

Determining the Fundamental Mass Parameters in SUSY SO(10) Models

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Introduction

- ILC has potential to determine SUSY masses and parameters to very high precision
- Allows exploration of physics near the GUT and Planck scales through RGE analysis
- SO(10) is the favoured GUT group
 - naturally incorporates the Seesaw mechanism of neutrino mass generation
→ Connection to neutrino physics

| | Mass, ideal | "LHC" | "LC" | "LHC+LC" |
|----------------------|-------------|-------|------|----------|
| $\tilde{\chi}_1^\pm$ | 179.7 | | 0.55 | 0.55 |
| $\tilde{\chi}_2^\pm$ | 382.3 | – | 3.0 | 3.0 |
| $\tilde{\chi}_1^0$ | 97.2 | 4.8 | 0.05 | 0.05 |
| $\tilde{\chi}_2^0$ | 180.7 | 4.7 | 1.2 | 0.08 |
| $\tilde{\chi}_3^0$ | 364.7 | | 3-5 | 3-5 |
| $\tilde{\chi}_4^0$ | 381.9 | 5.1 | 3-5 | 2.23 |
| \tilde{e}_R | 143.9 | 4.8 | 0.05 | 0.05 |
| \tilde{e}_L | 207.1 | 5.0 | 0.2 | 0.2 |
| $\tilde{\nu}_e$ | 191.3 | – | 1.2 | 1.2 |
| $\tilde{\mu}_R$ | 143.9 | 4.8 | 0.2 | 0.2 |
| $\tilde{\mu}_L$ | 207.1 | 5.0 | 0.5 | 0.5 |
| $\tilde{\nu}_\mu$ | 191.3 | – | | |
| $\tilde{\tau}_1$ | 134.8 | 5-8 | 0.3 | 0.3 |
| $\tilde{\tau}_2$ | 210.7 | – | 1.1 | 1.1 |
| $\tilde{\nu}_\tau$ | 190.4 | – | – | – |
| \tilde{q}_R | 547.6 | 7-12 | – | 5-11 |
| \tilde{q}_L | 570.6 | 8.7 | – | 4.9 |
| \tilde{t}_1 | 399.5 | | 2.0 | 2.0 |
| \tilde{t}_2 | 586.3 | | – | |
| \tilde{b}_1 | 515.1 | 7.5 | – | 5.7 |
| \tilde{b}_2 | 547.1 | 7.9 | – | 6.2 |
| \tilde{g} | 604.0 | 8.0 | – | 6.5 |
| h^0 | 110.8 | 0.25 | 0.05 | 0.05 |
| H^0 | 399.8 | | 1.5 | 1.5 |
| A^0 | 399.4 | | 1.5 | 1.5 |
| H^\pm | 407.7 | – | 1.5 | 1.5 |

SPS1a, Allanach et al., hep-ph/0403133

SO(10) GUT Model

- Yukawa sector

$$W = Y_d^{10} \mathbf{16}_F \cdot \mathbf{16}_F \cdot \mathbf{10}_{H_1} + Y_u^{10} \mathbf{16}_F \cdot \mathbf{16}_F \cdot \mathbf{10}_{H_2} + Y_{126}^{126} \mathbf{16}_F \cdot \mathbf{16}_F \cdot \mathbf{126}_H$$

