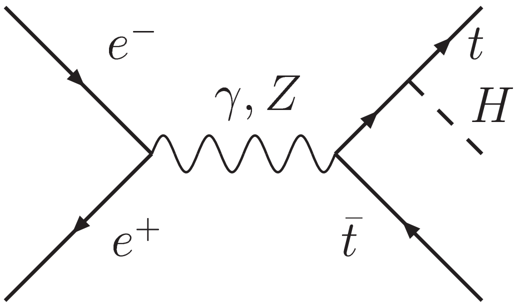
NLL QCD Corrections to $e^+e^- \to t\bar{t}H$ at 500 GeV



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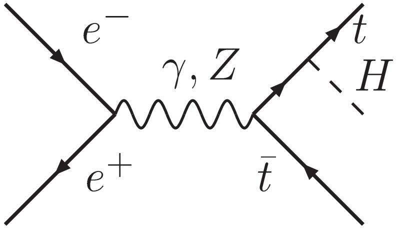
Outline

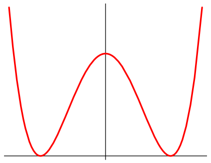
- Physical motivation
- Threshold effects of the top quarks, vNRQCD
- Results for $e^+e^- \longrightarrow t\bar{t}H$ at NLL order
- Analysis at $\sqrt{s} = 500$ GeV
- Preliminary: Effects of e^+e^- polarization

Physical Motivation

Why $e^+e^- \to t\bar{t}H$?



$\Rightarrow$ Top-Yukawa coupling Y_t



Test of the EWSB mechanism:

Higgs mechanism predicts $m_t = Y_t\, v$

Experimental accuracy at the ILC: $\delta Y_t / Y_t \sim \%$

$\Rightarrow$ Theoretical prediction: $\delta\sigma/\sigma \sim \%$

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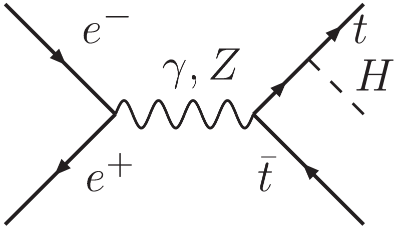

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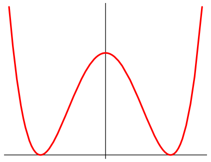

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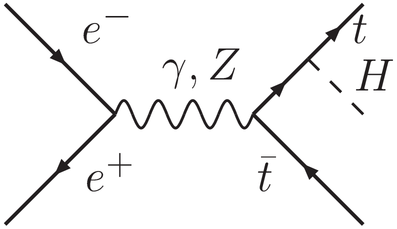

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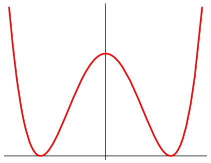

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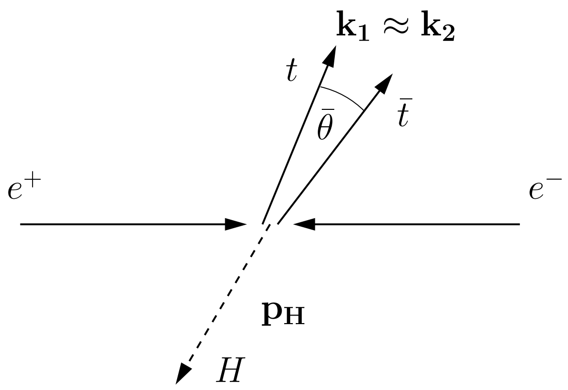
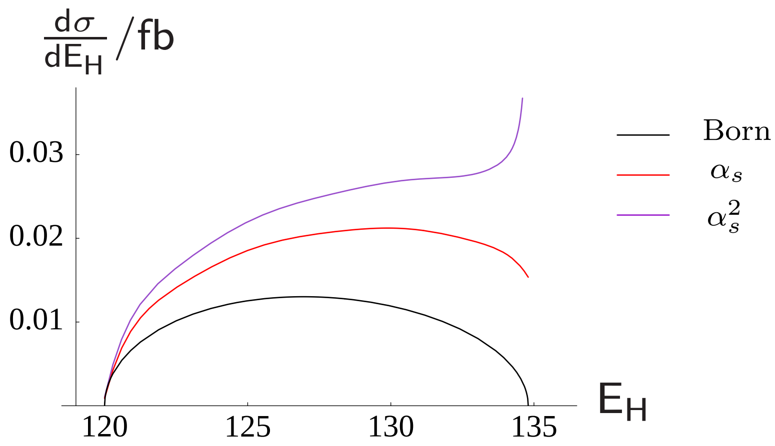
But: Interesting kinematical region in $d\sigma/dE_H$

The Limit of Large Higgs Energies





The top quarks become collinear

- Coulomb Singularities $\sim \left(\frac{\alpha_s}{v}\right)^n$ and
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- Fixed-order calculation breaks down

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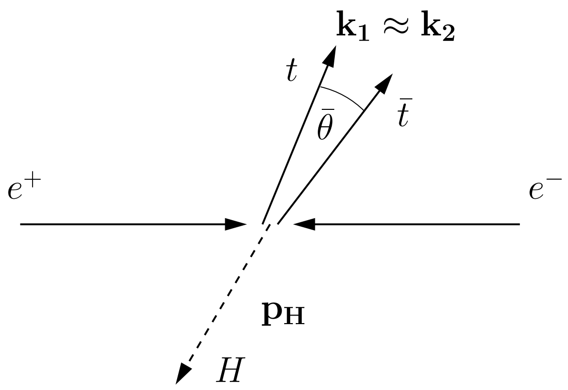

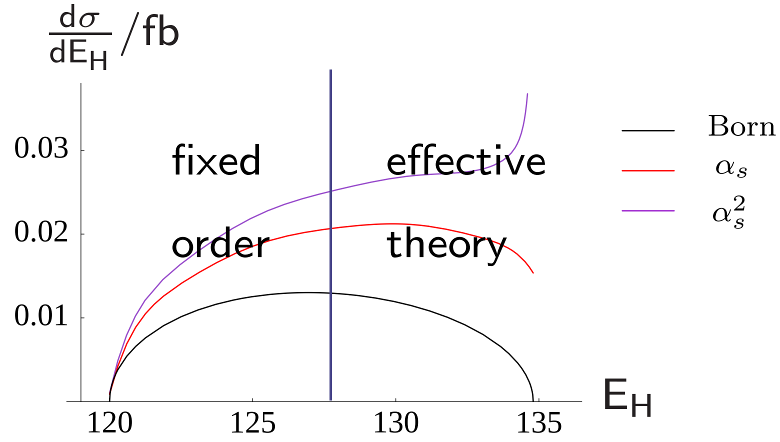

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$\Rightarrow$ Effective Theory

$$\frac{d\sigma}{dE_H} \sim v \sum_n \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^n \left(1\,(\mathrm{LL}) + \#\,\alpha_s\,(\mathrm{NLL})\right)$$

vNRQCD (velocity Non-Relativistic QCD)

[Luke, Manohar, Rothstein, Stewart, Hoang]

- Describes $t\bar{t}$ pairs near threshold
- Effective Theory with additional expansion parameter v
- Hierarchy:

$$m \gg mv \sim \vec{p} \gg mv^2 \sim E_{kin} \sim i\Gamma_t \gg \Lambda_{QCD}$$

- Width Γ_t as infrared regulator suppressing hadronization effects

vNRQCD

- Lagrangian includes the fields of all resonant degrees of freedom
  $\rightarrow$ Schrödinger equation in the CMS

- Effective currents for the creation and annihilation of the $t\bar{t}$ pair:

  $$I^{^1S_0} = (\psi^\dagger_{\vec{p}} \; \tilde{\chi}_{-\vec{p}}) \; c_0(\mu) \qquad |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

  $$I^{^3S_1} = (\psi^\dagger_{\vec{p}} \; \vec{\sigma} \; \tilde{\chi}_{-\vec{p}}) \; c_1(\mu) \qquad (\,|\uparrow\uparrow\rangle, \; |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \; |\downarrow\downarrow\rangle\,)$$

- Wilson coefficients contain the non-resonant contributions
  - RGE running: $c(\mu) = c(m_t) \cdot u(m_t, \mu)$

