Why July 4th is celebrated (not only in the US):

Towards the LC Precision:
Progress in the M_h Calculation in the MSSM

Sven Heinemeyer, IFCA (CSIC, Santander)

Tokyo, 11/2013

based on collaboration with
T. Hahn, W. Hollik, H. Rzehak, G. Weiglein

1. Introduction

2. Higgs boson masses and mixings at higher orders

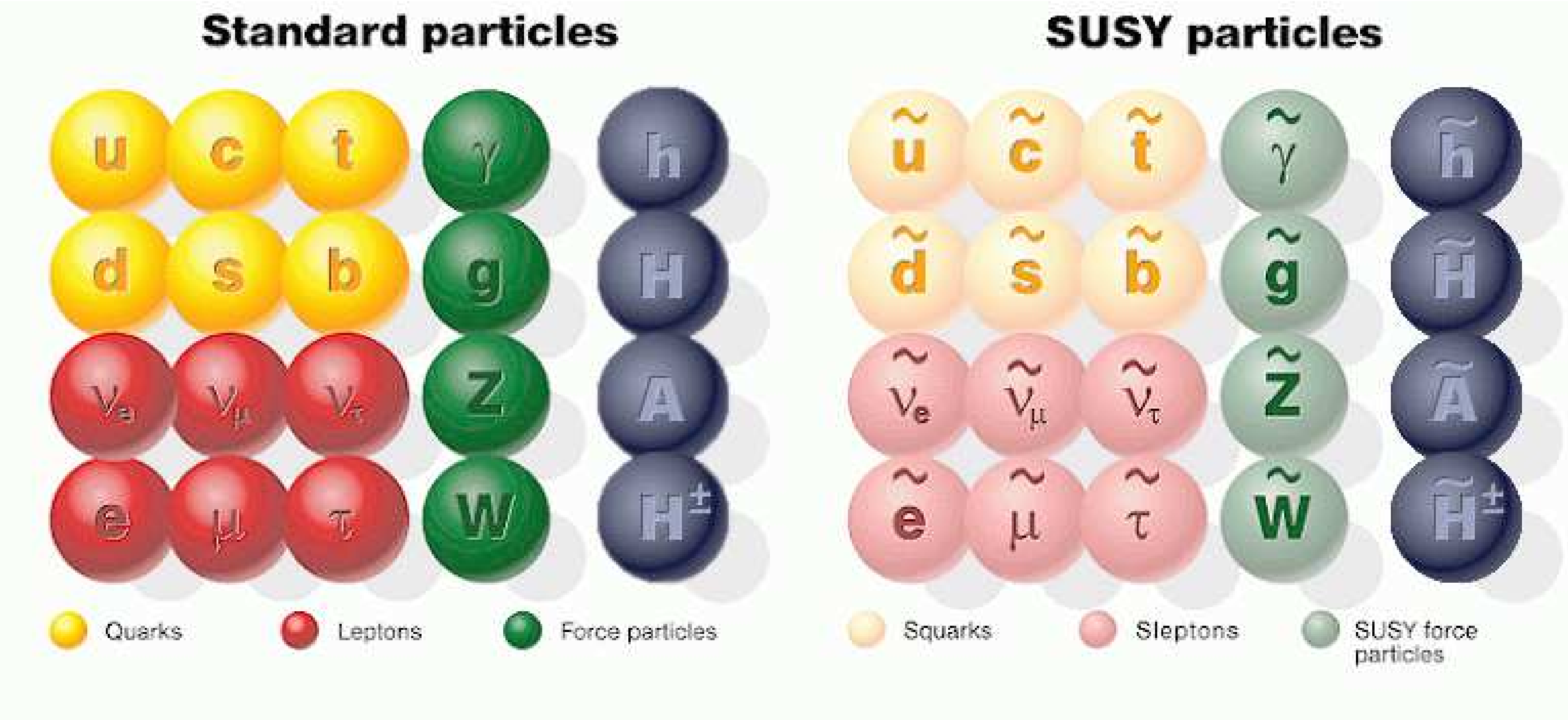
3. Numerical results

4. Conclusions

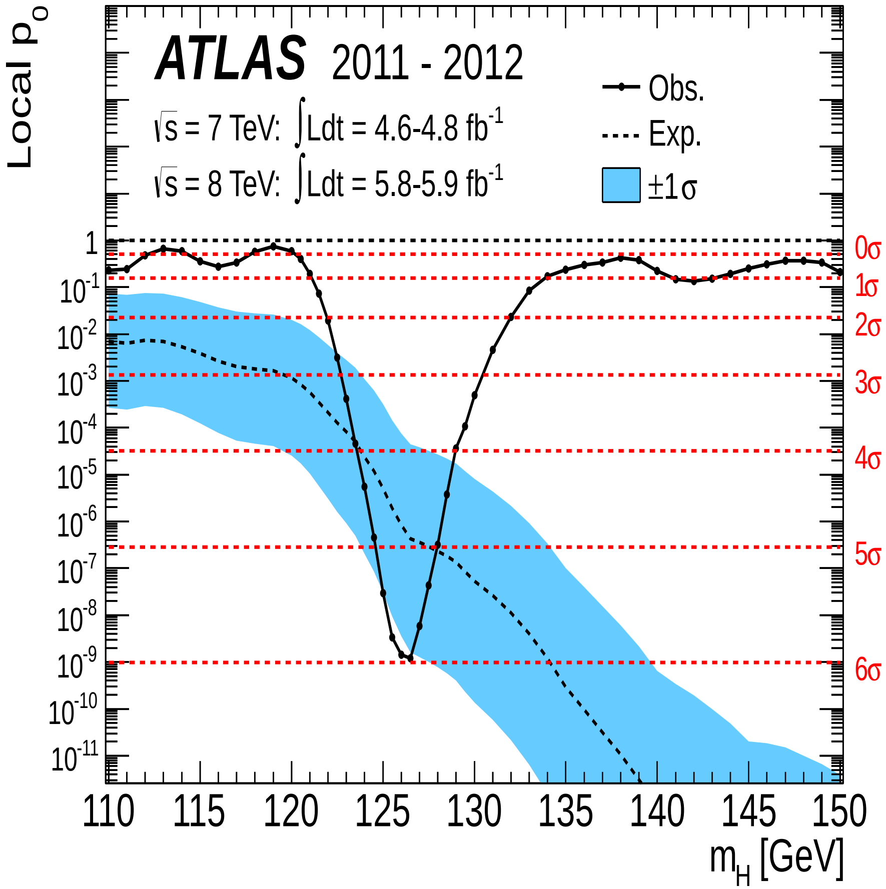
1. Introduction

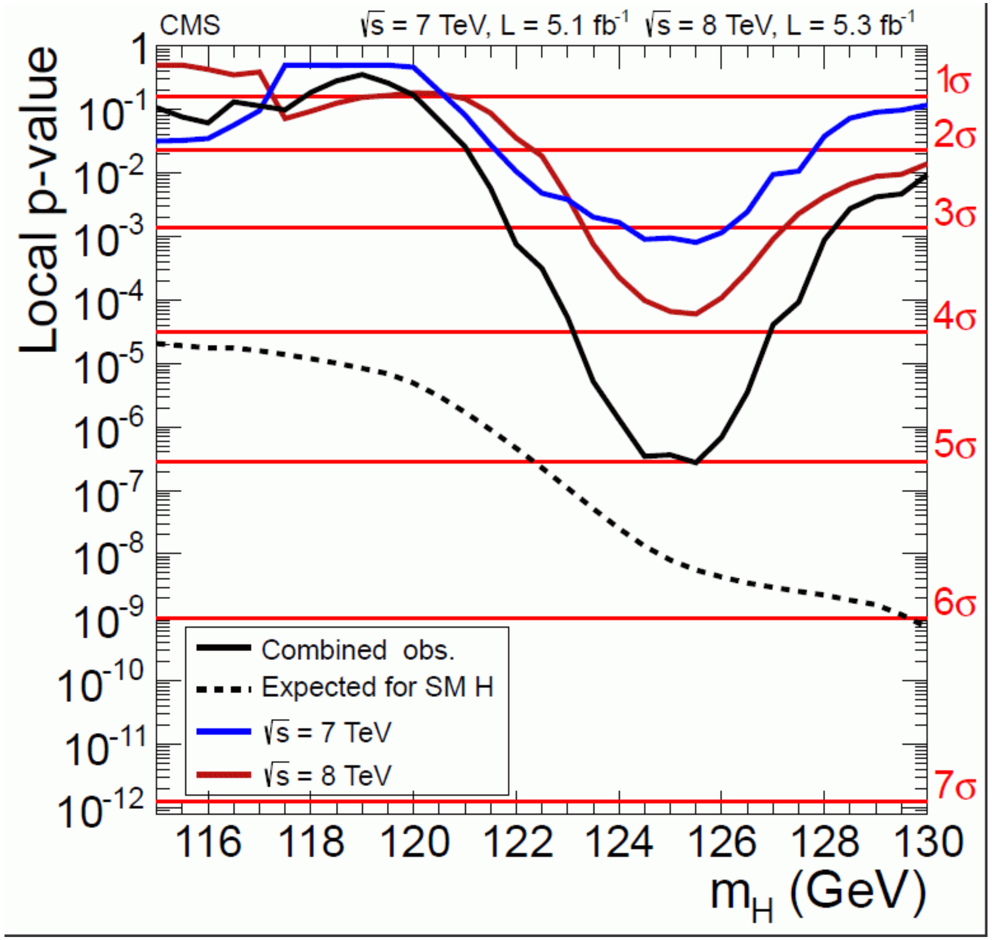
The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles



We have a discovery!





→ MSSM always predicted $M_h \lesssim 135$ GeV

→ MSSM predicts (over large parts of the parameter space) that the lightest Higgs is SM-like

⇒ discovery can be identified with the lightest MSSM Higgs boson!

MSSM: Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM $\Rightarrow m_h \leq M_Z$

