RG Analysis in NRQCD for Squark Pair Production

Pedro Ruiz-Femenía

in collaboration with André Hoang

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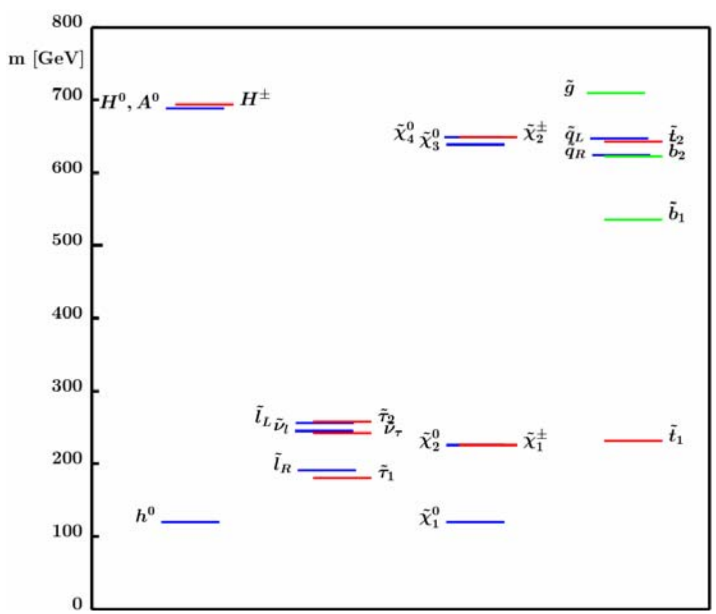
Max-Planck-Institute für Physik
(Werner-Heisenberg-Institut)

Outline

- Goal: NLL description of $e^+e^- \to \tilde{t}\bar{\tilde{t}}$ at threshold

SPS5



- mSugra scenario with relatively light scalar top quark

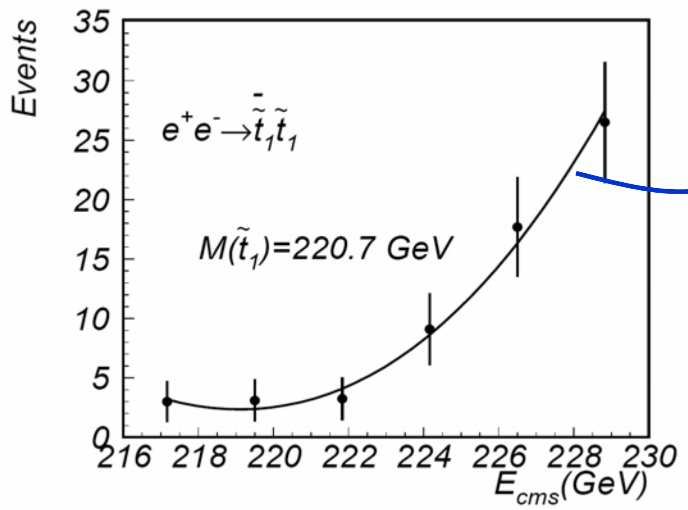
$$m_{\tilde{t}} \sim 250\,\mathrm{GeV} \qquad \Gamma_{\tilde{t}} \sim 40\,\mathrm{MeV}$$

- **SPS1a**

$$m_{\tilde{t}} \sim 400\,\mathrm{GeV} \qquad \Gamma_{\tilde{t}} \sim 2\,\mathrm{GeV}$$

- **Goal**: NLL description of $e^+e^- \to \tilde{t}\bar{\tilde{t}}$ at threshold

Threshold scan



talk by H.Nowak at ECFA Durham 2004

Fit to total cross section lineshape

$$\Delta m_{\tilde{t}} \sim 2\,\mathrm{GeV}$$

Improve theoretical uncertainty

→ Precise determinations of stop width, couplings, ...

N. Fabiano

Outline

- Goal: NLL description of $e^+e^- \to \tilde{t}\bar{\tilde{t}}$ at threshold

- Theoretical set-up

$$\sigma \propto \mathrm{Im}\left[\int d^4x\, e^{-i\hat{q}\cdot x}\left\langle 0\right| T\, J^\dagger_{\mathbf{p}}(0) J_{\mathbf{p}'}(x)\left|0\right\rangle\right] \propto \mathrm{Im}\left[\, c_0^2(\nu)\, G^1(0,0,v)\,\right]$$

This talk

QCD effects
- $\tilde{t}\bar{\tilde{t}}$ is a non-relativistic system
- EFT methods $\longrightarrow$ vNRQCD for scalar fields
- consistent matching, running...

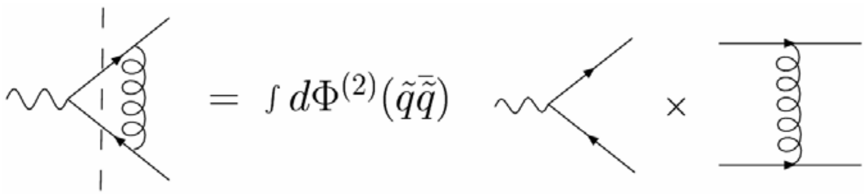
Electroweak (SUSY) effects
- finite width $\Gamma_{\tilde{t}}$
- phase space effects ...

Stop Physics at Threshold

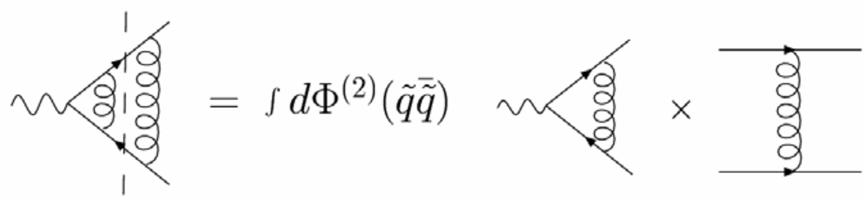
In the threshold region stop quarks move at small velocities

$$mv^2 \equiv \sqrt{s} - 2m \qquad \boxed{v \lesssim 0.2 \sim \alpha_s} \qquad E_{cm} \simeq 2m_{\tilde{t}} \pm 10\,\mathrm{GeV}$$

pQCD series has terms $\propto \left(\frac{\alpha_s}{v}\right)^n$



$$\alpha/v \;=\; v \;\times\; 1 \;\times\; \alpha/v^2$$



$$(\alpha/v)^2 = v \;\times\; \alpha/v \;\times\; \alpha/v^2$$

- ✓ count $\frac{\alpha_s}{v} \sim 1$ as LO
- ✓ perform expansion in v, α_s
- ✓ resummation of leading terms achieved by means of a Schrödinger field theory

