

Top Instability and Electroweak Effects at the $t\bar{t}$ Production Threshold

Christoph Reißer

Max-Planck-Institut für Physik, Munich

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Outline

- Motivation
- Status for $\sigma_{\text{tot}}(e^+e^- \rightarrow t\bar{t})$ at threshold
- Non-relativistic QCD (vNRQCD)
- Effects of the top instability
- (Usual) electroweak effects
- Numerical analysis
- Summary and Outlook

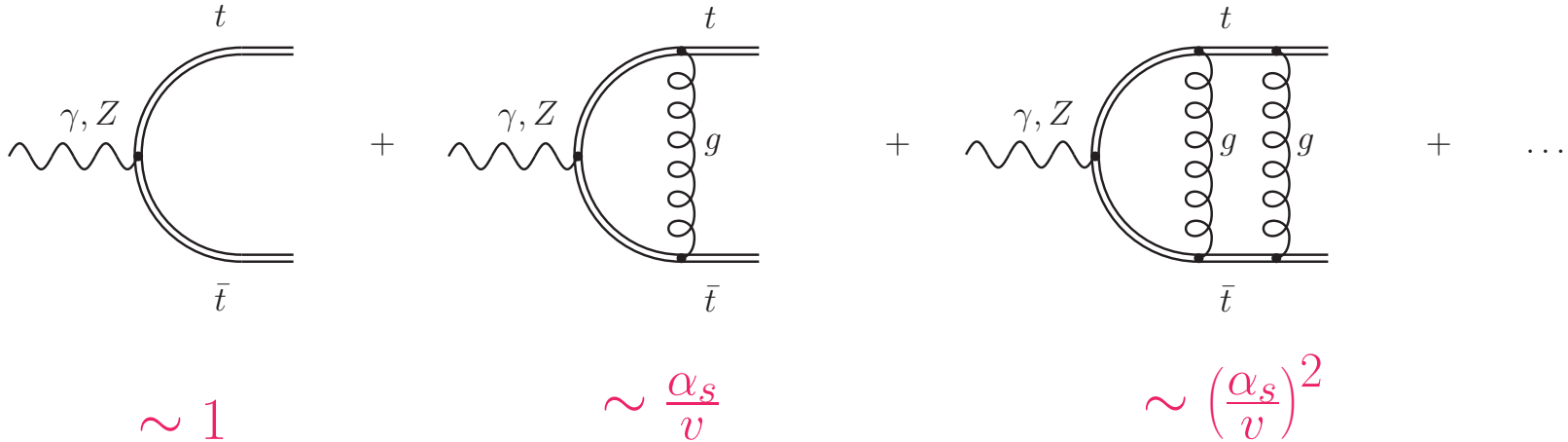


$t\bar{t}$ Production Threshold

e^+e^- collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are non-relativistic

$$v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1$$



⇒ Perturbation theory in α_s breaks down $v \sim \alpha_s$

⇒ Non-relativistic QCD \simeq Schrödinger theory at LO

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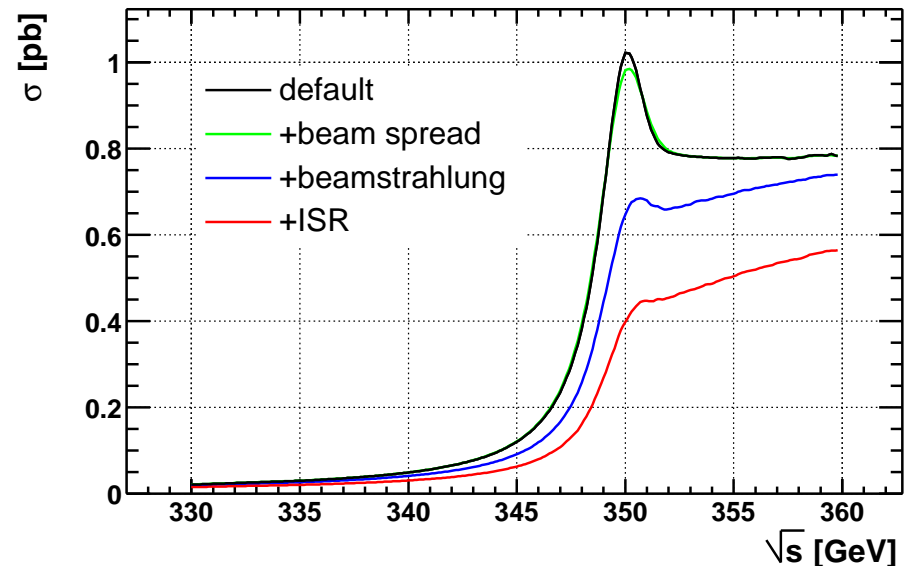
$$v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1$$

- Top quarks decay fast

$$\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}}$$

- ⇒ No bound states
- ⇒ Smooth line-shape
- ⇒ Non-perturbative effects suppressed

[Fadin,Khoze]



