Top Instability and Electroweak Effects at the $t\bar{t}$ Production Threshold

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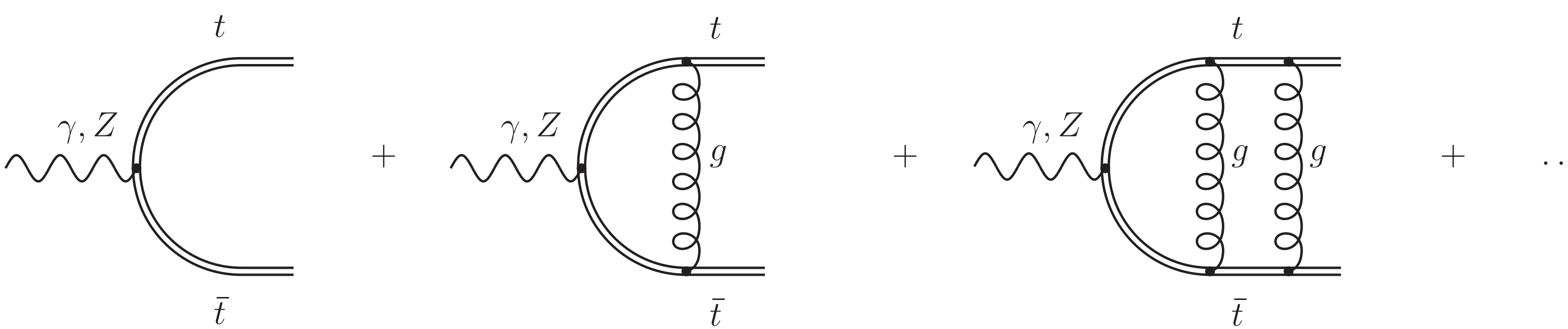
Outline

- Motivation
- Status for $\sigma_{\mathrm{tot}}(\mathrm{e}^+\mathrm{e}^- \to \mathrm{t}\bar{\mathrm{t}})$ at threshold
- Non-relativistic QCD (vNRQCD)
- Effects of the top instability
- (Usual) electroweak effects
- Numerical analysis
- Summary and Outlook

$t\bar{t}$ Production Threshold

<u>e^+e^- collisions:</u> c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are non-relativistic $\boxed{v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1}$



~ 1 $\qquad$ $\sim \frac{\alpha_s}{v}$ $\qquad$ $\sim \left(\frac{\alpha_s}{v}\right)^2$

$\Rightarrow$ Perturbation theory in α_s breaks down $\boxed{v \sim \alpha_s}$

$\Rightarrow$ Non-relativistic QCD $\simeq$ Schrödinger theory at LO

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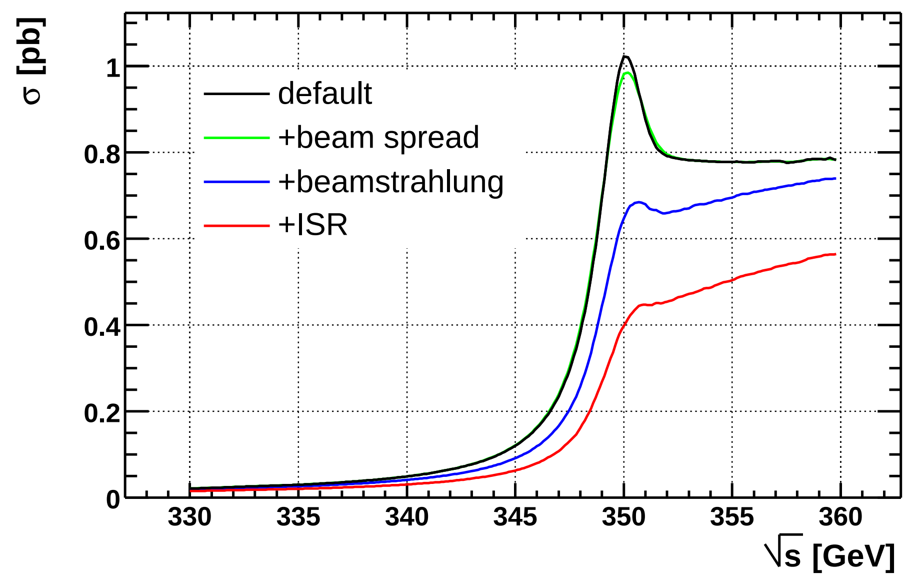
- Top quarks decay fast $\boxed{\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{\text{QCD}}}$

  $\Rightarrow$ No bound states

  $\Rightarrow$ Smooth line-shape

  $\Rightarrow$ Non-perturbative effects suppressed

  [Fadin,Khoze]



