Perturbative Aspects of Top Mass Reconstruction

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Based on work with:

Sean Fleming, Sonny Mantry and Iain Stewart (hep-ph/0703207)

… more work in progress

Outline

- Motivation. Why do we want a precision m_t ?
- Theory issues for reconstruction.
- Factorization theorem for t and $\bar{t}$ invariant mass distribution in electron-positron annihilation ($Q \gg m_t \gg \Gamma_t$)

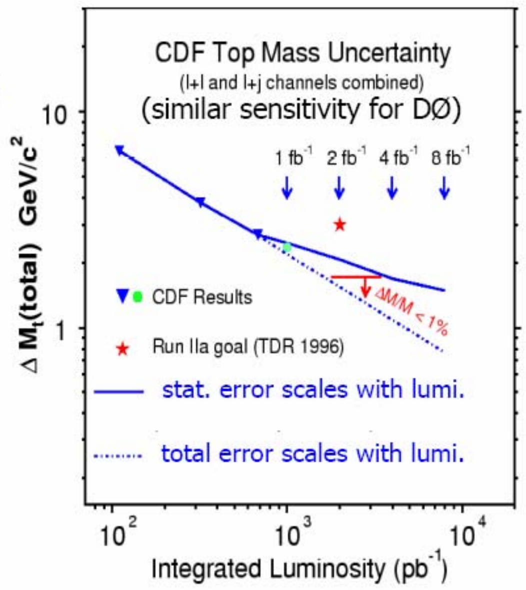
$$\boxed{\begin{aligned}\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\text{hemi}} &= \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) \\ &\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\text{hemi}}(\ell^+,\ell^-,\mu)\end{aligned}}$$

  - Effective Field Theories for top jets: SCET & HQET
  - Derivation Factorization at LO in $m/Q,\, \Gamma_t/m$
  - Phenomenology
  - Radiative Corrections
- Summary

Top Quark is Special !

- Heaviest known quark (related to SSB?)
- Important for quantum effects affecting many observables
- Very unstable, decays "before hadronization" $(\Gamma_t \approx 1.5\ \mathrm{GeV})$

Top Quark Mass at the Tevatron



Combined measurement (March 2007)

$$M_t = 170.9 \pm 1.8\ \mathrm{GeV}/c^2$$

1% precision !

How shall we theorists judge the error ?

What is the <u>theoretical</u> error ?

What mass is it ?

Methods at Tevatron

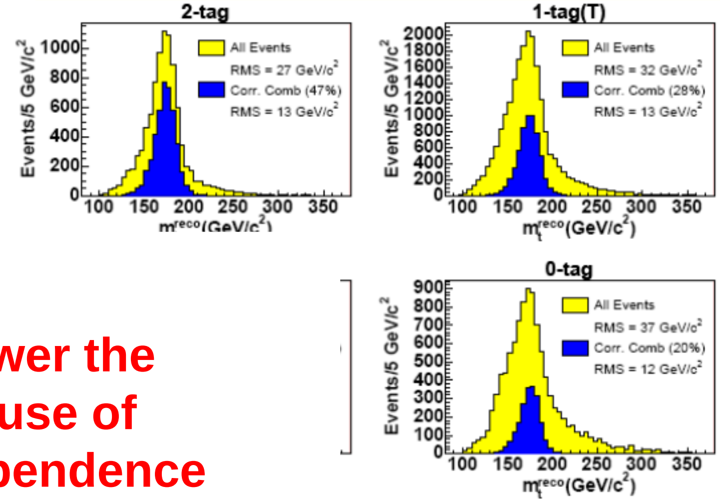
Template Method

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2}$$
$$+ \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2}$$

Usually pick solution with

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamics Metho

Not easy to answer the questions because of complicated dependence on the top quark mass

from Aurelio Juste

b^{-1})

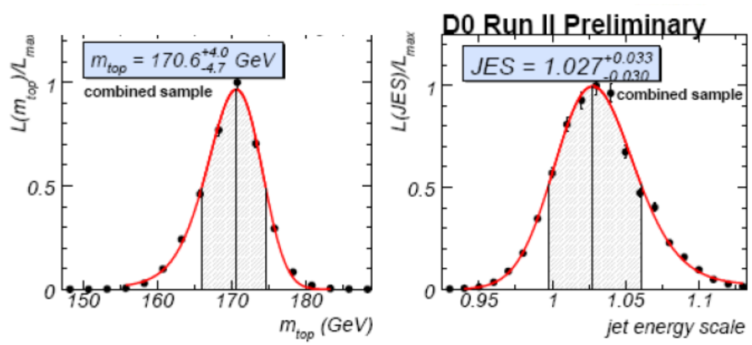
- Principle: compute eve
a function of m_t making
objects in the events (ir
Maximize sensitivity by:

$$P(x;m_t) = \frac{1}{\sigma} \int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1) f(q_2)\, W(x\,|\,y)$$

parton distribution functions

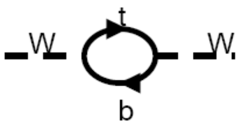
differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

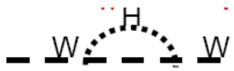


Need for a precise Top mass

Electroweak precision observables

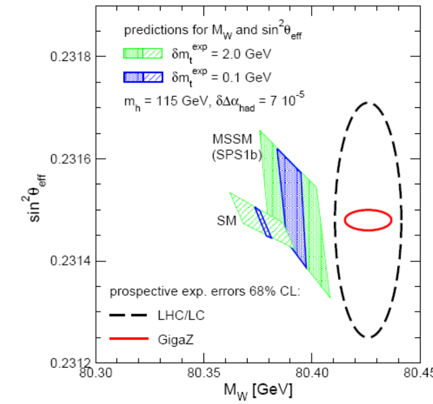


$\delta M_W \propto m_t^2$



$\delta M_W \propto \ln(M_H)$

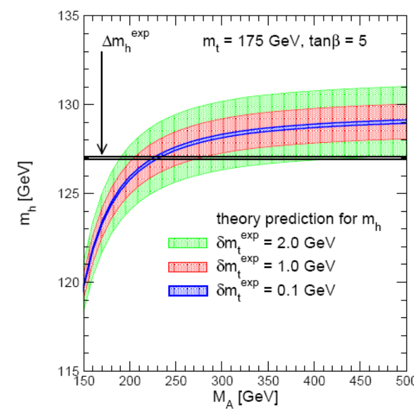
$$\sin\theta_W \times \left(1 + \delta(m_t, m_H, \ldots)\right)$$
$$= 1 - \frac{M_W^2}{M_Z^2}$$



Mass of Lightest MSSM Higgs Boson

	LHC	LC
δm_h	1 GeV	50 MeV
needed δm_t	4 GeV	0.2 GeV
expected δm_t	1-2 GeV	$\sim$ 0.1 GeV

$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$



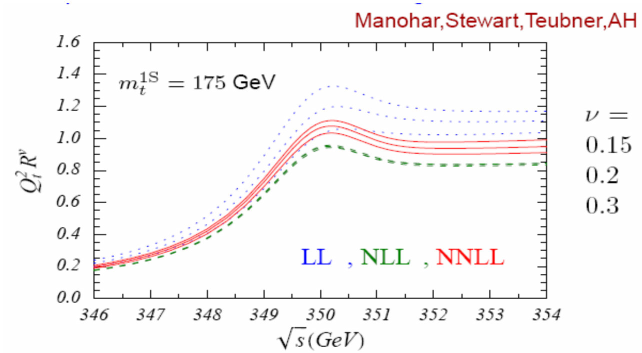
- Best precision possible wanted.
- Mass definition (with small error) needs to be well defined. (Which mass has 0.1 GeV uncertainty ?)