⟹ **NRQCD** Caswell, Lepage; Bodwin, Braaten

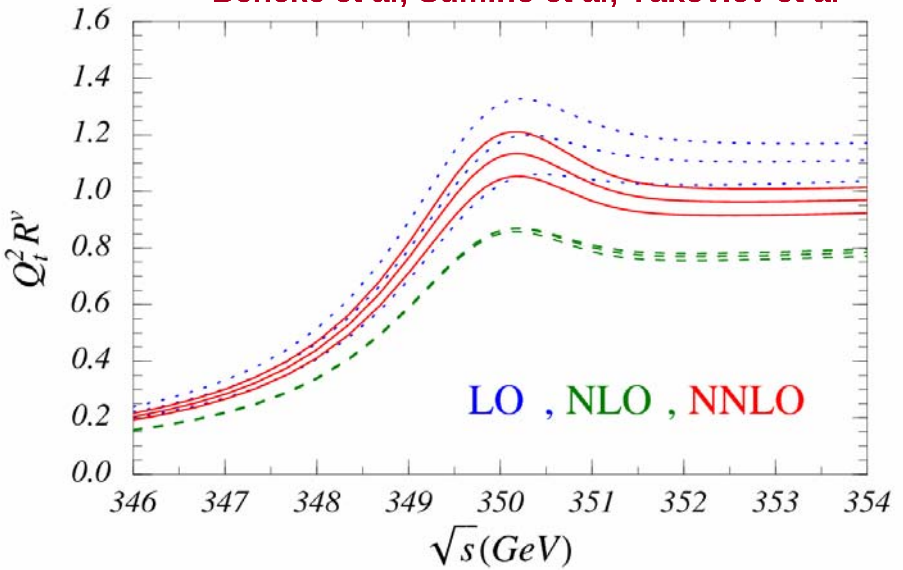
Learning from Top Physics at Threshold

Hoang,Teubner; Penin et al; Melnikov et al.
Beneke et al; Sumino et al; Yakovlev et al

fixed order scheme

$$\frac{\alpha_s}{v} \sim 1$$

$$\begin{aligned}
\text{LO} &\sim \left(\frac{\alpha_s}{v}\right)^n \\
\text{NLO} &\sim \{\alpha_s, v\} \times \left(\frac{\alpha_s}{v}\right)^n \\
\text{NNLO} &\sim \{\alpha_s^2, \alpha_s v, v^2\} \times \left(\frac{\alpha_s}{v}\right)^n
\end{aligned}$$



- large NNLO correction
- scale dependence ⟶ large uncertainty in normalization of cross section

$$m_t \sim 175\,\mathrm{GeV} \qquad \mathbf{p} \sim 25\,\mathrm{GeV} \qquad E \sim 4\,\mathrm{GeV}$$

⟶ NRQCD matrix elements, μ?

For example: $\alpha_s(m_t)\,\ln\left(\frac{m_t^2}{E^2}\right) \simeq 0.8$

Learning from Top Physics at Threshold

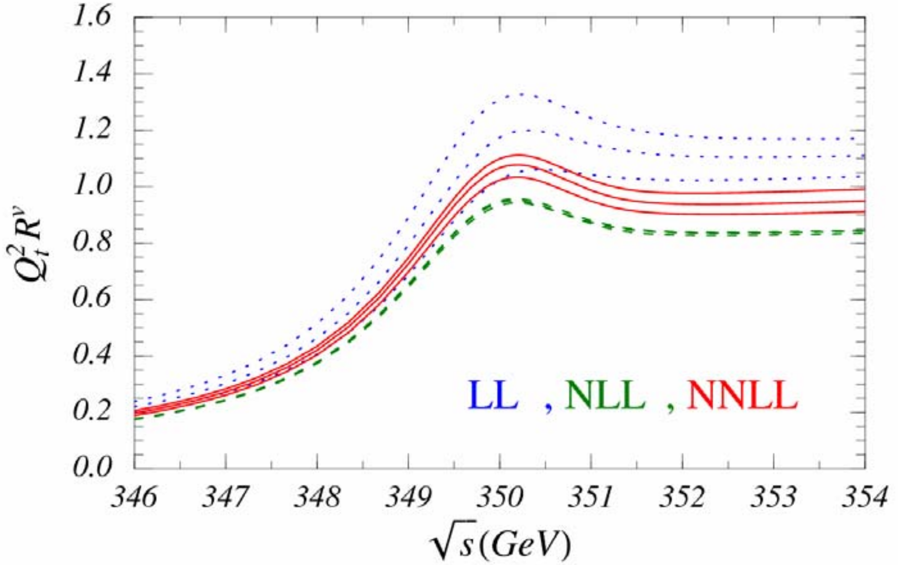
RG improved computations

$$\frac{\alpha_s}{v} \sim 1 \qquad \alpha_s \ln v \sim 1$$

$$\text{LL} \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$

$$\text{NLL} \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m \times \{\alpha_s, v\}$$

Hoang, Manohar, Stewart, Teubner



- ✓ log terms summed into coefficients through RGE
- ✓ reduced scale dependence

⇒ **vNRQCD** Luke, Manohar, Rothstein; Hoang, Stewart

- ✓ **EFT for NR heavy quark pairs**
- ✓ **consistent power counting in v**

For heavy $\tilde{q}\bar{\tilde{q}}$ pairs we need the scalar version

Effective Theory Framework

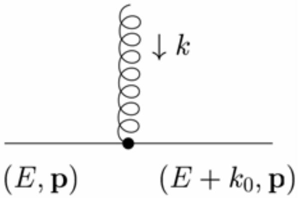
Scales in the non-relativistic $\tilde{q}\bar{\tilde{q}}$ system

$$m \quad \gg \quad \mathbf{p} \sim m\,v \quad \gg \quad E \sim m\,v^2 \quad > \quad \Lambda_{QCD}$$