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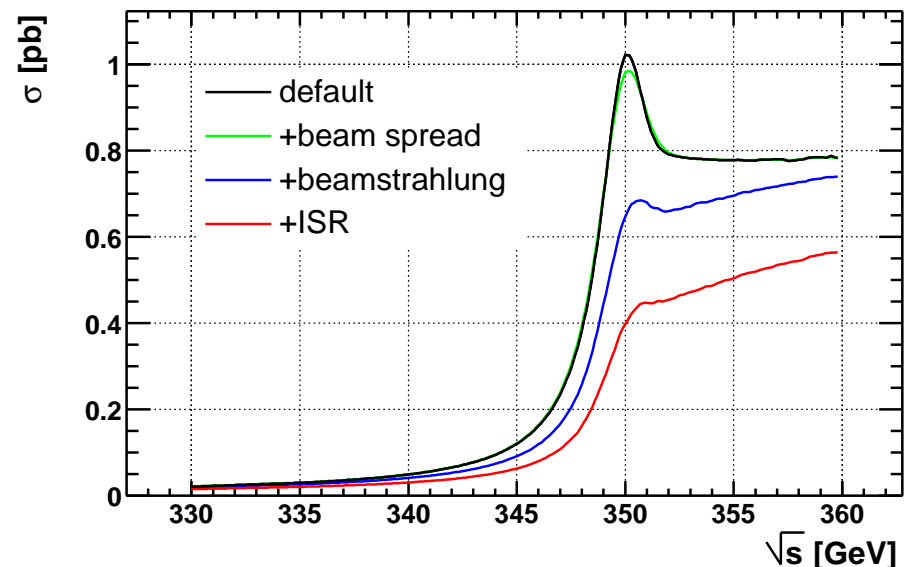
$$\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}}$$

- Measured cross section

$$\sigma^{\text{obs}}(s) = \int_0^1 dx \mathcal{L}(x) \sigma^{\text{theo}}(x^2 s)$$

contains

- beam spread
- beamstrahlung
- ISR



Measurements

Simulations of Threshold Scan ($\mathcal{L} \sim 300 \text{ fb}^{-1}$):
[Martinez, Miquel]

- Top quark mass

$$(\delta m_t)^{\text{exp}} \sim 50 \text{ MeV}$$

- Top Yukawa coupling

$$(\delta y_t / y_t)^{\text{exp}} \sim 0.35 \text{ (light Higgs)}$$

- Top decay width

$$(\delta \Gamma_t)^{\text{exp}} \sim 50 \text{ MeV}$$



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⇒ Theory needs

$$(\delta \sigma_{\text{tot}} / \sigma_{\text{tot}}) \leq 3\%$$



Theory Status

RGE methods to sum large logs $(\alpha_s \ln v)^m$

● QCD effects

LL ✓

NLL ✓

NNLL (nearly) $\rightarrow \left(\frac{\delta\sigma_{\text{tot}}}{\sigma_{\text{tot}}} \right)^{\text{NNLL}} \sim \pm 6\%$

$$\text{LL} \sim \left(\frac{\alpha_s}{v} \right)^n$$

$$\text{NLL} \sim \alpha_s \left(\frac{\alpha_s}{v} \right)^n$$

$$\text{NNLL} \sim \alpha_s^2 \left(\frac{\alpha_s}{v} \right)^n$$



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● Electroweak effects

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NLL (partly)

NNLL ?



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● Electroweak effects

LL ✓

NLL (partly) → New parametric NLL corrections

NNLL ? → NNLL top decay corrections



v NRQCD (stable quarks)

[Hoang, Luke, Manohar, Rothstein, Stewart]

Relevant scales

$$m_t \text{ (hard)} \gg \mathbf{p} \sim m_t v \text{ (soft)} \gg E \sim m_t v^2 \text{ (ultrasoft)}$$



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Lagrangian (LL) $\mathcal{L} = \mathcal{L}_{\text{bilinear}} + \mathcal{L}_{\text{potential}}$

$$\mathcal{L}_{\text{bilinear}} = \psi_{\mathbf{p}}^\dagger(\mathbf{x}) \left\{ iD^0 - \frac{\mathbf{p}^2}{2m_t} - \delta m_t \right\} \psi_{\mathbf{p}}(\mathbf{x}) + \dots$$

$$\mathcal{L}_{\text{potential}} = -\frac{\mathcal{V}_c(\mu)}{(\mathbf{p}-\mathbf{p}')^2} \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}} + \dots$$



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Lagrangian (LL)

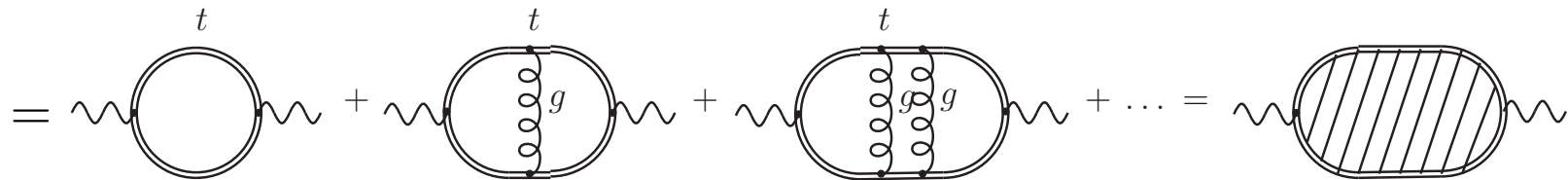
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⇒ Schrödinger equation with Coulomb potential

$$G^{\text{LL}}(0, 0, \sqrt{s}) = \frac{m_t^2}{4\pi} \left\{ i v - C_F \alpha_s \left[\frac{1}{4\epsilon} + \ln \left(\frac{-i m_t v}{\mu} \right) + \dots \right] \right\}$$



vNRQCD (stable quarks)