Direct Measurement Methods

Threshold Scan $Q \approx 2m_t$

Miquel, Martinez; Boogert, Gounaris

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood
  (renormalons, summations)



$\rightarrow \delta m_t^{\rm exp} \simeq$ 50 MeV

$\rightarrow \delta m_t^{\rm th} \simeq$ 100 MeV

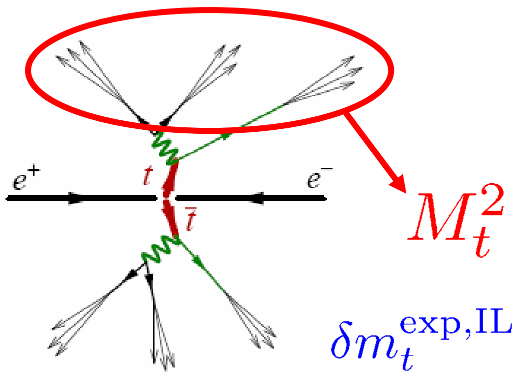
What mass?

$\sqrt{s}_{\rm rise} \sim 2m_t^{\rm thr}$ + pert.series

(short distance mass: 1S $\leftrightarrow$ $\overline{\rm MS}$)

Invariant Mass Reconstruction $Q \geq 2m_t$

Chekanov, Morgunov



$$\delta m_t^{\rm exp,ILC} \sim 0.3 \ {\rm GeV}$$

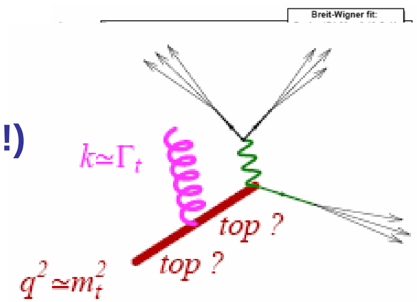
$$\delta m_t^{\rm exp,LHC} \sim 1 \ {\rm GeV}$$

What mass?

pole mass? (color triplet !)

$$\delta m_t \sim \Lambda_{\rm QCD}$$

$$\delta m_t \sim \alpha_s(\Gamma_t)\Gamma_t$$



The top quark does not live by itself !

Theory Issues for $pp \to t\bar{t} + X$

- definition of jet observables
- initial state radiation
- final state radiation
- underlying events
- color reconnection & soft gluon interactions
- beam remnant
- parton distributions
- summing large logs $Q \gg m_t \gg \Gamma_t$
- relation to Lagrangian short distance mass

First step: we will study

$$e^+e^- \to t\bar{t} + X$$

and the issues ★

Conceptual Goals

- relate top jet observables with a given Lagrangian mass (define suitable short-distance mass with good convergence properties → What mass is measured?)
- proof of factorization of dynamics at different length scales (→ What has to be computed by theorists ?)
- combined treatement of top production & decay
- separate perturbative from non-perturbative effects
- hopefully better understand & reduce theoretical & experimental uncertainties

