How conventional measures overestimate EW fine-tuning in SUSY theory, and why ILC must be built

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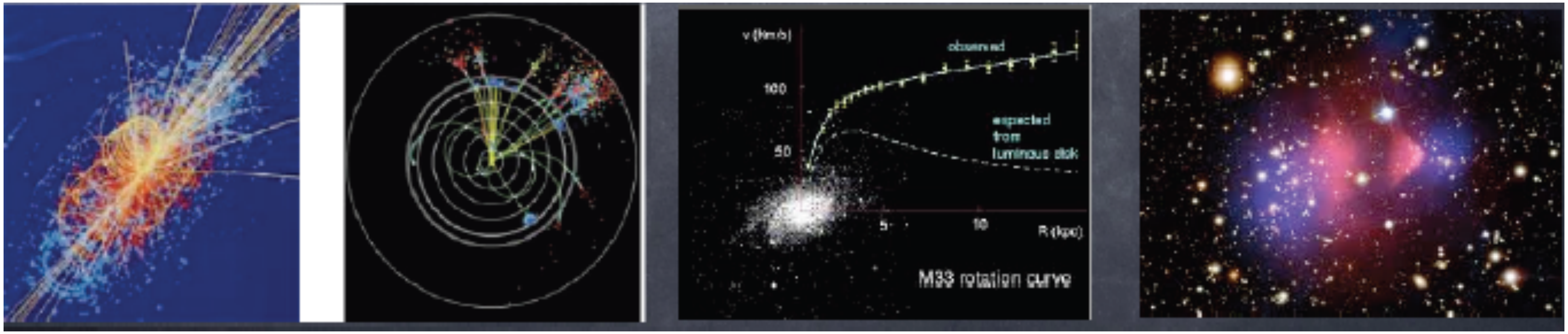
with Bae, Barger, Chun, Huang, Lessa, Mickelson, Mustafayev,
Padeffke-Kirkland, Sreethawong, Tata



- h(125.5+-0.5 GeV) discovered at LHC
- scalars need protective symmetry: SUSY
- m(h)~125.5 GeV falls within narrow MSSM expectation
- m(h) requires highly mixed TeV-scale stops
- LHC: no SUSY: m(glno)>1.3 TeV, m(sqrk)>1.7 TeV, t1 limits
- impression: then MSSM EW fine-tuned at .1%
- SUSY as expected likely wrong?
- need new features or new model?

- How does this perception arise?
- Why is it wrong?
- Overestimate of EWFT
- What does SUSY look like?
- How can we tell? Need ILC

Naturalness in the Standard Model

SM case: invoke a single Higgs doublet

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$m_h^2 = m_h^2|_{tree} + \delta m_h^2|_{rad}$$

$$m_h^2|_{tree} = \sqrt{2}\mu^2 \qquad \delta m_h^2|_{rad} = \frac{c}{16\pi^2}\Lambda^2$$

$m_h^2|_{tree}$ and $\delta m_h^2|_{rad}$ are *independent*,

$$\Delta_{SM} \equiv \delta m_h^2|_{rad}/(m_h^2/2)$$

$$\Delta_{SM} < 1 \Rightarrow \ \Lambda \sim 1\, TeV$$

MSSM case:

$$m_h^2 \simeq \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2|_{rad}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2X_t\right) \qquad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, mHu and running;
then we can integrate from mSUSY to Lambda

$$\delta m_{H_u}^2|_{rad} \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2)\ln\left(\Lambda^2/m_{SUSY}^2\right)$$

$\Delta \equiv \delta m_{H_u}^2/(m_h^2/2) \lesssim 10$ then $m_{\tilde{t}_{1,2}}, m_{\tilde{b}_1} \lesssim 200$ GeV and $m_{\tilde{g}} \lesssim 600$ GeV

apparently in violation of LHC constraints!

What's wrong with this argument?

In zeal for simplicity, have neglected that in SUSY

$m_{H_u}^2$ and $\delta m_{H_u}^2|_{rad}$ are not independent

the larger the value of $m_{H_u}^2(\Lambda)$, then the larger is the cancelling correction $\delta m_{H_u}^2|_{rad}$

The dependent terms should be grouped together

$$m_h^2|_{phys} = \mu^2 + \left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$$

where instead both μ^2 and $(m_{H_u}^2 + \delta m_{H_u}^2)$ should be comparable to $m_h^2|_{phys}$.

Such a re-grouping is used in Barbieri-Giudice measure:

$$\Delta_{BG} \equiv max_i\,[c_i] \quad \text{where} \quad c_i = \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right|$$

Here, the a_i are parameters of the theory

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \mu^2$$

↑

express weak scale value in terms of high scale parameters

Express m(Z) in terms of high scale parameters:

$$m_Z^2 \simeq -2m_{H_u}^2 - 2\mu^2$$

$$\begin{aligned}
-2\mu^2(m_{SUSY}) &= -2.18\mu^2 \\
-2m_{H_u}^2(m_{SUSY}) &= 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\
&\quad +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\
&\quad -0.025M_1A_t + 0.22A_t^2 + 0.004m_3A_b \\
&\quad -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\
&\quad +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\
&\quad +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\
&\quad +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2,
\end{aligned}$$

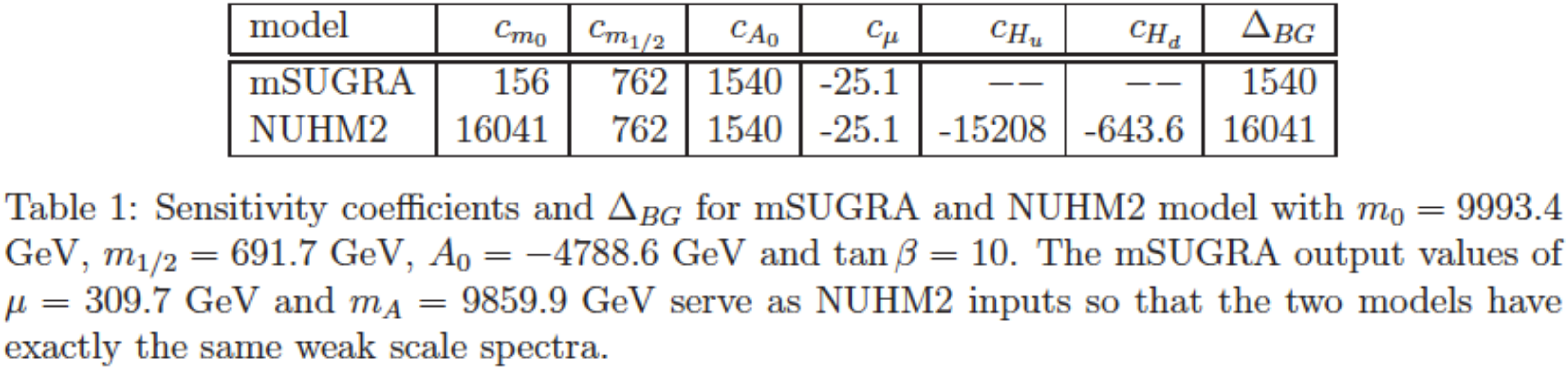
Abe, Kobayashi, Omura;
S. P. Martin

For generic parameter choices, $\Delta_{BG}\ is\ large$

But if: $m_{Q_{1,2}} = m_{U_{1,2}} = m_{D_{1,2}} = m_{L_{1,2}} = m_{E_{1,2}} \equiv m_{16}(1,2)$ then $\sim 0.007m_{16}^2(1,2)$

Even better: $m_{H_u}^2 = m_{H_d}^2 = m_{16}^2(3) \equiv m_0^2$ => $-0.017m_0^2$

For correlated parameters, EWFT collapses in 3rd gen. sector!



model	c_{m_0}	$c_{m_{1/2}}$	c_{A_0}	c_{μ}	c_{H_u}	c_{H_d}	Δ_{BG}
mSUGRA	156	762	1540	-25.1	−−	−−	1540
NUHM2	16041	762	1540	-25.1	-15208	-643.6	16041

Table 1: Sensitivity coefficients and Δ_{BG} for mSUGRA and NUHM2 model with $m_0 = 9993.4$ GeV, $m_{1/2} = 691.7$ GeV, $A_0 = -4788.6$ GeV and $\tan\beta = 10$. The mSUGRA output values of $\mu = 309.7$ GeV and $m_A = 9859.9$ GeV serve as NUHM2 inputs so that the two models have exactly the same weak scale spectra.

Lesson: the BG measure determines fine-tuning within particular effective theories.
Its value changes from theory to theory,
i.e. it is highly model-dependent,
as it must be since it depends on parameters

- most theorists hypothesize existence of an ultimate theory which describes nature
- perhaps MSSM with all correlated parameters is low E effective theory: UTH
- hope is that UTH is contained within more general multi-parameter effective theories which are popular in literature: mSUGRA, nuhm2,...
- The Δ_{BG} measures EWFT in the multi-parameter effective theories instead of UTH: leads to overestimate:
- example: mSUGRA serves as toy UTH for NUHM2 which contains more parameters
- need an EWFT measure which gives same value for effective theories as for UTH (i.e. model-independent)

What we really want to know is:
is nature is fine-tuned,
(and by implication the UTH which describes it),
and not whether-or-not
the more general effective theories
(which might contain the UTH)
are fine-tuned

Are we then to give up on naturalness
as a guide to SUSY models?

Model-independent EWFT measure: Δ_{EW}

No large uncorrelated cancellations in m(Z) or m(h)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

$\Delta_{EW} \equiv max_i \left|C_i\right| / (m_Z^2/2)$ with $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$ etc.

Since Δ_{EW} model-independent (within MSSM),

expect same value for Eff. theory as for UTH!

In order to achieve low Δ_{EW}, it is necessary that $-m_{H_u}^2$, μ^2 and $-\Sigma_u^u$ all be nearby to $m_Z^2/2$ to within a factor of a few[12, 13]:

1. μ is required to lie in the $100 - 300$ GeV range,
2. a value of $m_{H_u}^2(m_{GUT}) \sim (1.3 - 2.5)m_0$ may be chosen so that $m_{H_u}^2$ is driven radiatively to slightly negative at the weak scale, leading to $m_{H_u}^2(weak) \sim -m_Z^2/2$, and
3. with large stop mixing from $A_0 \sim \pm 1.6 m_0$, the top-squark radiative corrections are softened while m_h is raised to the ~ 125 GeV level.

