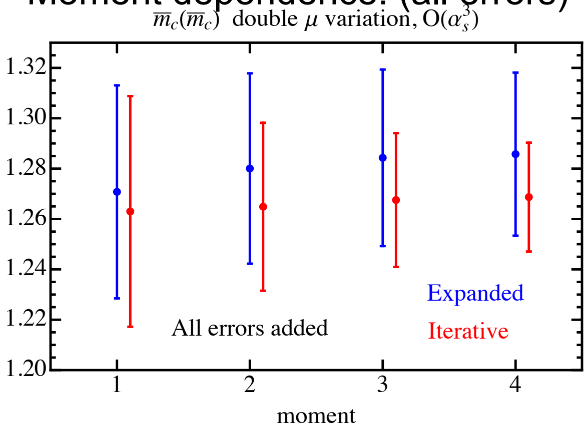
Preliminary pseudo scalar results:

(still no power corrections included, no detailed analysis of all ingredients)

1st moment (pert. error):



Moment dependence: (all errors)



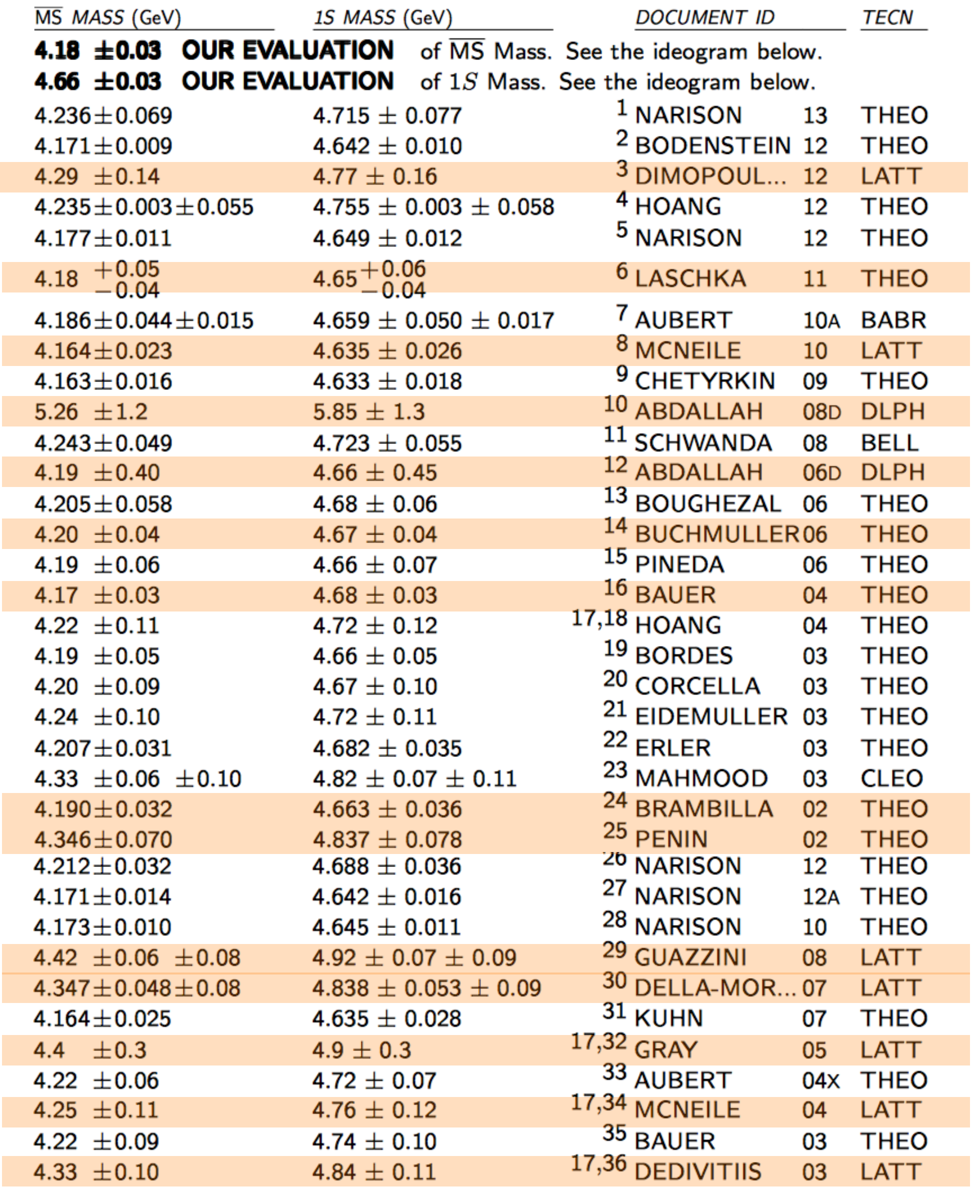
$$\frac{\bar{C}_n}{[4\bar{m}_c^2(\mu)]^n} = \bar{C}_n^{(0)}\left[\frac{R_{2n+1}}{m_\eta^{\text{exp}}}\right]^{2n}$$

1st moment result: (iterative) [very preliminary !]

$$\overline{m}_c(\overline{m}_c) = 1.263 \pm (0.041)_{\text{th}} \pm (0.006)_{\text{lat}} \pm (0.019)_{\alpha_s}\ \text{GeV}$$