- General structure of the $t\bar{t}$ production rate:

  $$\sigma \sim \mathrm{Im} \int e^{-iqx} d^4x \, \langle 0 | \, T j(x) j^\dagger(0) \, | 0 \rangle \sim c^2(\mu) \, \mathrm{Im}[G^0(\mu)]$$

Calculation

$$\frac{d\sigma}{dE_H}(E_H \approx E_H^{\max}) \sim \Big[f_0^2\, c_0^2(\mu, \sqrt{s}, m_t, m_H) + f_1^2\, c_1^2(\mu, \sqrt{s}, m_t, m_H) \Big] \,\mathrm{Im}\, G^{\mathrm{NLL}}_{\mathrm{Coulomb}}(\mu, \sqrt{s}, m_t)$$

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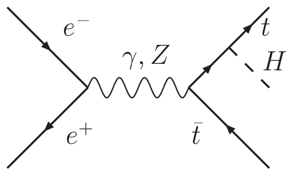
$f_{0,1}^2$: electroweak information in the endpoint

$$\sim \left(\frac{d\sigma_{0,1}}{dE_H}\right)_{\mathrm{Born}} \text{ for } E_H \to E_{H,\max}$$



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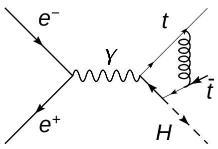
$c_{0,1}^2$: hard QCD corrections in the endpoint



$f_{0,1}^2$: electroweak information in the endpoint

Calculation

$$\frac{d\sigma}{dE_H}(E_H \approx E_H^{\max}) \sim \Big[f_0^2\, c_0^2(\mu, \sqrt{s}, m_t, m_H) + f_1^2\, c_1^2(\mu, \sqrt{s}, m_t, m_H)\Big]\, \mathrm{Im}\, G^{\mathrm{NLL}}_{\mathrm{Coulomb}}(\mu, \sqrt{s}, m_t)$$

Known: - Renormalization group running of $c_{0,1}$

- $G^{\mathrm{NLL}}_{\mathrm{Coulomb}}$

New: - Matching conditions $f_{0,1},\ c_{0,1}(\mu = m_t)$

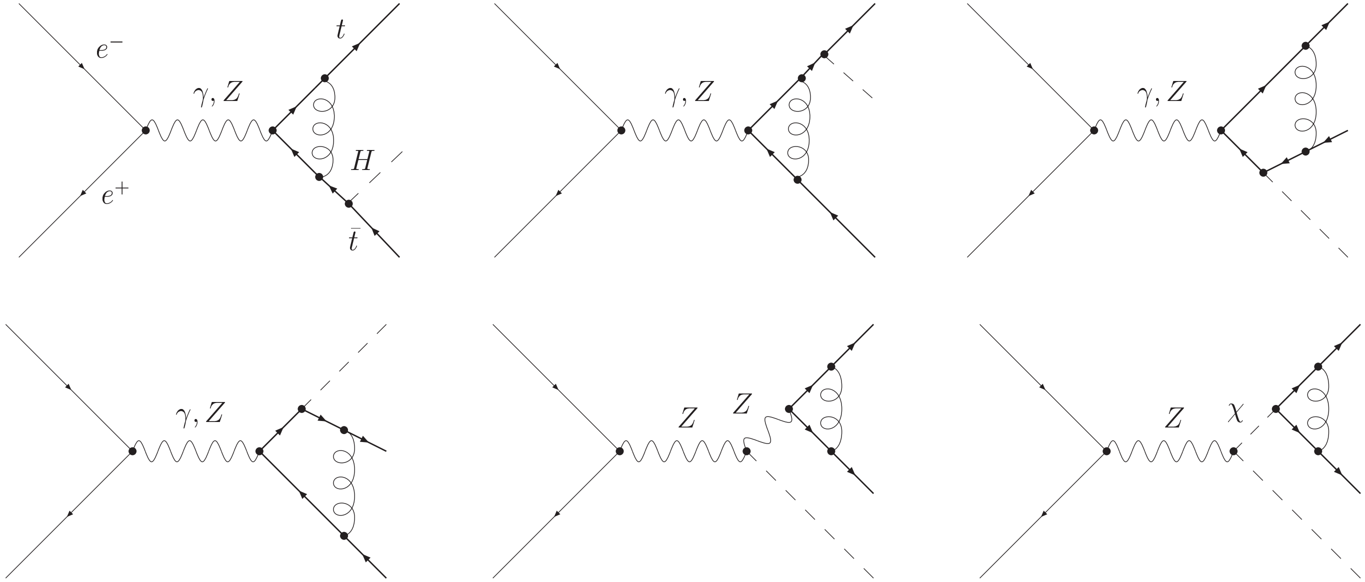
Ongoing: - Inclusion of e^+e^- polarization

Matching

$\mathcal{O}(\alpha_s)$ result in the limit $E_H \to E_H^{max}$

$\stackrel{!}{=}$ NLL result in the effective theory in $\mathcal{O}(\alpha_s)$

$\Rightarrow$ Numerical Matching Conditions of the Wilson coefficients $c_0(\mu = m_t)$, $c_1(\mu = m_t)$

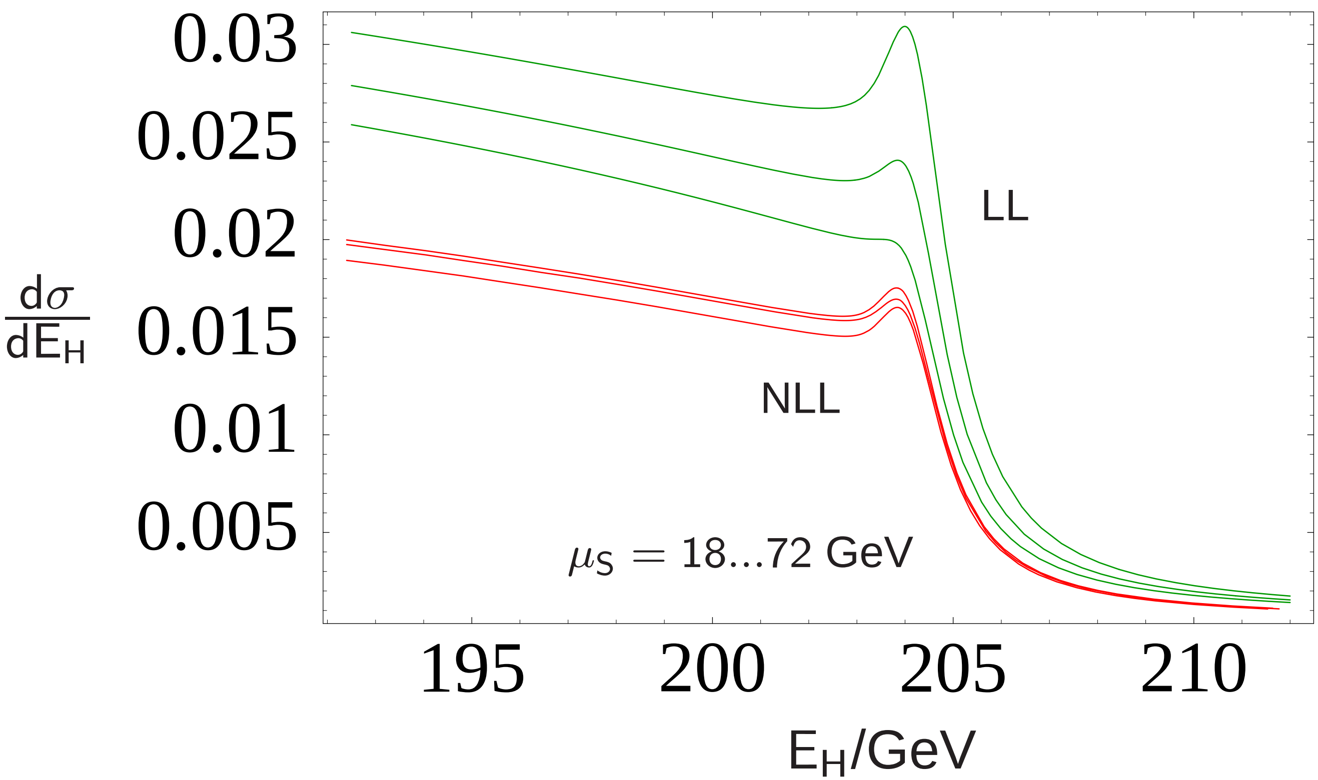


+ self-energy diagramms

[Denner, Dittmaier, Roth, Weber]