→ **Tool: Sequence of Effective Field Theories**

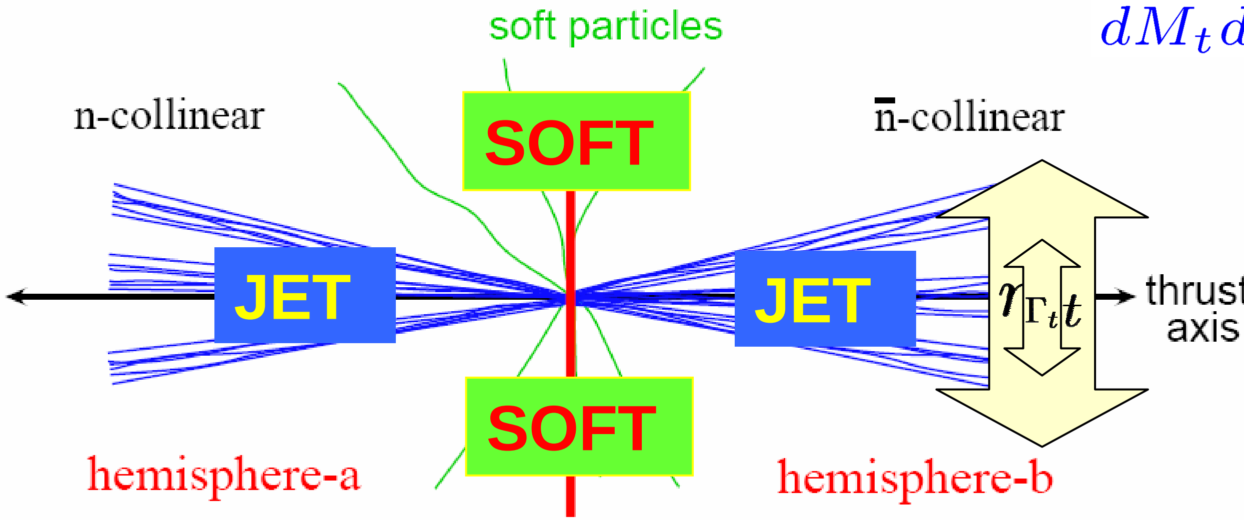
Basic Idea

Hemisphere invariant mass distribution

$$\frac{d^2\sigma}{dM_t\,dM_{\bar t}}(e^+e^- \to t\bar t + X)$$

$$M_t^2 = \left(\textstyle\sum_{i\,\epsilon\, a} p_i^\mu\right)^2$$

$$M_{\bar t}^2 = \left(\textstyle\sum_{i\,\epsilon\, b} p_i^\mu\right)^2$$



$$\boxed{Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}}$$

$$M_t - m_t \sim \Gamma_t$$
$$M_{\bar t} - m_t \sim \Gamma_t$$

Dijet Event

$1 - T \approx 0$

SCET

Jets with

$s_t - m_t \sim m_t$

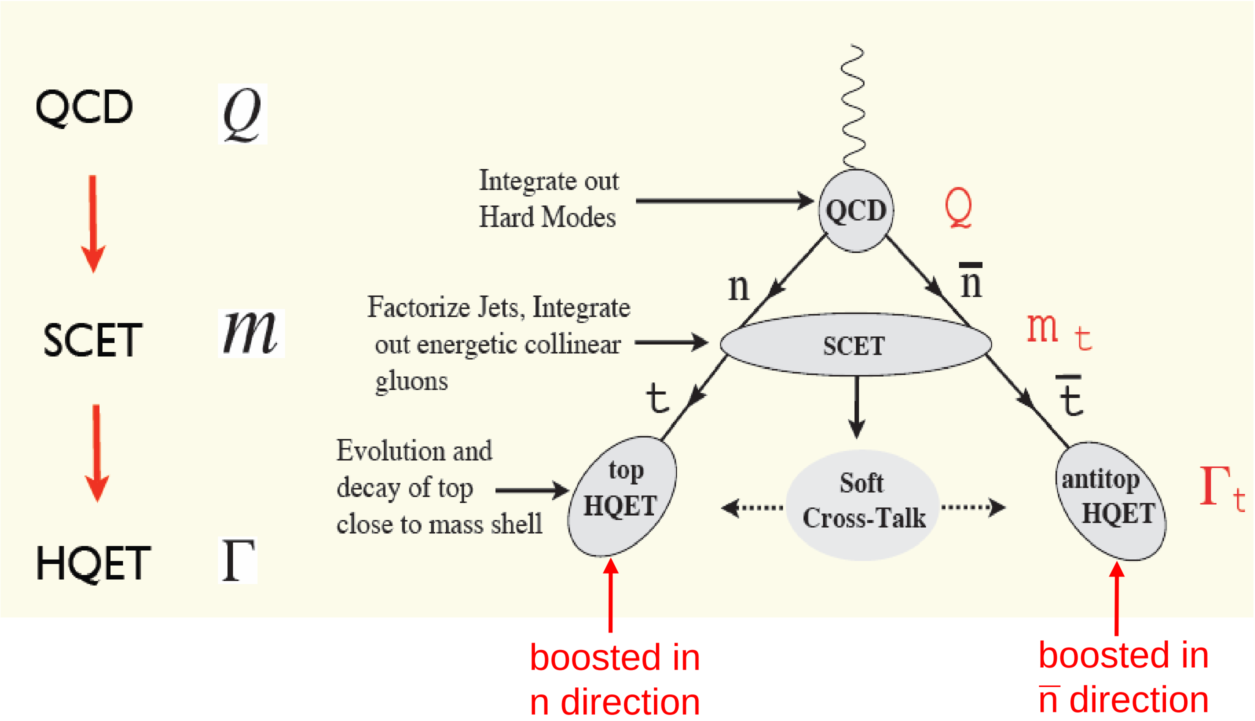
Integrate out m

boosted HQET Isgur, Wise

Jets with

$s_t - m_t \sim \Gamma_t$

Scheme of EFT's



Program technically analogous to combination of

threshold resummation ($M_t - m_t \to 0$) — Korshemsky, Sterman, etal.

&

method of unstable particle EFT — Fadin, Khoze; Beenacker etal., Beneke, etal.; Reisser, AH

Soft-Collinear Effective Theory

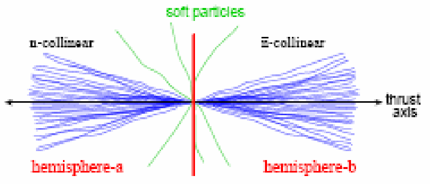
(Bauer, Fleming, Luke, Pirjol, Stewart)

Degrees of freedom

SCET $[\lambda \sim m/Q \ll 1]$		
	n-collinear $(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft $(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

quark fields

gluon fields



Light-cone coordinates

$$p^\mu = (+, -, \perp)$$

$$p_n^2 = p_n^+ p_n^- + p_\perp^2 \sim m^2 \ll Q^2$$

$$(p_n + p_s)^2 \sim m^2 \ll Q^2$$

Leading order Lagrangian (n-collinear)

$$\mathcal{L}_{qn}^{(0)} = \bar{\xi}_n \Big[in \cdot D_s + gn \cdot A_n + (i\not{D}_c^\perp - m) W_n \frac{1}{\bar{n} \cdot \mathcal{P}} W_n^\dagger (i\not{D}_c^\perp + m) \Big] \frac{\not{\bar{n}}}{2} \xi_n$$

collinear-soft coupling

collinear Wilson line

$$iD_s^\mu = i\partial^\mu + g A_s^\mu \qquad W_n(x) = P \exp\left(ig \int_0^\infty ds\, \bar{n} \cdot A_n(s\bar{n}) \right)$$

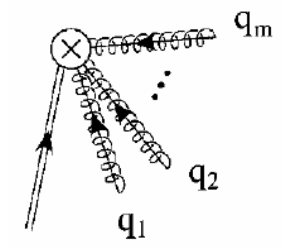
massive SCET:
Leibovich etal,
Rothstein

Soft-Collinear Effective Theory

Top pair production current

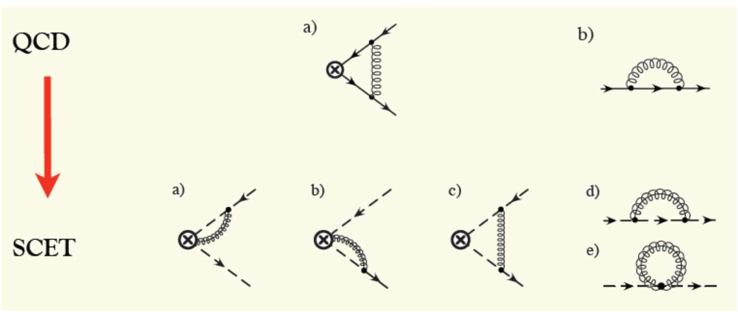
$$J_i^\mu(0) = \int d\omega\, d\bar\omega\, C(\omega,\bar\omega,\mu)\,\mathcal{J}_i^\mu(\omega,\bar\omega,\mu)$$

QCD Wilson Coeff. SCET



$$\mathcal{J}_i^\mu(\omega,\bar\omega,\mu) = \bar\chi_{n,\omega}(0)\Gamma_i^\mu\chi_{\bar n,\bar\omega}(0)\ , \quad \chi_{n,\omega}(0) = \delta(\omega-\bar{\mathcal{P}})(W^\dagger\xi_n)(0)$$

→ **one-loop matching**



- agrees with massless SCET
- known to $\mathcal{O}(\alpha_s^2)$ Gehrmann etal

$$C(\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3\ln\frac{-Q^2}{\mu^2} - \ln^2\frac{-Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

Soft-Collinear Effective Theory

QCD Cross Section

$$\sigma = \sum_X^{res.} (2\pi)^4\,\delta^4(q-p_X) \sum_{i=a,v} L^i_{\mu\nu}\,\langle 0|\mathcal{J}_i^{\nu\dagger}(0)|X\rangle\langle X|\mathcal{J}_i^{\mu}(0)|0\rangle$$

Integrate out hard fluctuations at Q

$$\mathcal{J}_i^{\mu}(0) = \int d\omega\, d\bar{\omega}\, C(\omega,\bar{\omega},\mu)\, J_i^{(0)\mu}(\omega,\bar{\omega},\mu)$$

SCET Cross Section (LO in m/Q)

$$\sigma = \sum_{\vec{n}} \sum_{X_n X_{\bar{n}} X_s}^{res.} (2\pi)^4\,\delta^4(q-P_{X_n}-P_{X_{\bar{n}}}-P_{X_s}) \sum_i L^{(i)}_{\mu\nu} \int d\omega\, d\bar{\omega}\, d\omega'\, d\bar{\omega}'$$

$$\times C(\omega,\bar{\omega})C^*(\omega',\bar{\omega}')\langle 0|\bar{\chi}_{\bar{n},\bar{\omega}'}\bar{\Gamma}^\nu_j \chi_{n,\omega'}|X_n X_{\bar{n}} X_s\rangle\langle X_n X_{\bar{n}} X_s|\overline{\chi}_{n,\omega}\Gamma^\mu_i \chi_{\bar{n},\bar{\omega}}|0\rangle$$

factorization of asymptotic final states

$$|X\rangle = |X_n X_{\bar{n}} X_s\rangle = |X_n\rangle \otimes |X_{\bar{n}}\rangle \otimes |X_s\rangle$$

