Karlsruhe Institute of Technology

# Measuring Unification

***[arXiv:1007.2190 [hep-ph]]***

**in collaboration with C. Adam, J.-L. Kneur, R. Lafaye, T. Plehn, D. Zerwas**

Michael Rauch | October 21, 2010

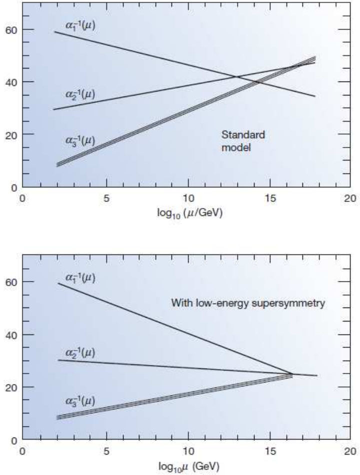
INSTITUTE FOR THEORETICAL PHYSICS

# Motivation & Outline

- Supersymmetry predicts accurate unification of gauge couplings
- High-scale models (like mSUGRA) also predict unification of soft-breaking masses
  ⇒ Test this!

Outline

- Low-scale setup:
  Experimental input
  Fit of general MSSM
  (20/22 parameters)
- Results for LHC and LHC+ILC
- Top-down vs. bottom-up
- Outlook: S-LHC and CLIC



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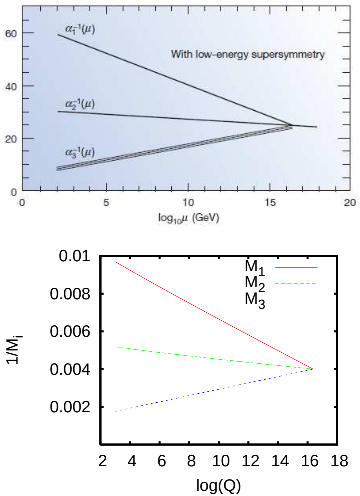

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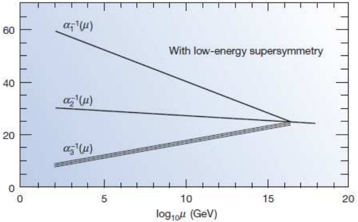
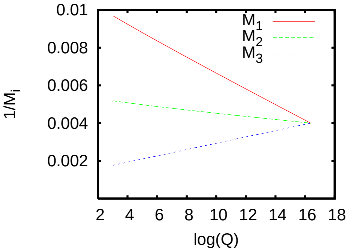
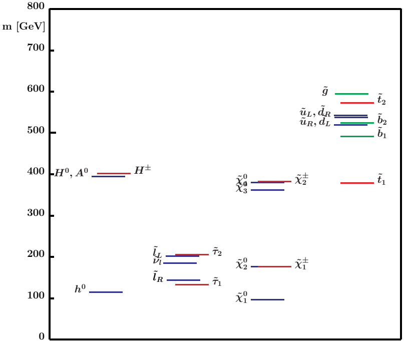


# Parameter Point

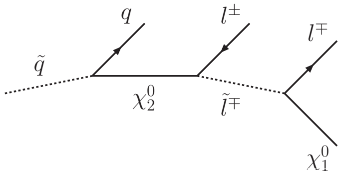
Benchmark Point: SPS1a (mSUGRA)
$m_0 = 100\,\mathrm{GeV}$ , $m_{1/2} = 250\,\mathrm{GeV}$ , $A_0 = -100\,\mathrm{GeV}$ , $\tan\beta = 10$, $\mathrm{sgn}\,\mu = +1$

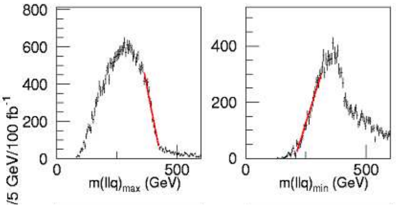



- light sleptons
- light ($\approx$ wino, bino) and heavy ($\approx$ higgsino) gauginos
- moderately heavy gluinos and squarks
- light Higgs around the LEP limit
- Spectrum and RGE running: Suspect [Kneur et al.]
  cross-checks: SoftSusy [Allanach]

