

Precision Measurement of the Stop Mass at the Linear Collider

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Publication in Preparation*

Introduction

- We have previously studied the light stop, with a small mass difference to the neutralino, in an attempt to understand EW baryogenesis the asymmetry matter anti-matter and its role in dark matter annihilation.

Phys. rev. D 72,115008(2005)

M. Carena, A. Finch, A. Freitas, C. Milstene, H. Nowak, A. Sopczak

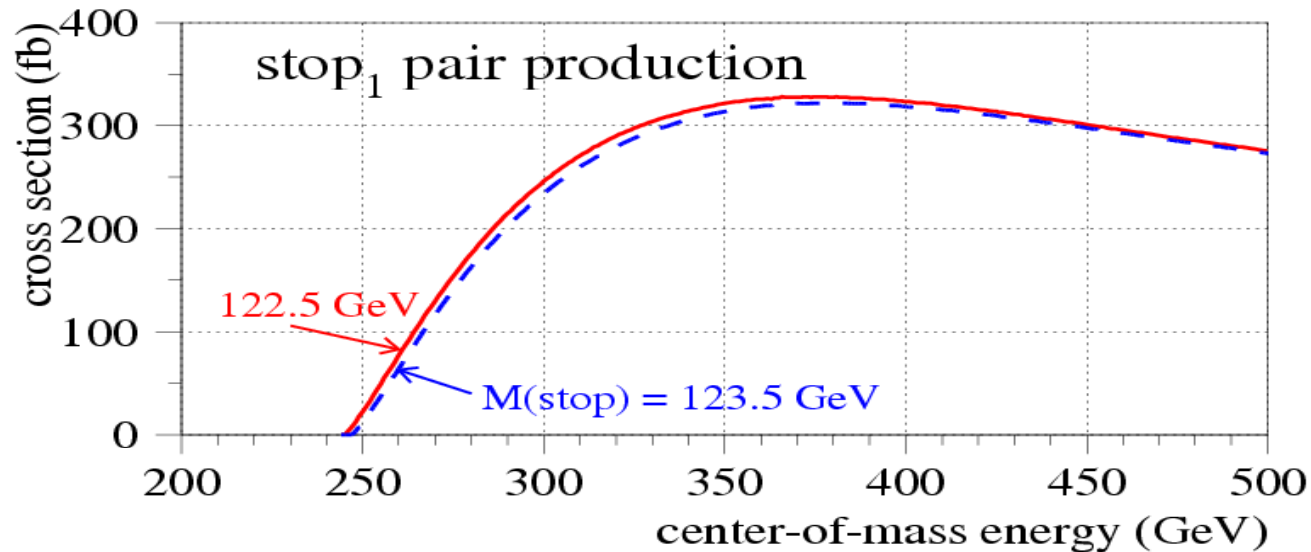
The mass precision measurement reached was $\delta m \sim 1.2 \text{ GeV}$.

This analysis aims at the minimization of the systematics while using more realistic data, stop hadronization/fragmentation included.

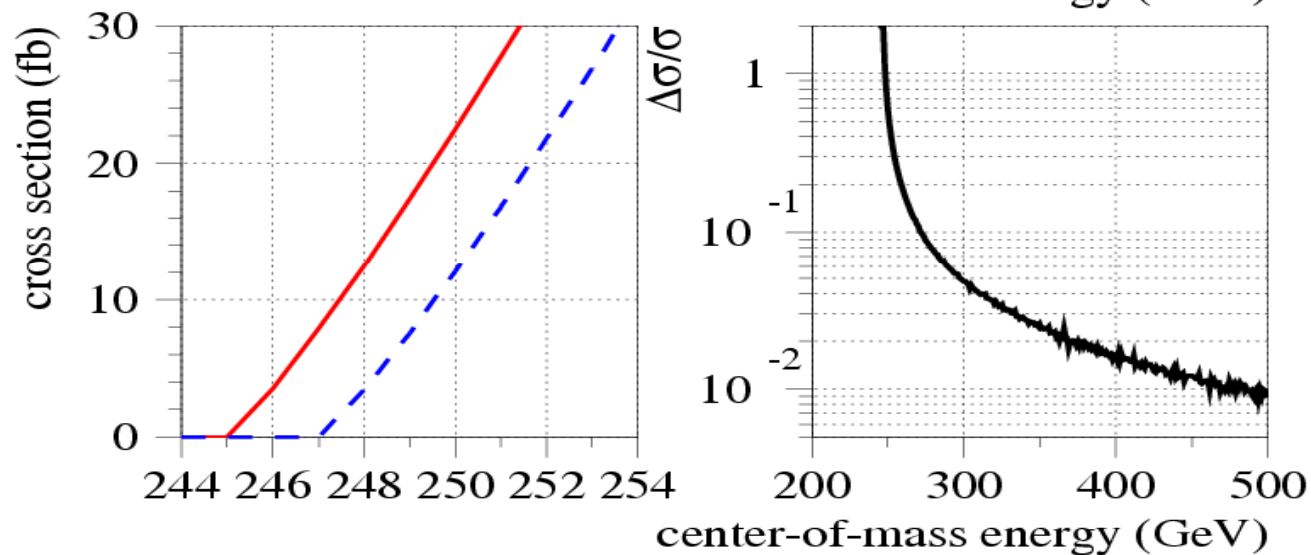
- The precision is improved in two ways:
 - a/ The systematic uncertainties are minimized by measuring the production cross-section at two energies \rightarrow cancellations .
 - b/ The 2nd energy point chosen at or close to the production energy threshold \rightarrow increased sensitivity to mass changes.
- The stop hadronization is included at production \rightarrow the c quark energy is spread out in the process of hadronization.
the final number jets increases- the c-tagging is now a necessity to identify the charm jets (bench-marking for the vertex detector)
- Two approaches are used, a cut based analysis, a multi-parameters optimization analysis IDA
- The polarization improves further the $\frac{\text{signal}}{\text{background}}$ ratio