Currents for $t\bar{t}$ production & annihilation

$$\mathbf{O}_{\mathbf{p}} = C(\mu) \cdot (\psi_{\mathbf{p}}^{\dagger} \boldsymbol{\sigma} \tilde{\chi}_{-\mathbf{p}}^*) \quad ({}^3S_1)$$



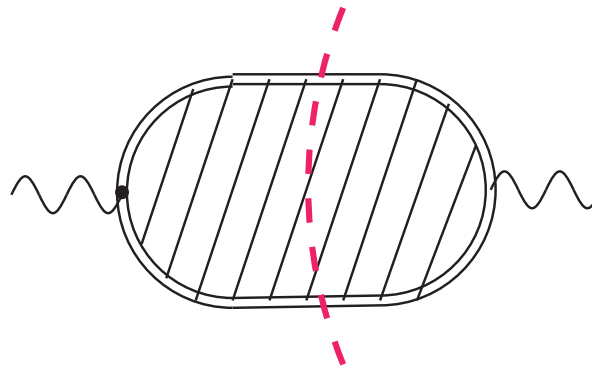
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Total cross section from $e^+e^- \rightarrow e^+e^-$ using the **Optical Theorem**
[Strassler, Peskin]

$$\begin{aligned} \sigma_{\text{tot}} &\propto \text{Im} \left[\int d^4x e^{-i\hat{q}\cdot x} \langle 0 | T \mathbf{O}_{\mathbf{p}}^{\dagger}(0) \mathbf{O}_{\mathbf{p}'}(x) | 0 \rangle \right] \\ &\propto \text{Im} [C(\mu)^2 G(0, 0, \sqrt{s})] \end{aligned}$$



v NRQCD (unstable quarks)

Optical Theory of absorptive medium

- Complex refractive indices in the Maxwell equations
- Can be derived systematically from microscopic processes



ν NRQCD (unstable quarks)

Optical Theory of **absorptive** medium

- Complex refractive indices in the Maxwell equations
- Can be derived systematically from microscopic processes

ν NRQCD accounting for $t \rightarrow Wb$

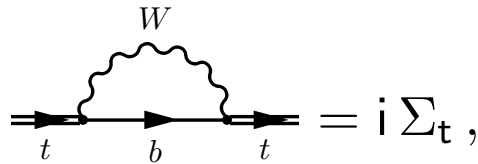
- Wilson coefficients become complex
- Obtained by matching effective theory to SM
- Effective Lagrangian non-Hermitian
→ Total cross section through the optical theorem



vNRQCD (unstable quarks)

Matching conditions accounting for $t \rightarrow Wb$

Bilinear quark operators



The diagram shows a top quark line (represented by a double arrow) entering from the left, splitting into a top quark and a bottom quark. The top quark and bottom quark are connected by a W boson loop (represented by a wavy line). The bottom quark then recombines with the top quark to form a top quark line exiting to the right. The diagram is labeled with 't' at the start and end, and 'b' and 'W' in the middle.

$$= i \Sigma_t,$$

$$\text{Im } \Sigma_t = \frac{1}{2} \Gamma_t$$

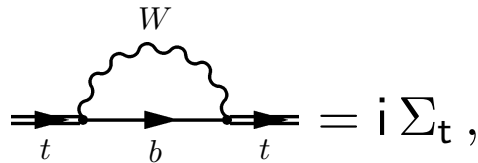
$$\Rightarrow \delta \mathcal{L} = \psi_{\mathbf{p}}^\dagger \left[i \frac{\Gamma_t}{2} \left(1 - \frac{\mathbf{p}^2}{2m_t^2} \right) \right] \psi_{\mathbf{p}}$$



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$$\text{Im } \Sigma_t = \frac{1}{2} \Gamma_t$$

$$\Rightarrow \delta \mathcal{L} = \psi_p^\dagger \left[\underset{\text{LL}}{\underbrace{i \frac{\Gamma_t}{2}}_{\text{LL}}} \left(1 - \frac{\mathbf{p}^2}{2m_t^2} \right) \right] \psi_p$$

NNLL

Power counting for ew. effects: $\Gamma_t \sim m_t g^2 \sim m_t \alpha_s^2 \Rightarrow g \sim g' \sim v \sim \alpha_s$

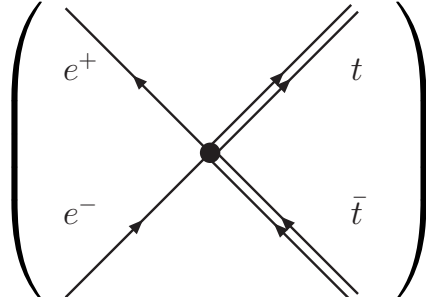
- $E \rightarrow E + i\Gamma_t$ replacement to include finite lifetime at **LL** [Fadin, Khoze] (not sufficient beyond LL order)
- Time dilatation is **NNLL**



vNRQCD (unstable quarks)

Matching conditions accounting for $t \rightarrow Wb$

Currents for $t\bar{t}$ production & annihilation

$$\mathbf{O}_p = \left[C^{\text{LL}} + C^{\text{NLL}} + C^{\text{NNLL}} + i C_{\text{abs}}^{\text{NNLL}} + \dots \right] \cdot \left(\begin{array}{c} e^+ \\ e^- \\ t \\ \bar{t} \end{array} \right) + \dots$$
A Feynman diagram enclosed in large parentheses. On the left side, two lines enter from the bottom: the top one is labeled e^+ and the bottom one is labeled e^- . On the right side, two lines exit: the top one is labeled t and the bottom one is labeled \bar{t} . The lines cross at a central black dot. The top-left and bottom-right lines are single lines, while the top-right and bottom-left lines are double lines.