hard soft ultrasoft

<u>Split heavy squark 4-momentum</u>

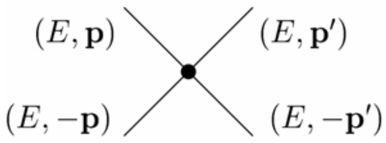
$$p^\mu = (m, 0) + \underbrace{(0, \mathbf{p})}_{\sim mv} + \underbrace{(k^0, \mathbf{k})}_{\sim mv^2}$$



$$\psi^*_{\mathbf{p}}\, A_\mu(x)\, \psi_{\mathbf{p}}$$

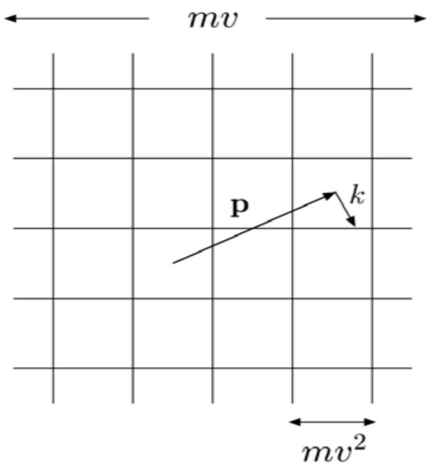
$$\boxed{\psi \to \sum_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}}\, \psi_{\mathbf{p}}(x)}$$

$$x \sim 1/mv^2$$



$$\frac{\psi^*_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^*_{-\mathbf{p}'}\chi_{-\mathbf{p}}}{(\mathbf{p}-\mathbf{p}')^2}$$

Effective Theory Framework



Recall HQET: $p^\mu = mv^\mu + k^\mu$ $\sim \Lambda_{QCD}$

$$\psi \to \sum_v e^{-iv\cdot x}\, \psi_v(x)$$

hard modes $(\, k_0 \,,\, \mathbf{k}\,) \sim (\, m \,,\, m\,)$ ⟶ **integrated out**

Resonant modes in the EFT

soft modes	A_q^μ		$\sim (\, mv \,,\, mv\,)$
potential modes	$\psi_\mathbf{p} \,,\, \chi_\mathbf{p}$		$\sim (\, mv^2 \,,\, mv\,)$
ultrasoft modes	A^μ		$\sim (\, mv^2 \,,\, mv^2\,)$

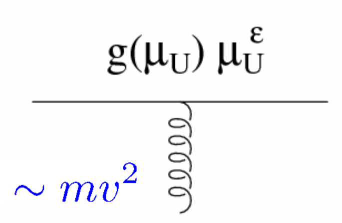
vNRQCD

Luke, Manohar, Rothstein; Hoang, Stewart

$$\mathcal{L} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}$$

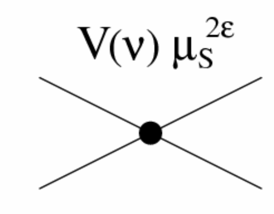
$$\mathcal{L}_{\text{us}} = \sum_{\mathbf{p}} \psi^*_{\mathbf{p}} \left[iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \frac{\mathbf{p}^4}{8m^3} + \dots \right] \psi_{\mathbf{p}}$$

$$D^\mu = \partial^\mu + i\mu_U^\epsilon g(\mu_U) A^\mu$$

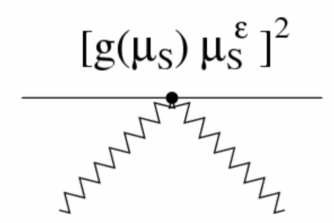


$k \sim mv^2$

$$\mathcal{L}_{\text{pot}} = -\mu_S^{2\epsilon} \sum_{\mathbf{p},\mathbf{p}'} \left[\frac{\mathcal{V}_c}{(\mathbf{p} - \mathbf{p}')^2} + \dots \right] \psi^*_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^*_{-\mathbf{p}'} \chi_{-\mathbf{p}}$$



$$\mathcal{L}_{\text{soft}} = -\mu_S^{2\epsilon} g_s^2(\mu_S) \sum_{\mathbf{p},\mathbf{p}',q,q',\sigma} \left[\frac{1}{2} \psi^*_{\mathbf{p}'} [A^\mu_{q'}, A^\nu_q] U^{(\sigma)}_{\mu\nu} \psi_{\mathbf{p}} + \dots \right]$$



$q \sim mv$

velocity Renormalization Group

Two kinds of α_s in vNRQCD

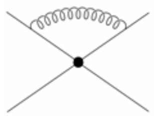
$$(\mu_S)^{2\epsilon}\alpha_s \;\rightarrow\; (\mu_S)^{2\epsilon}\alpha_s(\mu_S)$$
$$(\mu_U)^{2\epsilon}\alpha_s \;\rightarrow\; (\mu_U)^{2\epsilon}\alpha_s(\mu_U)$$

Correlation of scales

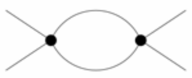
$$\mu_U = \mu_S^2/m \equiv m\nu^2$$

$$\nu \,\epsilon [0,1]$$

$$\text{ultrasoft loops:} \quad \mu_U^{4-D}\int \frac{d^Dk}{(2\pi)^D} \sim \ln\left(\mu_U/E\right) = \ln\frac{\nu^2}{v^2}$$



$$\text{potential and soft loops:} \quad \mu_S^{4-D}\int \frac{d^Dk}{(2\pi)^D} \sim \ln\left(\mu_S^2/\mathbf{p}^2\right) = \ln\frac{\nu^2}{v^2}$$



Choosing the renorm. point $\nu \sim v$ all logs are simultaneously small

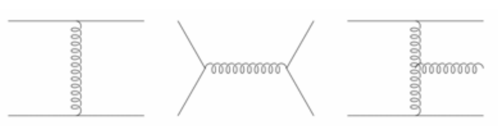
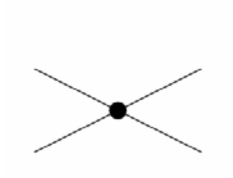
⇨ logs resumed in the Wilson coefficients of the EFT