Collinear: n Collinear: $\bar{n}$ Soft

Soft-Collinear Effective Theory

Factorization: soft-collinear decoupling

$$\sigma = \sum_{X_n X_{\bar n} X_s}^{res.} (2\pi)^4\, \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \sum_{ij} L_{\mu\nu}^{(ij)} \int d\omega\, d\bar\omega\, d\omega'\, d\bar\omega'$$
$$\times C(\omega,\bar\omega) C^\dagger(\omega',\bar\omega') \langle 0|\bar\chi_{n,\omega}\Gamma_i^\mu \chi_{\bar n,\bar\omega}|X_n X_{\bar n} X_s\rangle\langle X_n X_{\bar n} X_s|\bar\chi_{\bar n,\bar\omega'}\bar\Gamma_j^\nu \chi_{n,\omega'}|0\rangle$$

$$\mathcal{L}_{qn}^{(0)} = \bar\xi_n\, in.D_s\, \frac{\bar n}{2}\, \xi_n$$

Factorization and Soft field redefinition: soft-collinear decoupling

$$\xi_n \to Y_n \xi_n\,, \qquad A_n^\mu \to Y_n\, A_n^\mu\, Y_n^\dagger$$
$$Y_n(x) = \overline{\mathrm{P}}\, \exp\Big(- ig \int_0^\infty ds\, n\cdot A_s(ns+x)\Big)$$

$$\mathcal{L}_{qn}^{(0)} = \bar\xi_n\, in.\partial_s\, \frac{\bar n}{2}\, \xi_n$$

Hemisphere mass constraint

soft

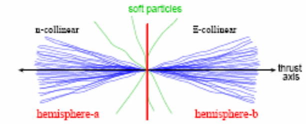
$$\sigma = \sum_{X_n X_{\bar n} X_s}^{res.} (2\pi)^4\, \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \sum_{ij} L_{\mu\nu}^{(ij)} \langle 0|Y_n^\dagger \Gamma_i^\mu Y_{\bar n}|X_s\rangle\langle X_s|\tilde Y_{\bar n}^\dagger \bar\Gamma_j^\nu \tilde Y_n|0\rangle$$
$$\times \int d\omega\, d\bar\omega\, |C(\omega,\bar\omega)|^2 \langle 0|\overline{\chi}_{n,\omega}|X_n\rangle\langle X_n|\chi_n|0\rangle\langle 0|\chi_{\bar n}|X_{\bar n}\rangle\langle X_{\bar n}|\overline{\chi}_{\bar n,\bar\omega}|0\rangle\,.$$

Hard Wilson coeff. Collinear: n Collinear: $\bar n$

SCET Cross Section

SCET factorization formula

$$s_{t,\bar t} = M^2_{t,\bar t} - m_t^2$$

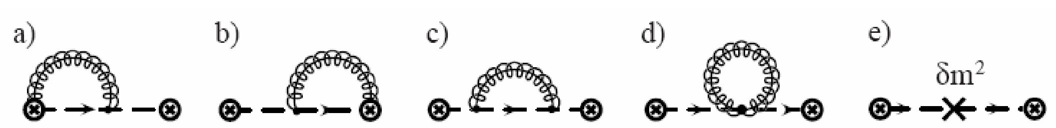


$$\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2} = \sigma_0\,H_Q(Q,\mu)\int_{-\infty}^{\infty} d\ell^+ d\ell^-\, J_n(s_t - Q\ell^+,\mu)\,J_{\bar n}(s_{\bar t} - Q\ell^-,\mu)\,S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

we find: Korchemsky, Sterman

Jet functions:

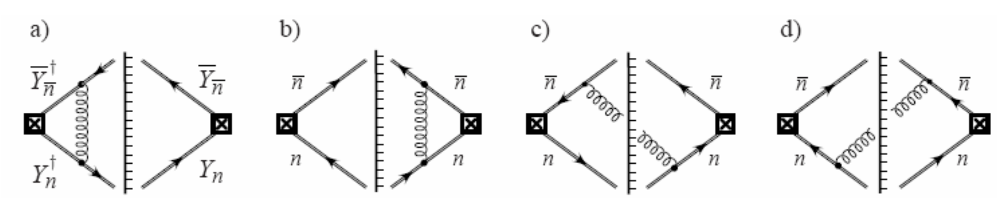
$$J_n(Qr_n^+ - m^2) = \frac{-1}{2\pi Q}\int d^4x\, e^{ir_n\cdot x}\,\mathrm{Disc}\,\langle 0|\mathrm{T}\{\overline{\chi}_{n,Q}(0)\bar{n}\!\!\!/\,\chi_n(x)\}|0\rangle$$



- perturbative
- depends on m_t, Γ_t
- insensitive to hemisphere constraints

Soft function:

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\,\delta(\ell^- - k_s^{-b})\,\langle 0|\overline{Y}_{\bar n}\,Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger(0)|0\rangle$$



- non-perturbative
- renormalized due to UV divergences
- governs massless dijet thrust and jet mass distributions
- depends on hemisph. constr.

Boosted HQET

SCET factorization formula $s_{t,\bar{t}} = M_{t,\bar{t}}^2 - m_t^2$

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, J_n(s_t - Q\ell^+, \mu)\, J_{\bar{n}}(s_{\bar{t}} - Q\ell^-, \mu)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Lower virtuality of jet function:

$m_t \;\longrightarrow\; \Gamma_t$

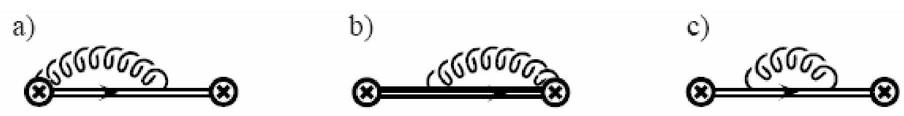
$$\mathcal{L}_{\pm}^{\rm bHQET} = \bar{h}_{v_\pm}(iv_\pm . D_\pm - \delta m + \tfrac{i}{2}\Gamma_t)h_{v_\pm}$$

$$J_n(m\hat{s}, \Gamma, \mu_m) = T_+(m, \mu_m)\; B_+(\hat{s}, \Gamma, \mu_m)$$

ultracollinear Jet functions:

$$B_+(2v_+\cdot k) \;=\; \frac{-1}{8\pi N_c m} \int d^4x\, e^{ik\cdot x}\, {\rm Disc}\, \langle 0|{\rm T}\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle$$

→ **one-loop matching**



$$T_\pm(\mu, m) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{m^2}{\mu^2} - \ln\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right)$$

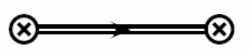
Only low energy fluctuations are relevant for the resonance region.

