



Study of the spatial resolution of a GEM based TPC

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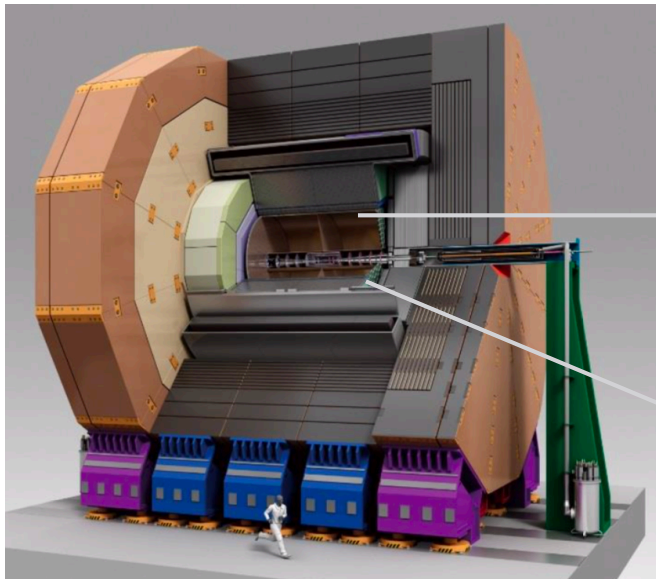
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9, March 2021

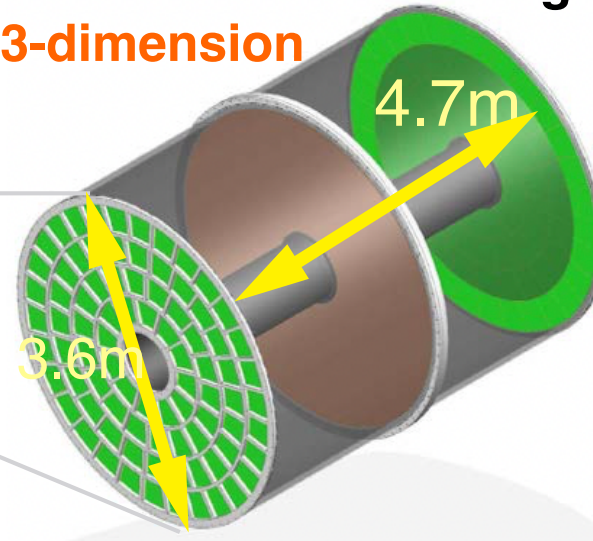
S O K E N D A I

Time Projection Chamber (TPC)

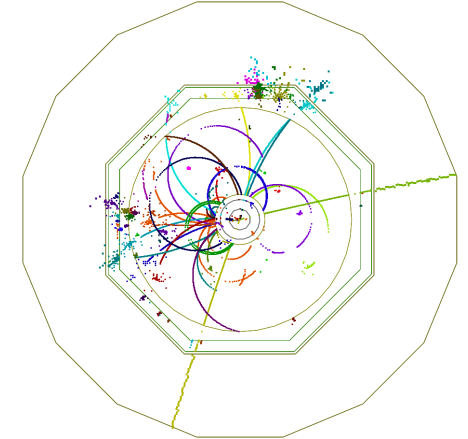
arXiv[1306.6329], ILC Technical Design Report: Volume 4, Part III



Reconstruct tracks of **charged** particles
in **3-dimension**



~200 hit points
Can see tracks
before reconstruction



Two main roles at the TPC:

- ❑ Momentum measurement

Measure the curvature radius of the tracks in $B=3.5\text{T}$ and drift length 2.2 m

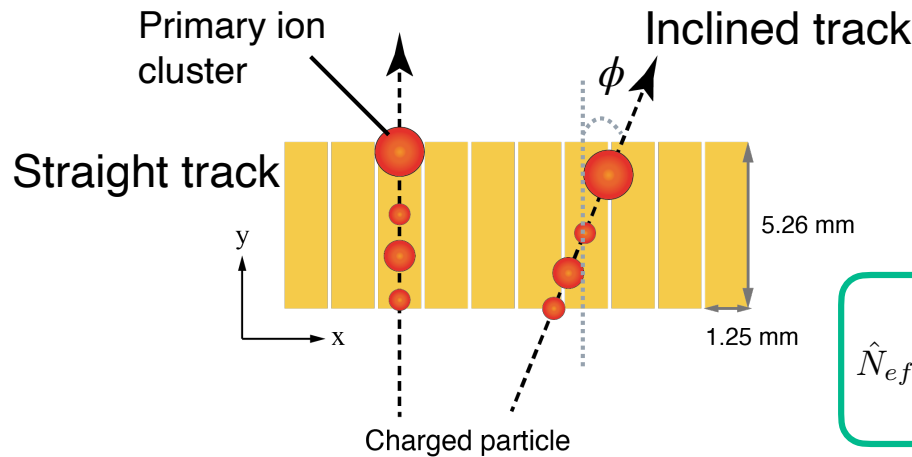
$$\frac{\sigma_{p_{\perp}}}{p_{\perp}} \simeq 1 \times 10^{-4} p_{\perp} [\text{GeV}/c]$$

➔ There are about **200** points of **position resolution** $\sigma_x \simeq 100[\mu\text{m}]$

- ❑ Particle identification measured by dE/dx

Incident angle effect on the spatial resolution

R.Yonamine,K.Fujii [<https://doi.org/10.1088/1748-0221/9/03/C03002>]



$$N_{eff} \approx \left[\left\langle \frac{1}{\sum_i k_i} \right\rangle_{N,k} \left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle_G \right]^{-1}$$

$$\hat{N}_{eff} \approx \left[\left\langle \sum_{i=1}^N \left\langle \left(\frac{\sum_{j=1}^{k_i} G_{ij}}{\sum_{i=1}^N \sum_{j=1}^{k_i} G_{ij}} \right)^2 \right\rangle_{G^{k_i, \sum_{i=1}^N k_i}} \right\rangle_{N,k} \right]^{-1}$$

$$\sigma_x^2(Z; w, L \tan \phi, C_d, N_{eff}, \hat{N}_{eff}, [f]) = [A] + \frac{1}{N_{eff}} [B] + [C] + \frac{1}{\hat{N}_{eff}} [D]$$

Hodoscope effect
(S-shape systematics)
important for the short
drift distance

Effects of diffusion
+ gas gain fluctuation
+ finite pad pitch

Electronic noise

Angular Pad effect L : Pad row pitch

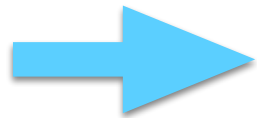
$$\frac{1}{\hat{N}_{eff}} [D] = \frac{L^2 \tan^2 \phi}{12 \hat{N}_{eff}}$$

My study: **estimate \hat{N}_{eff} from beam test and simulation**

Analytic expressions for \hat{N}_{eff}

R.Yonamine, K.Fujii [<https://doi.org/10.1088/1748-0221/9/03/C03002>]

$$\hat{N}_{eff} \approx \left[\left\langle \sum_{i=1}^N \left\langle \left(\frac{\sum_{j=1}^{k_i} G_{ij}}{\sum_{i=1}^N \sum_{j=1}^{k_i} G_{ij}} \right)^2 \right\rangle_{G, k_i, \sum_{i=1}^N k_i} \right\rangle_{N, k} \right]^{-1}$$



$$\hat{N}_{eff} \approx \left[\left\langle \frac{\sum_{i=1}^N k_i^2}{\left(\sum_{i=1}^N k_i \right)^2} \right\rangle + \frac{1}{N_{eff}} - \frac{1}{N_{eff}^{w/o G}} \right]^{-1}$$

