Modern vertex reconstruction methods

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Outline

- Vertex reconstruction – task definition
- Kalman filter – the classical method
- Adaptive vertex fitting
- Multi vertex fitting
- Gaussian sum vertex fitting
- Summary, outlook

Vertex reconstruction

Vertex reconstruction can be decomposed into:

- Vertex finding:

given a set of tracks, separate it into clusters of compatible tracks, i.e. vertex candidates

- Vertex fitting
  - **find the 3D point most compatible with a vertex candidate (i.e. a set of tracks).**
  - track smoothing: additional vertex information is used to re-estimate track momenta

This talk will focus on Vertex fitting.

Least squares methods

$$\hat{\beta}_{LS} = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^{n} r_i^2(\beta)$$

LinearVertexFitter

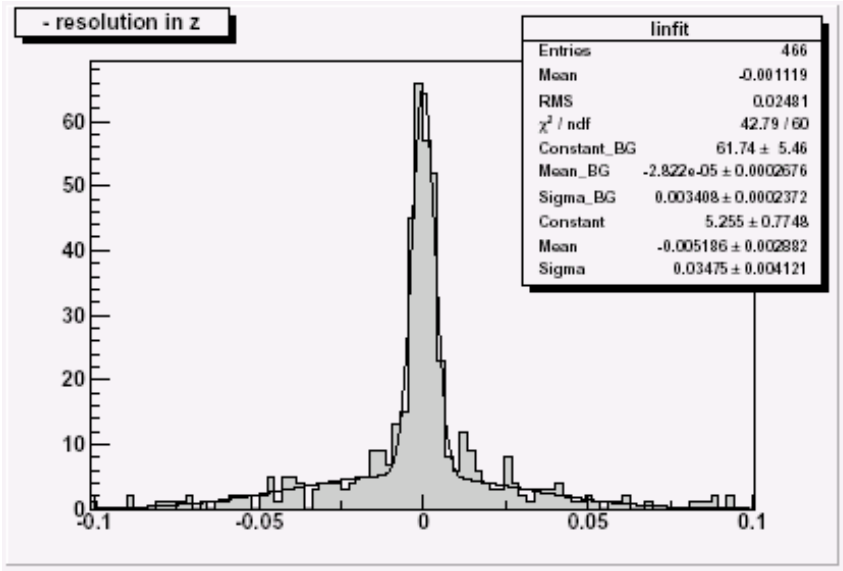
V.Karimäki, CMS Note 1997/051

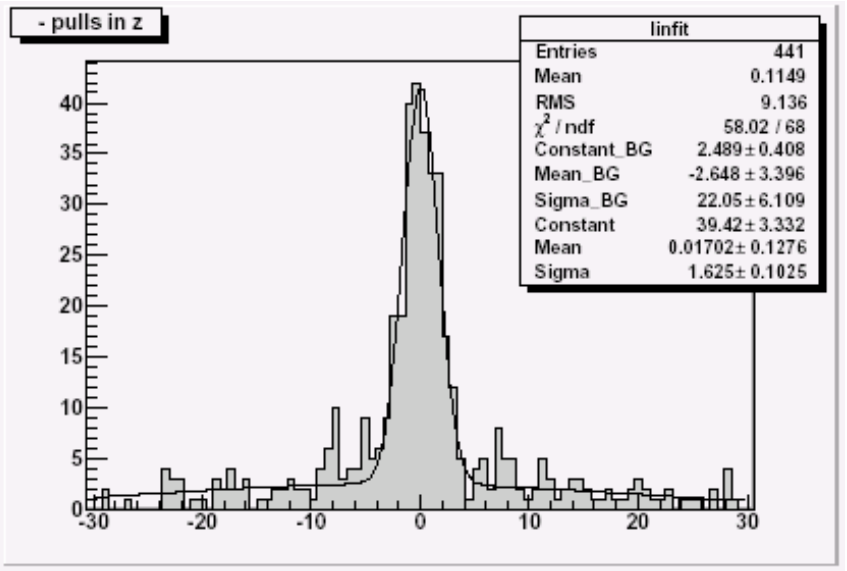
KalmanVertexFitter

R.Frühwirth et al., Computer Physics Comm. 96 (1991) 189-208

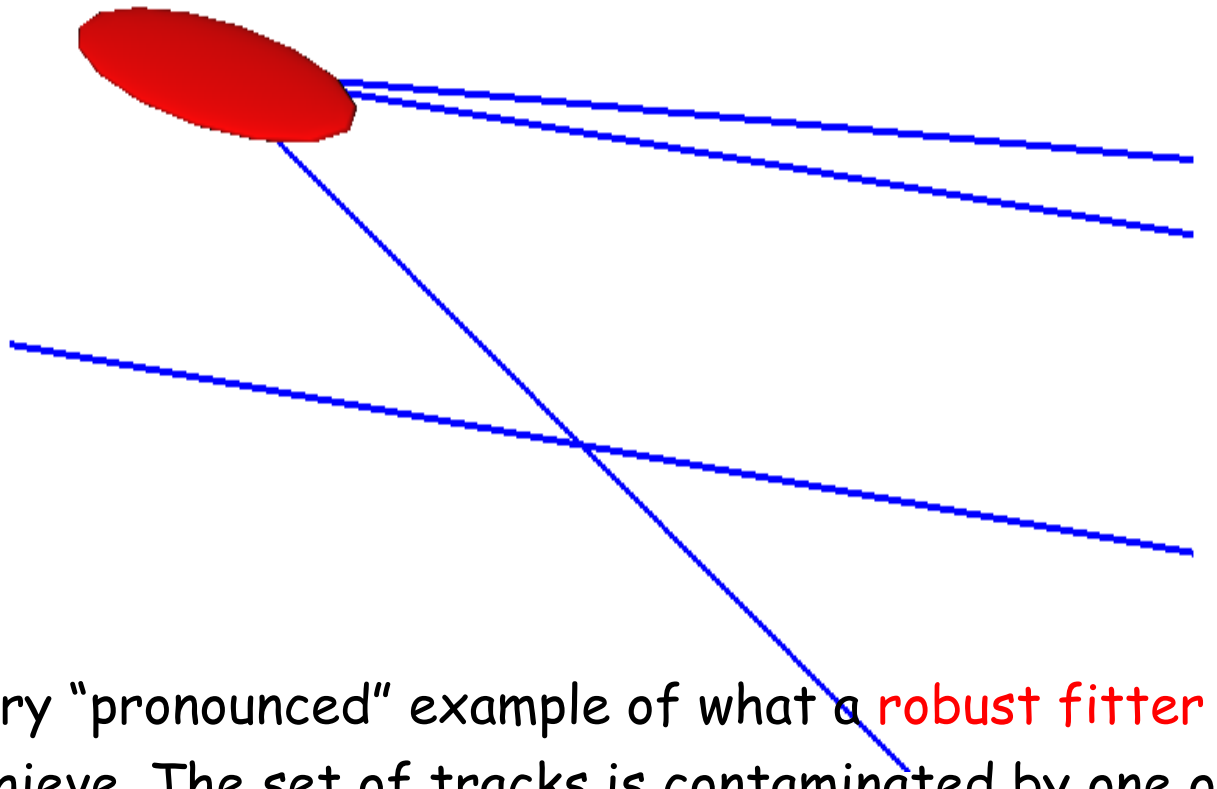
Classical methods are least squares methods. While being statistically efficient in the case of correctly described Gaussian errors and a correct linear track model, they are sensitive to outliers and mis-measured tracks.

cc, 100 GeV, $\eta \leq 1.4$





Robust fitting



Very "pronounced" example of what a robust fitter can achieve. The set of tracks is contaminated by one outlier. The vertex fitter (red ellipsoid) completely ignores the outlying track.

Trimming Vertex Fitter

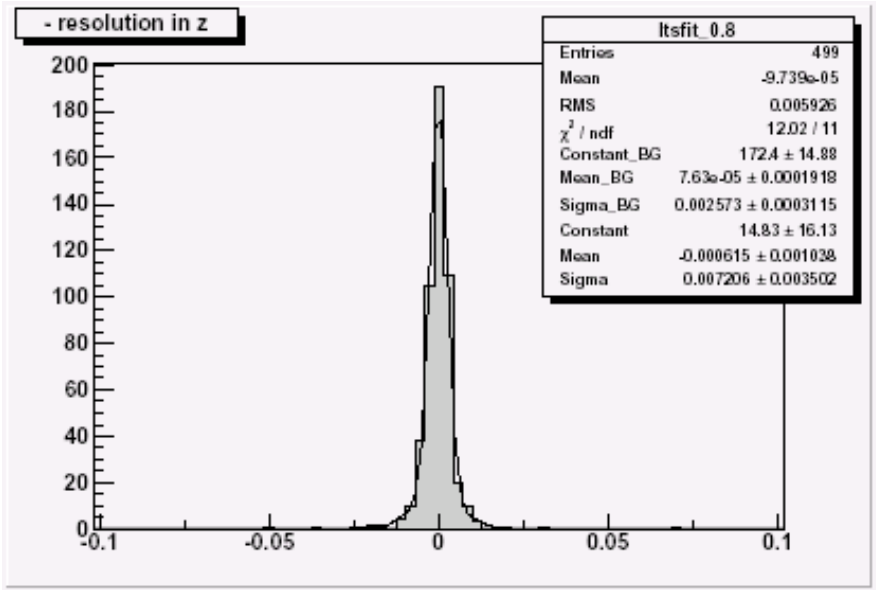
$$\hat{\beta}_{LTS} = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^{h \le n} r_i^2(\beta)$$