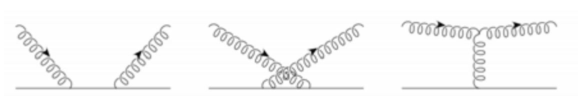
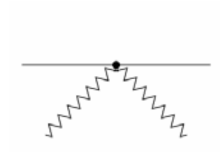
$$C_i(\nu) \sim \alpha_s \sum_k [\alpha_s \ln(\nu)]^k$$

vNRQCD – Matching

Coefficients in $\mathcal{L}_{\mathrm{vNRQCD}}$ are functions of renorm. parameter ν

Matching with QCD at the hard scale $(\mu_S = \mu_U = m)$

 $= \mathcal{V}(m) \times$ 

 $= U_{\mu\nu}^{(\sigma)} \times$ 

- ✓ Only assumes that $\alpha_s(m)$ is small
- ✓ No large logs in matching conditions

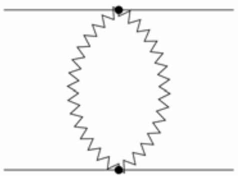
⇨ NNLO matching for scalar quarks

- ✓ Coulomb potential to $\mathcal{O}(\alpha_s^2)$ **Peter, Schröder**
- ✓ $v^2, \alpha_s v$ suppressed potentials at tree level
- ✓ soft vertices at NNLO

vNRQCD - Running

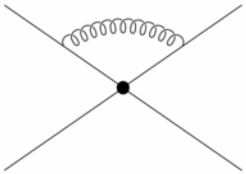
Running couplings

⇨ compute anomalous dimensions of the EFT operators



soft loops

μ_S



ultrasoft loops

μ_U

Matrix elements of the EFT $\sim \ln\left(\mu_S^2/\mathbf{p}^2\right) \quad \sim \ln\left(\mu_U^2/E^2\right)$

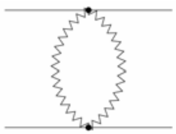
Correlation of scales $\mu_U = \mu_S^2/m \equiv m\nu^2$

✓ Running from $\nu = 1$ to $\nu \sim v$ sums large logs

✓ The scaling of the $C_i(\nu)$ obtained using the vRGE

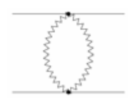
This work:
1-loop running of squark potentials

Running Potentials: An Example

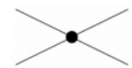


$$= -\frac{i\alpha_s^2(\mu_S)}{\mathbf{k}^2}\mu_S^{2\epsilon}\, T^A \otimes \bar{T}^A \left[\underbrace{\frac{11C_A - 4T_F n_f}{3}}_{\beta_0} \frac{1}{\epsilon} + \ldots \right]$$

$$i\mathcal{L}_{\text{pot}} = -i(T^A \otimes \bar{T}^A)\frac{\mathcal{V}_c^0}{\mathbf{k}^2} + \ldots \quad \Rightarrow$$



$+$



$$\delta\mathcal{V}_c = -\alpha_s^2(\mu_S)\beta_0\,\frac{1}{\epsilon}$$

= finite!

relation between renormalized and bare Wilson coef.: $\mathcal{V}_c^0 = \mu_S^{2\epsilon}\left(\mathcal{V}_c + \delta\mathcal{V}_c\right)$

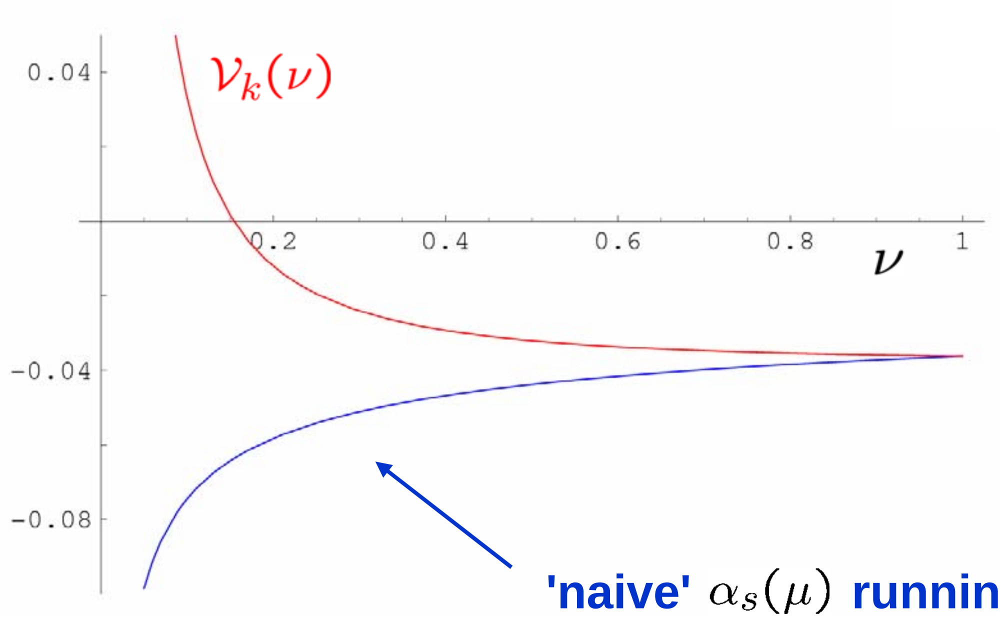
$$\text{vRGE } (\mu_S = m\nu): \quad \nu\frac{d}{d\nu}\mathcal{V}_c^0 = 0 \;\Rightarrow\; \nu\frac{d}{d\nu}\mathcal{V}_c(\nu) = -2\beta_0\alpha_s^2(m\nu)$$

boundary condition
$\mathcal{V}_c(m) = 4\pi\alpha_s(m) \;\rightarrow$ $\boxed{\mathcal{V}_c(\nu) = 4\pi\alpha_s(m\nu)}$ $\sim \alpha_s(m)\sum_k \left[\alpha_s(m)\,\ln(\nu)\right]^k$

<u>RG improved Coulomb pot. given by choosing</u> $\nu = |\mathbf{k}|/m\,, \quad \alpha_s = \alpha_s(|\mathbf{k}|)$

Running Potentials

$$\frac{\mathcal{V}_k\,\pi^2}{m\,\mathbf{k}}$$ potential



'naive' $\alpha_s(\mu)$ running

vNRQCD running

$$\mathcal{V}_k(\nu) = \frac{C_F}{2}(C_F - 2C_A)\,\alpha_s^2(m\nu) + \frac{8C_F C_A(C_A+2C_F)}{3\beta_0}\,\alpha_s^2(m\nu)\,\ln\frac{\alpha_s^2(m\nu^2)}{\alpha_s^2(m\nu)}$$

