Precise Higgs Mass Measurement
using π^0 Reconstruction

Brian van Doren

Department of Physics and Astronomy
University of Kansas

vandoren@ku.edu

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Outline

1. Motivation
2. Mass Measurements
   - Kinematic Constraint using π^0 Mass
   - Probability Density Approach
3. Higgs Mass using Reconstructed π^0's
   - "Strawman" Reconstruction
   - Using π^0 Reconstruction
4. Systematic Uncertainty
   - In situ HCAL Calibration
   - Matching Photons with π^0's
5. Conclusion

Motivation

Why π^0's and Higgs?

- High precision measurements test internal consistency of known laws of nature
- Dominant Higgs production at $\sqrt{s} = 1$ TeV: $e^+e^- \rightarrow \nu_e\bar{\nu}_e H$
- Particle Flow provides information about individual particles (full four-vector)
- Combined with prior information about how particles decay, can reconstruct π^0's from photons in an event to reduce measurement uncertainty.

Higgs Production at 1 TeV

- Higgs production: $e^+e^- \to \nu_e\bar{\nu}_e H$

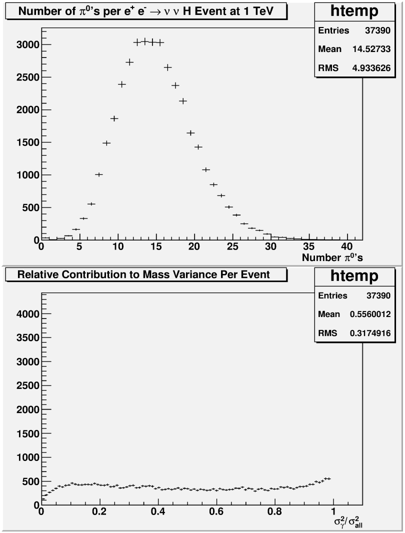
  Total cross section = 734 fb
  Assuming polarization -80% e^-, 20% e^+
  Will consider hadronic decays:
  $H \to q\bar{q}, H \to WW, H \to ZZ, H \to \gamma Z, H \to gg$
  Total luminosity of 1 ab^{-1} expect 396k events before cuts

- Cuts

  Semileptonic Heavy Flavor Decays $c \to sl\bar{\nu}$, $b \to cl\bar{\nu}$
  Exclude $H \to ZZ \to llll$ (four lepton)
  Total Higgs events approx 147k

Why Focus on π^0's in a Higgs Event?

- 95% of photons from π^0's
- 14.5 π^0's per event
- Approx 56% of energy *statistical* variance per event due to photon uncertainty. Large differences event-to-event.
- Using π^0 mass constraints can reduce variance of photon four-vectors



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Toy Monte-Carlo Environment

- Energy response

$$\sigma_{E-ECAL}/E = 16\%/\sqrt{E} \oplus 1\%$$
$$\sigma_{E-HCAL}/E = 45\%/\sqrt{E} \oplus 1.7\%$$

- Tracker response

$$\sigma_{1/p_{t-tracker}} = a \oplus b/(p_t \cdot \sin\theta)$$
$$a = 2 \times 10^{-5}, b = 1 \times 10^{-3}$$

- Angular response

$$\sigma_\theta = 0.5 mrad$$
$$\sigma_\phi = \sigma_\theta/\sin\theta$$

Kinematic Constraint using π^0 Mass

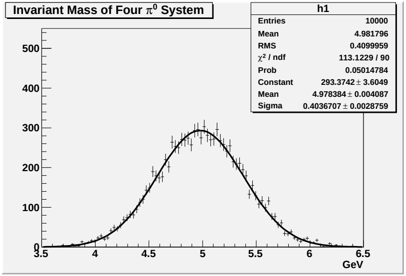
Traditional Method

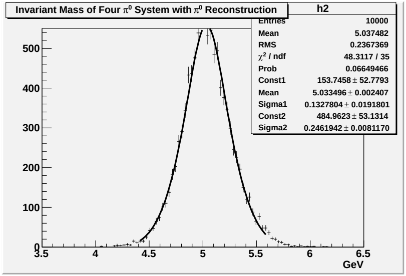
- $\sim$ 95% of photons are product of $\pi^0 \to \gamma\gamma$ process. Use this information to reconstruct the π^0's and improve uncertainty.
- Perform mass constrained fit by adjusting measured photon four-vectors to minimize χ^2 subject to the constraint C

