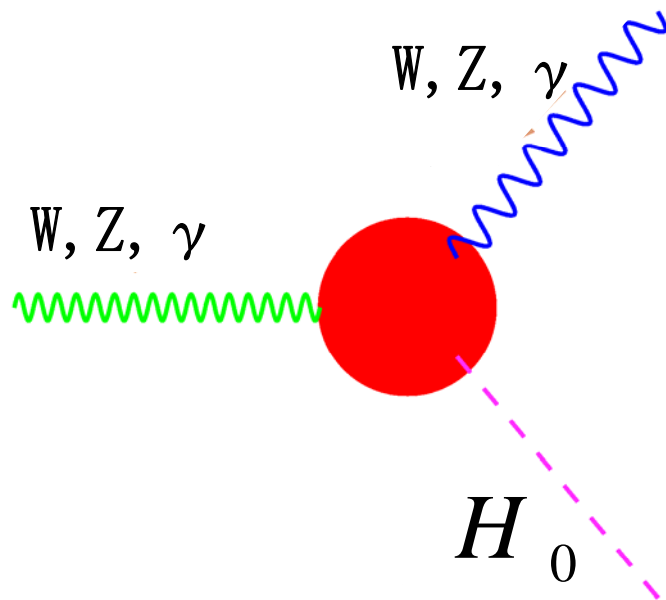


Precise measurement of the Higgs-boson electroweak couplings at Linear Collider and its physics impacts

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@TILC

2008, 3, 5

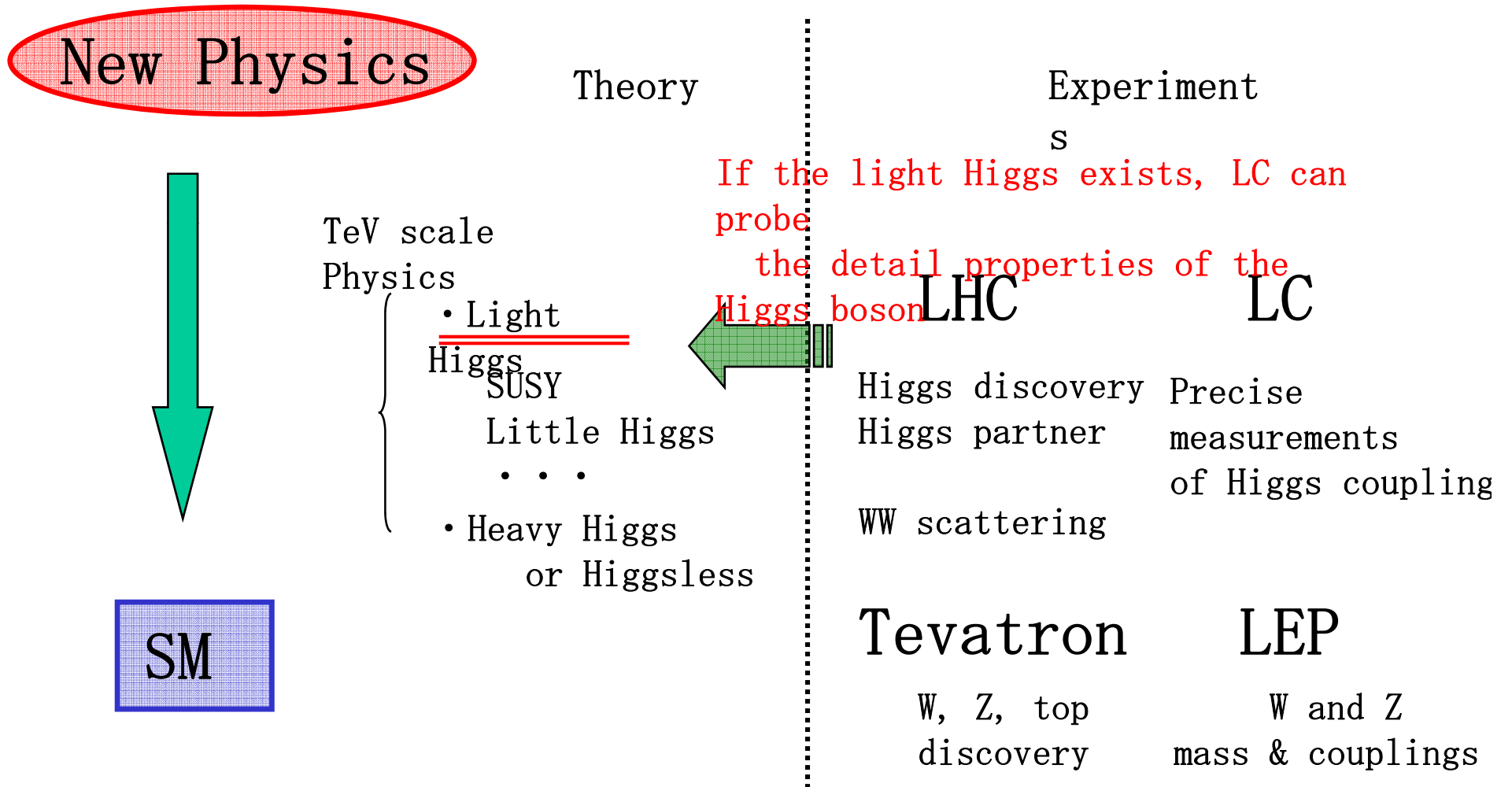


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1. Introduction
2. Analysis
3. Results
4. Conclusion

In collaboration with K. Hagiwara (KEK) and S. Dutta (Univ. of Delhi)

1-1. Motivation



1-2. Effective Lagrangian with Higgs doublet

New physics can be represented by higher mass dimension operators

$$L_{eff} = L_{SM} + L^{\text{dim5}} + L^{\text{dim6}} + L^{\text{dim7}} + L^{\text{dim8}} + \dots,$$

contribute to majorana neutrino mass

contribute to

{ four fermion coupling (Proton decay,
FCNC, etc)

{ purely gauge term (Triple gauge coupling, etc) ← We focused on this physics

Higgs electroweak couplings

We can write the effective Lagrangian including Higgs doublet as

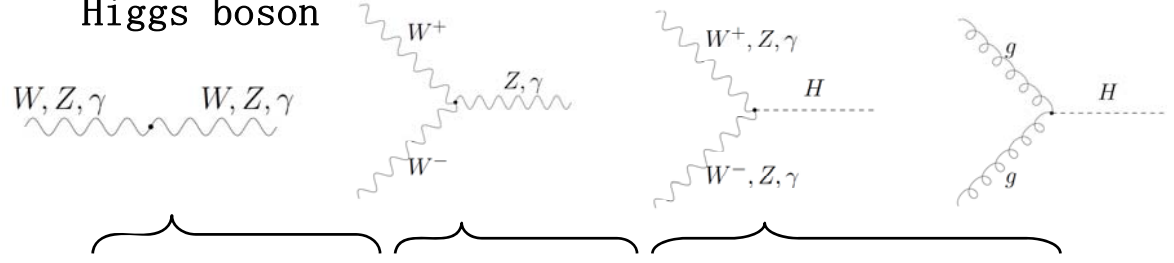
$$L_{eff} = L_{SM} + \sum_i \frac{f_i}{\Lambda^2} O_i^{(6)}$$

← New physics effects. Here we considered only dimension 6

and the operators are ...