Cross-Section Precision In Production

$$e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*$$



*Cross-sections [fb]
calculated using NLO
In MC software by
Freitas et al EPJ
C21(2001)361,
EPJ C34(2004)487*



The Method

$$\sigma = \frac{N - B}{\epsilon L}$$

$$Y(M_x \sqrt{s_{th}}) = \frac{N_{th} - B_{th}}{N_{pk} - B_{pk}} = \frac{\sigma(\sqrt{s_{th}})}{\sigma(\sqrt{s_{pk}})}$$

σ the cross-section [fb]

N the number of selected data events

B number of estimated background events

s Square of the energy in center of Mass

N_{th} , B_{th} , s_{th} at or close to production threshold

N_{pk} , B_{pk} , s_{pk} at peak value

ϵ total efficiency & acceptance

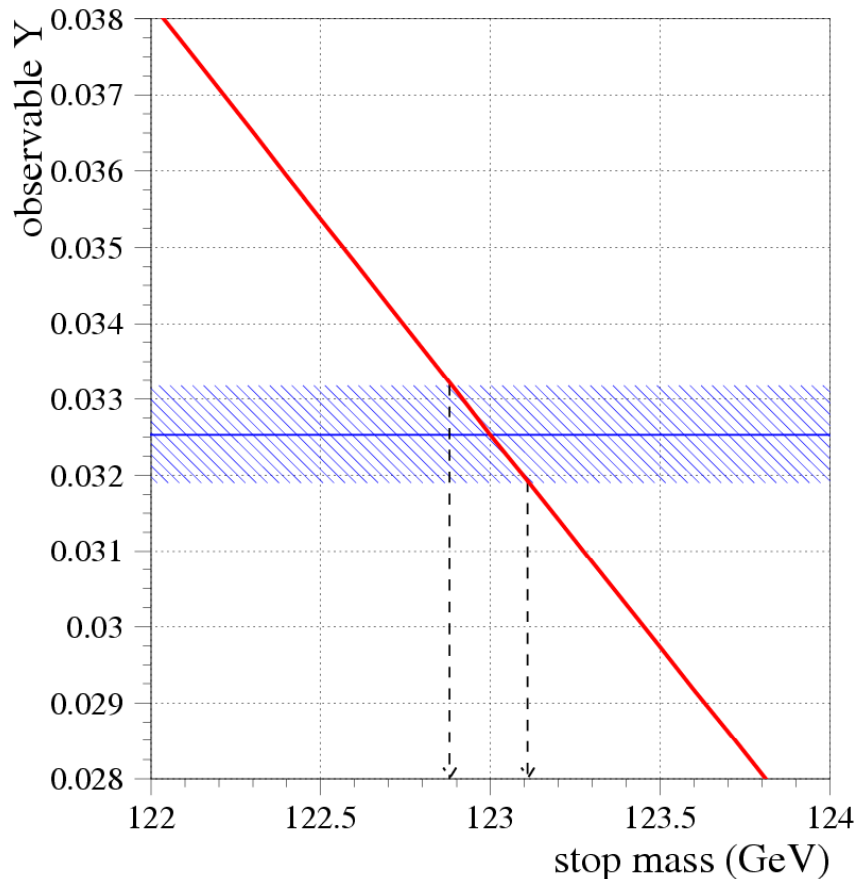
L Integrated luminosity

M_x : Mass to be determined with high precision.

Y ratio of cross-section σ_{th} and $\sigma_{pk} \rightarrow$ Allows Reduction of systematic uncertainty as well as uncertainties from L measurement.

Remark: yield close to threshold is very sensitive to $M_x \rightarrow$ choice of N_{th} and B_{th} ..

Determination of the Stop Mass



$Y=f(M_x)$ from the theoretical cross-section is drawn in Red
 Y from the data the blue line.

As an example, Assume 2% precision for Y ,
 The blue hashed region \rightarrow one obtains
 \rightarrow Precision $\Delta M_x = \pm 1\%$, the 2 vertical arrows

The Scenario depicted:
 $E_{CM}=260\text{GeV}$ with $\sigma=9.2\text{ fb}$ and $\sigma=77\text{fb}$
 at peak

Remark: Assumed luminosities
 $L_{th}=50\text{fb}^{-1}$ (260 GeV), $L_{pk}=500\text{fb}^{-1}$ (500 GeV)

$$e^+e^- \rightarrow \tilde{t}_1 \tilde{t}_1^- \rightarrow c\tilde{\chi}_0^1 \bar{c}\tilde{\chi}_0^1$$