# Experimental Input

LHC “experimental” data from cascade decays (best precision obtainable)
$\sqrt{S} = 14$ TeV, integrated luminosity $\mathcal{L} = 300\ \text{fb}^{-1}$




| Measurement | Value (GeV) | Errors (GeV) (stat) | (LES) | (JES) | (theo) |
|---|---|---|---|---|---|
| $(m_{llq}^{max})$:Edge($\tilde{q}_L$,$\chi_2^0$,$\chi_1^0$) | 449.08 | 1.4 | | 4.3 | 5.1 |
| $(m_{llq}^{min})$ :Thres($\tilde{q}_L$,$\chi_2^0$,$\tilde{\mu}_R$,$\chi_1^0$) | 216.00 | 2.3 | | 2.0 | 3.3 |
| $(m_{ll}^{max})$:Edge($\chi_2^0$,$\tilde{l}_R$,$\chi_1^0$) | 80.852 | 0.042 | 0.08 | | 1.2 |
| $m_h$ | 108.7 | 0.01 | 0.25 | | 2.0 |
| ... | ... | ... | | | |

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| ... | ... | ... | | | |

ILC data direct mass measurements
$\sqrt{S}$ up to 800 GeV, integrated luminosity $\mathcal{L} = 500\ \text{fb}^{-1}$

| Particle | Value (GeV) | Errors (GeV) (stat) | (theo) |
|---|---|---|---|
| $h$ | 108.7 | 0.05 | 2.0 |
| $\chi_1^0$ | 97.22 | 0.05 | 0.5 |
| $\tilde{t}_1$ | 398.93 | 2.0 | 4.0 |
| ... | ... | ... | |

# Experimental Input

LHC "experimental" data from cascade decays (best precision obtainable)
$\sqrt{S} = 14$ TeV, integrated luminosity $\mathcal{L} = 300$ fb$^{-1}$

ILC data direct mass measurements
$\sqrt{S}$ up to 800 GeV, integrated luminosity $\mathcal{L} = 500$ fb$^{-1}$

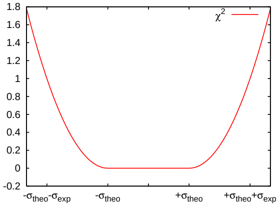
S-LHC: increased statistics
same measurements as LHC, systematic errors unchanged
$\sqrt{S} = 14$ TeV, integrated luminosity $\mathcal{L} = 3$ ab$^{-1}$

CLIC: higher center-of-mass energy $\Rightarrow$ complete squark sector visible
$\sqrt{S} = 3$ TeV, squark mass measurements with 0.5% statistical error

# Theoretical Errors

Treatment of errors:

- All experimental errors are Gaussian
  $$\sigma^2_{\text{exp}} = \sigma^2_{\text{stat}} + \sigma^2_{\text{syst}(j)} + \sigma^2_{\text{syst}(l)}$$
- Systematic errors from jet ($\sigma_{\text{syst}(j)}$) and lepton energy scale ($\sigma_{\text{syst}(l)}$) assumed 99% correlated each
- Theory error added as box-shaped (RFit scheme [Hoecker, Lacker, Laplace, Lediberder])




$$\Rightarrow -2\log L \equiv \chi^2 = \sum_{\text{measurements}} \begin{cases} 0 & \text{for } |x_{\text{data}} - x_{\text{pred}}| < \sigma_{\text{theo}} \\ \left(\frac{|x_{\text{data}} - x_{\text{pred}}| - \sigma_{\text{theo}}}{\sigma_{\text{exp}}}\right)^2 & \text{for } |x_{\text{data}} - x_{\text{pred}}| \geq \sigma_{\text{theo}} \end{cases}$$

