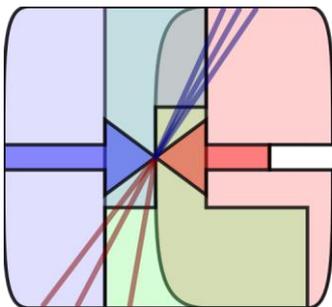
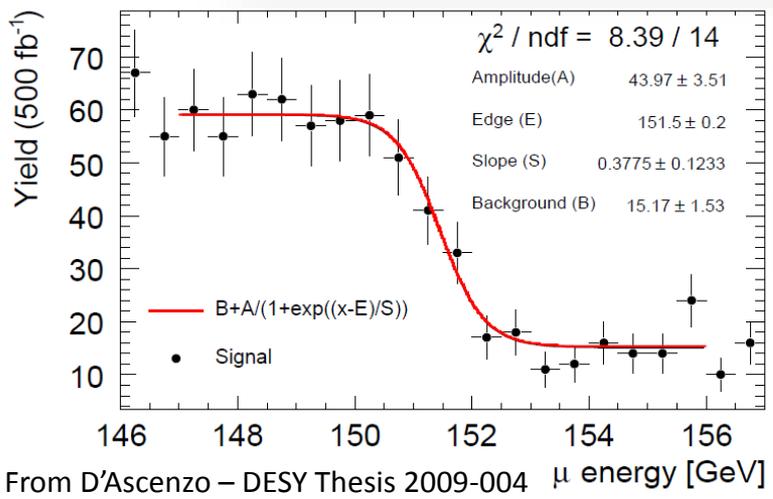
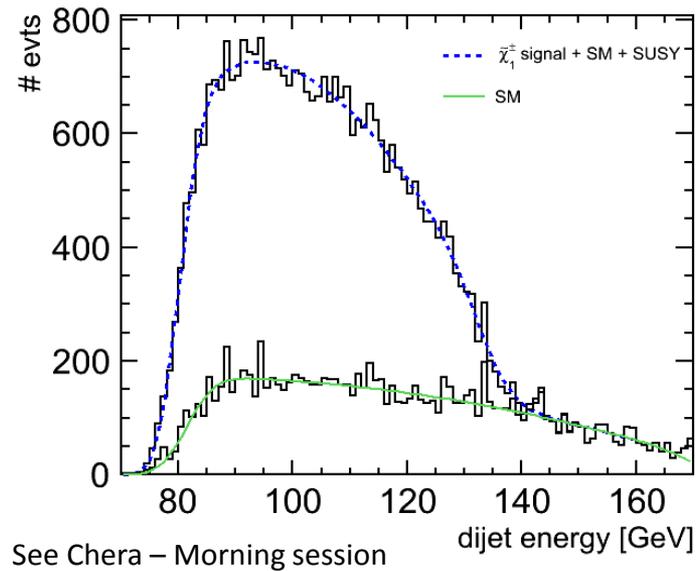
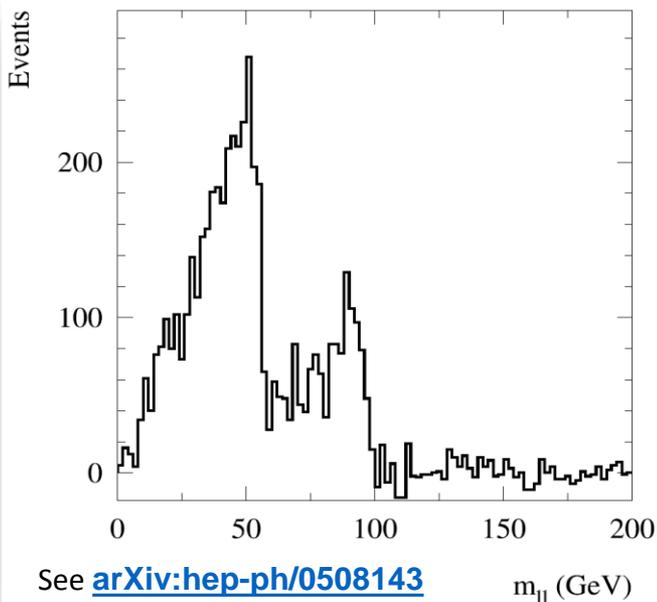


Computational edge detection on ILC SUSY measurements



Sabato Stefano Caiazza,
J. List, M. Berggren
29 May 2013, ECFA 2013





Often defining feature of a distribution

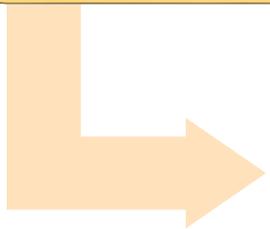
- Edge position due to a kinematic constraint

Common in SUSY

- Decay Cascades
- S-fermions decays
- Neutralino/Chargino decays

$$f(x; t_0, b_0, \sigma_1, \gamma) = f_{SM} + \int_{t_0}^{t_1} (b_2 t^2 + b_1 t + b_0) V(x - t, \sigma(t), \gamma) dt$$

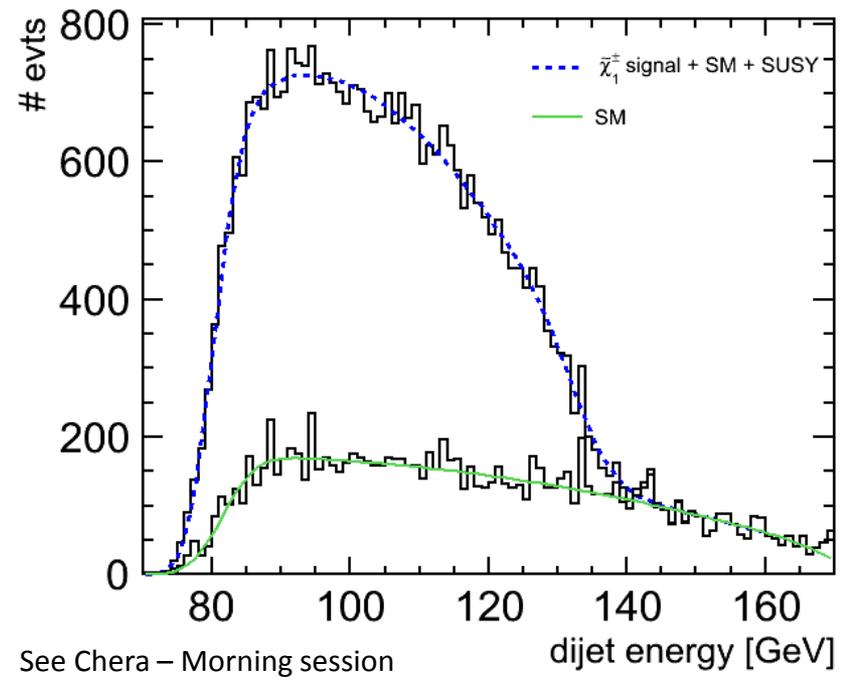
One function describes the whole distribution