$$\begin{aligned}
\chi^2 &= \left(\frac{E_1^f - E_1^m}{\sigma_{E_1}}\right)^2 + \left(\frac{E_2^f - E_2^m}{\sigma_{E_2}}\right)^2 + \left(\frac{\theta_1^f - \theta_1^m}{\sigma_{\theta_1}}\right)^2 \\
&\quad + \left(\frac{\theta_2^f - \theta_2^m}{\sigma_{\theta_2}}\right)^2 + \left(\frac{\phi_1^f - \phi_1^m}{\sigma_{\phi_1}}\right)^2 + \left(\frac{\phi_2^f - \phi_2^m}{\sigma_{\phi_2}}\right)^2 \\
C &= (\mathbf{p}_{\gamma_1} + \mathbf{p}_{\gamma_2})^2 - m_{\pi^0}^2 = 0
\end{aligned}$$

How Does it Perform?

Consider an example: Four 1.25 GeV π^0's with invariant mass 5 GeV and zero total momentum.





- Naive analysis using double gaussians shows a decrease in mass uncertainty from 4.09 MeV to 2.41 MeV
- Note that both are biased (π^0 fitting especially so)

Improvement: Event-by-Event Weighting

A detector like ILD can estimate the measurement uncertainty for each identified particle. Propagate this uncertainty to the mass calculation

$$M^2 = \sum_{i=1}^{n} m_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_i E_j - \cos\theta_{ij}\sqrt{E_i^2 - m_i^2}\sqrt{E_j^2 - m_j^2}$$

$$\sigma_M = \frac{1}{2M}\sqrt{\sum_i^n \left(\frac{d(M^2)}{dE_i}\sigma_{E_i}\right)^2}$$

Then perform inverse variance weighted mean of the mass measurements from each event

$$M_{avg} = \frac{\sum M_i/\sigma_{M_i}^2}{\sum 1/\sigma_{M_i}^2} \qquad \sigma_{M_{avg}} = \sum \frac{1}{1/\sigma_{M_i}^2}$$

Improvement: Event-by-Event Weighting

Performing weighted mean allows improved precision *and* cross-checks that the results are acceptable

Standard Reconstruction

- Nearly same σ_m: $4.09 \to 4.12$ MeV
- $\chi^2/df = 9915/9999$ (-0.61σ)

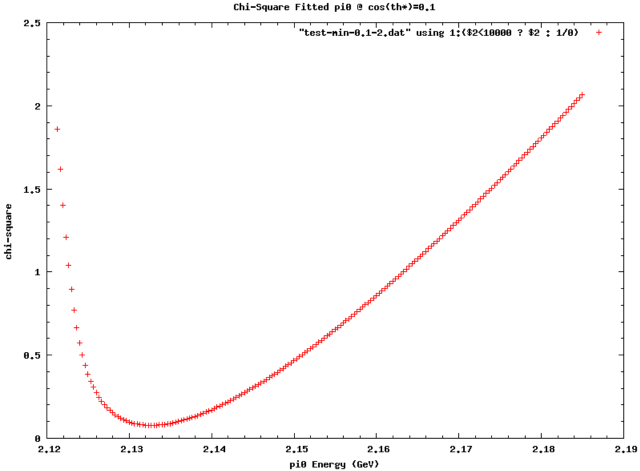
Using π^0 Reconstruction

- Improved σ_m: $2.41 \to 1.91$ MeV
- $\chi^2/df = 10353/9999$ $(+2.49\sigma)$

π^0 reconstruction benefits from weighted mean due to event-by-event variation of uncertainty. Note that χ^2/df for π^0 fitting is suspect. Why?