$$\overline{m}_c(\overline{m}_c) = 1.263 \pm 0.047\ \text{GeV}$$

Bottom Mass



$\overline{\text{MS}}$ *MASS* (GeV)	*1S MASS* (GeV)	*DOCUMENT ID*	*TECN*
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4.66 ±0.03 OUR EVALUATION of $1S$ Mass. See the ideogram below.			
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4.29 ± 0.14	4.77 ± 0.16	[3] DIMOPOUL... 12	LATT
$4.235 \pm 0.003 \pm 0.055$	$4.755 \pm 0.003 \pm 0.058$	[4] HOANG 12	THEO
4.177 ± 0.011	4.649 ± 0.012	[5] NARISON 12	THEO
$4.18\ ^{+0.05}_{-0.04}$	$4.65^{+0.06}_{-0.04}$	[6] LASCHKA 11	THEO
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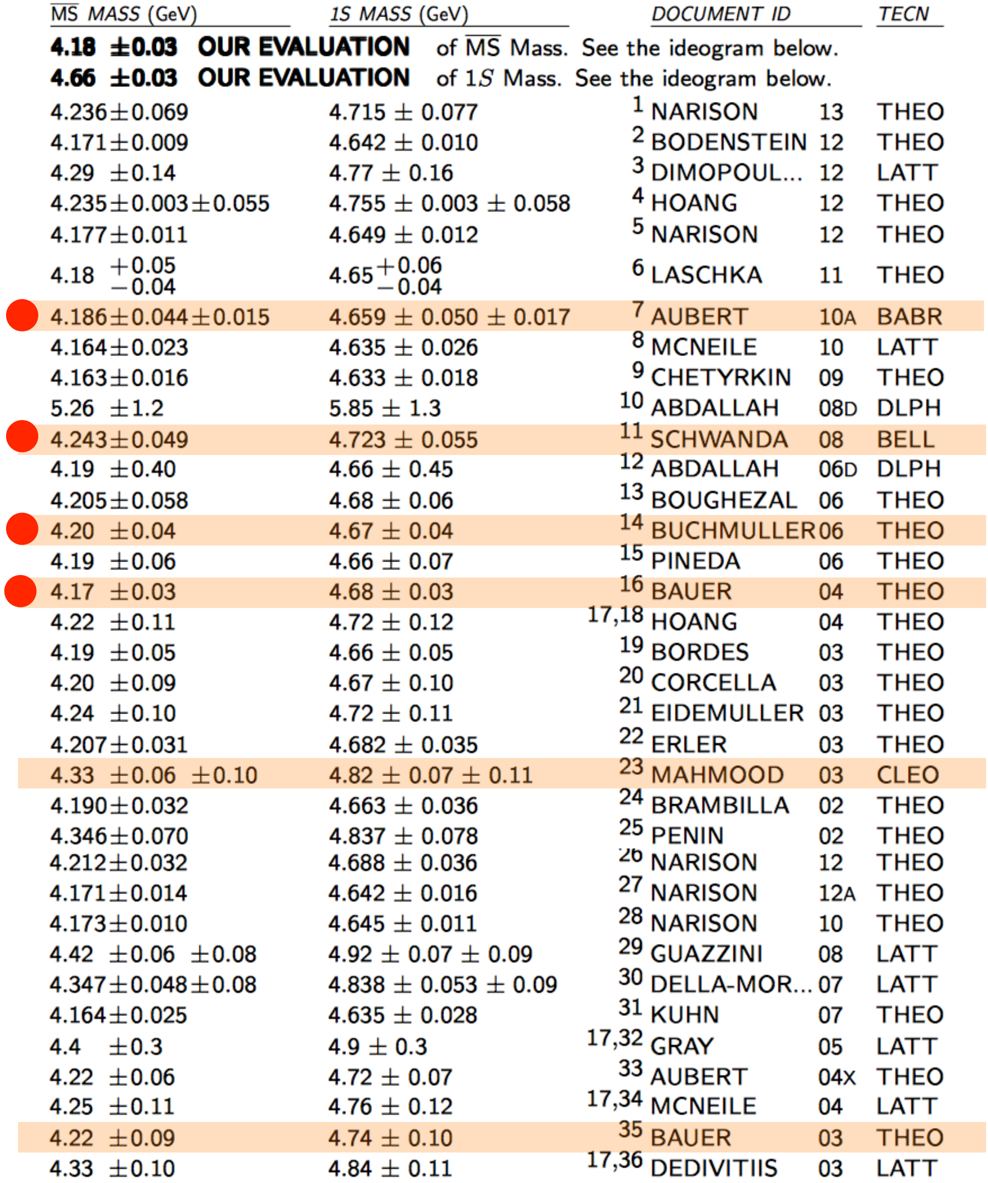
Will not discuss:

- heavy-to-light sum rules
- Pert. Upsilon spectrum calculations, Ups(1S)
- $e^+e^- \to$ 3 jets (big errors)
- lattice

Inclusive B-Decays



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Method:

- $B \to X_c l \nu$ and $B \to X_s \gamma$
- Moments of lepton energy spectrum
- Moments of hadron invariant mass spectrum
- Dependence on exp. cuts
- pQCD + OPE: in HQET or heavy mass expansion

● at $O(\alpha_s^2 \beta_0)$

Inclusive B-Decays

Main aim: V_{cb} and V_{ub}

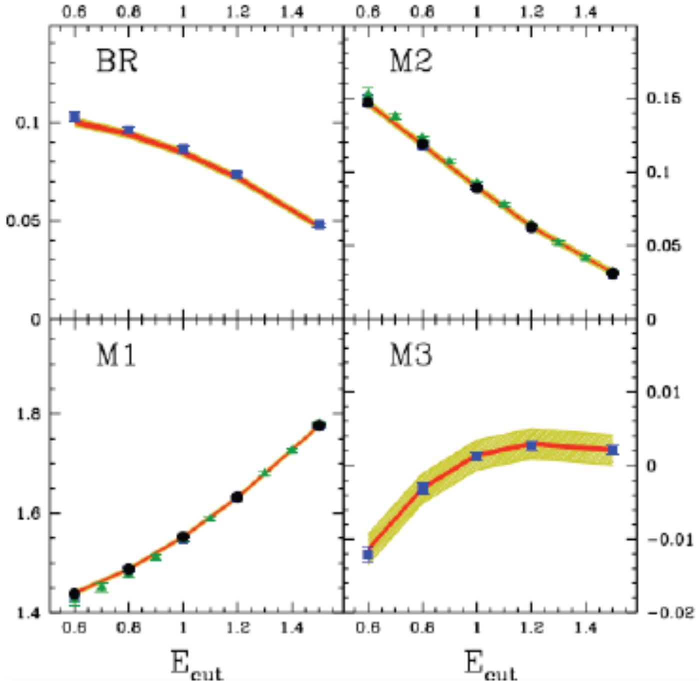
Experimental Data:

moments in semileptonic decays

E_l : lepton energy spectrum in $B \to X_c l \nu$ (BaBar Belle CLEO DELPHI)

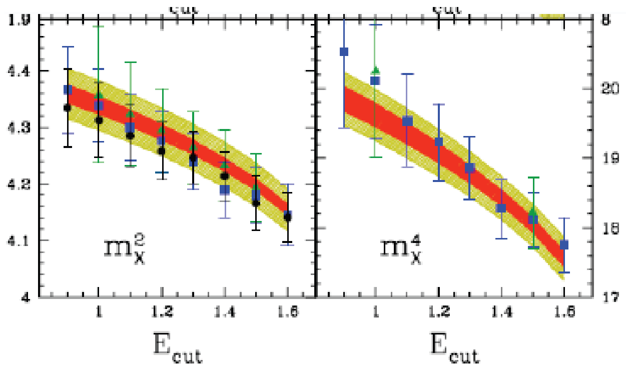
$M_X{}^2$: hadronic mass spectrum in $B \to X_c l \nu$ (BaBar CDF CLEO DELPHI)