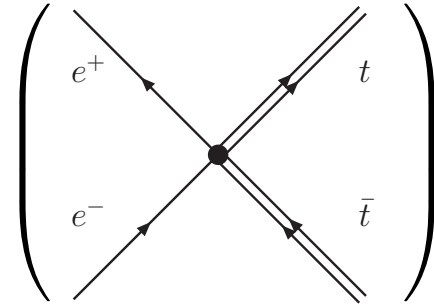
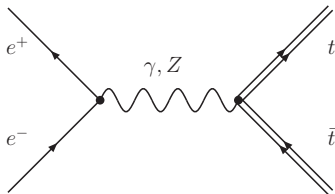


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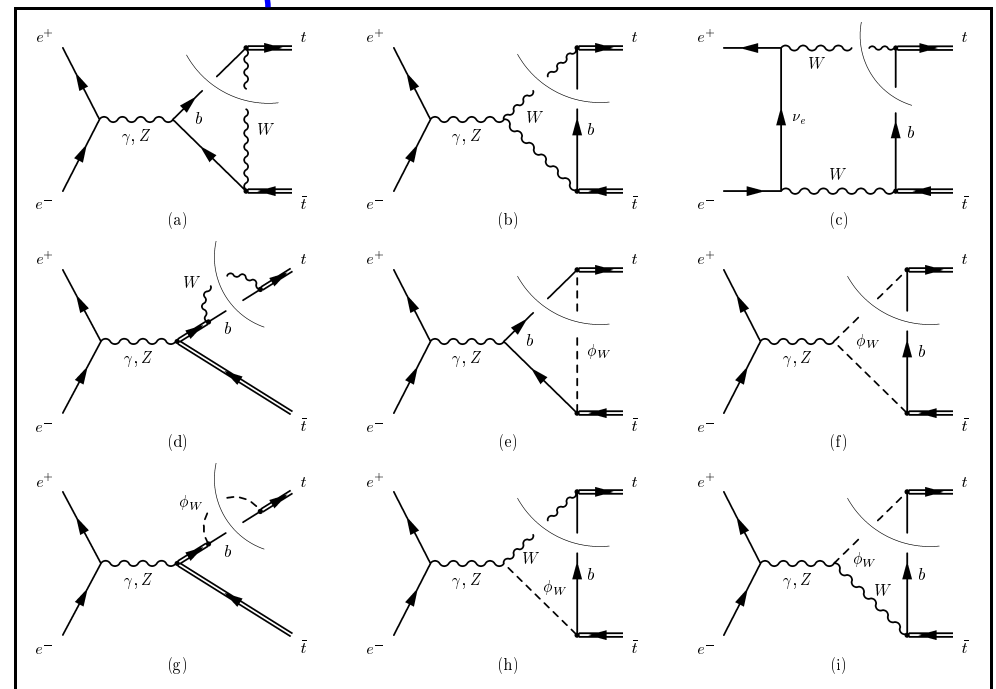
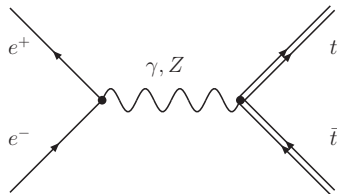


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- > Wb-cuts account for Wb final states
- > Wb-cuts are gauge invariant



vNRQCD (unstable quarks)

Interactions in vNRQCD constrained by **symmetries**

Ward Identity

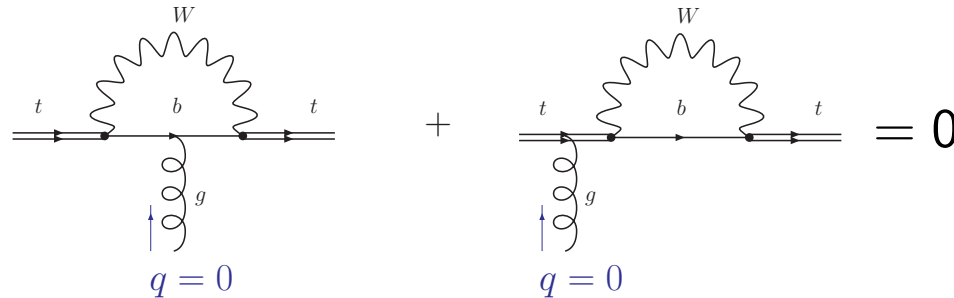
The diagram shows two Feynman diagrams representing the Ward identity. Each diagram features a top quark line (t) with a bottom quark loop (b) and a gluon emission (g) from the loop. The gluon momentum is $q = 0$. The diagrams are summed to zero, as indicated by the equation $= 0$.



vNRQCD (unstable quarks)