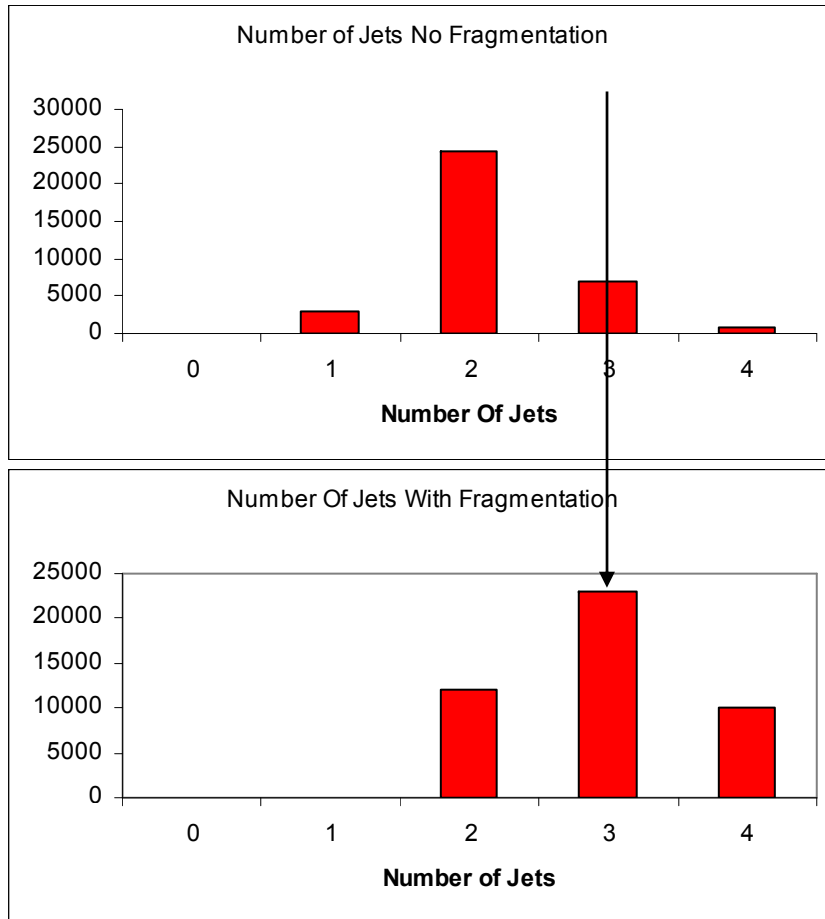
- Analysis uses N-tuple tool incorporating jet finding algorithm (T. Kuhl)
- Soft Multi-jets in the final state
- Stop Hadronization \rightarrow the final state jets smeared :
due to gluon radiation + fragmentation
- At ECM=260 GeV mostly 2 jets, carry the charm.
- At ECM=500 GeV 2jets \rightarrow 2,3,4 jets (more energy available in the CM)
 \rightarrow the Charm tagging a necessary tool to identify
the charm jets (Vertex bench-marking)

Simulation Characteristics

- Signal and Background generated with: Pythia (6.129)
Simdet (4-0-3)– Circe(1.0)
 - Hadronisation of the c quark and the \tilde{t} from the Lund string fragmentation
Pythia uses Peterson fragmentation
(*Peterson et al PR D27:105*)
 - The \tilde{t} fragmentation is simulated using Torbjorn code
[//http://www.thep.lu.se/torbjorn/pythia/main73.f](http://www.thep.lu.se/torbjorn/pythia/main73.f)

The \tilde{t}_1 quark is **set stable** until **after fragmentation** where it is
Allowed to **decay again** as described in (*Kraan, EPJ C37:91*)
- Signal and Background are generated in each channel for the given luminosity in conjunction to the cross-sections

Jet Multiplicity – Without/With Fragmentation



- Stop fragmentation simulated using Torbjorn code

<http://www.thep.lu.se/torbjorn/pythia/main73.f>

- The stop fragmentation parameter is set relative to the bottom fragmentation parameter

$$\tilde{\epsilon}_t = \epsilon_b \cdot m_b^2 / m_t^2$$

And $\epsilon_b = -0.0050 \pm 0.0015$

following (OPAL, EPJ C6:225)