- Addition of individual theory errors linearly (not in quadrature!) and without correlations (conservative), *e.g.* for kinematic edges
- Numerical value: 1% for gluino and squark masses,
  0.5% for all other sparticles
  2 GeV for $m_{h^0}$ (unknown higher-order terms)

# MSSM parameter space

20 (LHC) / 22 (ILC) parameters in total
(no flavour violation, no CP violation)

Trilinear couplings basically undetermined at the LHC
$\Rightarrow$ fix $A_b$, $A_\tau$ to central value of allowed range
$A_b$, $A_\tau$ = 0 GeV for LHC
$\longrightarrow$ introduced bias small

Some sectors only partly constrained
$\longrightarrow$ only $\tilde{\tau}_R$, no $\tilde{t}$, no heavy Higgs at LHC
light-Higgs loop corrections introduce dependence
$\Rightarrow$ free parameters in the fit (and large errors as result)

Two relevant SM parameters for later RGE running:

- $\alpha_s$
- $m_t$

$\Rightarrow$ add corresponding measurement and fit as parameter

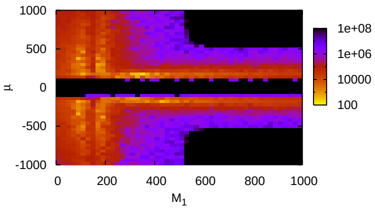
$\tan\beta$
$\mu$
$m_A$
$M_1$
$M_2$
$M_3$
$M_{\tilde{\tau}_L}$
$M_{\tilde{\tau}_R}$
$M_{\tilde{\mu}_L}$
$M_{\tilde{\mu}_R}$
$M_{\tilde{e}_L}$
$M_{\tilde{e}_R}$
$M_{\tilde{q}_{3L}}$
$M_{\tilde{t}_R}$
$M_{\tilde{b}_R}$
$M_{\tilde{q}_L}$
$M_{\tilde{q}_R}$
$A_t$
$A_b$
$A_\tau$
$m_t$, $\alpha_s$

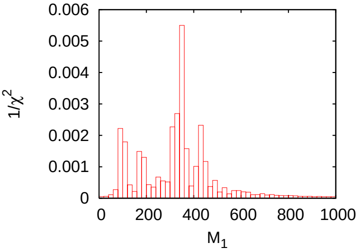
# Multiple solutions

$\chi_3^0$ and both charginos unobserved
$\Rightarrow$ Hierarchy of $M_1$, $M_2$ and $|\mu|$ and sgn($\mu$) undetermined
$\Rightarrow$ 8 discrete solutions (not 12, as $|\mu|$ cannot be the smallest)
All solutions have equal $\chi^2$ (ILC can later resolve this ambiguity)

[SFitter 2007]

| | DS1 | DS2 | DS3 | DS4 | DS7 (SPS1a-like) | DS8 | DS9 | DS10 |
|---|---|---|---|---|---|---|---|---|
| $\tan\beta$ | 12.3±5.6 | 12.4±5.0 | 14.9±9.8 | 8.9±5.9 | 13.8±7.5 | 12.6±7.9 | 19.2±14.3 | 23.0±15.6 |
| $M_1$ | 102.7±7.1 | 189.5±6.2 | 107.2±9.2 | 383.2±9.1 | 105.0±6.9 | 191.7±6.6 | 116.3±7.5 | 380.9±9.3 |
| $M_2$ | 185.5±7.0 | 96.±6.4 | 356.9±8.7 | 114.2±10.7 | 194.7±7.3 | 105.5±7.3 | 354.0±8.2 | 137.2±9.1 |
| $\mu$ | -362.7±7.8 | -364.7±6.8 | -186.0±8.5 | -167.0±9.6 | 353.0±7.7 | 357.1±8.3 | 188.9±7.1 | 172.8±8.7 |
| $\Delta\chi^2_{\rm ILC}$ | 73 | 22000 | 1700 | 25000 | 0.4 | 22000 | 2000 | 24000 |
| ILC | $\tilde{\tau}_1$ | $\chi_1^\pm$ | $\chi_3^0$ | $\chi_1^\pm$ | $\tilde{\tau}_1$ | $\chi_1^\pm$ | $\chi_3^0$ | $\chi_1^\pm$ |