Asymmetric χ^2 and Errors

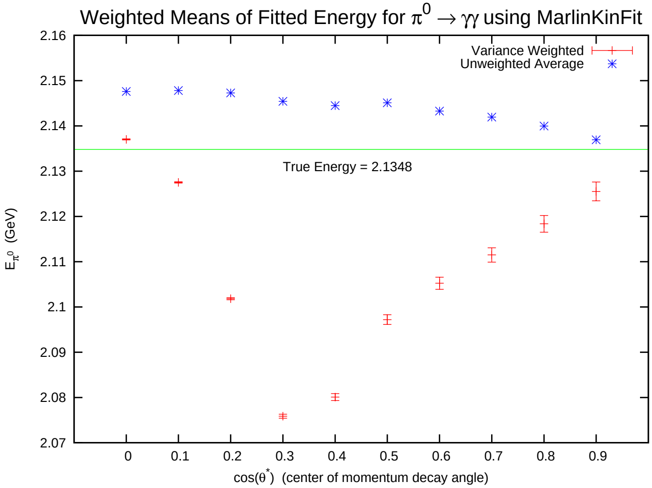
Minimization packages assume a χ^2 that is symmetric and report errors assuming a parabolic minimum. This is clearly not the general case for π^0's.



χ^2 Minimum is Biased but not Consistent

Unweighted average π^0 energy is biased high

Inverse variance weighted π^0 energy is biased low



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Reduce Number of Variables

Simplify the π^0 mass constraint and χ^2 expressions by introducing new variable:

$$\begin{aligned} z &= 2(1 - \cos\Psi_{12}) \\ &= 2(1 - \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\Delta\phi) \end{aligned}$$

Where Ψ_{12} is the opening angle of the photons. The measured information now is E_1, E_2, and z. The constraint is expressed as

$$m^2_{\pi^0} = E_1 E_2 z$$

Event Mass as Probability Distribution

- To combat asymmetric errors and inconsistent bias, express the per event mass measurement as a probability distribution

$$p(m) = \int \delta[m - M(\mathbf{x})] f(\mathbf{x}) d\mathbf{x}$$

- where

  $p(m)$ is the probability density of a particular mass value

  $\mathbf{x}$ is the vector of all particle four-vector tuples

  $f(\mathbf{x})$ is the joint probability distribution of all four-vector tuples in the event

  $M(\mathbf{x})$ is the calculated mass due to $\mathbf{x}$

- Use these per event pdf's of $p(m)$ to calculate Higgs mass and uncertainty.

Bayesian Model for Kinematic Constraints

For a single π^0 characterized by E_1, E_2, z drawn from normal distributions:

$$E_1 \sim N(\mu_1, \sigma_1), \quad E_2 \sim N(\mu_2, \sigma_2), \quad z \sim N(\mu_z, \sigma_z)$$

What is the probability that the model parameters produce this measurement (what is posterior distribution of the model given the data)?

$$P(\mu_1, \mu_2, \mu_z | E_1, E_2, z) = \frac{P(E_1, E_2, z | \mu_1, \mu_2, \mu_z) P(\mu_1, \mu_2, \mu_z)}{P(E_1, E_2, z)}$$

where the probability of the measurement given the model is

$$P(E_1, E_2, z | \mu_1, \mu_2, \mu_z) = N(E_1 - \mu_1, \sigma_1) N(E_2 - \mu_2, \sigma_2) N(z - \mu_z, \sigma_z)$$

and the prior probability of the model is the mass contraint

$$P(\mu_1, \mu_2, \mu_z) = \delta(\mu_1 \mu_2 \mu_z - m_{\pi^0}^2)$$

Combine π^0 PDF with other particles

- Contributions to event $p(m)$ from each particle are independent. Can separate π^0 from non-π^0 joint pdf's

$$f(\mathbf{x}) = f_{\pi^0}(\mathbf{x}_{\pi^0}) f_{other}(\mathbf{x}_{other})$$

- Non-π^0 contributions are normally distributed and can be modeled as single gaussian.