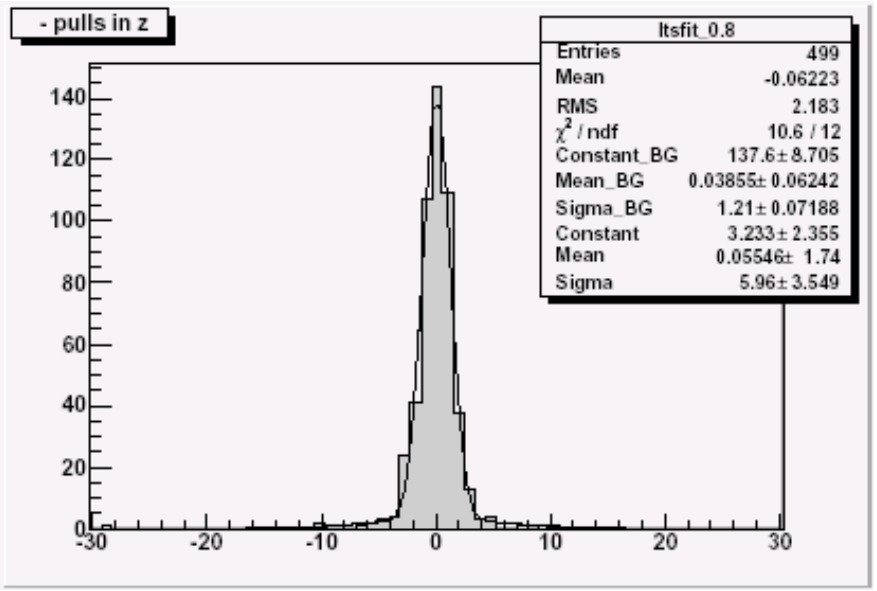
The trimming vertex fitter iteratively discards the least compatible tracks.

- algorithm: Fast-LTS (iterative) *P.J. Rousseuw, 1999*

- user can choose trimming fraction
  - e.g. 3-prong τ, 4 tracks in cone
    - choose *h/N* = 0.75

$c\bar{c}$, 100 GeV, $\eta \le 1.4$
LTSVertexFitter (80%)