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

The lightest MSSM Higgs boson

MSSM predicts upper bound on M_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}$, $\frac{e\, m_t^2}{M_W s_W}$, ...

$\Rightarrow$ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

> The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Present status of M_h prediction in the MSSM:

Complete 1L, 'almost complete' 2L available, also very leading 3L ...

$\tilde{t}/\tilde{b}$ sector of the MSSM:

Stop, sbottom mass matrices ($X_t = A_t - \mu/\tan\beta$, $X_b = A_b - \mu\tan\beta$):

$$\mathbf{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathbf{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

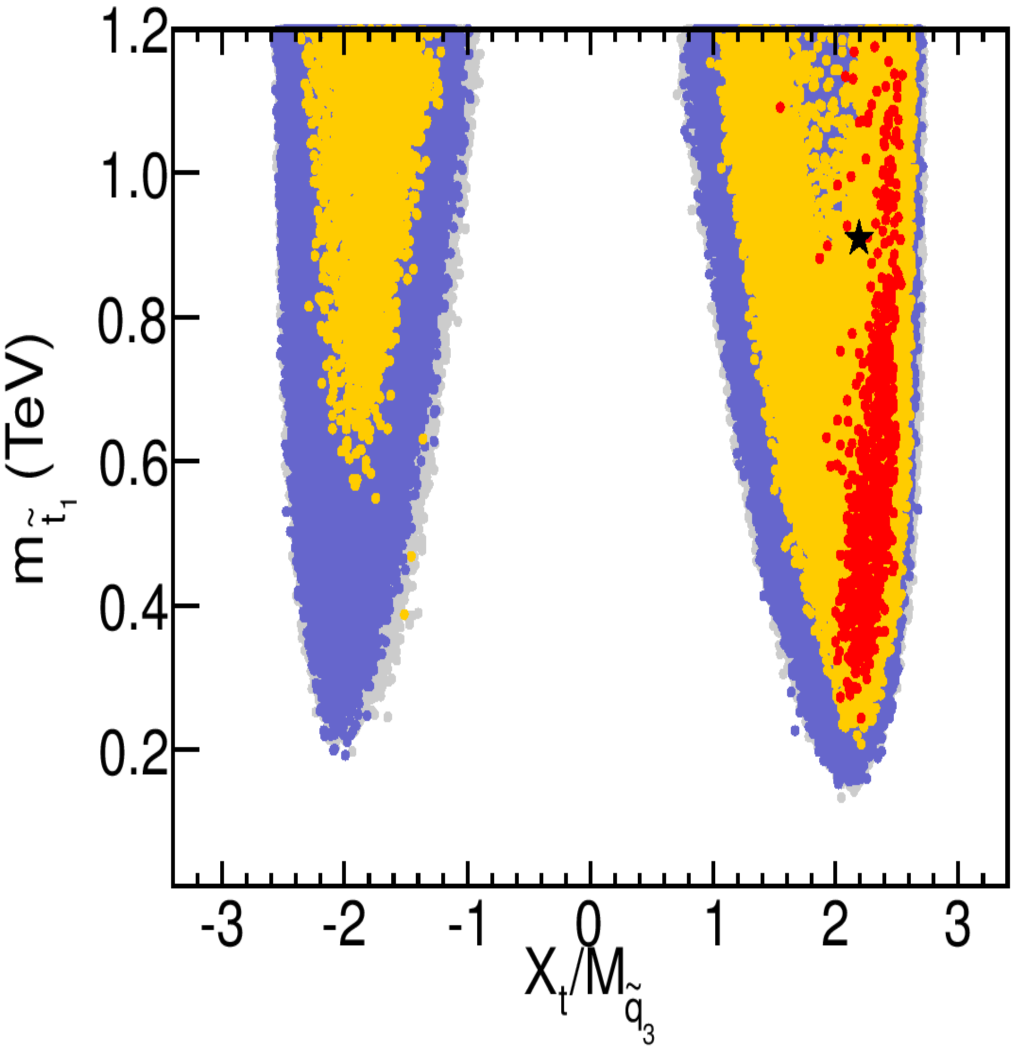
mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