E_γ : photon energy spectrum in $B \to X_s \gamma$ (Babar Belle CLEO)



$$B \to X_c l \nu: \qquad \int_{E_{\rm cut}} dE_l \frac{d\Gamma}{dE_l} (E_l)^n \ , \qquad \int_{E_{\rm cut}} dm_X^2 \frac{d\Gamma}{dm_X^2} (m_X^2)^n$$

$$B \to X_s \gamma: \qquad \int_{E_{\rm cut}} dE_\gamma \frac{d\Gamma}{dE_\gamma} (E_\gamma)^n$$



Inclusive B-Decays

Theoretical Moments:

- theory input: perturbative QCD $\mathcal{O}(\alpha_s, \alpha_s^2\beta_0)$

  power corrections $\mathcal{O}(\Lambda_{\rm QCD}^2/m^2, \Lambda_{\rm QCD}^3/m^3)$

A) Gambino, Uraltsev (2004)

B) Bauer, Ligeti, Luke, Manohar, Trott (2004)

- expansion scheme: A) $1/m_b$ expansion $\Rightarrow |V_{cb}|, m_b, m_c, \lambda_{1,2}, \rho_{1,2}$

  B) $1/m_b$ & $1/m_c$ expansion $\Rightarrow |V_{cb}|, m_b, \lambda_1, \rho_1, \tau^*_{1,2,3}$

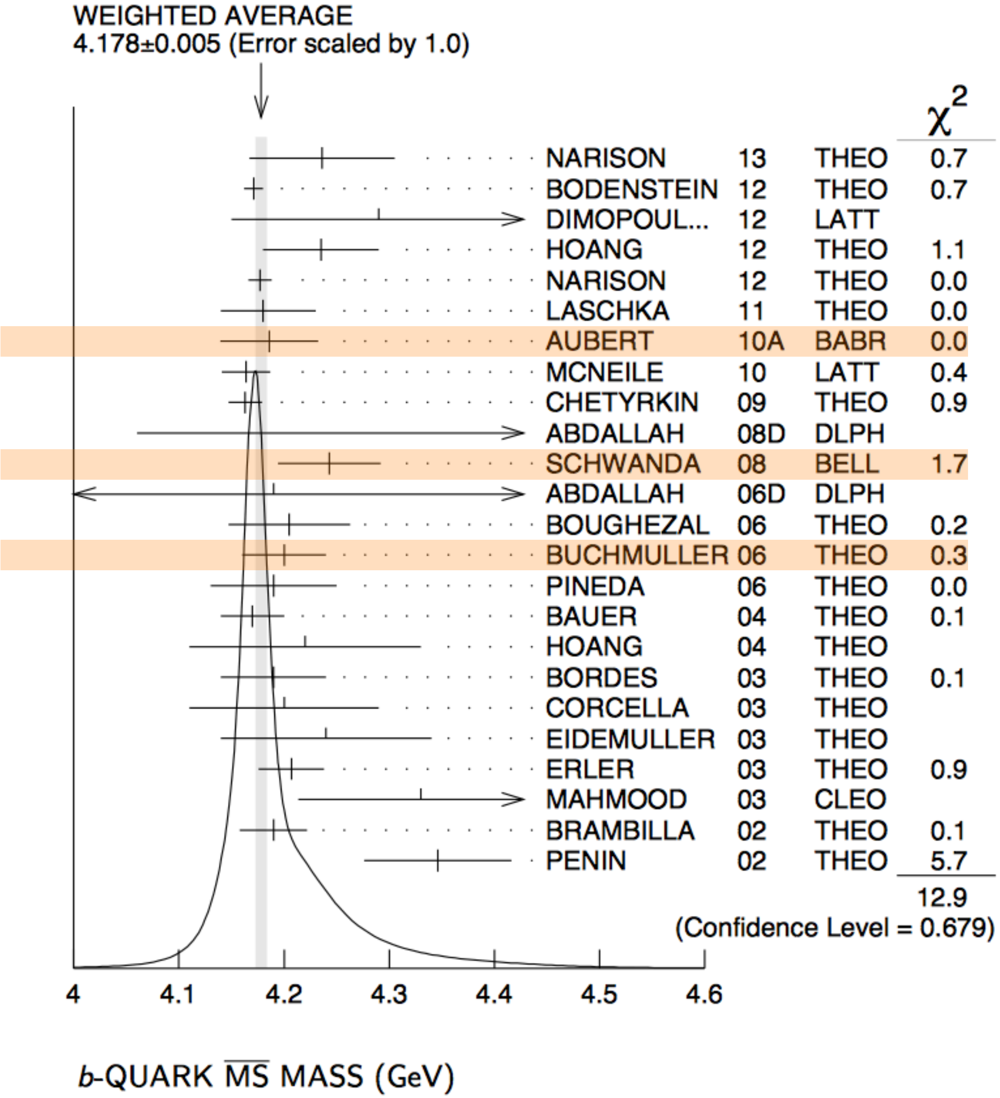
  input: meson mass difference

- mass scheme: m_b : threshold mass ((A) kinetic, (B) 1S)

  m_c : A) short-distance mass (MSbar)

  B) eliminated (m_b - m_c = m_B – m_D) + power corr.

Inclusive B-Decays



WEIGHTED AVERAGE
4.178±0.005 (Error scaled by 1.0)

			χ^2
NARISON	13	THEO	0.7
BODENSTEIN	12	THEO	0.7
DIMOPOUL...	12	LATT	
HOANG	12	THEO	1.1
NARISON	12	THEO	0.0
LASCHKA	11	THEO	0.0
AUBERT	10A	BABR	0.0
MCNEILE	10	LATT	0.4
CHETYRKIN	09	THEO	0.9
ABDALLAH	08D	DLPH	
SCHWANDA	08	BELL	1.7
ABDALLAH	06D	DLPH	
BOUGHEZAL	06	THEO	0.2
BUCHMULLER	06	THEO	0.3
PINEDA	06	THEO	0.0
BAUER	04	THEO	0.1
HOANG	04	THEO	
BORDES	03	THEO	0.1
CORCELLA	03	THEO	
EIDEMULLER	03	THEO	
ERLER	03	THEO	0.9
MAHMOOD	03	CLEO	
BRAMBILLA	02	THEO	0.1
PENIN	02	THEO	5.7
			12.9

(Confidence Level = 0.679)

Current Status:

- Results consistent
- Exp error ~ 40 MeV
- Syst error ≲ stat error
- theory error ≲ exp. error
- Significant correlations
- Combined analyses smaller errors (~30-40 MeV)

Upcoming Developments:

- Theory to $O(\alpha_s^2)$ Biswas, Melnikov Pak, Czarnecki
- Moderate error reduction feasible, but still one order away from sum rule precision.