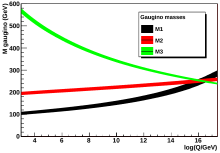
# Testing Unification

Procedure:

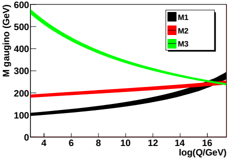
- 5000 toy experiments at the electro-weak scale
- Use renormalization group equations (RGEs) to evolve to high scale
- Test each point of the eight-fold solution separately
- Unification determined by
  $\chi^2_{\text{avg}}(Q^2) = \sum_{i,j}^{N} (M_i - m_U)(C_p^{-1})_{ij}(M_j - m_U) = \min$
  ($Q$ unification scale  $m_U$ unified mass  $M_j$ MSSM mass parameter
  $C$ covariance matrix of the parameters)

Gaugino masses:

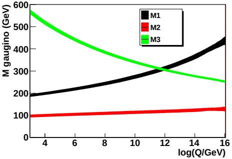
Correct solution (DS7)




Flipped sign of $\mu$ (DS1)




Wrong solution (DS2)



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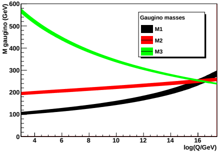
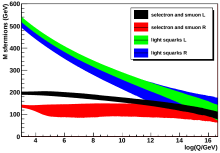
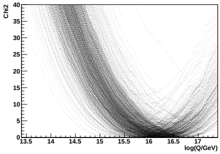



scalar masses (1+2 gen)

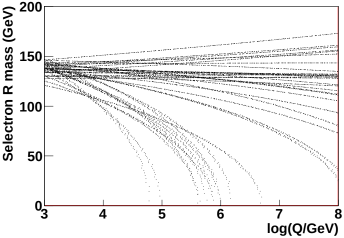



$\chi^2_{\text{avg}}$



# Tachyonic solutions

Not many solutions survive:




RGE for $M_{\tilde{e}_R}$:

$$\frac{d}{dt}M^2_{\tilde{e}_R} = \frac{1}{2\pi}\frac{3}{5}\alpha_1\left(\mathrm{Tr}[Ym^2] - 4M_1^2\right) \text{ with}$$

$$\begin{aligned}\mathrm{Tr}[Ym^2] = {} & M^2_{H_2} - M^2_{H_1} \\ & + M^2_{\tilde{q}1_L} - M^2_{\tilde{e}_L} - 2M^2_{\tilde{u}_R} + M^2_{\tilde{d}_R} + M^2_{\tilde{e}_R} \\ & + M^2_{\tilde{q}2_L} - M^2_{\tilde{\mu}_L} - 2M^2_{\tilde{c}_R} + M^2_{\tilde{s}_R} + M^2_{\tilde{\mu}_R} \\ & + M^2_{\tilde{q}3_L} - M^2_{\tilde{\tau}_L} - 2M^2_{\tilde{t}_R} + M^2_{\tilde{b}_R} + M^2_{\tilde{\tau}_R} \\ & \overset{\text{mSUGRA/tree-level}}{=} 0\end{aligned}$$

Undetermined masses spoil evolution

$\Rightarrow$ Solutions turn tachyonic

$\rightarrow$ remove set

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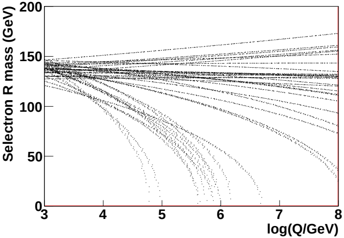