Model-independent EWFT measure: Δ_{EW}

No large uncorrelated cancellations in m(Z) or m(h)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

For the top squark contributions, we find

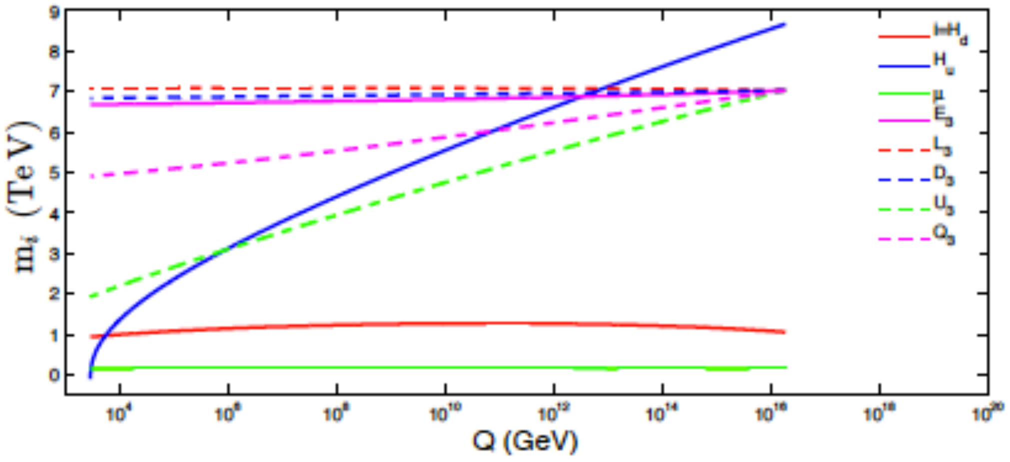
$$\Sigma_u^u(\tilde t_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde t_{1,2}}^2)\left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde t_2}^2 - m_{\tilde t_1}^2}\right]$$

where $\Delta_t = (m_{\tilde t_L}^2 - m_{\tilde t_R}^2)/2 + M_Z^2\cos 2\beta(\frac{1}{4} - \frac{2}{3}x_W)$ and $x_W \equiv \sin^2\theta_W$

where

$$F(m^2) = m^2\left(\log\frac{m^2}{Q^2} - 1\right)$$

with the optimized scale choice $Q^2 = m_{\tilde t_1} m_{\tilde t_2}$.



Large At drives down $\Sigma_u^u(\tilde t_{1,2})$ while lifting m(h) to 125 GeV!

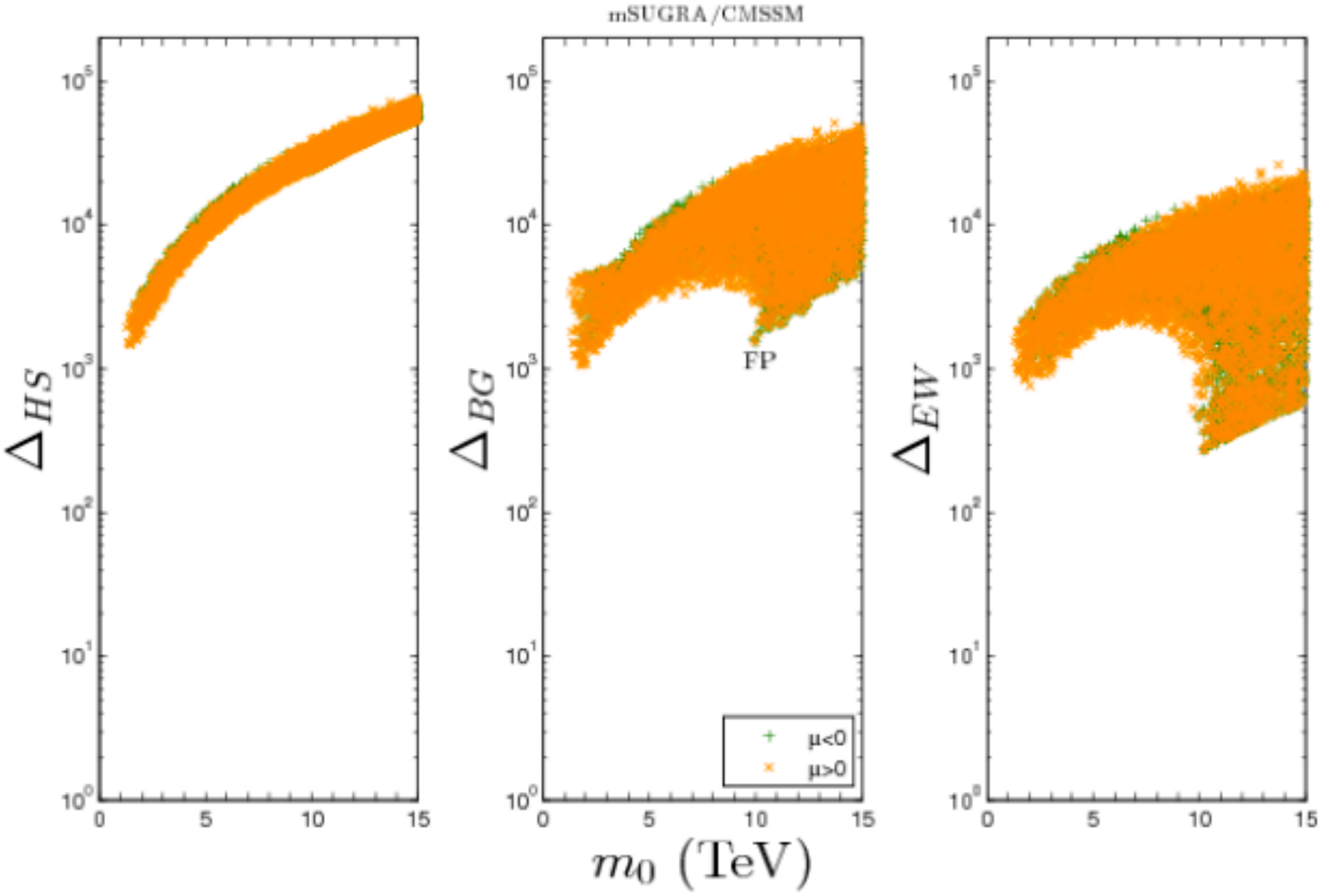
Radiatively-driven natural SUSY:

H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, *Phys. Rev. Lett.* **109** (2012) 161802.

H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, *Phys. Rev.* **D 87** (2013) 115028 [`arXiv:1212.2655` [hep-ph]].

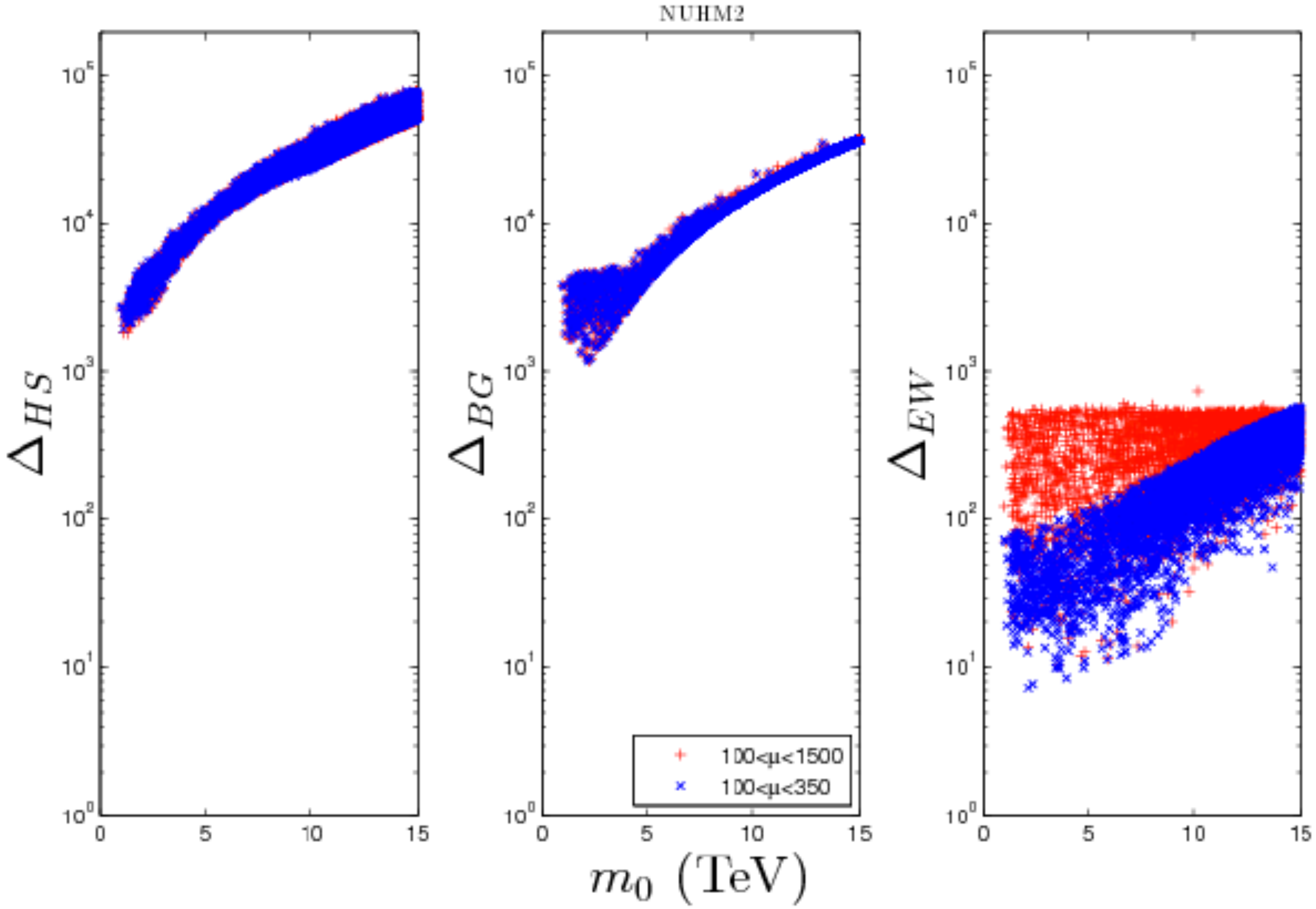
The mSUGRA/CMSSM model is fine-tuned under all three measures:

this effective theory is unlikely to contain the UTH



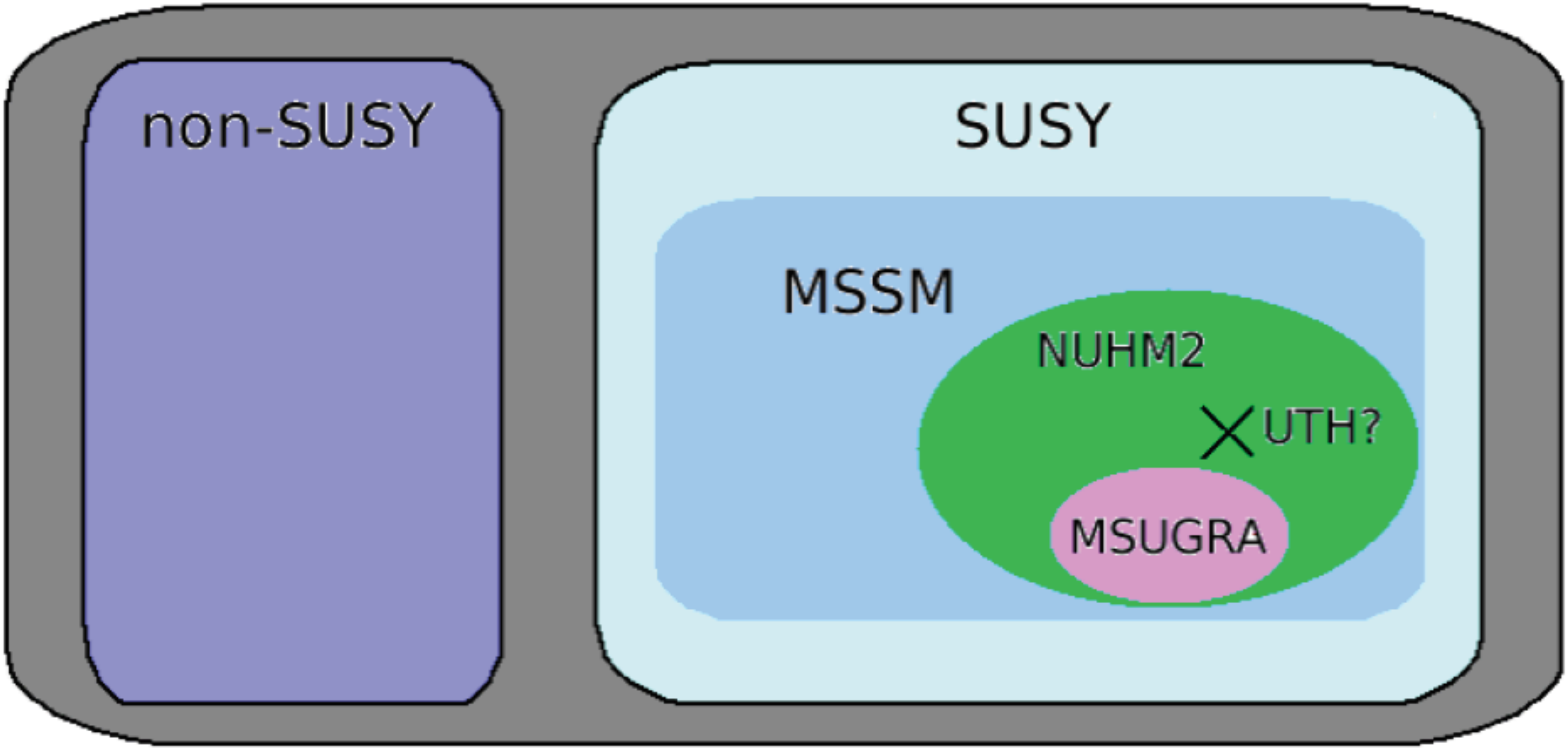
HB, Barger, Mickelson: arXiv:1309.2984

The NUHM2 model allows for not-too-heavy stops at 1-3 TeV with large mixing and m(h)~125 GeV while maintaining low mu~100-200 GeV: it allows for EWFT at just ~10% level, thus it may well contain the UTH



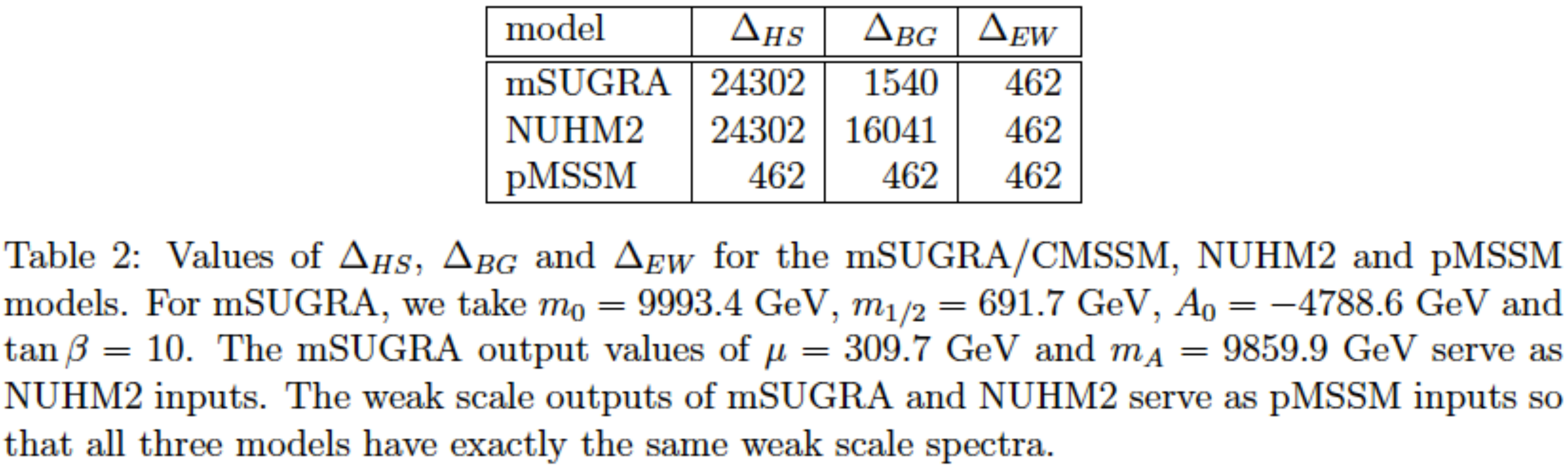
HB, Barger, Mickelson: arXiv:1309.2984

THEORY SPACE



How conventional measures overestimate electroweak fine-tuning in supersymmetric theory

HB, Barger, Mickelson: arXiv:1309.2984



model	Δ_{HS}	Δ_{BG}	Δ_{EW}
mSUGRA	24302	1540	462
NUHM2	24302	16041	462
pMSSM	462	462	462

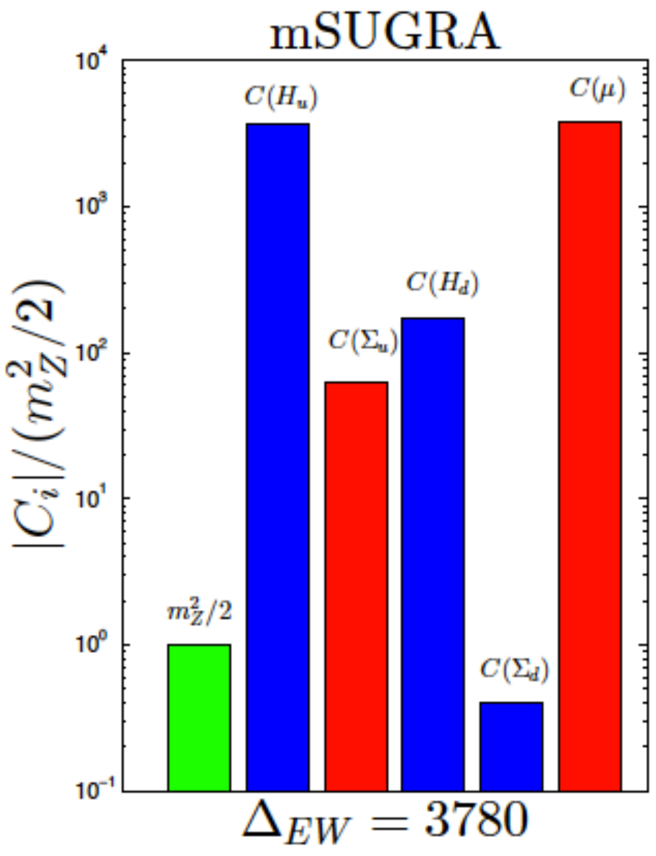
Table 2: Values of Δ_{HS}, Δ_{BG} and Δ_{EW} for the mSUGRA/CMSSM, NUHM2 and pMSSM models. For mSUGRA, we take $m_0 = 9993.4$ GeV, $m_{1/2} = 691.7$ GeV, $A_0 = -4788.6$ GeV and $\tan\beta = 10$. The mSUGRA output values of $\mu = 309.7$ GeV and $m_A = 9859.9$ GeV serve as NUHM2 inputs. The weak scale outputs of mSUGRA and NUHM2 serve as pMSSM inputs so that all three models have exactly the same weak scale spectra.

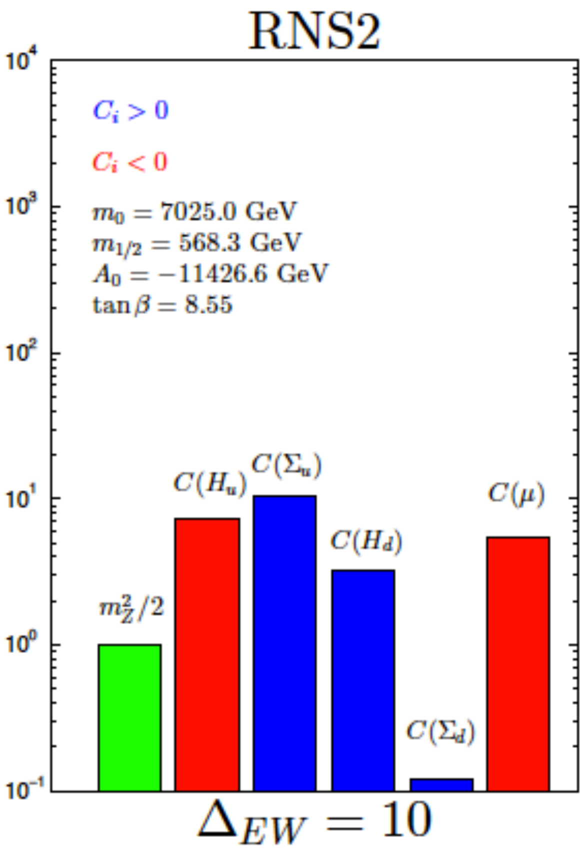
$$\lim_{\Lambda \to m_{SUSY}} \Delta_{HS} = \Delta_{EW}$$

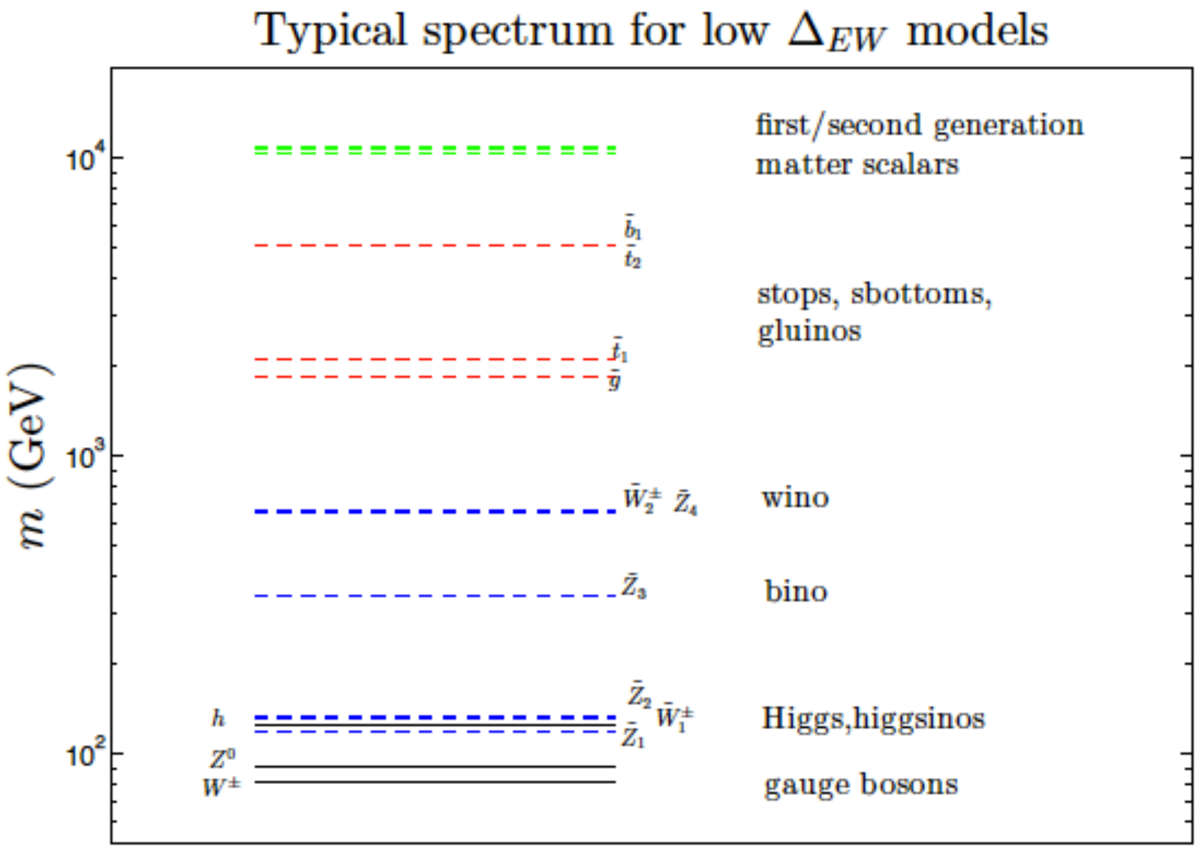
$$\lim_{\Lambda \to m_{SUSY}} \Delta_{BG} \sim \Delta_{EW}$$

$$\Delta_{EW} < \Delta_{BG} \lesssim \Delta_{HS}$$

All contributions to m(Z) and m(h) are comparable to m(Z) and m(h): model is natural in EW sector!