Inclusive B-Decays

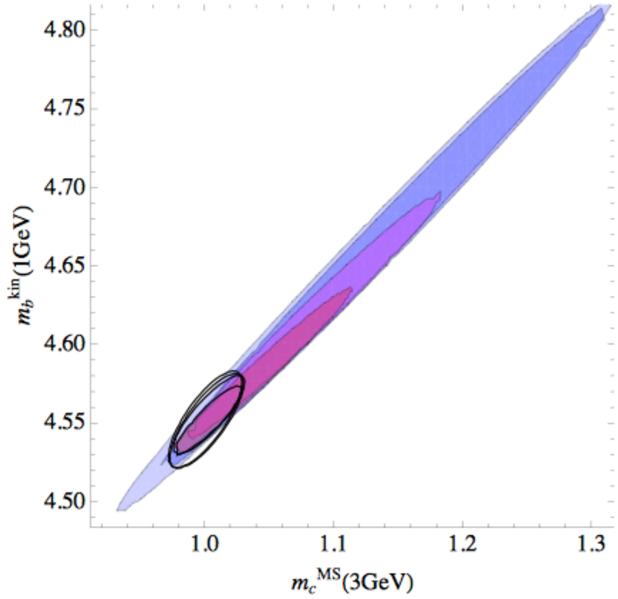
Gambino, Schwanda 2013

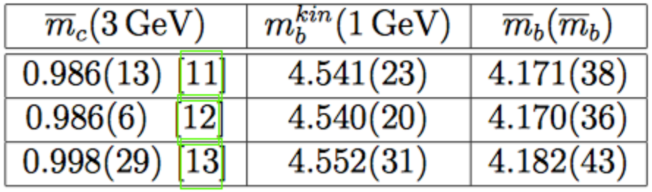
Analysis at O(α_s^2):

→ very strong degeneracy m_c vs. m_b in their heavy quark expansion scheme.

Simple fit for charm and bottom masses difficult.

→ take m_c as input: determine m_b





	$\overline{m}_c(3\,\mathrm{GeV})$		$m_b^{kin}(1\,\mathrm{GeV})$	$\overline{m}_b(\overline{m}_b)$
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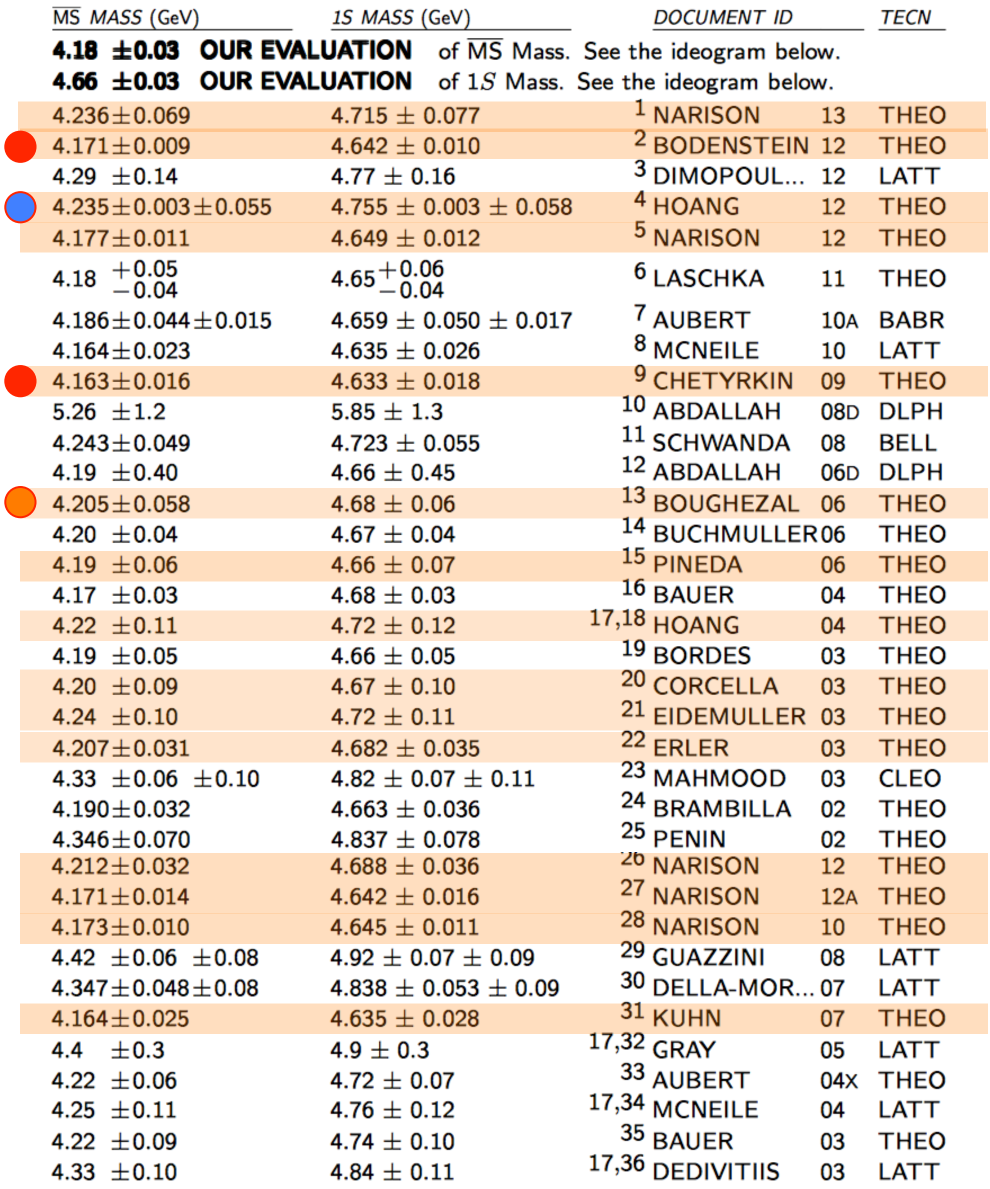
Allison etal. →	0.986(6)	[12]	4.540(20)	4.170(36)
Dehnadi etal. →	0.998(29)	[13]	4.552(31)	4.182(43)

Table 2: b mass resulting from different m_c determinations. All masses are expressed in GeV.