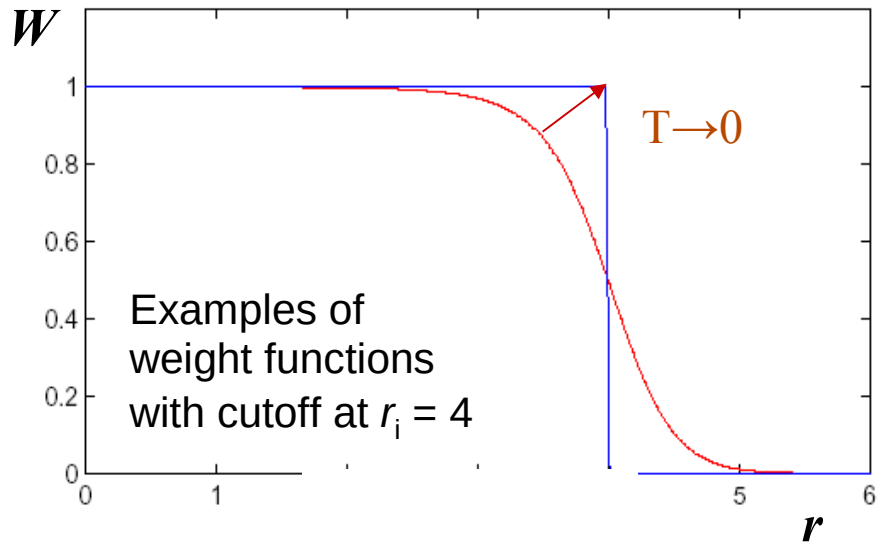


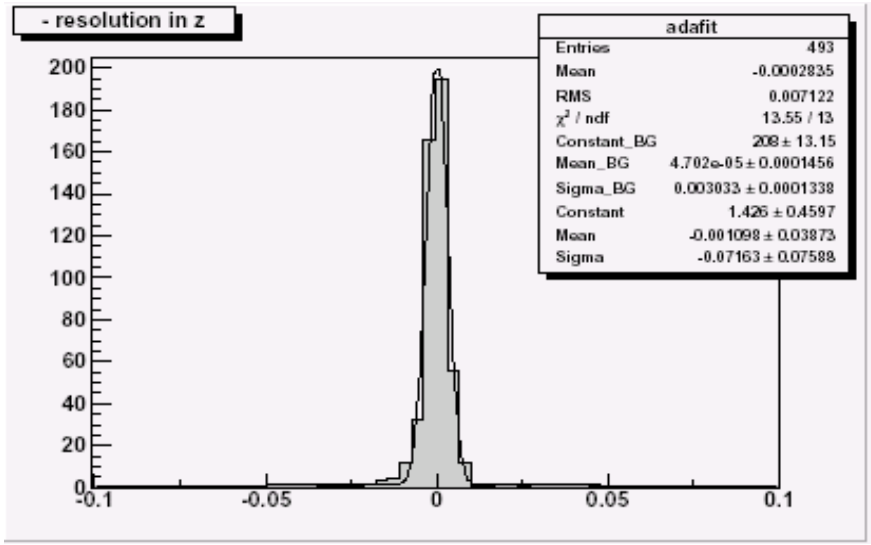
AdaptiveVertexFitter

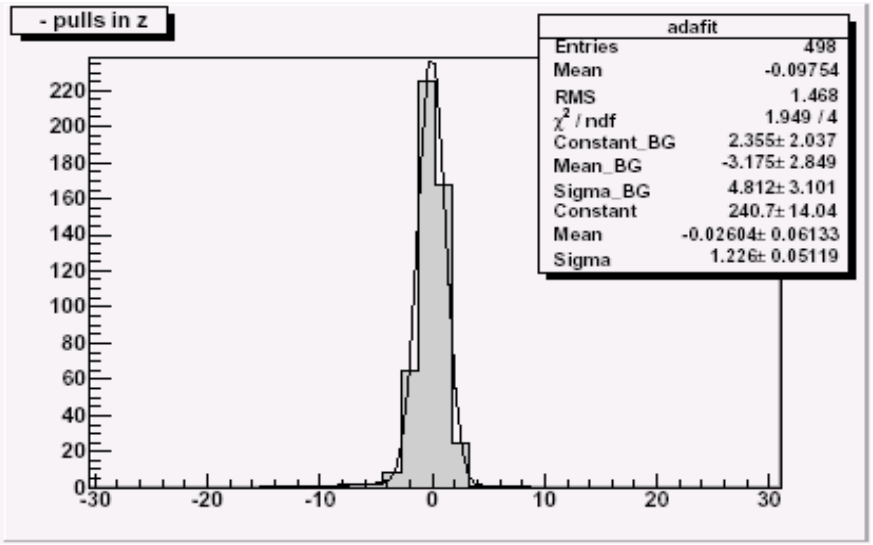
$$\hat{\beta}_{Adaptive} = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^{n} \left(w_i \cdot r_i^2(\beta) \right)$$

The adaptive method iteratively assigns weights to tracks before fitting the weighted tracks to a vertex.

An deterministic annealing schedule is employed in order to avoid falling into a local non-optimal minimum.



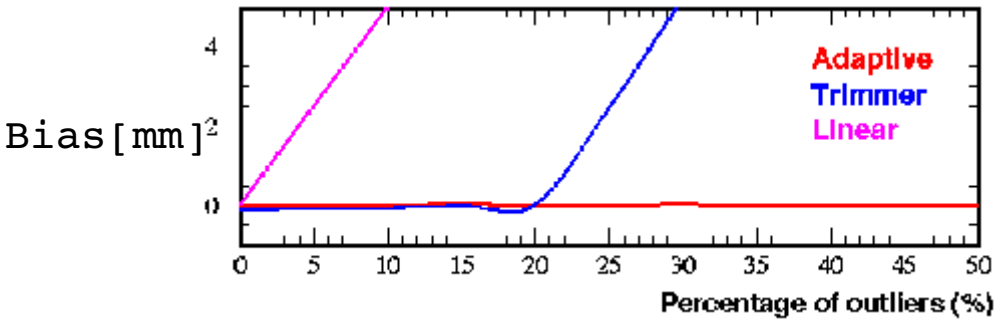




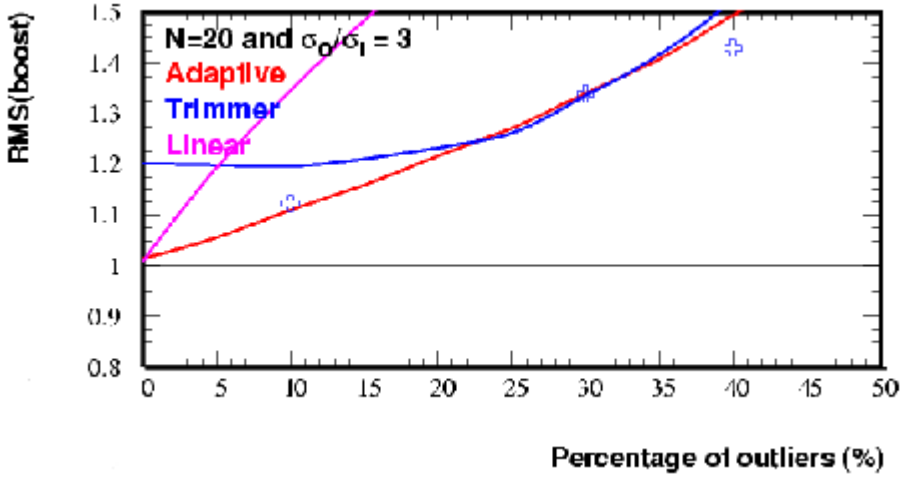
Statistical properties of the fitters

Simulation experiment that shows the statistical properties of the different fitters. A vertex was contaminated with more and more outlying tracks.