w/ gas gain fluctuation
w/o gas gain fluctuation

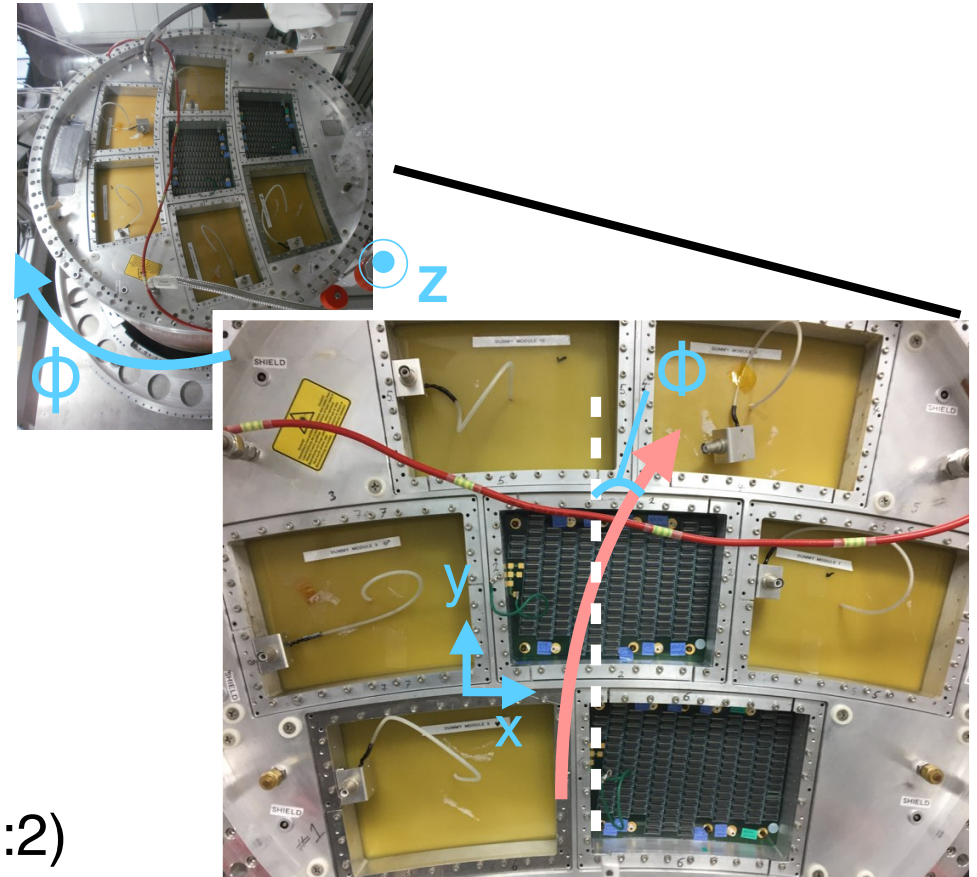
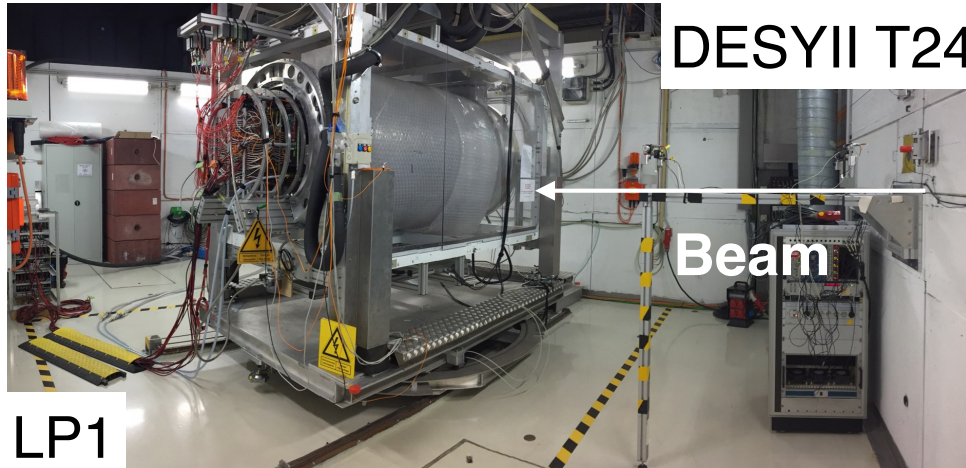
N : the number of primary ionization **clusters** G : Gas gain k_i : the number of seed electrons originating from the i -th primary cluster

$$\left(N_{eff}^{w/o G} \approx \left[\left\langle \frac{1}{\sum_i k_i} \right\rangle_{N, k} \right]^{-1} \right)$$

\hat{N}_{eff} can be decomposed into two parts, due to **cluster size fluctuation** and **gas gain fluctuation**.

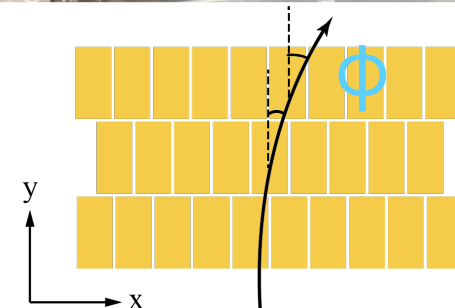
2016 beam test of Asian GEM modules

The original purpose of the beam test was to compare performance of the Asian modules with and without the gating foil.



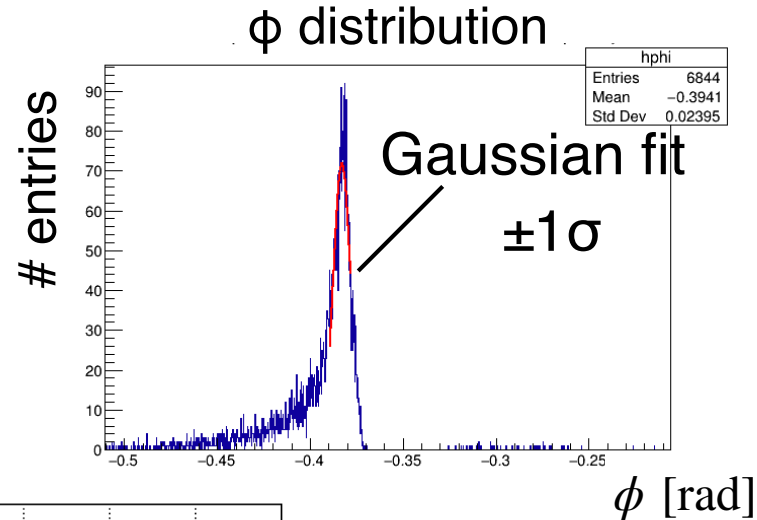
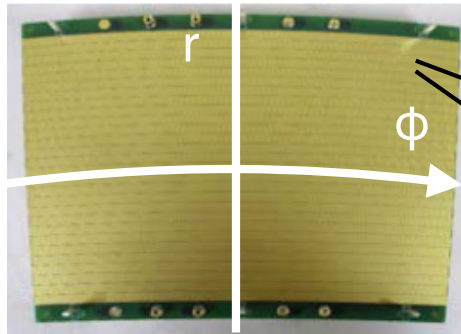
Set up

- ▶ Electron Beam = 5 GeV
- ▶ $B = 1$ T
- ▶ T2K gas (Ar:CF₄:iso-C₄H₁₀ = 95:3:2)
- ▶ Analysis frame work : Marlin TPC
- ▶ Data set used : $\phi = -20^\circ, 0^\circ, 10^\circ, 20^\circ$

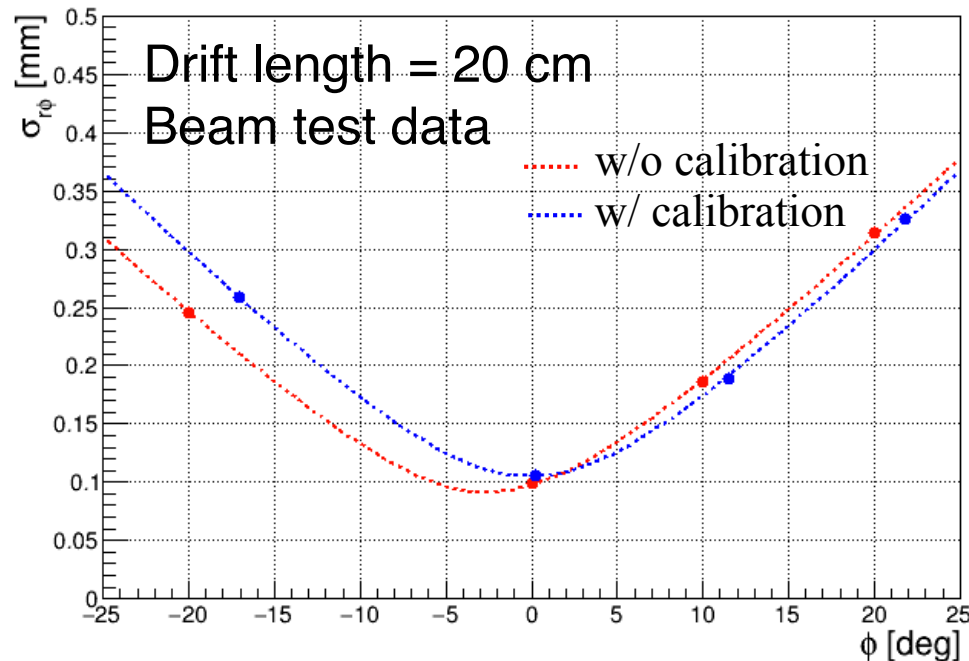


ϕ angle calibration

The readout pad rows are curved to cover the **fan-shaped** anode plane of each module