$SU(2)$ relation $\Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Stop masses:

[*P. Bechtle, S.H., O. Stål, T. Stefaniak, G. Weiglein, L. Zeune '12*]



$M_h = 125.5 \pm 3$ GeV

$\star$: best-fit point

red: $\Delta\chi^2 < 2.3$

orange: $\Delta\chi^2 < 5.99$

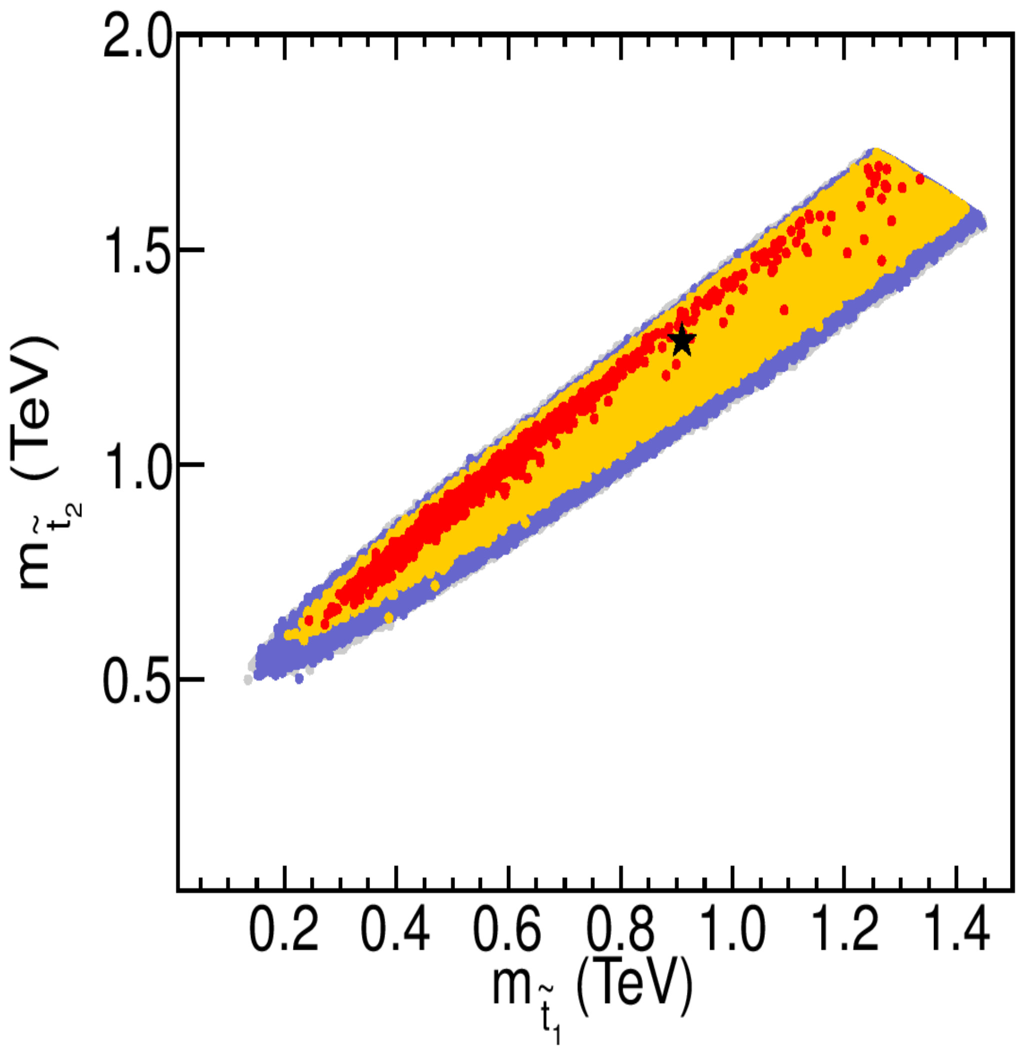
blue: all points HiggsBounds allowed

gray: all scan points

$\Rightarrow$ light and heavy stops compatible with $M_h \simeq 125.5$ GeV

Stop masses:

[*P. Bechtle, S.H., O. Stål, T. Stefaniak, G. Weiglein, L. Zeune '12*]