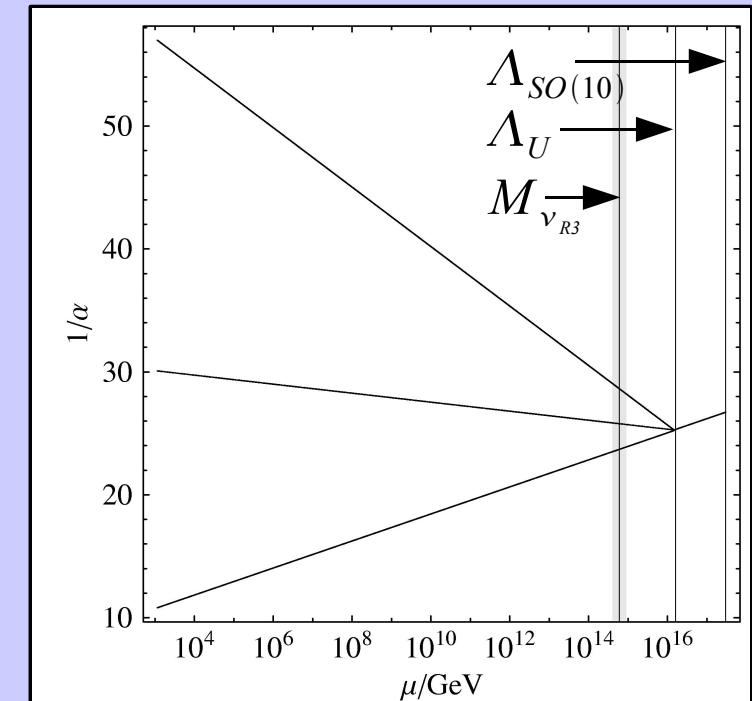
$$\begin{aligned} M^u &= \mathbf{M}_{\mathbf{10}_2}^5 + M_{126}^5 \\ M_{LR}^\nu &= \mathbf{M}_{\mathbf{10}_2}^5 - 3 M_{126}^5 \\ M^d &= \mathbf{M}_{\mathbf{10}_1}^5 + M_{126}^{45} \\ M^e &= \mathbf{M}_{\mathbf{10}_1}^5 - 3 M_{126}^{45} \\ M_{LL}^\nu &= M_{126}^{15} \\ M_{RR}^\nu &= \mathbf{M}_{\mathbf{126}}^1 \end{aligned}$$

- Model Choices

- Not our goal: realistic mass relations for all fermions
- Top unification $Y_\nu = Y_u$
- Dominance of $M_{126}^1 \rightarrow$ Seesaw Type I
- Two Higgs-10 fields to avoid large $\tan\beta$

- SO(10) breaking

- Direct: SO(10) \rightarrow SM
- Two-Step: SO(10) \rightarrow SU(5) \rightarrow SM



Neutrino Sector

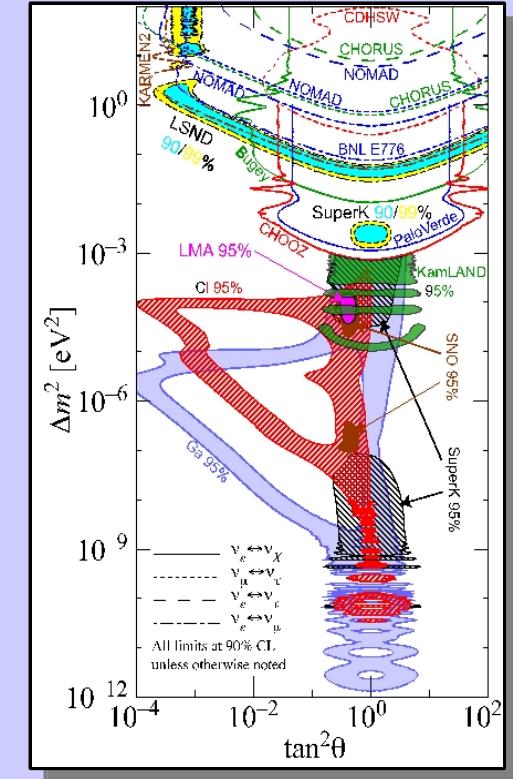
- Normal hierarchy of light neutrino masses

$$m_{\nu_1}, \quad m_{\nu_2} = \sqrt{m_{\nu_1} + \Delta m_{12}^2}, \quad m_{\nu_3} = \sqrt{m_{\nu_1} + \Delta m_{13}^2}, \\ \Delta m_{12}^2 \approx 8 \cdot 10^{-5} \text{ eV}^2, \quad \Delta m_{13}^2 \approx 2 \cdot 10^{-3} \text{ eV}^2$$

- Tri-bimaximal neutrino mixing matrix

$$U_\nu = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \end{pmatrix}$$

- Heavy neutrino masses via Seesaw Relation



$$M_{\nu_R} = Y_\nu \cdot m_\nu^{-1} \cdot Y_\nu^T \nu_u^2 \approx \text{diag}(m_{u_i}) \cdot U_\nu \cdot \text{diag}(m_{\nu_i}) \cdot U_\nu^T \cdot \text{diag}(m_{u_i})$$

$Y_\nu = Y_u$ Yukawa unification
neglect small quark mixing

Heavy Neutrino Mass Spectrum

- Strongly hierarchical masses

$$M_{\nu_{R1}} : M_{\nu_{R2}} : M_{\nu_{R3}} \approx m_u^2 : m_c^2 : m_t^2$$

$$M_{\nu_{R1}} = \frac{m_{\nu_1} + 2m_{\nu_2} + 3m_{\nu_3}}{3m_{\nu_1}m_{\nu_2} + 2m_{\nu_1}m_{\nu_3} + m_{\nu_2}m_{\nu_3}} m_u^2$$