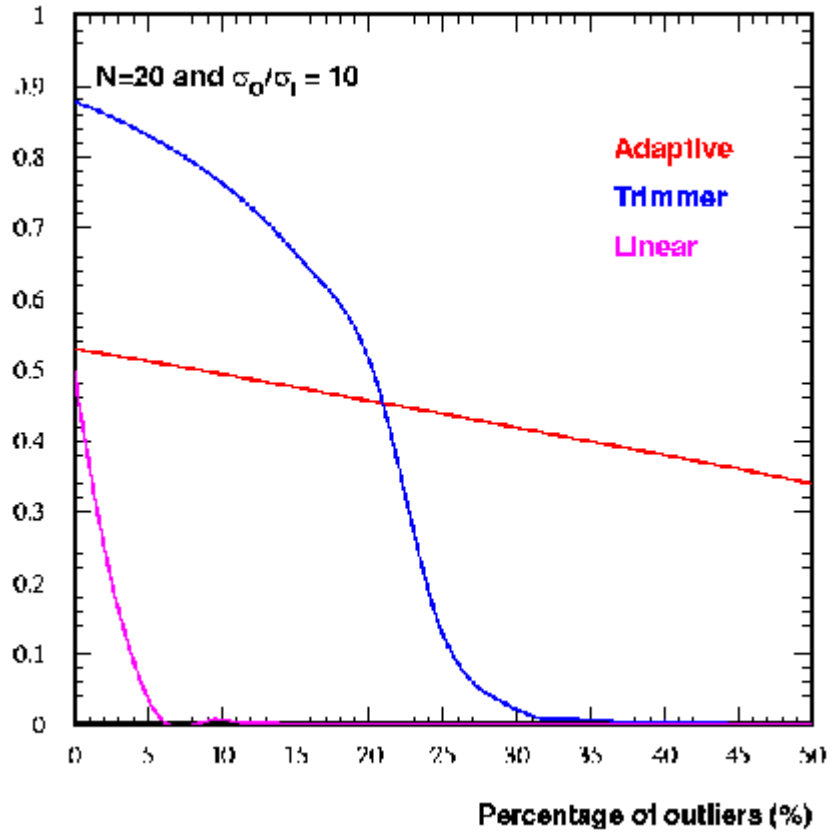
"Bias" of the estimate



RMS of the standardized residuals



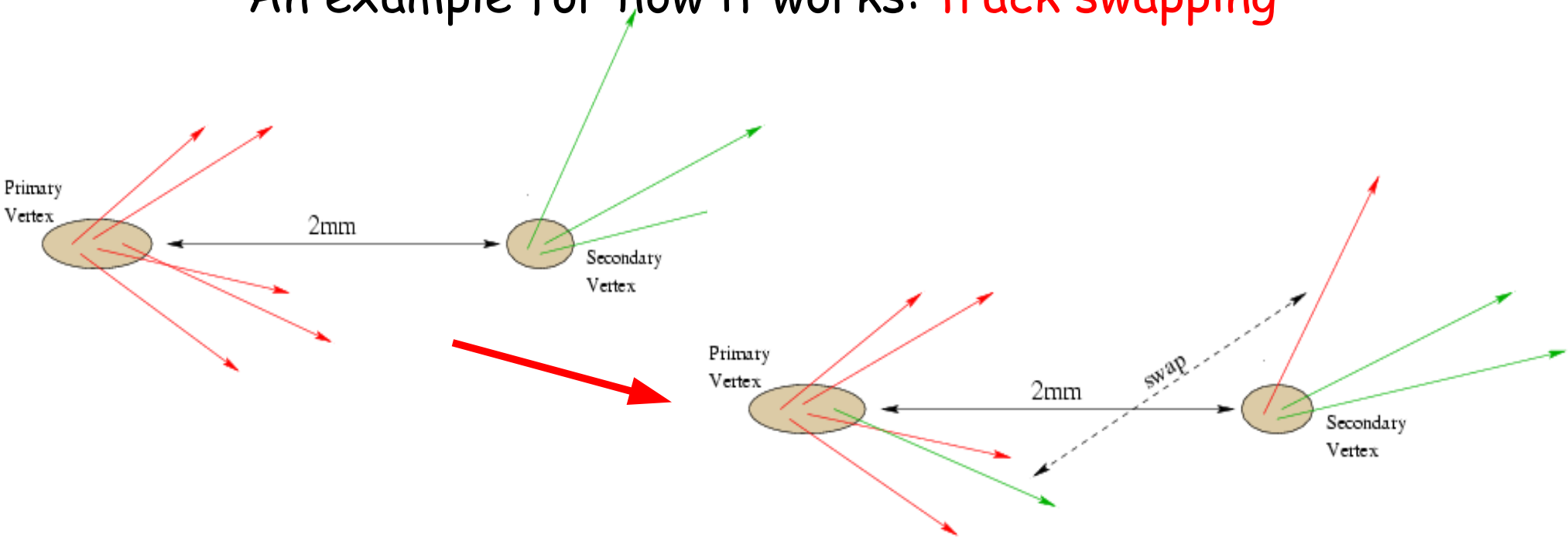
Average of the χ^2-probabilities



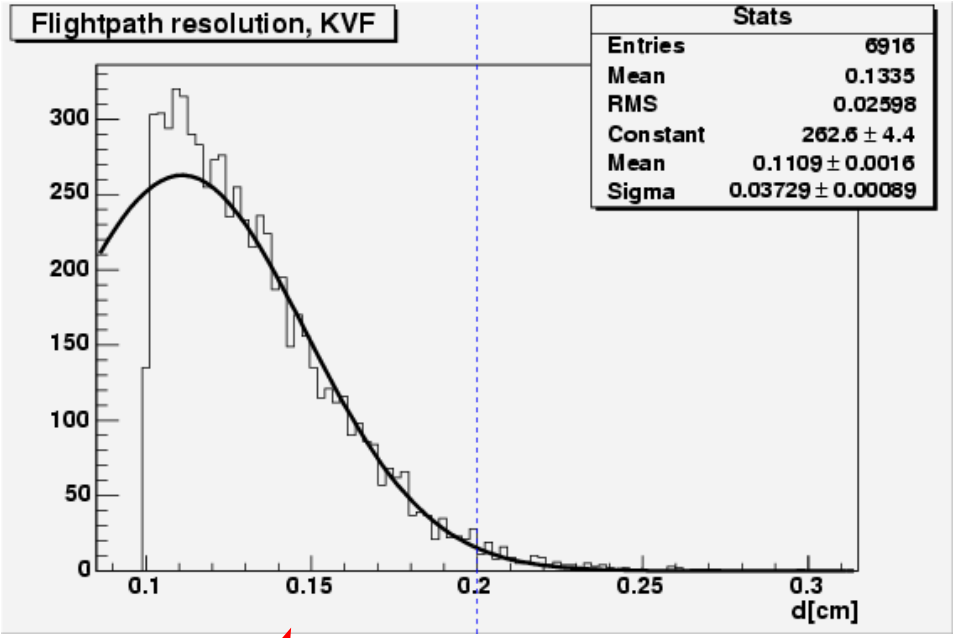
The Multi Vertex Fitter