Match potential at $\mu = m$

$$\mathcal{V}_k(m) = \frac{C_F}{2}(C_F - 2C_A)\,\alpha_s^2(m)$$

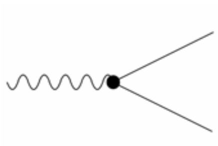
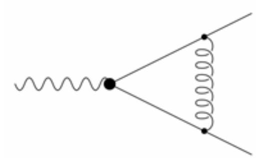
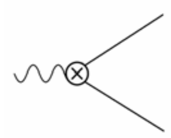
$$\mathcal{V}_k(\nu) = \frac{C_F}{2}(C_F - 2C_A)\,\alpha_s^2(m\nu)$$

Production of Heavy Scalars

P-wave current (NLL)

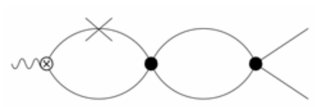
$$J_{\mathbf{p}} = c_P(\nu)\, \psi_{\mathbf{p}}^* \, \mathbf{p} \, \chi_{-\mathbf{p}}^* + \ \ldots \qquad e^+e^- \to \tilde{t}\,\bar{\tilde{t}}\ \left({}^1P_1\right)$$

Matching with sQCD at NLO

 $+$  $= c_P(1) \times$  $\Rightarrow \ c_P(1) = 1 - C_F \dfrac{\alpha_s(m)}{\pi}$

Evolution of $c_P(\nu)$ at NLL

✓ anomalous dim at two-loops

$\gamma_{c_P}^{\mathrm{NLL}}$  $+ \ldots$

✓ solve the renormalization group eq.

$$\nu \frac{d}{d\nu} c_P(\nu) = \gamma_{c_P}^{\mathrm{NLL}} c_P(\nu)$$

NLL Running of Wilson Coeficients

$m_{stop} = 220$ GeV

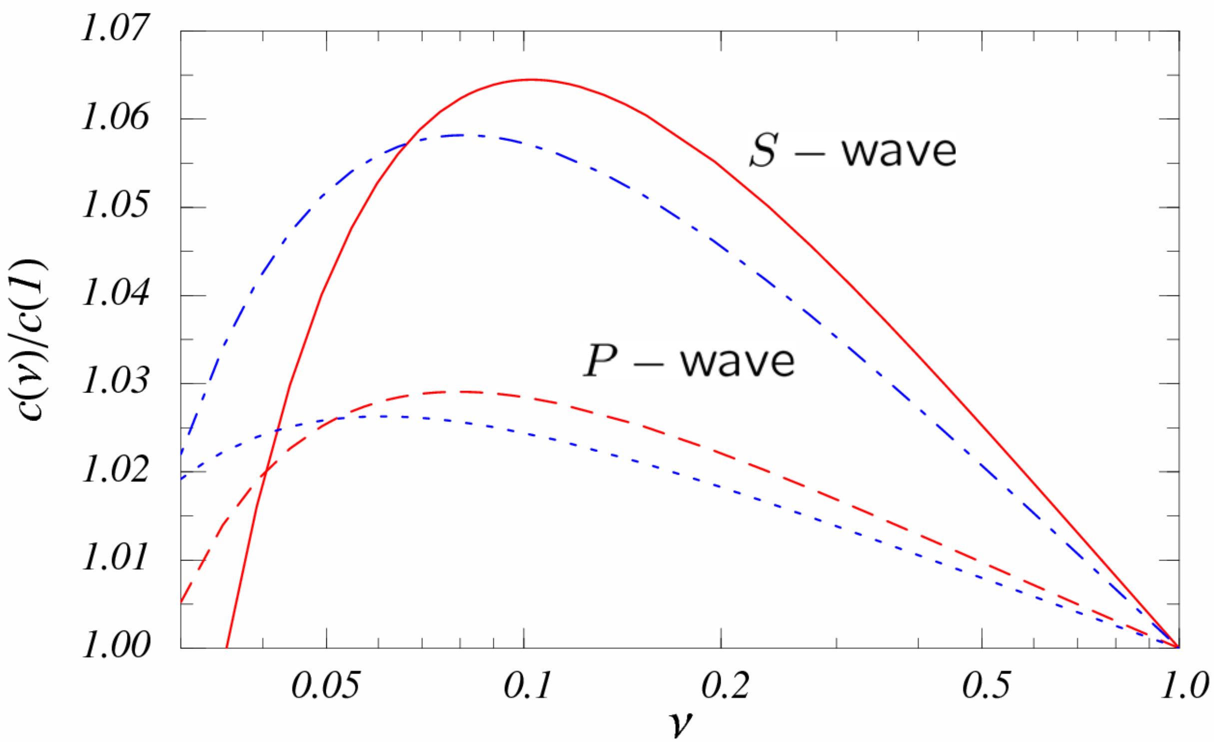
$m_{stop} = 500$ GeV

$$J(^1P_1) = c_P(\nu)\,\psi^*_{\mathbf{p}}\,\mathbf{p}\,\chi^*_{-\mathbf{p}} + \;\ldots$$

$$e^+e^- \to \tilde{t}\,\bar{\tilde{t}}\;\;(^1P_1)$$

$$J(^1S_0) = c_S(\nu)\,\psi^*_{\mathbf{p}}\,\chi^*_{-\mathbf{p}} + \;\ldots$$

$$\gamma\gamma \to \tilde{t}\,\bar{\tilde{t}}\;\;(^1S_0)$$



Outlook

- **vNRQCD** allows for RG-improved computations of bound state energies and $\sigma(e^+e^-, \gamma\gamma \to \tilde{q}\,\bar{\tilde{q}})$

- Complete NLL analysis of $\sigma(e^+e^- \to \tilde{t}\,\bar{\tilde{t}})$ at threshold on the way

$$\sigma \propto \mathrm{Im}\left[\int d^4x\, e^{-i\hat{q}\cdot x} \left\langle 0|T\, J^\dagger_{\mathbf{p}}(0) J_{\mathbf{p}'}(x)\,|0\right\rangle\right] \propto \mathrm{Im}\left[\, c_0^2(\nu)\, G^1(0,0,v)\,\right]$$

Wilson coefficient $\quad c_0(\nu) \sim \sum_k [\alpha_s \ln(\nu)]^k \times \{\alpha_s, v\}$

Coulomb Green´s function $\quad G^1(0,0,v) \sim \left(\frac{\alpha_s}{v}\right)^n$

- Next step: EW effects ...