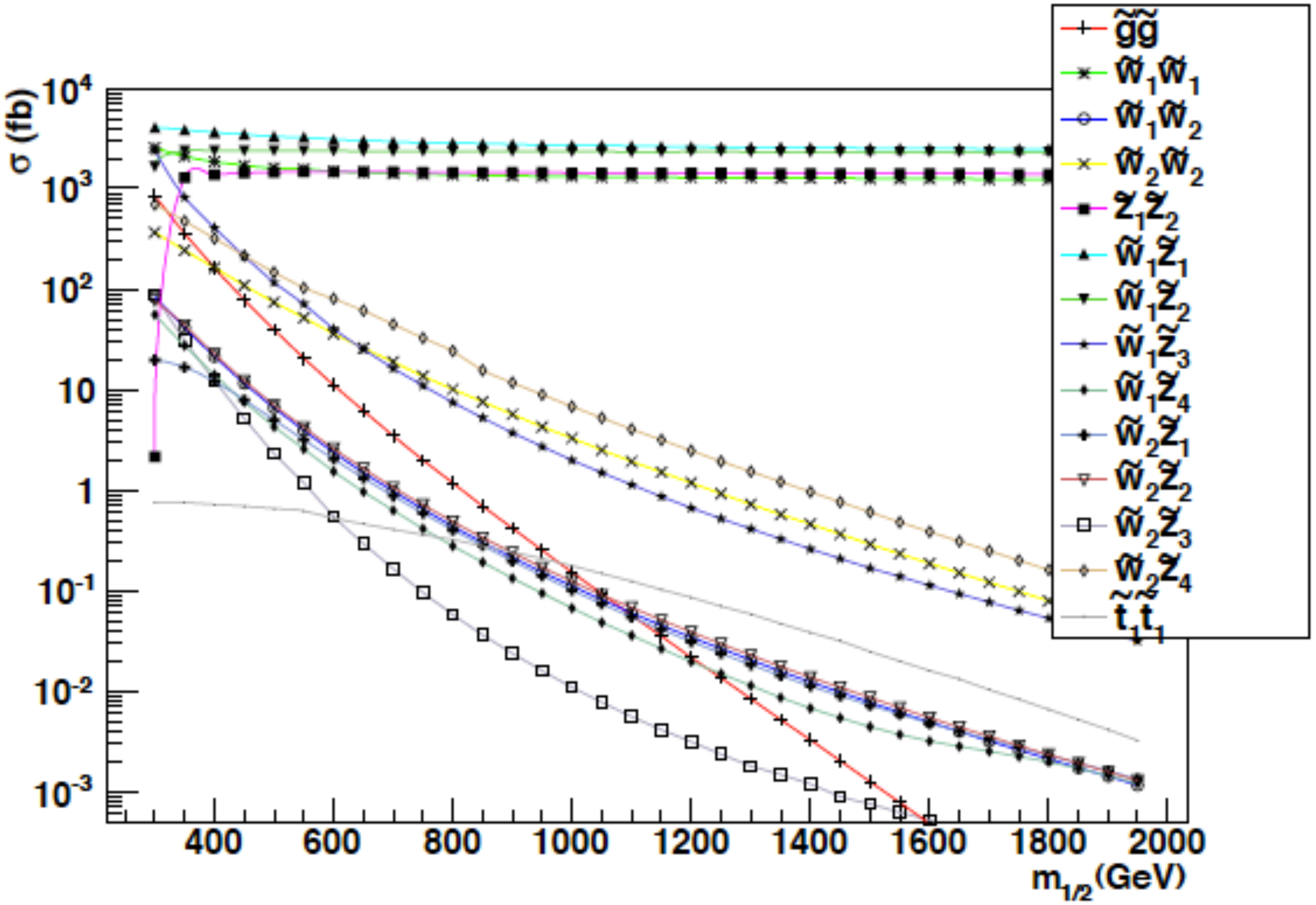




There is a Little Hierarchy, but it is no problem

Sparticle production along RNS model-line:

LHC14

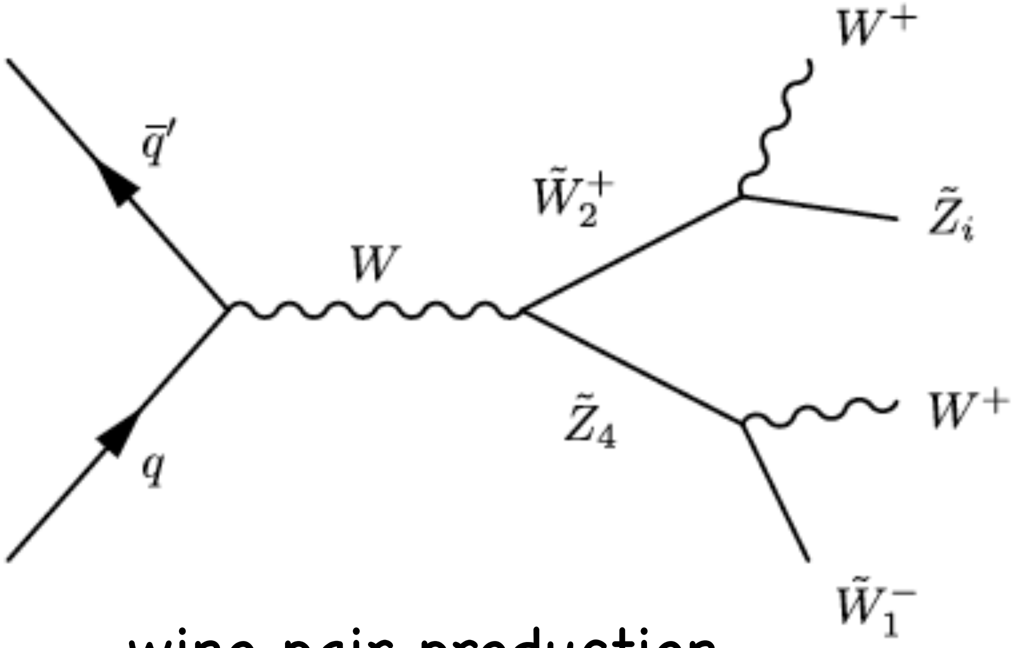


*higgsino pair production dominant-but only soft visible energy release from higgsino decays

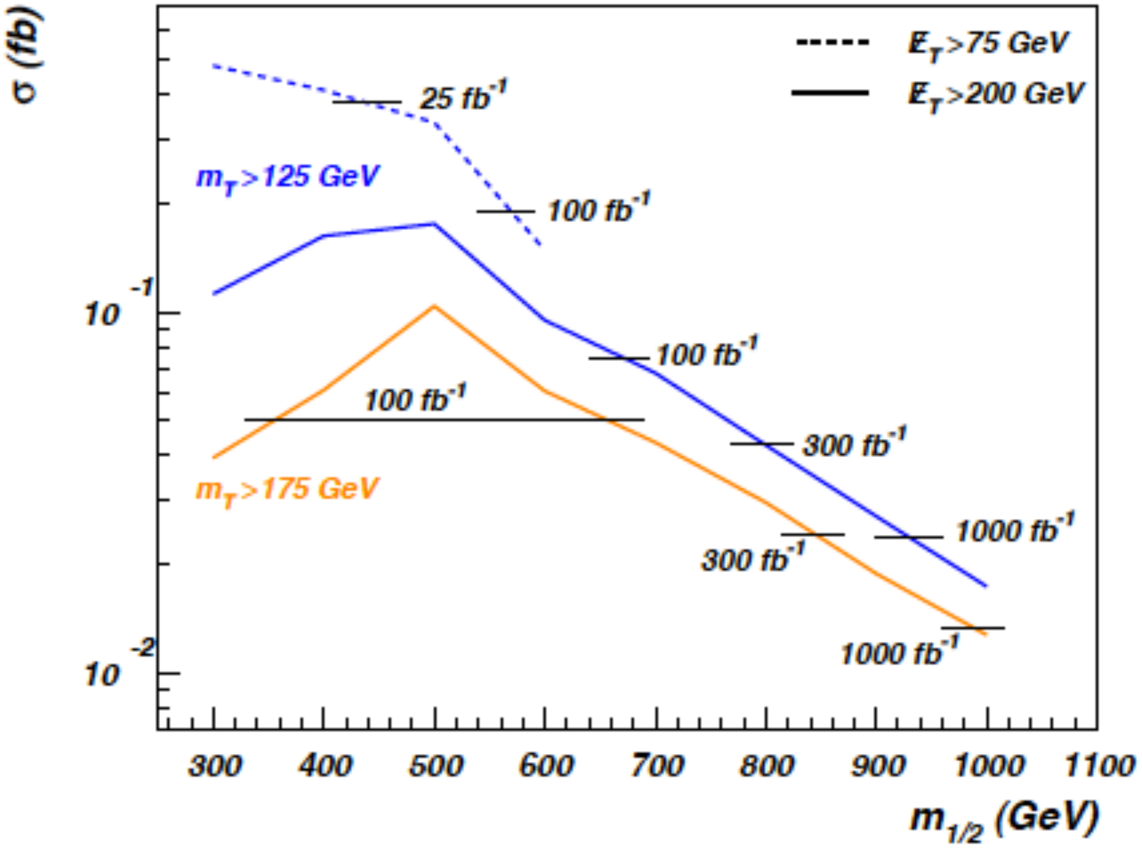
*largest visible cross section: wino pairs