Υ Sum Rules



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🔵 NNLL nonrelativistic SR

🔴 Relativistic SR
New Babar data and O(α_s^3)

🟠 O(α_s^3)

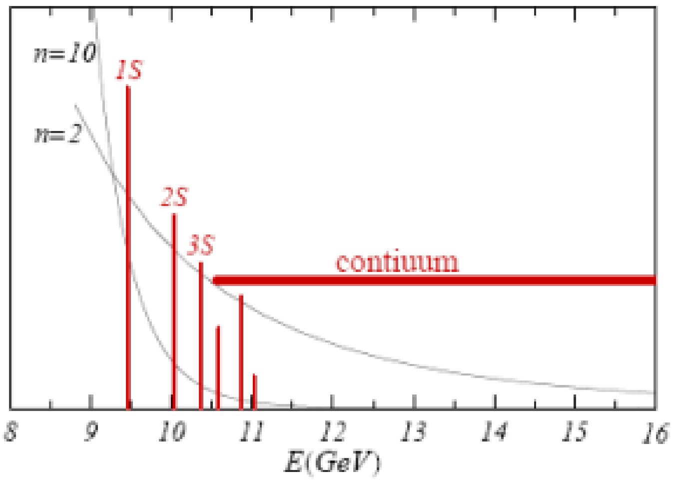
ϒ Sum Rules

- Moments:

$$P_n = \int_{M_{\Upsilon(1S)}}^{\infty} dE\, \frac{R_b(E)}{E^{2n+1}} \sim \frac{1}{m_b^{2n}}$$

$$= \left(\frac{d}{dq^2}\right)^n \int d^4 e^{iqx} \langle 0|T j(x) J(0)|0\rangle \Big|_{q^2=0}$$

duality bound $n \lesssim 10$



→ Generic form: $P_n^{\text{theory}} \sim \frac{1}{m_b^{2n}} \sum_i \left(\alpha_s(m_b/n)\sqrt{n}\right)^i$

A) relativistic: $n \lesssim 4$ ⇒ $\overline{\text{MS}}$ mass

→ usual loop expansion in powers of α_s ,

B) non-relativistic: $4 \lesssim n \lesssim 10$ ⇒ threshold masses

→ $(\alpha_s\sqrt{n})$ terms must be summed using NRQCD

nonperturbative
power corrections
very small

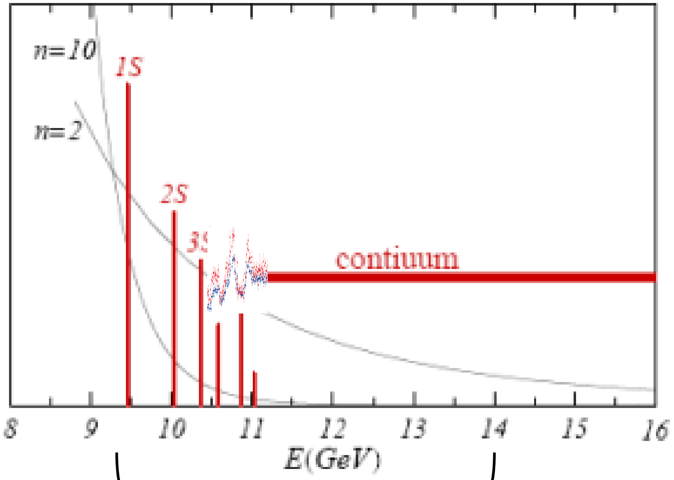
Υ Sum Rules

- Moments:

$$P_n = \int_{M_{\Upsilon(1S)}}^{\infty} dE\, \frac{R_b(E)}{E^{2n+1}} \sim \frac{1}{m_b^{2n}}$$

$$= \left(\frac{d}{dq^2}\right)^n \int d^4 e^{iqx} \langle 0|T j(x) J(0)|0\rangle\Big|_{q^2=0}$$

duality bound $n \lesssim 10$



Only data on resonances and between 10.6 GeV and 11.2 GeV (BaBar), ISR corrections needed

→ Generic form: $P_n^{\text{theory}} \sim \frac{1}{m_b^{2n}} \sum_i \left(\alpha_s(m_b/n)\sqrt{n}\right)^i$

A) relativistic: $n \lesssim 4$ ⇒ $\overline{\text{MS}}$ mass

→ usual loop expansion in powers of α_s ,

⇒ Use model for continuum, higher moments less sensitive (MSbar mass)

B) non-relativistic: $4 \lesssim n \lesssim 10$ ⇒ threshold masses

→ $(\alpha_s\sqrt{n})$ terms must be summed using NRQCD

⇒ Low-E data dominates, model irrelevant (threshold mass)

Υ Sum Rules

Non-relativistic Sum Rules (n>4):

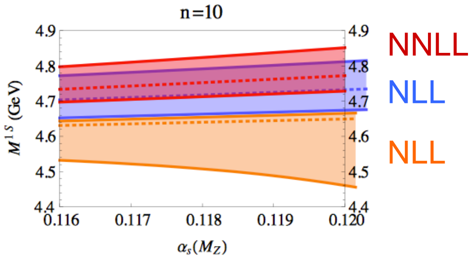
- All NNLO fixed-order analyses with O(200 MeV) errors. Special treatments needed to achieve 30 / 50 / 100 MeV). [hard scale ?]
- Only RG-improved NNLL analyses with small errors. Only one analysis with full NNLL order at this time.