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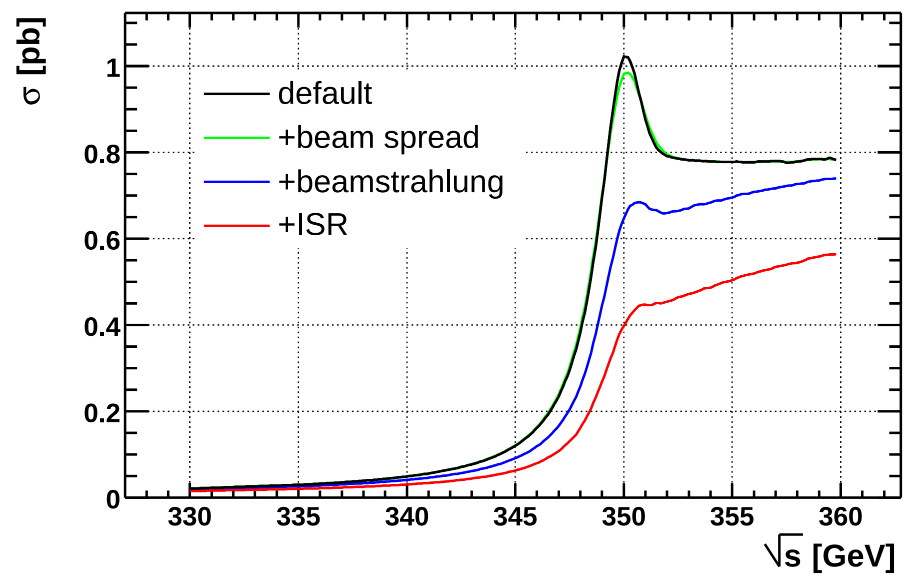
- Top quarks decay fast $\boxed{\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{QCD}}$
- Measured cross section $\boxed{\sigma^{obs}(s) = \int_0^1 dx\, \mathcal{L}(x) \sigma^{theo}(x^2 s)}$ contains
  - beam spread
  - beamstrahlung
  - ISR



Measurements

Simulations of Threshold Scan ($\mathcal{L} \sim 300\,\mathrm{fb}^{-1}$):
[Martinez, Miquel]

- Top quark mass

  $(\delta m_t)^{\mathrm{exp}} \sim 50$ MeV

- Top Yukawa coupling

  $(\delta y_t / y_t)^{\mathrm{exp}} \sim 0.35$ (light Higgs)

- Top decay width

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$\Rightarrow$ Theory needs

$$\boxed{(\delta\sigma_{\mathrm{tot}}/\sigma_{\mathrm{tot}}) \leq 3\,\%}$$

Theory Status

RGE methods to sum large logs $(\alpha_s \ln v)^m$

- QCD effects

LL ✓ | LL $\sim \left(\frac{\alpha_s}{v}\right)^n$

NLL ✓ | NLL $\sim \alpha_s \left(\frac{\alpha_s}{v}\right)^n$

NNLL (nearly) $\rightarrow \left(\frac{\delta\sigma_{\mathrm{tot}}}{\sigma_{\mathrm{tot}}}\right)^{\mathrm{NNLL}} \sim \pm 6\,\%$ | NNLL $\sim \alpha_s^2 \left(\frac{\alpha_s}{v}\right)^n$

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  NLL (partly)

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- Electroweak effects

LL ✓

NLL (partly) $\rightarrow$ New parametric NLL corrections

NNLL ? $\rightarrow$ NNLL top decay corrections

vNRQCD (stable quarks)

[Hoang, Luke, Manohar, Rothstein, Stewart]

Relevant scales

$$m_t \text{ (hard)} \quad \gg \quad \mathbf{p} \sim m_t v \text{ (soft)} \quad \gg \quad E \sim m_t v^2 \text{ (ultrasoft)}$$

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Lagrangian (LL) $\quad \mathcal{L} = \mathcal{L}_{\text{bilinear}} + \mathcal{L}_{\text{potential}}$

$$\mathcal{L}_{\text{bilinear}} = \psi_{\mathbf{p}}^{\dagger}(x)\left\{iD^0 - \frac{\mathbf{p}^2}{2m_t} - \delta m_t\right\}\psi_{\mathbf{p}}(x) + \ldots$$

$$\mathcal{L}_{\text{potential}} = -\frac{\mathcal{V}_c(\mu)}{(\mathbf{p}-\mathbf{p}')^2}\psi_{\mathbf{p}'}^{\dagger}\psi_{\mathbf{p}}\chi_{-\mathbf{p}'}^{\dagger}\chi_{-\mathbf{p}} + \ldots$$

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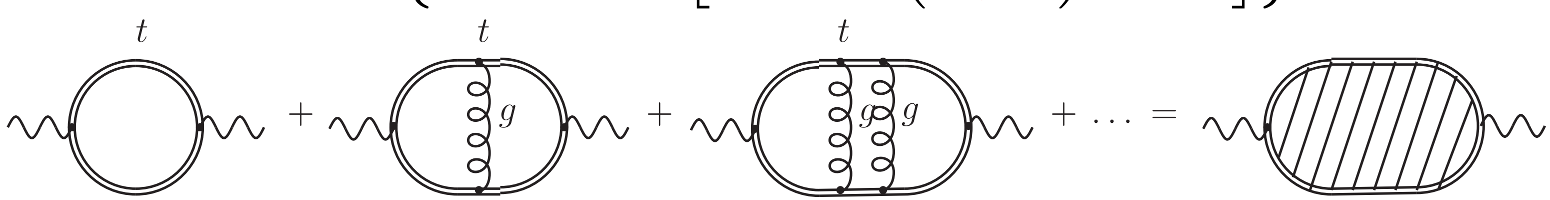
$\Rightarrow$ Schrödinger equation with Coulomb potential

$$G^{\text{LL}}(0,0,\sqrt{s}) = \frac{m_t^2}{4\pi} \left\{ i\,v - C_F \alpha_s \left[\frac{1}{4\epsilon} + \ln\left(\frac{-i\, m_t v}{\mu} \right) + \ldots \right] \right\}$$