#20156,
w/ field shaper,
 $\phi=20^\circ$,
B=1T,
row =16



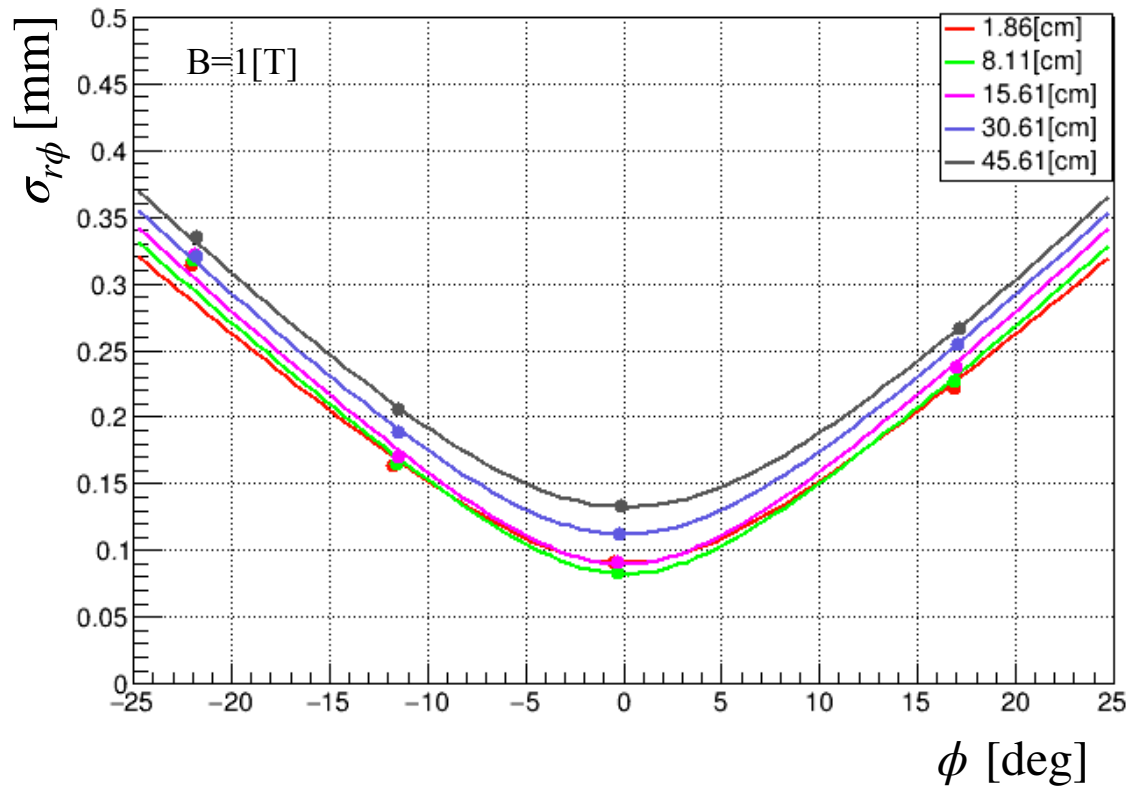
Fitting function

$$\sigma_{r\phi} = \sqrt{\sigma_0^2 + \sigma_{ang}^2 \tan^2(\phi - \phi_0)}$$

After calibration, minimum at $\phi_0 = -0.04 \pm 0.12^\circ$ from $-2.5 \pm 0.1^\circ$

Spatial Resolution at **different drift distances** as a function of ϕ (beam test data)

Spatial Resolution at different drift distances (z)
for module3, row16



Fitting function

$$\sigma_{r\phi} = \sqrt{\sigma_0^2 + \sigma_{ang}^2 \tan^2(\phi - \phi_0)}$$

$$\sigma_{ang}^2 \equiv \frac{L^2}{12 \left(\hat{N}_{eff}(\phi = 0) / \cos \phi \right)}$$

The function fits the data reasonably well.

→ ϕ dependance is consistent with the theory.

We get \hat{N}_{eff} from fit results.

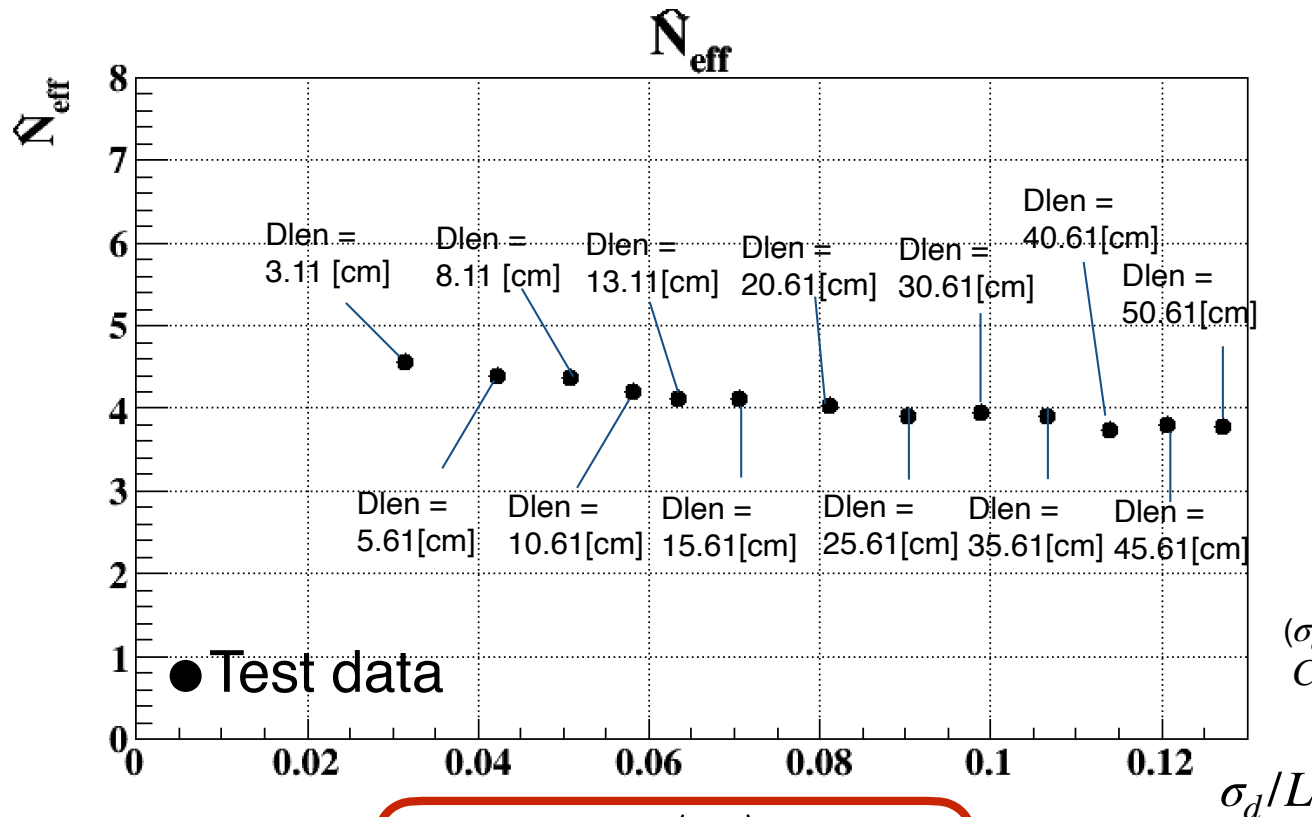
Beam test data results- \hat{N}_{eff}

\hat{N}_{eff} value defined by the fitting function

$$\hat{N}_{eff}(\phi = 0) = \frac{L^2 \tan^2 \phi \cos \phi}{12\sigma_{ang}^2}$$

the effective number of
primary clusters

► L(Pad row pitch) = 0.536 [cm]



($\sigma_d = C_d \sqrt{\text{Drift length [cm]}}$,
 C_d : diffusion constant)