NNLL vNRQCD:

Hoang, Stahlhofen 2012

$$P_n^{th,\mathrm{NNLL}} = \frac{3\,N_c\,Q_b^2\,\sqrt{\pi}}{4^{n+1}(M_b^{\mathrm{pole}})^{2n}\,n^{3/2}}\left\{c_1(h,\nu)^2\,\varrho_{n,1}(h,\nu) + 2\,c_1(h,\nu)c_2(h,\nu)\,\varrho_{n,2}(h,\nu)\right\}$$

- Full NNLL (missing NNLL soft mixing log terms small)
- Charm mass effects still uncalculated (effects about -30 MeV)
- Double scale variation (hard, soft-ultrasoft)
- Convergence good but not excellent (dependence on usoft scale)
- Charm mass effects (neglected) lead to negative shift ~ 30 MeV



$$\begin{aligned} M_b^{1S} &= 4.755 \pm 0.057_{\mathrm{pert}} \pm 0.009_{\alpha_s} \pm 0.003_{\mathrm{exp}}\ \mathrm{GeV} \\ \overline{m}_b(\overline{m}_b) &= 4.235 \pm 0.055_{\mathrm{pert}} \pm 0.003_{\mathrm{exp}}\ \mathrm{GeV} \end{aligned}$$

Consistent with Pineda etal. (NNLL corrections in c_1 were neglected)

ϒ Sum Rules

Relativistic Sum Rules (n<4):

- Theory input as for charmonium sum rules Chetyrkin etal.
- Finite-energy sumrules: cut off integration at s_0 (analytic implementation: designed linear combination of different moments)
- Regular moments: needs model input for missing data above 11.2 GeV
- Different treatment of strong coupling uncertainties

→ All analyses used same renormalization scale for $\alpha_s(\mu)$ and $m(\mu)$.

→ Issue concerning different expansion expected should be less severe ($m_b > m_c$).

5 GeV < μ < 15 GeV

Finite energy: $\overline{m}_b(\overline{m}_b) = 4.171 \pm 0.009$ Bodenstein etal.

→ Insensitivity to different continuum models eliminated by design, tested in analysis.

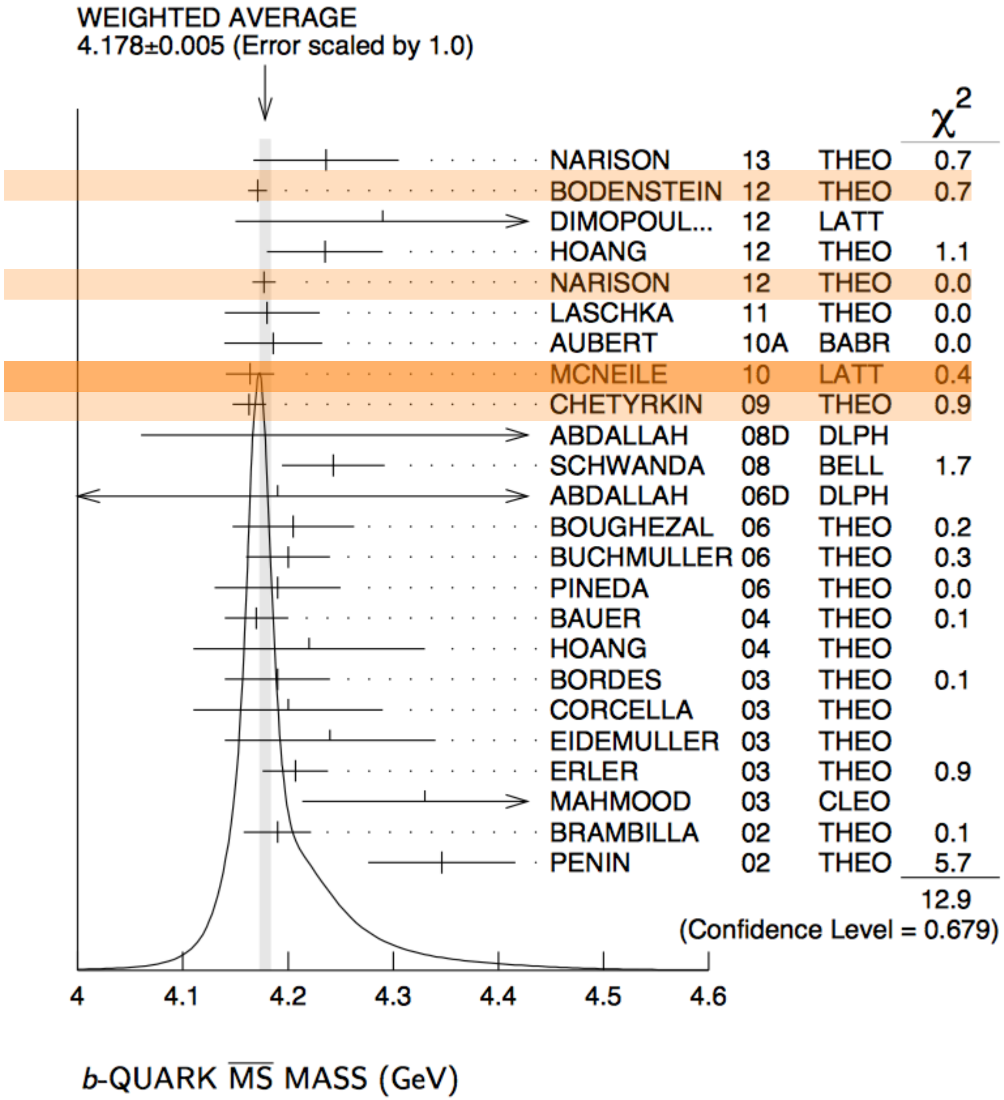
Regular: $\overline{m}_b(\overline{m}_b) = 4.177 \pm 0.014$ Narison (μ ?)

$\overline{m}_b(\overline{m}_b) = 4.163 \pm 0.016$ Chetyrkin etal.

→ Continuum models = pQCD prediction

4 GeV < μ < 10 GeV (?)

ϒ Sum Rules



- Very good consistency with lattice

ϒ Sum Rules

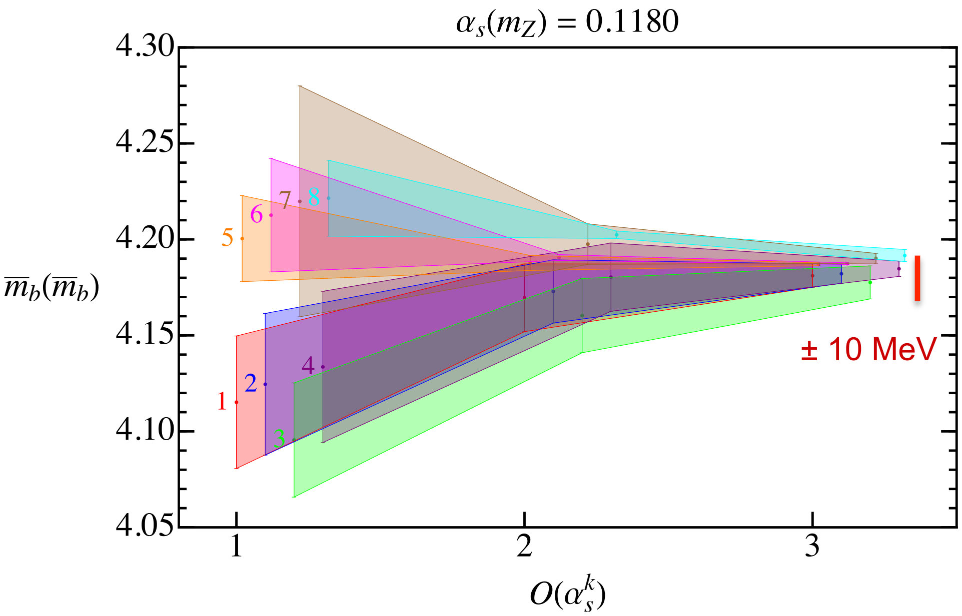
n	$m_b(10\,\mathrm{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