- The jet Multiplicity without Fragmentation
Upper figure

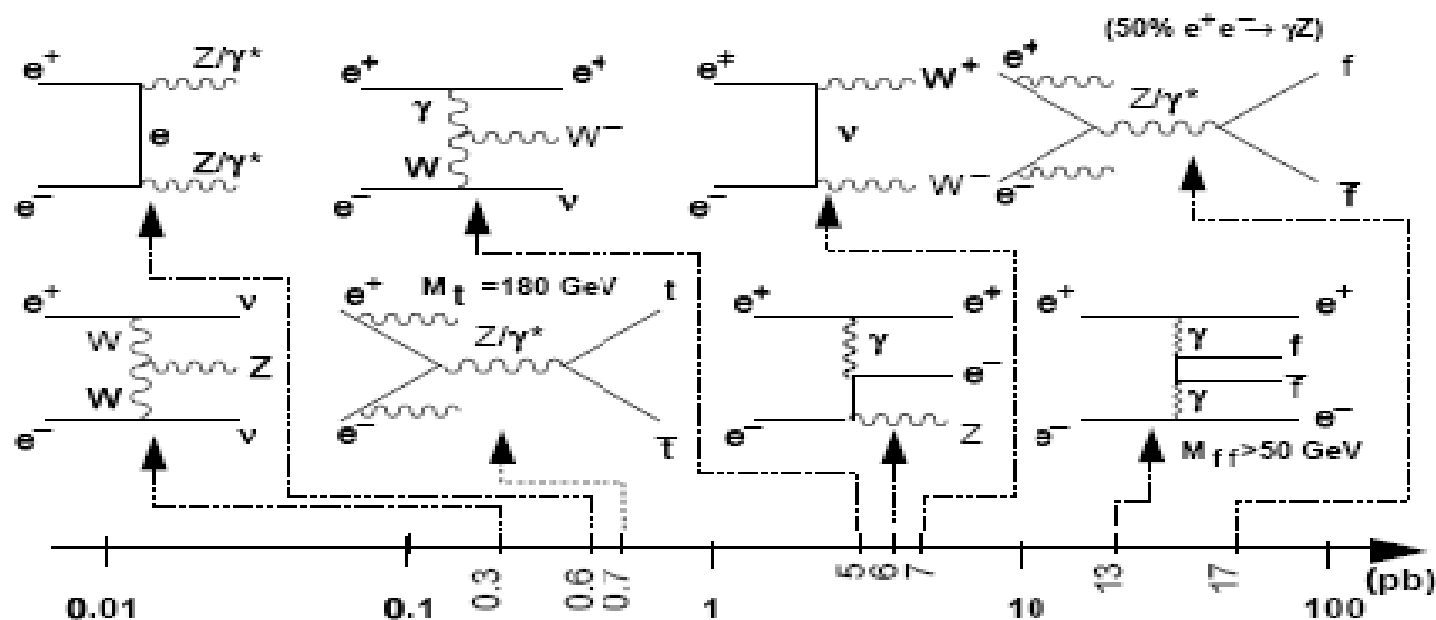
~ 70% 2 jets

- The jet Multiplicity with \tilde{t} Fragmentation
Lower Figure

~ 50% 3 jets

& bigger admixture of 4jets

Background- Channels @500 GeV



Z Phys. C 76 (1997) 549- A.Bartl, H. Eberl, S. Kraml, W.Majerotto, W.Porod, A. Sopczak

The cross-sections

Process	$\sigma[\text{pb}]$ at ECM=260GeV			$\sigma[\text{pb}]$ at ECM=500GeV		
P(e-)/ P(e+)	0/0	-80%/+60%	+80%/-60%	0/0	-80%/+60%	+80%/-60%
$\tilde{t}_1 \tilde{t}_1^*$	0.032	0.017	0.077	0.118	0.072	0.276
W W	16.9	48.6	1.77	8.6	24.5	0.77
Z Z	1.12	2.28	0.99	0.49	1.02	0.44
Wenu	1.73	3.04	0.50	6.14	10.6	1.82
eeZ	5.1	6.0	4.3	7.5	8.5	6.2
qq, qq \neq tt	49.5	92.7	53.1	13.1	25.4	14.9
tt	0.0	0.0	0.0	0.55	1.13	0.50
2 γ ($p_t > 5$ GeV)	786			936		

Table 1

A. Freitas et al EPJ C21(2001)361, EPJ C34(2004)487 and GRACE and COMPHEP -Next to leading order, assuming a stop mixing angle (0.01)

Selection Cuts at $E_{CM}=260, 500 \text{ GeV}$

Variable	ECM 260 GeV	ECM 500 GeV
Number of jets	$N_{\text{jets}}=2$	$N_{\text{jets}} \geq 2 \ \& \ E_n < 25 \text{ GeV}$ $n=3,4,..$
Transverse Momentum p_t Thrust T $\cos\theta_{\text{Thrust}}$ Visible Energy E_{vis} Acoplanarity Φ_{acop} Invariant mass of jet pair m_{jj} Charm tagging likelihood P_c	$p_t > 10 \text{ GeV}$ - $ \cos\theta_{\text{Thrust}} < 0.7$ $E_{\text{vis}} < 0.175 * ECM$ $ \Phi_{\text{acop}} < 0.9$ $25.5 \text{ GeV} < m_{jj} < 90 \text{ GeV}$ $P_c > 40\%$	$p_t > 12 \text{ GeV}$ $T > 0.8$ $ \cos\theta_{\text{Thrust}} < 0.7$ $E_{\text{vis}} < 0.4 * ECM$ $ \Phi_{\text{acop}} < 0.9$ $60 \text{ GeV} < m_{jj} < 90 \text{ GeV}$ $P_c > 40\%$

Table 2

In order to optimize the cancellation of the systematics we aim to have a selection as similar as possible at the two energies. (cancellation in Y)
The two-photons background did require a 5GeV p_t cut.

Events Generated and After Sequential cuts

	L=50fb ⁻¹ at ECM=260GeV			L= 500fb ⁻¹ at ECM=500GeV		
P (e-)/ P(e+)	Generated	0/0	+80%/-60%	Generated	0/0	+80%/-60%
$\tilde{t}_1 \tilde{t}_1^*$	50000	382	921 (<u>24%eff.</u>)	50000	11300	26430 (<u>19%eff.</u>)
WW	180000	<5	<1	210000	102	9
ZZ	30000	<2	<2	30000	250	224
Wenu	210000	36	4	210000	10102	2994
eeZ	210000	<1	<1	210000	<18	<15
qq, q≠t	350000	<7	<8	350000	19	22
tt	-	0	0	180000	21	19
2-Photons	1.6 10 ⁶	12	12	8.5x10 ⁶	120	120