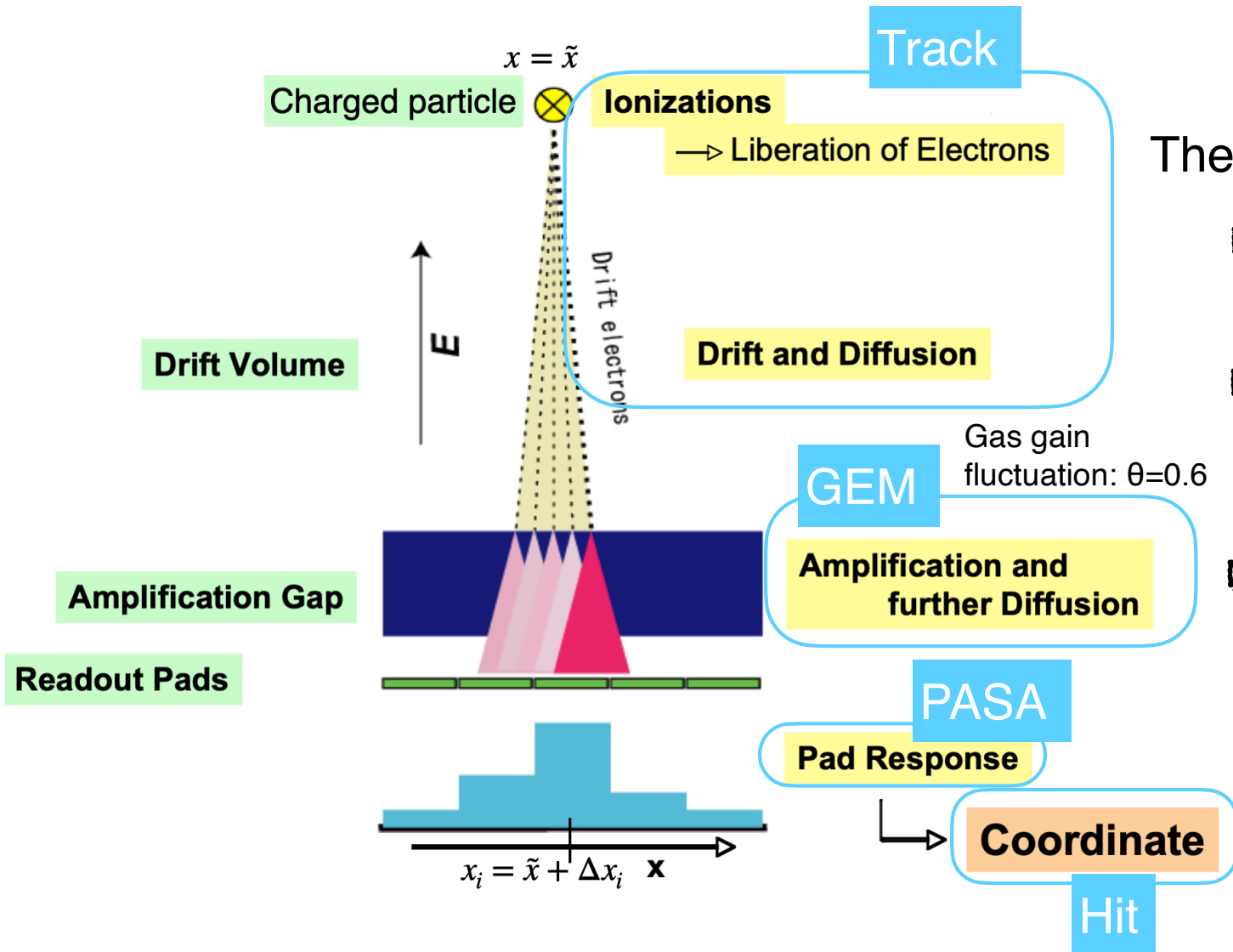
$$\hat{N}_{eff} \ll \langle \hat{N} \rangle \simeq 20$$

$\langle \hat{N} \rangle$: The average number of cluster

TPC simulator

* C++

We have developed a new TPC simulator



The implemented effects

- finite pad size effect (S-shape systematics)
- finite Pad Response Function width (hodoscope effect)
- the magnetic field effect (curling effect of primary electrons)

Comparison of an **approximate formula** with the \hat{N}_{eff} value as defined by the fitting function

Approximate formula **ignoring the pad size**

$$\hat{N}_{eff} \approx \left[\left\langle \sum_{i=1}^N \left\langle \left(\frac{\sum_{j=1}^{k_i} G_{ij}}{\sum_{i=1}^N \sum_{j=1}^{k_i} G_{ij}} \right)^2 \right\rangle_{G}^{k_i, \sum_{i=1}^N k_i} \right\rangle_{N,k} \right]^{-1}$$