$=$ 

vNRQCD (stable quarks)

Currents for $\mathrm{t\bar{t}}$ production & annihilation

$$\mathbf{O_p} = \mathsf{C}(\mu) \cdot (\psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}\, \tilde{\chi}^*_{-\mathbf{p}}) \qquad ({}^3\mathsf{S}_1)$$

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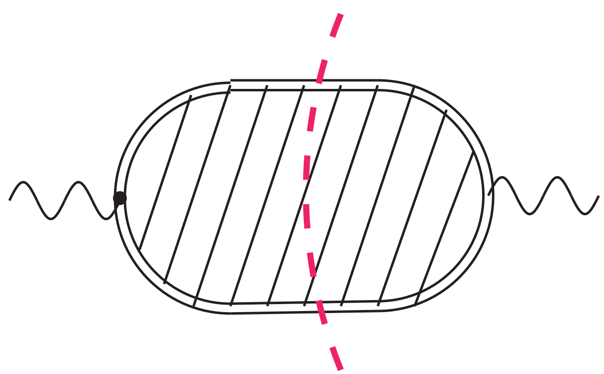
Total cross section from $e^+e^- \rightarrow e^+e^-$ using the Optical Theorem [Strassler, Peskin]

$$\sigma_{\rm tot} \quad \propto \quad \mathrm{Im}\left[\int d^4x\, e^{-i\hat{q}\cdot x} \left\langle 0 \left| T\, \mathbf{O}^\dagger_{\mathbf{p}}(0)\, \mathbf{O}_{\mathbf{p}'}(x) \right| 0 \right\rangle\right]$$

$$\propto \quad \mathrm{Im}\left[C(\mu)^2\, G_(0, 0, \sqrt{s})\right]$$



vNRQCD (unstable quarks)

Optical Theory of absorptive medium

- Complex refractive indices in the Maxwell equations
- Can be derived systematically from microscopic processes

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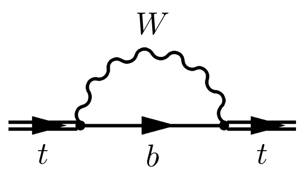
vNRQCD accounting for $t \to Wb$

- Wilson coefficients become complex
- Obtained by matching effective theory to SM
- Effective Lagrangian non-Hermitian
  $\to$ Total cross section through the optical theorem

vNRQCD (unstable quarks)

Matching conditions accounting for $t \rightarrow Wb$

Bilinear quark operators



$$= i\,\Sigma_t\,, \qquad\qquad \mathrm{Im}\,\Sigma_t = \frac{1}{2}\Gamma_t$$

$$\Rightarrow \quad \delta\mathcal{L} = \psi_{\mathbf{p}}^{\dagger}\left[i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)\right]\psi_{\mathbf{p}}$$

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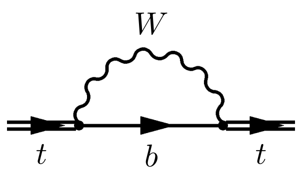

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Power counting for ew. effects: $\Gamma_t \sim m_t g^2 \sim m_t \alpha_s^2 \Rightarrow \boxed{g \sim g' \sim v \sim \alpha_s}$

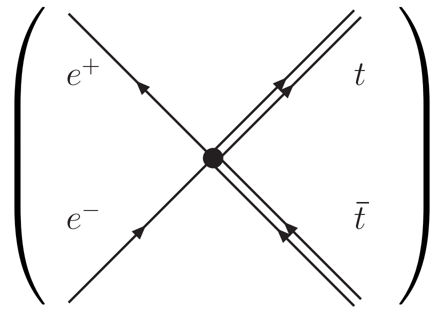
- $E \to E + i\Gamma_t$ replacement to include finite lifetime at LL [Fadin, Khoze] (not sufficient beyond LL order)
- Time dilatation is NNLL

vNRQCD (unstable quarks)

Matching conditions accounting for $t \to Wb$

<u>Currents</u> for $t\bar{t}$ production & annihilation

$$\mathbf{O_p} = \Big[\, C^{\mathrm{LL}} + C^{\mathrm{NLL}} + C^{\mathrm{NNLL}} + i\, C^{\mathrm{NNLL}}_{\mathrm{abs}} + \ldots \Big] \cdot$$



$$+ \ldots$$

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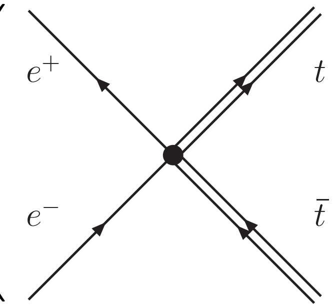