Factorization Theorem

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m)\, H_m\!\left(m,\frac{Q}{m},\mu_m,\mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\!\left(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\right) B_-\!\left(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\right) S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Everything computable in perturbation theory except for S_hemi (which is known from massless event shapes)

Ultracollinear jet functions:

- **at tree level:** 

$$B_\pm(\hat s,\Gamma_t) = \frac{1}{\pi m_t}\,\frac{\Gamma_t}{\hat s^2 + \Gamma_t^2}$$

Breit-Wigner only at tree level !

- **describes perturbative contributions to the invariant mass distributions**
- **top width acts as IR-cutoff.**
- **computable for any given Lagrangian mass**

How general is the approach?

Why Q>>m is crucial

- top and antitop boosted in opposite directions,

➡ top and antitop jet axes $\vec{n}$ and $\vec{\bar{n}}$ can be defined

➡ allows factorization $(\mathrm{jet}_n \times \mathrm{jet}_{\bar{n}}) \otimes \mathrm{soft}$

- combinatorial background, wrong assignment suppressed by $\left(\frac{m}{Q}\right)^2$

ILC: ok for $Q \sim 0.5 - 1\ \mathrm{TeV}$

LHC: probably ok for tops with $p_T > 200\ \mathrm{GeV}$

Tev: ? $Q \sim 2m_t$

Other Mass Prescriptions

We don't have to assume a hemisphere mass definition:

- any jet algorithm that combines soft particles with the hard jets from the top decay
- wide cone definition $R > m/Q$ that contains the top/antitop jet axes and top decay products (collinear radiation off top)

➡ **Different soft function, same factorization formula**

[The soft functions for most cases at unknown at this time and migh need to be fitted together with the top mass, determined from MC's or by other means.]

$$\boxed{\begin{aligned}\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} &= \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) \\ &\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)\end{aligned}}$$

Summation of logs

- low energy scales of jet functions and soft function can differ
- RG-evolution of jet and soft functions individually involve plus functions and convolutions
- different possibilities to set up RG-evolution

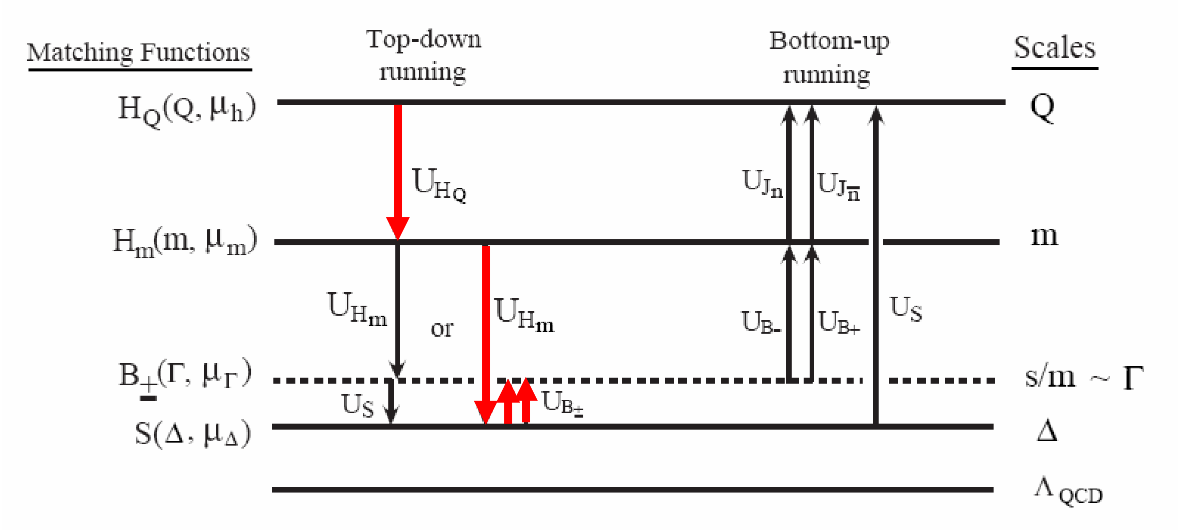
$$B_\pm(\hat{s},\mu) = \int d\hat{s}' \; U_{B_\pm}(\hat{s}-\hat{s}',\mu,\mu_\Gamma) \; B_\pm(\hat{s}',\mu_\Gamma)$$

$$\mu\frac{d}{d\mu}B_\pm(\hat{s},\mu) = \int d\hat{s}' \; \gamma_{B_\pm}(\hat{s}-\hat{s}') \; B_\pm(\hat{s}',\mu)$$

$$\gamma_{B_\pm}(\hat{s}-\hat{s}',\mu) = -\frac{\alpha_s C_F}{\pi}\left\{\frac{2}{\kappa_3}2\left[\frac{\kappa_3\theta(\hat{s}'-\hat{s})}{\hat{s}'-\hat{s}}\right]_+ - \left[2\ln\left(\frac{\mu}{\kappa_3}\right)+1\right]\delta(\hat{s}'-\hat{s})\right\}$$

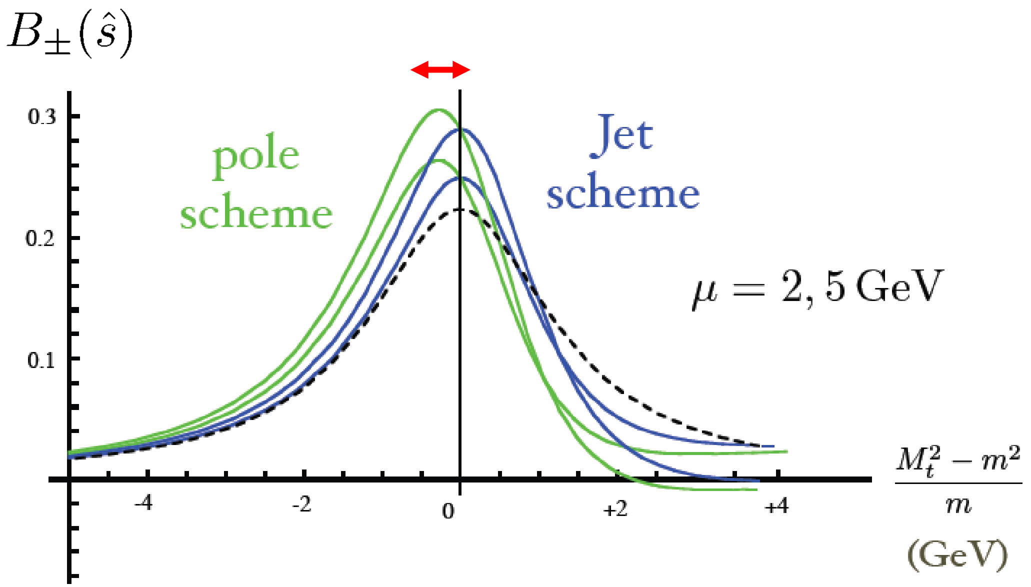
$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \int d\ell'^+ d\ell'^- \; U_S(\ell^+-\ell'^+,\ell^- -\ell'^-,\mu,\mu_m) \; S_{\rm hemi}(\ell'^+,\ell'^-,\mu_m)$$