1-3. dimension 6 operators including Higgs

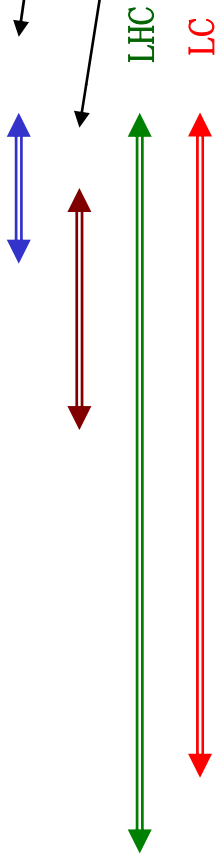
2 point function, TGC, vertices include Higgs boson



Precision measurement (SLC, LEP, Tevatron)
 triple gauge couplings (LEP2, Tevatron)

Dimension 6 operators

\mathcal{O}	WW	ZZ	Z γ	$\gamma\gamma$	WW γ	WWZ	HWW	HZZ	HZ γ	H $\gamma\gamma$	Hgg
$\mathcal{O}_{\phi,1} = [(D_\mu \Phi)^\dagger \Phi] [\Phi^\dagger (D^\mu \Phi)]$		✓					✓	✓			
$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}^{\mu\nu} \hat{W}_{\mu\nu} \Phi$		✓	✓	✓	✓	✓	✓	✓	✓	✓	
$\mathcal{O}_W = (D^\mu \Phi)^\dagger \hat{W}_{\mu\nu} (D^\nu \Phi)$					✓	✓	✓	✓	✓		
$\mathcal{O}_B = (D^\mu \Phi)^\dagger \hat{B}_{\mu\nu} (D^\nu \Phi)$					✓	✓		✓	✓		
$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}^{\mu\nu} \hat{W}_{\mu\nu} \Phi$	-	-	-	-	-	-	✓	✓	✓	✓	
$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}^{\mu\nu} \hat{B}_{\mu\nu} \Phi$		-	-	-				✓	✓	✓	
$\mathcal{O}_{\phi,4} = (\Phi^\dagger \Phi) (D_\mu \Phi)^\dagger (D^\mu \Phi)$	-	-					✓	✓			
$\mathcal{O}_{\phi,2} = \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)$							✓	✓			
$\mathcal{O}_{gg} = \Phi^\dagger \hat{G}^{\mu\nu} \hat{G}_{\mu\nu} \Phi$											✓



1-4. Operators and Vertices, Form Factors

We exchange the operators into HVV interaction vertices as the experimental observables



$$\begin{aligned}
 L_{eff} = L_{SM} + \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i^{(6)} = & \underbrace{(1 + a_{ZZ}) \frac{g_Z m_Z}{2} H Z_\mu Z^\mu}_{\text{red dotted}} + (1 + a_{WW}) g m_W H W_\mu^+ W^{-\mu} \\
 & + \frac{g_Z}{m_Z} \left[\frac{b_{ZZ}}{2} H Z_{\mu\nu} Z^{\mu\nu} + \frac{c_{ZZ}}{2} ((\partial_\mu H) Z_\nu - (\partial_\nu H) Z_\mu) Z^{\mu\nu} \right] \\
 & + \frac{g_Z}{m_Z} \left[b_{Z\gamma} H Z_{\mu\nu} A^{\mu\nu} + c_{Z\gamma} ((\partial_\mu H) Z_\nu - (\partial_\nu H) Z_\mu) A^{\mu\nu} \right] \\
 & + \frac{g_Z}{m_Z} \left[b_{WW} H W_{\mu\nu}^+ W^{-\mu\nu} + \frac{c_{WW}}{2} (((\partial_\mu H) W_\nu^- - (\partial_\nu H) W_\mu^-) W^{+\mu\nu} + h.c.) \right] + \dots
 \end{aligned}$$

※ $(a_Z, b_Z, c_Z, b_\gamma, c_\gamma, b_{\gamma\gamma}, a_W, b_W, d_W)$

are the linear function of

f_i

$$\left\{ \begin{aligned}
 a_{ZZ} &= \frac{v^2}{4\Lambda^2} (3f_\phi + 3f_{\phi 4} - 2f_{\phi 2}), \\
 b_{ZZ} &= \frac{m_Z^2}{\Lambda^2} (-s_W^4 f_{BB} - s_W^2 c_W^2 f_{BW} - c_W^4 f_{WW}), \\
 b_{Z\gamma} &= \frac{m_Z^2}{\Lambda^2} (s_W^2 f_{BB} + \frac{1}{2}(c_W^2 - s_W^2) f_{BW} - c_W^2 f_{WW}) s_W c_W, \\
 c_{ZZ} &= \frac{m_Z^2}{2\Lambda^2} (-s_W^2 f_B - c_W^2 f_W), \\
 c_{Z\gamma} &= \frac{m_Z^2}{4\Lambda^2} (f_B - f_W) s_W c_W, \\
 b_{\gamma\gamma} &= \frac{m_Z^2}{\Lambda^2} (-f_{BB} + f_{BW} - f_{WW}) c_W^2 s_W^2,
 \end{aligned} \right.$$

  : EW precision, S and T parameter

 • Triple Gauge Couplings

1-5. Optimal observable method

The differential cross section can be expressed by using non-SM couplings

$$\frac{d\sigma}{d\Omega} = \Sigma_{SM}(\Omega) + \sum_i c_i \underline{\Sigma_i(\Omega)} \quad \leftarrow c_i = (a_{WW}, a_{ZZ}, b_{WW}, b_{ZZ}, b_{Z\gamma}, b_{\gamma\gamma}, c_{WW}, c_{ZZ}, c_{Z\gamma})$$

Ω is 3-body phase space

$$\begin{cases} N_{EXP}^k = \underline{L\Sigma_{SM}(\Omega_k)\Delta\Omega_k}, \\ N_{TH}^k = L\Sigma_{SM}(\Omega_k)\Delta\Omega_k + L\sum_i c_i \Sigma_i(\Omega_k)\Delta\Omega_k \end{cases} \quad \leftarrow \begin{array}{l} \text{number of event in the k-th bin} \\ \text{for experiment and theory} \end{array}$$