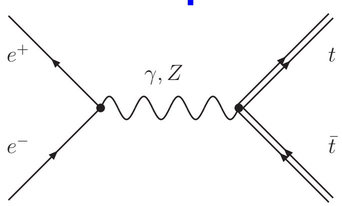

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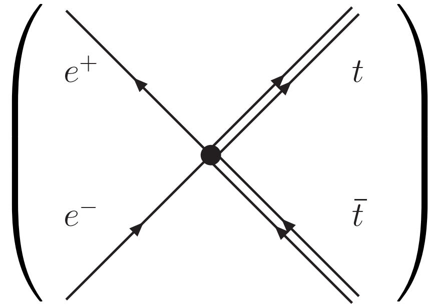
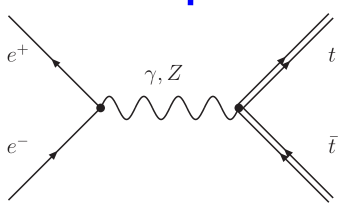
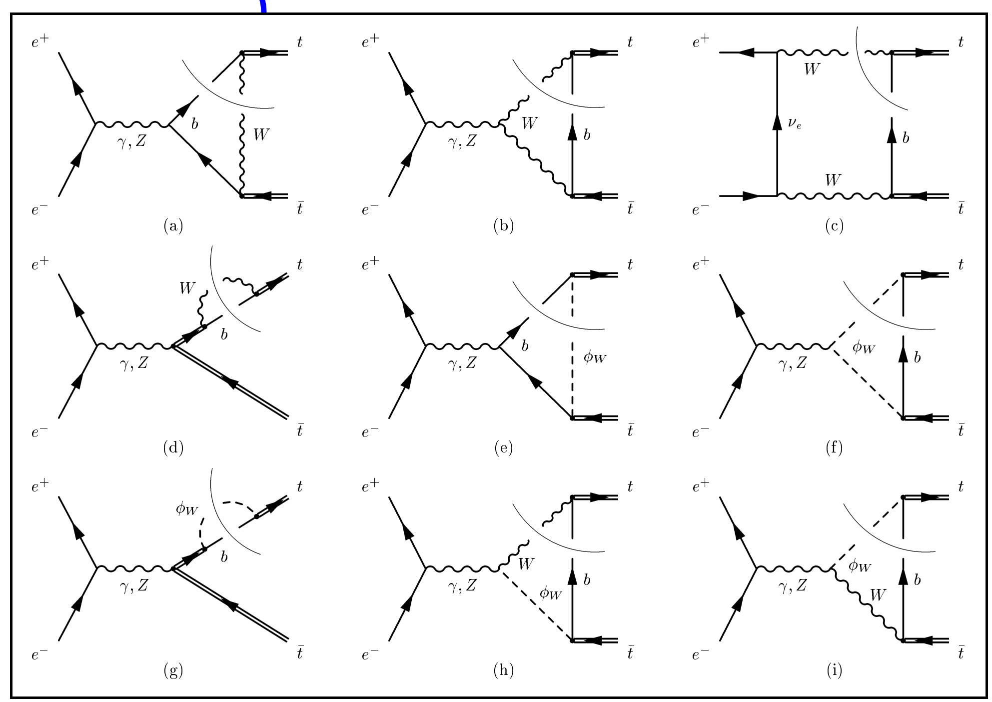




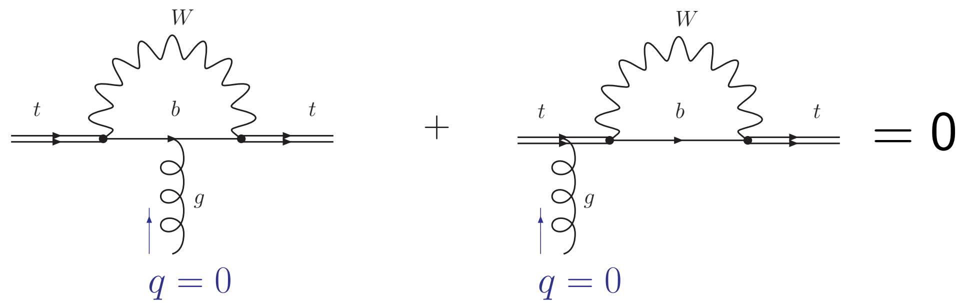


- Wb-cuts account for Wb final states
- Wb-cuts are gauge invariant

vNRQCD (unstable quarks)

Interactions in vNRQCD constrained by symmetries

Ward Identity



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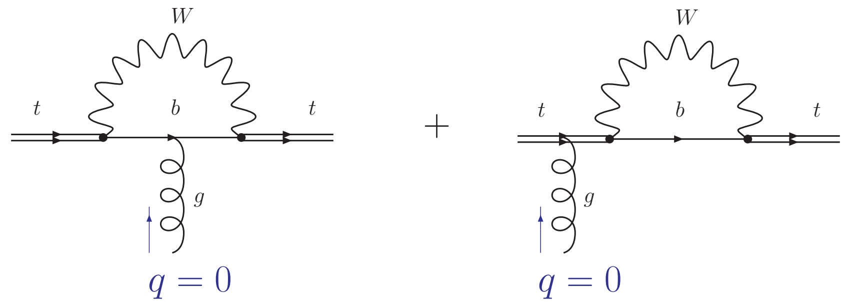