Table 3

0/0 polarization beam → Unambiguous discovery

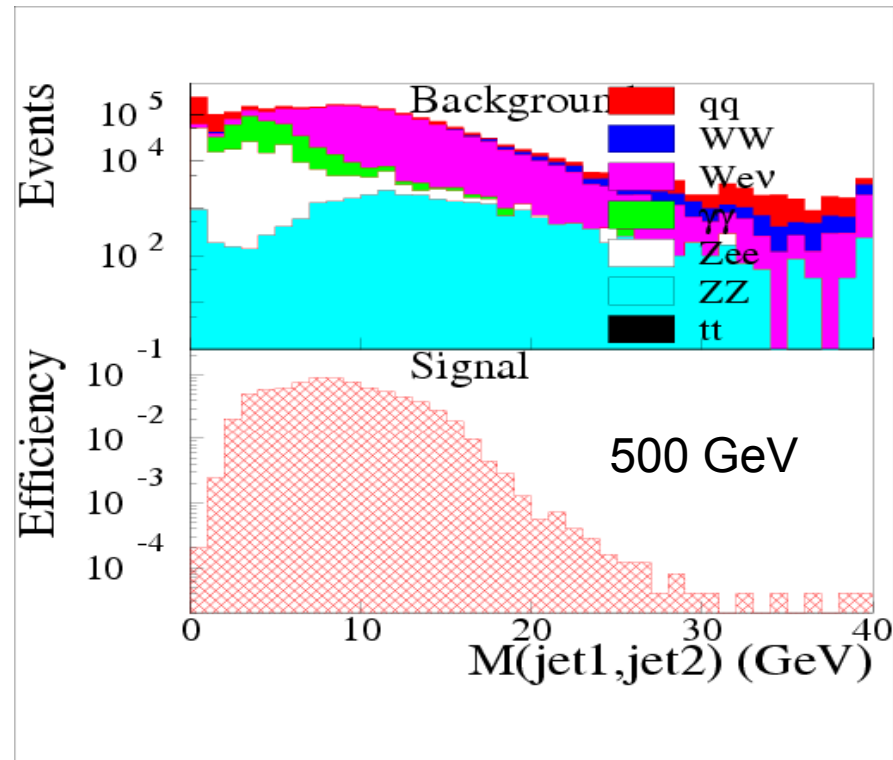
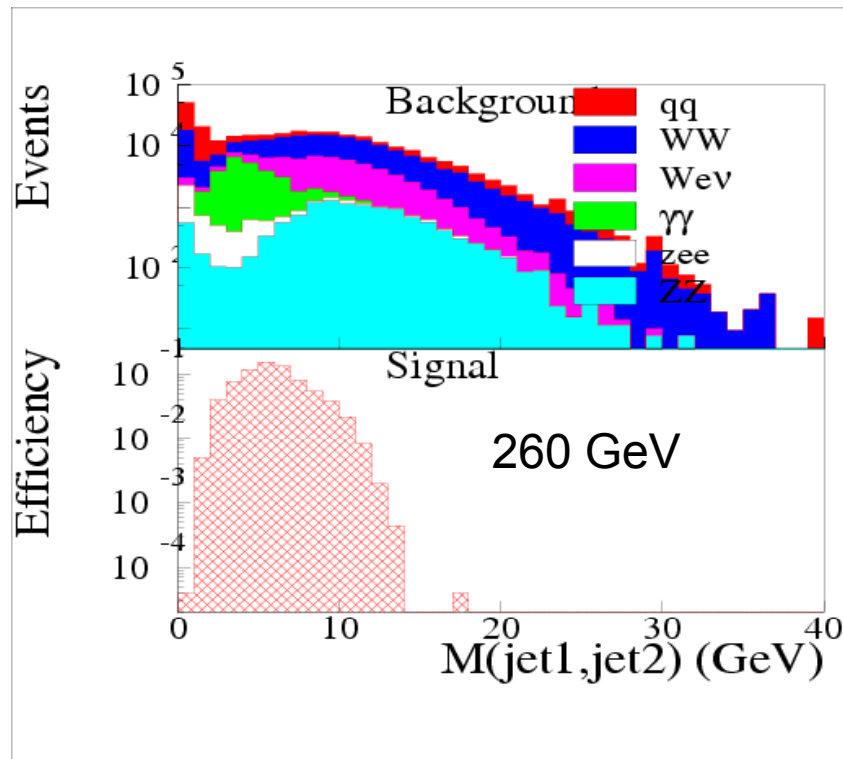
+80%/-60% polarization → Precision Measurement