*gluino pairs sharply dropping

Characteristic same-sign diboson (SSdB) signature from SUSY models with light higgsinos:



wino pair production

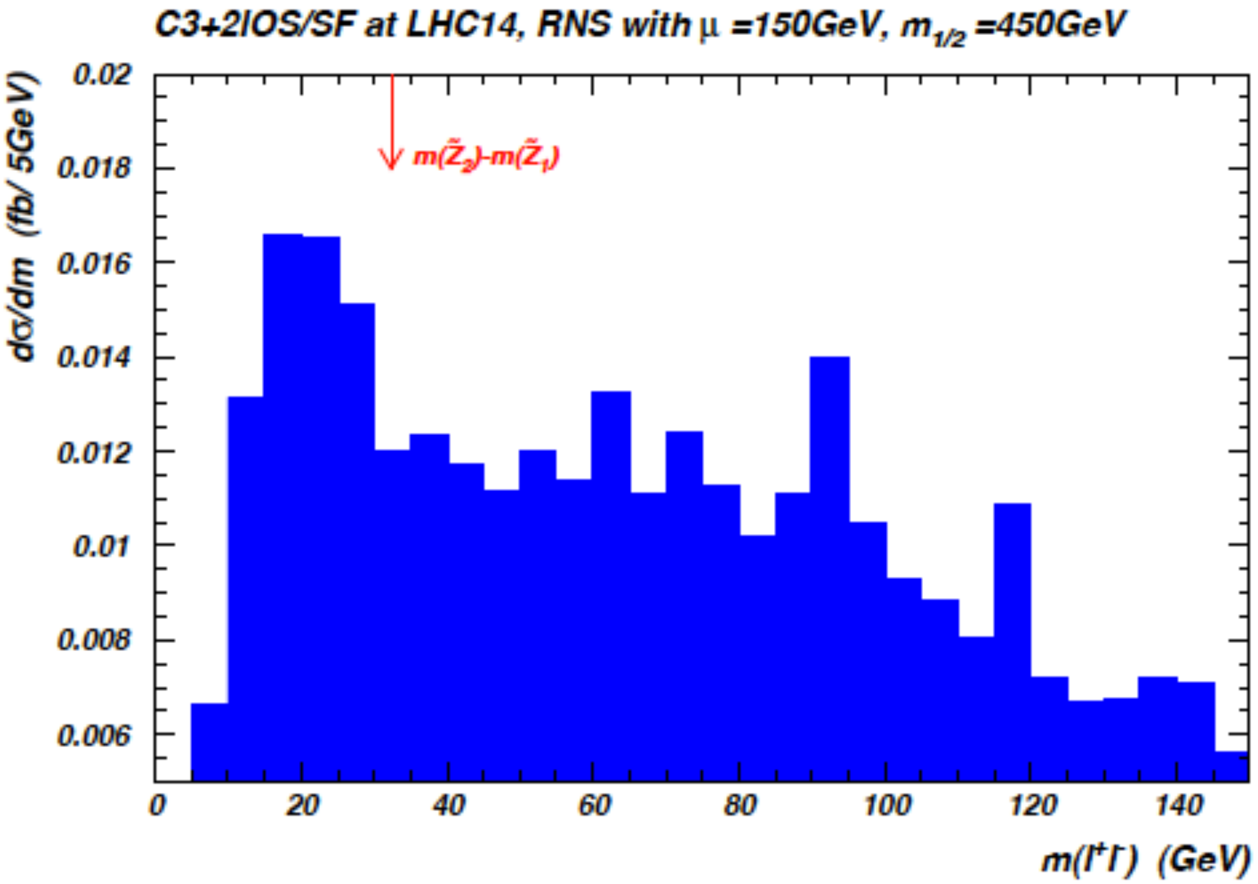


H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, *Phys. Rev. Lett.* **110** (2013) 151801.

This channel offers best reach of LHC14 for RNS; it is also indicative of wino-pair prod'n followed by decay to higgsinos

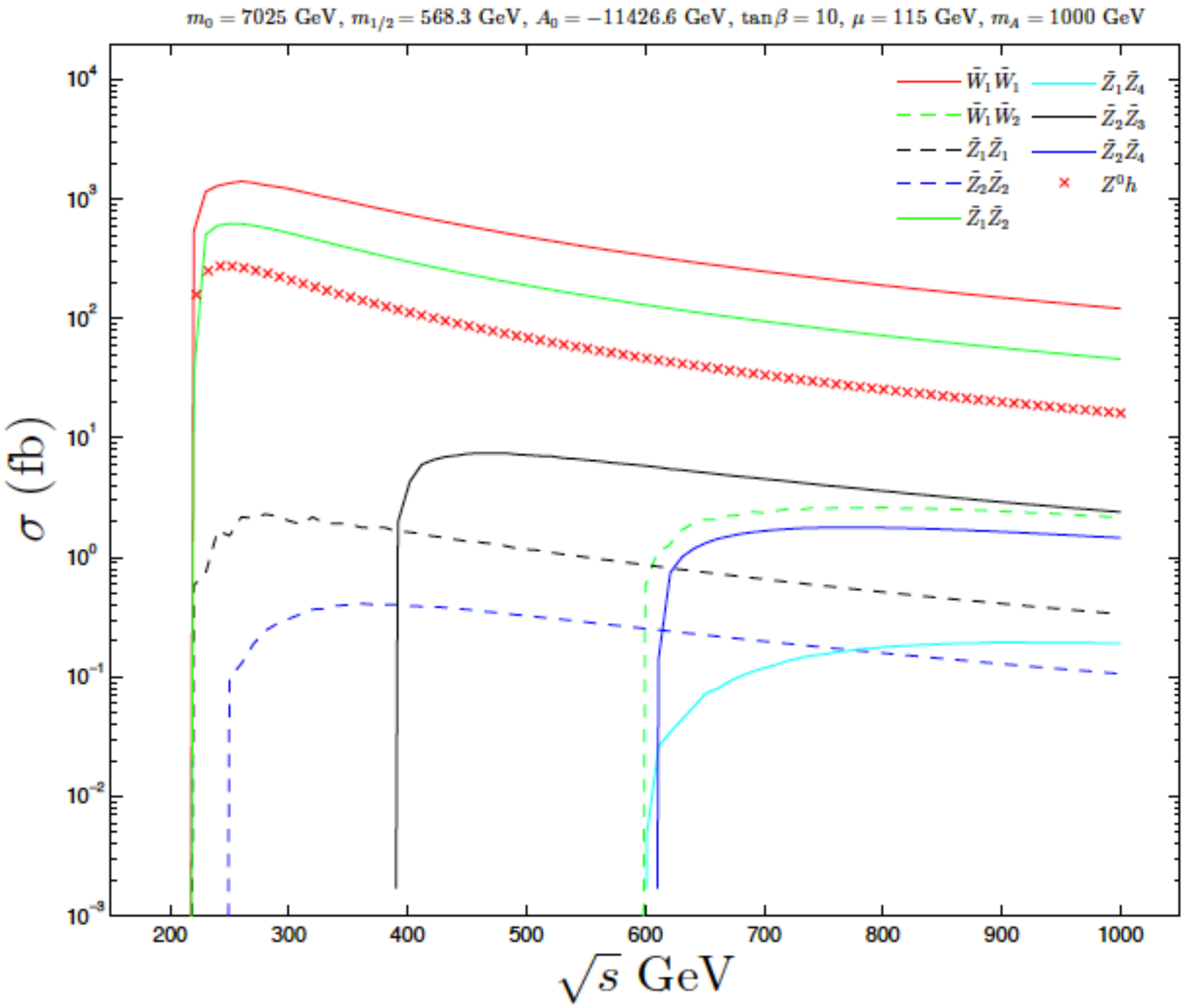
LHC14 has some reach for RNS; if a signal is seen, should be characteristic

Int. lum. (fb^{-1})	$\tilde{g}\tilde{g}$	SSdB	$WZ \to 3\ell$	4ℓ
10	1.4	–	–	–
100	1.6	1.6	–	~ 1.2
300	1.7	2.1	1.4	$\gtrsim 1.4$
1000	1.9	2.4	1.6	$\gtrsim 1.6$



OS/SF dilepton mass edge apparent from cascade decays with z2->z1+l+lbar

Smoking gun signature:
light higgsinos at ILC:
ILC is Higgs/higgsino factory!