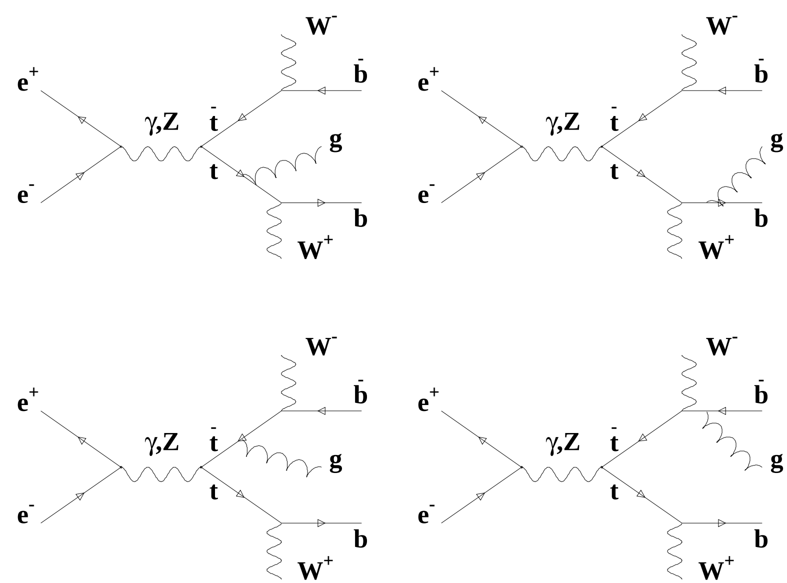
<u>Potentials:</u>

$\longrightarrow$ no NNLL electroweak corrections

<u>Soft, ultrasoft interactions:</u>

$\longrightarrow$ no NNLL electroweak corrections

$\longrightarrow$ also: no non-factorizable effects from ultrasoft gluons [Melnikov, Yakovlev]



Total Cross Section

Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\text{tot}} = 2\,\mathsf{N_c}\,\mathrm{Im}\left[\,\mathsf{C}(\mu)^2\,\mathsf{G}(0,0,\sqrt{\mathsf{s}})\,\right]}$

- NNLL finite lifetime correction:

$$\Delta^{\Gamma,1}\sigma_{\text{tot}} = 2\,\mathsf{N_c}\,\mathrm{Im}\left[2\,\mathsf{C}^{\text{LL}}\,\mathsf{i}\mathsf{C}^{\text{NNLL}}_{\text{abs}}\,\mathsf{G}^{\text{LL}}(0,0,\sqrt{\mathsf{s}}) + \left(\mathsf{C}^{\text{LL}}\right)^2\delta\mathsf{G}^{\text{NNLL}}(0,0,\sqrt{\mathsf{s}})\right]$$

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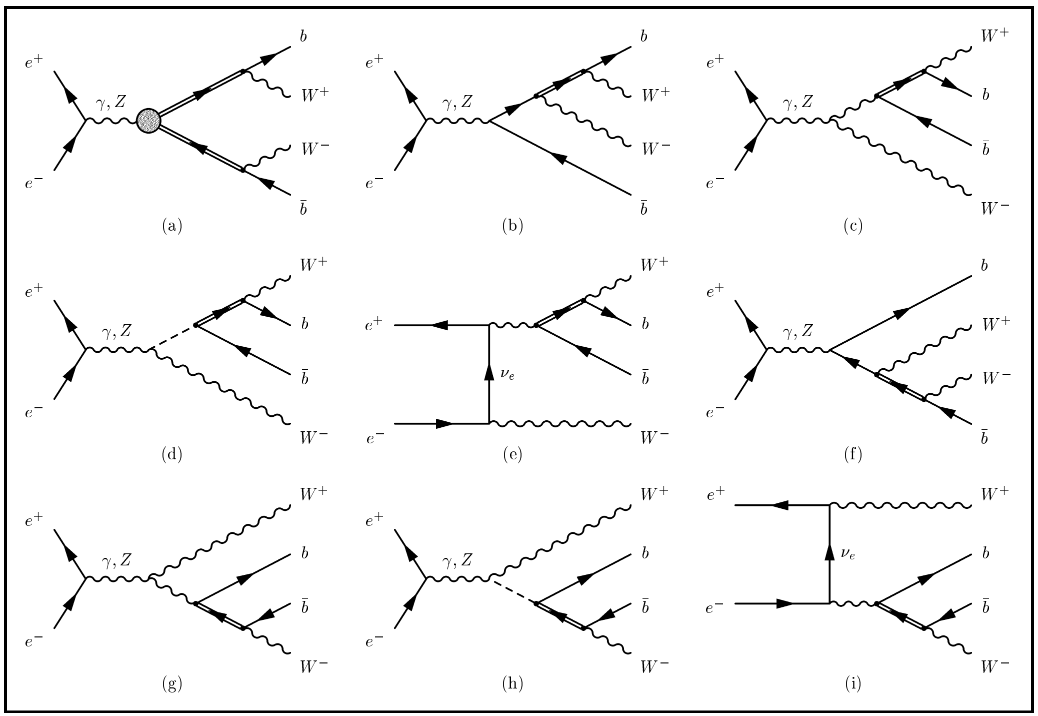

dilatation of the lifetime

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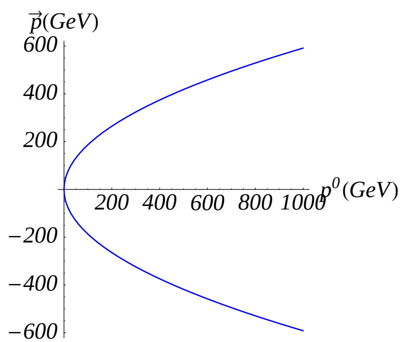
- NLL mixing effect: $\Delta^{\Gamma,1}\sigma_{\rm tot}$ contains