$M_h = 125.5 \pm 3$ GeV

$\star$: best-fit point

red: $\Delta\chi^2 < 2.3$

orange: $\Delta\chi^2 < 5.99$

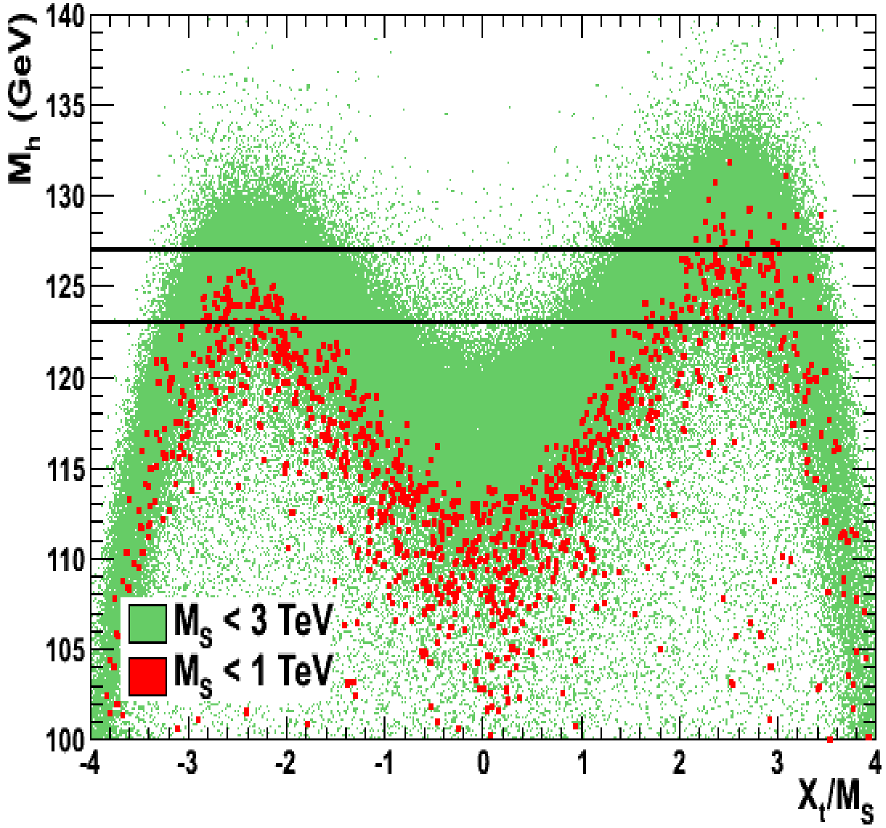
blue: all points HiggsBounds allowed

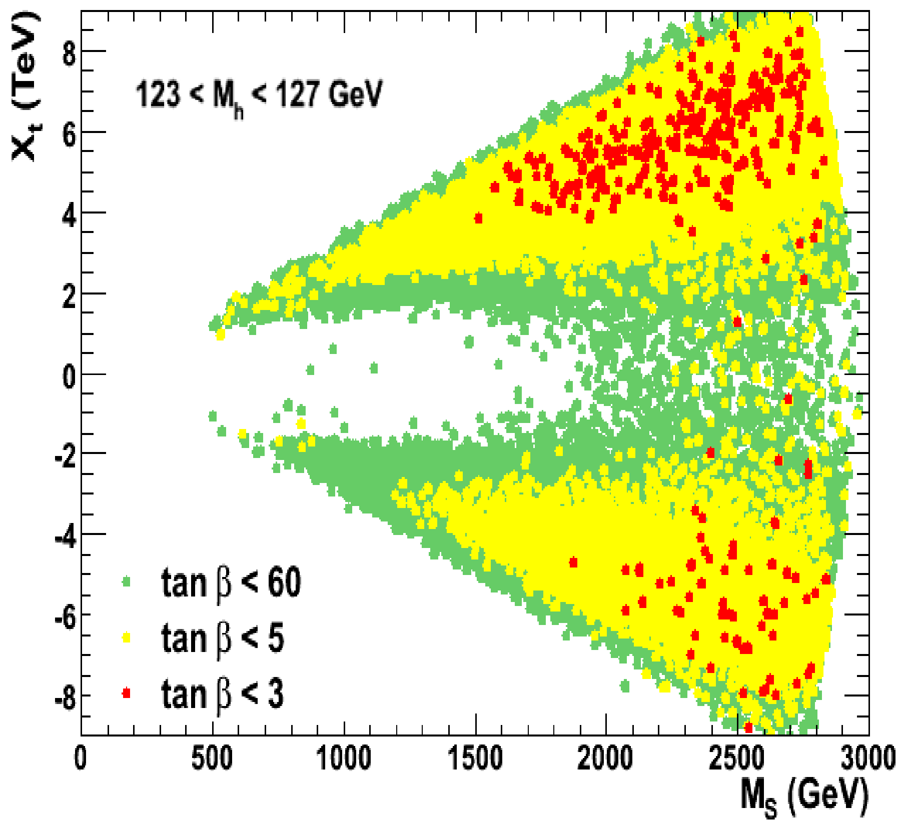
gray: all scan points

$\Rightarrow$ light and heavy stops compatible with $M_h \simeq 125.5$ GeV

Stop masses for $M_h = 125.5$ GeV

[*A. Arbey et al., '11*]



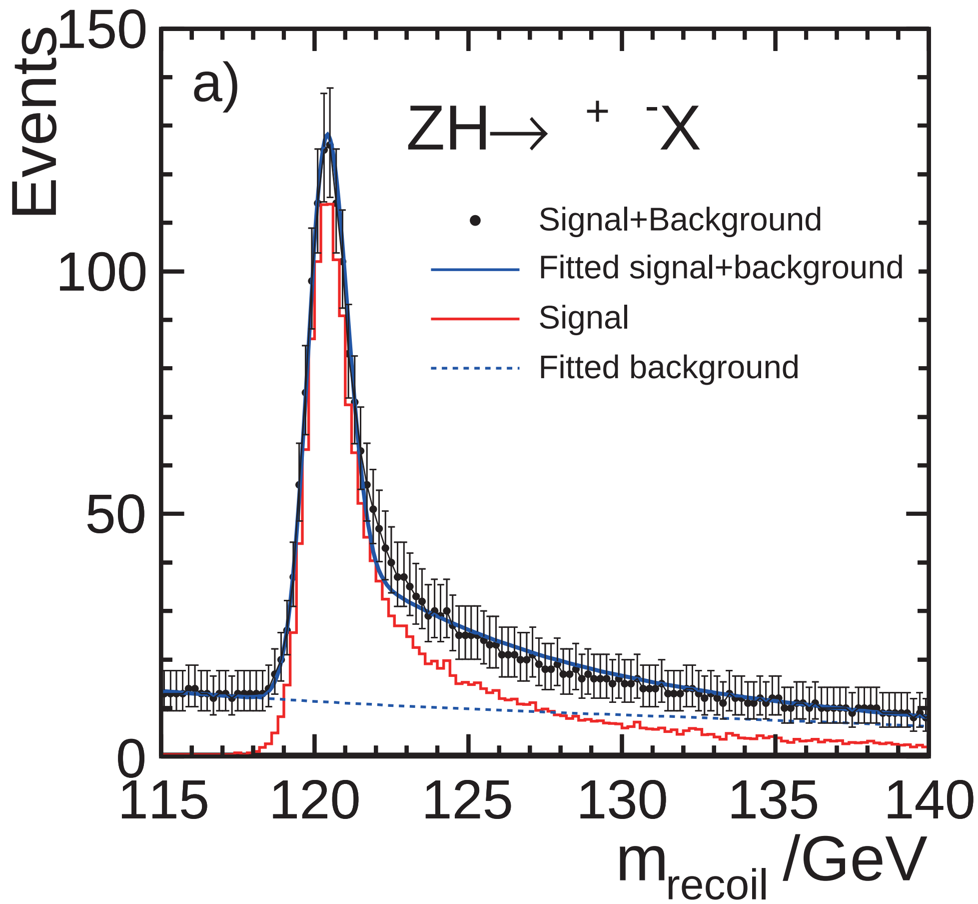


$\Rightarrow M_h \sim 125.5$ GeV requires large X_t and/or large M_{SUSY}

Model independent mass and cross section measurement at the LC

[*ILC TDR '13*]

Z-recoil method: $e^+e^- \to ZH \to \mu^+\mu^- X$



⇒ crucial for a model independent coupling measurement! $\delta M_H^{\rm exp} \lesssim 0.05$ GeV

2. Higgs boson masses and mixings at higher orders

Predictions for m_h, m_H from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \quad \leftarrow \text{ Diagonalization}, \alpha$$

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

$$m^{2,\text{tree}}_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

Higher-order corrections in the Feynman diagrammatic method:

Propagator/Mass matrix at tree-level:

$$\begin{pmatrix} q^2 - m_H^2 & 0 \\ 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
($\to$ Feynman-diagrammatic approach):

$$M_{hH}^2(q^2) = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H)$: renormalized Higgs self-energies

$\mathcal{CP}$-even fields can mix

$\Rightarrow$ complex roots of $\det(M_{hH}^2(q^2))$: $\mathcal{M}_{h_i}^2 (i = 1, 2)$: $\mathcal{M}^2 = M^2 - iM\Gamma$