Scale Dependence

$\sqrt{s} = 600$ GeV, $m_H = 120$ GeV, $m_t = 180$ GeV



$\mathrm{LL} \sim \left(\frac{\alpha_s}{v}\right)^n$

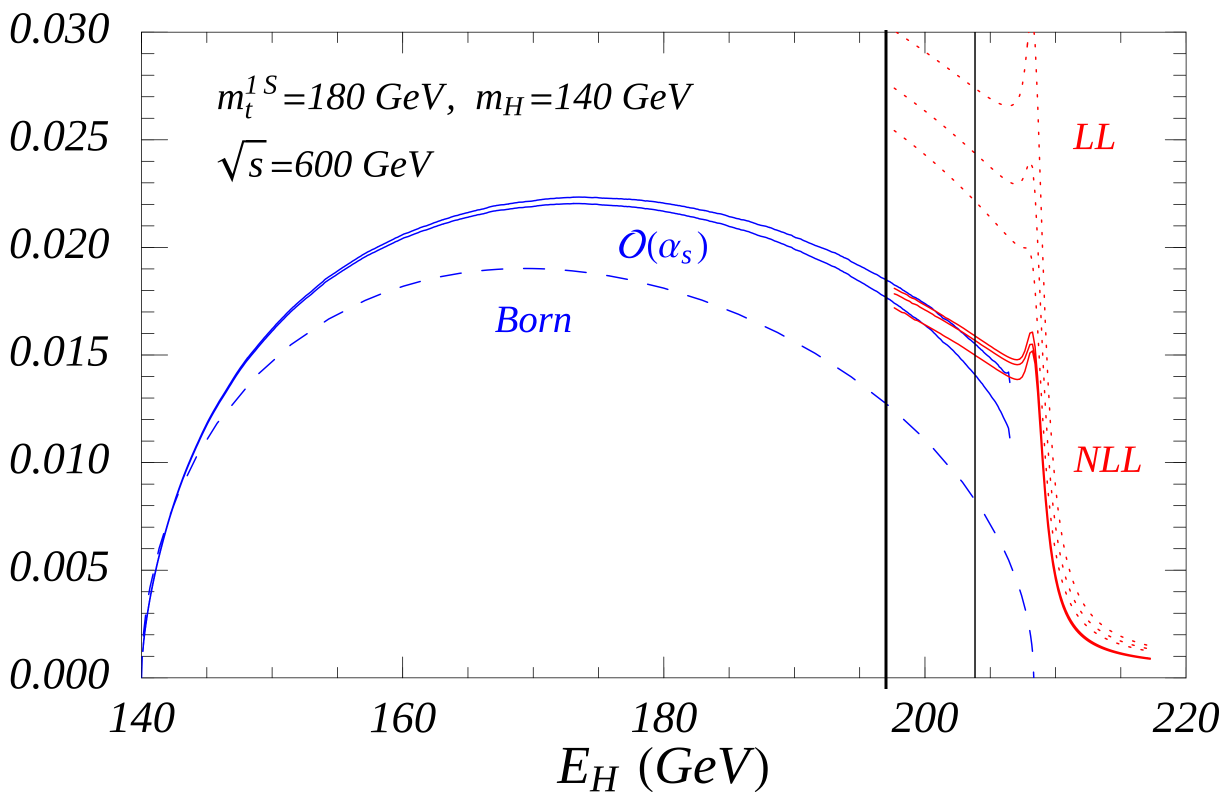
$\mathrm{NLL} \sim \alpha_s \left(\frac{\alpha_s}{v}\right)^n$