$\boxed{\Gamma_t\,\alpha_s\,\frac{1}{\epsilon}}$ (logarithmic UV phase space divergence due to finite lifetime)

Phase space:

stable tops

$$\frac{1}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\epsilon} \longrightarrow$$



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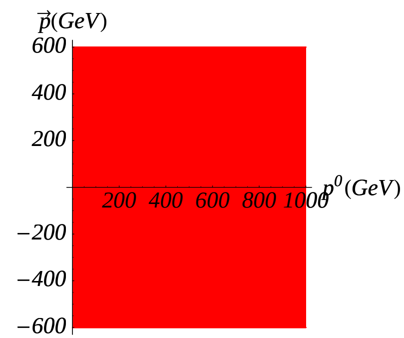

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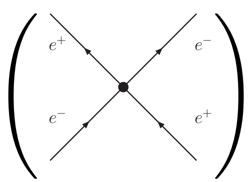
$\Rightarrow$ Anomalous dimension for $(e^+e^-)(e^+e^-)$ operator:

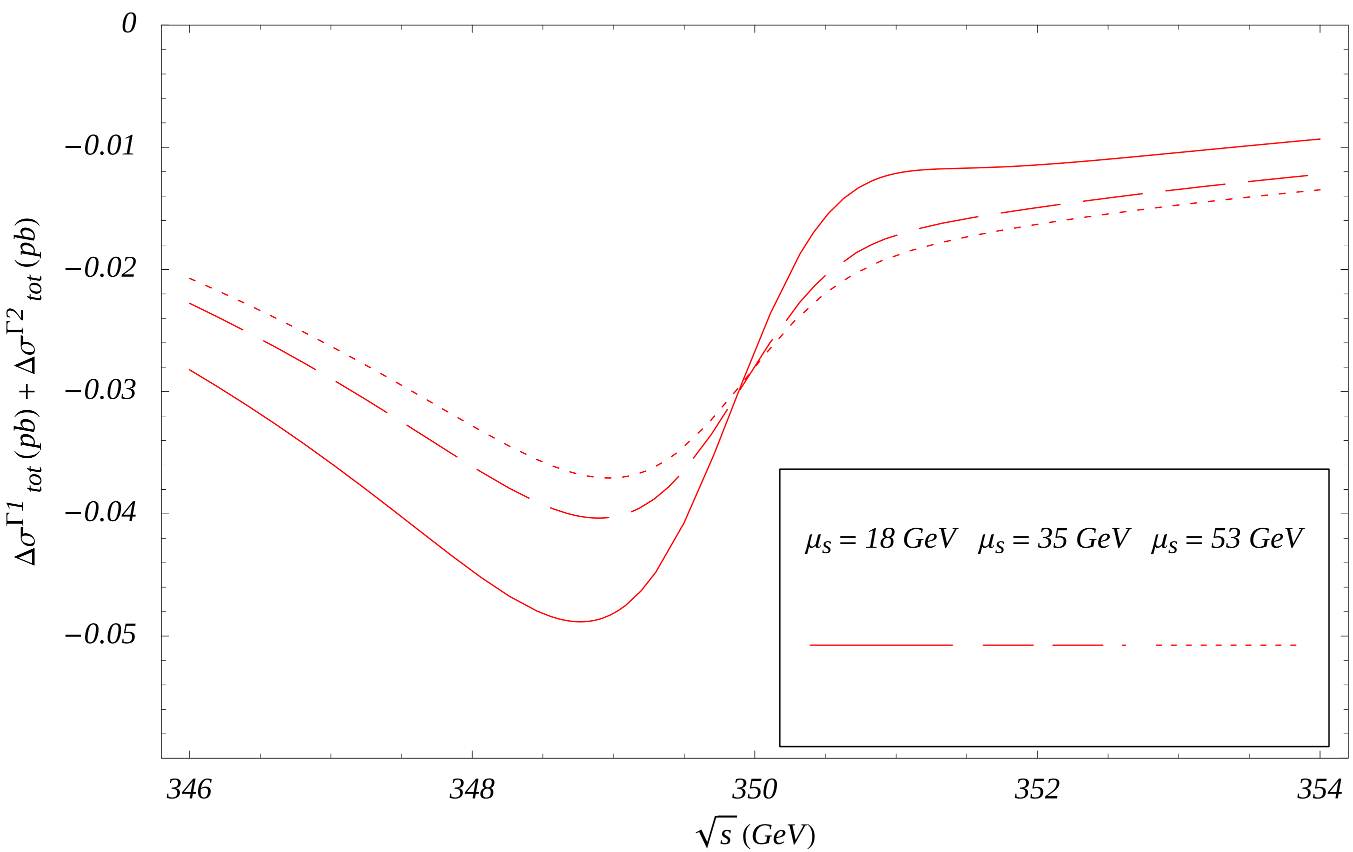
$i\,\tilde{C}^{eeee}(\mu)\cdot$ 

$\Rightarrow$ Correction $\Delta^{\Gamma,2}\sigma_{\rm tot}$

- $\sqrt{s}$-independent
- scale-dependent

* Matching coefficient $\tilde{C}^{eeee}(\mu = m_t)$ not yet determined $\rightarrow$ w.i.p.