Building up the Effective Theory - vNRQCD

Luke, Manohar, Rothstein; Hoang, Stewart

$$\mathcal{L}_{\rm QCD} = D_\mu \phi^* D^\mu \phi - m^2 \phi^* \phi$$

Example: tree-level 2-point function in momentum space

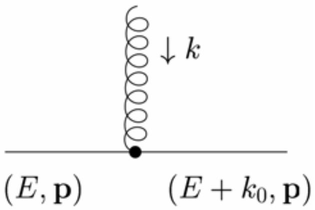
$$\phi = \frac{\psi}{\sqrt{2m}} \qquad \frac{1}{2m}\psi^*\left(p^2 - m^2\right)\psi = \psi^*\left(k^0 - \frac{\mathbf{p}^2}{2m} - \frac{\mathbf{p}\cdot\mathbf{k}}{m} + \frac{\mathbf{p}^4}{8m^3} + \ldots\right)\psi$$

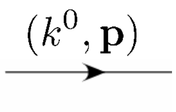
$$p = (m + k^0, \mathbf{p} + \mathbf{k})$$

$$\mathcal{L}_{\rm us} = \sum_{\mathbf{p}} \psi^*_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \frac{\mathbf{p}^4}{8m^3} + \ldots\right]\psi_{\mathbf{p}}(x) + (\psi \to \chi)$$

$$-\frac{1}{4}G^{\mu\nu}G_{\mu\nu} + \ldots ,$$

$$D^\mu = \partial^\mu + igA^\mu$$



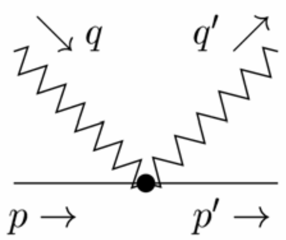


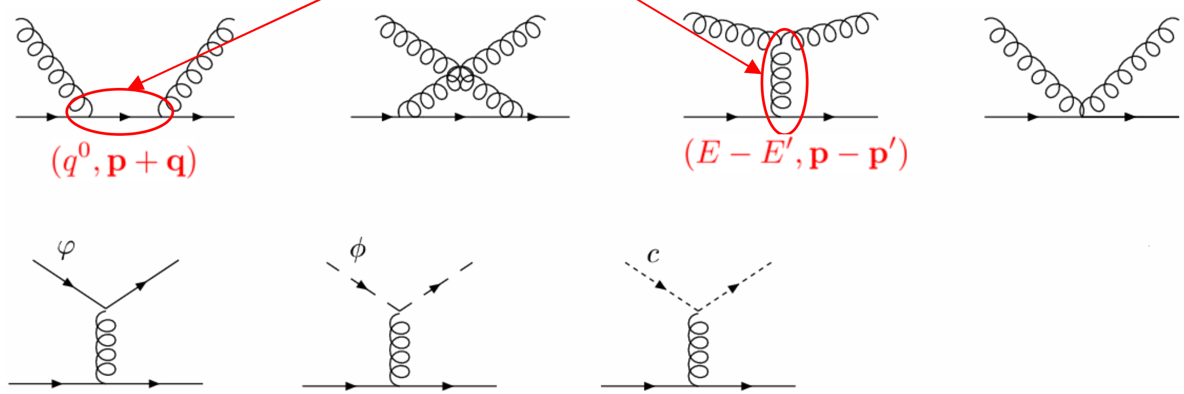
$$\frac{1}{k^0 - \frac{\mathbf{p}^2}{2m} + i\epsilon}$$

Interactions with Soft Gluons

soft gluons : $q \sim mv$

Off-shell modes





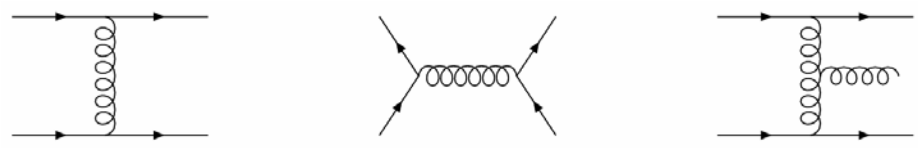
$$\mathcal{L}_{\text{soft}} = \{\text{soft kin. terms}\} - g^2 \sum_{\mathbf{p},\mathbf{p}',q,q'} \Bigg[\frac{1}{2}\psi^*_{\mathbf{p}'}[A^\mu_{q'}, A^\nu_q] U^{(\sigma)}_{\mu\nu}\psi_{\mathbf{p}} + \dots$$

$$+\psi^*_{\mathbf{p}'}[\bar{c}_{q'}, c_q] Y^{(\sigma)}\psi_{\mathbf{p}} + \sum_{i=1}^{n_f}(\psi^*_{\mathbf{p}'} T^B Z^{(\sigma)}_\mu \psi_{\mathbf{p}})\,(\bar{\varphi}_{i,q'}\gamma^\mu T^B \varphi_{i,q}) + \dots \Bigg]$$

$U^{(\sigma)}_{\mu\nu}$, $Y^{(\sigma)}$, $Z^{(\sigma)}_\mu$ $\dots$ functions of $(\mathbf{p}, \mathbf{p}', q, q')$ of order v^σ