Fit the data with a background correction



Find the edges from the fit parameters



Use whole data set

Implies a strong expectation on the signal features

Requires a strong understanding of background and systematics

Roughly detect the edge location

Define an ad-hoc function

Fit only in the area around the edge

Only uses a limited amount of data

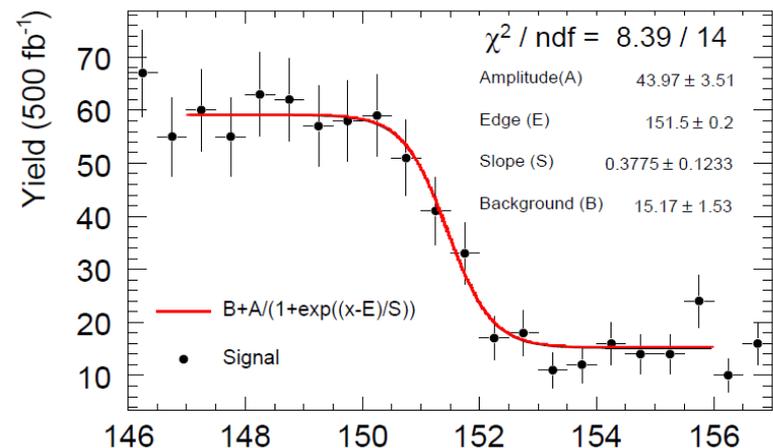
- Ignores the distribution bulk

Requires a limited a-priori knowledge

- Only of a limited part of the distribution

Machine and detector effects more relevant

- Compared to the global fit scenario
- This effects can be more difficult to evaluate



From D'Ascenzo – DESY Thesis 2009-004 μ energy [GeV]

Computational detection

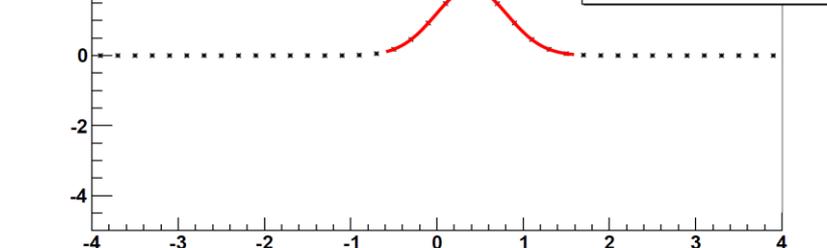
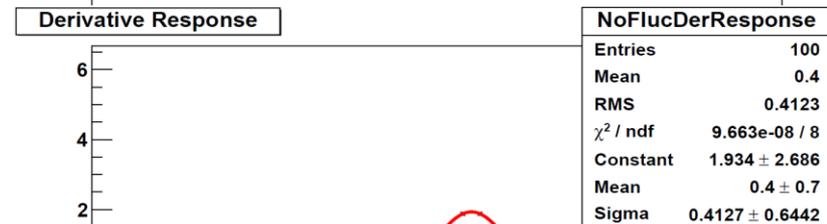
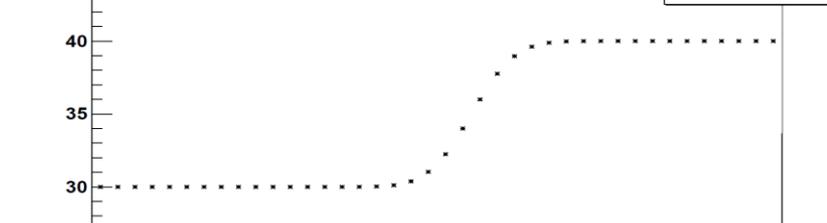
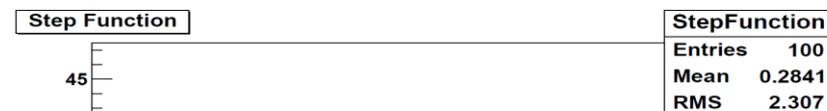
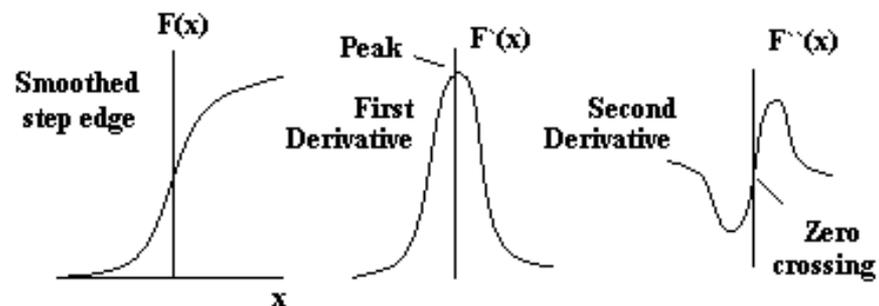
- Defining an algorithm weakly dependent from the edge shape
- Defining your measurement as a function of the algorithm response
- Eventually describe the edge parameters from the response

Continuous function

- Calculate the derivative
- Take the turning point

Discrete domain

- $F'(x) = (F(x+h) - F(x-h))/2h$
- Find the maximum



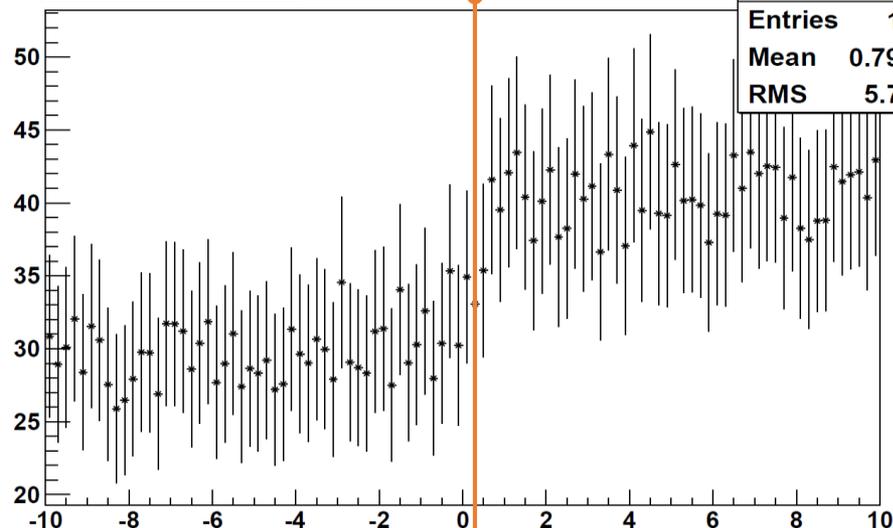
Noisy smeared edge

- Edge Position at 0.37
- Gaussian smearing with $\sigma = 2$
- Poisson fluctuation on background and signal
- \sqrt{n} errors