Interactions in vNRQCD constrained by **symmetries**

Ward Identity



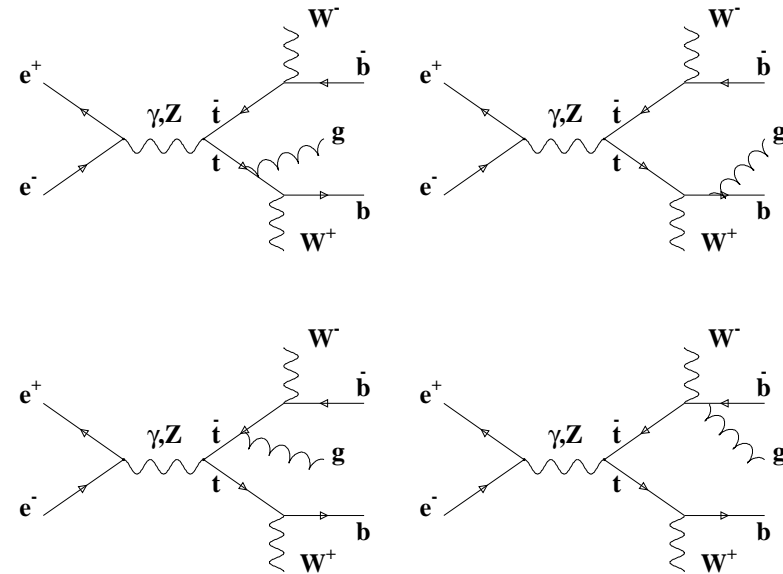
Potentials:

→ no NNLL electroweak corrections

Soft, ultrasoft interactions:

→ no NNLL electroweak corrections

→ also: no non-factorizable effects from ultrasoft gluons [Melnikov, Yakovlev]



Total Cross Section

Optical Theorem \Rightarrow $\sigma_{\text{tot}} = 2 N_c \text{Im} [C(\mu)^2 G(0, 0, \sqrt{s})]$

- NNLL finite lifetime correction:

$$\Delta^{\Gamma,1} \sigma_{\text{tot}} = 2 N_c \text{Im} \left[2 C^{\text{LL}} i C_{\text{abs}}^{\text{NNLL}} G^{\text{LL}}(0, 0, \sqrt{s}) + (C^{\text{LL}})^2 \delta G^{\text{NNLL}}(0, 0, \sqrt{s}) \right]$$

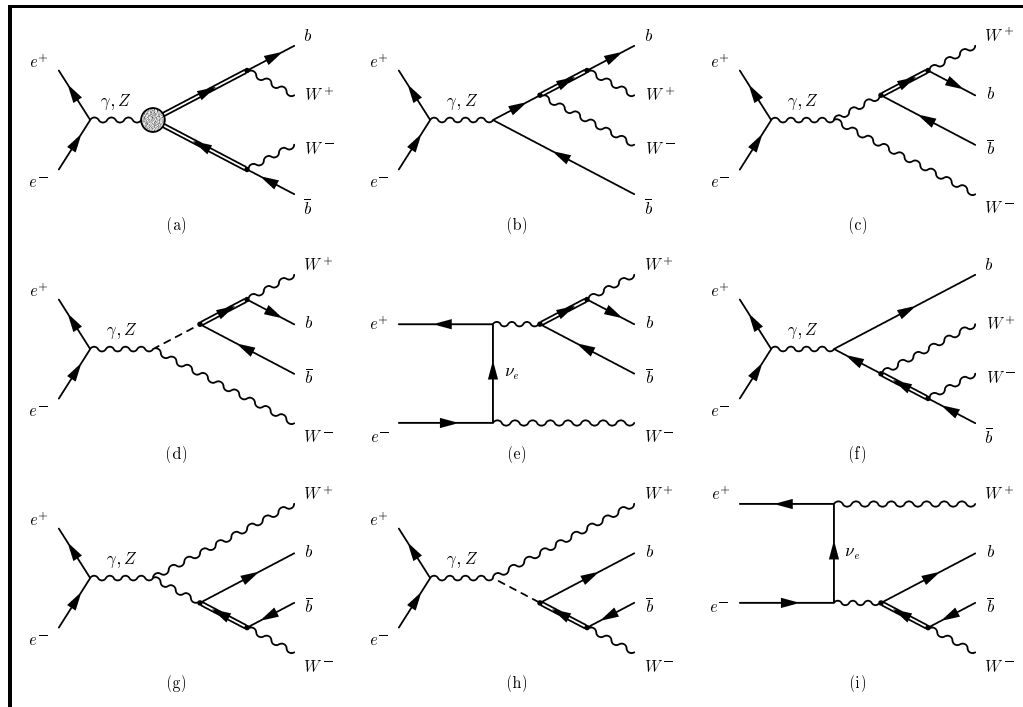


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dilatation of the lifetime



Total Cross Section

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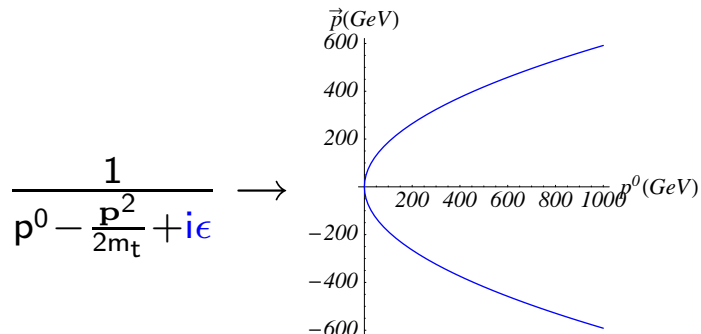
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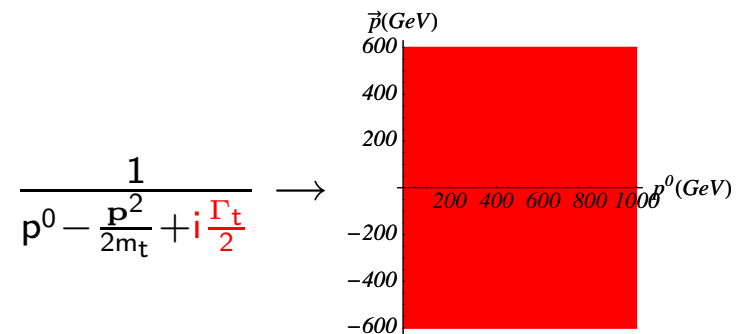
- NLL mixing effect: $\Delta^{\Gamma,1} \sigma_{\text{tot}}$ contains