χ^2 can be expressed in terms of non-SM couplings

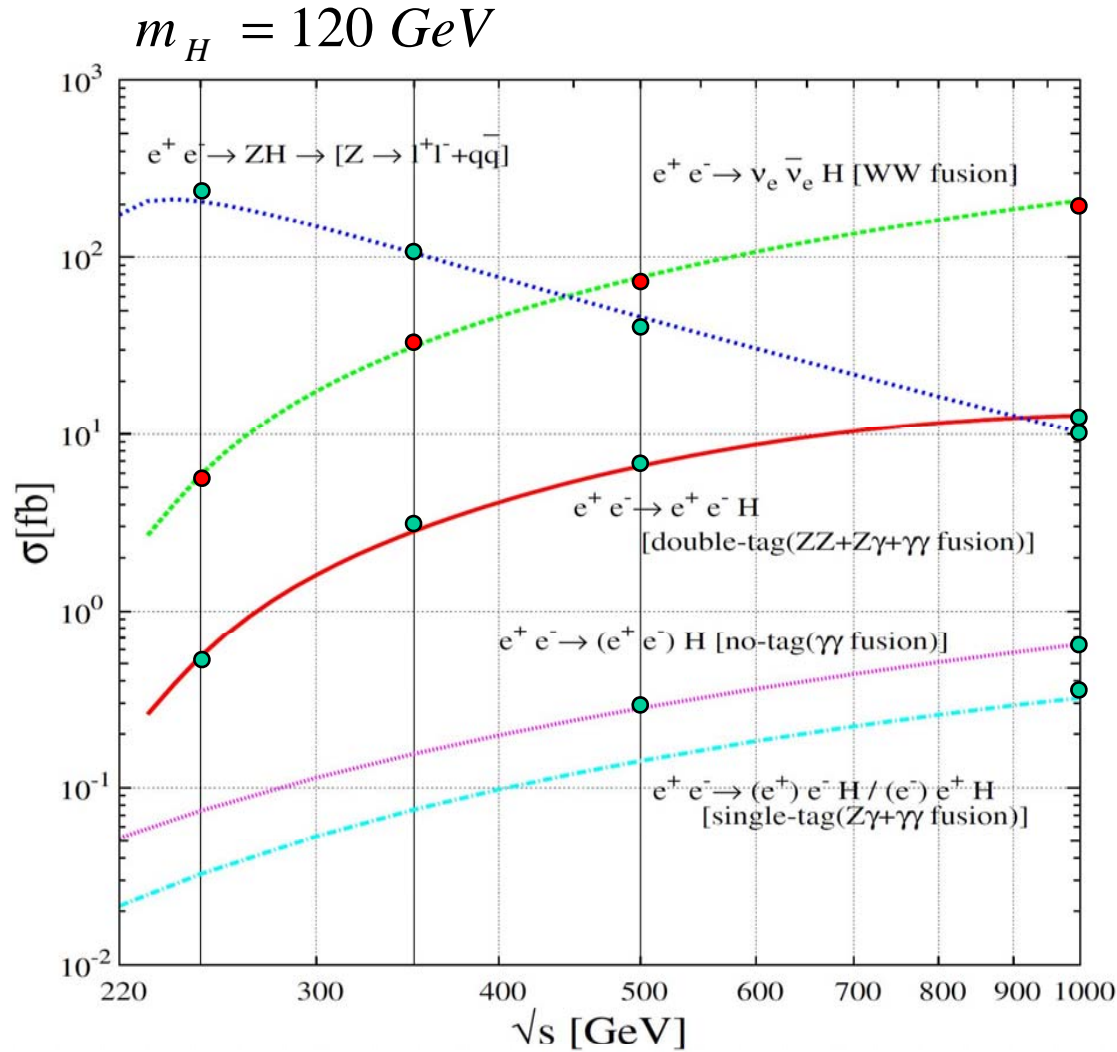
$$\begin{aligned} \chi^2(c_1, \dots, c_n) &= \sum_k \left(\frac{N_{EXP}^k - N_{TH}^k(c_i)}{\sqrt{N_{EXP}^k}} \right)^2 = \sum_k \left(\frac{L\sum_i c_i \Sigma_i(\Omega_k)\Delta\Omega_k}{\sqrt{L\Sigma_{SM}(\Omega_k)\Delta\Omega_k}} \right)^2 \\ &= \sum_{i,j} c_i c_j L \sum_k \frac{\Sigma_i(\Omega_k)\Sigma_j(\Omega_k)}{\Sigma_{SM}(\Omega_k)} \Delta\Omega_k = \sum_{i,j} c_i c_j L \int \frac{\Sigma_i(\Omega)\Sigma_j(\Omega)}{\Sigma_{SM}(\Omega)} d\Omega \equiv \sum_{i,j} c_i \underline{(V^{-1})_{ij}} c_j \end{aligned}$$

if V^{-1} is given, we can calculate Δc_i

$$\Delta c_i = \sqrt{V_{ii}}$$

The large discrepancy between Σ_{SM} and $\Sigma_i(\Omega)$ makes V^{-1} large \rightarrow errors become small

2-0. Cross section



\sim ILC phase I
phase II

• a_{WW}, b_{WW}, c_{WW} measurement

2-1 : WW-fusion
 \rightarrow 250GeV-

1000GeV

• $a_{ZZ}, b_{ZZ}, c_{ZZ}, b_{Z\gamma}, c_{Z\gamma}$

2-2: ZH production
 \rightarrow 250GeV - 1000GeV

• $a_{ZZ}, b_{ZZ}, c_{ZZ}, b_{Z\gamma}, c_{Z\gamma}, b_{\gamma\gamma}$

2-3: ZZ-fusion
 \rightarrow

• $b_{\gamma\gamma}$ 250GeV, 1000GeV

2-4: $\gamma\gamma$ -fusion
 \rightarrow

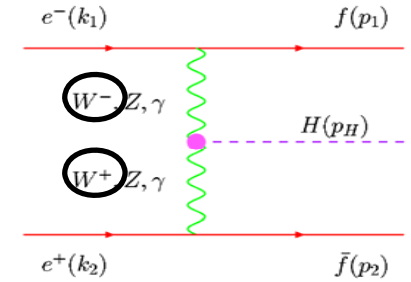
• $b_{Z\gamma}, c_{Z\gamma}, b_{\gamma\gamma}$ 500GeV, 1000GeV

Z γ -fusion

\rightarrow very small effect

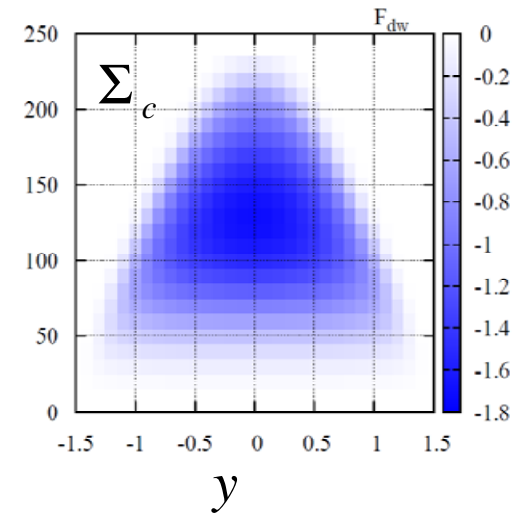
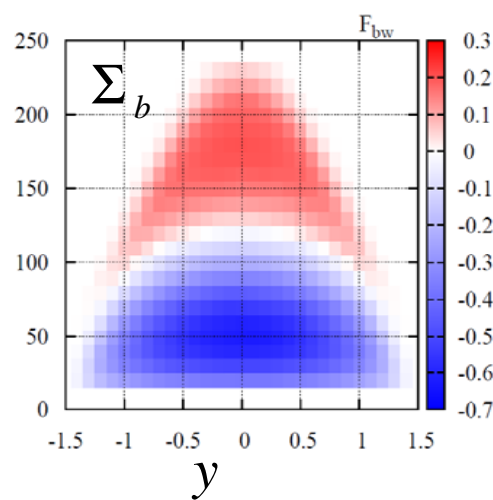
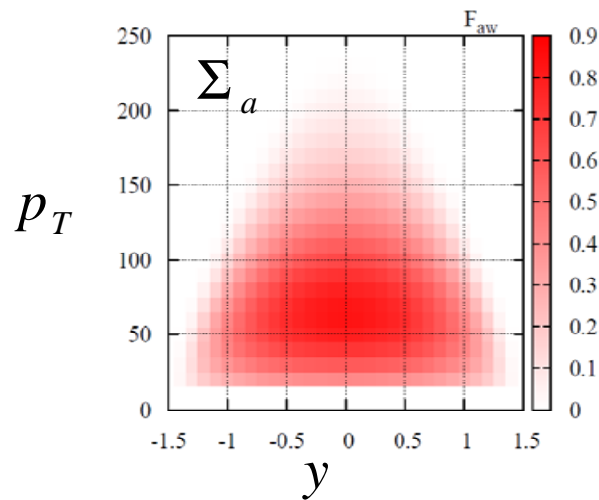
2-1. WW fusion process

In WW fusion process, the out going fermions are neutrinos.