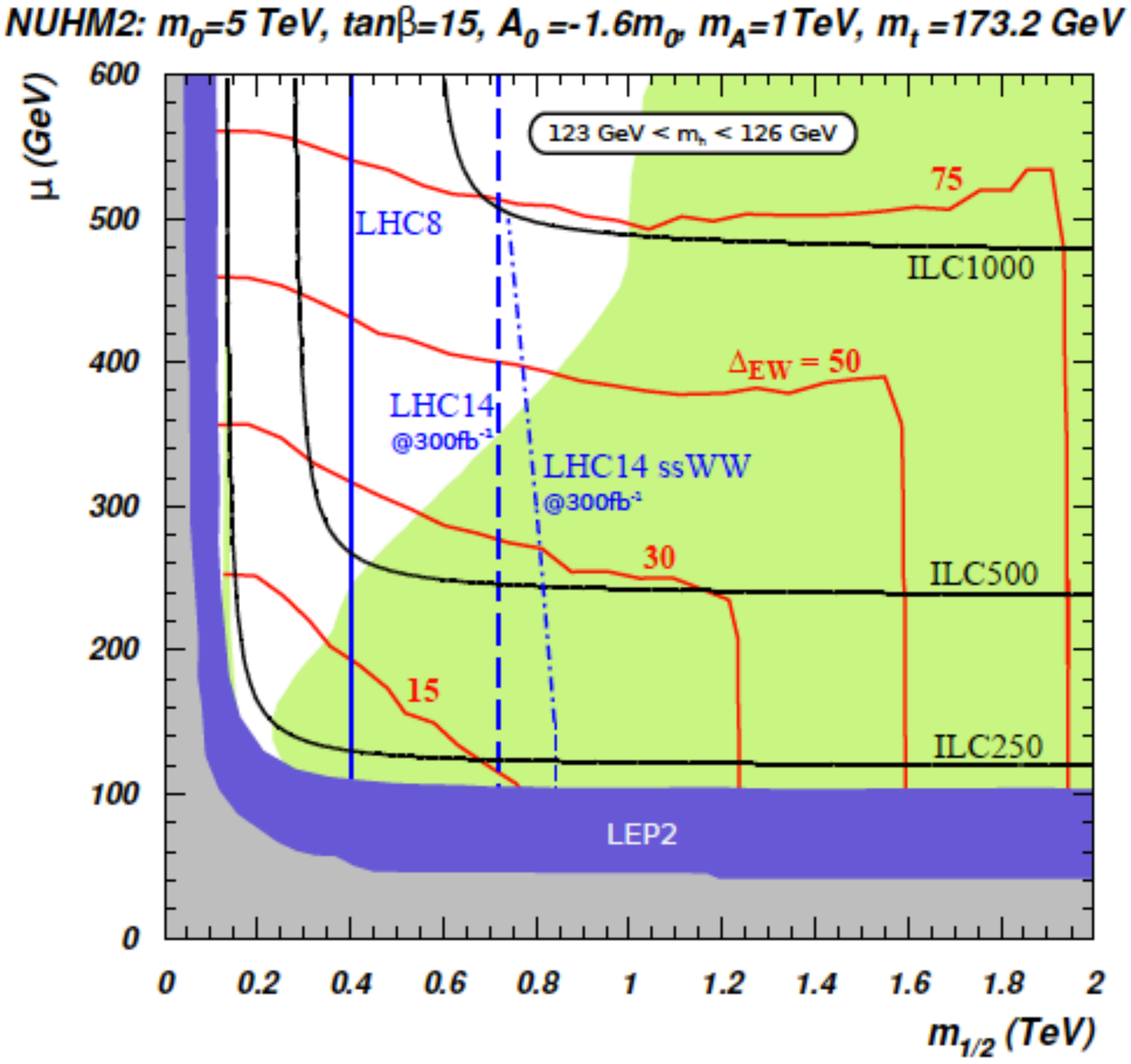


$$\sigma(higgsino) \gg \sigma(Zh)$$

10-15 GeV higgsino mass
gaps no problem
in clean ILC environment

ILC either sees light higgsinos or natural SUSY dead

LHC/ILC complementarity



NUHM2: m_0=5 TeV, tanβ=15, A_0=-1.6m_0, m_A=1TeV, m_t=173.2 GeV

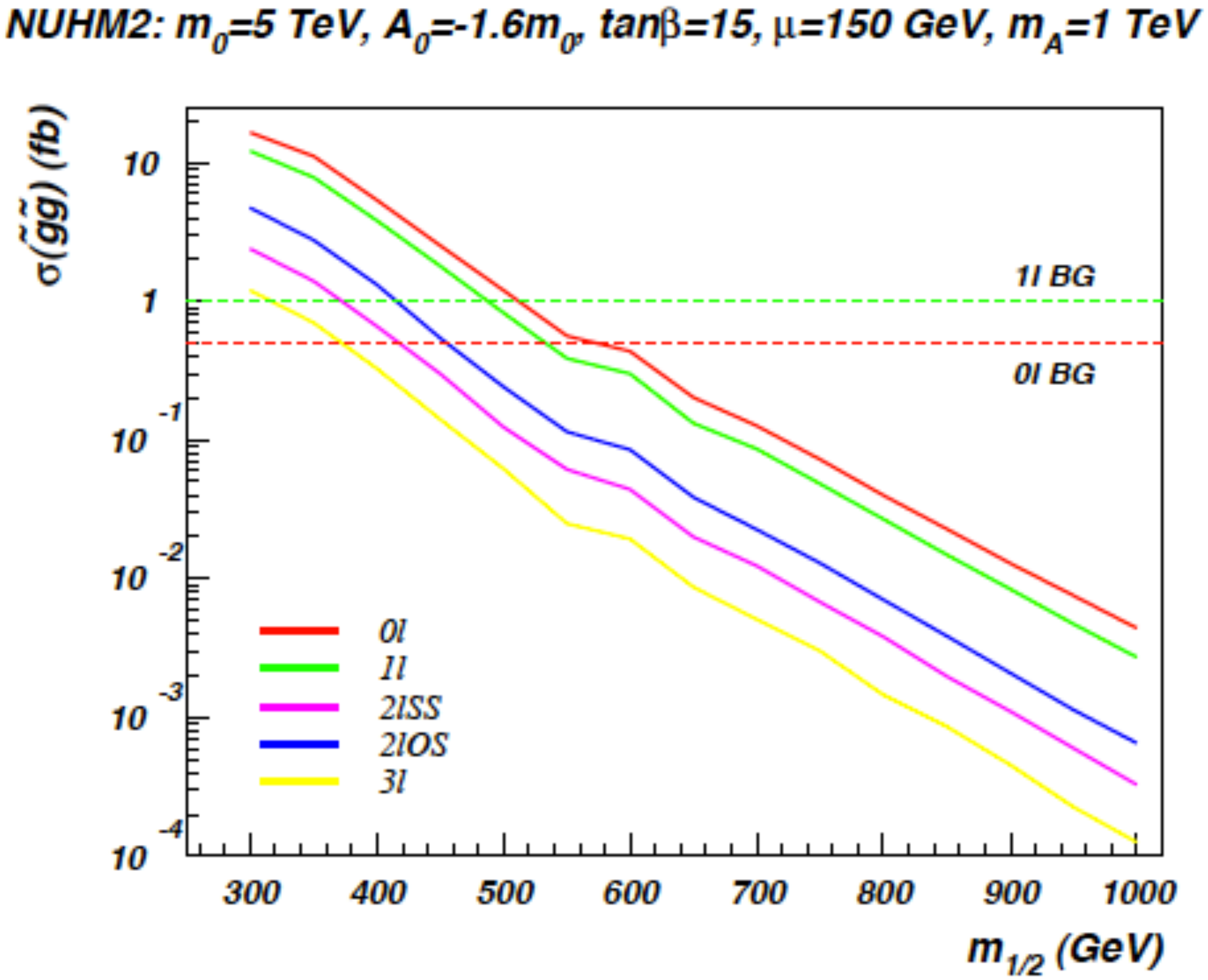
When to give up on naturalness in SUSY?

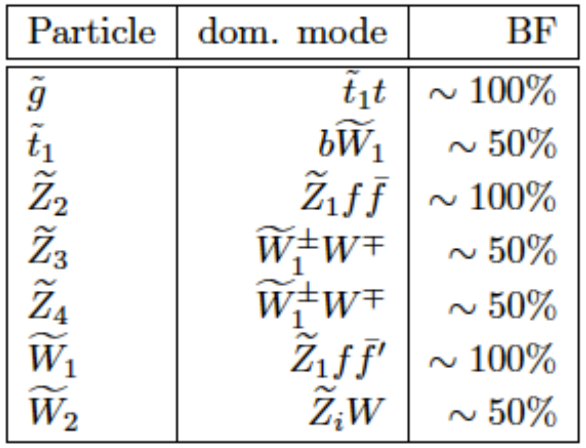
If ILC(500-600 GeV) sees no light higgsinos

Conclusions: status of SUSY post LHC8

- SUSY EWFT non-crisis: EWFT allowed at 10% level in radiatively-driven natural SUSY: SUSY GUT paradigm is just fine
- naturalness maintained for mu~100-200 GeV; t1~1-2 TeV, t2~2-4 TeV, highly mixed; m(glno)~1-5 TeV
- RNS spectra characterized by mainly higgsino-like WIMP: standard relic underabundance
- LHC14 w/ 300 fb^-1 can see about half of RNS parameter space
- e+e- collider with sqrt(s)~500-600 GeV needed to find predicted light higgsino states
- Also address strong CP problem via axion-axino-saxion
- DFSZ invisible axion model: solves mu problem while allowing for mu~m(Z)
- Expect mainly axion CDM with 5-10% higgsino-like WIMPs over much of p-space
- Direct detect both axion and higgsino-like WIMP

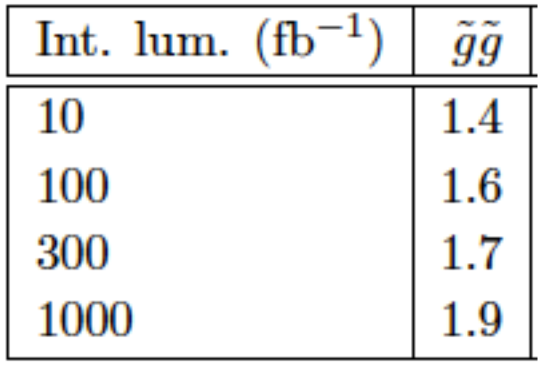
gluino pair cascade decay signatures





Particle	dom. mode	BF
$\tilde{g}$	$\tilde{t}_1 t$	$\sim 100\%$
$\tilde{t}_1$	$b\widetilde{W}_1$	$\sim 50\%$
$\widetilde{Z}_2$	$\widetilde{Z}_1 f\bar{f}$	$\sim 100\%$
$\widetilde{Z}_3$	$\widetilde{W}_1^{\pm} W^{\mp}$	$\sim 50\%$
$\widetilde{Z}_4$	$\widetilde{W}_1^{\pm} W^{\mp}$	$\sim 50\%$
$\widetilde{W}_1$	$\widetilde{Z}_1 f\bar{f}'$	$\sim 100\%$
$\widetilde{W}_2$	$\widetilde{Z}_i W$	$\sim 50\%$

Table 1: Dominant branching fractions of various sparticles along the RNS model line for $m_{1/2} =$ 1 TeV.



Int. lum. (fb^{-1})	$\tilde{g}\tilde{g}$
10	1.4
100	1.6
300	1.7
1000	1.9

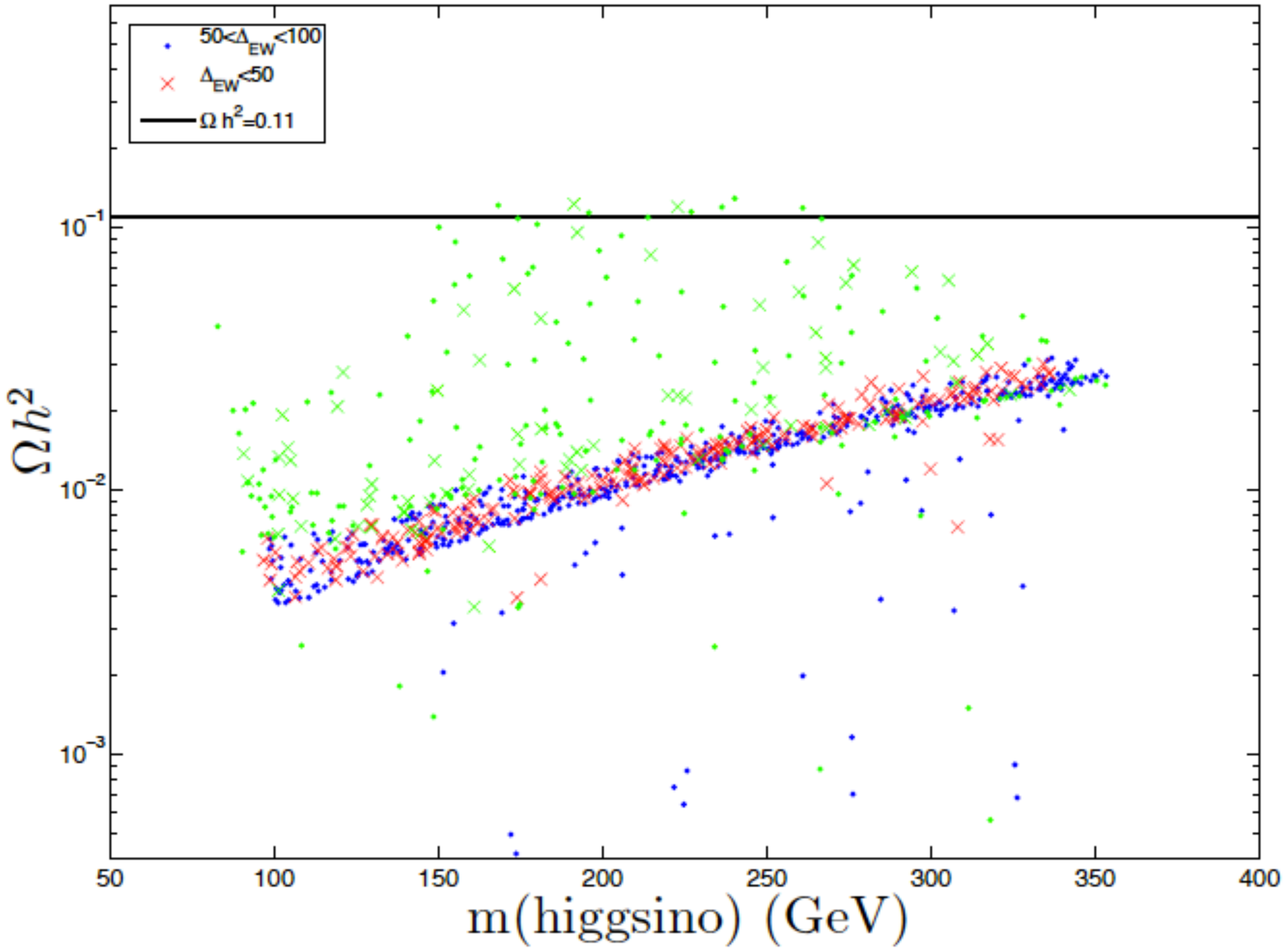
LHC14 reach
in m(gluino) (TeV)

since m(gluino) extends to ~5 TeV,
LHC14 can see about half the low EWFT
parameter space in these modes

dark matter in natural SUSY

- (see talk by KJ Bae)
- thermal WIMP (higgsino) abundance low by 10-15
- solve ``strong fine-tuning” via axion
- tame SUSY mu problem via Kim-Nilles/DFSZ
- get 90-95% axion CDM plus 5-10% higgsinos over bulk of parameter space
- reduced abundance of higgsinos still seeable at ton-scale WIMP detectors
- may see axion as well, e.g. ADMX