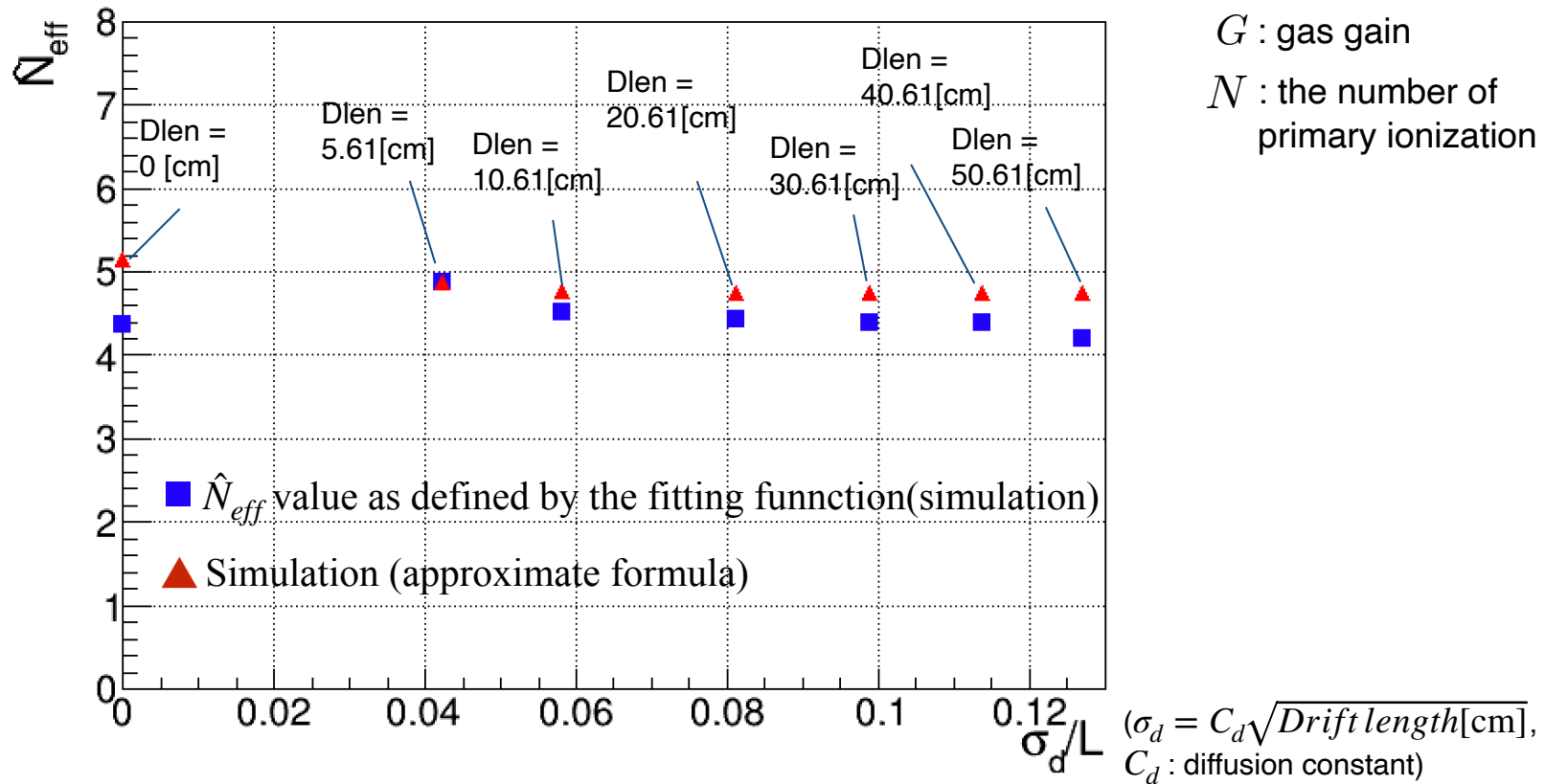
i : primary cluster

j : electron

k_i : the number of seed electrons originating from the i -th primary cluster

G : gas gain

N : the number of primary ionization **clusters**

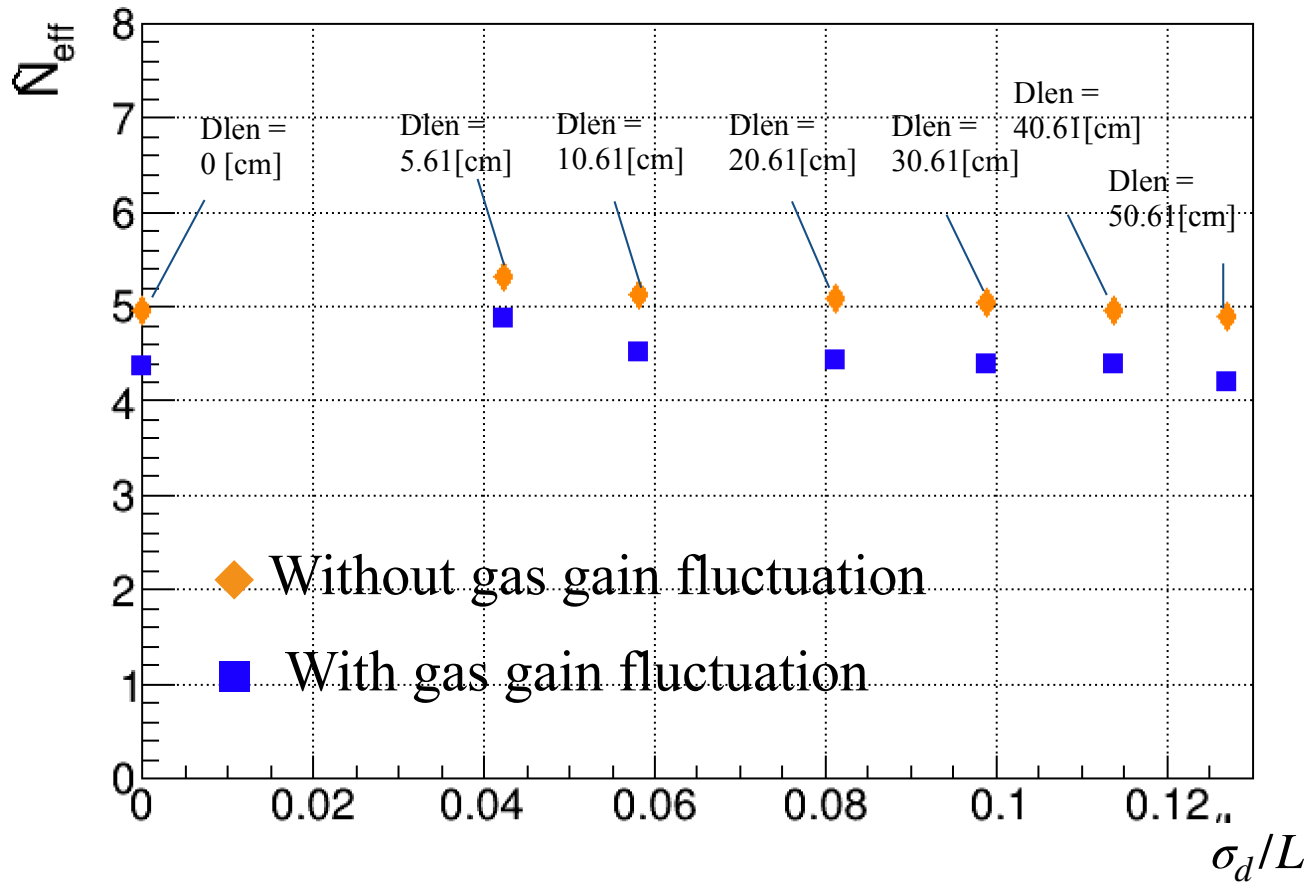


Approximate formula reproduces the fitted result quite well

\hat{N}_{eff} w/ and w/o gas gain fluctuation

Contribution from gas gain fluctuation

$$\hat{N}_{eff} \simeq \left[\left\langle \frac{\sum_{i=1}^N k_i^2}{\left(\sum_{i=1}^N k_i\right)^2} \right\rangle + \frac{1}{N_{eff}} - \frac{1}{N_{eff}^{w/o G}} \right]^{-1}$$

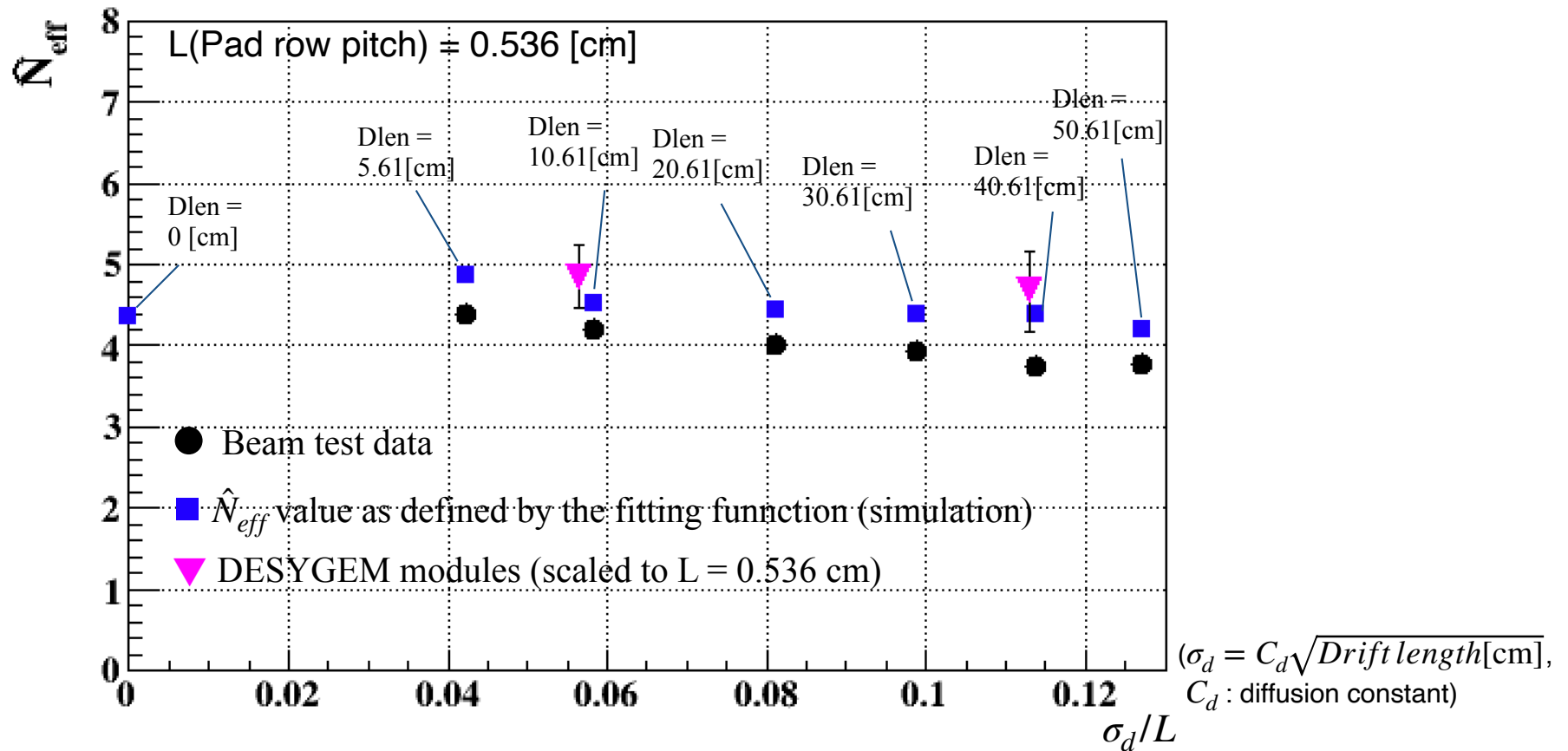


The gas gain fluctuation significantly reduces \hat{N}_{eff} at all distances

Comparison beam test with simulation - \hat{N}_{eff}

\hat{N}_{eff} value defined by the fitting function

$$\sigma_{ang}^2 = \frac{L^2 \tan^2 \phi}{12 \left(\hat{N}_{eff}(\phi = 0) / \cos \phi \right)} \quad \rightarrow \quad \hat{N}_{eff}(\phi = 0) = \frac{L^2 \tan^2 \phi \cos \phi}{12 \sigma_{ang}^2}$$



The measured and simulated \hat{N}_{eff} agree to about 5~10%.