The observable distributions should be

$$\frac{d\sigma}{dydp_T} = \Sigma_{SM}(y, p_T) + a_{WW}\Sigma_a(y, p_T) + b_{WW}\Sigma_b(y, p_T) + c_{WW}\Sigma_c(y, p_T)$$



The eigenvectors and eigenvalues $\forall f$ are

$$\sqrt{s} = 500 GeV$$

$$.24a_{WW} + .045b_{WW} - .97c_{WW} = \pm .0019$$

$$.69a_{WW} - .71b_{WW} + .13c_{WW} = \pm .0056$$

$$.68a_{WW} + .70b_{WW} + .20c_{WW} = \pm .14$$

$$\sqrt{s} = 1000 GeV$$

$$.14a_{WW} + .038b_{WW} - .99c_{WW} = \pm .00075$$

$$.71a_{WW} - .70b_{WW} + .072c_{WW} = \pm .0032$$

$$.69a_{WW} + .71b_{WW} + .12c_{WW} = \pm .066$$

2-1-2. Combined Results (1)

The results combining each collision energy with beam polarization $\rho_{e^-e^+} = 0$ are

$$\sqrt{s} = 250 \text{ GeV}, L = 100 \text{ fb}^{-1}$$

$$+ \sqrt{s} = 350 \text{ GeV}, L = 100 \text{ fb}^{-1}$$

$$+ \sqrt{s} = 500 \text{ GeV}, L = 500 \text{ fb}^{-1}$$

$$\begin{aligned}
 a_{WW} &= \pm 0.025 \\
 b_{WW} &= \pm 0.026 \\
 c_{WW} &= \pm 0.0074
 \end{aligned}
 \begin{pmatrix}
 1 & & \\
 .991 & 1 & \\
 .993 & .998 & 1
 \end{pmatrix}
 \begin{aligned}
 .24a_{WW} + .042b_{WW} - .97c_{WW} &= \pm 0.00084 \\
 .70a_{WW} - .70b_{WW} + .14c_{WW} &= \pm 0.0024 \\
 .68a_{WW} + .71b_{WW} + .20c_{WW} &= \pm 0.037
 \end{aligned}$$

There are strong correlation because there is a combination of anomalous couplings which has a large error at each energy experiment

$$\sqrt{s} = 250 \text{ GeV}, L = 100 \text{ fb}^{-1}$$

$$+ \sqrt{s} = 350 \text{ GeV}, L = 100 \text{ fb}^{-1}$$

$$+ \sqrt{s} = 500 \text{ GeV}, L = 500 \text{ fb}^{-1}$$

$$+ \sqrt{s} = 1 \text{ TeV}, L = 500 \text{ fb}^{-1}$$

$$\begin{aligned}
 a_{WW} &= .0072 \\
 b_{WW} &= .0076 \\
 c_{WW} &= .0014
 \end{aligned}
 \begin{pmatrix}
 1 & & \\
 .97 & 1 & \\
 .97 & .96 & 1
 \end{pmatrix}
 \begin{aligned}
 .15a_{WW} + .039b_{WW} - .99c_{WW} &= \pm 0.00031 \\
 .72a_{WW} - .69b_{WW} + .082c_{WW} &= \pm 0.0012 \\
 .68a_{WW} + .72b_{WW} + .13c_{WW} &= \pm 0.010.
 \end{aligned}$$

this means HWW coupling can be measured with 0.72% accuracy

$$\begin{aligned}
 c_{WW} &= .0014 \\
 (a_{WW} - b_{WW})/\sqrt{2} &= .0013 \\
 (a_{WW} + b_{WW})/\sqrt{2} &= .010
 \end{aligned}
 \begin{pmatrix}
 1 & & \\
 -.14 & 1 & \\
 .97 & -.21 & 1
 \end{pmatrix}$$

There are still strong correlation

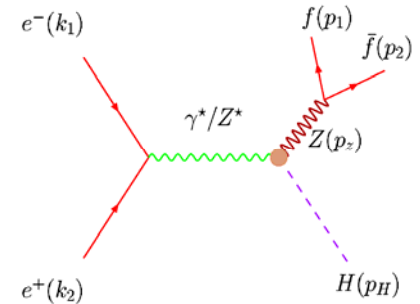
2-2. ZH production

process

The differential cross section is expressed as

$$\frac{d\sigma}{d\Omega} = \Sigma_{SM}(\Omega) + \sum_i c_i \Sigma_i(\Omega) \quad \text{for } Z \rightarrow l\bar{l}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} [\Sigma_{SM}(\Omega) + \Sigma_{SM}(\tilde{\Omega})] + \sum_i c_i \frac{1}{2} [\Sigma_i(\Omega) + \Sigma_i(\tilde{\Omega})] \quad \text{for } Z \rightarrow q\bar{q}$$



$$\sqrt{s} = 250 \text{ GeV}$$

$$.0097a_{ZZ} + .056b_{ZZ} + .027c_{ZZ} + .53b_{Z\gamma} + \textcircled{.85c_{Z\gamma}} = 0 \pm \underline{\underline{.00024}}$$

$$.15a_{ZZ} + \textcircled{.89b_{ZZ}} + .42c_{ZZ} - .032b_{Z\gamma} - .052c_{Z\gamma} = 0 \pm .00049$$

$$.013a_{ZZ} - .094b_{ZZ} + .20c_{ZZ} - \textcircled{.83b_{Z\gamma}} + .51c_{Z\gamma} = 0 \pm .0034$$

$$.055a_{ZZ} - .42b_{ZZ} + \textcircled{.88c_{ZZ}} + .19b_{Z\gamma} - .12c_{Z\gamma} = 0 \pm .0043$$

$$\sqrt{s} = 500 \text{ GeV}$$

$$.0016a_{ZZ} + .015b_{ZZ} + .035c_{ZZ} + .18b_{Z\gamma} + \textcircled{.98c_{Z\gamma}} = 0 \pm \underline{\underline{.000098}}$$

$$.042a_{ZZ} + .38b_{ZZ} + \textcircled{.92c_{ZZ}} - .0085b_{Z\gamma} - .037c_{Z\gamma} = 0 \pm .00028$$

$$.0046a_{ZZ} + .24b_{ZZ} - .097c_{ZZ} + \textcircled{.95b_{Z\gamma}} - .17c_{Z\gamma} = 0 \pm .00086$$

$$.017a_{ZZ} + \textcircled{.89b_{ZZ}} - .37c_{ZZ} - .25b_{Z\gamma} + .046c_{Z\gamma} = 0 \pm .0010$$

here $\begin{cases} \Omega(\theta_j, \varphi_j = \theta_q, \varphi_q) \\ \tilde{\Omega}(\theta_j, \varphi_j = \theta_{\bar{q}}, \varphi_{\bar{q}}) \end{cases}$

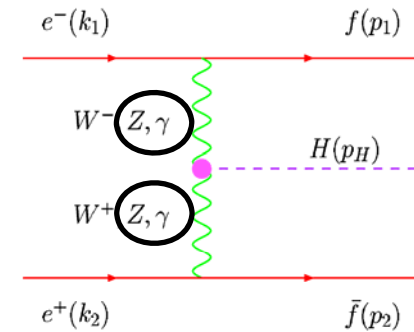
- 4 constraints for 5 couplings
- a_Z cannot be measured independently
- $c_{Z\gamma}$ is most sensitive because its coefficient includes s
- $c_{ZZ}, c_{Z\gamma}$ become more sensitive when \sqrt{s} is increased
- other couplings does not change so much because cross section decrease

2-3. eeH double-tag Z, Zγ, γ γ fusion process

Final fermions are electron. we can use full information of $\Sigma_{SM}(\Omega)$

$$\frac{d\sigma}{d\Omega} = \Sigma_{SM}(\Omega) + \sum_i c_i \Sigma_i(\Omega)$$

Double e+ e- tag conditions:
 $\sqrt{s} = 500 \text{ GeV}$
 $|\cos\theta_{e^\pm}| < 0.995$ $p_T(e^\pm) > 1 \text{ GeV}$