Summation of logs



$$
\begin{aligned}
\frac{d^2\sigma}{dM_t^2 dM_{\bar t}^2} &= \sigma_0\, H_Q(Q,\mu_m)\, H_m\Big(m,\frac{Q}{m},\mu_m,\mu_\Delta\Big) \\
&\times \int_{-\infty}^{\infty} d\hat s_t'\, d\hat s_{\bar t}'\; U_{B_+}(\hat s_t,\hat s_t',\mu_\Delta,\mu_\Gamma)\, U_{B_-}(\hat s_{\bar t},\hat s_{\bar t}',\mu_\Delta,\mu_\Gamma) \\
&\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\Big(\hat s_t' - \frac{Q\ell^+}{m},\Gamma,\mu_\Gamma\Big)\, B_-\Big(\hat s_{\bar t}' - \frac{Q\ell^-}{m},\Gamma,\mu_\Gamma\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu_\Delta)
\end{aligned}
$$

Short-distance Top Jet Mass



- **One-loop: shift in the pole scheme 300 MeV**
- **shift in the pole scheme contains $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon**
- **<u>jet mass scheme:</u> defined such that peak located at the mass to all orders**

$$m_J(\mu) = m_{\rm pole} - \Gamma \frac{\alpha_s(\mu)}{3} \left[\ln\left(\frac{\mu}{\Gamma}\right) + \frac{3}{2} \right]$$

What mass is measured?

Answer: the one that gives the best convergence in the theoretical expansion.

LO Numerical Analysis

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

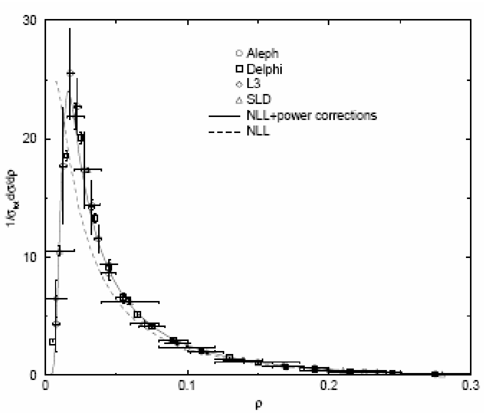
Jet functions:

$$B_\pm(\hat s, \Gamma_t) = \frac{1}{\pi m_t} \frac{\Gamma_t}{\hat s^2 + \Gamma_t^2}$$

Soft function:

$$S_{\rm hemi}^{\rm M1}(\ell^+, \ell^-) = \theta(\ell^+)\theta(\ell^-) \frac{\mathcal{N}(a,b)}{\Lambda^2} \Big(\frac{\ell^+\ell^-}{\Lambda^2}\Big)^{a-1} \exp\Big(\frac{-(\ell^+)^2 - (\ell^-)^2 - 2b\ell^+\ell^-}{\Lambda^2}\Big)$$

$$a = 2\,, \quad b = -0.4$$
$$\Lambda = 0.55\ \text{GeV}$$



Fit to heavy jet mass distribution

Korchemsky, Tafat
hep-ph/0007005

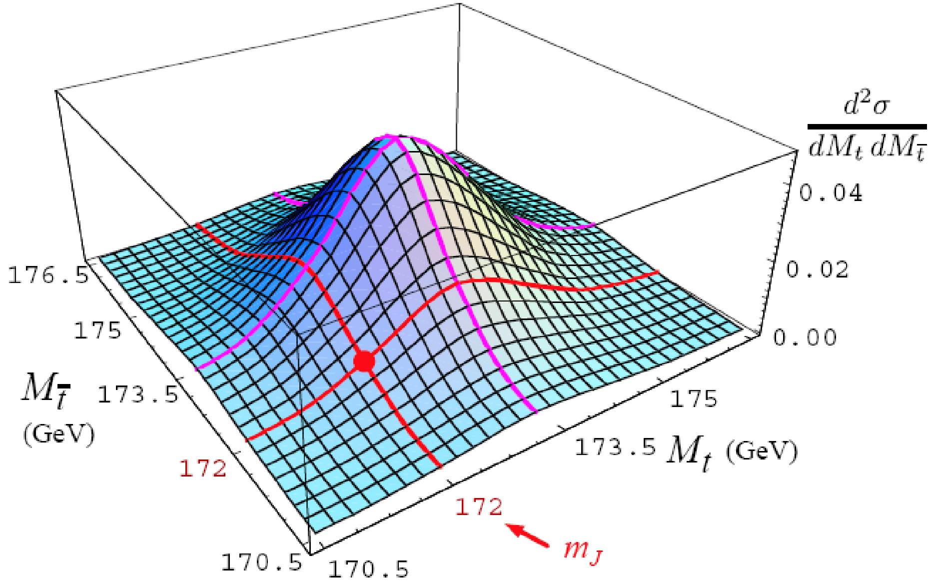
LO Numerical Analysis

Double differential invariant mass distribution:

$$Q = 745 \text{ GeV}$$

$$\Gamma = 1.43 \text{ GeV}$$

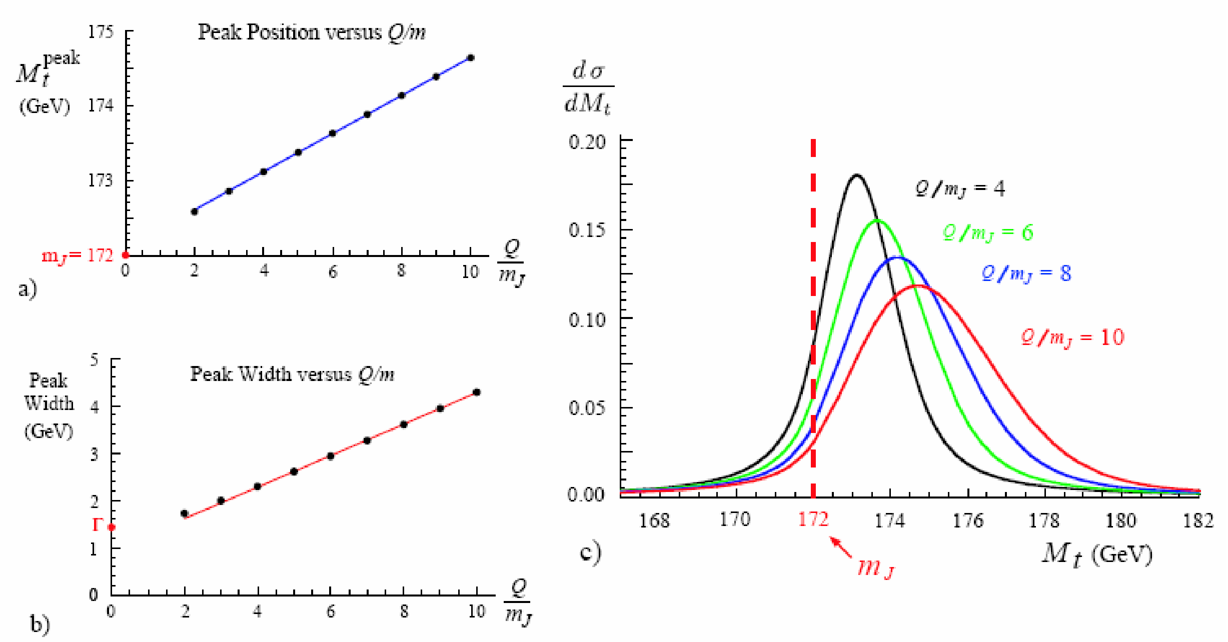
$$m_J = 172 \text{ GeV}$$



Non-perturbative effects shift the peak to higher energies and broaden the distribution.