$\Rightarrow$ NLL: scale dependence of the order of 3%

$\Rightarrow$ Error estimate of 5%

Differential Cross Section

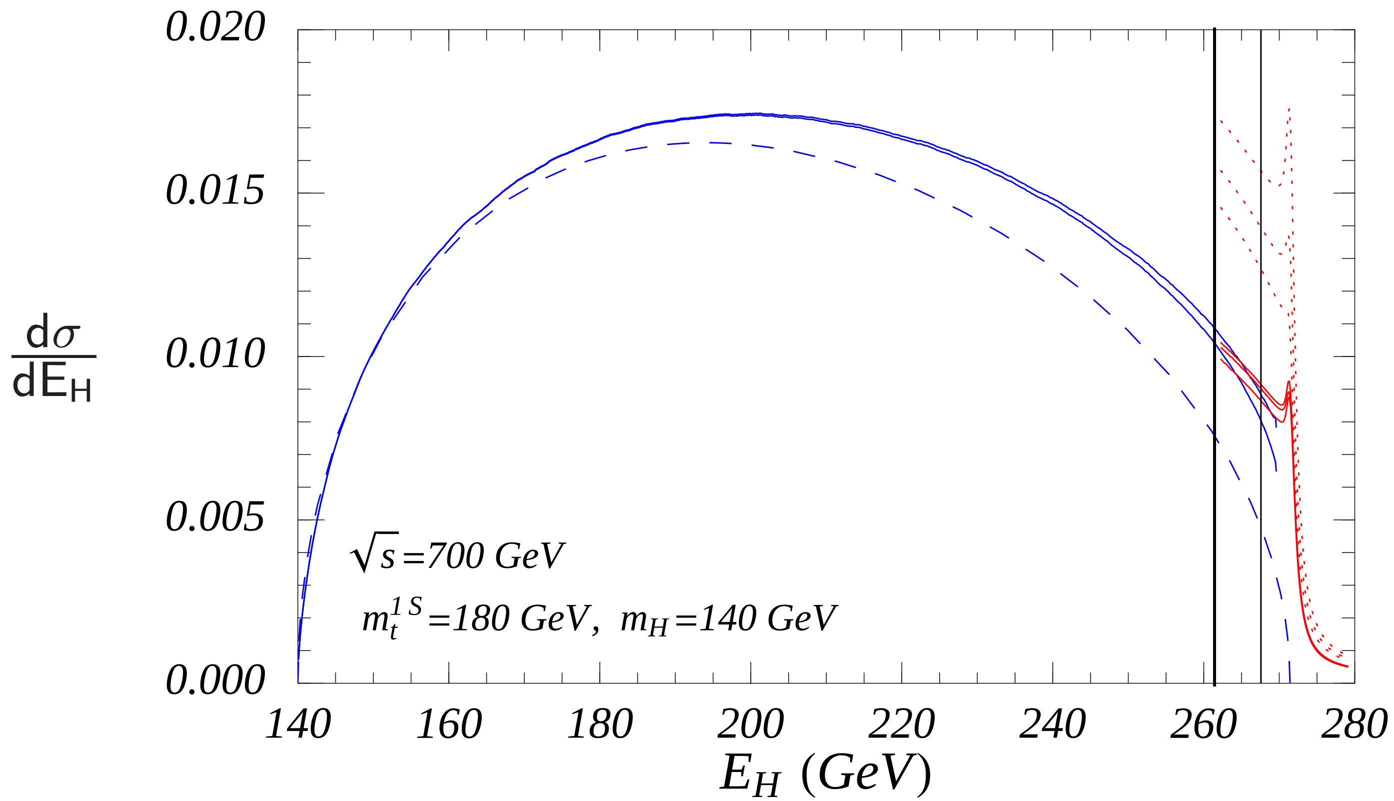
$\sqrt{s} = 600$ GeV



- In subleading order: Good transition around $\beta = 0.2$

Differential Cross Section

$\sqrt{s} = 700$ GeV



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Differential Cross Section

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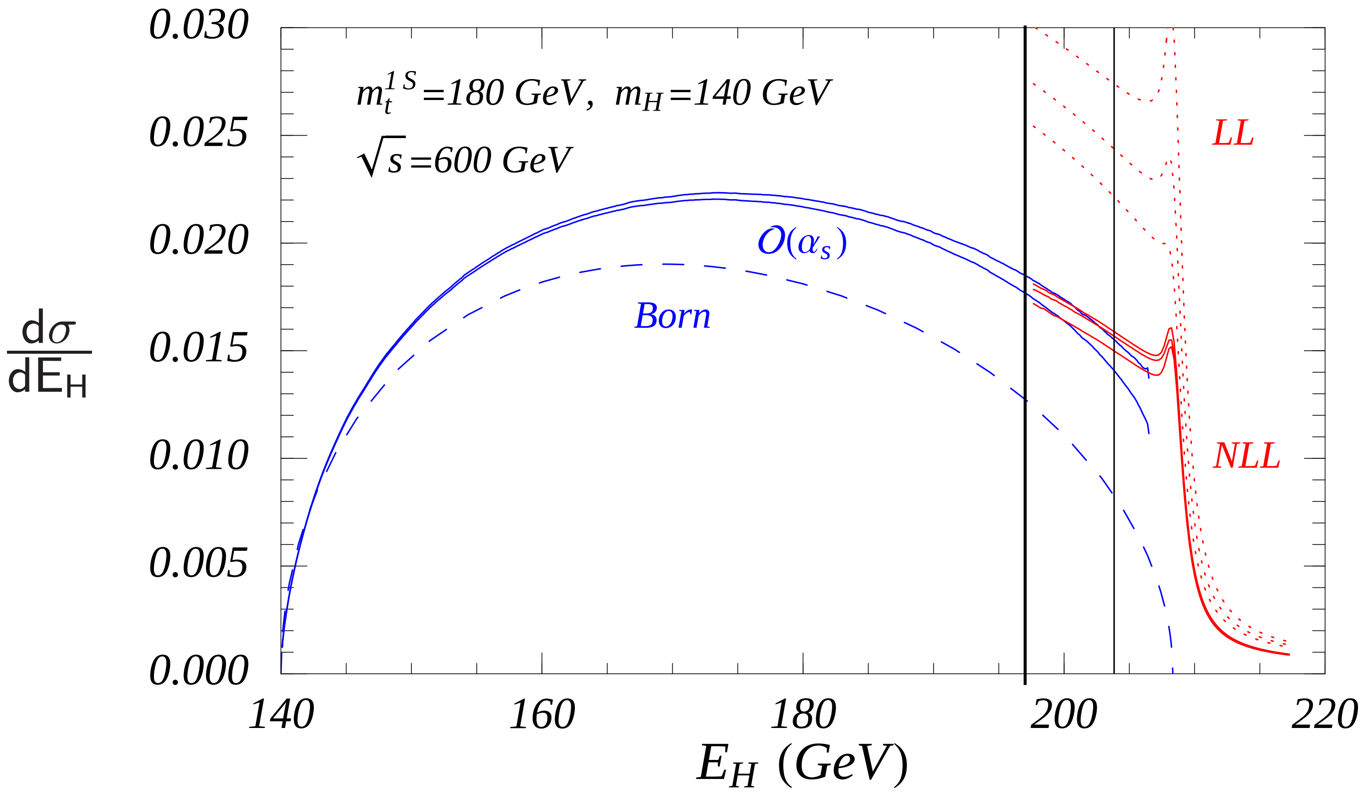

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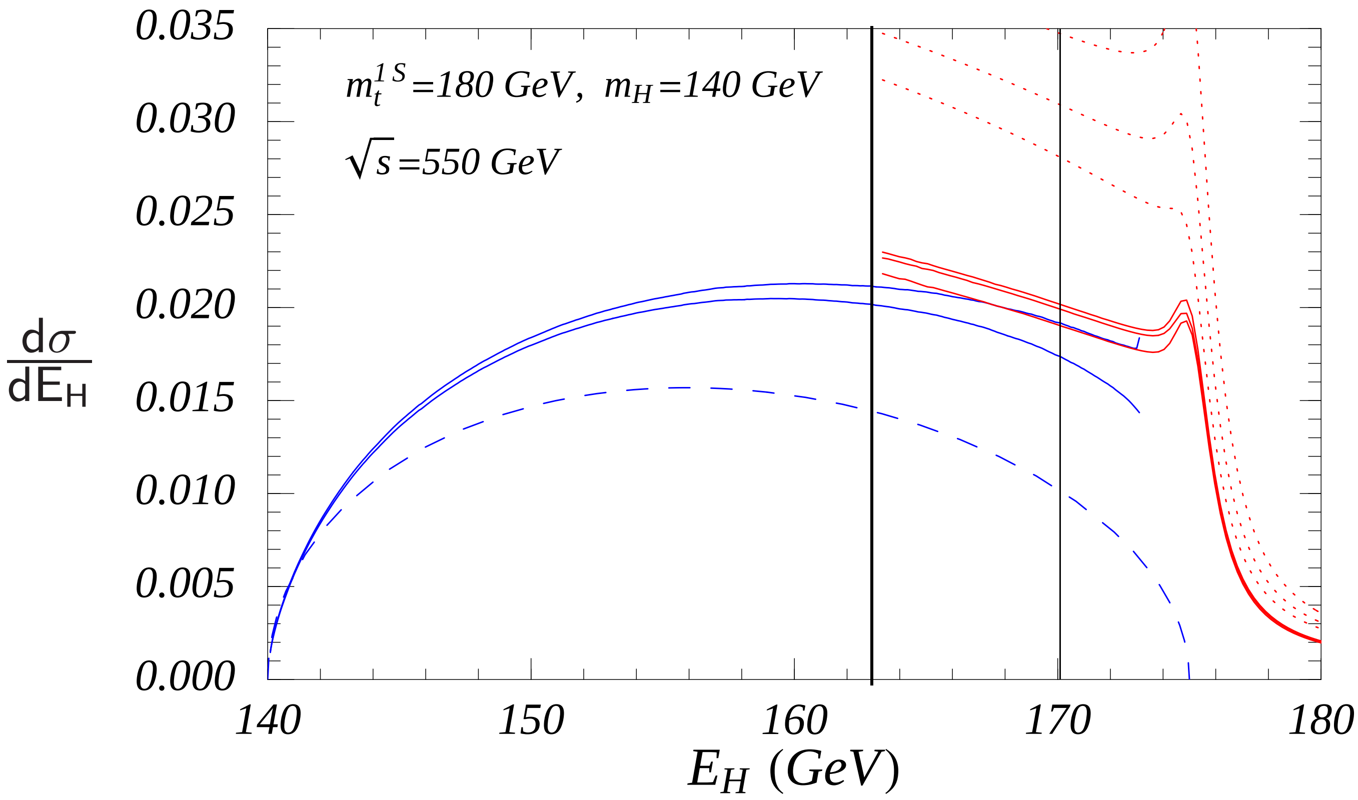
Differential Cross Section