The Multi Vertex Fitter is a natural generalisation of the adaptive method; several vertices are estimated at once. The method is competitive – the vertices compete for the tracks. A good seeding is essential for this method to function.

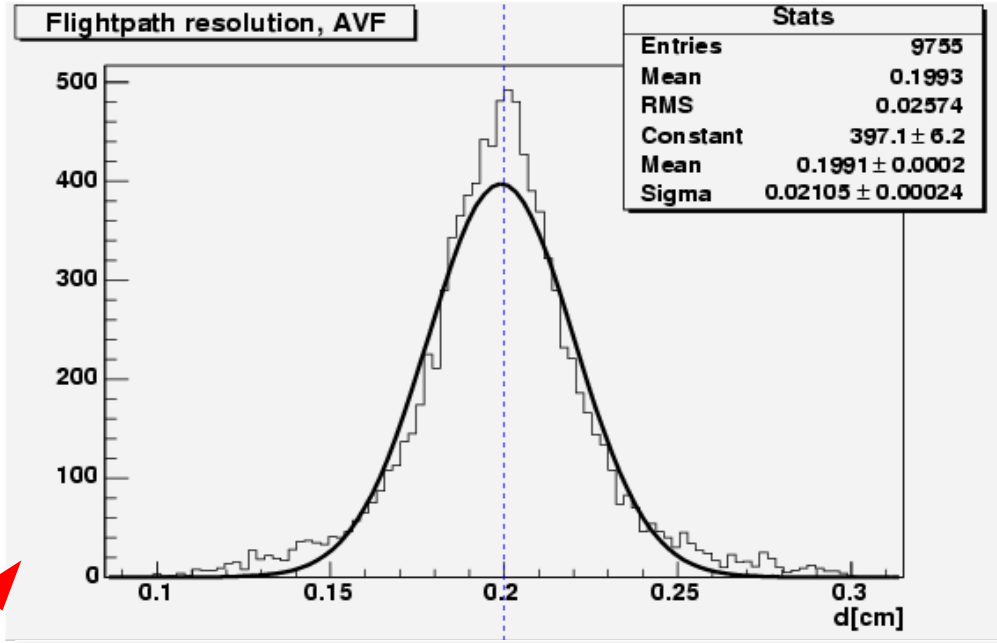
An example for how it works: track swapping