LO Numerical Analysis

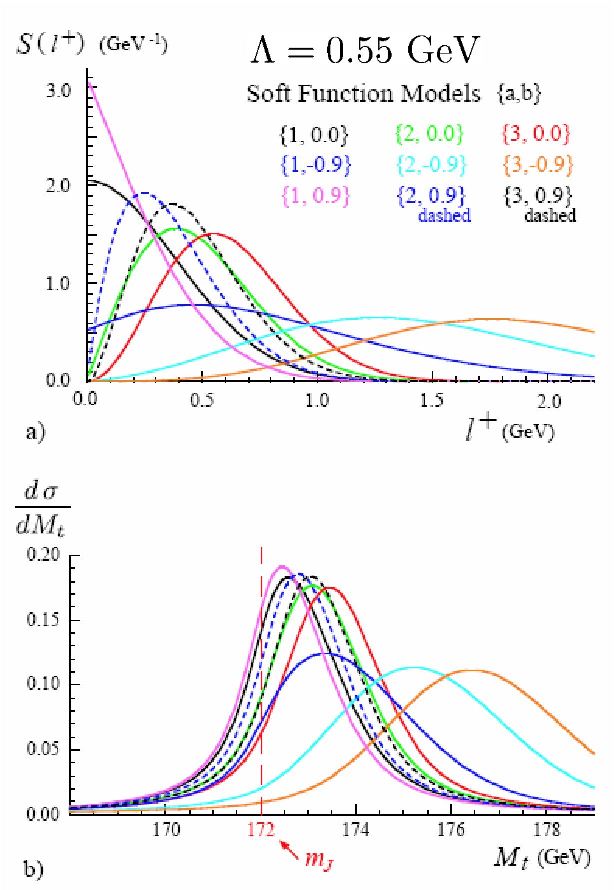
Single differential distribution:

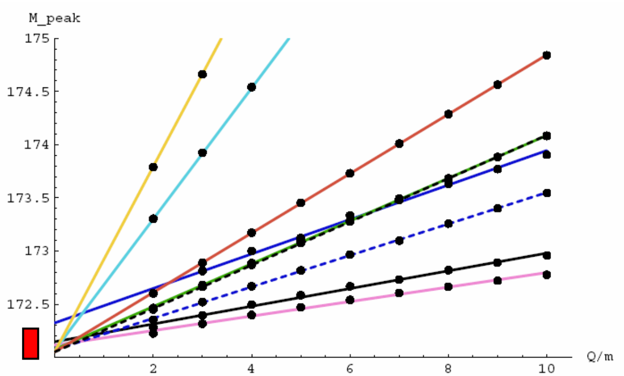


Non-perturbative effects shift the peak to higher energies and broaden the distribution.

LO Numerical Analysis

Different invariant mass prescriptions/soft functions:

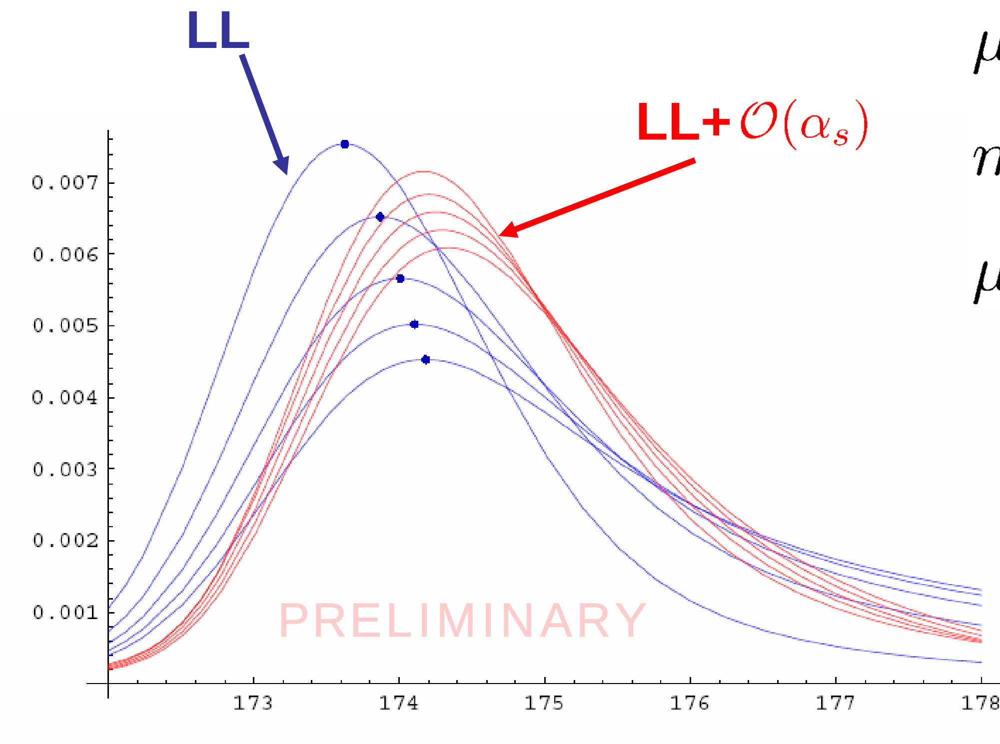




$$M_t^{\text{peak}} \approx m_J + \frac{Q}{m_J}\,\text{const}$$

Fairly precise determination of jet mass from determination of Q-dependence of the peak position and extrapolation Q to zero

Perturbative corrections



$$\mu_\Delta = 1 \text{ GeV}$$

$$m_J(2 \text{ GeV}) = 172 \text{ GeV}$$

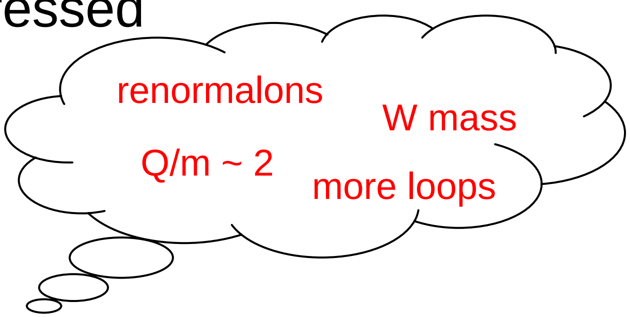
$$\mu_\Gamma = 2, 4, 6, 8, 10 \text{ GeV}$$

Summary & Outlook

- established factorization theorem for invariant mass distributions: separation of perturbative and non-perturbative effects
- applicable for many other systems and setups: (any colored unstable particle, W mass reconstruction, etc..)
- exact and systematic relation of peak to a Lagrangian mass: What mass is measured ? “Jet-mass”
- resummation of large logarithms $Q \gg m_t \gg \Gamma_t$
- soft gluon color reconnection power suppressed
- Here: $e^+e^- \to t\bar{t} + X_{\rm soft}$
- Planned: $pp \to t\bar{t} + X_{\rm soft}$ (large p_T)