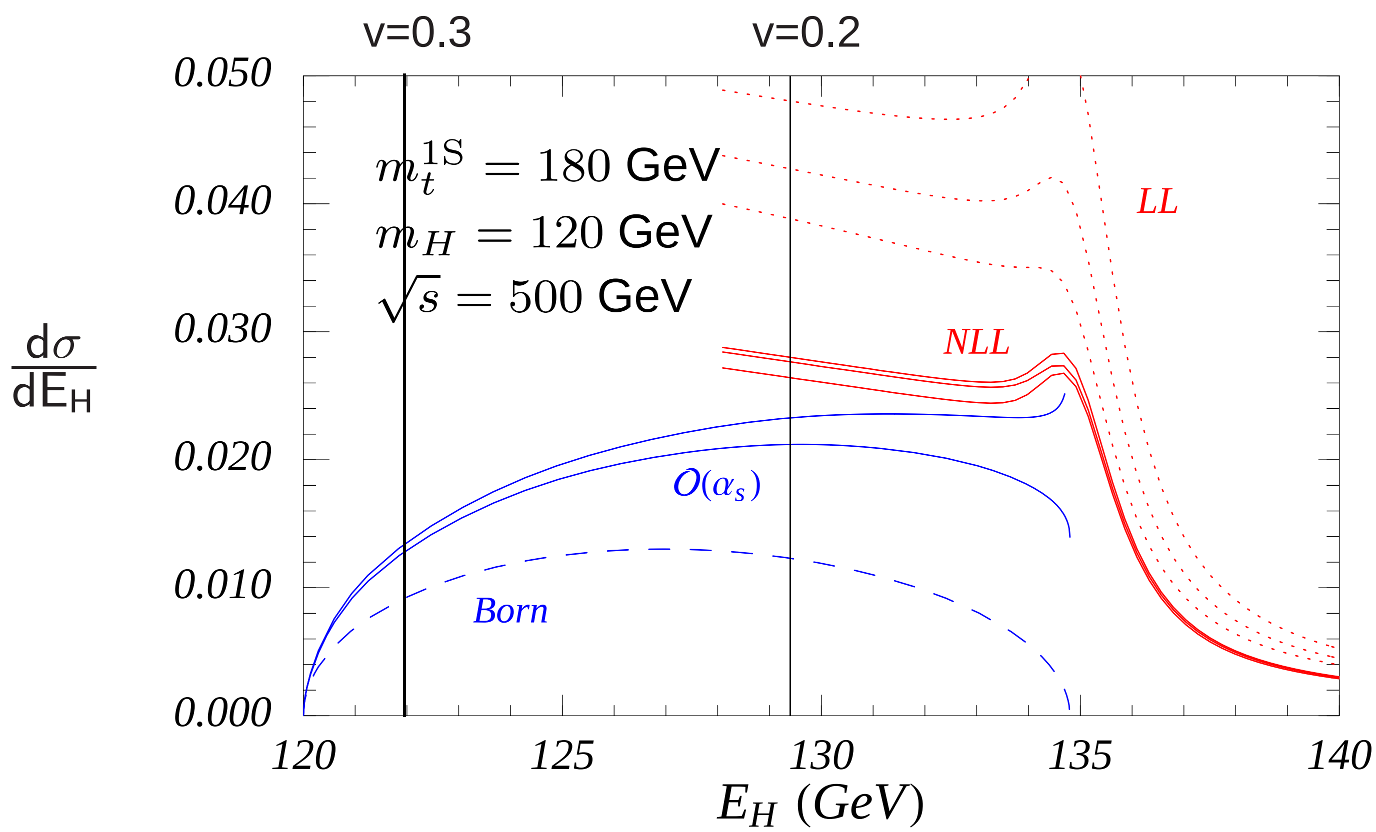
$\sqrt{s} = 550$ GeV



- In subleading order: Good transition around $\beta = 0.2$

Differential Cross Section

$\sqrt{s} = 500$ GeV



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ILC at 500 GeV

Maximum cms energy $\sqrt{s} = 500$ GeV

Little phase space $(2m_t + m_H \gtrsim 465\,\text{GeV})$

Dedicated analysis using the Born cross section:

$\delta Y_t / Y_t \approx 25\%$ (A. Juste, 2002)

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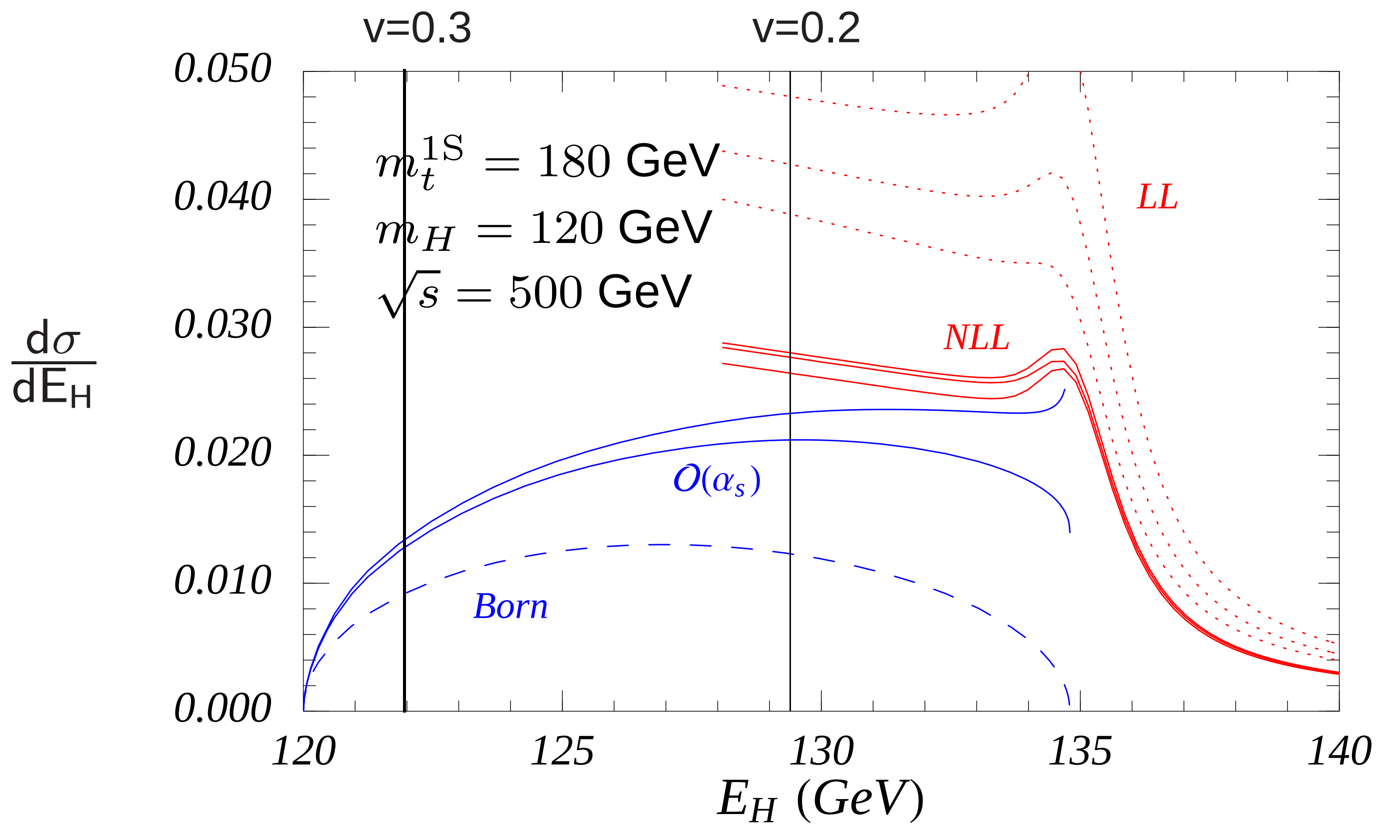
$$\left(\frac{d\sigma}{dE_H}\right)_{E_H \approx E_H^{\max}} \sim \left[f_0^2\, c_0^2(\mu, \sqrt{s}, m_t, m_H) + f_1^2\, c_1^2(\mu, \sqrt{s}, m_t, m_H)\right] \mathrm{Im}\, G^{\mathrm{NLL}}_{\mathrm{Coulomb}}(\mu, \sqrt{s}, m_t$$

$$\left(\frac{d\sigma_{\mathrm{Born}}}{dE_H}\right)_{E_H \rightarrow E_{H,\,\max}} \longrightarrow \frac{d\sigma_{\mathrm{Born}}}{dE_H}$$

$\Rightarrow$ Inclusion of the kinematical information in the Born CS

Differential Cross Section

$\sqrt{s} = 500$ GeV



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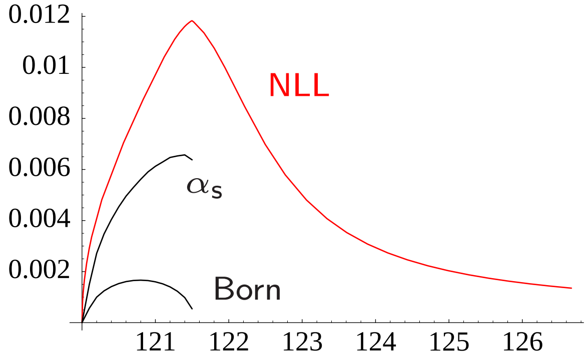
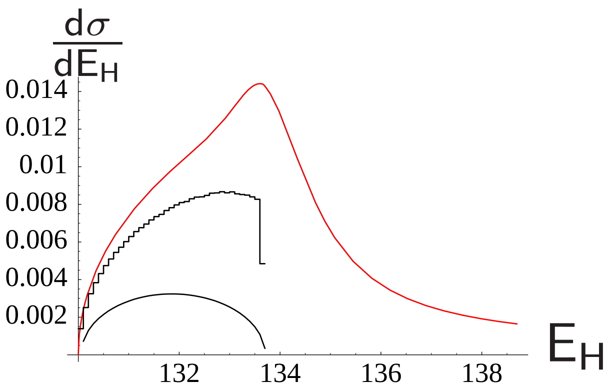
ILC at 500 GeV