$$\Gamma_t \alpha_s \frac{1}{\epsilon} \quad (\text{logarithmic UV phase space divergence due to finite lifetime})$$

Phase space:
stable tops



unstable tops



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- NLL mixing effect: $\Delta^{\Gamma,1} \sigma_{\text{tot}}$ contains

$$\Gamma_t \alpha_s \frac{1}{\epsilon} \quad (\text{logarithmic UV phase space divergence due to finite lifetime})$$

\Rightarrow Anomalous dimension for $(e^+ e^-)(e^+ e^-)$ operator:

$$i \tilde{C}^{\text{eeee}}(\mu) \cdot \left(\begin{array}{cc} e^+ & e^- \\ e^- & e^+ \end{array} \right)$$

\Rightarrow Correction $\Delta^{\Gamma,2} \sigma_{\text{tot}}$

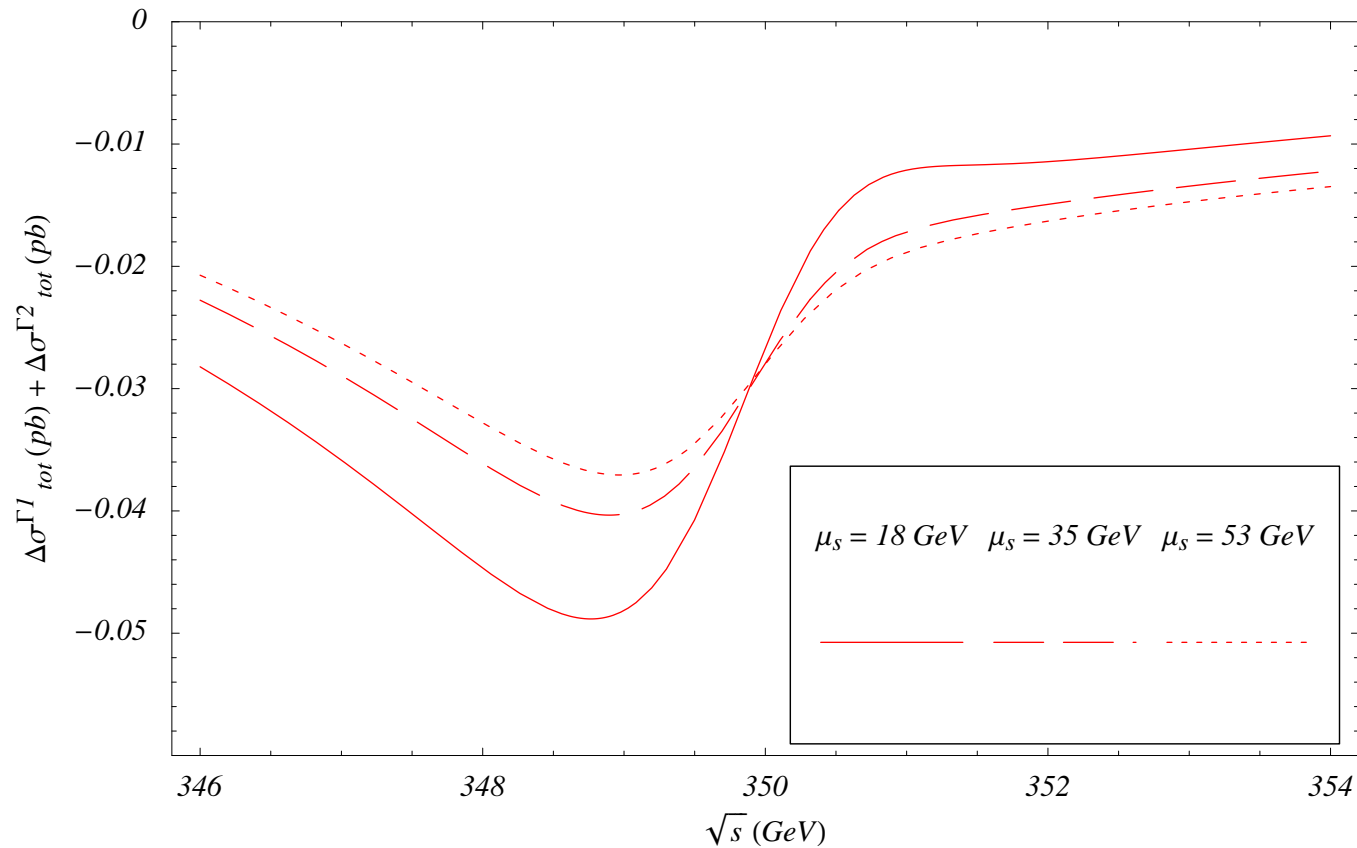
- \sqrt{s} -independent
- scale-dependent

* Matching coefficient

$\tilde{C}^{\text{eeee}}(\mu = m_t)$ not yet determined \rightarrow w.i.p.