Remark: \tilde{t}_1 fragmentation → the separation from the Wenu more difficult

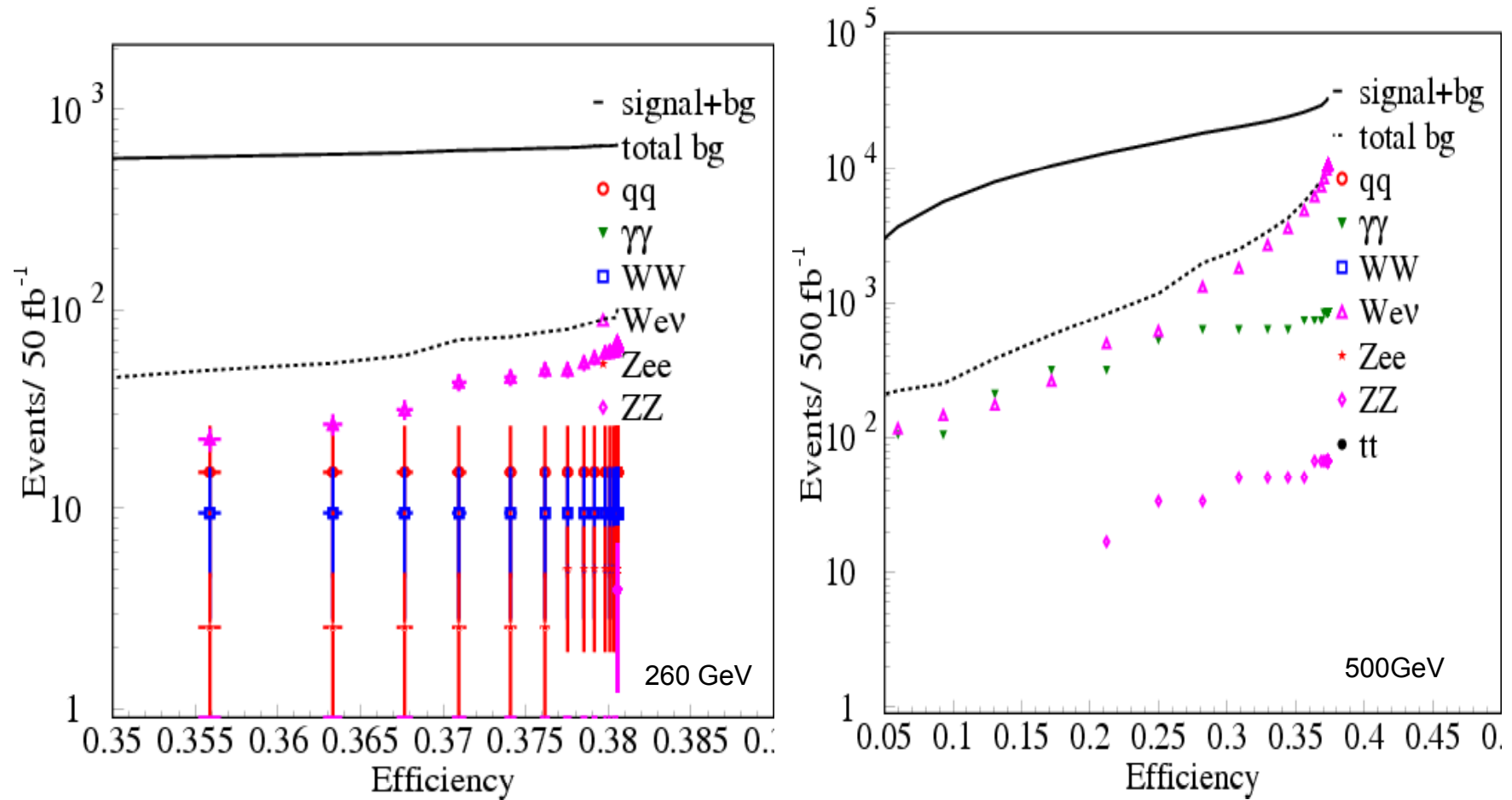
Iterative Discriminant Analysis (IDA)

- Improves even more the precision in the \tilde{t}_1 mass measurement an Iterative Discriminant Analysis (IDA) is used. (modified Fisher Disc. Analysis)
- IDA combines the kinematic variables in parallel. The same variables and simulated events are used than in the cut based analysis . A non linear discriminant function followed by iterations are enhancing the separation between signal and background.
- Both the signal and background have been divided in two equally sized samples, one sample is used for training, the other as data.
- Two IDA steps have been performed, with a cut after the 1st IDA iteration keeping 99% of the signal efficiency.
- The performance is shown in the two next figures at 260 and 500 GeV.

Invariant Mass Di-Jets Before Final IDA



IDA Performance



Events Generated and After IDA Selection

	L=50fb ⁻¹ at ECM=260GeV		L= 500fb ⁻¹ at ECM=500GeV	
P (e-)/ P(e+)	0/0	+80%/-60%	0/0	+80%/-60%
$\tilde{t}_1 \tilde{t}_1^*$	610	1470 (38%eff.)	21240	49700 (<u>36%eff.</u>)
WW	19	2	<41	<4
ZZ	7	7	67	60
Wenu	68	39	10640	3155
eeZ	10	8	<36	<30
qq, q≠t	30	32	<38	<43
tt	0	0	<3	<3
2-Photons	<25	<25	840	840

Table 4

The efficiencies improves from 24% ,19% cut based → 38% ,36% IDA, while the background is of the same order of magnitude.

Systematic Uncertainty in Kinematics Cuts Variables

Variable	Error on variable	Error on Y
p_t	2%	0.28%
$\cos\theta_{\text{Thrust}}$	1.8%	0.18%
E_{vis}	2%	0
Φ_{acop}	1%	0.08%
m_{jj}	4%	0.61%

Table 5

- All cuts are applied to hadronic and jet observables→ Calibration quantities are jet energy scale & jet angle.
- Based on LEP, we assume 2% calibration error for jets, 1 deg for jet angle
- Effect on signal efficiency: Partial cancellation between 260 and 500 GeV
- We assume cancellation in total luminosity in Y between 260&500GeV

Effect of Stop and Charm Fragmentation

Comparison of the signal generated with and without gluon radiation

→ The signal efficiency changes due to jet number cut is 2.5%

→ We assume an error of 1% for the number of jets

Charm fragmentation parameters assumed as precise as for LEP/OPAL

→ $\epsilon_c = -0.0031 \pm 0.011$

Stop fragmentation is set relative to bottom fragmentation, $\epsilon_{\tilde{t}1} = \epsilon_b (m_b/m_t)^2$