Standard derivative fails

- The signal is flooded by the noise

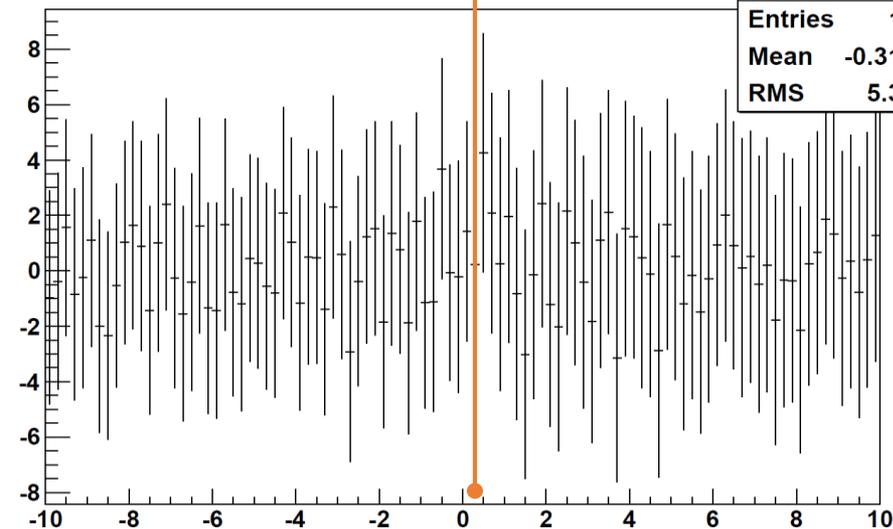
Step Function realistic



StepFunctionFluct

Entries	100
Mean	0.7999
RMS	5.728

Derivative Response



FlucDerivResponse

Entries	100
Mean	-0.3165
RMS	5.324

Computational edge detection

- Common problem in image processing
- Problem solution available (Canny Filter)

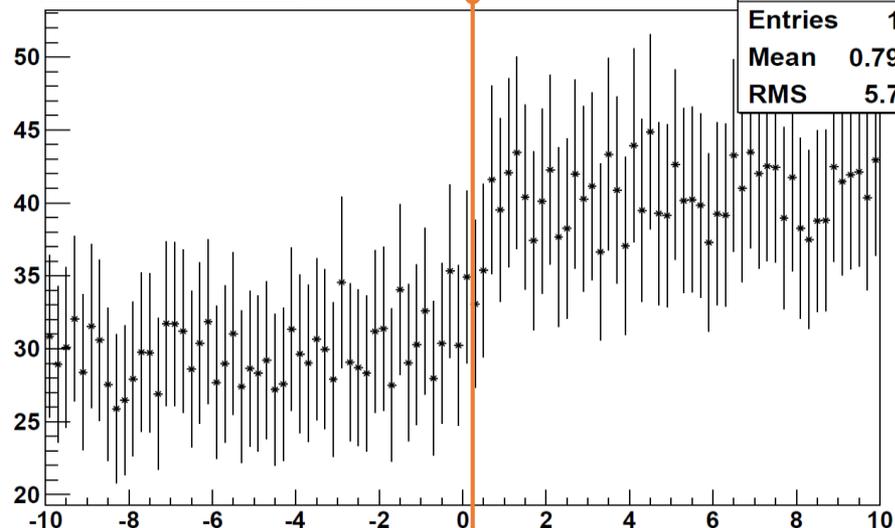
How does it work

- Convolution with a specific weighting function
- Weighting function is optimized for the feature to detect (Steps, Ridges, Roofs)
- See: [John Canny – A Computational approach to edge Detection \(1986\)](#)

Algorithm test

- Same test function as before
- The response shows a clear peak at the right place
- The ideal response should be gaussian
- Expected value: 0.37
- Detected value: 0.39 ± 0.18

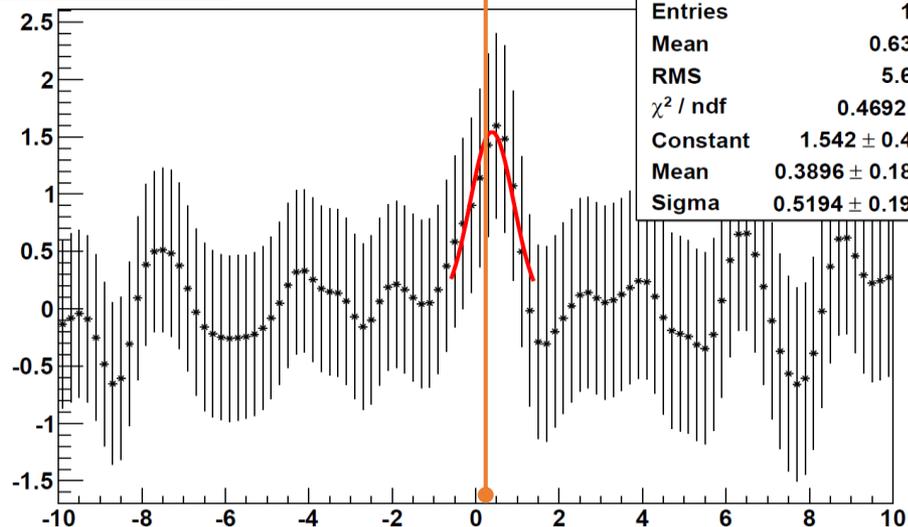
Step Function realistic



StepFunctionFluct

Entries	100
Mean	0.7999
RMS	5.728

FDOG Response



FlucFDOGResponse

Entries	100
Mean	0.6352
RMS	5.628
χ^2 / ndf	0.4692 / 7
Constant	1.542 ± 0.472
Mean	0.3896 ± 0.1839
Sigma	0.5194 ± 0.1994

General case

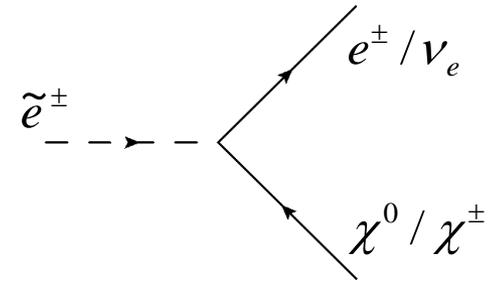
- Electron + any Neutralino
- Neutrino + any Chargino
- Start of a potentially long decay chain with many potential kinematic edges

Special case

- Electron + LSP (TDR4 benchmark scenario)
- Very clean signature and potentially high cross section
- Two body decay of a boosted scalar particle