Absorptive Parts

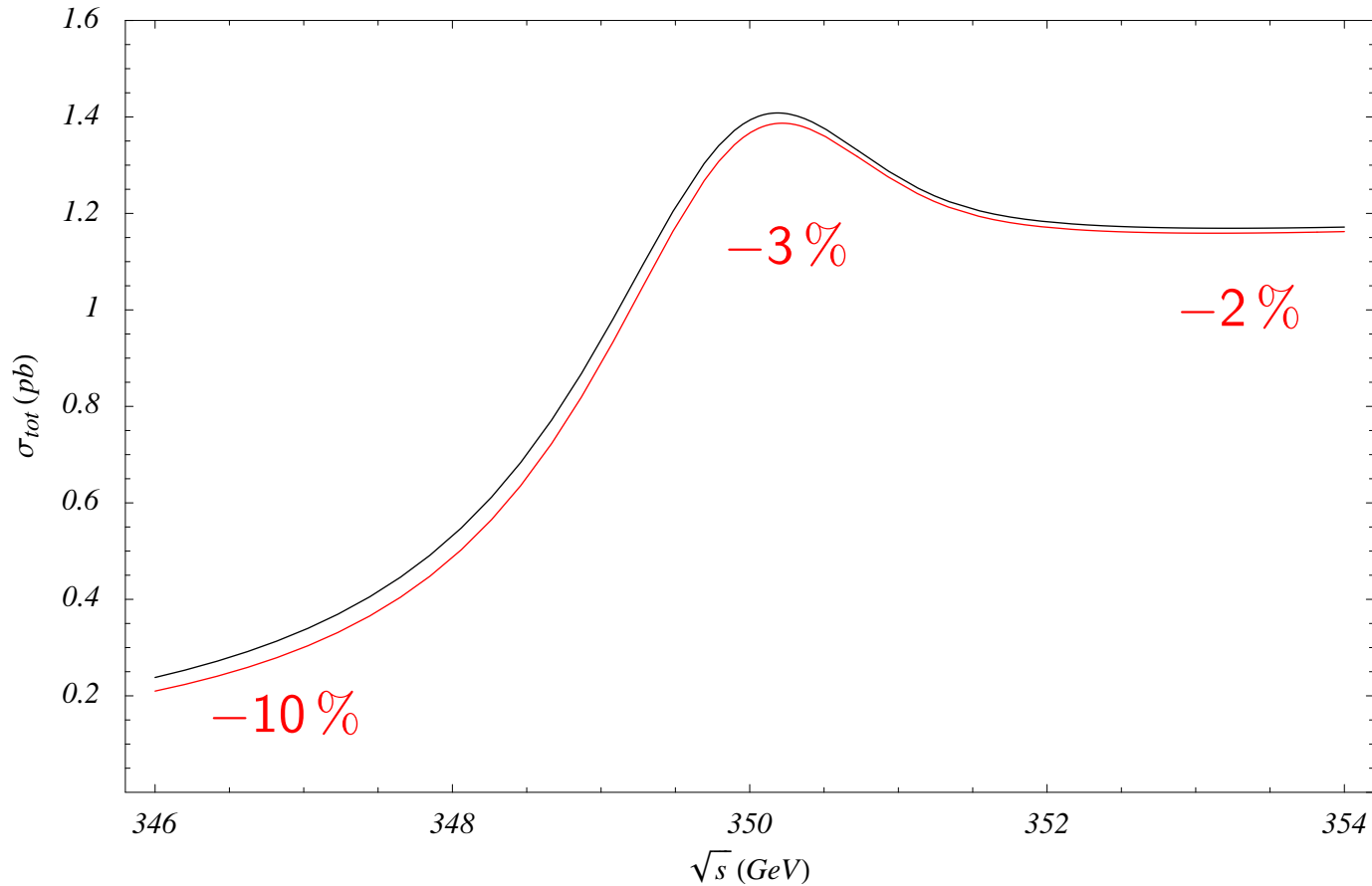


⇒ Comparable to NNLL QCD corrections

⇒ LL peak position shifted by 30 – 50 MeV



Absorptive Parts



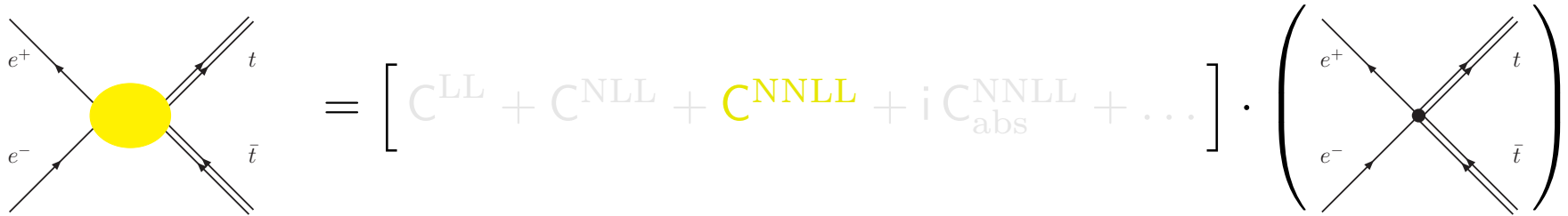
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Electroweak Corrections (preliminary)