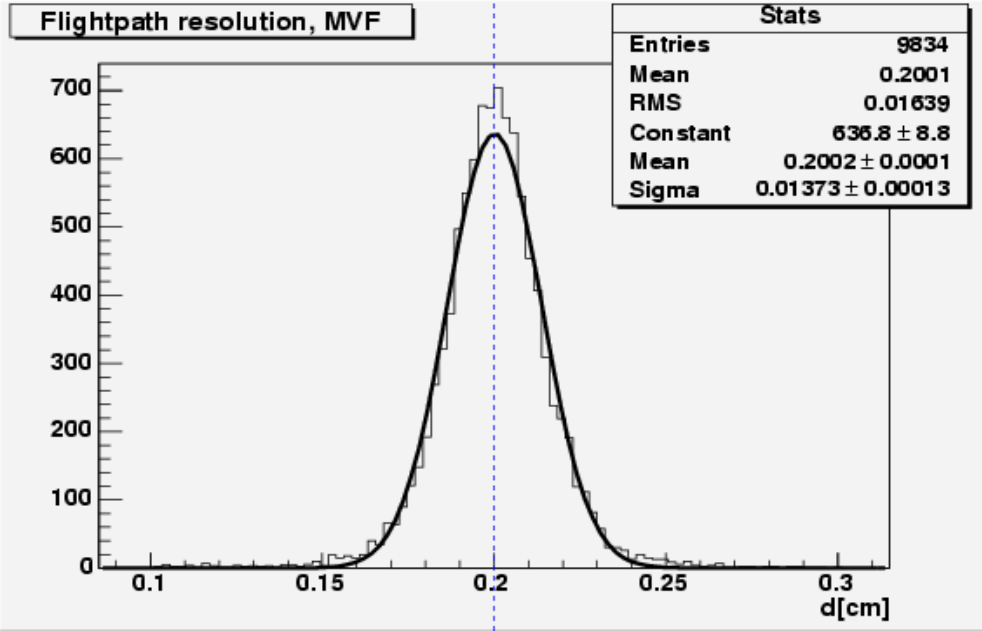
The Multi Vertex Fitter(2)



Linear Method, both vertices fitted separately



Adaptive Fitter, both vertices fitted separately



Multi Vertex Fitter, both vertices fitted at once

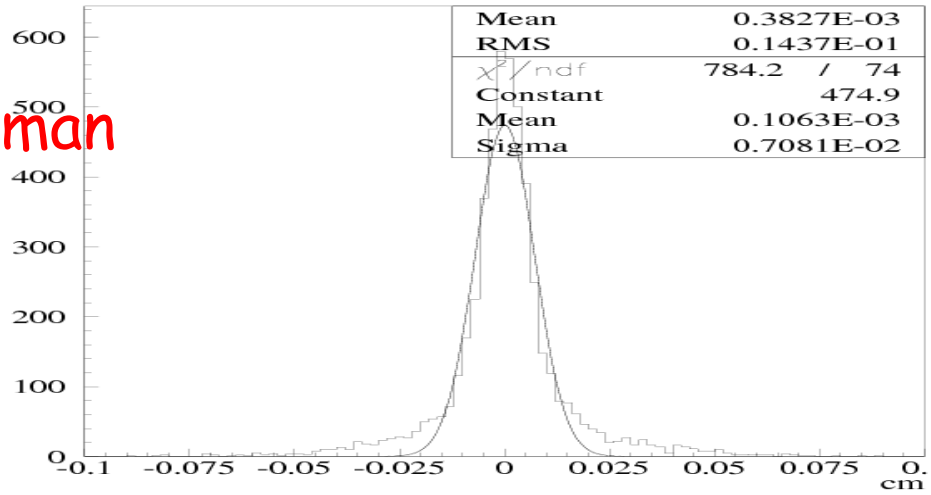
Gaussian Sum Vertex Fitter

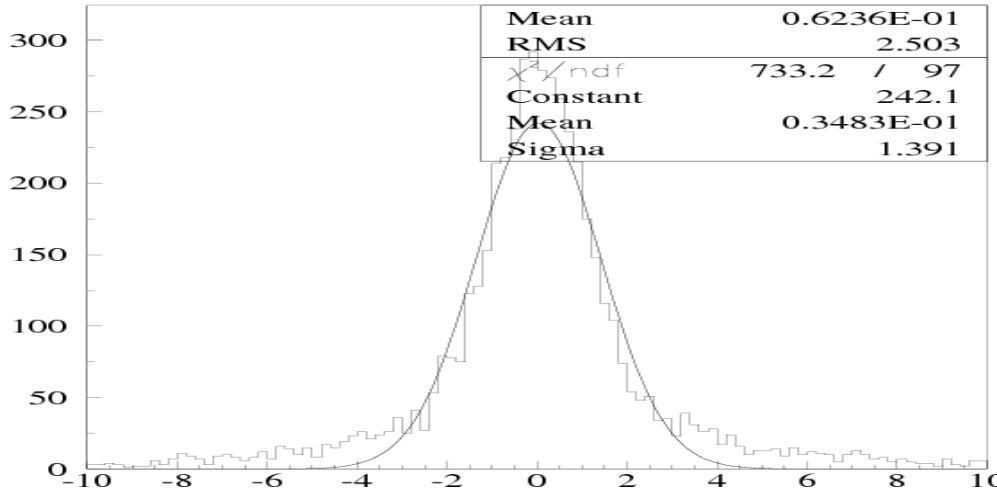
Tracks must be modeled by a mixture of Gaussians.
The Gaussian Sum Vertex Fitter is then a weighted sum of several Kalman filters run in parallel.
For every multi-Gaussian track, every component of the vertex candidate is updated with every component of the track. -> Combinatorial explosion! Trimming is necessary.