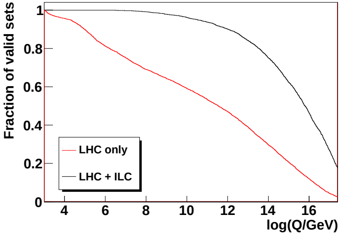


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\end{aligned}$$

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$\rightarrow$ remove set

LHC: only 7% survive up to GUT scale
ILC: adding measurements helps
38% remaining at GUT scale

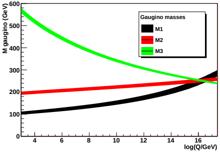
# LHC plus ILC

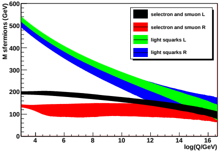
ILC data significantly improves precision

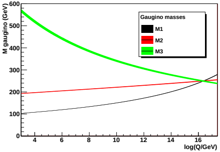
Upper row: LHC alone
Lower row: LHC + ILC

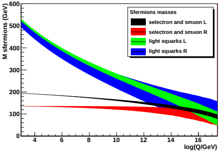
Gaugino

Sfermion (1+2 gen)




Results:

| | LHC |
|---|---|
| $m_{1/2}$ | $251.9 \pm 5.9$ |
| $m_0$ | $98.5 \pm 10.5$ |
| $\log(Q/\mathrm{GeV})$ | $16.23 \pm 0.29$ |
| $Q$ ($10^{16}$ GeV) | $1.7 \pm 1.1$ |

| | LHC+ILC |
|---|---|
| $m_{1/2}$ | $249.5 \pm 1.8$ |
| $m_0$ | $105.3 \pm 9.1$ |
| $A_0$ | $-164 \pm 182$ |
| $\log(Q/\mathrm{GeV})$ | $16.37 \pm 0.05$ |
| $Q$ ($10^{16}$ GeV) | $2.33 \pm 0.28$ |

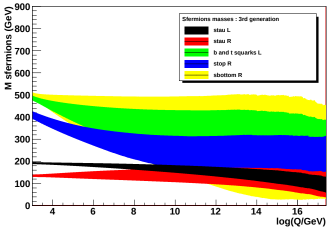
# Top-down vs. Bottom-up

Top-down approach

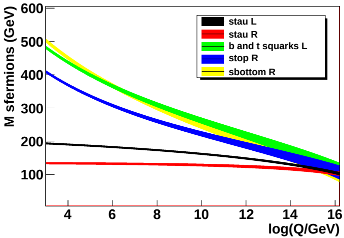
- Use high-scale MSSM with all parameters defined at the GUT scale
- Evolve parameters down to electro-weak scale
- Compare evolved parameters with experimental measurements

Sfermion (3rd generation):

Bottom-up (LHC+ILC)




Top-down




$\longrightarrow$ Huge differences between both approaches

# Top-down vs. Bottom-up

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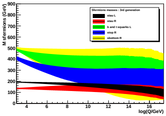
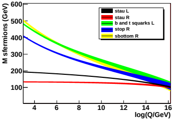
Sfermion (3rd generation):

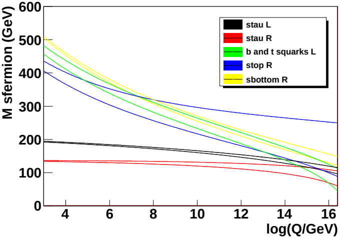
Bottom-up




Top-down




Single data-set




Tachyonic data sets:

- bottom-up: considered up to scale where first entry becomes tachyonic
- top-down: non-existent by construction

Alternative solutions:

due to badly constrained parameters best-fitting solution far off possible

- bottom-up: smooth transition from true solution
  easy to find for fitting algorithms
  large blow-up only in RGE running
- top-down: solutions separated by huge valley of unlikely points
  very hard for fitting algorithms
  find sub-optimal solution close to true one instead
  $\Rightarrow$ underestimate parameter errors