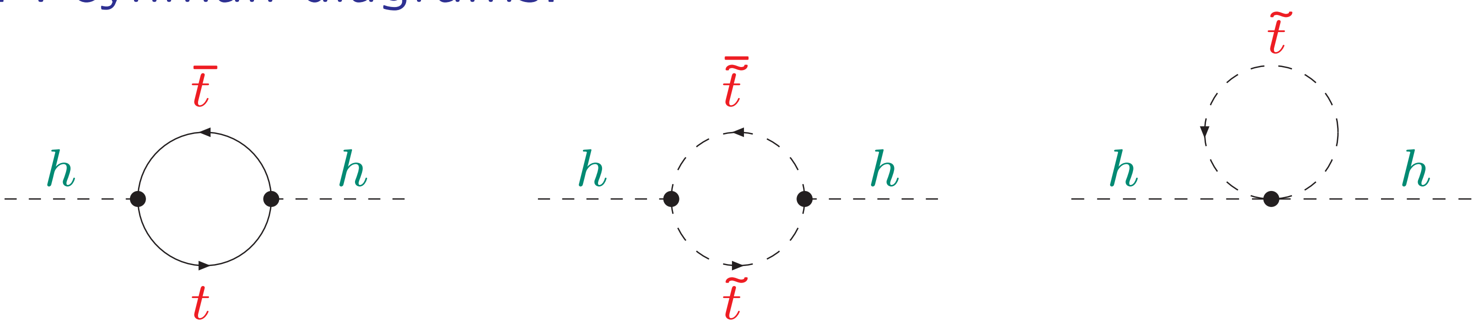
Calculation of renormalized Higgs boson self-energies:

$\hat{\Sigma}(q^2) = \hat{\Sigma}^{(1)}(q^2) + \hat{\Sigma}^{(2)}(q^2) + \dots$

all MSSM particles contribute

main contribution: $t/\tilde{t}$ sector ($\tilde{t}$: scalar top, SUSY partner of the t)

1-Loop: Feynman diagrams:



Dominant 1-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

size of the corrections: $\mathcal{O}$(50 GeV)

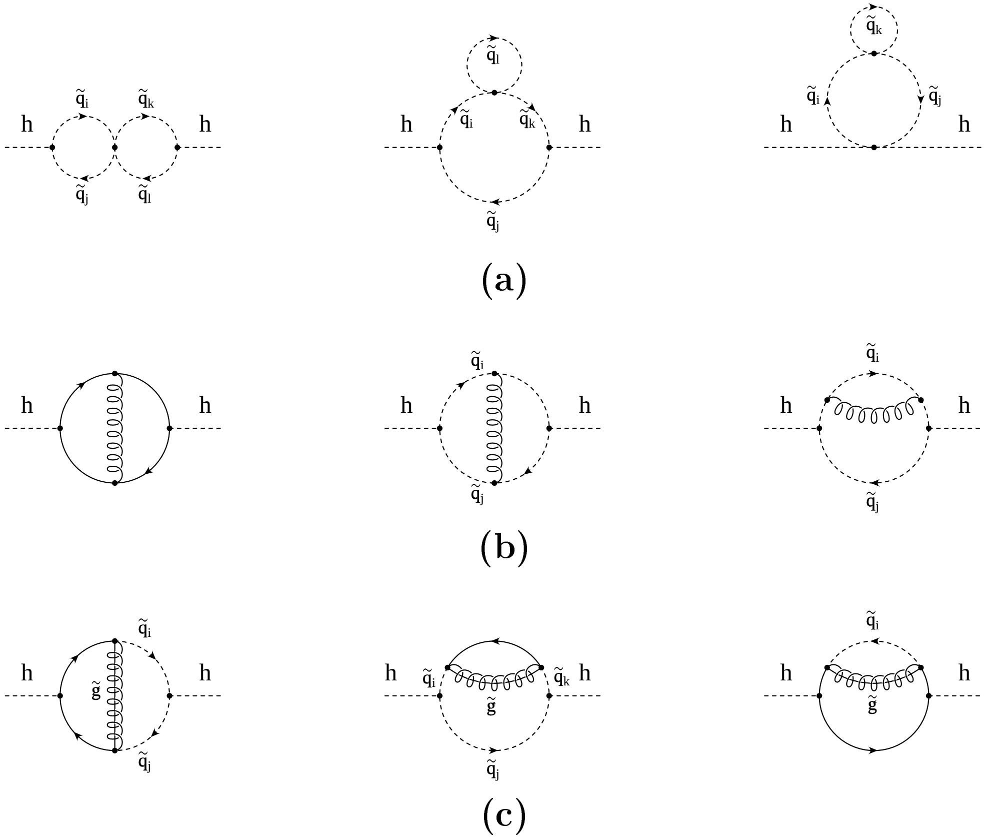
$\Rightarrow$ 2-Loop calculation necessary!

Example for two-loop: $\hat{\Sigma}^{(2)}(0)$

[*S. H., W. Hollik, G. Weiglein '98*]

dominant contributions of $\mathcal{O}(\alpha_t \alpha_s)$:

(a) pure scalar diagrams
(b) diagrams with gluonexchange
(c) diagrams with gluinoexchange



Quite complicated calculation . . .
$\Rightarrow$ Need for computer algebra programs

To avoid large corrections:
On-shell renormalization of the scalar top sector $\Rightarrow X_t^{\rm OS}$

$$\sim m_t^4 \left[\log^2 \left(\frac{m_{\tilde{t}}}{m_t} \right) + \log \left(\frac{m_{\tilde{t}}}{m_t} \right) \right]$$

['98 - '13:] $\Rightarrow$ many more corrections calculated!

Our code:

FeynHiggs

`www.feynhiggs.de`

[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weigleín '98 – '13*]

→ all Higgs masses, couplings, BRs, XSs (easy to link, easy to use :-)

– full one-loop (also for complex parameters)
– leading and subleading two-loop: $\mathcal{O}\left(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$
– running top mass (to minimize three-loop)

Remaining theoretical uncertainties in prediction for M_h in the MSSM:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

– From unknown higher-order corrections: $\Rightarrow \Delta M_h \approx 3$ GeV

– From uncertainties in input parameters $\Delta m_t \approx 1$ GeV $\Rightarrow \Delta M_h \approx 1$ GeV

One-loop:

$$\Delta M_h^2 \sim m_t^2 \alpha_t \left[L + L^0\right] \; , \qquad L := \log\left(\frac{m_{\tilde t}}{m_t}\right)$$

Two-loop:

$$\Delta M_h^2 \sim m_t^2 \left\{\alpha_t \alpha_s \left[L^2 + L + L^0\right] + \alpha_t^2 \left[L^2 + L + L^0\right]\right\}$$

Three-loop:

$$\begin{aligned}\Delta M_h^2 \sim m_t^2 \{ \quad & \alpha_t \alpha_s^2 \left[L^3 + L^2 + L + L^0\right] \\ & + \alpha_t^2 \alpha_s \left[L^3 + L^2 + L + L^0\right] \\ & + \alpha_t^3 \left[L^3 + L^2 + L + L^0\right] \}\end{aligned}$$