Mainly higgsino-like WIMPs thermally underproduce DM



green: excluded;
red/blue:allowed

Factor of 10-15 too low

But so far we have addressed only Part 1 of fine-tuning problem:

In QCD sector, the term $\frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu}\tilde{F}_A^{\mu\nu}$ must occur

But neutron EDM says it is not there: strong CP problem

(frequently ignored by SUSY types)

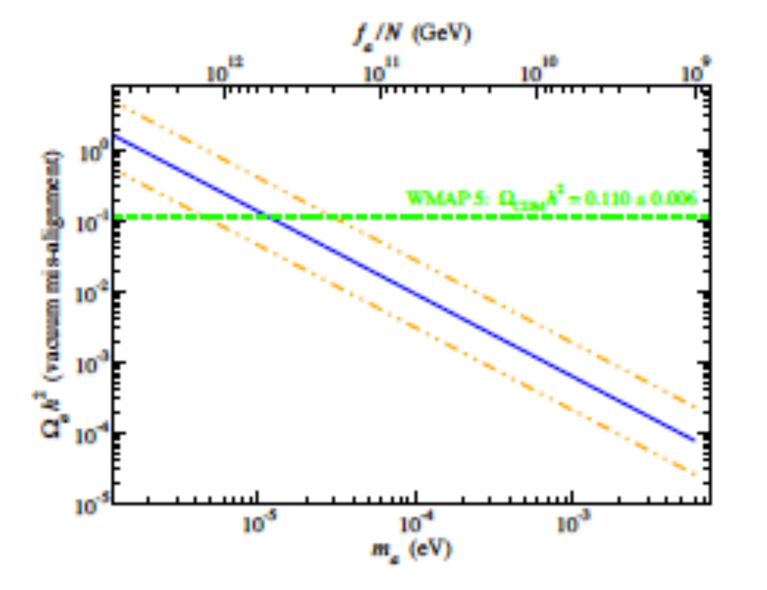
Best solution after 35 years: PQWW invisible axion

In SUSY, axion accompanied by axino and saxion

Changes DM calculus:
expect mixed WIMP/axion DM (2 particles)

Axion cosmology

- ★ Axion field eq'n of motion: $\theta = a(x)/f_a$
  - – $\ddot{\theta} + 3H(T)\dot{\theta} + \frac{1}{f_a^2}\frac{\partial V(\theta)}{\partial \theta} = 0$
  - – $V(\theta) = m_a^2(T) f_a^2 (1 - \cos\theta)$
  - – Solution for T large, $m_a(T) \sim 0$:
    $\theta = const.$
  - – $m_a(T)$ turn-on ~ 1 GeV
- ★ a(x) oscillates,
  creates axions with $\vec{p} \sim 0$:
  production via vacuum mis-alignment
- ★ $\Omega_a h^2 \sim \frac{1}{2}\left[\frac{6\times 10^{-6} eV}{m_a}\right]^{7/6} \theta_i^2 h^2$
- ★ astro bound: stellar cooling $\Rightarrow f_a \gtrsim 10^9 GeV$



Axino/saxion decays

Decays very model-dependent;
also depend on KSVZ or DFSZ model

axino-> particle+sparticle: augment LSP abundance but
also provide late-time entropy injection

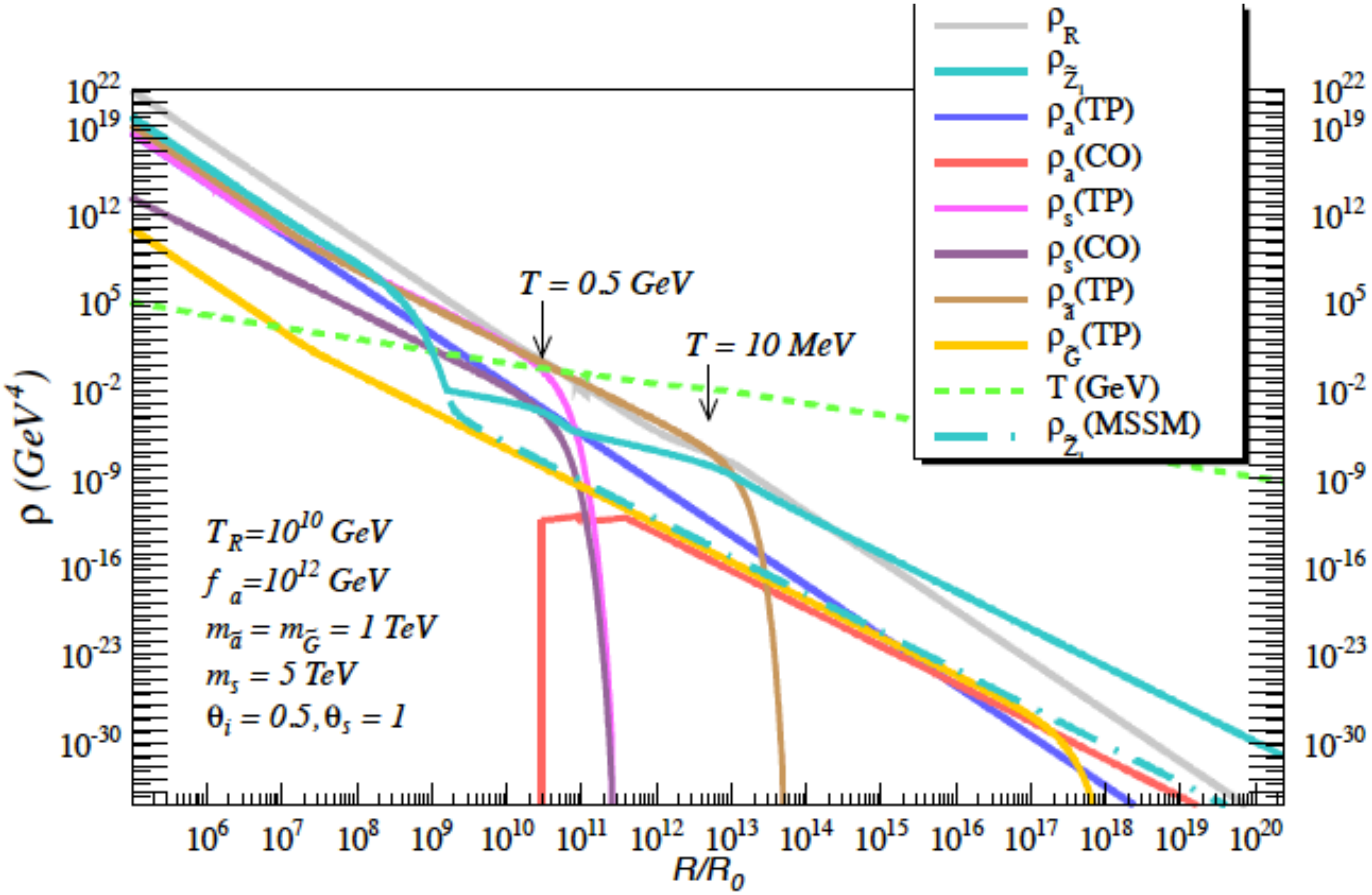
saxion-> gg, hh, etc SM particles (entropy dilution)

saxion-> glno+gno, hgno+hgno, etc (SUSY particles, augment)

saxion->aa, dark radiation, $\Delta N_{eff}\ bounds$

Bae, HB, Lessa, JCAP1304 (2013) 041

Coupled Boltzmann KSVZ $\xi = 0$



HB, Lessa, Sreethawong

mu problem: why is mu~m(weak) and not Mp?

$$W_{MSSM} \ni \mu H_u H_d$$

Kim-Nilles solution to SUSY mu problem

SUSY DFSZ model

(Dine-Fischler-Srednicki-Zhitnitsky, 1983)

Higgs fields Hu and Hd carry PQ charge: mu term forbidden

Field S carries PQ charge and contains axion-axino-saxion

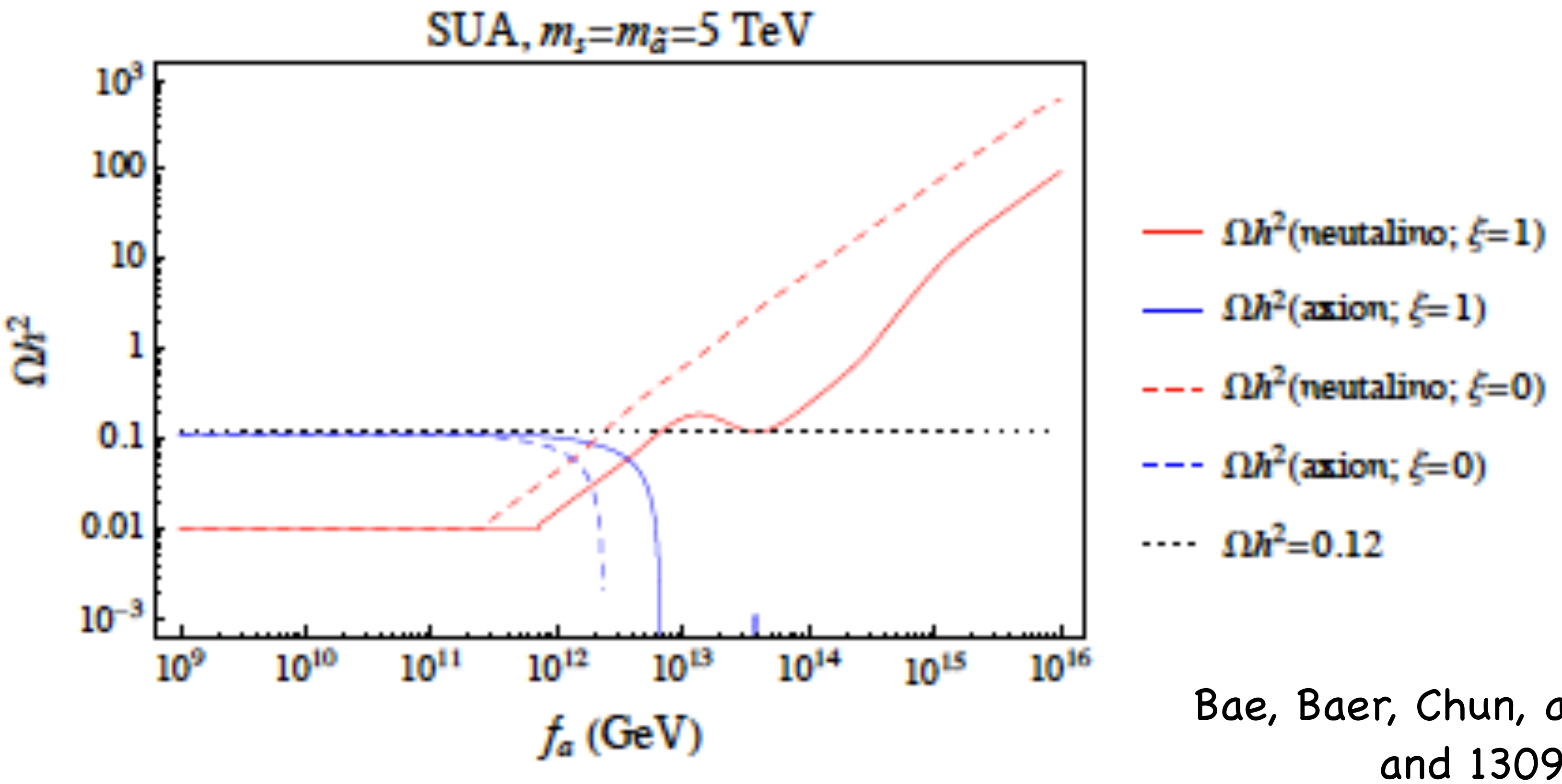
$$W_{\rm DFSZ} \ni \lambda \frac{S^2}{M_P} H_u H_d$$