# Top-down vs. Bottom-up

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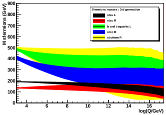
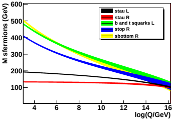
Sfermion (3rd generation):

Bottom-up




Top-down

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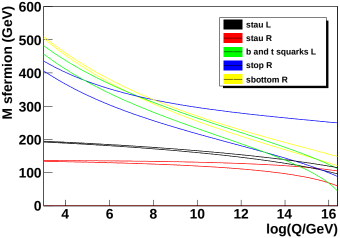


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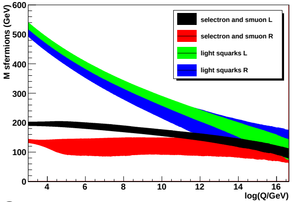
  $\Rightarrow$ **underestimate parameter errors**

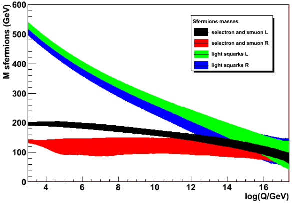
# Further into the future . . .

S-LHC:
More statistics for $\tilde{q}_R$ and $\tilde{q}_{3L}$ measurements
but mostly systematics-dominated already at the end of LHC
$\longrightarrow$ not much improvement

LHC




S-LHC




| | LHC | S-LHC |
|---|---|---|
| $\tan\beta$ | $13.8 \pm 7.4$ | $13.4 \pm 7.3$ |
| $\mu$ | $353.1 \pm 7.7$ | $352.6 \pm 7.1$ |
| $M_1$ | $105.0 \pm 6.9$ | $104.8 \pm 6.9$ |
| $M_{\tilde{q}_{3L}}$ | $491.4 \pm 16.2$ | $491.0 \pm 15.4$ |
| $M_{\tilde{q}_R}$ | $509.0 \pm 16.4$ | $508.9 \pm 12$ |
| % remaining | 7% | 7% |

# Further into the future ...

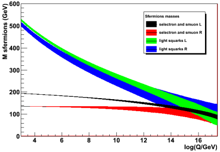
CLIC: Higher center-of-mass energy (3 TeV)

- squark sector kinematically accessible
- first measurement of $\tilde{t}_2$
- no improvement on parameters already well-measured by ILC

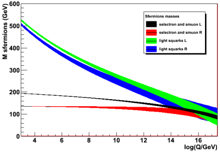
| | S-LHC | S-LHC+ILC | S-LHC+CLIC |
|---|---|---|---|
| $\tan\beta$ | 13.4 $\pm$ 7.3 | 11.2 $\pm$ 3.3 | 11.1 $\pm$ 3.2 |
| $M_1$ | 104.8 $\pm$ 6.9 | 103.05 $\pm$ 0.7 | 103.1 $\pm$ 0.6 |
| $M_{\tilde{q}_R}$ | 508.9 $\pm$ 12.0 | 507.8 $\pm$ 10.0 | 507.2 $\pm$ 5.8 |
| $M_{\tilde{q}_{3L}}$ | 491.0 $\pm$ 15.4 | 486.6 $\pm$ 10.8 | 485.2 $\pm$ 8.1 |
| $M_{\tilde{t}_R}$ | 494 $\pm$ 234 | 408.8 $\pm$ 17.1 | 407.7 $\pm$ 12.8 |
| $A_t$ | -387 $\pm$ 372 | -497.4 $\pm$ 65 | -492.2 $\pm$ 38 |
| $m_0$ | | 96.0 $\pm$ 8.6 | 94.1 $\pm$ 7.7 |
| $m_{1/2}$ | | 249.3 $\pm$ 1.7 | 249.3 $\pm$ 1.66 |
| $\log Q/\mathrm{GeV}$ | | 16.4 $\pm$ 0.05 | 16.36 $\pm$ 0.05 |
| % remaining | 7% | 43% | 50% |