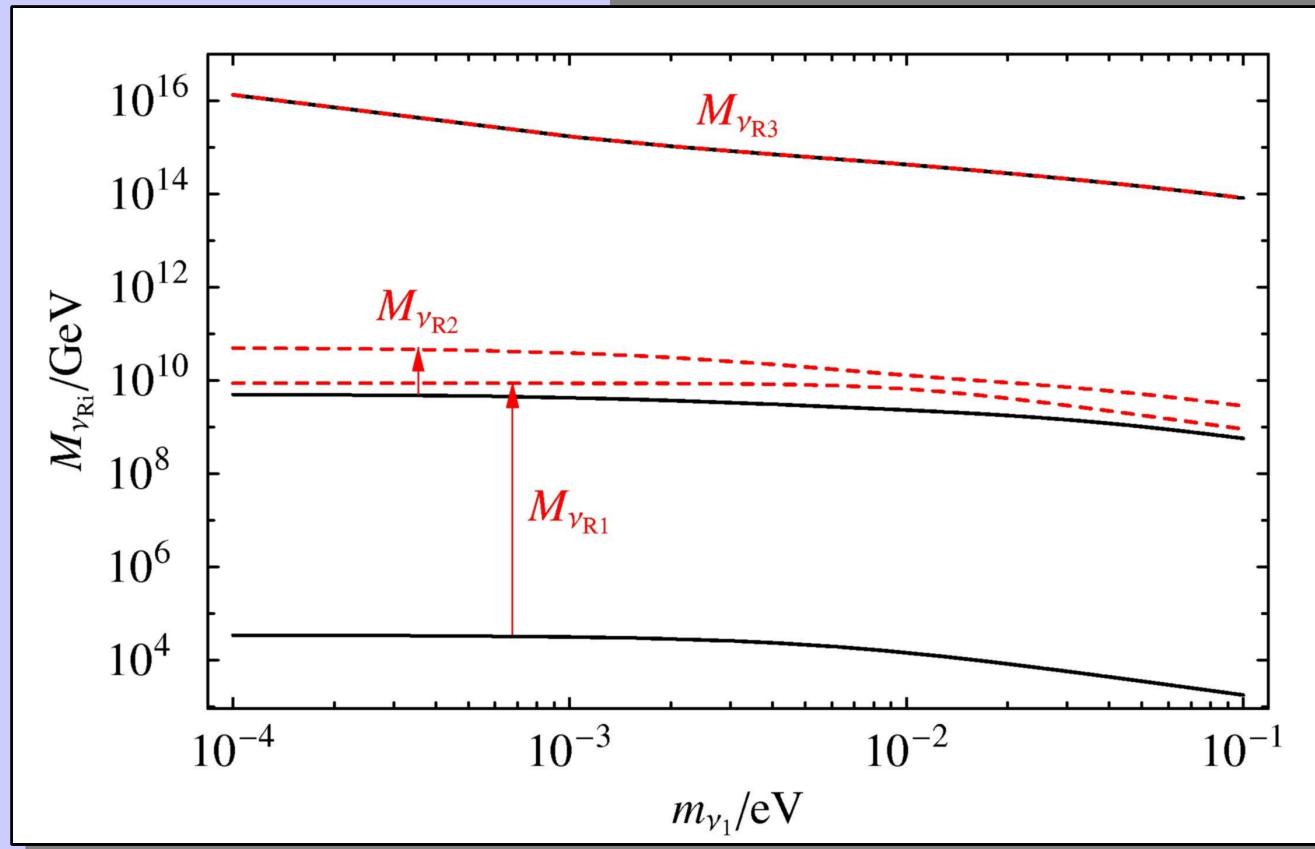
$$M_{\nu_{R2}} = \frac{4m_{\nu_1} + 2m_{\nu_2}}{3m_{\nu_1}m_{\nu_2} + 2m_{\nu_1}m_{\nu_3} + m_{\nu_2}m_{\nu_3}} m_c^2$$

$$M_{\nu_{R3}} = \frac{3m_{\nu_1}m_{\nu_2} + 2m_{\nu_1}m_{\nu_3} + m_{\nu_2}m_{\nu_3}}{6m_{\nu_1}m_{\nu_2}m_{\nu_3}} m_t^2$$