Electron momentum box distribution

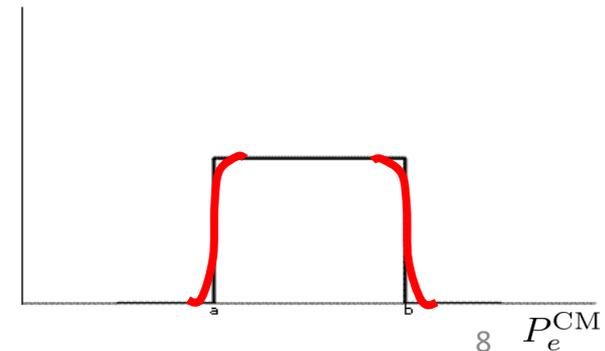
- Kinematically constrained electron momentum
- Uniform distribution between two edges
- Edge smearing due to beam-strahlung and detector effects



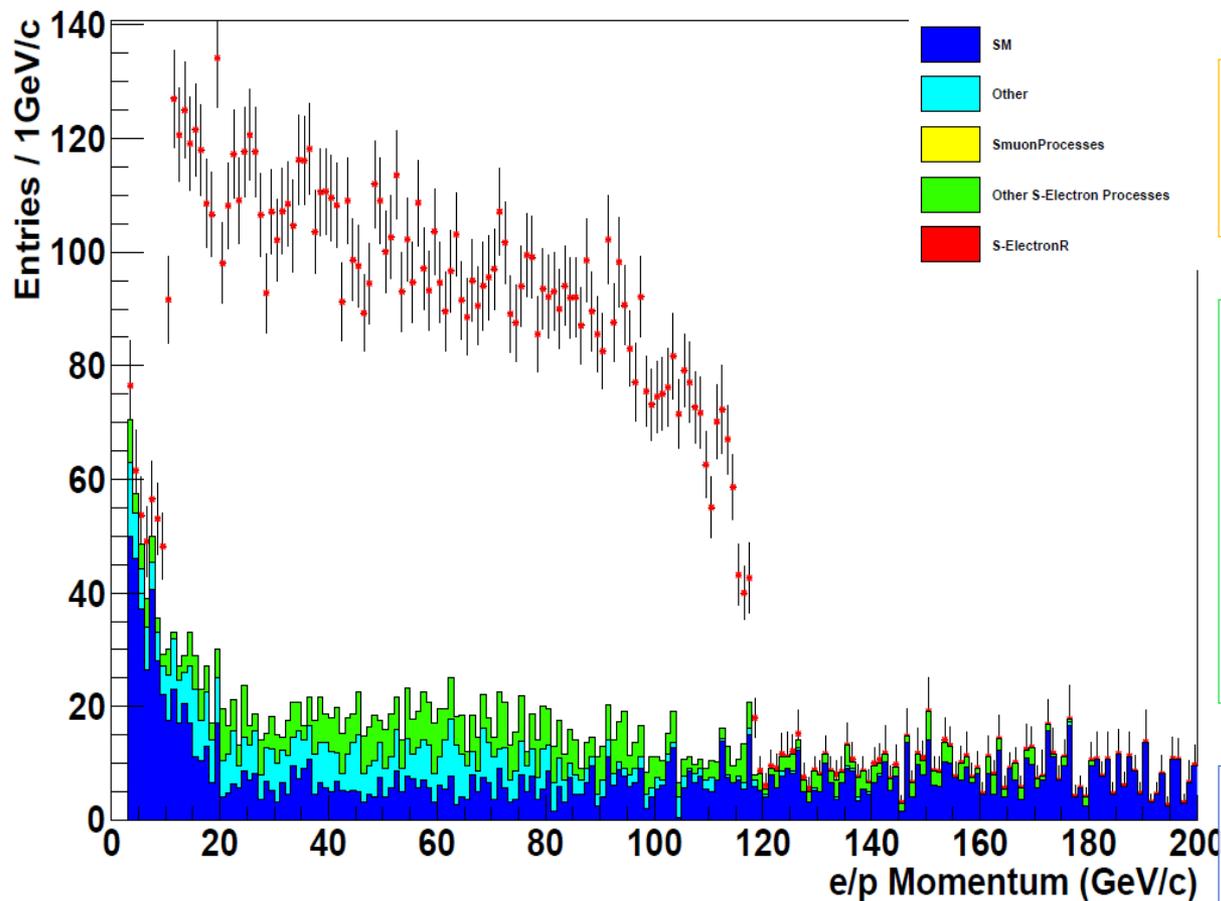
$$P_e^{\tilde{e}} = \frac{M_{\tilde{e}}^2 - M_{\chi}^2}{2M_{\tilde{e}}^2}$$

$$\text{Min}(P_e^{\text{CM}}) = P_e^{\tilde{e}}(\gamma - \beta\gamma)$$

$$\text{Max}(P_e^{\text{CM}}) = P_e^{\tilde{e}}(\gamma + \beta\gamma)$$



e/p Jet Momenta Distribution 10 fb⁻¹



10 fb⁻¹ ILC Data

- Equivalent to 1 week of full luminosity

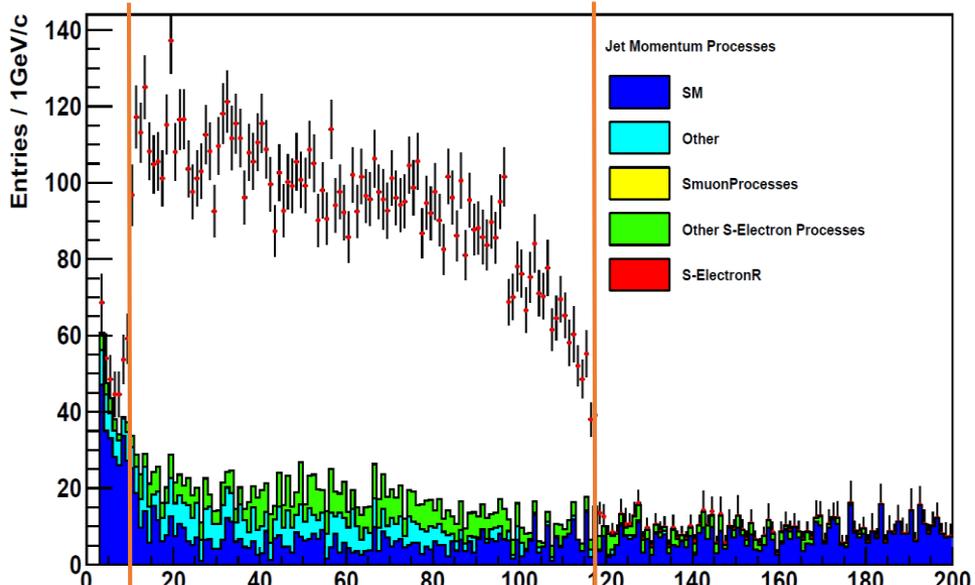
Simple selection

- Model independent cuts applied based on the LEP constraints
- 2 isolated high energy electrons with > 90GeV missing energy and low activity around the beam pipe

Fit Methods

- Strong machine and detector effects distort the distribution
- Difficult to define a theoretical valid template

e/p Jet Momentum Distribution 10 fb-1



Sensitivity

- The edge is clearly identifiable in the response function

Accuracy

- Fitting a Gaussian to the peak reveals the peak position

High Edge Position:

- Expected: 117.64 GeV/c
- Measured 117.5 ± 0.1 GeV/c

Low Edge Position:

- Expected: 10.11 GeV/c
- Measured 10.07 ± 0.10 GeV/c

TwoElectronMomentumDistribution_Convol	
Entries	420
Mean	72.2
RMS	57.5
χ^2 / ndf	4.721 / 9
Constant	5.275 ± 0.36
Mean	10.07 ± 0.10
Sigma	1.232 ± 0.095

Using 1 ILC-week data set

Model independent and not optimized cuts

Without calibration/corrections

Distribution center

- Expected: 63.88 GeV/c
- Measured: 63.78 ± 0.07 GeV/c

Distribution Width

- Expected: 107.53 GeV/c
- Measured: 107.43 ± 0.14 GeV/c

Neutralino Mass

- Expected: 94.4 GeV/c²
- Calculated: 94.3 ± 0.7 GeV/c²

S-Electron Mass

- Expected: 135.0 GeV/c²
- Calculated: 134.8 ± 0.7 GeV/c²

Refine the edge detection

- Understand better the systematics of the algorithm
- Use the response to measure the feature distortions

Calibration around the TDR4 benchmark point

- Verify the stability of the measurement
- Improve the measurement precision on the full 250 fb^{-1} data set

Compare with a different technique

- Compare with a local fit technique

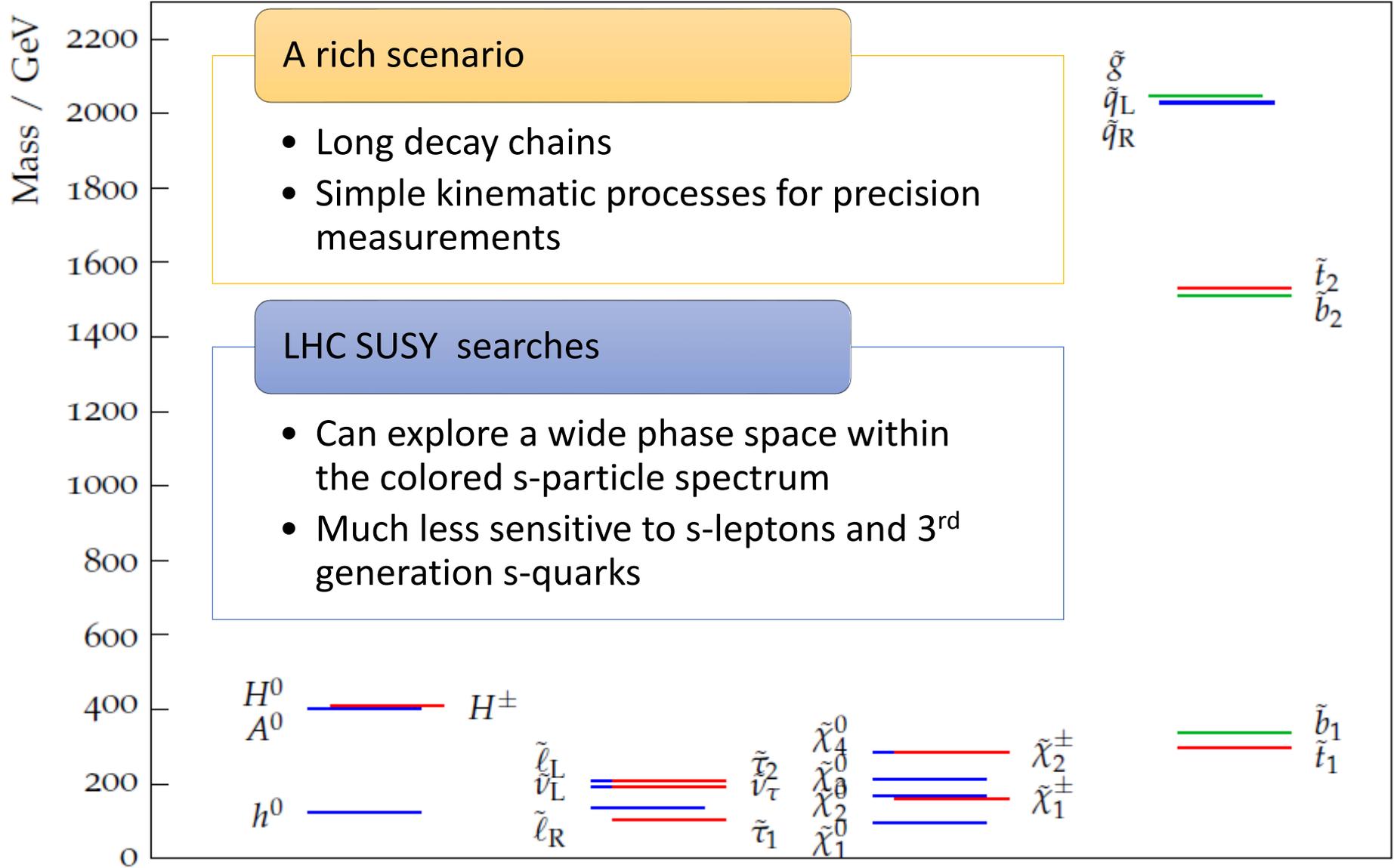
Use the same method to detect other type of features

- For example the peak of a triangle distribution (roof detection)



BACKUP





S-Channel production

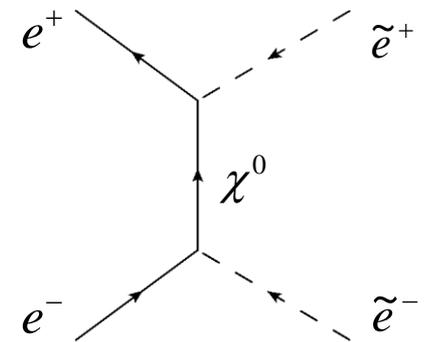
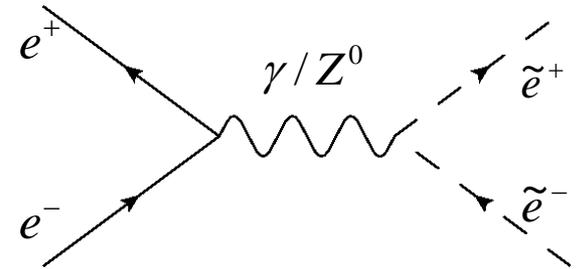
- Common to all s-fermions
- Same cross section for \tilde{e} and $\tilde{\mu}$ (if mass degenerate)
- Isotropic angular distribution

Massive T-Channel exchange

- Only possible for \tilde{e}
- Strongly enhance the \tilde{e} production (from 116,7 fb to 641,9 fb in TDR4)
- Forward peaking angular distribution