Sfermions 1st+2nd gen

S-LHC+ILC




S-LHC+CLIC



# Further into the future . . .

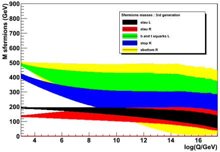
CLIC: Higher center-of-mass energy (3 TeV)

- squark sector kinematically accessible
- first measurement of $\tilde{t}_2$
- no improvement on parameters already well-measured by ILC

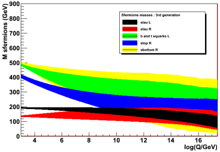
| | S-LHC | S-LHC+ILC | S-LHC+CLIC |
|---|---|---|---|
| $\tan\beta$ | 13.4 ± 7.3 | 11.2 ± 3.3 | 11.1 ± 3.2 |
| $M_1$ | 104.8 ± 6.9 | 103.05 ± 0.7 | 103.1 ± 0.6 |
| $M_{\tilde{q}_R}$ | 508.9 ± 12.0 | 507.8 ± 10.0 | 507.2 ± 5.8 |
| $M_{\tilde{q}_{3L}}$ | 491.0 ± 15.4 | 486.6 ± 10.8 | 485.2 ± 8.1 |
| $M_{\tilde{t}_R}$ | 494 ± 234 | 408.8 ± 17.1 | 407.7 ± 12.8 |
| $A_t$ | -387 ± 372 | -497.4 ± 65 | -492.2 ± 38 |
| $m_0$ | | 96.0 ± 8.6 | 94.1 ± 7.7 |
| $m_{1/2}$ | | 249.3 ± 1.7 | 249.3 ± 1.66 |
| $\log Q/\text{GeV}$ | | 16.4 ± 0.05 | 16.36 ± 0.05 |
| % remaining | 7% | 43% | 50% |

Sfermions 3rd gen

S-LHC+ILC




S-LHC+CLIC



# Conclusions

- Test SUSY unification at the GUT scale
- LHC
  - Some MSSM parameters with large errors at low scale and 8-fold degeneracy (for SPS1a-like parameter points)
  - Unification for two solutions (sgn $\mu$)
  - Determine $m_{1/2}$ and unification scale to 2%
- ILC
  - Measures all kinematically accessible masses with high precision $\Rightarrow$ Significant improvement in parameter determination
  - Degenerate solutions disappear
- S-LHC and CLIC
  - only marginal improvement at S-LHC
  - Precise measurements of parameters at CLIC (mostly 0.5% to 1%)
  - Limitation $\longrightarrow$ Theory error
  - Improvement over LHC+ILC not dramatic
    $\longleftarrow$ Precise masses of light particles and well-known edges perform pretty well for *e.g.* squark masses