Absorptive Parts



⇒ Comparable to NNLL QCD corrections

⇒ LL peak position shifted by 30 – 50 MeV

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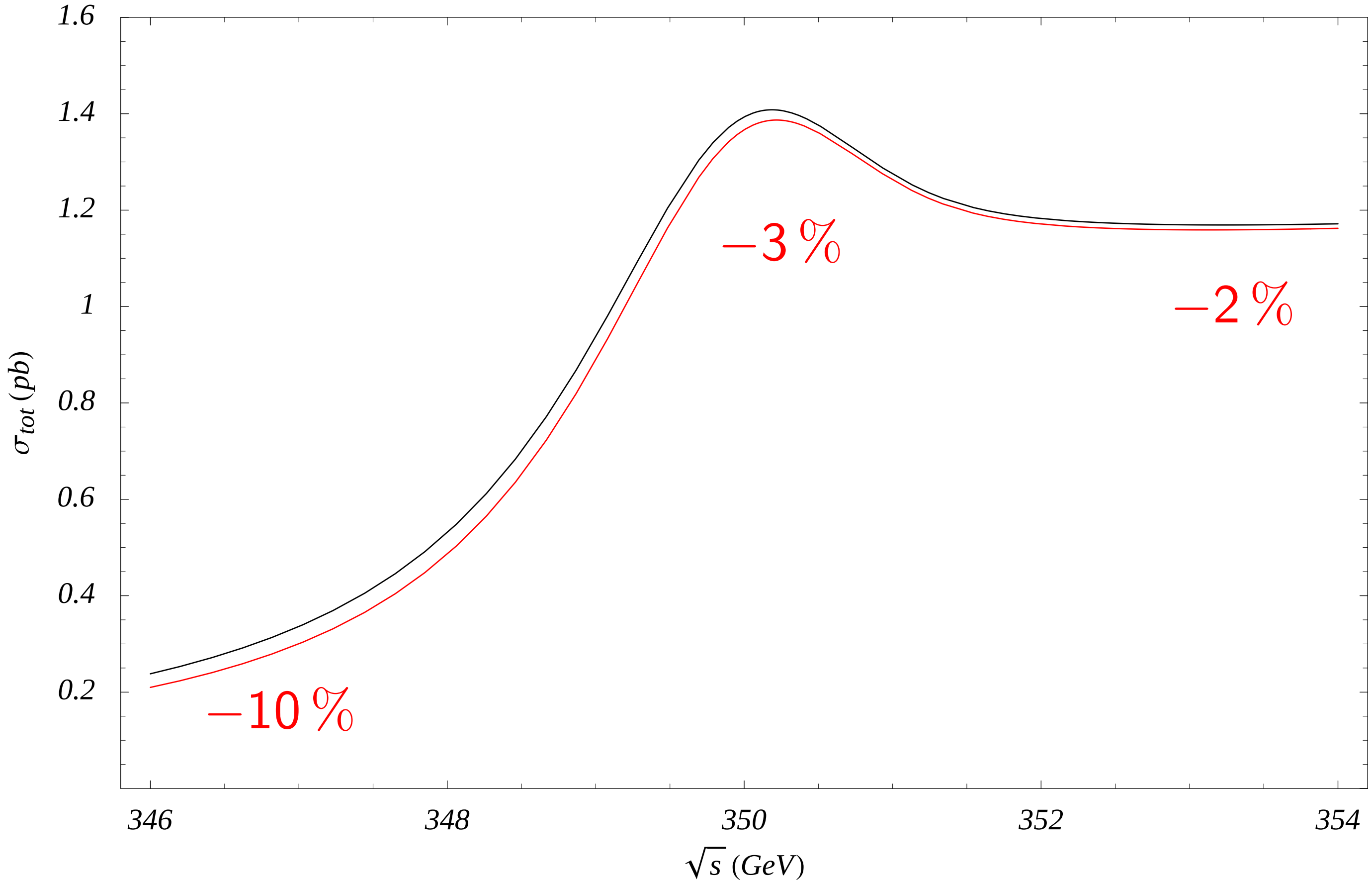

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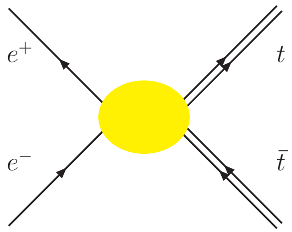
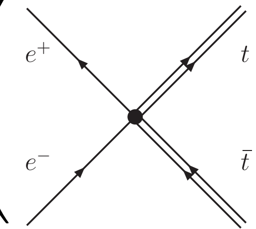
Electroweak Corrections (preliminary)