$$p(m) = \int \delta(m' - M(\mathbf{x}_{\pi^0}, \mathbf{x}_{other^*})) f(\mathbf{x}_{\pi^0}) p_{other}(m - m') d\mathbf{x}_{\pi^0} dm'$$

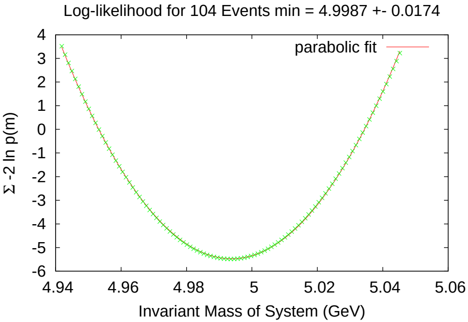
- $p_{\pi^0}(m)$ determined via Monte-Carlo integration. Final $p(m)$ is convolution of the two groups

$$p(m) = \int p_{\pi^0}(m') p_{other}(m - m') dm'$$

* $\mathbf{x}_{other}$ set to measured values

Minimum Log-Likelihood

- Each event is assigned a likelihood function $p_i(m)$
- Minimum log-likelihood is found for subsets of the events
- This allows cross-check for consistent bias



How Does This Perform?

Four π^0 event with invariant mass of 5 GeV (10,000 events):

Reconstruction	σ_M (MeV)	χ^2/df
Double Gaussian	4.12	
Dbl Gaussian w/ π^0 Reco	2.41	
Var Weighted w/ π^0 Reco	1.91	101353/9999
Log-Likelihood w/ π^0 Reco	1.75	199.4/196

Note that χ^2/df for π^0 log-likelihood is now acceptable

Outline

Higgs Mass

Hadronic decay of: $e^+e^- \rightarrow \nu_e\bar{\nu}_e H$ at 1 TeV

- $H \rightarrow q\bar{q}$
- $H \rightarrow WW$
- $H \rightarrow ZZ$
- $H \rightarrow \gamma Z$
- $H \rightarrow gg$

Cut heavy flavor semileptonic decays

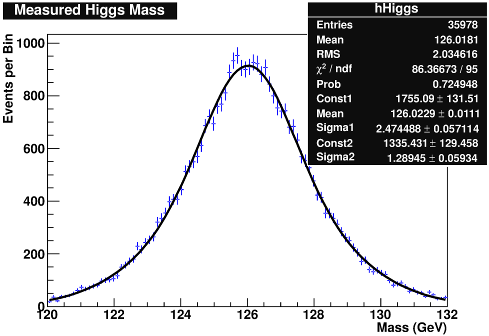


Figure: "Strawman" reconstruction $e^+e^- \rightarrow \nu_e\bar{\nu}_e H$, statistical uncertainty only, assuming perfect reconstruction. $\sigma_M = 11.1$ MeV per 250 fb^{-1}

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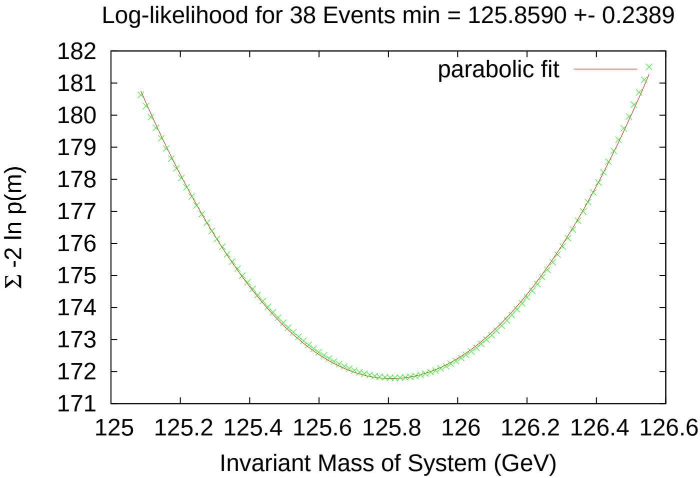
Effects of π^0 Reco on Higgs Mass