Electroweak short distance corrections: Real parts C^{NNLL}



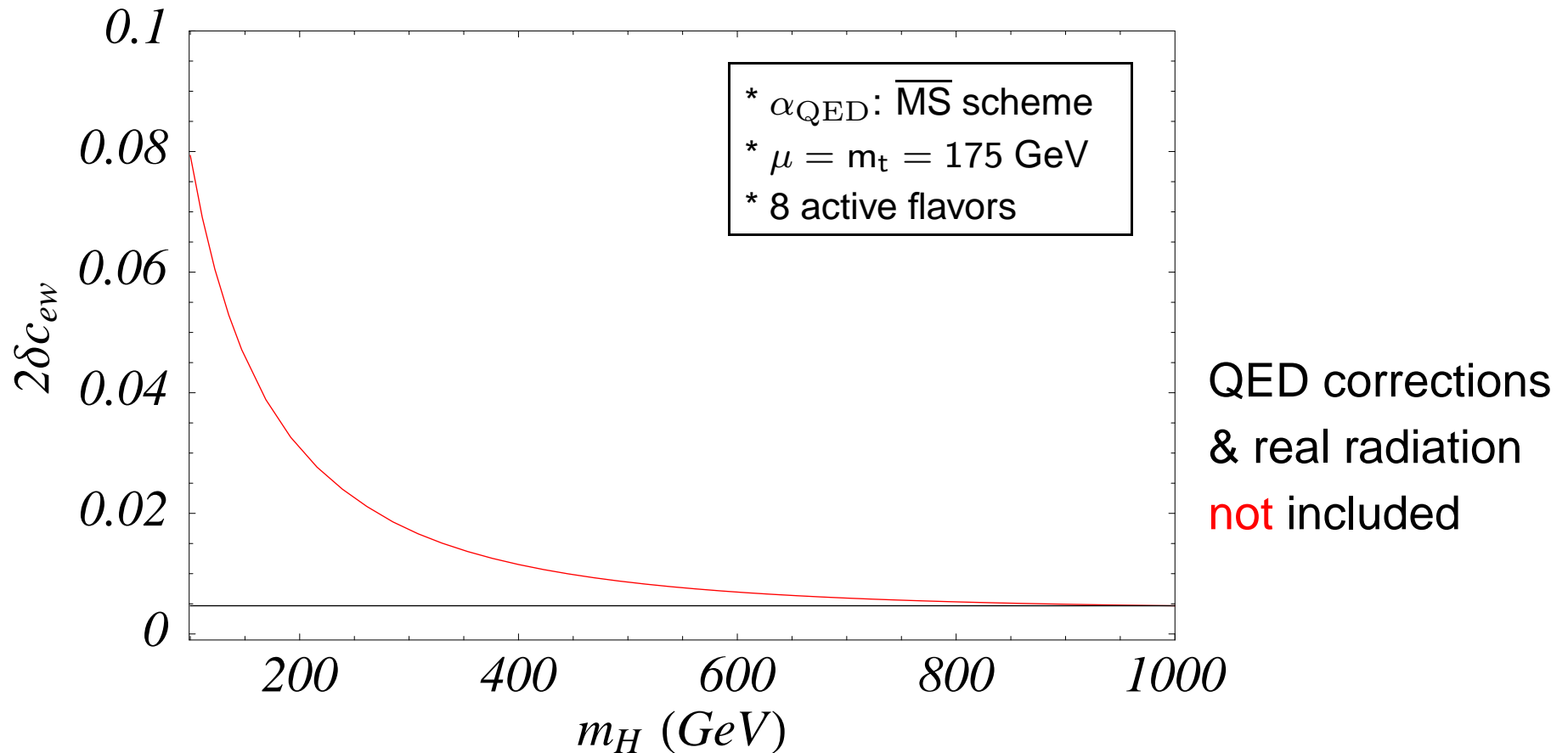
The diagrammatic equation shows a scattering process with incoming particles e^+ and e^- and outgoing particles t and \bar{t} . On the left, a yellow circle represents the full electroweak theory one-loop diagrams. This is equated to a bracketed sum of coefficients: $C^{\text{LL}} + C^{\text{NLL}} + C^{\text{NNLL}} + iC_{\text{abs}}^{\text{NNLL}} + \dots$. The C^{NNLL} term is highlighted in yellow. This sum is multiplied by a diagram in large parentheses, which is a tree-level diagram with a black dot at the vertex, representing the leading-order process.

- Full electroweak theory one-loop diagrams $O(\alpha_{\text{em}}) \rightarrow$
NNLL
- Sizable shift of total cross section normalization
- **Pure QED & ISR diagrams** require separate treatment (SCET) \rightarrow w.i.p.



Electroweak Corrections (preliminary)

Correction to the total cross section normalization: $(1 + 2\delta c_{ew}) \sigma_{tot}$



Summary and Outlook

Summary

- Threshold scan allows for precise m_t, y_t, Γ_t determination
- Effective theory approach (unstable particles)
 - UV divergencies from instability
 - Interference of resonant and non-resonant diagrams
 - Symmetries restrict interactions
 - Short distance electroweak corrections
- Corrections comparable to QCD

Outlook

- Phase space matching $\tilde{C}^{eeee}(\mu = m_t)$
- NNLL running of $\tilde{C}^{eeee}(\mu)$
- QED contributions: ISR, beamstrahlung, Coulomb singularities