Chetyrkin, Kuhn, Meier, Meierhofer, Marquard Steinhauser (2009)

- $m_b(10\,\mathrm{GeV}) = 3610 \pm 16\,\mathrm{MeV}$
- $m_b(m_b) = 4163 \pm 16\,\mathrm{MeV}$

Our check of different expansions: → pert. error ± 10 MeV expected

Dehnadi, AH, Mateu, w.i.p.

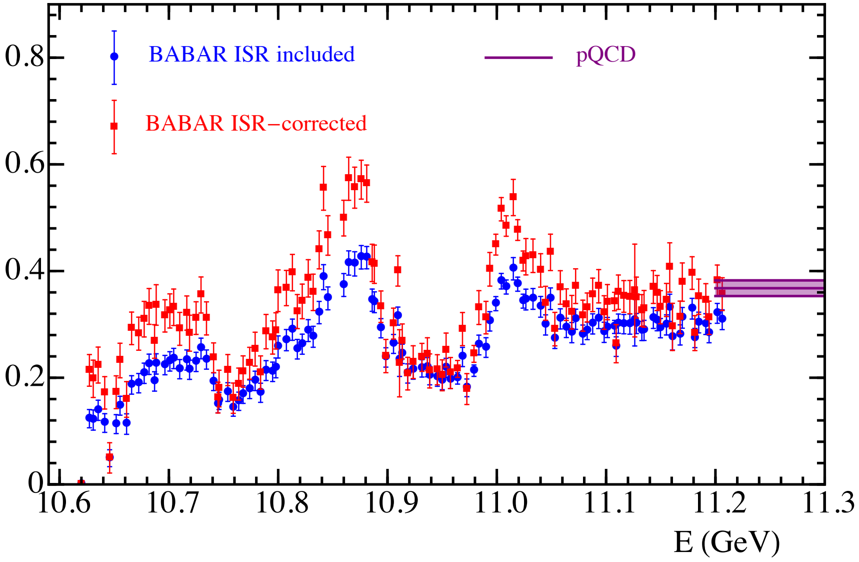


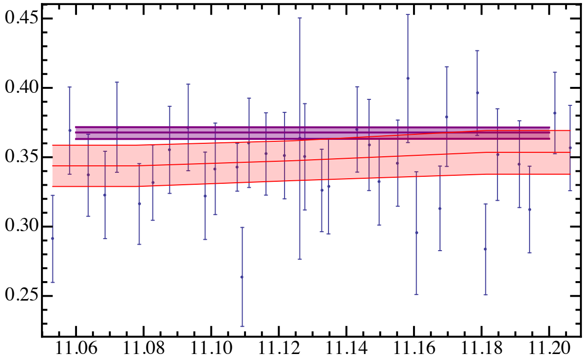
Υ Sum Rules

Dehnadi, AH, Mateu, w.i.p.

Very preliminary

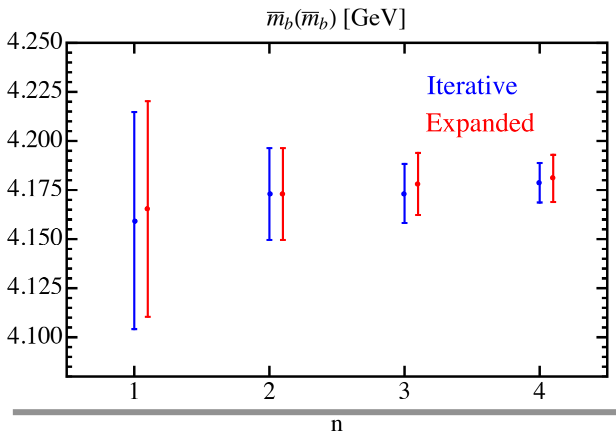
Continuum (11.06 – 11.21 GeV):