Minimum log-likelihood applied to $e^+e^- \rightarrow \nu_e\bar{\nu}_e H$:



Statistical Higgs Mass Uncertainty

Similarly to the four π^0 example, the best Higgs mass resolution (statistical) can be obtained with log-likelihood fit and the worst resolution is from a double gaussian

Reconstruction	σ_M (MeV)	per 1 ab^{-1}
Double Gaussian	11.1	5.5
Dbl Gaussian w/ π^0 Reco	9.84	4.92
Var Weighted	8.65	4.32
Var Weighted w/ π^0 Reco	7.55	3.78
Log-Likelihood w/ π^0 Reco	7.32	3.66

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In situ HCAL Calibration

- Statistical uncertainty is of similar order to the expected systematic uncertainty and can be improved with in situ HCAL calibration
- Example: Adjust the HCAL response linearly by minimizing:

$$\chi^2(M,g) = \sum_i \left(\frac{m_i(g) - M}{\sigma_{m_i}} \right)^2$$

g is the linear adjustment to the HCAL energy response (near 1.0)

$m_i(g)$ is the calculated mass adjusted by the HCAL response

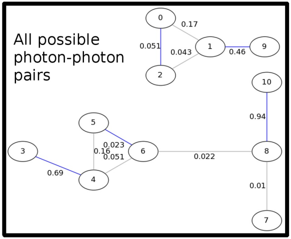
- Method applicable to Higgs, Z, and W (with proper width convolution)

Skew HCAL	$\Delta\sigma^2_M$ percent	Skew M_H GeV	g_{fit} ± 0.0014
1.01	$+0.6$	$+0.056$	1.0084
1.02	$+1.2$	$+0.106$	1.0185
1.04	$+2.4$	$+0.204$	1.0381

Results are per 250 fb^{-1} of the Higgs sample. Potential many-fold increase when considering all clean hadronic decaying events.

Outline

Matching Photons with π^0's



Evaluate all solutions and select the "best"

Final solution uses each photon at most once

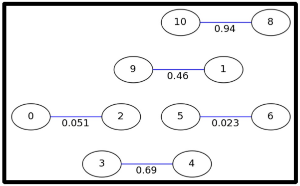


Figure: Example photon pairing solution. Segments between photons represent fit probability. Blue segments are correct.

Combinatorics of pairing photons with parent π^0's is tractable

Attempt fit on all pairs of photons per event

Select solution that minimizes sum of fit χ^2 and maximizes number of π^0's

In full simulation 92 GeV $Z^0 \rightarrow q\bar{q}$, 75 - 80% of π^0 energy is correctly recovered

Incorrect pairings increase bias but increase uncertainty only moderately ($\sim$ 6%)

Room for improvement of statistical treatment

Conclusion

- Measurements of Higgs (and other) mass in future e^+e^- colliders will be very precise
- Techniques to reduce statistical uncertainties

  Event-by-Event Likelihood Fitting
  Intermediate Particle (π^0) Reconstruction

Method	σ_{M_H} (MeV)
Double-Gaussian	± 5.5
Variance Weighted	± 4.32
χ^2 Minimum π^0 Reco	± 3.78
Log-likelihood π^0 Reco	± 3.66

- Acceptance, backgrounds, and reconstruction efficiency will degrade performance, but previous full simulation Z^0 study shows improvements can be achieved.
- Future Work

  Full simulation w/ log-likelihood
  Improvements to photon pairing algorithm