$.0017a_{ZZ}$	$+.00029b_{ZZ}$	$-.0067c_{ZZ}$	$-.0041b_{Z\gamma}$	$-.9999c_{Z\gamma}$	$-.0054b_{\gamma\gamma}$	$= \pm .00080$
$.034a_{ZZ}$	$-.20b_{ZZ}$	$-.064c_{ZZ}$	$-.61b_{Z\gamma}$	$-.0070c_{Z\gamma}$	$-.76b_{\gamma\gamma}$	$= \pm .0041$
$.044a_{ZZ}$	$-.28b_{ZZ}$	$-.030c_{ZZ}$	$+.78b_{Z\gamma}$	$+.00c_{Z\gamma}$	$-.55b_{\gamma\gamma}$	$= \pm .0050$
$.19a_{ZZ}$	$+.31b_{ZZ}$	$-.93c_{ZZ}$	$+.045b_{Z\gamma}$	$-.0066c_{Z\gamma}$	$-.029b_{\gamma\gamma}$	$= \pm .0095$
$.34a_{ZZ}$	$-.84b_{ZZ}$	$-.23c_{ZZ}$	$-.098b_{Z\gamma}$	$-.00048c_{Z\gamma}$	$+.33b_{\gamma\gamma}$	$= \pm .019$
$.92a_{ZZ}$	$+.27b_{ZZ}$	$+.28c_{ZZ}$	$+.012b_{Z\gamma}$	$-.00010c_{Z\gamma}$	$-.063b_{\gamma\gamma}$	$= \pm .044$

$\sqrt{s} = 1000 \text{ GeV}$

$.0020a_{ZZ}$	$+.0016b_{ZZ}$	$-.020c_{ZZ}$	$-.019b_{Z\gamma}$	$+.9996c_{Z\gamma}$	$-.0061b_{\gamma\gamma}$	$= \pm .00057$
$.0078a_{ZZ}$	$+.033b_{ZZ}$	$-.15c_{ZZ}$	$-.95b_{Z\gamma}$	$-.022c_{Z\gamma}$	$-.26b_{\gamma\gamma}$	$= \pm .0025$
$.089a_{ZZ}$	$+.12b_{ZZ}$	$-.97c_{ZZ}$	$+.19b_{Z\gamma}$	$-.017c_{Z\gamma}$	$-.091b_{\gamma\gamma}$	$= \pm .0031$
$.035a_{ZZ}$	$+.67b_{ZZ}$	$+.018c_{ZZ}$	$+.17b_{Z\gamma}$	$+.00012c_{Z\gamma}$	$-.72b_{\gamma\gamma}$	$= \pm .0036$
$.15a_{ZZ}$	$-.72b_{ZZ}$	$-.17c_{ZZ}$	$-.17b_{Z\gamma}$	$-.0019c_{Z\gamma}$	$+.64b_{\gamma\gamma}$	$= \pm .0082$
$.98a_{ZZ}$	$+.12b_{ZZ}$	$+.11c_{ZZ}$	$+.011b_{Z\gamma}$	$+.00c_{Z\gamma}$	$-.063b_{\gamma\gamma}$	$= \pm .027$

6 constraints for 6

→ all couplings can be measured

Because the distribution of is same to SM

2-4. (ee)H no-tag $\gamma\gamma$ fusion process

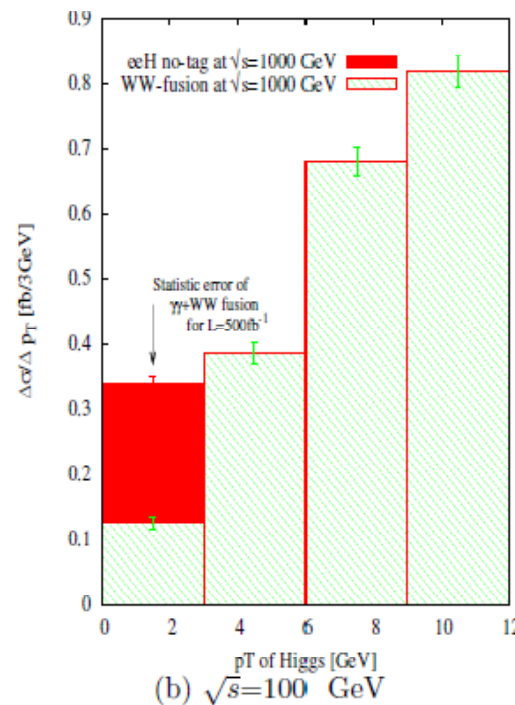
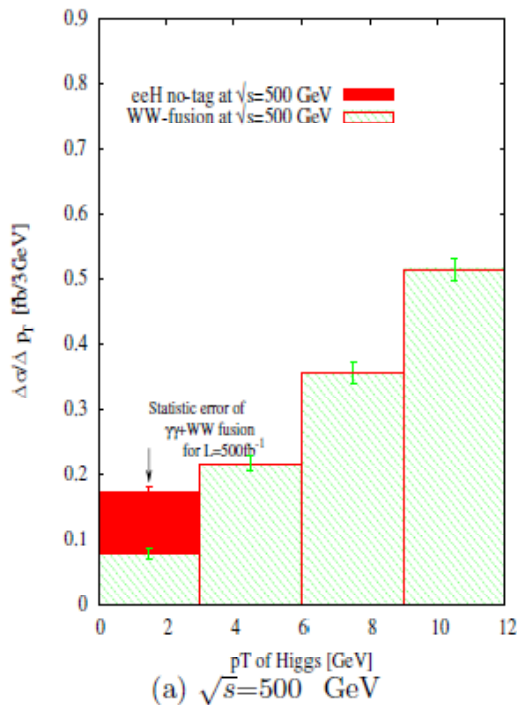
Parametrize the cross section by using the equivalent real photon fusion

$$\sigma^{ee \rightarrow (ee)H}(\sqrt{s} = 500 \text{ GeV}) = 150 \left(\frac{b_{\gamma\gamma}^{\text{SM}}}{2} + b_{\gamma\gamma} \right) [\text{fb}]$$

$$\sigma^{ee \rightarrow (ee)H}(\sqrt{s} = 1 \text{ TeV}) = 345 \left(\frac{b_{\gamma\gamma}^{\text{SM}}}{2} + b_{\gamma\gamma} \right) [\text{fb}]$$

here $b_{\gamma\gamma}^{\text{SM}} = \frac{\alpha}{4\pi} |I| = 0.0038$.

is SM 1-loop contribution



event contamination
from WW fusion
process

p_{TH} resolution is set to 3 GeV

$$150b_{\gamma\gamma} = \pm 0.0318, \rightarrow b_{\gamma\gamma} = 0 \pm 0.00022 \quad \text{for } \sqrt{s} = 500 \text{ GeV}$$

$$345b_{\gamma\gamma} = \pm 0.0324, \rightarrow b_{\gamma\gamma} = 0 \pm 0.000094 \quad \text{for } \sqrt{s} = 1 \text{ TeV.}$$

2-5. Combined results (2)