  $pp \to t\bar{t} + {\rm jet} + X_{\rm soft}$

  different mass definitions (cone, k_T)



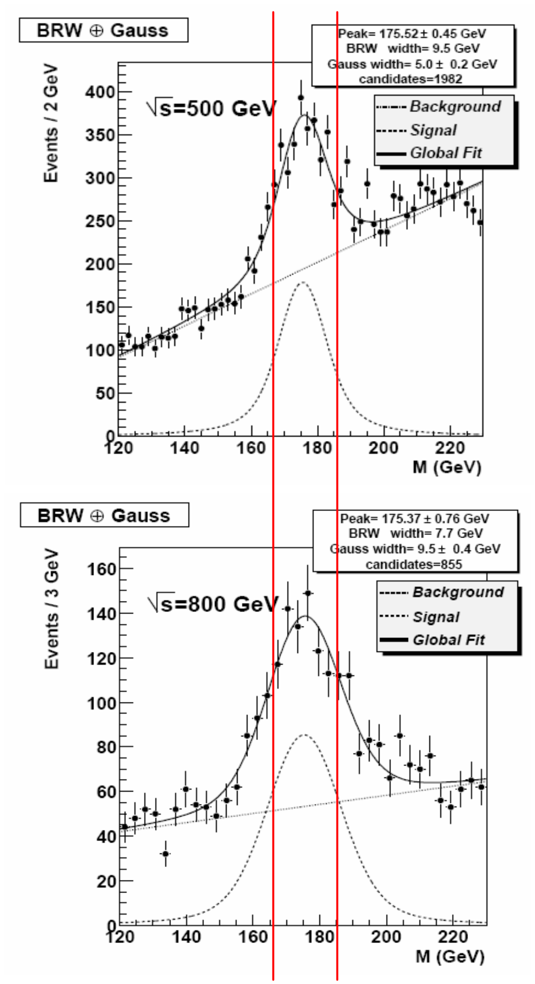
Backup Slides

ILC Simulations

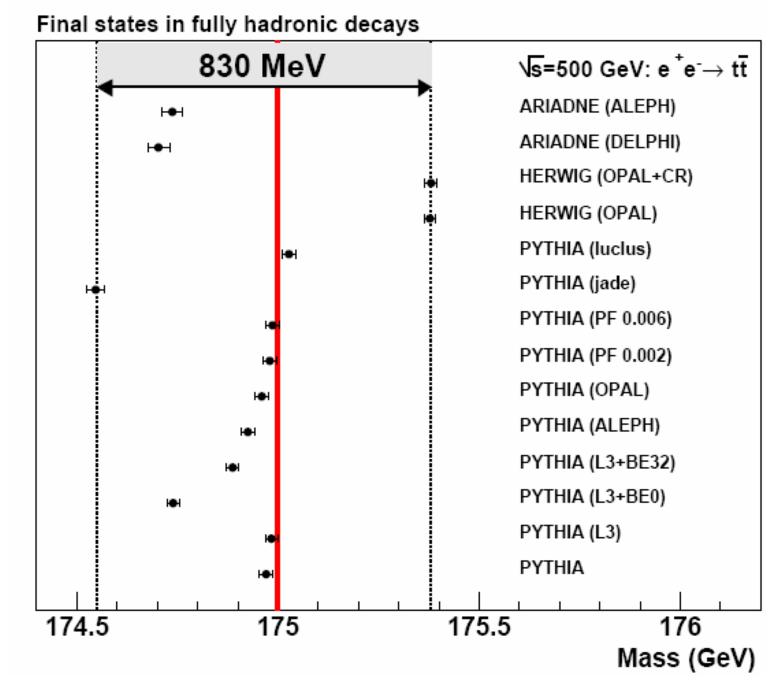
Reconstruction for all-hadronic events at the LC

Chekanov; Morgunov

hep-ex/0301014



Q dependence of observed width



Not relevant once soft function is known

Dependence on parton shower, hadronization, jet algorithms

Event Shapes

e.g. Thrust distribution on the peak region:

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

$$\frac{d\sigma}{dT} = \sigma_0^H(\mu) \int_{-\infty}^{\infty} ds_t\, ds_{\bar t}\; \tilde{B}_+\Big(\frac{s_t}{m_J}, \Gamma, \mu\Big)\, \tilde{B}_-\Big(\frac{s_{\bar t}}{m_J}, \Gamma, \mu\Big)\, S_{\rm thrust}\Big(1 - T - \frac{(2m_J^2 + s_t + s_{\bar t})}{Q^2}, \mu\Big)$$

$$S_{\rm thrust}(\tau, \mu) = \int_0^{\infty} d\ell^+\, d\ell^-\, \delta\Big(\tau - \frac{(\ell^+ + \ell^-)}{Q}\Big) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

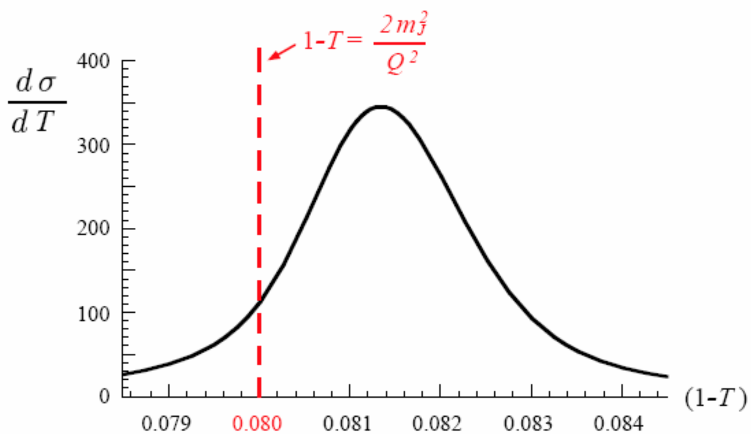
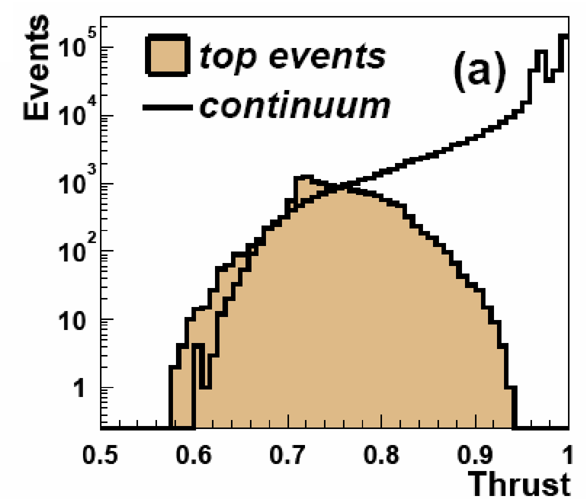


FIG. 8: Plot of the thrust distribution, $d\sigma/dT$ in units of σ_0^H, for top-initiated events in the peak region. We use $Q/m_J = 5$, $m_J = 172\,\text{GeV}$ and the soft function parameters in Eq. (115).



Chekanov; Morgunov

hep-ex/0301014