Choosing the product chirality

- \tilde{e}_R and \tilde{e}_L are different particles
- ILC beams can be polarized
- Allows the production of s-channel forbidden combinations ($\tilde{e}_R + \tilde{e}_L$)



General case

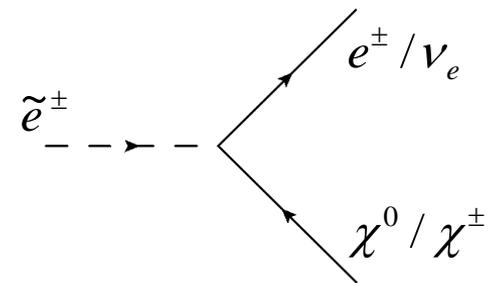
- Electron + any Neutralino
- Neutrino + any Chargino
- Start of a potentially long decay chain

Special case

- Electron + LSP
- Very clean signature and potentially high cross section
- It can be the only available decay
- Two body decay of a boosted scalar particle

Kinematics parameters

- Neutralino Mass
- S-Electron mass
- Center of mass energy



2-Body decay in the parent rest frame

- Daughters' momentum fixed

Boosting the parent rest frame

- Daughters' momentum depend on the angle between the decay and the boost direction.

Scalar parent

- The particle decay isotropically
- The momentum is distributed uniformly

Experimental parameters

- Width and mean of the momentum distribution
- Position of the edges of the distribution
- Distribution smearing due to instability of E_{CM} and other uncertainties

$$M_e = 0 \quad \gamma = E_{CM}/2M_{\tilde{e}}$$

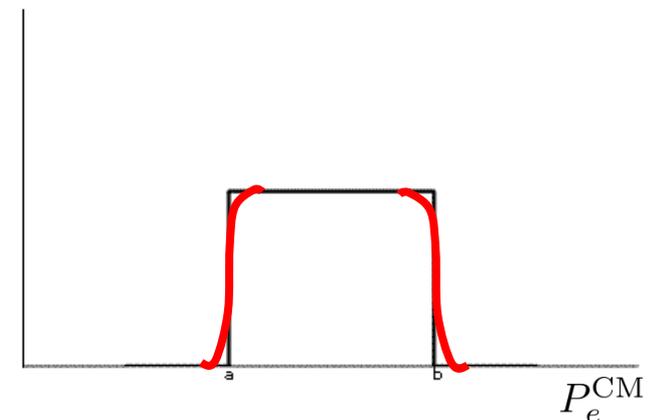
$$P_e^{\tilde{e}} = \frac{M_{\tilde{e}}^2 - M_x^2}{2M_{\tilde{e}}^2}$$

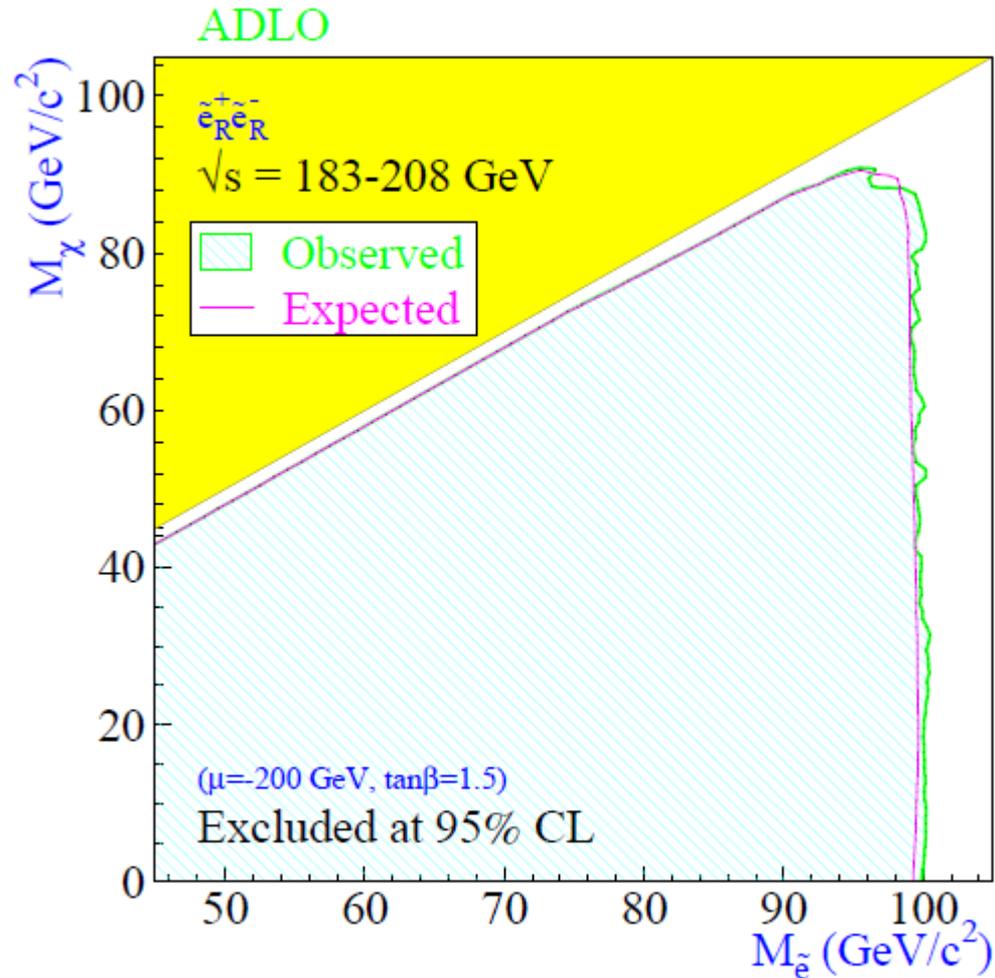
$$\text{Min}(P_e^{CM}) = P_e^{\tilde{e}}(\gamma - \beta\gamma)$$