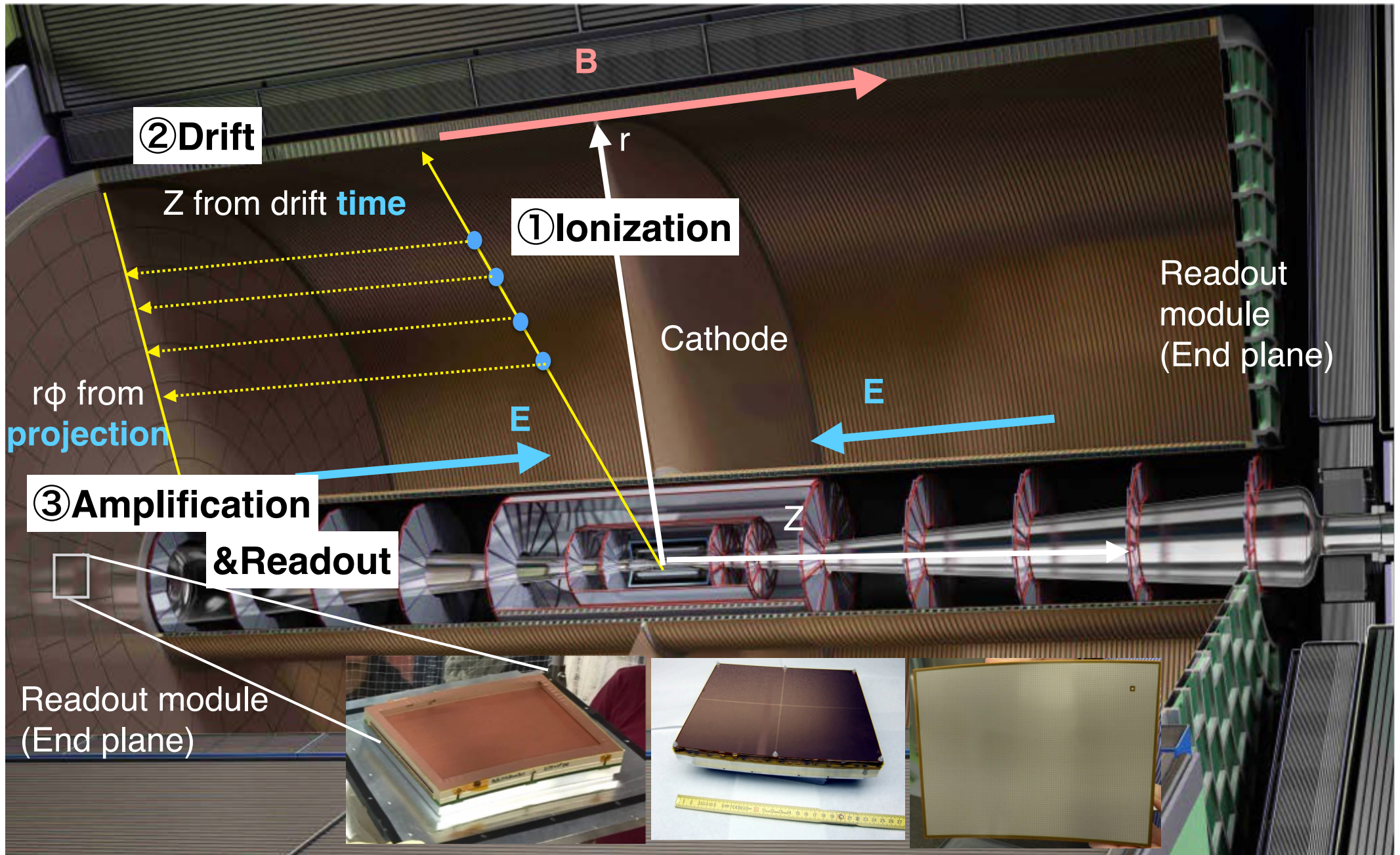
Summary

- ☆ We used Asian GEM beam test data to confirm the **angular pad effect**.
→the ϕ dependence became consistent with the theory after calibration
- ☆ The TPC simulation was improved to take into account the some effects
→the **gas gain fluctuation** has significant effect on \hat{N}_{eff}
- ☆ The measured and simulated \hat{N}_{eff} agree to about 5~10%.

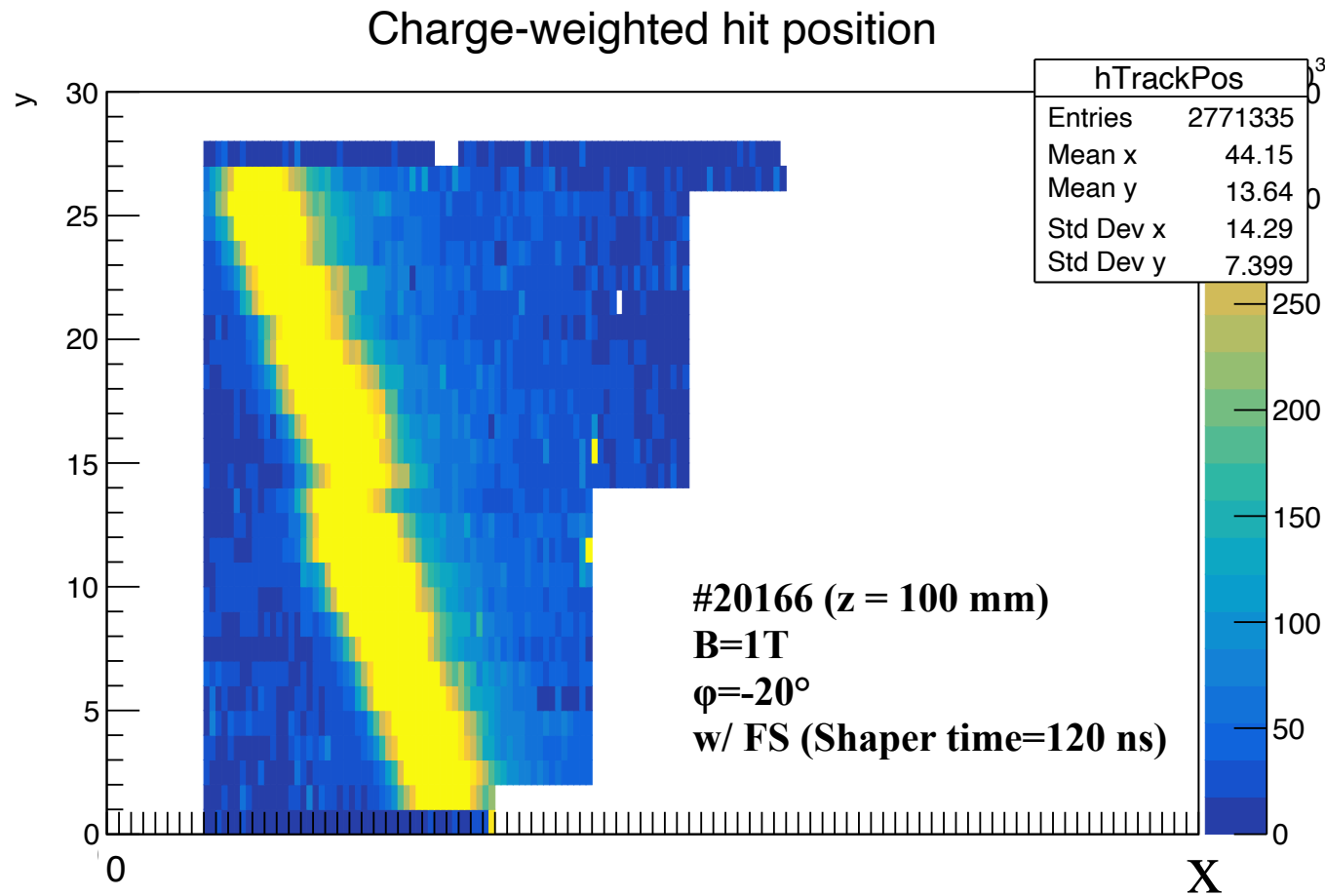
Next step

- ☆ The electrons ionisation process is carried out in Heed, a Garfield++ package.
(←implemented, but undergoing minor modifications.)