- Can be adjusted by introducing a small violation to Yukawa unification

$$Y_\nu = Y_u + O\left(\frac{m_c}{M_Z}\right)$$

- Leptogenesis
- Realistic fermion mass relations
- would not affect our analysis



Direct Breaking $\text{SO}(10) \rightarrow \text{SM}$

- Input at GUT = $\text{SO}(10)$ scale Λ_U :

$$\begin{aligned} m_{16} &= m_{10_1} = m_{10_2}, \\ M_{1/2}, A_0, \tan \beta, \text{sign } \mu, \\ D_U \end{aligned}$$

SSB masses at Λ_U

$$\begin{aligned} m_L^2 &= m_{16}^2 - 3D_U \\ m_E^2 &= m_{16}^2 + D_U \end{aligned}$$

D term associated with breaking rank-5 $\text{SO}(10)$ to rank-4 SM:

$$D_U \simeq g_U^2 O(m_{16}^2)$$

- Λ_U to M_Z : MSSM Slepton mass evolution

first (+second) generation

$$\begin{aligned} m_{\tilde{e}_L}^2 &= m_L^2 + \alpha_L M_{1/2}^2 - (1 - 2s_W^2) D_{EW} \\ m_{\tilde{\nu}_{eL}}^2 &= m_L^2 + \alpha_L M_{1/2}^2 + D_{EW} \\ m_{\tilde{e}_R}^2 &= m_E^2 + \alpha_R M_{1/2}^2 - 2s_W^2 D_{EW} \end{aligned}$$

third generation,

$$\begin{aligned} m_{\tilde{\tau}_L}^2 &= m_{\tilde{e}_L}^2 + m_\tau^2 - \Delta_\tau - \Delta_{\nu_\tau} \\ m_{\tilde{\nu}_{\tau L}}^2 &= m_{\tilde{\nu}_{eL}}^2 - \Delta_\tau - \Delta_{\nu_\tau} \\ m_{\tilde{\tau}_R}^2 &= m_{\tilde{e}_R}^2 + m_\tau^2 - 2\Delta_\tau \end{aligned}$$

is shifted due to strong Yukawa couplings, e.g.

$$\Delta_{\nu_\tau} \approx \frac{m_t^2(\Lambda_U)}{4\pi^2 v_u^2} (3m_{16}^2 + A_0^2) \log \frac{\Lambda_U^2}{M_{\nu_{R3}}^2}$$

Two-Step Breaking $SO(10) \rightarrow SU(5) \rightarrow SM$

- Input at $SO(10)$ scale Λ_O :

$$\begin{aligned} m_{16} &= m_{10_1} = m_{10_2}, \\ M_{1/2}, A_0, \tan \beta, \text{sign } \mu, \\ D_O \end{aligned}$$

SSB masses at Λ_O

$$\begin{aligned} m_{10}^2 &= m_{16}^2 + D_O \\ m_{\bar{5}}^2 &= m_{16}^2 - 3D_O \end{aligned}$$

\rightarrow **16** decomposes into
 $SU(5)$ **10** and **$\bar{5}$**

- Λ_O to Λ_U : Sfermion mass evolution

first (third) generation

$$\begin{aligned} m_{10}^2 &\rightarrow m_{10}^2 + \alpha'_R M_{1/2}^2 (+\Delta'_{t'}) \\ m_{\bar{5}}^2 &\rightarrow m_{\bar{5}}^2 + \alpha'_L M_{1/2}^2 (+\Delta'_{b'}) \end{aligned}$$

\rightarrow Caveat: Running depends on $SU(5)$
Higgs content breaking $SU(5) \rightarrow SM$.
We use minimal $SU(5)$ with Higgs- **24**

- At Λ_U :