Partial results: *[S. Martin '07]* *[R. Harlander, P. Kant, L. Mihaila, M. Steinhauser '08]*

Large $m_{\tilde t}$ $\Rightarrow$ large L $\Rightarrow$ resummation of logs necessary

SUSY mass scale: $M_{\mathsf{SUSY}} = M_{\mathsf{S}} \sim m_{\tilde{t}}$

Above M_{SUSY}: MSSM
Below M_{SUSY}: SM

Relevant SM parameters:
- quartic coupling λ
- top Yukawa coupling h_t ($\alpha_t = h_t^2/(4\,\pi)$)
- strong coupling constant g_s ($\alpha_s = g_s^2/(4\,\pi)$)

Procedure:

1. Take: $h_t(m_t), g_s(m_t)$

   SM RGEs for h_t, g_s: $h_t, g_s(m_t) \to h_t, g_s(M_{\mathsf{S}})$

2. Take $\lambda(M_{\mathsf{S}}), h_t(M_{\mathsf{S}}), g_s(M_{\mathsf{S}})$

   SM RGEs for λ, h_t, g_s: $\lambda, h_t, g_s(M_{\mathsf{S}}) \to \lambda, h_t, g_s(m_t)$

3. Evaluate M_h^2

$$M_h^2 \sim 2\lambda(m_t)v^2$$

SM one-loop RGEs:

$$16\pi^2\beta_\lambda = 6(\lambda^2 + \lambda - h_t^4) \; ,$$
$$16\pi^2\beta_{h_t^2} = h_t^2(9/2h_t^2 - 8g_s^2) \; ,$$
$$16\pi^2\beta_{g_s^2} = -7g_s^4$$

$\Rightarrow$ at n-loop order: L^n

Our procedure:

– SM two-loop RGEs

– one-loop threshold correction for $\lambda(M_{\mathrm{SUSY}})$:

$$\lambda(M_{\mathrm{SUSY}}) = \frac{3\,h_t^4}{8\,\pi^2} x_t^2 \left[1 - 1/12\,x_t^2\right] \; , \quad x_t = X_t^{\overline{\mathrm{MS}}}/M_{\mathrm{S}}$$

$\Rightarrow$ at n-loop order: $L^n + L^{n-1}$

– add correction ($\times 1/\sin^2\beta$) to $\hat{\Sigma}_{\phi_2\phi_2}$

– subtract leading and subleading logs at one- and two-loop (with X_t^{OS}) to avoid double counting

$\Rightarrow$ combination of best FD result with resummed LL, NLL corrections for large $m_{\tilde{t}}$

$\Rightarrow$ most precise M_h prediction for large $m_{\tilde{t}}$ $\Rightarrow$ FeynHiggs 2.10.0

3. Numerical results

[FeynHiggs 2.10.0 - PRELIMINARY]

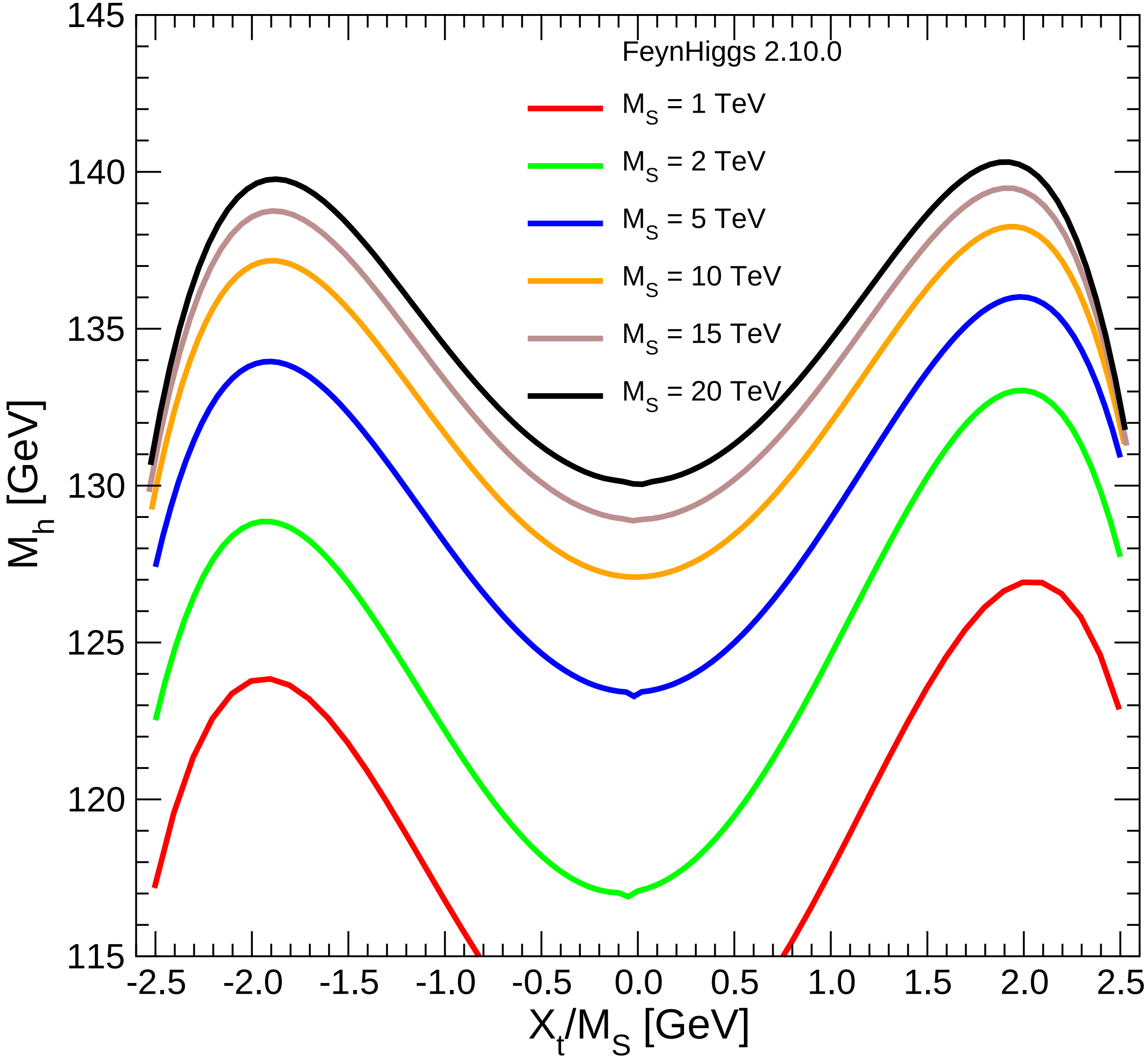
Parameters:

$$
\begin{aligned}
M_S &= \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \\
M_A &= 1000 \text{ GeV} \\
\mu &= 1000 \text{ GeV} \\
M_2 &= 1000 \text{ GeV} \\
m_{\tilde{g}} &= 1600 \text{ GeV} \\
\tan\beta &= 10
\end{aligned}
$$