$$\text{Max}(P_e^{CM}) = P_e^{\tilde{e}}(\gamma + \beta\gamma)$$

$$\Delta E = E_{\text{max}} - E_{\text{min}} = 2P_e^{\tilde{e}}\beta\gamma$$

$$C = \frac{E_{\text{max}} + E_{\text{min}}}{2} = P_e^{\tilde{e}}\gamma$$





LEP constraints

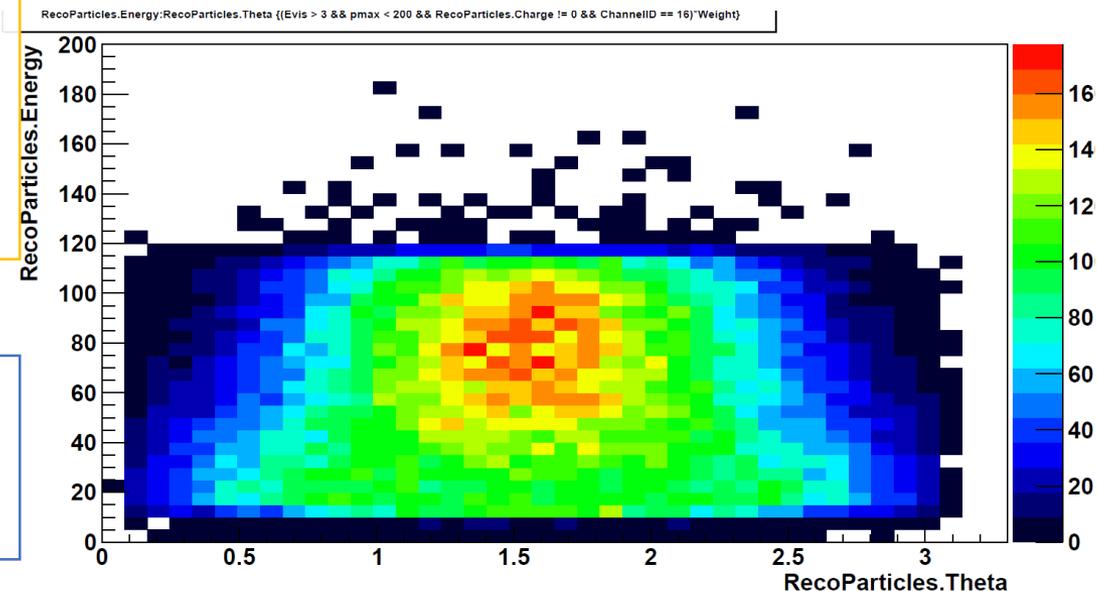
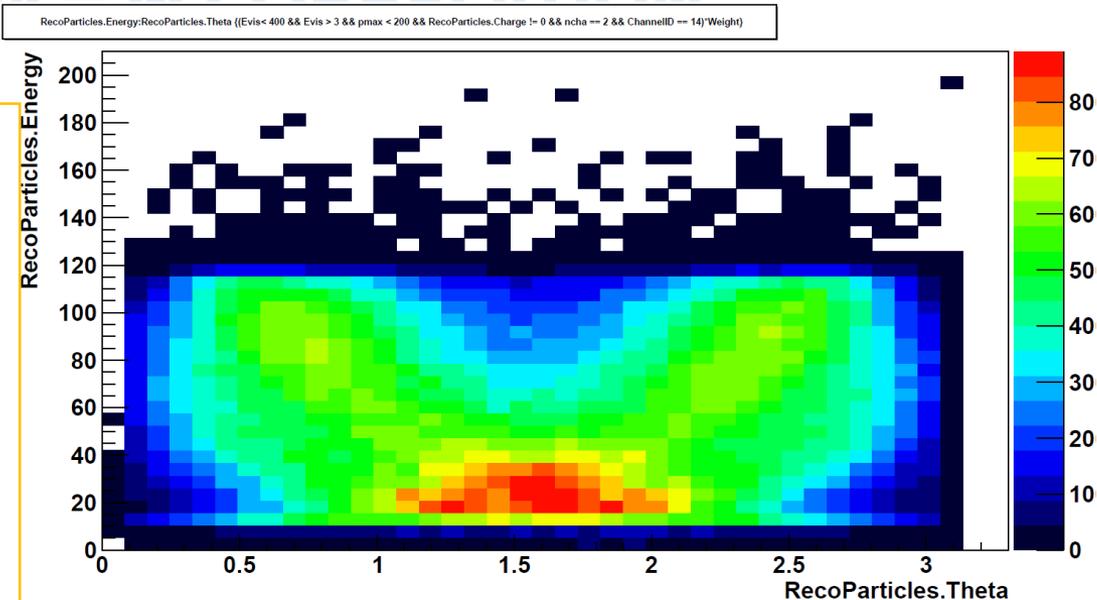
- Neutralino heavier than 45 GeV
- S-Electron / S-muon heavier than 95 GeV

S-Electrons

- Momentum transfer limited by Neutralino mass (in T-channel)
- Forward peaking with a minimum emission angle (0.465 rad for TDR4)
- Maximum electron energy at that angle

Other scalars

- Isotropic production



Using all data available

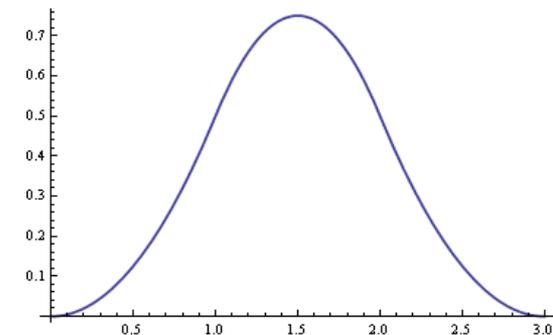
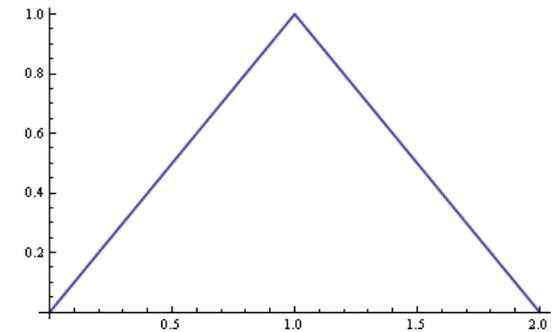
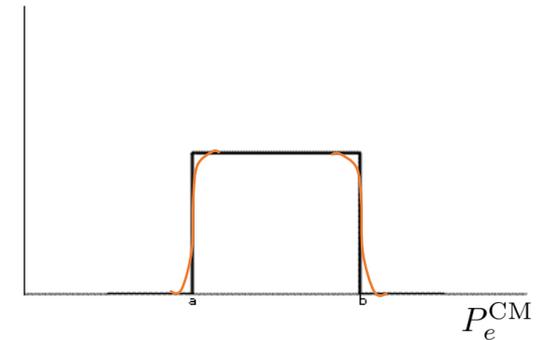
- Edge detection techniques only use a limited amount of data near the feature

Central Limit Theorem

- The mean of a sufficiently large number of independent random variables, each with a well-defined mean and well-defined variance, will be approximately normally distributed
- The momentum of each selected electron is an independent random variable sampled from a box like distribution

Parameter determination

- Average a random sample of electron momenta and fit the results with a gaussian
- Derive the mean and the width of the initial distribution
- Quite effective with large amounts of data