Fundamental principle of TPC



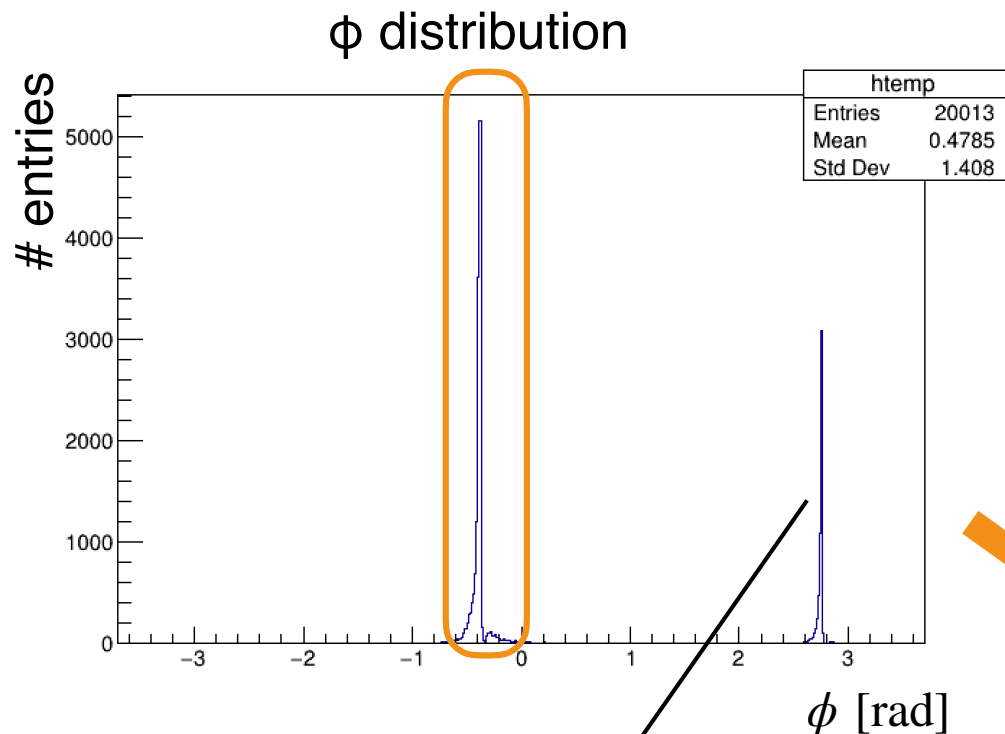
Charge-weighted hit position



The small number of dead readout channels

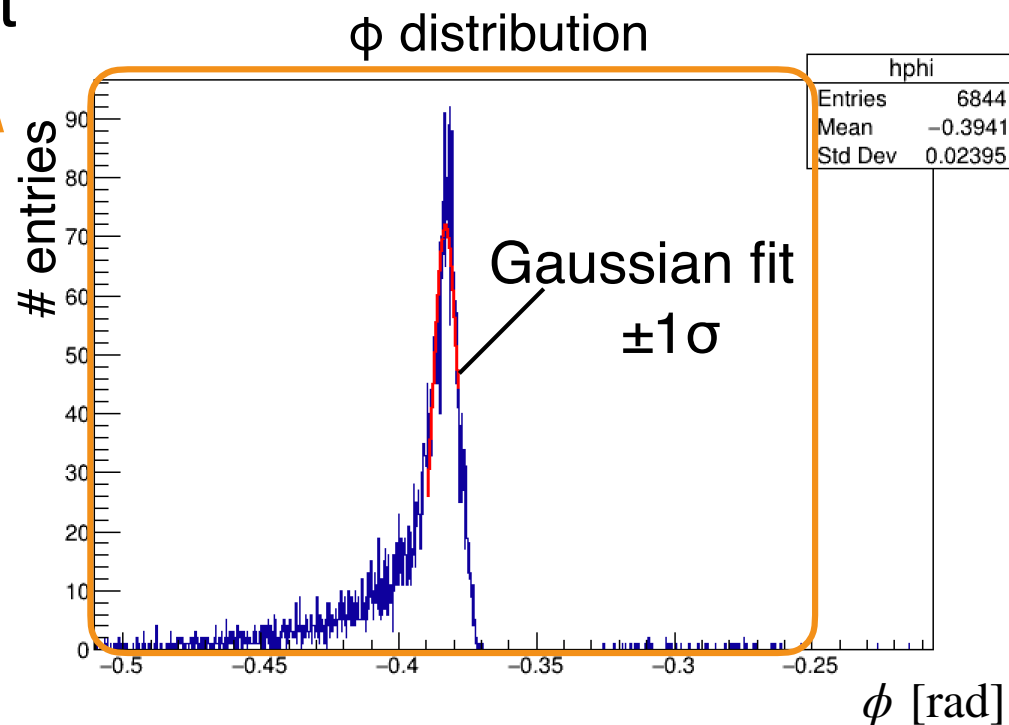
How to determine ϕ

#20156, w/o gating foil, $\phi=20^\circ$, $B=1\text{T}$, row =16



$-0.8 < 1/p_T(e^-) < 0$

Cut

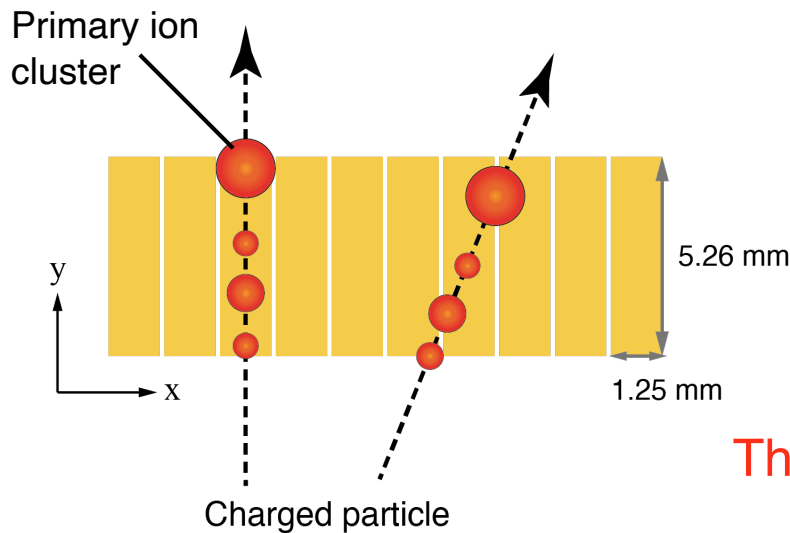


Positrons from bremsstrahlung
photon conversions before entering TPC

Incident angle effect on the spatial resolution

$$\sigma_x^2(Z; w, L \tan \phi, C_d, N_{eff}, \hat{N}_{eff}, [f]) = [A] + \frac{1}{N_{eff}} [B] + [C] + \frac{1}{\hat{N}_{eff}} [D]$$

Hodoscope effect and S-shape for the short drift distance



$$[A] = \int_{-1/2}^{1/2} d\left(\frac{\tilde{x}}{w}\right) \sum_{N=1}^{\infty} P_{PI}(N; \bar{N}) \prod_{i=1}^N \left[\sum_{k_i=0}^{\infty} \bar{P}_{SI}(k_i) \right]$$

Primary ion cluster

Secondary ion cluster

$$\times \left\{ \left(\sum_a (aw) \sum_{i=1}^N \langle \langle F_a \rangle_{\Delta x}^y \rangle^{k_i} \left\langle \frac{\sum_{j=1}^{k_i} G_{ij}}{\sum_{i=1}^N \sum_{j=1}^{k_i} G_{ij}} \right\rangle_G^{k_i, \sum_{i=1}^N k_i} - \tilde{x} \right)^2 \right\}$$

Diffusion averaged & cluster position average charge centroid

Systematic

The diffusion (Gas gain fluctuation) + finite pad pitch

$$[B] = \int_{-1/2}^{1/2} d\left(\frac{\tilde{x}}{w}\right) \left\langle \left(\sum_a (aw) F_a(\tilde{x} + \Delta x) - \sum_a (aw) \langle F_a(\tilde{x} + \Delta x) \rangle_{\Delta x} \right)^2 \right\rangle_{\Delta x}$$