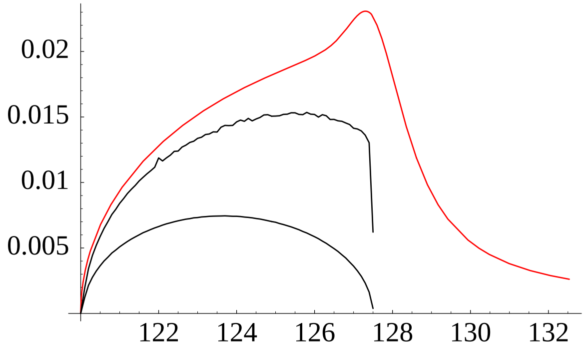
$\sqrt{s} = 482$ GeV, $m_H = 120$ GeV



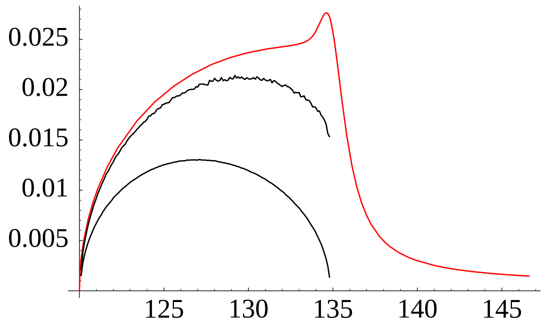
$\sqrt{s} = 500$ GeV, $m_H = 130$ GeV



$\sqrt{s} = 490$ GeV, $m_H = 120$ GeV



$\sqrt{s} = 500$ GeV, $m_H = 120$ GeV

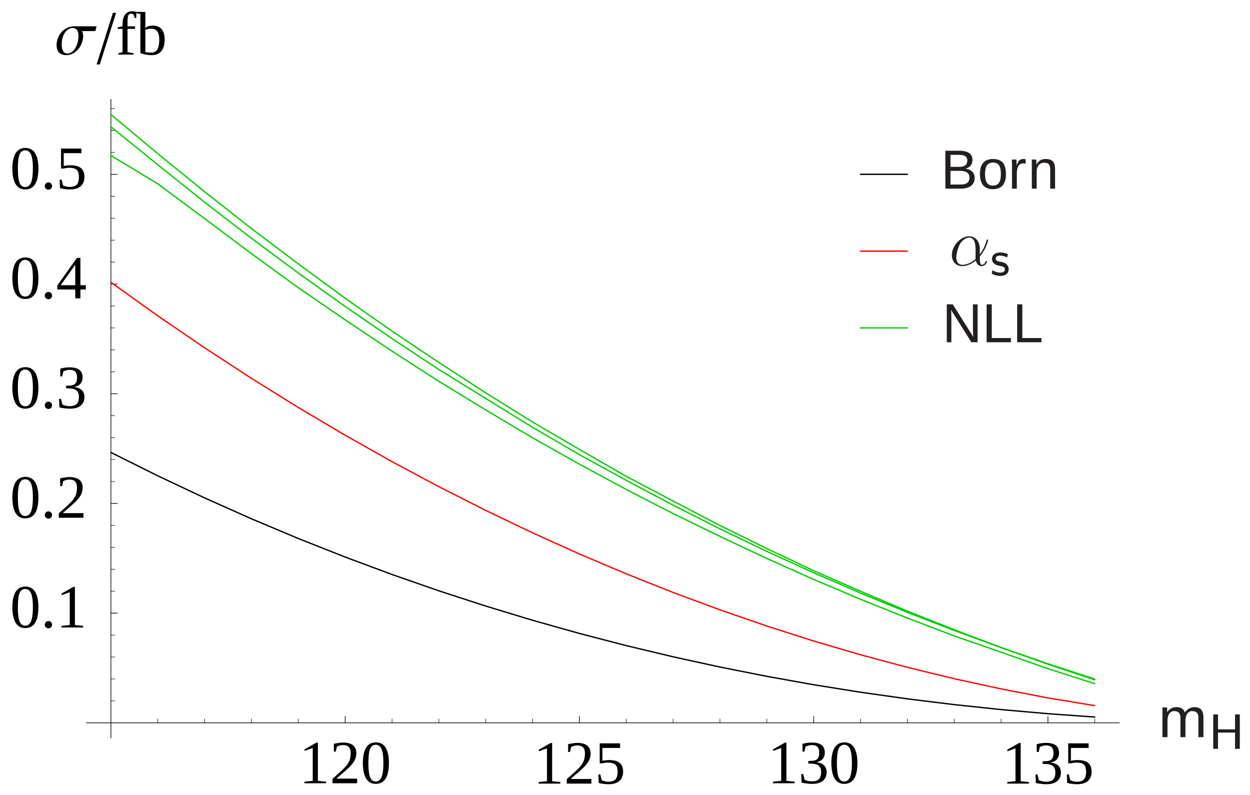


$\Rightarrow$ Behavior far from threshold is well reproduced

$\Rightarrow$ Increase of total cross section

ILC at 500 GeV

$\sqrt{s} = 500$ GeV, $m_t = 180$ GeV

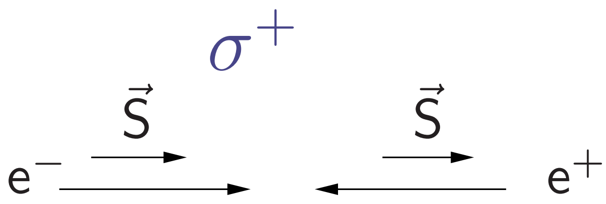


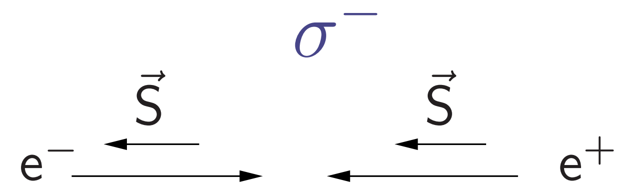
Cross section: NLL 2-4 times larger than Born

⇒ Measurement of Y_t is possible
for all accessible Higgs masses!

e^+e^- Polarization (preliminary)

Cross section depends on the helicity of e^+ and e^-:





Total cross section:

$$\begin{aligned}\sigma_{\text{pol}} &= \frac{1}{4}(1+P_+)(1-P_-)\,\sigma^+ + \frac{1}{4}(1-P_-)(1+P_+)\,\sigma^- \\ &= \sigma_{\text{unpol}}\left[1 - P_-\,P_+ - A_{LR}(P_+ - P_-)\right]\end{aligned}$$

$P_\pm$: degree of $e^\pm$ polarization

left-right asymmetry: $A_{LR} = \dfrac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}$

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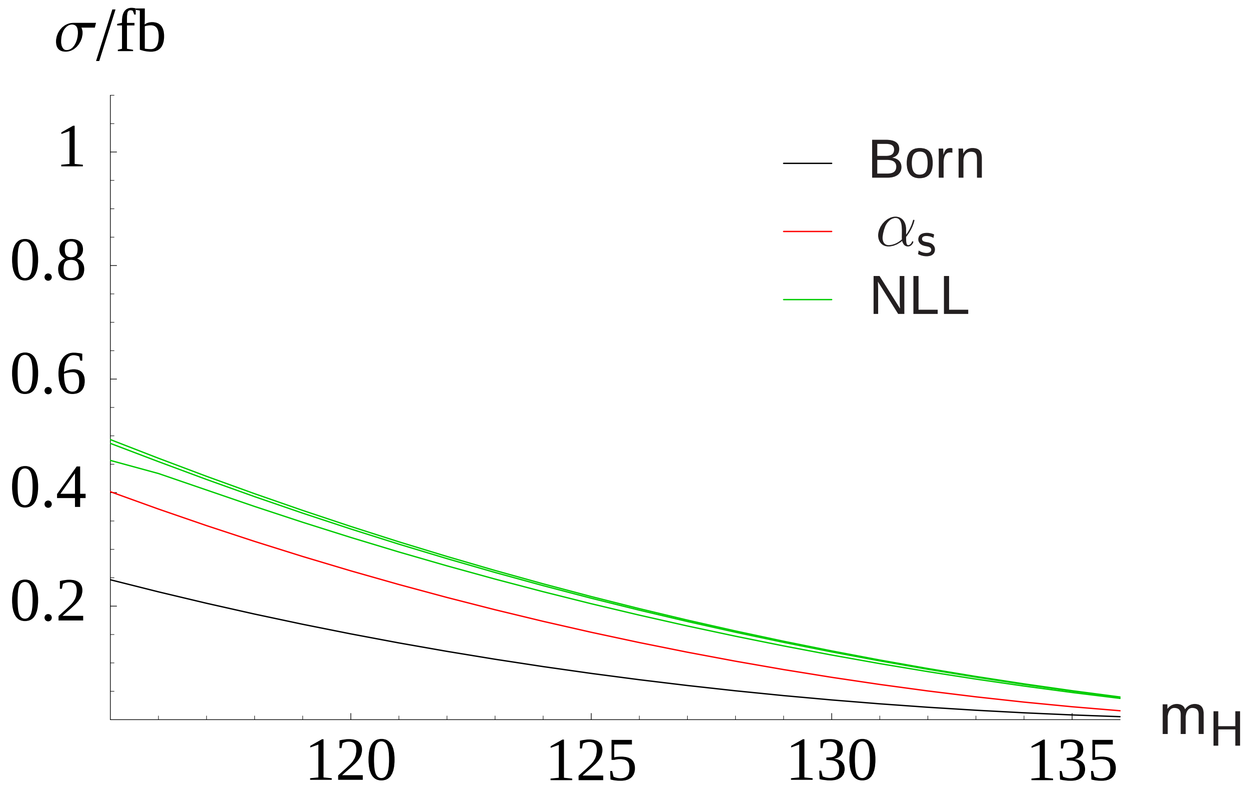
$\Rightarrow$ Increase of σ_{tot} by polarization possible