Combined results with ZH process and eeH double-tag, (ee)H single-tag process

$$|P_{e^-}|=0.8, |P_{e^+}|=0.0$$

with beam polarization

$$\begin{aligned} &\sqrt{s} = 250\text{GeV}, L = 100\text{fb}^{-1} \\ &+ \sqrt{s} = 350\text{GeV}, L = 100\text{fb}^{-1} \\ &+ \sqrt{s} = 500\text{GeV}, L = 500\text{fb}^{-1} \end{aligned}$$

$$\sqrt{s} = 250\text{GeV}, L = 100\text{fb}^{-1}$$

$$\begin{aligned} &+ \sqrt{s} = 350\text{GeV}, L = 100\text{fb}^{-1} \\ &+ \sqrt{s} = 500\text{GeV}, L = 500\text{fb}^{-1} \\ &+ \sqrt{s} = 1\text{TeV}, L = 500\text{fb}^{-1} \end{aligned}$$

$$\begin{aligned} a_{ZZ} &= 0 \pm .0045 \\ b_{ZZ} &= 0 \pm .00048 \\ c_{ZZ} &= 0 \pm .00020 \\ b_{Z\gamma} &= 0 \pm .00030 \\ c_{Z\gamma} &= 0 \pm .000072 \\ b_{\gamma\gamma} &= 0 \pm .000095 \end{aligned} \left(\begin{array}{cccccc} 1 & & & & & \\ & \color{red}{-0.74} & & & & \\ & & 1 & & & \\ & & & -0.40 & -0.16 & 1 \\ & & & & & & 0.0036 & -0.064 & 0.066 & 1 \\ & & & & & & & & & & -0.0036 & 0.047 & -0.082 & \color{red}{-0.81} & 1 \\ & & & & & & & & & & & 0.0090 & -0.0080 & -0.0023 & -0.0017 & 0.0015 & 1 \end{array} \right)$$

$$\begin{aligned} a_{ZZ} &= 0 \pm \color{blue}{.0032} \\ b_{ZZ} &= 0 \pm .00034 \\ c_{ZZ} &= 0 \pm \color{blue}{.000063} \\ b_{Z\gamma} &= 0 \pm .00018 \\ c_{Z\gamma} &= 0 \pm \color{blue}{.000023} \\ b_{\gamma\gamma} &= 0 \pm \color{blue}{.000038} \end{aligned} \left(\begin{array}{cccccc} 1 & & & & & \\ & \color{red}{-0.74} & & & & \\ & & 1 & & & \\ & & & -0.22 & -0.20 & 1 \\ & & & & & & -0.00095 & -0.059 & 0.053 & 1 \\ & & & & & & & & & & -0.0013 & 0.033 & -0.089 & \color{red}{-0.60} & 1 \\ & & & & & & & & & & & 0.0050 & -0.0048 & -0.00032 & -0.0013 & 0.00085 & 1 \end{array} \right)$$

The effect of ZH production process @250GeV

$1a_{ZZ}$ cannot be measured independently at one energy experiment

2. New physics term is proportional to $(b_{Z\gamma} + c_{Z\gamma})$

Correlation goes small by combining other processes

It is possible to measure HZZ

coupling with 0.32% accuracy

2-5. Combined results (3)

Comparison with polarization $|P_{e^-}|=0.0, |P_{e^+}|=0$ $|P_{e^-}|=0.8, |P_{e^+}|=0.0$

and
 ■ Combine up to $\sqrt{s}=1TeV$ $|P_{e^-}|=0.0, |P_{e^+}|=0.0$

$$\begin{array}{l}
 a_{ZZ} = 0 \pm .0032 \\
 b_{ZZ} = 0 \pm .00043 \\
 c_{ZZ} = 0 \pm .00025 \\
 b_{Z\gamma} = 0 \pm .0027 \\
 c_{Z\gamma} = 0 \pm .00098 \\
 b_{\gamma\gamma} = 0 \pm .000038
 \end{array}
 \left(
 \begin{array}{cccccc}
 1 & & & & & \\
 -.59 & 1 & & & & \\
 -.0044 & -.29 & 1 & & & \\
 .00039 & -.58 & .61 & 1 & & \\
 -.049 & .27 & -.98 & -.64 & 1 & \\
 .0048 & -.0044 & -.0023 & .00 & .0023 & 1
 \end{array}
 \right)$$

- Strong correlation disappears
 → all couplings are measured independently
- $a_{ZZ}, b_{\gamma\gamma}$ dose not change by beam polarization

■ Combine up to $\sqrt{s}=1TeV$ $|P_{e^-}|=0.8, |P_{e^+}|=0.0$

$$\begin{array}{l}
 a_{ZZ} = 0 \pm .0032 \\
 b_{ZZ} = 0 \pm .00034 \\
 c_{ZZ} = 0 \pm .000063 \\
 b_{Z\gamma} = 0 \pm .00018 \\
 c_{Z\gamma} = 0 \pm .000023 \\
 b_{\gamma\gamma} = 0 \pm .000038
 \end{array}
 \left(
 \begin{array}{cccccc}
 1 & & & & & \\
 -.74 & 1 & & & & \\
 -.22 & -.20 & 1 & & & \\
 -.00095 & -.059 & .053 & 1 & & \\
 -.0013 & .033 & -.089 & -.60 & 1 & \\
 .0050 & -.0048 & -.00032 & -.0013 & .00085 & 1
 \end{array}
 \right)$$

- The error of $c_{ZZ}, b_{Z\gamma}, c_{Z\gamma}$ become much smaller when polarized beam is applied
- $$c_{ZZ} \propto s_W^2 f_B + c_W^2 f_W \rightarrow 1/4$$
- $$b_{Z\gamma} \propto s_W^2 f_{BB} - c_W^2 f_{WW} \rightarrow 1/15$$
- $$c_{Z\gamma} \propto f_B - f_W \rightarrow 1/40$$

■ Each couplings are measured with below accuracy

$$\Delta(a_{WW}, b_{WW}, c_{WW}) \approx (0.72, 0.76, 0.14)\%$$

$$\Delta(a_{ZZ}, b_{ZZ}, c_{ZZ}) \approx (0.3, 0.03, 0.006)\%$$

$$\Delta(b_{Z\gamma}, c_{Z\gamma}, b_{\gamma\gamma}) \approx (0.02, 0.002, 0.0004)\%$$

3-2. Luminosity uncertainty

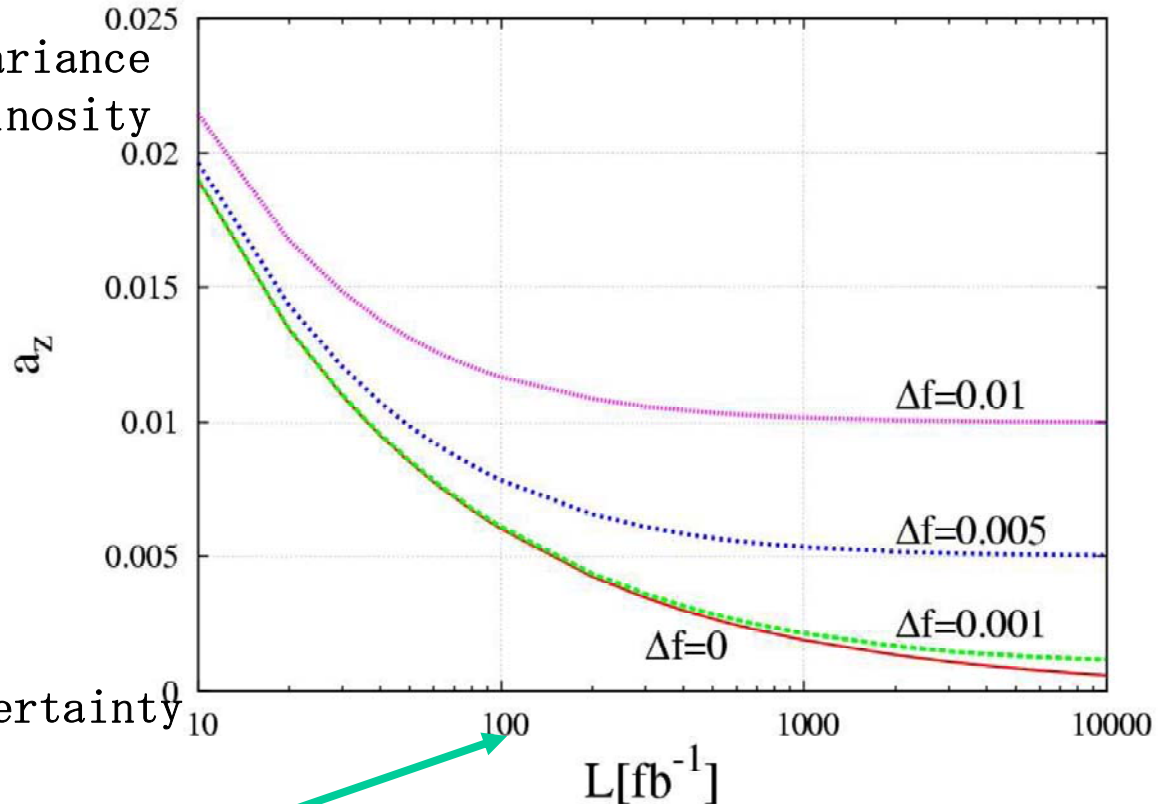
Redefine inverse of the covariance matrix with considering Luminosity uncertainty (=df)