Diffusion-averaged charge centroid

Displacement due to diffusion for a single electron

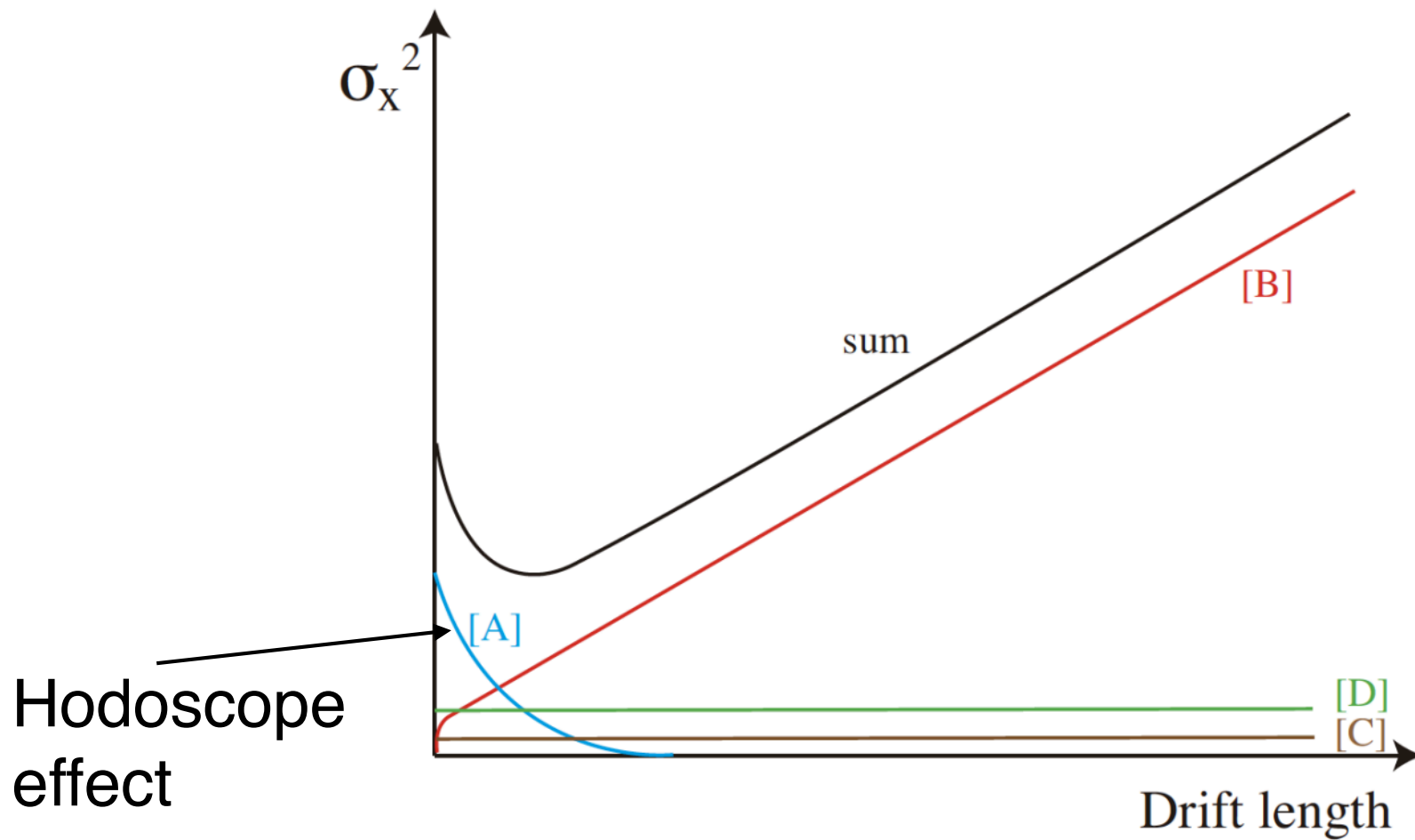
Electric noise

$$[C] = \left(\frac{\sigma_E}{G}\right)^2 \left\langle \frac{1}{N^2} \right\rangle_N \sum_a (aw)^2$$

Angular Pad effect

$$\frac{1}{\hat{N}_{eff}} [D] = \frac{L^2 \tan^2 \phi}{12 \hat{N}_{eff}} \quad L : \text{Pad height}$$

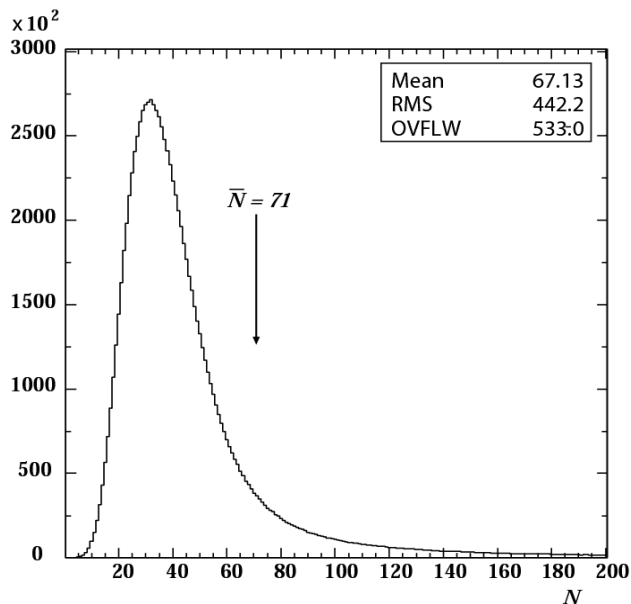
Schematic image of the spatial resolution as a function of drift length



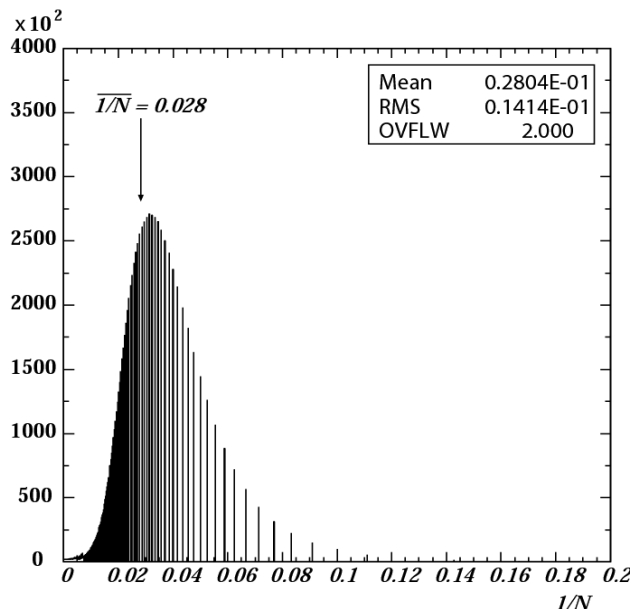
N_{eff} in typical model

4 GeV pion, pad pitch 6mm, pure Ar

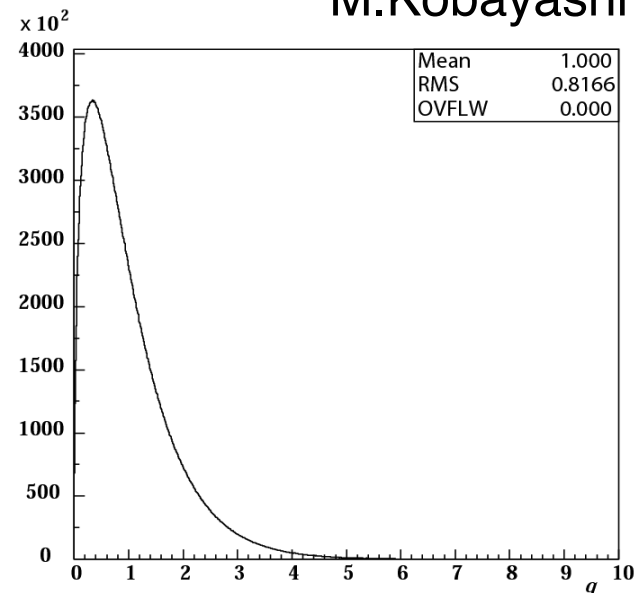
M.Kobayashi



Distribution of N
($\langle N \rangle = 71$)



Distribution of $1/N$
($\langle 1/N \rangle = 0.028$)



Distribution of Q
($K = 0.67$)

$$\left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle = 1 + \left(\frac{\sigma_G}{\bar{G}} \right)^2 \equiv 1 + K$$

$$N_{eff} := \left[\left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}} \right)^2 \right\rangle \right]^{-1} = 21 < \frac{1}{\langle 1/N \rangle} = 36 < \langle N \rangle = 71$$