e^+e^- Polarization (preliminary)

Unpolarized e^+e^--beams:

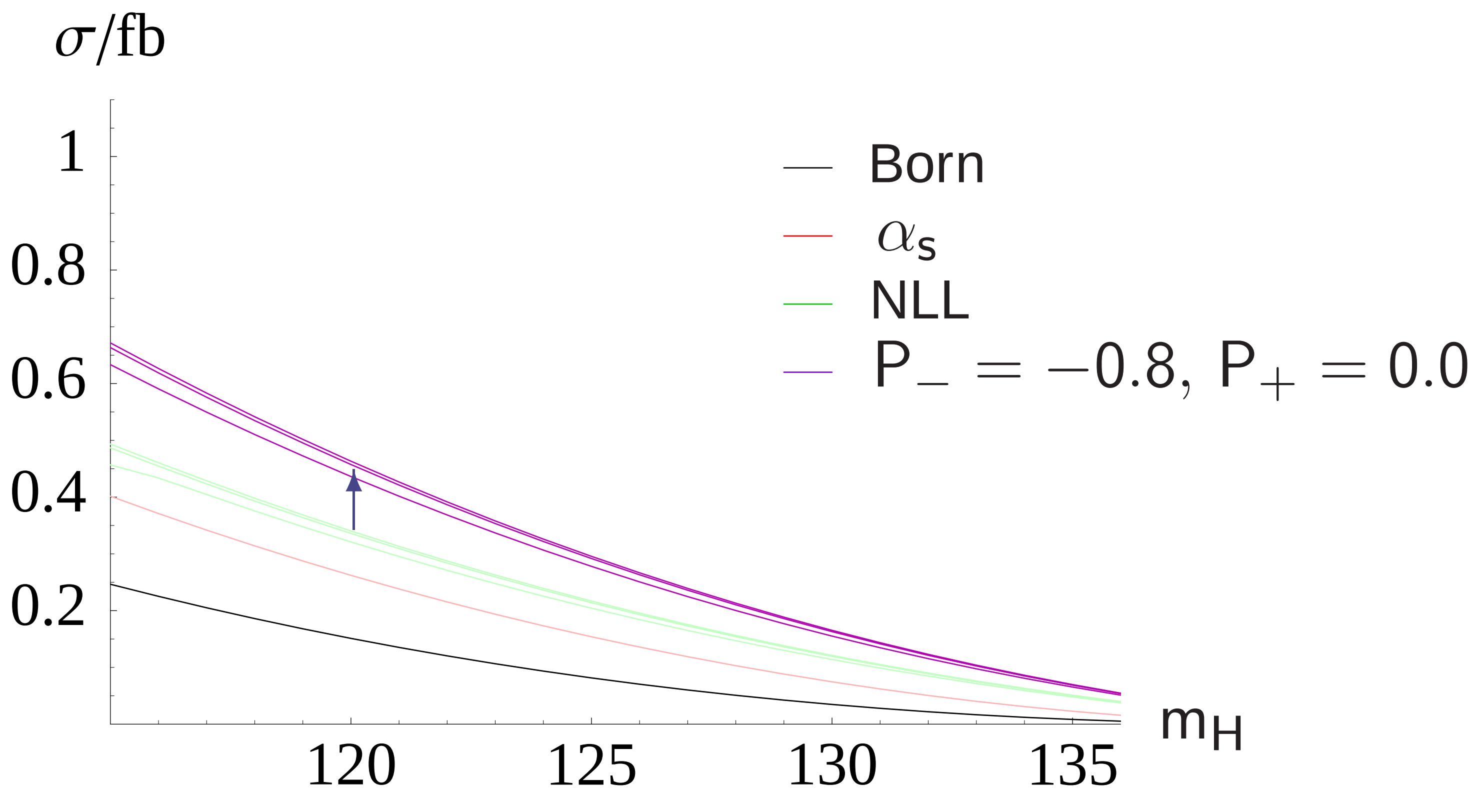
$\sqrt{s} = 500$ GeV, $m_t = 180$ GeV



e^+e^- Polarization (preliminary)

Polarized electron beam:

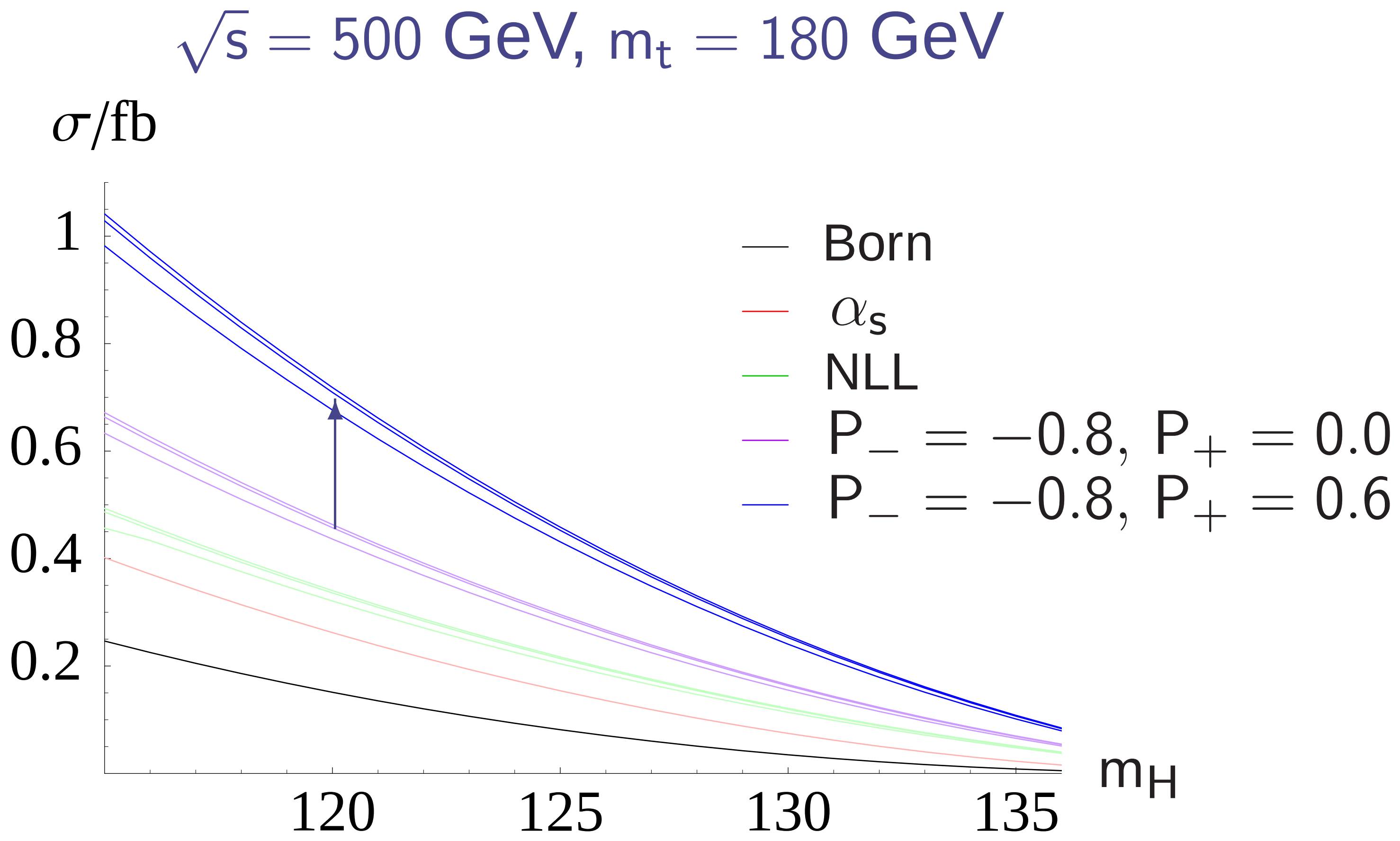
$\sqrt{s} = 500$ GeV, $m_t = 180$ GeV



- Increase of σ_{tot} by $\sim 40\%$

e^+e^- Polarization (preliminary)