Potentials in vNRQCD

Squark-antisquark scattering interactions



$$\frac{1}{(p'-p)^2} = -\frac{1}{(\mathbf{p}'-\mathbf{p})^2} + \dots$$

$$\mathcal{L}_{\text{pot}} = -\sum_{\mathbf{p},\mathbf{p}'} V(\mathbf{p},\mathbf{p}')\,\psi^*_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^*_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \dots$$

$$\begin{aligned} V(\mathbf{p},\mathbf{p}') &= (T^A\otimes\bar{T}^A)\left[\frac{\mathcal{V}_c^{(T)}}{\mathbf{k}^2} + \frac{\mathcal{V}_k^{(T)}\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r^{(T)}(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2^{(T)}}{m^2} + \dots\right] \\ &+ (1\otimes 1)\left[\frac{\mathcal{V}_c^{(1)}}{\mathbf{k}^2} + \frac{\mathcal{V}_k^{(1)}\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_2^{(1)}}{m^2} + \dots\right] \end{aligned}$$

$$\mathcal{V}_c^{(T)}(m) = 4\pi\alpha_s(m)$$

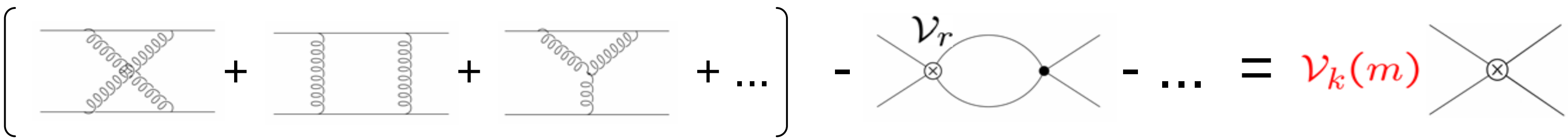
$$\mathcal{V}_2^{(T)}(m) = -\pi\alpha_s(m)$$

$$\mathcal{V}_r^{(T)}(m) = 4\pi\alpha_s(m)$$

$$\mathbf{k} = \mathbf{p}' - \mathbf{p}$$

1-loop Potentials

$\dfrac{\mathcal{V}_k\,\pi^2}{m\,\mathbf{k}}$ potential is first generated at 1-loop



$$\mathcal{V}_k^{(T)}(m) = \alpha_s^2(m)\left(\frac{5C_A}{4} - C_F\right) \qquad \mathcal{V}_k^{(T)}(1) = \frac{\alpha_s^2(m)}{2}\left(\frac{C_F C_A}{2} - C_F^2\right)$$

Power counting in the Lagrangian

velocity scaling of the fields

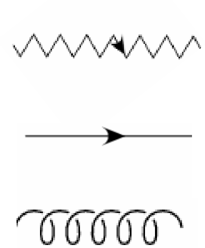
$$S^{(0)} = \int d^4x \, \mathcal{L}_{\text{kin}}(x) + \ldots = \int d^4x \left(\sum_{\mathbf{p}} \psi^*_{\mathbf{p}}(x) \left(iD^0 - \frac{\mathbf{p}^2}{2m} \right) \psi_{\mathbf{p}}(x) \right.$$

$$\left. -\frac{1}{4} G^{\mu\nu} G_{\mu,\nu} + \sum_q \left| q^\mu A^\nu_q - q^\nu A^\mu_q \right|^2 \right) + \ldots \sim v^0$$

gluon kin. term: $\partial^\mu \sim v^2 \, , \int d^4x \sim v^{-8} \longrightarrow A^\mu(x) \sim v^2$

scalar kin. term: $\int d^4x \sum_{\mathbf{p}} \sim v^{-2} v^{-3} \longrightarrow \psi_{\mathbf{p}} \sim v^{3/2}$

$\mathbf{p}$	$\psi_{\mathbf{p}}, \ \chi_{\mathbf{p}}$	A^μ_p	D^0	$\mathbf{D}$	A^μ
v	$v^{3/2}$	v	v^2	v^2	v^2



$\sim v^{-4}$

$\sim v^{-5}$

$\sim v^{-8}$

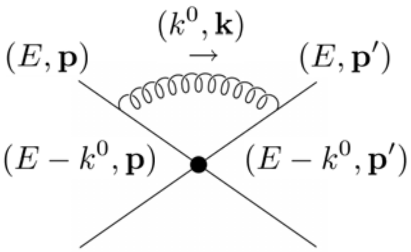
vertices $\sim v^k$

$$\frac{\psi^*_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^*_{-\mathbf{p}'} \chi_{-\mathbf{p}}}{(\mathbf{p} - \mathbf{p}')^2} \sim v^4$$

Power counting – Loops in the EFT

ultrasoft loops

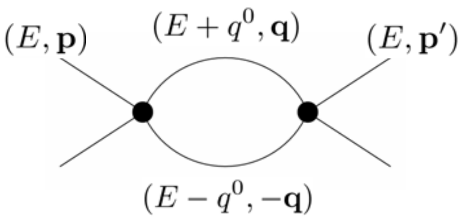
$$\int d^4k \, \frac{1}{E - k^0 - \mathbf{p}^2/2m} \, \frac{1}{(k^0)^2 - \mathbf{k}^2} \, \frac{1}{E - k^0 - \mathbf{p}'^2/2m}$$



$$k^0 \sim E \, , \; \mathbf{k} \sim k^0 \;\longrightarrow\; \int d^4k \sim v^8$$

potential loops

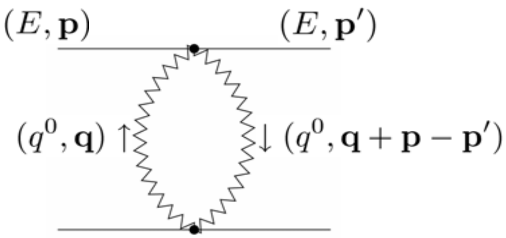
$$\int d^4q \, \frac{1}{(\mathbf{p} - \mathbf{q})^2} \, \frac{1}{q^0 + E - \mathbf{q}^2/2m} \, \frac{1}{-q^0 + E - \mathbf{q}^2/2m} \, \frac{1}{(\mathbf{p}' - \mathbf{q})^2}$$



$$q^0 \sim E \, , \; \mathbf{q} \sim \sqrt{mE} \;\longrightarrow\; \int d^4q \sim v^5$$

soft loops

$$\int d^4q \, \frac{1}{(q^0)^2 - \mathbf{q}^2} \, \frac{1}{(q^0)^2 - (\mathbf{q} + \mathbf{p} - \mathbf{p}')^2}$$



$$q^0 \sim \mathbf{q} \sim mv \;\longrightarrow\; \int d^4q \sim v^4$$

Power counting – General formula

general diagram is of order $v^{\delta}\alpha_s^{n}$

Luke, Manohar, Rothstein

$$\delta = 5 + \sum_k \left[(k-8)V_k^{(U)} + (k-5)V_k^{(P)} + (k-4)V_k^{(S)} \right] - N_S$$

$$\boxed{\delta' = \delta - 5}$$

<u>Some examples:</u>

Coulomb potential

$$\psi^*_{\mathbf{p}'}\psi_{\mathbf{p}}\frac{\alpha_s}{(\mathbf{p}-\mathbf{p}')^2}\chi^*_{-\mathbf{p}'}\chi_{-\mathbf{p}} \longrightarrow \delta' = -1$$