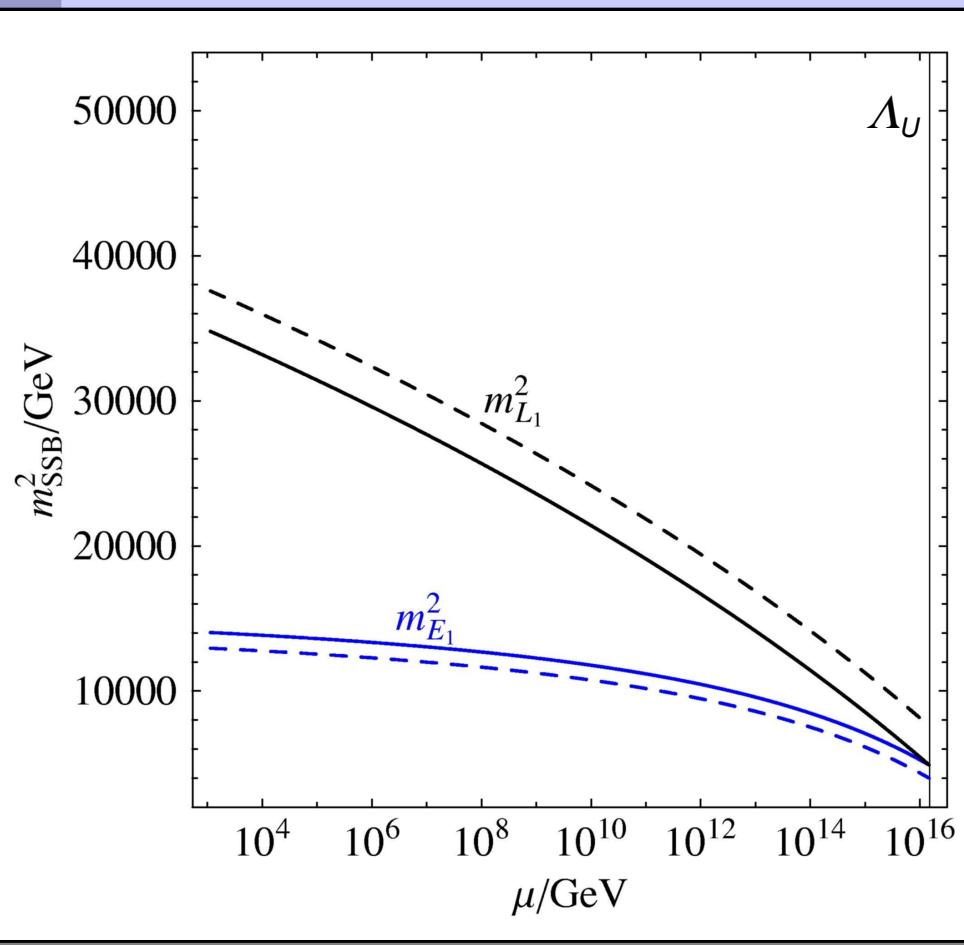
$$\begin{aligned} m_L^2 &= m_{\bar{5}}^2 \\ m_E^2 &= m_{10}^2 \end{aligned}$$