Vary M_S, X_t to analyze effects

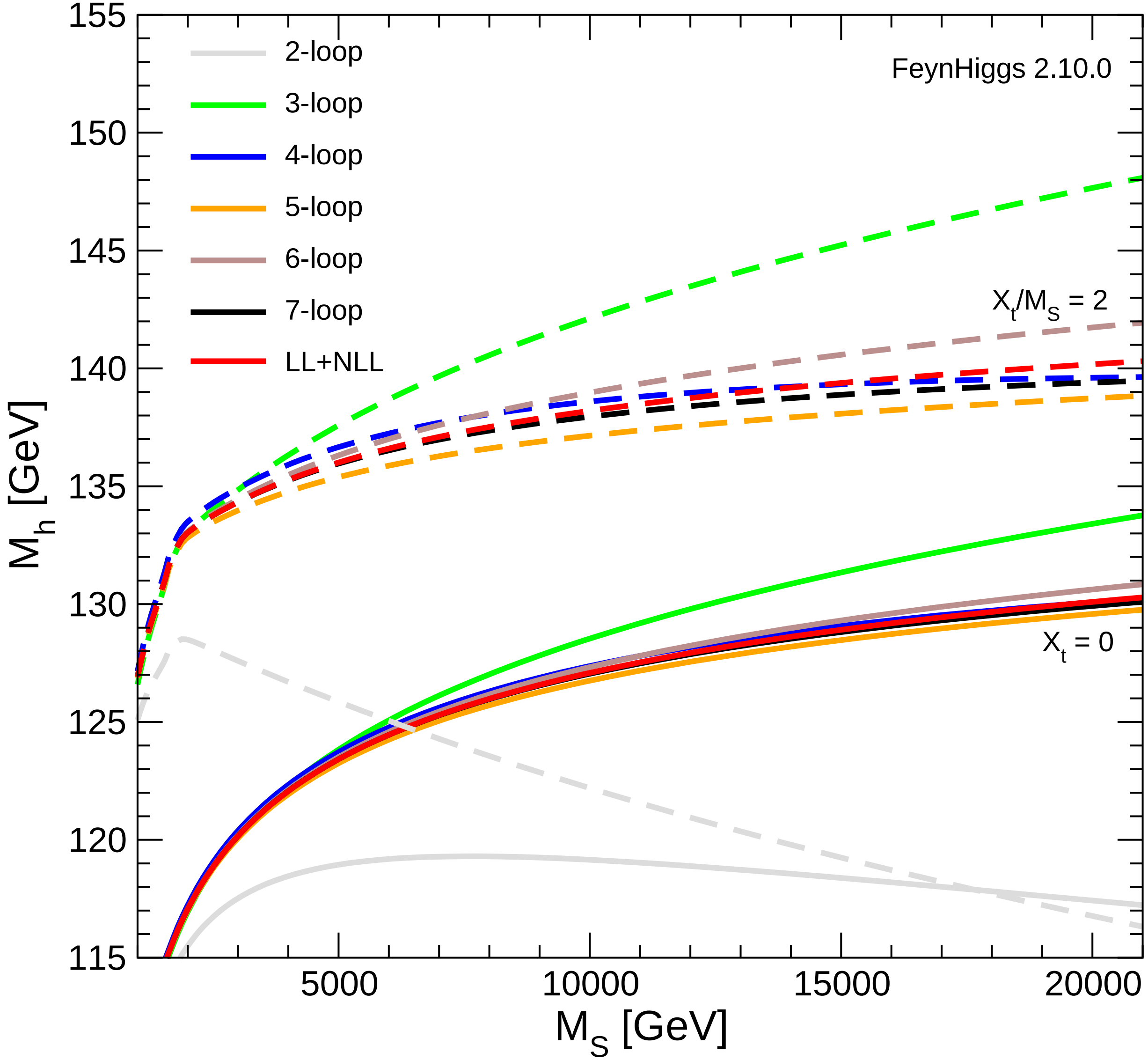
$M_h(X_t/M_S)$:

[*FeynHiggs 2.10.0 - PRELIMINARY*]



$\Rightarrow$ increase with M_S, maxima at $X_t/M_S = \pm 2$

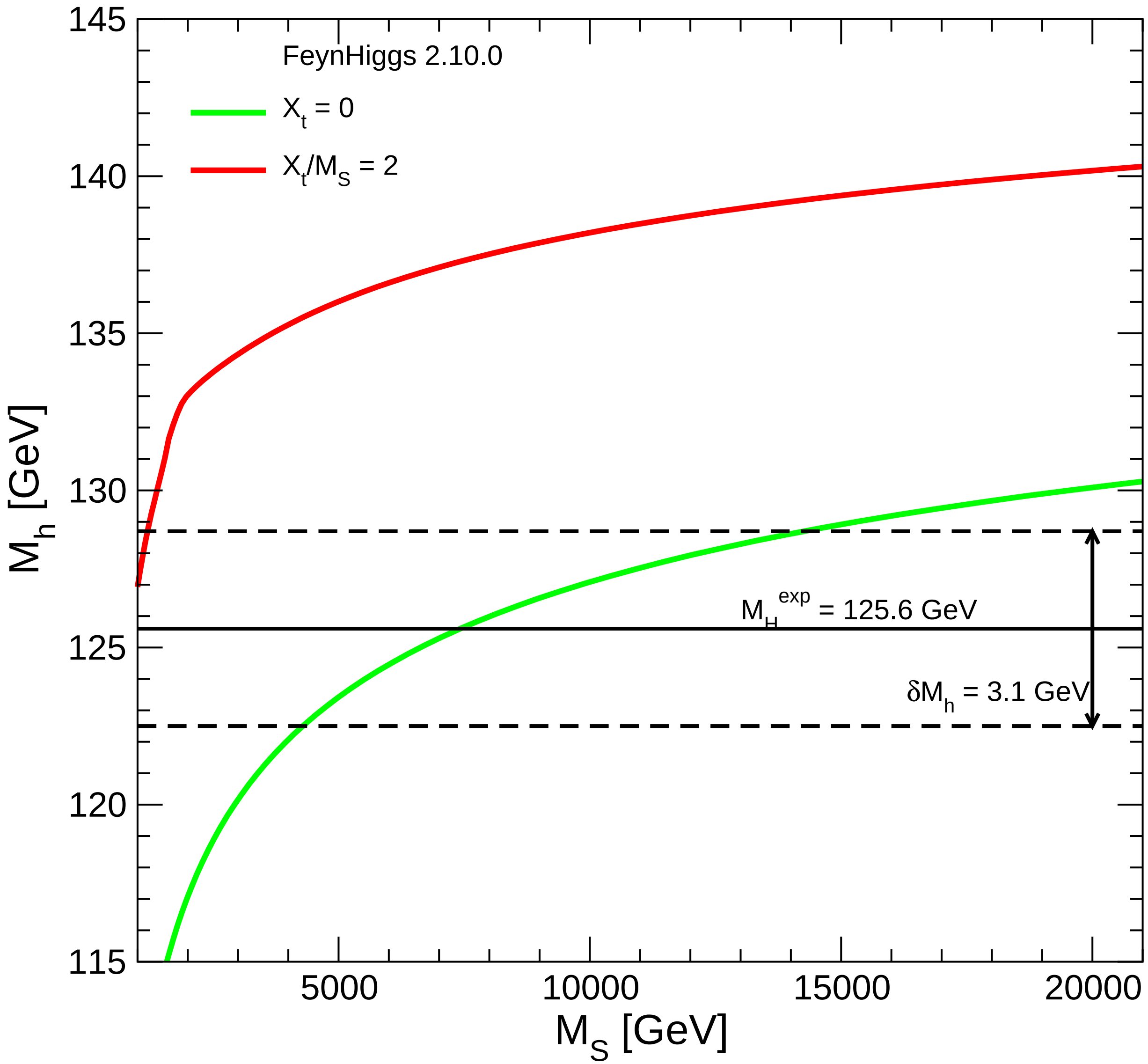
$M_h(M_S)$ for various approximations: [*FeynHiggs 2.10.0 - PRELIMINARY*]