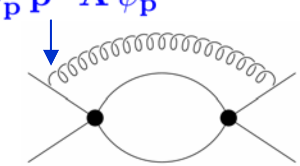
each Coulomb pot. gives $\frac{\alpha_s}{v} \sim v^0$

$\alpha_s \sim v \implies$ ***must be summed up to all orders in α_s !***

QED Lamb shift

$-e\,\psi^*_{\mathbf{p}}\,\mathbf{p}\cdot\mathbf{A}\,\psi_{\mathbf{p}}$

$V_6^{(P)} = 2\ ,\ V_4^{(P)} = 2$



$\sim \alpha^3 v^5$

$\mathcal{O}(\alpha\, v^2) \sim \mathcal{O}(\alpha^3)$ correction

The vNRQCD Lagrangian in D dimensions

Regularize the EFT in $D = 4 - 2\epsilon$

$$S^{(0)} = \int d^D x \, \mathcal{L}_{\rm kin}(x) + \ldots \sim v^0 \quad \Longrightarrow \quad \begin{array}{l} \psi_{\mathbf{p}} \sim (mv)^{3/2-\epsilon} \\ A^\mu \sim (mv^2)^{1-\epsilon} \\ A_q^\mu \sim (mv)^{1-\epsilon} \end{array}$$

$\Rightarrow D^\mu \sim mv^2 \longrightarrow gA^\mu$ must be multiplied by $(\mu_U)^\epsilon \sim (mv^2)^\epsilon$

$$\Rightarrow \text{ 4-squark operators } \longrightarrow (\mu_S)^{2\epsilon} \sum_{\mathbf{p},\mathbf{p}'} V(\mathbf{p},\mathbf{p}')\, \psi^*_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^*_{-\mathbf{p}'} \chi_{-\mathbf{p}} \qquad \mu_S \sim mv$$

$$\Rightarrow \text{ Interactions with soft fields } \longrightarrow (\mu_S)^{2\epsilon} g^2 \sum_{\mathbf{p},\mathbf{p}',q,q'} \frac{1}{2} \psi^*_{\mathbf{p}'} [A^\mu_{q'}, A^\nu_q] U^{(\sigma)}_{\mu\nu} \psi_{\mathbf{p}} + \ldots$$

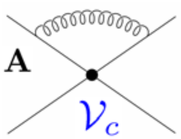
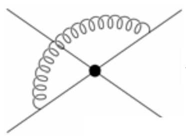
Two kinds of α_s in vNRQCD

$$\begin{array}{lll} (\mu_S)^{2\epsilon}\alpha_s & \rightarrow & (\mu_S)^{2\epsilon}\alpha_s(\mu_S) \\ (\mu_U)^{2\epsilon}\alpha_s & \rightarrow & (\mu_U)^{2\epsilon}\alpha_s(\mu_U) \end{array}$$

Correlation of scales

$$\mu_U = \mu_S^2/m \equiv m\nu^2$$

Running Potentials: Mixing

$\mathbf{p}\cdot\mathbf{A}$  $+$  $+\ldots$

$$= i\mu_S^{2\epsilon}\,\mathcal{V}_c(\nu)\,\alpha_s(\mu_U)\left[(T^A\otimes\bar{T}^A)\frac{\mathbf{p}^2+\mathbf{p}'^2}{2m^2\mathbf{k}^2}\,\frac{\#}{\epsilon}+(1\otimes 1)\frac{1}{m^2}\,\frac{\#}{\epsilon}+\ldots\right]$$

$\delta\mathcal{V}_r$

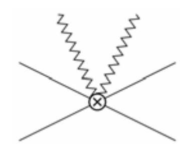
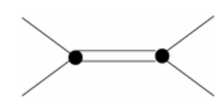
$\delta\mathcal{V}_2$

v^2 - suppressed potentials

$$i\mathcal{L}_{\rm pot} = -i(T^A\otimes\bar{T}^A)\left[\ldots+\frac{\mathcal{V}_r^0(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2}+\ldots\right] +(1\otimes 1)\left[\ldots+\frac{\mathcal{V}_2^0}{m^2}+\ldots\right]$$

$$\boxed{\nu\frac{d}{d\nu}\mathcal{V}_{r,2}(\nu) = 2\#\,\mathcal{V}_c(\nu)\,\alpha_s(m\nu^2)+\ldots}$$

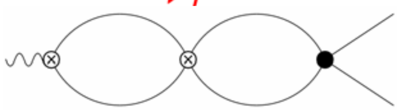
$\mathcal{V}_2(\nu)$ vanishes at the hard scale $(\nu=1)$, but is generated by mixing

Other operators generated by usoft renormalization $\longrightarrow$  

$C_{2i}^{(2)}(\nu)\,\mathcal{O}_{2i}^{(2)}$ $\quad$ $\mathcal{V}_{ki}^{(2)}(\nu)\,\mathcal{O}_{ki}^{(2)}$

NLL Running of Wilson Coeficients

LL values of 4-squark potentials needed for $c_P(\nu)$ at NLL



vRGE:

$$\frac{\partial}{\partial\nu}\ln[c_P(\nu)] = -\frac{\mathcal{V}_c^{(s)}(\nu)}{48\pi^2}\left[\frac{\mathcal{V}_c^{(s)}(\nu)}{4}+\mathcal{V}_r^{(s)}(\nu)\right] + \frac{\mathcal{V}_k^{(s)}(\nu)}{6} + \alpha_s^2(m\nu)\left[\mathcal{V}_{k1}^{(s)}(\nu)+\frac{2}{3}\mathcal{V}_{k2}^{(s)}(\nu)\right]$$

$$\Rightarrow \ln\frac{c_P(\nu)}{c_P(1)} = d_2\,\pi\alpha_s(m)\,(1-z) + d_0\,\alpha_s(m)\left[z-1-w^{-1}\ln(w)\right]$$

$$z=\frac{\alpha_s(m\nu)}{\alpha_s(m)} \quad , \quad w=\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)}$$