\rightarrow no D-term

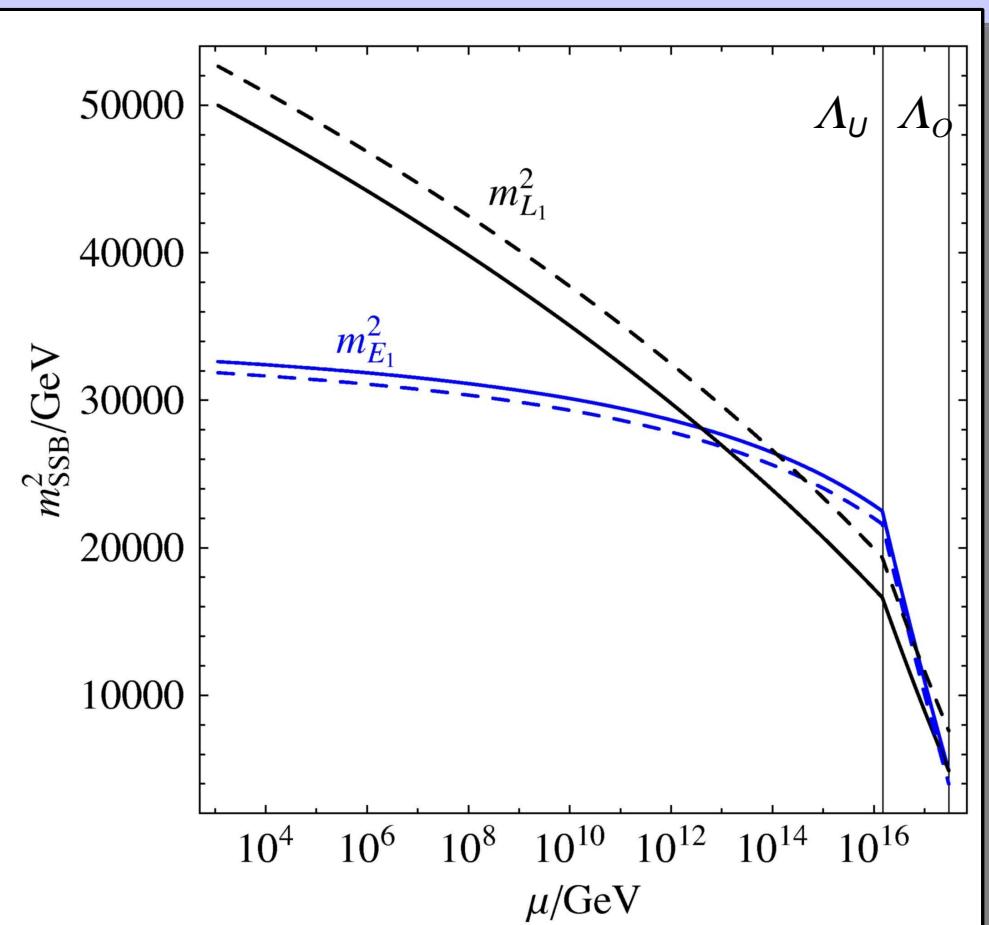
- Λ_U to M_Z : Slepton mass evolution

RG Evolution, 1st generation

- One-Step



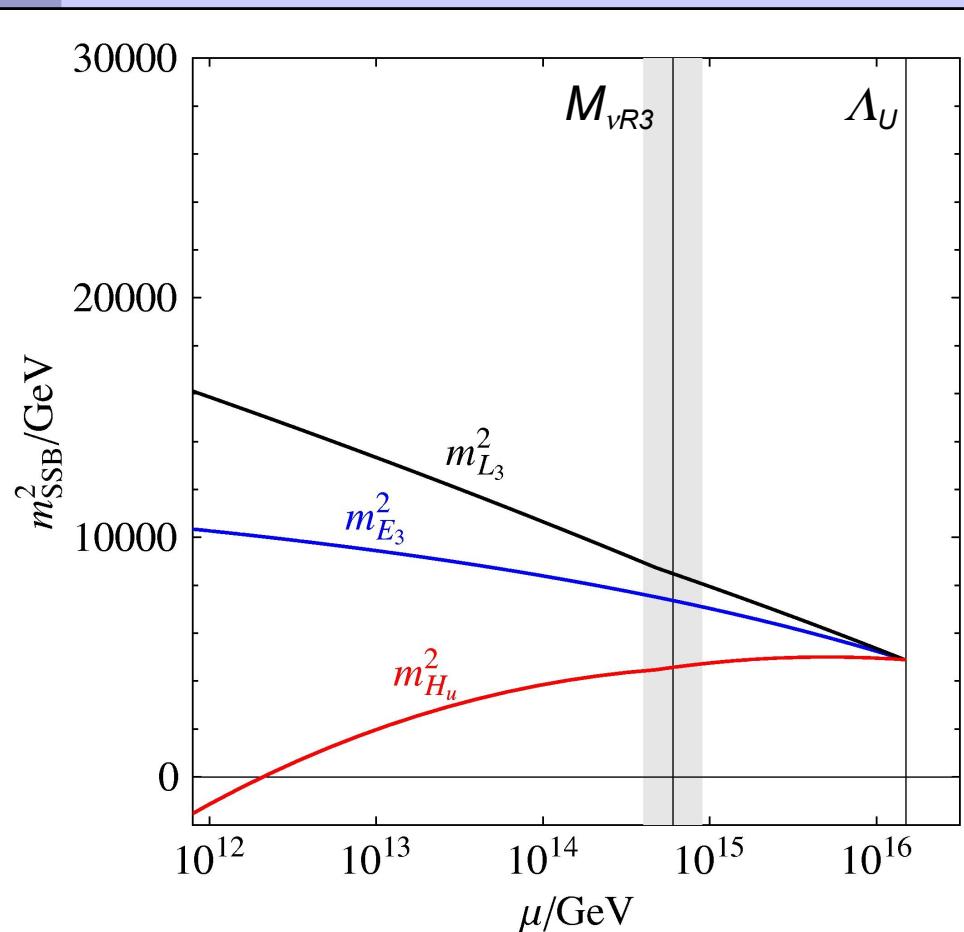
- Two-Step



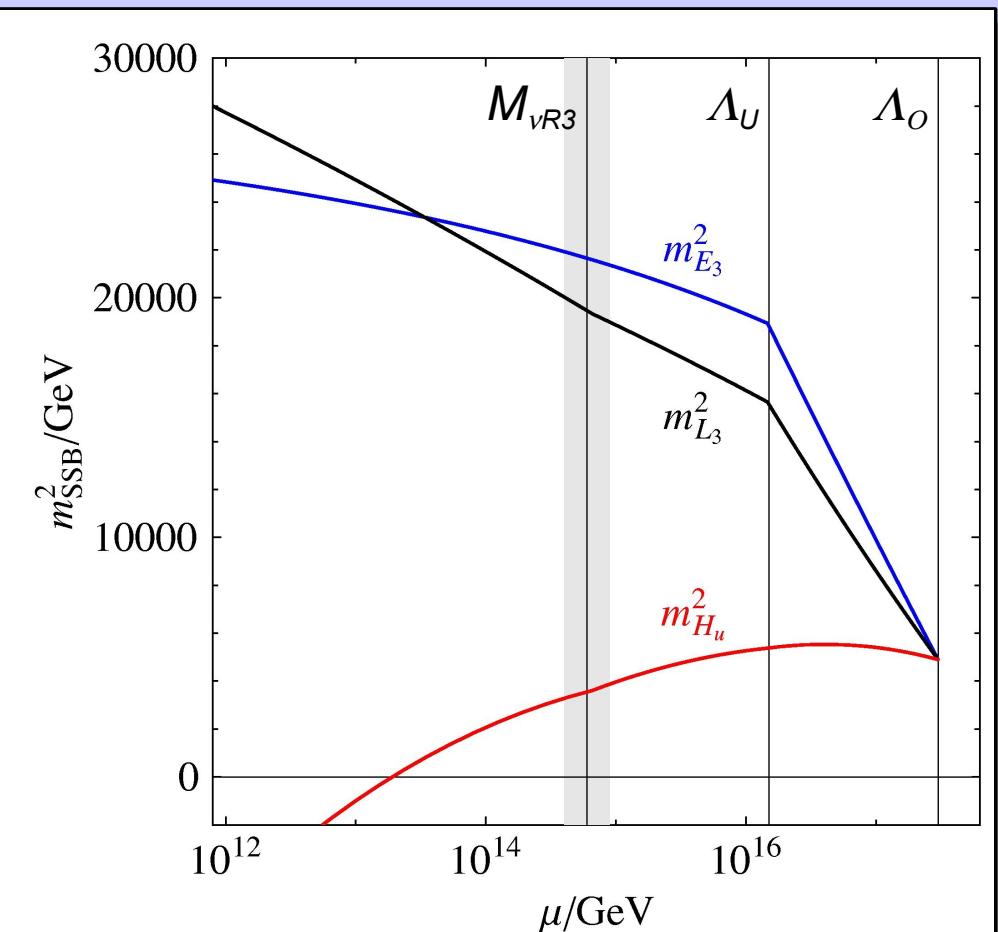
Solid: $D_U=0$
Dashed: $D_U=-30^2 \text{ GeV}^2$

RG Evolution, 3rd generation

- One-Step



- Two-Step



kink at $M_{\nu_{R3}}$ due to Δ_{ν_τ}

Determination of $M_{\nu_{R3}}$ and m_{ν_1}

- RG evolution (One-Step)

$$\Delta_{\nu_\tau} \approx \frac{m_t^2(\Lambda_U)}{4\pi^2 v_u^2} (3m_{16}^2 + A_0^2) \log \frac{\Lambda_U^2}{M_{\nu_{R3}}^2} \quad (3m_{16}^2 + A_0^2) = (3 \cdot 70^2 + 300^2 \pm 160^2) \text{ GeV}^2$$