$$M_{ij} \equiv L \int d\Phi F_i(\Phi) F_j(\Phi) \left[\frac{d\sigma^{SM}}{d\Phi} \right]^{-1}$$

$$\widetilde{M}_{ij} \equiv M_{ij} - \frac{M_{i1} M_{1j} (c_1^{SM})^2}{1 + M_{11} (c_1^{SM} \Delta f)^2} (\Delta f)^2$$

※ If $df \rightarrow 0$ ($\widetilde{M}_{ij} \rightarrow M_{ij}$)

The luminosity error will dominate the statistic uncertainty



L ≈ 100 fb⁻¹ region

df < 1% is required
to get the error of a_z around 1%

L ≥ 100 fb⁻¹ region

df become dominant error.
To make statistic error and df to be same contribution, df < 0.005 is required especially at ab-1 scale.

Comparison with polarization $|P_{e^-}|=0.0, |P_{e^+}|=0.0$ $|P_{e^-}|=0.8, |P_{e^+}|=0.0$
 and

$\sqrt{s} = 250 GeV$ with $|P_{e^-}|=0.0, |P_{e^+}|=0.0$

$$\text{Re} (.078a_Z + .90b_Z + .42c_Z + .049b_\gamma + .079c_\gamma) = 0 \pm .00024$$

$$\text{Re} (.028a_Z - .43b_Z + .90c_Z - .019b_\gamma + .076c_\gamma) = 0 \pm .0021$$

$$\text{Re} (-.0087a_Z - .061b_Z - .088c_Z + .54b_\gamma + .83c_\gamma) = \underline{\underline{0 \pm .0053}}$$

$$\text{Re} (.0017a_Z - .023b_Z + .052c_Z + .84b_\gamma - .54c_\gamma) = \underline{\underline{0 \pm .072}}$$

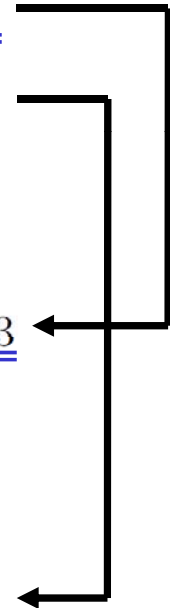
$\sqrt{s} = 250 GeV$ with $|P_{e^-}|=0.8, |P_{e^+}|=0.0$

$$\text{Re} (.0027a_Z + .044b_Z + .13c_Z + .15b_\gamma + .98c_\gamma) = \underline{\underline{0 \pm .000083}}$$

$$\text{Re} (.020a_Z + .33b_Z + .94c_Z - .022b_\gamma - .13c_\gamma) = 0 \pm .00012$$

$$\text{Re} (.0099a_Z + .94b_Z - .33c_Z + .097b_\gamma - .015c_\gamma) = 0 \pm .00057$$

$$\text{Re} (-.00096a_Z - .093b_Z + .033c_Z + .98b_\gamma - .15c_\gamma) = \underline{\underline{0 \pm .0097}}$$



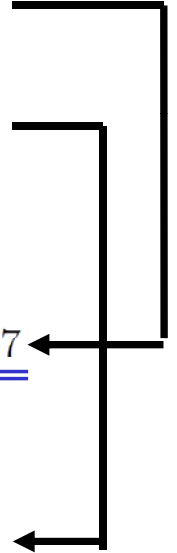
Comparison with polarization $|P_{e^-}|=0.0, |P_{e^+}|=0$ $|P_{e^-}|=0.8, |P_{e^+}|=0$
and

$\sqrt{s}=1TeV$ with $|P_{e^-}|=0.0, |P_{e^+}|=0.0$

$$\begin{aligned}
\text{Re}(\quad .042a_Z \quad -.11b_Z \quad \color{red}{+.94c_Z} \quad +.032b_\gamma \quad -.28c_\gamma \quad +.17b_{\gamma\gamma}) &= \pm.0015 \\
\text{Re}(\quad -.015a_Z \quad +.68b_Z \quad -.052c_Z \quad -.049b_\gamma \quad +.00012c_\gamma \quad \color{green}{+.73b_{\gamma\gamma}}) &= \pm.0018 \\
\text{Re}(\quad -.069a_Z \quad +.70b_Z \quad +.16c_Z \quad -.18b_\gamma \quad -.13c_\gamma \quad -.65b_{\gamma\gamma}) &= \pm.0039 \\
\text{Re}(\quad -.018a_Z \quad +.064b_Z \quad +.30c_Z \quad -.018b_\gamma \quad \color{blue}{+.95c_\gamma} \quad -.040b_{\gamma\gamma}) &= \underline{\underline{\pm.0077}} \\
\text{Re}(\quad .99a_Z \quad +.077b_Z \quad +.055c_Z \quad +.12b_\gamma \quad -.0035c_\gamma \quad -.039b_{\gamma\gamma}) &= \pm.027 \\
\text{Re}(\quad -.13a_Z \quad +.16b_Z \quad -.0056c_Z \quad \color{blue}{+.98b_\gamma} \quad +.0038c_\gamma \quad -.083b_{\gamma\gamma}) &= \underline{\underline{\pm.043}}
\end{aligned}$$

$\sqrt{s}=1TeV$ with $|P_{e^-}|=0.8, |P_{e^+}|=0.0$

$$\begin{aligned}
\text{Re}(\quad .0022a_Z \quad +.0035b_Z \quad -.045c_Z \quad -.019b_\gamma \quad \color{blue}{+.99c_\gamma} \quad -.013b_{\gamma\gamma}) &= \underline{\underline{\pm.00057}} \\
\text{Re}(\quad -.043a_Z \quad -.095b_Z \quad \color{red}{+.96c_Z} \quad +.084b_\gamma \quad +.049c_\gamma \quad +.22b_{\gamma\gamma}) &= \pm.0015 \\
\text{Re}(\quad -.0087a_Z \quad +.62b_Z \quad -.13c_Z \quad +.15b_\gamma \quad +.0050c_\gamma \quad \color{green}{+.76b_{\gamma\gamma}}) &= \pm.0018 \\
\text{Re}(\quad .029a_Z \quad -.42b_Z \quad -.15c_Z \quad \color{blue}{+.88b_\gamma} \quad +.013c_\gamma \quad +.14b_{\gamma\gamma}) &= \underline{\underline{\pm.0025}} \\
\text{Re}(\quad -.068a_Z \quad +.65b_Z \quad +.16c_Z \quad +.44b_\gamma \quad +.0054c_\gamma \quad -.59b_{\gamma\gamma}) &= \pm.0044 \\
\text{Re}(\quad .99a_Z \quad +.058b_Z \quad +.056c_Z \quad +.0097b_\gamma \quad -.00032c_\gamma \quad -.028b_{\gamma\gamma}) &= \pm.027
\end{aligned}$$