Electroweak short distance corrections: Real parts C^{NNLL}



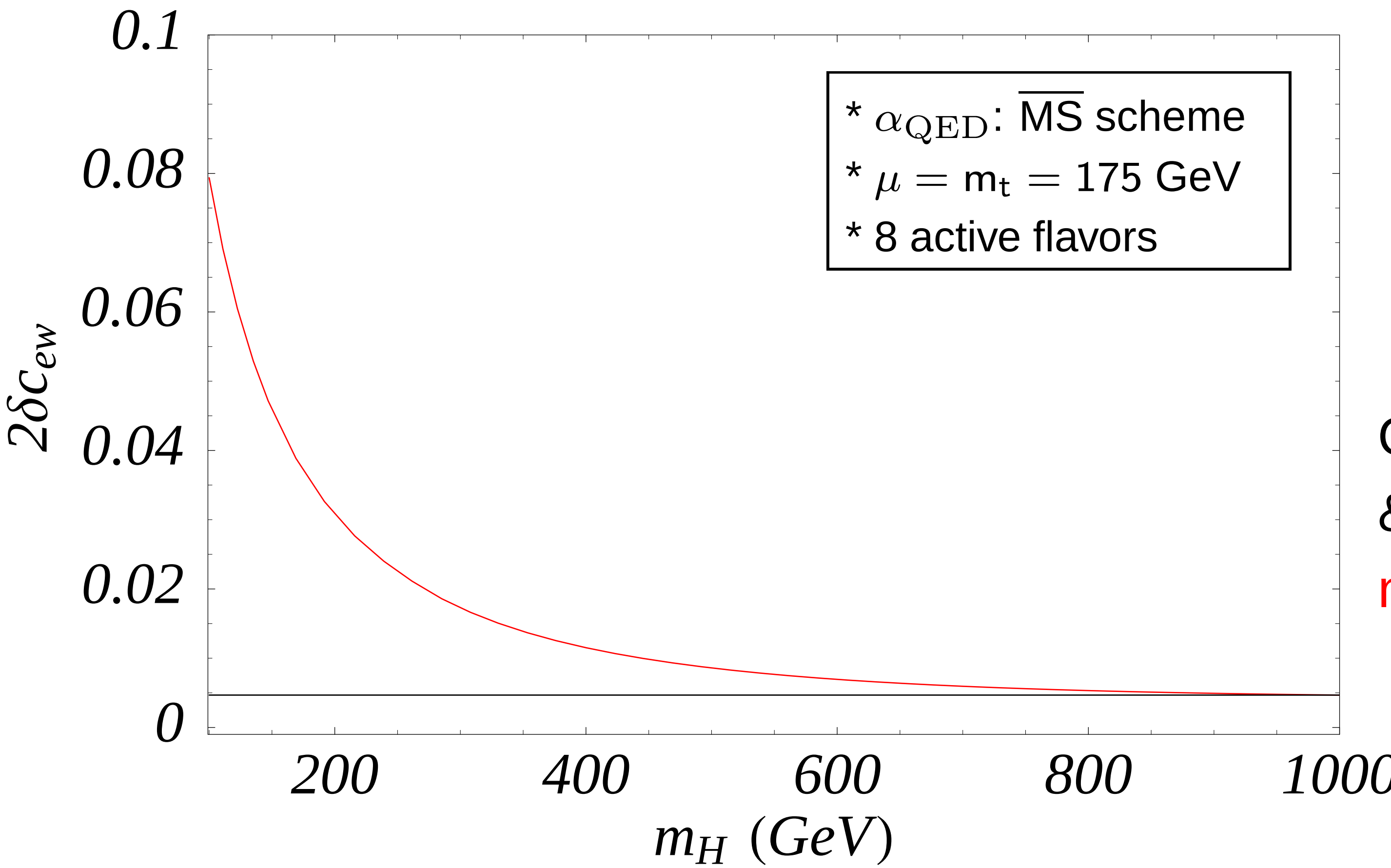
$$= \left[C^{\mathrm{LL}} + C^{\mathrm{NLL}} + C^{\mathrm{NNLL}} + i\, C^{\mathrm{NNLL}}_{\mathrm{abs}} + \ldots \right] \cdot \Bigg(\quad \Bigg)$$



- ≫ Full electroweak theory one-loop diagrams $O(\alpha_{\mathrm{em}}) \rightarrow$ NNLL
- ≫ Sizable shift of total cross section normalization
- ≫ Pure QED & ISR diagrams require separate treatment (SCET) $\rightarrow$ w.i.p.

Electroweak Corrections (preliminary)

Correction to the total cross section normalization: $(1 + 2\delta c_{\mathrm{ew}})\,\sigma_{\mathrm{tot}}$



QED corrections
& real radiation
not included

Summary and Outlook

Summary

- Threshold scan allows for precise $\mathrm{m_t}, \mathrm{y_t}, \Gamma_\mathrm{t}$ determination
- Effective theory approach (unstable particles)
  - UV divergencies from instablility
  - Interference of resonant and non-resonant diagrams
  - Symmetries restrict interactions
  - Short distance electroweak corrections
- Corrections comparable to QCD

Outlook

- Phase space matching $\tilde{\mathrm{C}}^{\mathrm{eeee}}(\mu = \mathrm{m_t})$
- NNLL running of $\tilde{\mathrm{C}}^{\mathrm{eeee}}(\mu)$
- QED contributions: ISR, beamstrahlung, Coulomb singularities