- Low energy stau mass measurement

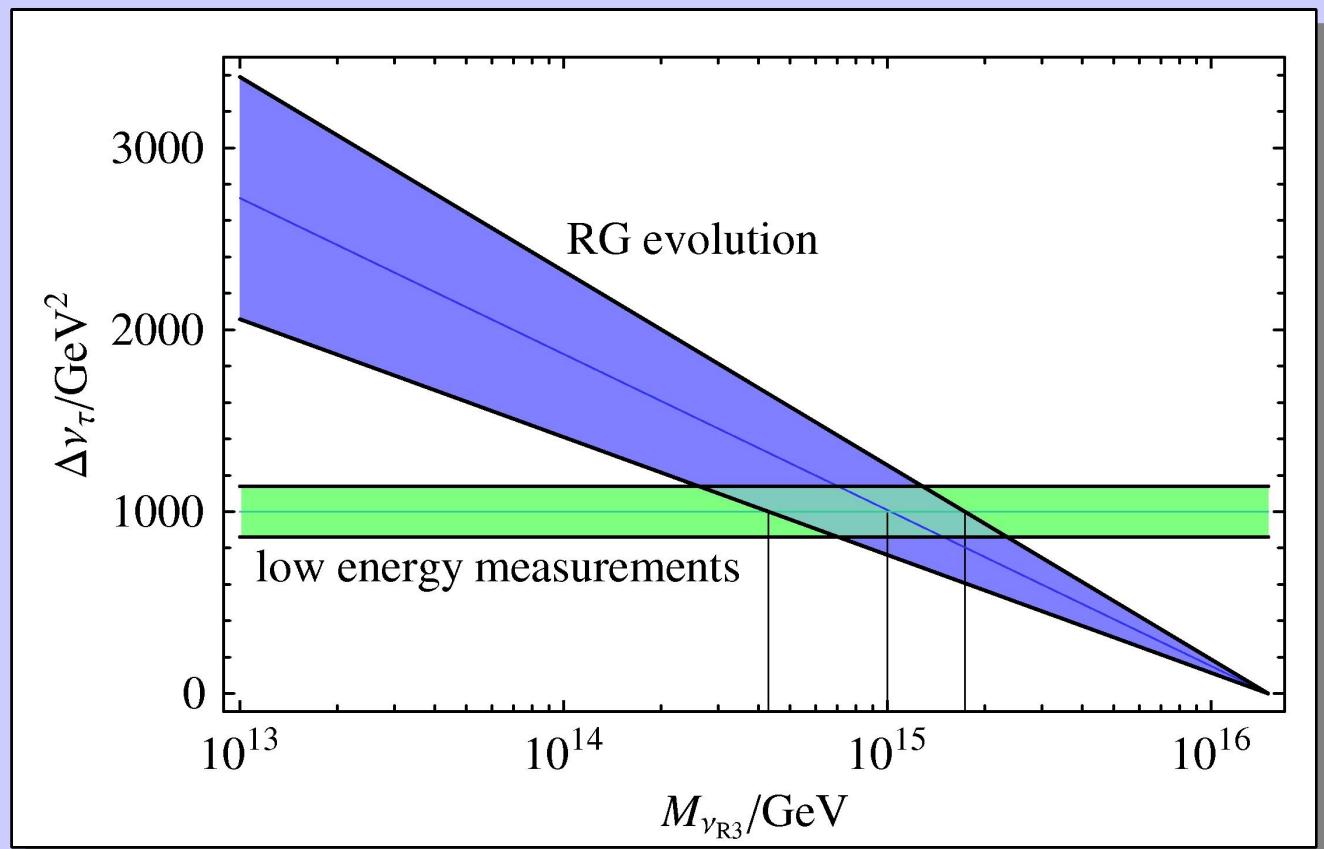
$$\Delta_{\nu_\tau} = (1.0 \pm 0.14) \cdot 10^3 \text{ GeV}^2$$

- Heavy neutrino mass

$$M_{\nu_{R3}} = (1.0 \pm 0.6) \cdot 10^{15} \text{ GeV}$$

- Light neutrino mass

$$m_{\nu_1} = (3.0^{+10}_{-2.0}) \cdot 10^{-3} \text{ eV}$$



Conclusion

- Analysis of a minimal SUSY SO(10) model
- Fundamental GUT and Planck parameters can be extracted from precision SUSY mass measurements at ILC
- Determination of heaviest and lightest neutrino mass possible

| | |
|-------------------------|----------------|
| SU(5) unification scale | Λ_U |
| SO(10) scale | Λ_O |
| Matter scalar mass | m_{16}^2 |
| Higgs scalar mass | $m_{10_1}^2$ |
| Higgs scalar mass | $m_{10_2}^2$ |
| GUT D-term | $D_{U,O}$ |
| Heaviest neutrino mass | $M_{\nu_{R3}}$ |
| Lightest neutrino mass | m_{ν_1} |