2-5. $e(e)H$ single-tag $Z\gamma$ fusion process

Parametrize the cross section by using the equivalent real photon for untagged side

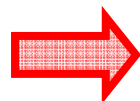
$$\sigma^{(e)eH} = \frac{1}{4} \left(\frac{1 + |P|}{2} \sigma^{e_R^-(e^+)H} + \frac{1 - |P|}{2} \sigma^{e_L^-(e^+)H} \right) + \frac{1}{4} \left(\frac{1 - |P|}{2} \sigma^{e_R^-(e^+)H} + \frac{1 + |P|}{2} \sigma^{e_L^-(e^+)H} \right) + \frac{1}{4} \sigma^{(e^-)e_R^+H} + \frac{1}{4} \sigma^{(e^-)e_L^+H}.$$

For 500 GeV

$$\left\{ \begin{aligned} \sigma_{(e_R^-)e^+H} &= 0.0461 + 36.0 \underline{b_{\gamma\gamma}} + 7284 \underline{b_{\gamma\gamma}^2} \\ &\quad - 3.89 \underline{(b_\gamma - c_\gamma)} + 104 \underline{(b_\gamma - c_\gamma)^2} - 1366 \underline{b_{\gamma\gamma} (b_\gamma - c_\gamma)} [\text{fb}] \\ \sigma_{(e_L^-)e^+H} &= 0.0121 + 17.1 \underline{b_{\gamma\gamma}} + 7290 \underline{b_{\gamma\gamma}^2} \\ &\quad + 1.17 \underline{(b_\gamma - c_\gamma)} + 140 \underline{(b_\gamma - c_\gamma)^2} + 1593 \underline{b_{\gamma\gamma} (b_\gamma - c_\gamma)} [\text{fb}] \end{aligned} \right.$$

For 1 TeV

$$\left\{ \begin{aligned} \sigma_{(e_R^-)e^+H} &= 0.099 + 68.1 \underline{b_{\gamma\gamma}} + 11901 \underline{b_{\gamma\gamma}^2} \\ &\quad - 11.1 \underline{(b_\gamma - c_\gamma)} + 340 \underline{(b_\gamma - c_\gamma)^2} - 3640 \underline{b_{\gamma\gamma} (b_\gamma - c_\gamma)} [\text{fb}] \\ \sigma_{(e_L^-)e^+H} &= 0.0097 + 17.3 \underline{b_{\gamma\gamma}} + 11868 \underline{b_{\gamma\gamma}^2} \\ &\quad + 1.95 \underline{(b_\gamma - c_\gamma)} + 460 \underline{(b_\gamma - c_\gamma)^2} + 4209 \underline{b_{\gamma\gamma} (b_\gamma - c_\gamma)} [\text{fb}] \end{aligned} \right.$$

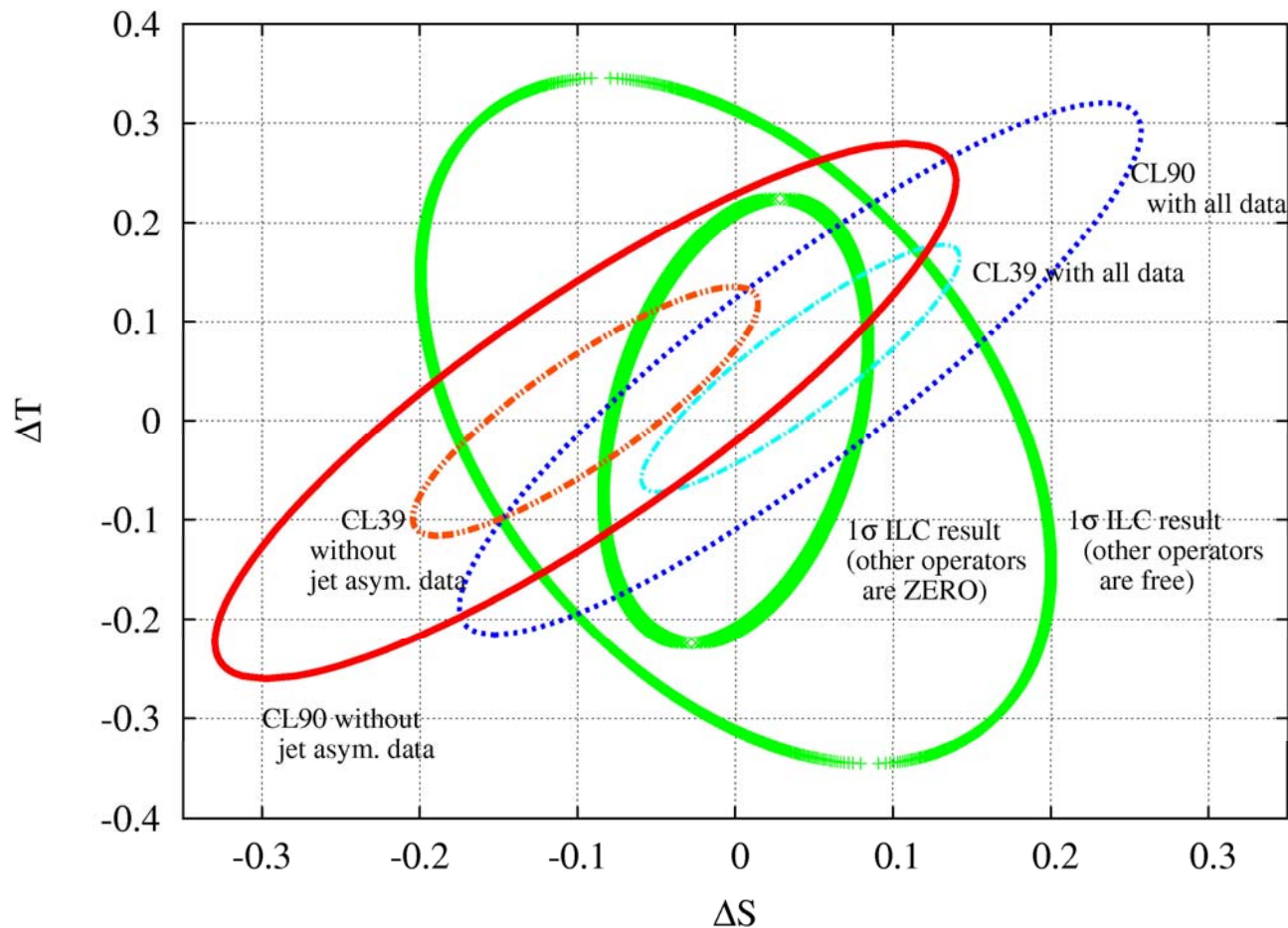


Add the contribution of this process to the V_{total}^{-1}

total $b_{\gamma\gamma}$ $(b_\gamma - c_\gamma)$

Single-tag process is sensitive to $b_{\gamma\gamma}$ and $(b_\gamma - c_\gamma)$

3-3. ILC measurement Contribution to the T_z and S_z



■ When the ILC measurements is combined,
it gives more strong constraint on the dS_z
and dT_z