⇒ 3-loop good for $M_S \lesssim 2$ TeV, 7-loop: $\Delta \sim 1$ GeV for $M_S = 20$ TeV

$M_h(M_S)$:

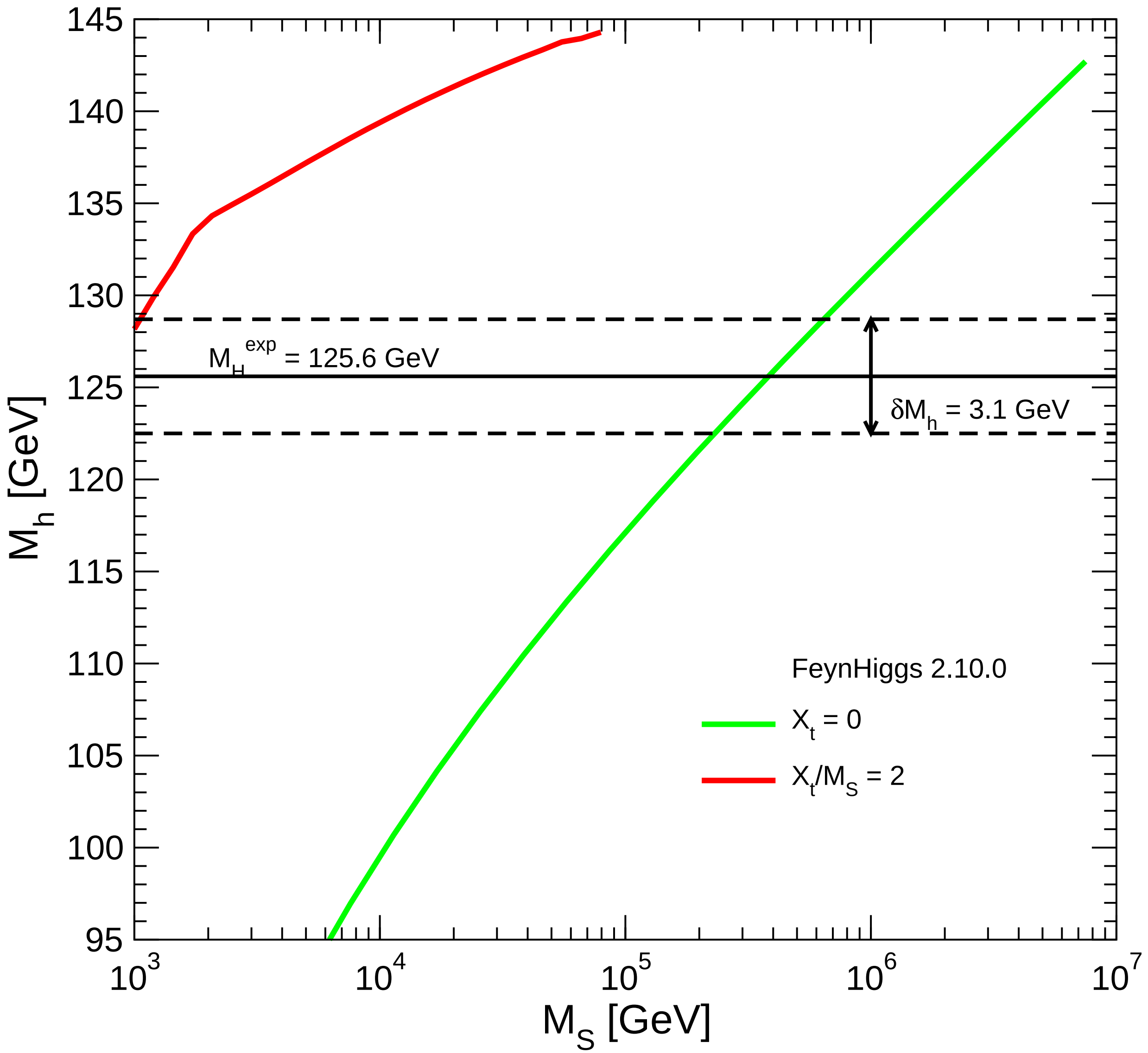
[FeynHiggs 2.10.0 - PRELIMINARY]



⇒ upper bound on M_S?

$M_h(M_S)$ for $\tan\beta = 1\,(X_t = 0)$ or $\tan\beta = 40\,(X_t/M_S = 2)$:

[*FeynHiggs 2.10.0 - PRELIMINARY*]



$\Rightarrow$ "upper bound": $M_S \lesssim 650$ TeV $\Rightarrow$ needs refinement!

4. Conclusinos

- $\delta M_h^{\text{ILC}} \lesssim 0.05$ GeV $\Leftrightarrow$ $\delta M_h^{\text{intr,current}} \sim 3$ GeV
  $\Rightarrow$ strong reduction of theory uncertainty necessary!

- Feynman-diagrammatic result as included in FeynHiggs (so far):
  – full one-loop (also for complex parameters)
  – leading and subleading two-loop: $\mathcal{O}\left(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$
  – running top mass (to minimize three-loop)

- SM RGEs at two-loop (plus one-loop threshold correction:)
  $\Rightarrow$ M_h^2 at n-loop order: $L^n + L^{n-1}$

- Combination of best FD result with
  resummed LL, NLL corrections for large $m_{\tilde{t}}$
  $\Rightarrow$ most precise M_h prediction for large $m_{\tilde{t}}$

  FeynHiggs 2.10.0 – www.feynhiggs.de

- Numerical results:
  – stable behavior for very large stop masses
  – 3-loop good for $M_S \lesssim 2$ TeV
  – 7-loop: $\Delta \sim 1$ GeV for $M_S = 20$ TeV