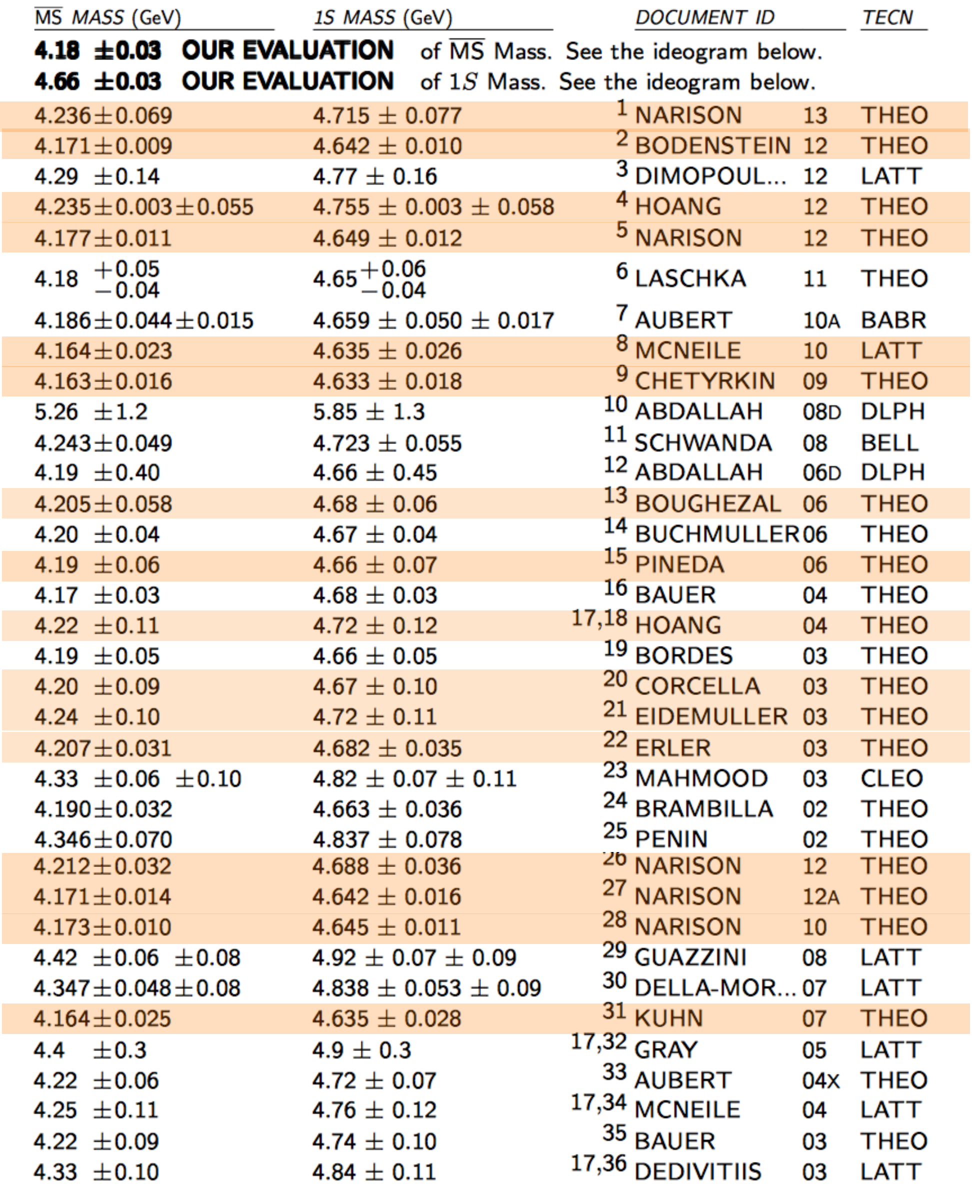
Agreement (averaged) data vs. theory : 4%

→ conservative continuum model: R_b(theory) ± 4%



1st moment strongly affected by continuum model error.

Υ Sum Rules



$\overline{\rm MS}$ MASS (GeV)	1S MASS (GeV)	DOCUMENT ID	TECN
4.18 ±0.03	**OUR EVALUATION** of $\overline{\rm MS}$ Mass. See the ideogram below.		
4.66 ±0.03	**OUR EVALUATION** of $1S$ Mass. See the ideogram below.		
4.236±0.069	4.715 ± 0.077	[1] NARISON 13	THEO
4.171±0.009	4.642 ± 0.010	[2] BODENSTEIN 12	THEO
4.29 ±0.14	4.77 ± 0.16	[3] DIMOPOUL... 12	LATT
4.235±0.003±0.055	4.755 ± 0.003 ± 0.058	[4] HOANG 12	THEO
4.177±0.011	4.649 ± 0.012	[5] NARISON 12	THEO
$4.18\ ^{+0.05}_{-0.04}$	$4.65\ ^{+0.06}_{-0.04}$	[6] LASCHKA 11	THEO
4.186±0.044±0.015	4.659 ± 0.050 ± 0.017	[7] AUBERT 10A	BABR
4.164±0.023	4.635 ± 0.026	[8] MCNEILE 10	LATT
4.163±0.016	4.633 ± 0.018	[9] CHETYRKIN 09	THEO
5.26 ±1.2	5.85 ± 1.3	[10] ABDALLAH 08D	DLPH
4.243±0.049	4.723 ± 0.055	[11] SCHWANDA 08	BELL
4.19 ±0.40	4.66 ± 0.45	[12] ABDALLAH 06D	DLPH
4.205±0.058	4.68 ± 0.06	[13] BOUGHEZAL 06	THEO
4.20 ±0.04	4.67 ± 0.04	[14] BUCHMULLER 06	THEO
4.19 ±0.06	4.66 ± 0.07	[15] PINEDA 06	THEO
4.17 ±0.03	4.68 ± 0.03	[16] BAUER 04	THEO
4.22 ±0.11	4.72 ± 0.12	[17,18] HOANG 04	THEO
4.19 ±0.05	4.66 ± 0.05	[19] BORDES 03	THEO
4.20 ±0.09	4.67 ± 0.10	[20] CORCELLA 03	THEO
4.24 ±0.10	4.72 ± 0.11	[21] EIDEMULLER 03	THEO
4.207±0.031	4.682 ± 0.035	[22] ERLER 03	THEO
4.33 ±0.06 ±0.10	4.82 ± 0.07 ± 0.11	[23] MAHMOOD 03	CLEO
4.190±0.032	4.663 ± 0.036	[24] BRAMBILLA 02	THEO
4.346±0.070	4.837 ± 0.078	[25] PENIN 02	THEO
4.212±0.032	4.688 ± 0.036	[26] NARISON 12	THEO
4.171±0.014	4.642 ± 0.016	[27] NARISON 12A	THEO
4.173±0.010	4.645 ± 0.011	[28] NARISON 10	THEO
4.42 ±0.06 ±0.08	4.92 ± 0.07 ± 0.09	[29] GUAZZINI 08	LATT
4.347±0.048±0.08	4.838 ± 0.053 ± 0.09	[30] DELLA-MOR... 07	LATT
4.164±0.025	4.635 ± 0.028	[31] KUHN 07	THEO
4.4 ±0.3	4.9 ± 0.3	[17,32] GRAY 05	LATT
4.22 ±0.06	4.72 ± 0.07	[33] AUBERT 04X	THEO
4.25 ±0.11	4.76 ± 0.12	[17,34] MCNEILE 04	LATT
4.22 ±0.09	4.74 ± 0.10	[35] BAUER 03	THEO
4.33 ±0.10	4.84 ± 0.11	[17,36] DEDIVITIIS 03	LATT

Current Status:

- Double scale variation avoids accidentally small scale variation
- Excellent convergence observed. More loops will decrease error

Oportunities:

- Full O($\alpha_s{}^4$) moments ?
- Lattice "exp" moments for bottom case ?

Conclusions

- Charmonium and bottomonium sum rules rule for pQCD methods
  - “simple” calculations and “simple” concept
  - Only calculational issue
  - Status: NNNLO → O(10-20 MeV)

- Other methods: NNLO
  - Precision consistent with NNLO → O(30-50 MeV)
  - Improvements toward NNNLO feasible but much harder because more issues than just Feynman diagrams need to be resolved at the same time
  - Provide important cross checks

- Comparison with lattice important cross check