LEP Limits

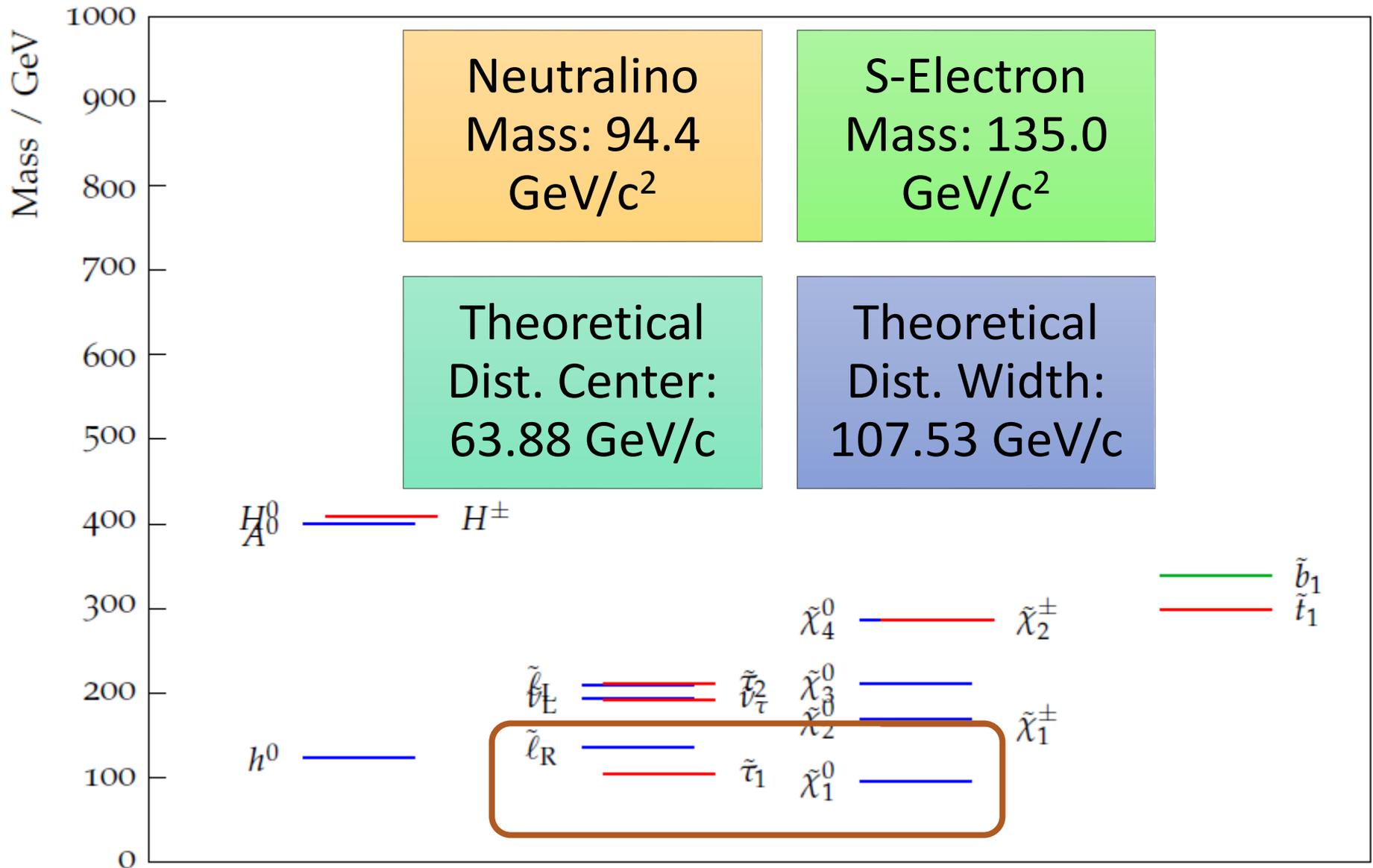
- S-Electron mass > 95 GeV
- Neutralino mass > 45 GeV

ILC Parameters

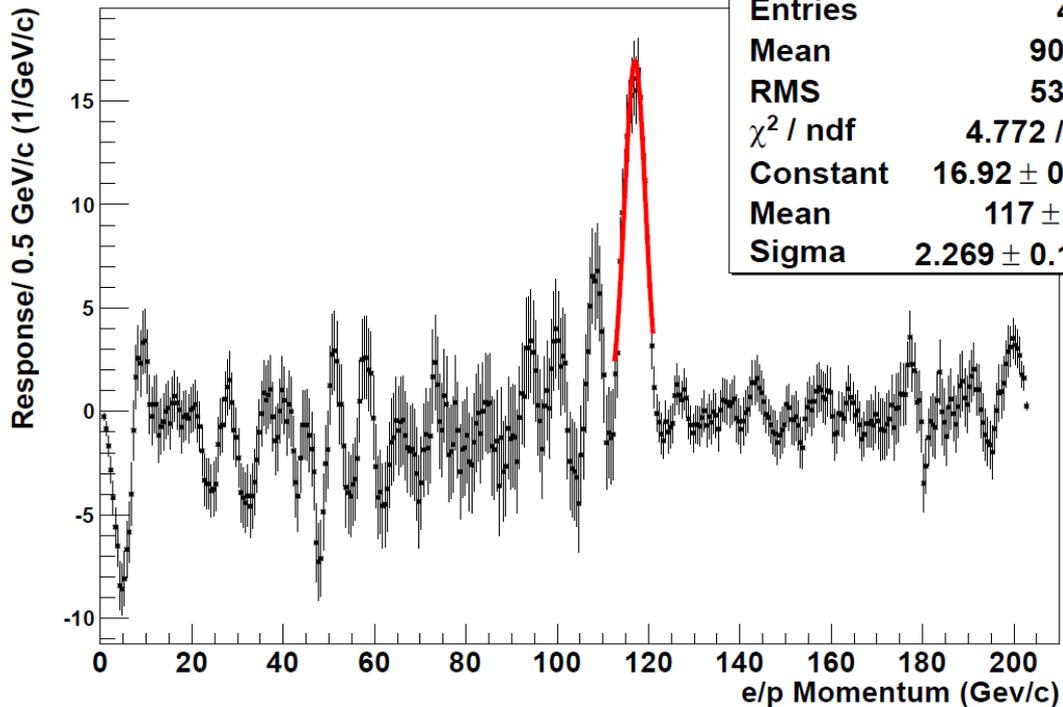
- CM energy 500 GeV
- Max electron polarization 80%
- Max positron polarization 30%

Model Independent signature

- Missing energy > 90 GeV
- 1 electron and 1 positron only in the final state
- Each e/p momentum < 200 GeV
- Low activity around the beam pipe



Edge Detector Response



High Edge Position:

- Expected: 117.64 GeV/c
- Measured 117 ± 0.1 GeV/c

Low Edge Position:

- Expected: 10.11 GeV/c
- Measured: 10.22 ± 0.07 GeV/c

High Edge Position

- Expected: 117.64 GeV/c
- Measured 117.5 ± 0.1 GeV/c

Low Edge Position

- Expected: 10.11 GeV/c
- Measured: 10.07 ± 0.10 GeV/c

Symmetric offset

- Some systematics yet to understand
- It could be calibrated

Distribution center

- Expected: 63.88 GeV/c
- Measured: 63.78 ± 0.07 GeV/c