Polarized e^+e^- - beams:

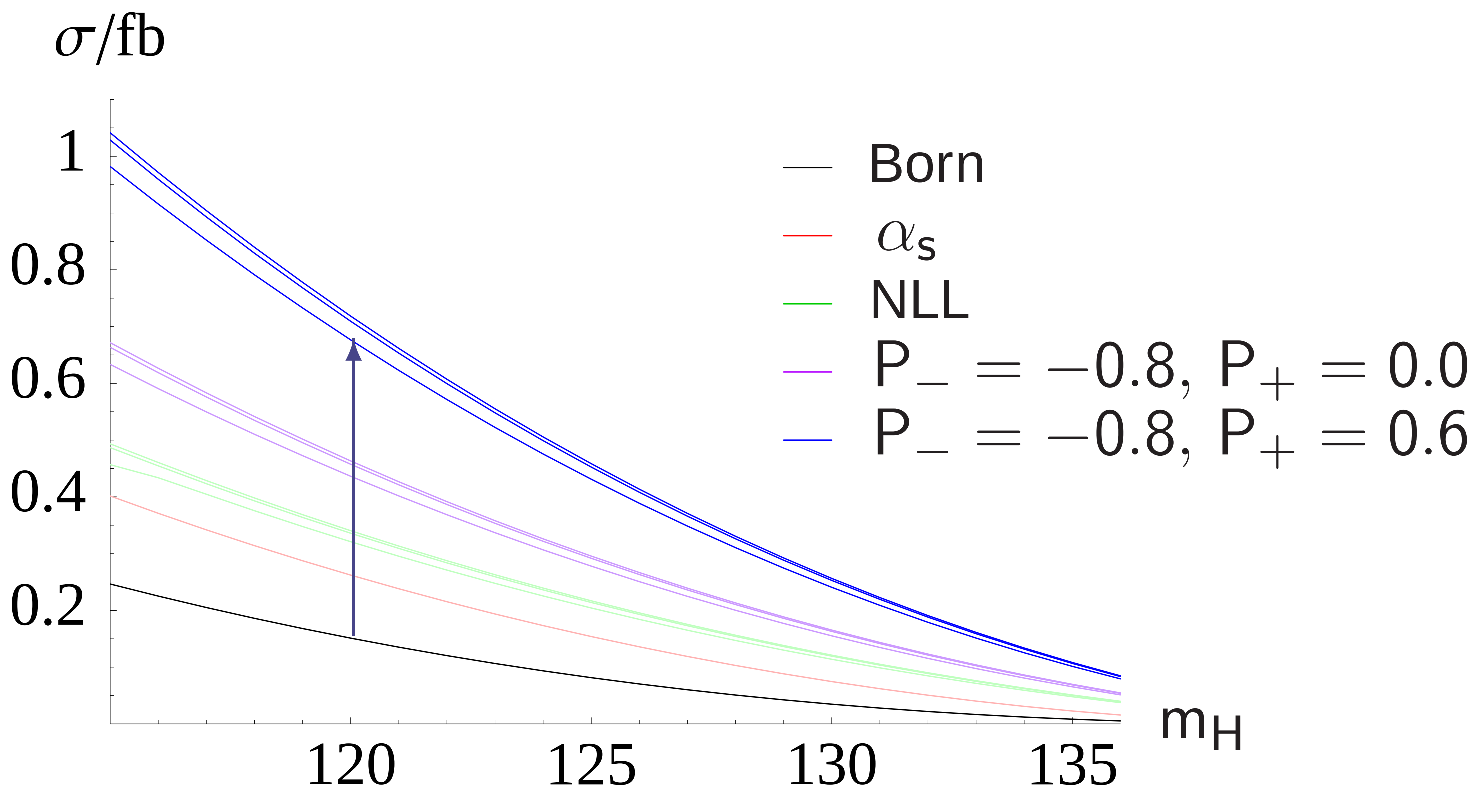


- Increase of σ_{tot} by $\sim$ 100% vs. $P_+ = 0$

e^+e^- Polarization (preliminary)

Polarized e^+e^-- beams:

$\sqrt{s} = 500$ GeV, $m_t = 180$ GeV



- Increase of σ_{tot} by $\sim$ 400% vs. Born Cross section
  $\Rightarrow$ Decrease of statistical error by 50% to $\sim$ 12%

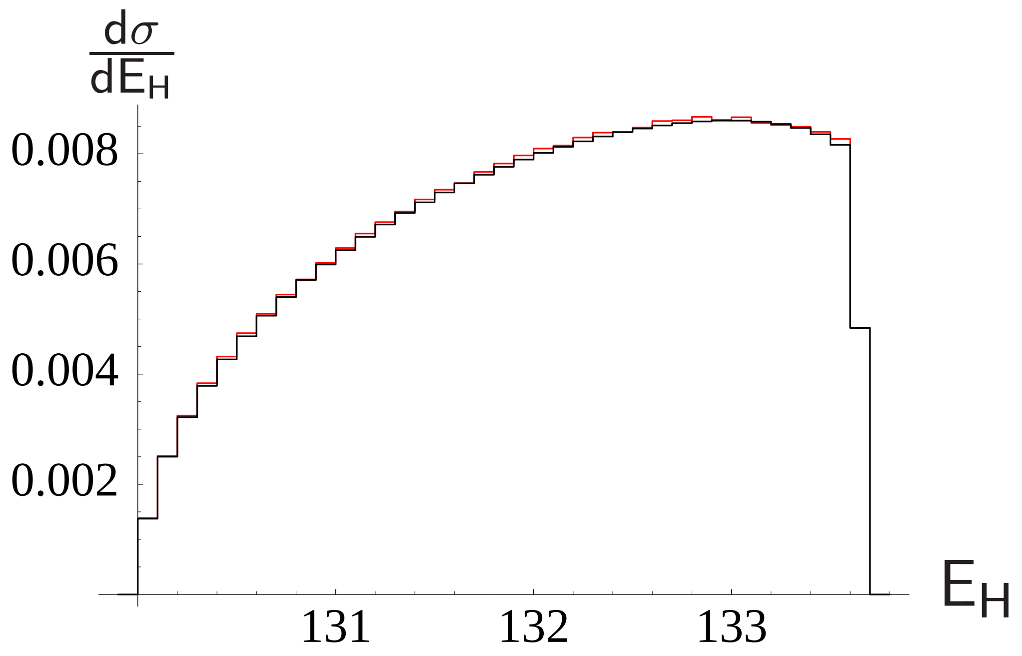
Conclusion

- Measurement of the top-Yukawa coupling for the verification of the Higgs mechanism
- First phase of the ILC
  - At energies of 500 GeV resummation must be included
  $\Rightarrow$ vNRQCD
- Calculation at NLL order
  - Scale dependence of the order of 3%
  - Large enhancement of the cross section
- Outlook: Inclusion of e^+e^- polarization
  - Increase of the total cross section
  - Y_t might be measurable to 10-15% at 500 GeV

Reproduction of the $\mathcal{O}(\alpha_s)$ Cross Section

Comparison: - Full $\mathcal{O}(\alpha_s)$ result
- NLL result at $\mathcal{O}(\alpha_s^1\, v^1)$

$\sqrt{s} = 495$ GeV , $m_t = 180$ GeV, $m_H = 125$ GeV



Difference: $\Rightarrow$ At threshold: $\sim$ Permille
$\Rightarrow$ Up to $\sqrt{s} = 700$ GeV: maximally $\sim 1.5\%$