# LHC measurements

| | type of measurement | nominal value | stat. | LES error | JES error | theo. |
|---|---|---|---|---|---|---|
| $m_h$ | | 108.7 | 0.01 | 0.25 | | 2.0 |
| $m_t$ | | 171.20 | 0.01 | | 1.0 | |
| $m_{\tilde{l}_L} - m_{\chi_1^0}$ | | 102.38 | 2.3 | 0.1 | | 1.1 |
| $m_{\tilde{g}} - m_{\chi_1^0}$ | | 511.38 | 2.3 | | 6.0 | 6.1 |
| $m_{\tilde{q}_R} - m_{\chi_1^0}$ | | 446.39 | 10.0 | | 4.3 | 5.5 |
| $m_{\tilde{g}} - m_{\tilde{b}_1}$ | | 89.01 | 1.5 | | 1.0 | 8.0 |
| $m_{\tilde{g}} - m_{\tilde{b}_2}$ | | 62.93 | 2.5 | | 0.7 | 8.2 |
| $m_{ll}^{\max}$: | three-particle edge($\chi_2^0,\tilde{l}_R,\chi_1^0$) | 80.852 | 0.042 | 0.08 | | 1.2 |
| $m_{llq}^{\max}$: | three-particle edge($\tilde{q}_L,\chi_2^0,\chi_1^0$) | 449.08 | 1.4 | | 4.3 | 5.1 |
| $m_{lq}^{\mathrm{low}}$: | three-particle edge($\tilde{q}_L,\chi_2^0,\tilde{l}_R$) | 326.32 | 1.3 | | 3.0 | 5.2 |
| $m_{ll}^{\max}(\chi_4^0)$: | three-particle edge($\chi_4^0,\tilde{l}_L,\chi_1^0$) | 277.36 | 3.3 | 0.3 | | 2.0 |
| $m_{\tau\tau}^{\max}$: | three-particle edge($\chi_2^0,\tilde{\tau}_1,\chi_1^0$) | 83.21 | 5.0 | | 0.8 | 1.0 |
| $m_{lq}^{\mathrm{high}}$: | four-particle edge($\tilde{q}_L,\chi_2^0,\tilde{l}_R,\chi_1^0$) | 390.18 | 1.4 | | 3.8 | 5.0 |
| $m_{llq}^{\mathrm{thres}}$: | threshold($\tilde{q}_L,\chi_2^0,\tilde{l}_R,\chi_1^0$) | 216.00 | 2.3 | | 2.0 | 3.3 |
| $m_{llb}^{\mathrm{thres}}$: | threshold($\tilde{b}_1,\chi_2^0,\tilde{l}_R,\chi_1^0$) | 198.41 | 5.1 | | 1.8 | 3.1 |

# ILC measurements

| particle | $m_{\mathrm{SPS1a}}$ value | ±stat.err. | ±theo.err. |
| --- | --- | --- | --- |
| $h$ | 108.7 | $\pm$ 0.05 | $\pm$ 2.0 |
| $H$ | 395.34 | $\pm$ 1.5 | $\pm$ 2.0 |
| $A$ | 394.9 | $\pm$ 1.5 | $\pm$ 2.0 |
| $H+$ | 403.5 | $\pm$ 1.5 | $\pm$ 2.0 |
| $\chi_1^0$ | 97.22 | $\pm$ 0.05 | $\pm$ 0.5 |
| $\chi_2^0$ | 180.44 | $\pm$ 1.2 | $\pm$ 0.9 |
| $\chi_3^0$ | 355.45 | $\pm$ 4.0 | $\pm$ 1.8 |
| $\chi_4^0$ | 375.09 | $\pm$ 4.0 | $\pm$ 1.9 |
| $\chi_1^\pm$ | 179.79 | $\pm$ 0.55 | $\pm$ 0.9 |
| $\chi_2^\pm$ | 375.22 | $\pm$ 3.0 | $\pm$ 1.9 |
| $\tilde{t}_1$ | 398.93 | $\pm$ 2.0 | $\pm$ 4.0 |
| $\tilde{e}_L$ | 199.59 | $\pm$ 0.2 | $\pm$ 1.0 |
| $\tilde{e}_R$ | 142.68 | $\pm$ 0.05 | $\pm$ 0.7 |
| $\tilde{\mu}_L$ | 199.59 | $\pm$ 0.5 | $\pm$ 1.0 |
| $\tilde{\mu}_R$ | 142.68 | $\pm$ 0.2 | $\pm$ 0.7 |
| $\tilde{\tau}_1$ | 133.36 | $\pm$ 0.3 | $\pm$ 0.7 |
| $\tilde{\tau}_2$ | 203.62 | $\pm$ 1.1 | $\pm$ 1.0 |
| $\tilde{\nu}_e$ | 183.72 | $\pm$ 1.2 | $\pm$ 0.9 |