Gaussian Sum Vertex Fitter (2)

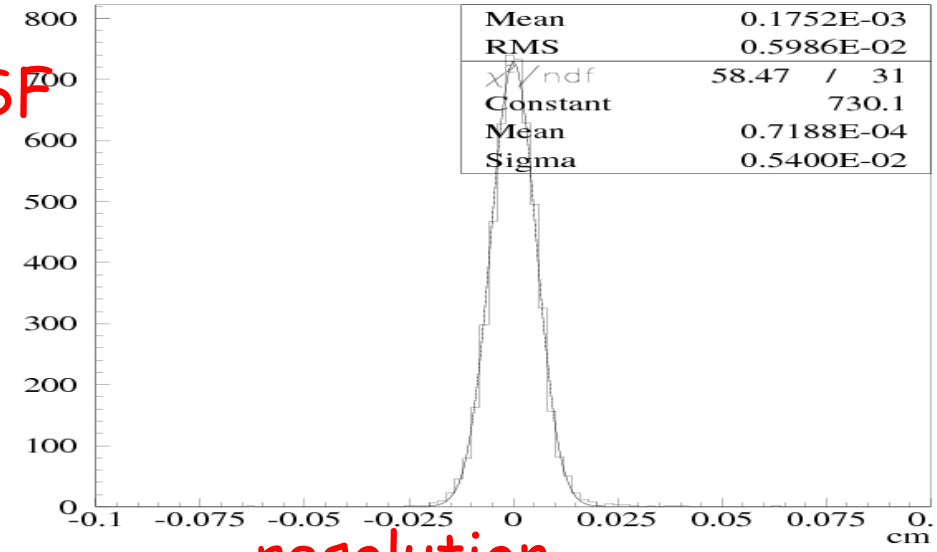
Task: Fit 4 tracks with non-Gaussian Tails

Kalman

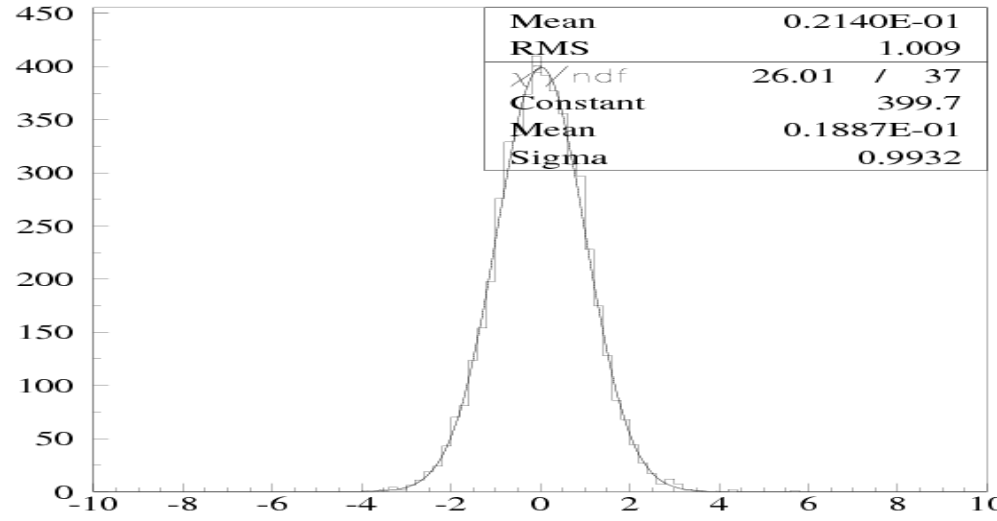




GSF



resolution



standardized residuals ("pulls")

Adaptive Gaussian Sum Vertex Fitter

The adaptive method and the Gaussian sum method can be mixed. Trivial, if the code is designed well.

Summary

The following powerful algorithms must be added to the list of classical vertex fitting methods:

- The adaptive method can deal with contamination and mis-measured tracks.
- The multi vertex fitter fits several vertices at once, resulting in a local optimum of track-to-vertex association.
- The Gaussian sum vertex fitter can deal with multi-Gaussian tracks – it can exploit information beyond the standard Gaussian distributed errors.