$\epsilon_{\tilde{t}1} = -0.0050 \pm 0.015$

They don't cancel between the 2 energies but are small

Including the effects of the fragmentation at both energy points

$\delta\epsilon_c = \pm 35\% \rightarrow \text{Error } \delta Y = +1.2\% - 0.2\%$

$\delta\epsilon_{\tilde{t}1} = \pm 30\% \rightarrow \text{Error } \delta Y = +0.4\% + 2.4\%$

→ contribute an error $O(\text{few}\%)$

Theoretical Uncertainties

- Precise cross-section calculations are needed
- Stop production receive large corrections from QCD gluon exchange
Between the final state stops (bigger @Threshold) \rightarrow Coulomb corr.
- NLO- QCD corrections $\sim 100\%$ @threshold down to 10% at high energies are included here
- NNLO-QCD corrections are expected to be same order than NLO based on the results for the top quark. The missing higher order correction $\sim 7\%$ @260GeV, 2.5% @500 GeV
- It is expected that theoretical uncertainties can be brought down by a factor 2
- Here we assume an uncertainty of 3.5% @260GeV and 1% @500 GeV
- The EW corrections : NLO \sim several %, the NNLO $\sim 1\%$
- Combined $\rightarrow \sim 4\%$ @260 GeV and 1.5% @500GeV $\rightarrow \delta Y = 5.5\%$

Combined Statistic and systematic Errors

Error source for Y	Cut-based Analysis
Statistical	4.1%
Detector Effects	1.15%
Jet number	1%
Charm Fragmentation	1.2%
Stop Fragmentation	2.4%
Charm tagging algorithm	<0.5%
Sum of Experimental Errors	5.2%
Theory for signal σ	5.5%
Theory for background σ	0.5%
Total error δY	7.2%

For IDA the determination of *systematic uncertainties* in progress.

Table 6

Results

Combining the statistical and systematic errors Table 6(*)

$\delta Y = 7.2\% \rightarrow \delta m_{\tilde{t}_1} \sim 0.3 \text{ GeV}$ – a factor 4 better (*Phys. rev. D 72,115008(2005)*)
(dominated by the theory, expected to improve for signal and background)

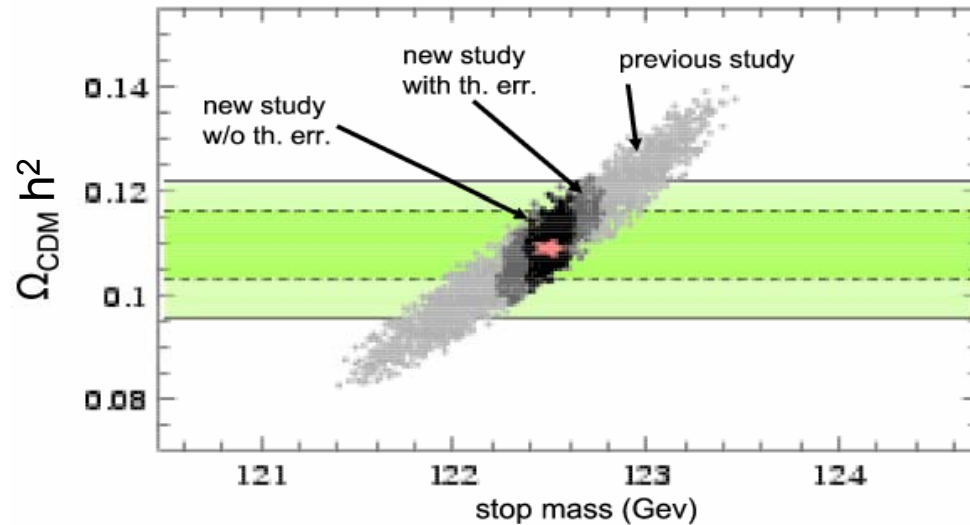
$\delta Y = 5.2\% \rightarrow \delta m_{\tilde{t}_1} \sim 0.2 \text{ GeV}$ (cut based experimental errors alone)

$\delta Y = 4.2\% \rightarrow \delta m_{\tilde{t}_1} \sim 0.15 \text{ GeV}$ (experimental errors & IDA) (expected)

→ Improvements in dark matter relic density due to improvement in $\delta m_{\tilde{t}_1}$ is shown in the next figure.

Other limiting factors start to interplay, e.g. the precision on the neutralino mass $\delta m_{\chi_0^1} \sim 0.3 \text{ GeV}$,(hep-ph/0608255, M.Carena, A.Freitas)