If S develops vev ~ f_a, then weak scale mu generated!

$$\mu \sim \lambda f_a^2/M_P \sim \lambda m_{3/2}$$

Tree level axion superfield couplings to higgs/higgsinos: axino/saxion decay before WIMP freezeout for f_a<10^12 GeV

Then usual WIMP abundance obtains but supplemented by axion CDM!



Bae, Baer, Chun, arXiv:1309.0519
and 1309.5365

Get 90-95% axion CDM plus 5-10% higgsino-like WIMPs

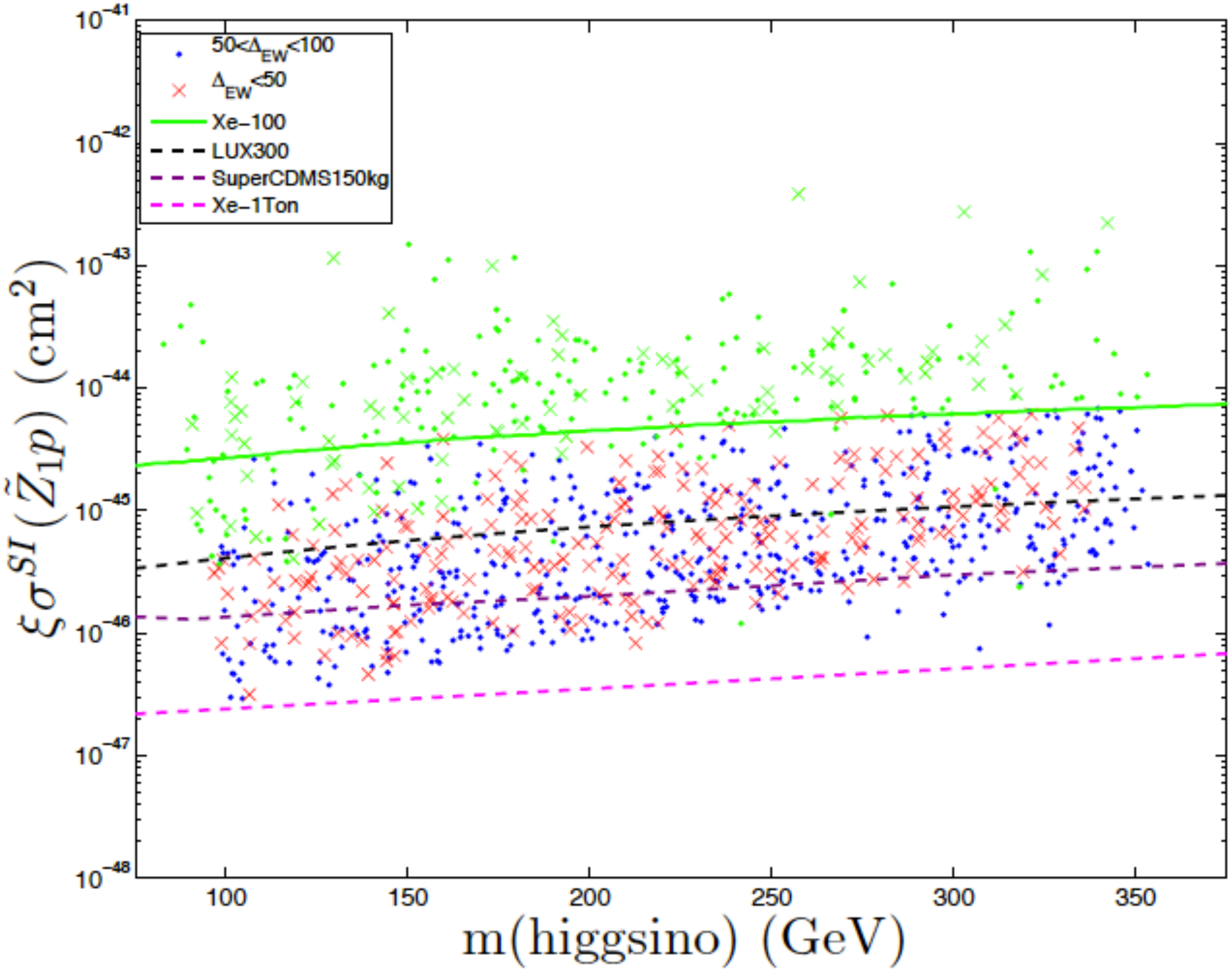
Detection of mixed a/Z1 DM in natural SUSY with DFSZ axion

detection of axion as usual: range of PQ scale f_a:
10^9-10^12 favored in SUSY DFSZ

detection of WIMPs same as usual but theory projections should be scaled to account for WIMPs making only a fraction of total DM density

use Bottino, Fornengo et al. $\xi \equiv \Omega_\chi h^2/0.12$ rescaling factor

Direct higgsino detection rescaled for minimal local abundance



HB, Barger, Mickelson
arXiv:1303.3816

$$\mathcal{L} \ni -X_{11}^h \overline{\widetilde{Z}}_1 \widetilde{Z}_1 h$$

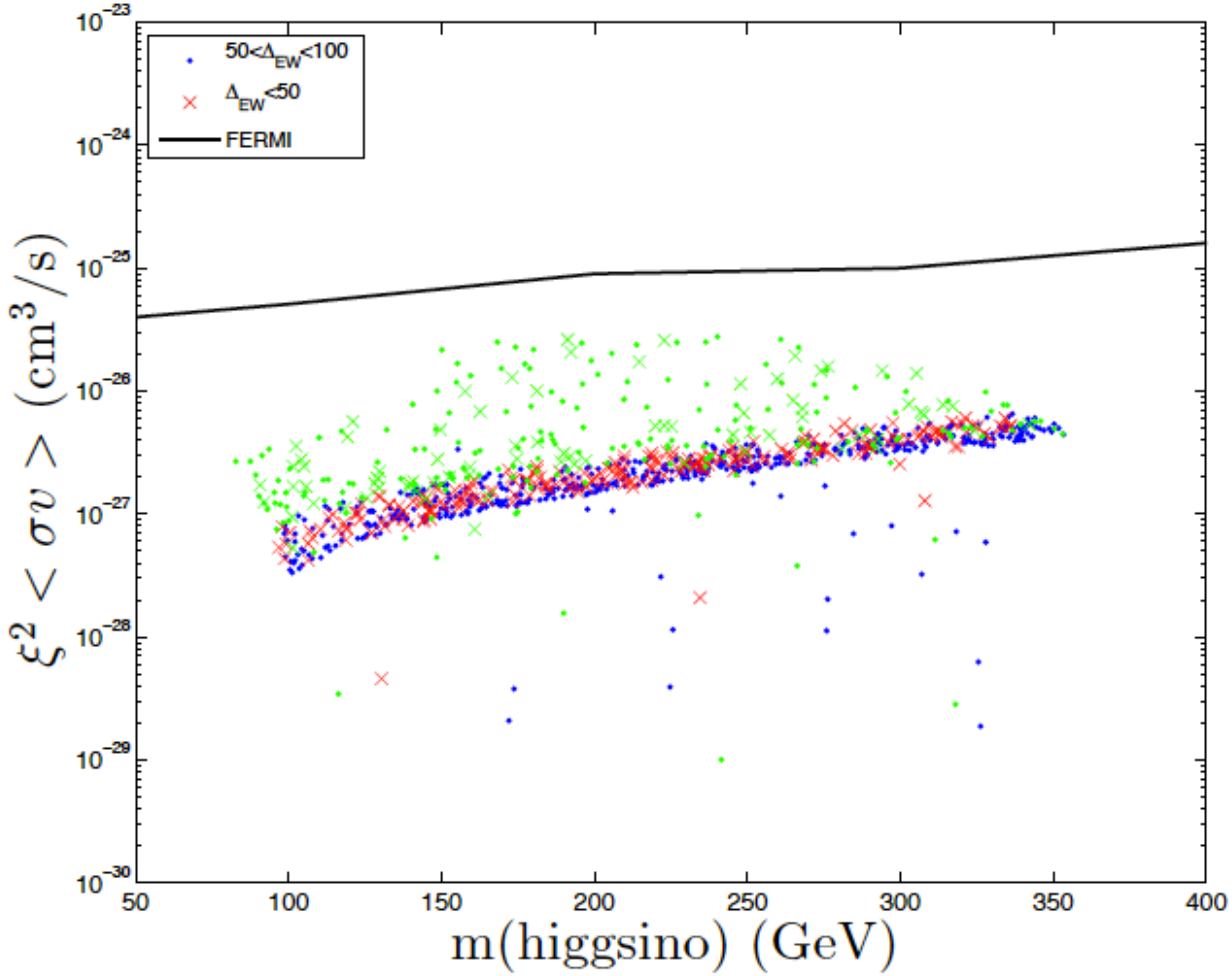
$$X_{11}^h = -\frac{1}{2}\left(v_2^{(1)}\sin\alpha - v_1^{(1)}\cos\alpha\right)\left(g v_3^{(1)} - g' v_4^{(1)}\right)$$

new LUX results

Deployment of Xe-1ton
coming soon!

Can test completely with ton scale detector or equivalent (subject to minor caveats)

Higgsino detection via halo annihilations:



green: excluded by Xe-100

annihilation rate is high but rescaling is squared

Gamma-ray sky signal is factor 10-20 below current limits