Dark Matter Relic Abundance= $f(m_{\text{stop}})$



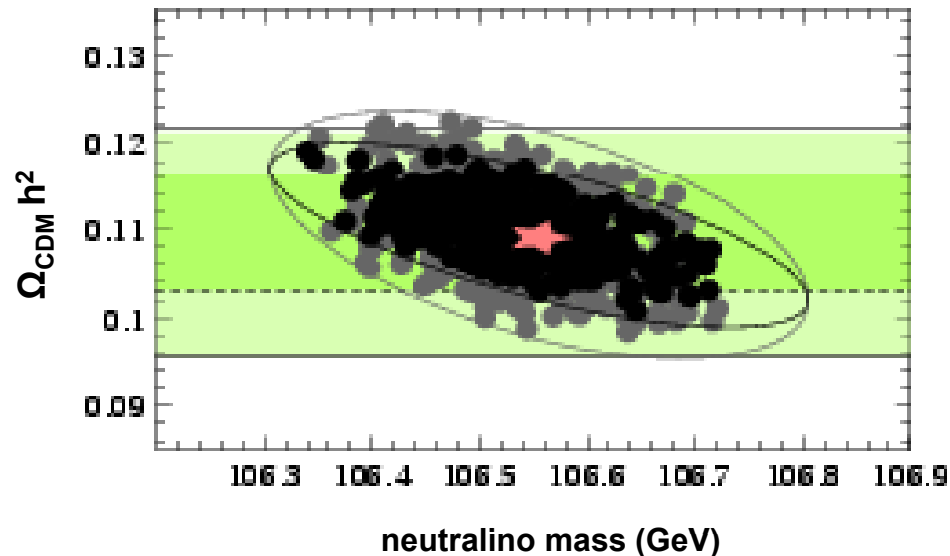
Dark Matter relic density accounting
The estimated experimental errors
For stop, Chargino, neutralino and
Higgs sector –(scan over 1σ)
versus m_{st}^1 for
 $\delta m_{\tilde{\tau}_1} = 1.2$ GeV light gray dot
Previous study
 $\delta m_{\tilde{\tau}_1} = 0.3$ GeV dark gray dot
Now this study
 $\delta m_{\tilde{\tau}_1} = 0.15$ GeV black dots
Expected this study
with IDA

$$\delta m_{\text{st}}^1 = 0.3 \text{ GeV} \rightarrow \Omega_{\text{CDM}} h^2 = 0.109 + 0.0013 - 0.010$$

$$\delta m_{\text{st}}^1 = 0.2 \text{ GeV} \rightarrow \Omega_{\text{CDM}} h^2 = 0.109 + 0.0012 - 0.009$$

$$\delta m_{\text{st}}^1 = 0.15 \text{ GeV} \rightarrow \Omega_{\text{CDM}} h^2 = 0.109 + 0.0011 - 0.009$$

Relic Abundance as Function of $m_{\chi_0^1}$



Dark Matter relic density as a function of the neutralino mass accounting for the estimated experimental errors as before but as function of the

Lightest neutralino mass $m_{\chi_0^1}$

Gray dots for $\delta m_{\tilde{\tau}_1} = 0.3$ This study
Errors from Experiment+theory

Black dots for $\delta m_{\tilde{\tau}_1} = 0.15$ This Study
Experiment. Err. and IDA

$$\delta m_{\tilde{\tau}_1} = 0.3 \text{ GeV} \rightarrow \Omega_{\text{CDM}} h^2 = 0.109 \pm 0.0013 \text{ (stat)} \pm 0.010 \text{ (th)}$$

$$\delta m_{\tilde{\tau}_1} = 0.15 \text{ GeV} \rightarrow \Omega_{\text{CDM}} h^2 = 0.109 \pm 0.0011 \text{ (stat)} \pm 0.009 \text{ (th)}$$

$$\text{WMAP: } \Omega_{\text{CDM}} h^2 = 0.1106 \pm 0.0056 \text{ (stat)} \pm 0.0075 \text{ (th)}$$

Conclusion

- More realistic data were produced including hadronization/fragmentation
- The precision, however, improved by a factor three on our previous analysis with $\delta m_{st}^1 = 0.3 \text{ GeV}$.
- This method could be applied to other particles e.g. to measure the Higgs mass
- The method improves the precision to the mass determination in two ways
a/ by reducing the systematics in Y- cancellation between the two energy points.
b/ by choosing the energy at threshold, Y extremely sensitive to the mass
- The polarization separates the right-handed signal \tilde{t}_1 from background.
- Due to hadronization and fragmentation the c-tagging was a necessary tool to identify the charm jets at $E_{CM}=500 \text{ GeV}$ (benchmark for the vertex detector)
- Systematics in progress for the IDA a multi-parameters analysis, expected improvement to $\delta m_{st}^1 = 0.15 \text{ GeV}$
- Progress in the theoretical calculations is expected and partly accounted for
- With that precision we become limited by other factors.
- With this mass precision, the calculated relic density is in accordance with WMAP and SLOAN ,
 $\delta m_{st}^1 = 0.15 \text{ GeV} \rightarrow \Omega_{CDM} h^2 = 0.109 \pm 0.0011 \pm 0.009$
WMAP: $\Omega_{CDM} h^2 = 0.1106 \pm 0.0056 \pm 0.0075$

Backup slides

A Sample Parameter Point

- $m_{\tilde{U}_3}^2 = -99^2 \text{ GeV}^2$
- $A_t = -1050 \text{ GeV}$
- $M_1 = 112.6 \text{ GeV}$
- $M_2 = 225 \text{ GeV}$
- $|\mu| = 320 \text{ GeV}$
- $\Phi_\mu = 0.2$
- $\tan \beta = 5$

Which gives:

$m_{\tilde{t}_1} = 122.5 \text{ GeV}; m_{\tilde{t}_2} = 4203 \text{ GeV};$

$m_{\tilde{\chi}_1^0} = 107.2 \text{ GeV}; m_{\tilde{\chi}_1^\pm} = 194.3 \text{ GeV}; m_{\tilde{\chi}_2^0} = 196.1 \text{ GeV}$

$m_{\tilde{\chi}_3^0} = 325.0 \text{ GeV}; m_{\tilde{\chi}_2^\pm} = 359.3 \text{ GeV}$

$\cos\theta_{\tilde{t}} = 0.0105 \sim \tilde{t} \text{ right handed}$

$\rightarrow \Delta m = 15.2 \text{ GeV}$